1 Introduction

As an example of Metropolis method, in this assignment, I simulated 2-D Ising model in a rectangular lattice with periodic boundary condition.

$$energy = -J \sum_{j} \sum_{i<j} \sigma_i \sigma_j \tag{1}$$

where σ can be -1 or +1.
Probability has shown as Boltzmann function $e^{-\beta E}$ where β is proportional to inverse of temperature.

2 Algorithm

1. **initial state**:
   $grid = Rand((-1, 1), size = (l * l)$
2. **Monte-Carlo steps**:

   (a) Randomly choose one element(S) in the grid for a change in its energy.

   (b) $E(S) = -E(S)$

   (c) check if this change can be accepted.
   if $\Delta E < 0$ then accept.
   else $Rand(0, 1) < \frac{e^{-\beta E(s)}}{e^{-\beta E(i)}}$ then accept.
3. **Loop (2)**.

In this assignment I simulate 2-D Ising model using mentioned algorithm, and calculate correlation length, magnetization, heat capacity, and magnetic susceptibility .
To speed up the algorithm, assume we want to change the energy of S_{ij} from E to -E. In this situation, $\Delta E = -2S_{ij}(S_{i+1j} + S_{i-1j} + S_{ij+1} + S_{ij-1})$. So, ΔE will be divisible to 2 and must be $\Delta E = (-8, -4, 0, 4, 8)$. We can calculate the Boltzmann factor for these five cases and call them during the main loop.