Reglunarfræði T 501

Sýnidæmi kafli 3

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E3.3 Obtain a state variable matrix for a system with a differential equation

$$\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = 20u(t).$$

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$$\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = 20u(t).$$

E3.3 The system in phase variable form is

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}u \\ y &= \mathbf{C}\mathbf{x}\end{aligned}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -6 & -4 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

E3.4 A system can be represented by the state vector differential equation of Equation (3.16), where

$$\mathbf{A} = \begin{bmatrix} 0 & 4 \\ -1 & -4 \end{bmatrix}.$$

Find the characteristic roots of the system.

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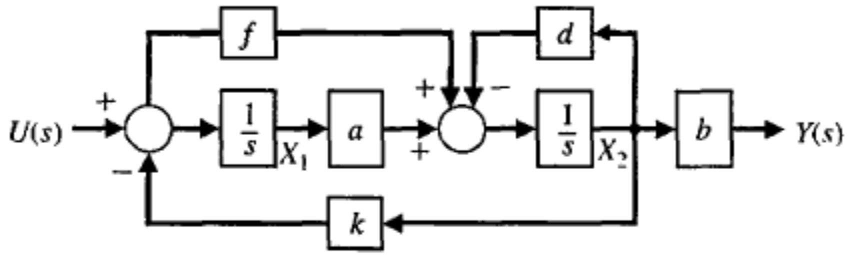
E3.4 The charactersitic roots, denoted by λ, are the solutions of $\det(\lambda\mathbf{I}-\mathbf{A}) = 0$. For this problem we have

$$\det(\lambda\mathbf{I}-\mathbf{A}) = \det\left(\begin{bmatrix} \lambda & -4 \\ 1 & \lambda+4 \end{bmatrix}\right) = \lambda(\lambda+4)+4 = \lambda^2+4\lambda+4 = 0\ .$$

Therefore, the characteristic roots are

$$\lambda_1 = -2 \quad \text{and} \quad \lambda_2 = -2\ .$$

E3.5 A system is represented by a block diagram as shown in Figure E3.5. Write the state equations in the form of Equations (3.16) and (3.17).



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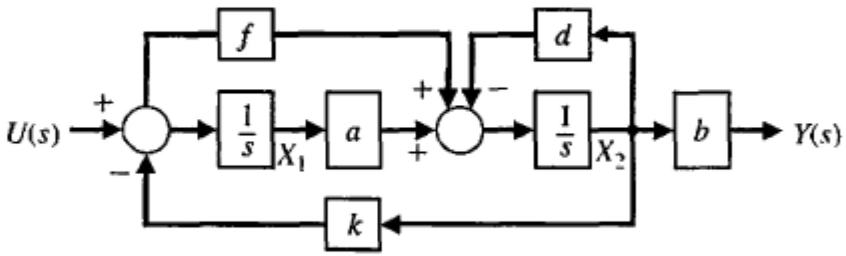

FIGURE E3.5 Block diagram.

E3.5 From the block diagram we determine that the state equations are

$$\dot{x}_2 = -(fk + d)x_2 + ax_1 + fu$$
$$\dot{x}_1 = -kx_2 + u$$

and the output equation is

$$y = bx_2 \ .$$

Therefore,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x} + \mathbf{D}u \ ,$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & -k \\ a & -(fk + d) \end{bmatrix} , \quad \mathbf{B} = \begin{bmatrix} 1 \\ f \end{bmatrix} , \quad \mathbf{C} = \begin{bmatrix} 0 & b \end{bmatrix} \text{ and } \quad \mathbf{D} = [0] \ .$$

E3.9 A multi-loop block diagram is shown in Figure E3.9. The state variables are denoted by x_1 and x_2. (a) Determine a state variable representation of the closed-loop system where the output is denoted by $y(t)$ and the input is $r(t)$. (b) Determine the characteristic equation.

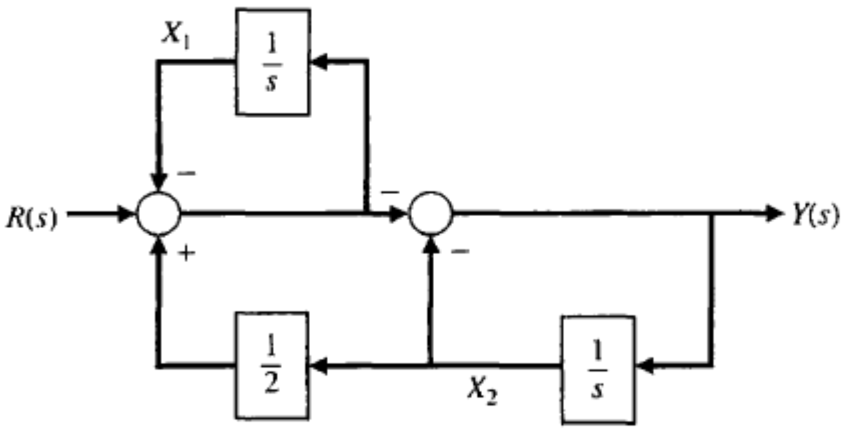

FIGURE E3.9 Multi-loop feedback control system.

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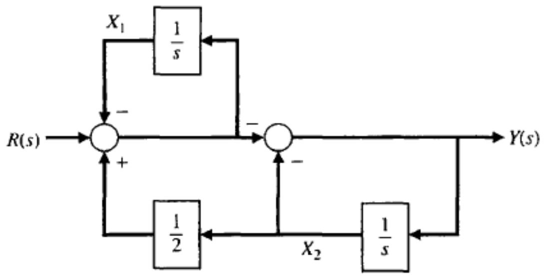

FIGURE E3.9 Multi-loop feedback control system.

E3.9 Analyzing the block diagram yields

$$\dot{x}_1 = -x_1 + \frac{1}{2}x_2 + r$$
$$\dot{x}_2 = x_1 - \frac{3}{2}x_2 - r$$
$$y = x_1 - \frac{3}{2}x_2 - r.$$

In state-variable form we have

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & \frac{1}{2} \\ 1 & -\frac{3}{2} \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} r \,, \quad y = \begin{bmatrix} 1 & -\frac{3}{2} \end{bmatrix}\mathbf{x} + \begin{bmatrix} -1 \end{bmatrix} r \,.$$

The characteristic equation is

$$s^2 + \frac{5}{2}s + 1 = (s+2)(s+\frac{1}{2}) = 0 \,.$$

E3.10 A hovering vehicle control system is represented by two state variables, and [13]

$$A = \begin{bmatrix} 0 & 12 \\ -1 & -7 \end{bmatrix}.$$

(a) Find the roots of the characteristic equation.
(b) Find the state transition matrix $\Phi(t)$.

E3.10 A hovering vehicle control system is represented by two state variables, and [13]

$$A = \begin{bmatrix} 0 & 6 \\ -1 & -5 \end{bmatrix}.$$

(a) Find the roots of the characteristic equation.
(b) Find the state transition matrix $\Phi(t)$.

E3.10 (a) The characteristic equation is

$$\det[\lambda \mathbf{I} - \mathbf{A}] = \det \begin{bmatrix} \lambda & -12 \\ 1 & (\lambda+7) \end{bmatrix} = \lambda(\lambda+7)+12 = (\lambda+4)(\lambda+3) = 0 \,.$$

So, the roots are $\lambda_1 = -4$ and $\lambda_2 = -3$.

(b) We note that

$$\Phi(s) = [s\mathbf{I} - \mathbf{A}]^{-1} = \begin{bmatrix} s & -12 \\ 1 & s+7 \end{bmatrix}^{-1} = \frac{1}{(s+4)(s+3)} \begin{bmatrix} s+7 & 12 \\ -1 & s \end{bmatrix} .$$

Taking the inverse Laplace transform yields the transition matrix

$$\Phi(t) = \begin{bmatrix} 4e^{-3t} - 3e^{-4t} & 12e^{-3t} - 12e^{-4t} \\ -e^{-3t} + e^{-4t} & -3e^{-3t} + 4e^{-4t} \end{bmatrix} .$$

E3.18 Consider a system represented by the following differential equations:

$$Ri_1 + L_1\frac{di_1}{dt} + v = v_a$$

$$L_2\frac{di_2}{dt} + v = v_b$$

$$i_1 + i_2 = C\frac{dv}{dt}$$

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E3.18 The governing equations of motion are

$$Ri_1 + L_1\frac{di_1}{dt} + v = v_a$$

$$L_2\frac{di_2}{dt} + v = v_b$$

$$i_L = i_1 + i_2 = C\frac{dv}{dt} \ .$$

Let $x_1 = i_1, x_2 = i_2, x_3 = v, u_1 = v_a$ and $u_2 = v_b$. Then,

$$\dot{\mathbf{x}} = \begin{bmatrix} -\frac{R}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & 0 & -\frac{1}{L_2} \\ \frac{1}{C} & \frac{1}{C} & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \\ 0 & 0 \end{bmatrix} \mathbf{u}$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \mathbf{x} + [0]\,\mathbf{u} \ .$$

E3.22 Consider the system in state variable form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x} + \mathbf{D}u$$

with

$$\mathbf{A} = \begin{bmatrix} 3 & 2 \\ 3 & 4 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \mathbf{C} = [1 \quad 0], \text{ and } \mathbf{D} = [0].$$

(a) Compute the transfer function $G(s) = Y(s)/U(s)$. (b) Determine the poles and zeros of the system. (c) If possible, represent the system as a first-order system

$$\dot{x} = ax + bu$$
$$y = cx + du$$

where a, b, c, and d are scalars such that the transfer function is the same as obtained in (a).

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E3.22 The transfer function is

$$G(s) = \frac{s-6}{s^2 - 7s + 6} \; .$$

The poles are at $s_1 = 1$ and $s_2 = 6$. The zero is at $s = 6$. So, we see that there is a pole-zero cancellation. We can write the system in state variable form as

$$\dot{x} = x - \sqrt{2}u$$
$$y = -\frac{\sqrt{2}}{2}x$$

and the transfer function is

$$G(s) = \frac{1}{s-1} \; .$$