1 Introduction

In this assignment, I simulate RC circuit and simple harmonic oscillator using ODE, and compare some ODE methods.

2 RC circuit: First Ordinary Differential Equation

RC circuit equation:

$$R\frac{dq}{dt} + \frac{q}{C} = V \tag{1}$$

For simplicity, R = C = V = 1

$$\frac{dq}{dt} + q = 1 \tag{2}$$

$$\int_0^Q \frac{dq}{1-q} = \int_0^t dt \tag{3}$$

$$q(t) = 1 - e^{-t} \tag{4}$$

Euler algorithm:

$$\frac{dq}{dt} = f(q,t) = 1 - q \tag{5}$$

$$q_{n+1} = q_n + f(q_n)h = q_n + (1 - q_n)h \tag{6}$$

At the first part of code, I compared analytical and statistical solution with each other. Relative error is calculated, too.

3 Algorithm Instability

In this section, I show the instability of $q_{n+1} = q_{n-1} + 2f(q_n)h = q_{n-1} + 2(1 - q_n)h$, while h = 0.01.

4 Simple Harmonic Oscillator: Second Ordinary Differential Equation

Simple harmonic oscillator:

$$-kx = m\ddot{x} \tag{7}$$

For simplicity:

$$\frac{dx}{dt} = v \tag{8}$$

$$a = \frac{dv}{dt} = -x \tag{9}$$

Analytical Solution: $x_0 = 1, v_0 = 0, x = cost.$
This equation is analyzed with Euler, Euler-Cromer, Frog Leaping, Verlet, Velocity-Verlet and Beeman algorithms.