1 Introduction

This file provides a brief introduction to two Monte Carlo methods for approximating integrals: **simple sampling** and **importance sampling**.

2 Simple Sampling

Simple sampling is a basic technique that uses random samples from a given distribution to estimate the expected value of a function. For example, suppose we want to compute the integral

$$I = \int_a^b f(x)dx \tag{1}$$

where $f(x)$ is a continuous function and a and b are finite constants. We can rewrite this integral as

$$I = (b-a)\int_a^b \frac{f(x)}{b-a}dx = (b-a)E[f(X)] \tag{2}$$

where X is a uniform random variable on $[a, b]$. To approximate I, we can generate n independent samples $X_1, X_2, ..., X_n$ from the uniform distribution and use the sample mean as an estimator:

$$\hat{I}_n = (b-a)\frac{1}{n}\sum_{i=1}^{n} f(X_i) \tag{3}$$

3 Importance Sampling

Importance sampling is a variance reduction technique that uses random samples from a different distribution to estimate the expected value of a function. The idea is to choose a distribution that is more similar to the function and gives more weight to the regions where the function is large. For example, suppose we want to compute the integral

$$I = \int_a^b f(x)dx \tag{4}$$

where $f(x)$ is a continuous function and a and b are finite constants. We can rewrite this integral as

$$I = \int_a^b \frac{f(x)}{g(x)}g(x)dx = E[f(X)w(X)] \tag{5}$$

where X is a random variable with density $g(x)$ and $w(x) = 1/g(x)$ is the importance function. To approximate I, we can generate n independent samples

$X_1, X_2, ..., X_n$ from the density $g(x)$ and use the weighted sample mean as an estimator:

$$\hat{I}_n = \frac{1}{n}\sum_{i=1}^{n} f(X_i)w(X_i) \tag{6}$$

in this assignment, I use simple and importance sampling to calculate $f(x) = \int_a^b e^{-x^2}dx$. I use $g(x) = e^{-x}$ for importance sampling. Statistical and absolute error of each way are shown.