Reglunarfræði T 501

Reykjavik University

Æfingardæmi kafli 5
21. sept. 2015

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E5.2 The engine, body, and tires of a racing vehicle affect the acceleration and speed attainable [9]. The speed control of the car is represented by the model shown in Figure E5.2. (a) Calculate the steady-state error of the car to a step command in speed. (b) Calculate overshoot of the speed to a step command.
Answer: (a) $e_{ss} = A/11$; (b) $P.O. = 36\%$

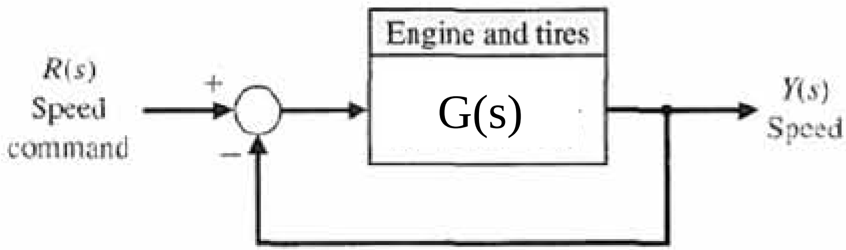

FIGURE E5.2 Racing car speed control.

Yfirfærslufall framrásarinnar er:

$$G(s) = \frac{100}{(s + 2)(s + 5)}$$

E5.2 (a) The closed-loop transfer function is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{100}{(s+2)(s+5)+100} = \frac{100}{s^2+2\zeta\omega_n s+\omega_n^2} \ .$$

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The steady-state error is given by

$$e_{ss} = \frac{A}{1+K_p} \ ,$$

where $R(s) = A/s$ and

$$K_p = \lim_{s\to 0} G(s) = \frac{100}{10} = 10 \ .$$

Therefore,

$$e_{ss} = \frac{A}{11} \ .$$

(b) The closed-loop system is a second-order system with natural frequency

$$\omega_n = \sqrt{110} \ ,$$

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and damping ratio

$$\zeta = \frac{7}{2\sqrt{110}} = 0.334 \ .$$

Since the steady-state value of the output is 0.909, we must modify the percent overshoot formula which implicitly assumes that the steady-state value is 1. This requires that we scale the formula by 0.909. The percent overshoot is thus computed to be

$$P.O. = 0.909(100e^{-\pi\zeta/\sqrt{1-\zeta^2}}) = 29\% \ .$$

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E5.3 A feedback system with negative unity feedback has a loop transfer function

$$L(s) = G_c(s)G(s) = \frac{2(s+8)}{s(s+4)}.$$

(a) Determine the closed-loop transfer function $T(s) = Y(s)/R(s)$. (b) Find the time response, $y(t)$, for a step input $r(t) = A$ for $t > 0$. (c) Using Figure 5.13(a), determine the overshoot of the response. (d) Using the final-value theorem, determine the steady-state value of $y(t)$.

Answer: (b) $y(t) = 1 - 1.07e^{-3t}\sin\left(\sqrt{7}t + 1.2\right)$

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Lausn:

E5.3 (a) The closed-loop transfer function is

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$$T(s) = \frac{G(s)}{1+G(s)} = \frac{2(s+8)}{s^2+6s+16} \ .$$

(b) We can expand $Y(s)$ in a partial fraction expansion as

$$Y(s) = \frac{2(s+8)}{(s^2+6s+16)} \frac{A}{s} = A\left(\frac{1}{s} - \frac{s+4}{s^2+6s+16}\right) \ .$$

Taking the inverse Laplace transform (using the Laplace transform tables), we find

$$y(t) = A[1 - 1.07e^{-3t}\sin(\sqrt{7}t + 1.21)] \ .$$

(c) Using the closed-loop transfer function, we compute $\zeta = 0.75$ and $\omega_n = 4$. Thus,

$$\frac{a}{\zeta\omega_n} = \frac{8}{3} = 2.67 \ ,$$

where $a = 8$. From Figure 5.13(a) in Dorf & Bishop, we find (approximately) that $P.O. = 4\%$.

(d) This is a type 1 system, thus the steady-state error is zero and $y(t) \to A$ as $t \to \infty$.

Dæmi E5,3   b) lítur

Laplace vörpun útmerkisins er: $Y(s) = \dfrac{2(s+8)}{s^2+6s+16} \cdot \dfrac{A}{s}$

Þá má finna stofnbrotin með því að rita:

$$Y(s) = \left[\frac{k_0}{s} + \frac{k_1}{s-p_1} + \frac{k_1^*}{s-p_1^*} \right] \qquad (\text{veljum } A = 1)$$

þar sem rætur kennijöfnunnar eru tvinntölur: $\begin{cases} p_1 = -3 + j\sqrt{7} \\ p_1^* = -3 - j\sqrt{7} \end{cases}$

Þá finnst að:

$$k_0 = sY(s)\Big|_{s=0} = \frac{16}{16} = 1 \; ; \; k_1 = (s-p_1)Y(s)\Big|_{s=p_1}$$

Hinsvegar er einfaldara að rita:

$$Y(s) = \left[\frac{k_0}{s} + \frac{\alpha_1 s + \alpha_0}{s^2+6s+16} \right]$$

þar sem finna má α_1 og α_0 þannig:

$$\alpha_1 s + \alpha_0 \Big| = (s^2+6s+16)Y(s)\Big| = \frac{2(s+8)}{}\Big|$$

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fæst:

$$s+\alpha_0\Big]_{s=-3+j\sqrt{7}} = \Big[2+\frac{16}{s}\Big]_{s=-3+j\sqrt{7}}$$

gefur

$$\alpha_1 + j\alpha_1\sqrt{7} + \alpha_0 = 2+(-3-j\sqrt{7}) = -1-j\sqrt{7}$$

$$\underline{\alpha_1 = -1} \qquad \underline{\alpha_0 = -1-3 = -4}$$

æst:

$$Y(s) = \left[1 - \frac{s+4}{s^2+6s+16}\right]$$

má finna andhverfu Laplace vörpunina:

$$y(t) = 1 - 1{,}07\, e^{-3t} \sin\left(\sqrt{7}\, t + 1{,}21\right)$$

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E5.6 Consider the block diagram shown in Figure E5.6 [16].
(a) Calculate the steady-state error for a ramp input.
(b) Select a value of K that will result in zero overshoot to a step input. Provide the most rapid response that is attainable.

Plot the poles and zeros of this system and discuss the dominance of the complex poles. What overshoot for a step input do you expect?

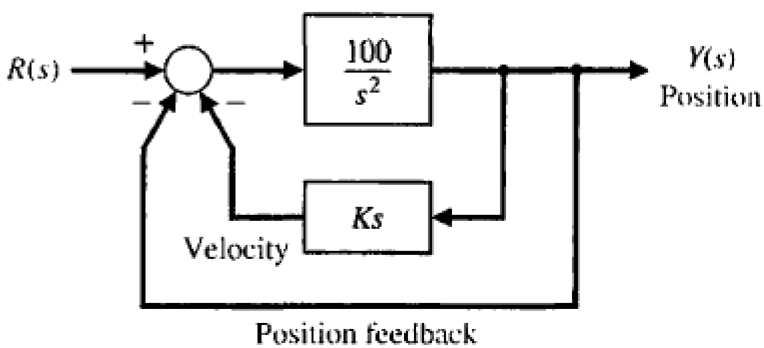

FIGURE E5.6 Block diagram with position and velocity feedback.

Lausn:

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E5.6 (a) The closed-loop transfer function is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + GH(s)} = \frac{100}{s^2 + 100Ks + 100} ,$$

where $H(s) = 1 + Ks$ and $G(s) = 100/s^2$. The steady-state error is computed as follows:

$$\begin{aligned} e_{ss} &= \lim_{s \to 0} s[R(s) - Y(s)] = \lim_{s \to 0} s[1 - T(s)] \frac{A}{s^2} \\ &= \lim_{s \to 0} \left[1 - \frac{\frac{100}{s^2}}{1 + \frac{100}{s^2}(1 + Ks)} \right] \frac{A}{s} = KA . \end{aligned}$$

E5.6...

(b) From the closed-loop transfer function, $T(s)$, we determine that $\omega_n = 10$ and

$$\zeta = \frac{100K}{2(10)} = 5K \ .$$

We want to choose K so that the system is critically damped, or $\zeta = 1.0$. Thus,

$$K = \frac{1}{5} = 0.20 \ .$$

The closed-loop system has no zeros and the poles are at

$$s_{1,2} = -50K \pm 10\sqrt{25K^2 - 1} \ .$$

The percent overshoot to a step input is

$$P.O. = 100e^{\frac{-5\pi K}{\sqrt{1-25K^2}}} \quad \text{for} \quad 0 < K < 0.2$$

and $P.O. = 0$ for $K > 0.2$.