Reglunarfræði T
501

Sýnidæmi úr 7. kafla

Þorgeir Pálsson

E7.5 Consider a unity feedback system with a loop transfer function

$$G_c(s)G(s) = \frac{s^2 + 2s + 10}{s^4 + 38s^3 + 515s^2 + 2950s + 6000}.$$

(a) Find the breakaway points on the real axis. (b) Find the asymptote centroid. (c) Find the values of K at the breakaway points.

```
r =

  -15.0000
  -10.0000
   -8.0000
   -5.0000
```

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E7.5 (a) The root locus is in Figure E7.5. The breakaway points are

$$\sigma_{b1} = -13.0 \ , \quad \sigma_{b2} = -5.89 \ .$$

(b) The asymptote centroid is

$$\sigma_{cent} = -18 \ ,$$

and

$$\phi_{asym} = \pm 90^o \ .$$

(c) The gains are $K_1 = 1.57$ and $K_2 = 2.14$ at the breakaway points.

```
r =

  -15.0000
  -10.0000
   -8.0000
   -5.0000
```

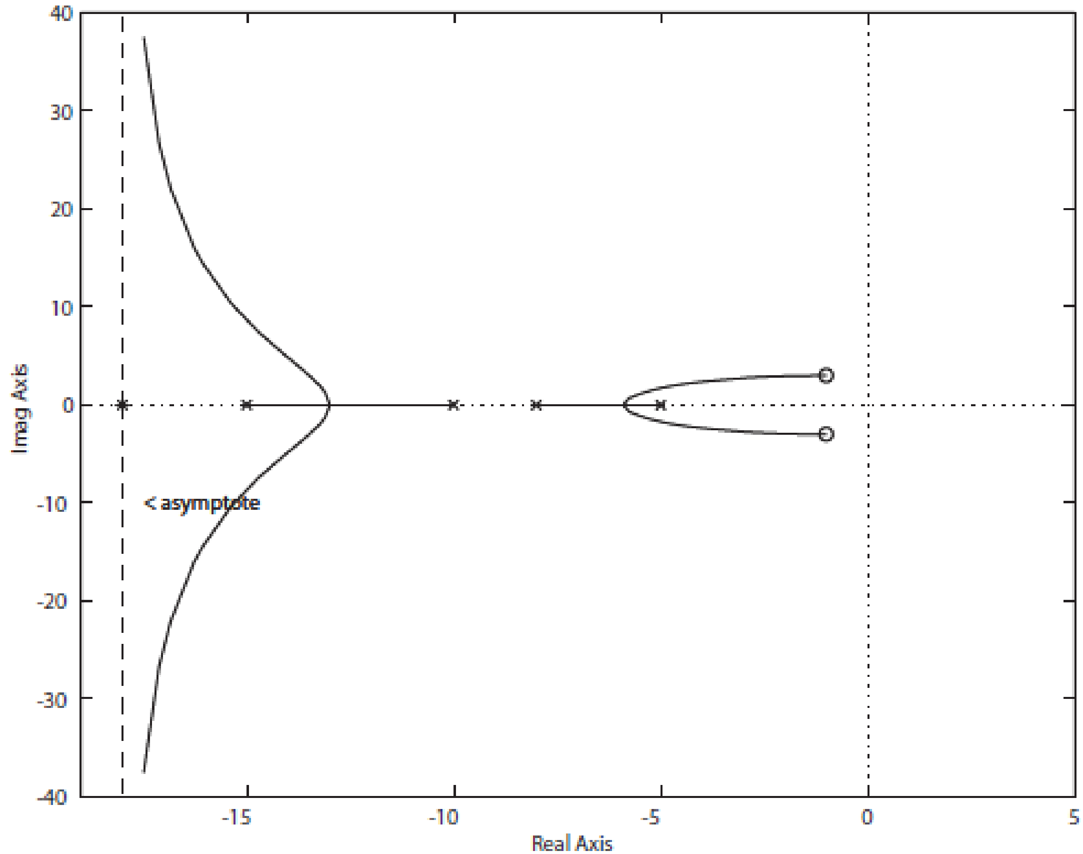

FIGURE E7.5
Root locus for $1 + K\frac{s^2+2s+10}{(s^4+38s^3+515s^2+2950s+6000)} = 0.$

E7.9 The world's largest telescope is located in Hawaii. The primary mirror has a diameter of 10 m and consists of a mosaic of 36 hexagonal segments with the orientation of each segment actively controlled. This unity feedback system for the mirror segments has the loop transfer function

$$L(s) = G_c(s)G(s) = \frac{K}{s(s^2 + 2s + 5)}.$$

(a) Find the asymptotes and draw them in the s-plane.
(b) Find the angle of departure from the complex poles.
(c) Determine the gain when two roots lie on the imaginary axis.
(d) Sketch the root locus.

E7.9 The characteristic equation is

$$1 + K\frac{1}{s(s^2+2s+5)} = 0$$

or

$$s^3 + 2s^2 + 5s + K = 0 \ .$$

(a) The system has three poles at $s = 0$ and $-1 \pm j2$. The number of asymptotes is $n_p - n_z = 3$ centered at $\sigma_{cent} = -2/3$, and the angles are ϕ_{asymp} at $\pm 60^o, 180^o$.

(b) The angle of departure, θ_d, is $90^o + \theta_d + 116.6^o = 180^o$, so $\theta_d = -26.6^o$.

(c) The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & 5 \\ s^2 & 2 & K \\ s^1 & b & \\ s^o & K & \end{array}$$

where $b = 5 - K/2$. So, when $K = 10$ the roots lie on the imaginary axis. The auxilary equation is

$$2s^2 + 10 = 0 \quad \text{which implies} \quad s_{1,2} = \pm j\sqrt{5} \,.$$

(d) The root locus is shown in Figure E7.9.

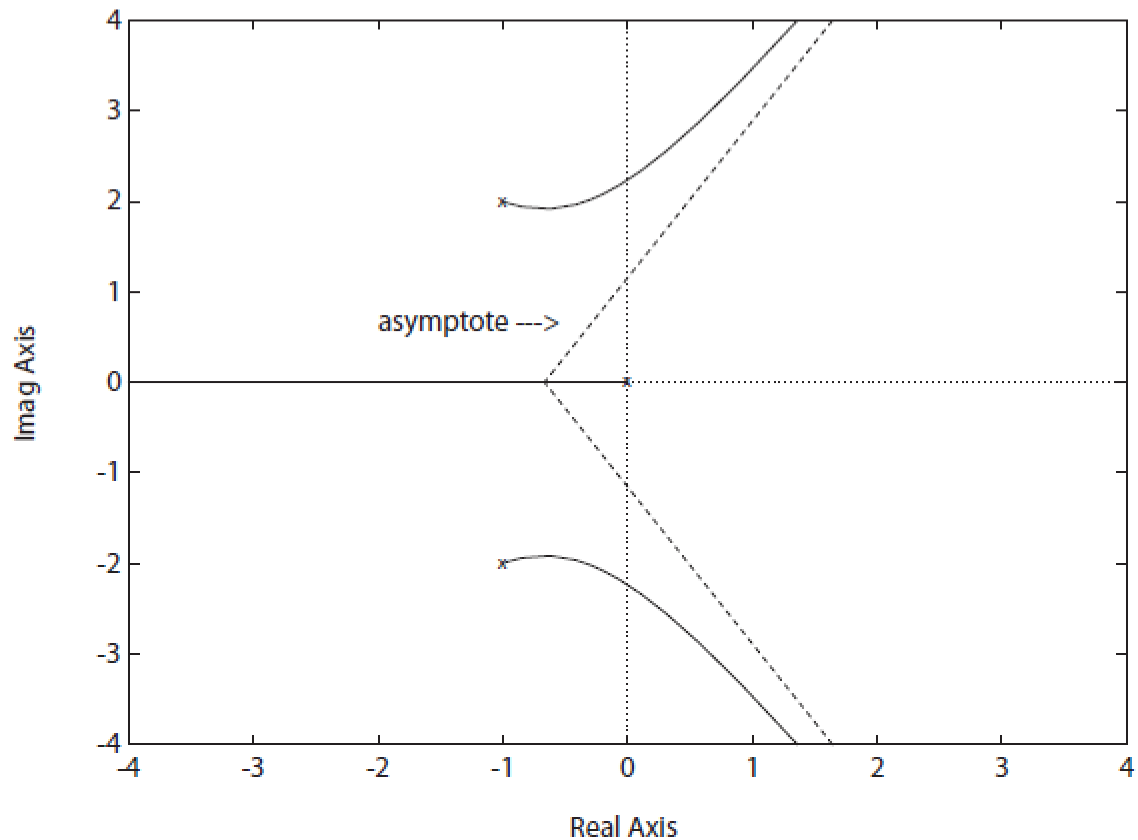

FIGURE E7.9
Root locus for $1 + K\frac{1}{s(s^2+2s+5)} = 0$.

E7.12 A unity feedback system has a loop transfer function

$$L(s) = KG(s) = \frac{K(s + 1)}{s(s^2 + 6s + 18)}.$$

(a) Sketch the root locus for $K > 0$. (b) Find the roots when $K = 10$ and 20. (c) Compute the rise time, percent overshoot, and settling time (with a 2% criterion) of the system for a unit step input when $K = 10$ and 20.

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E7.12 (a) The root locus is shown in Figure E7.12 for the characteristic equation

$$1 + \frac{K(s+1)}{s(s^2+6s+18)} = 0 .$$

(b) The roots of the characteristic equation are

(i) $K = 10$: $s_{1,2} = -2.8064 \pm 4.2368j$ and $s_3 = -0.3872$

(ii) $K = 20$: $s_{1,2} = -2.7134 \pm 5.2466j$ and $s_3 = -0.5732$

(c) The step response performance of the system is summarized in Table E7.12.

K	10	20
T_s (sec)	9.0	5.5
$P.O.$	0	0
T_r (sec)	4.8	2.6

TABLE E7.12 System performance when $K = 10$ and $K = 20$.

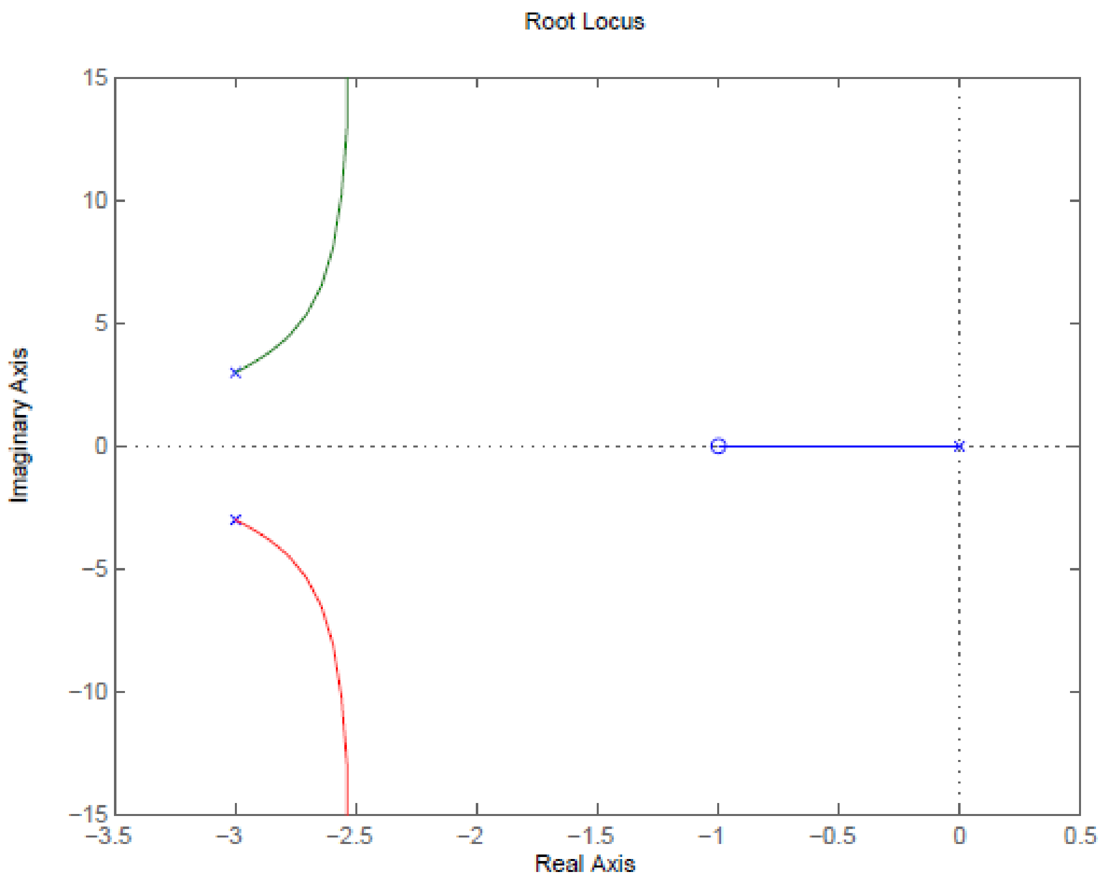

FIGURE E7.12
Root locus for $1 + \frac{K(s+1)}{s(s^2+6s+18)} = 0$.

E7.15 (a) Plot the root locus for a unity feedback system with loop transfer function

$$L(s) = G_c(s)G(s) = \frac{K(s + 10)(s + 2)}{s^3}.$$

(b) Calculate the range of K for which the system is stable. (c) Predict the steady-state error of the system for a ramp input.

Answers: (a) $K > 1.67$; (b) $e_{ss} = 0$

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E7.15 (a) The characteristic equation

$$1 + K\frac{(s+10)(s+2)}{s^3} = 0$$

has the root locus in Figure E7.15.

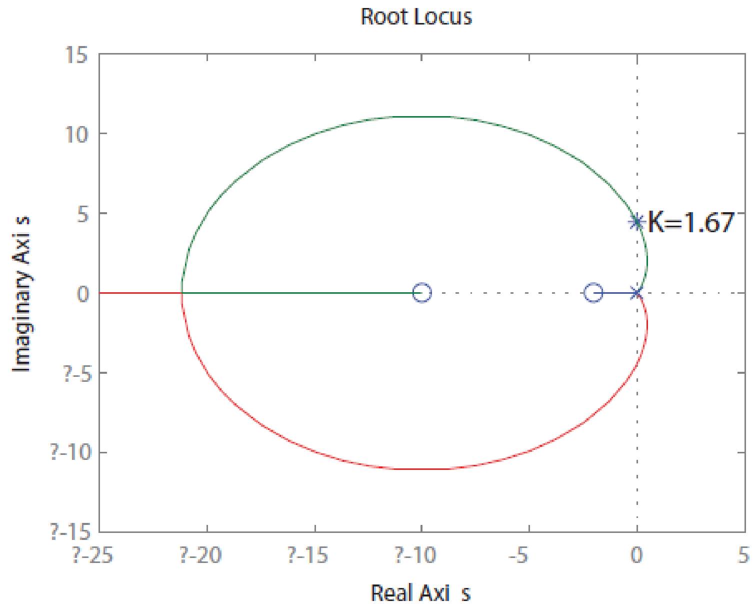

FIGURE E7.15
Root locus for $1 + \frac{K(s+10)(s+2)}{s^3} = 0$.

(b) The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & 12K \\ s^2 & K & 20K \\ s^1 & b & \\ s^o & 20K & \end{array}$$

when $b = 12K - 20$. For stability, we require all elements in the first column to be positive. Therefore, to be unstable

$$0 < K < 1.66 \; .$$

(c) When $K > 3/4$, we have

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \frac{1}{1 + GH(s)} \cdot \frac{1}{s^2} = \lim_{s \to 0} \frac{s^2}{s^3 + K(s+1)(s+3)} = 0 \, .$$

E7.19 A unity feedback system has a loop transfer function

$$L(s) = G_c(s)G(s) = \frac{K}{s(s + 3)(s^2 + 6s + 64)}.$$

(a) Determine the angle of departure of the root locus at the complex poles. (b) Sketch the root locus. (c) Determine the gain K when the roots are on the $j\omega$-axis and determine the location of these roots.

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E7.19 The characteristic equation is

$$1 + \frac{K}{s(s+3)(s^2+6s+64)} = 0 ,$$

and the root locus is shown in Figure E7.19. When $K = 1292.5$, the roots are

$$s_{1,2} = \pm j4.62 \quad \text{and} \quad s_{3,4} = -4.49 \pm j6.36 .$$

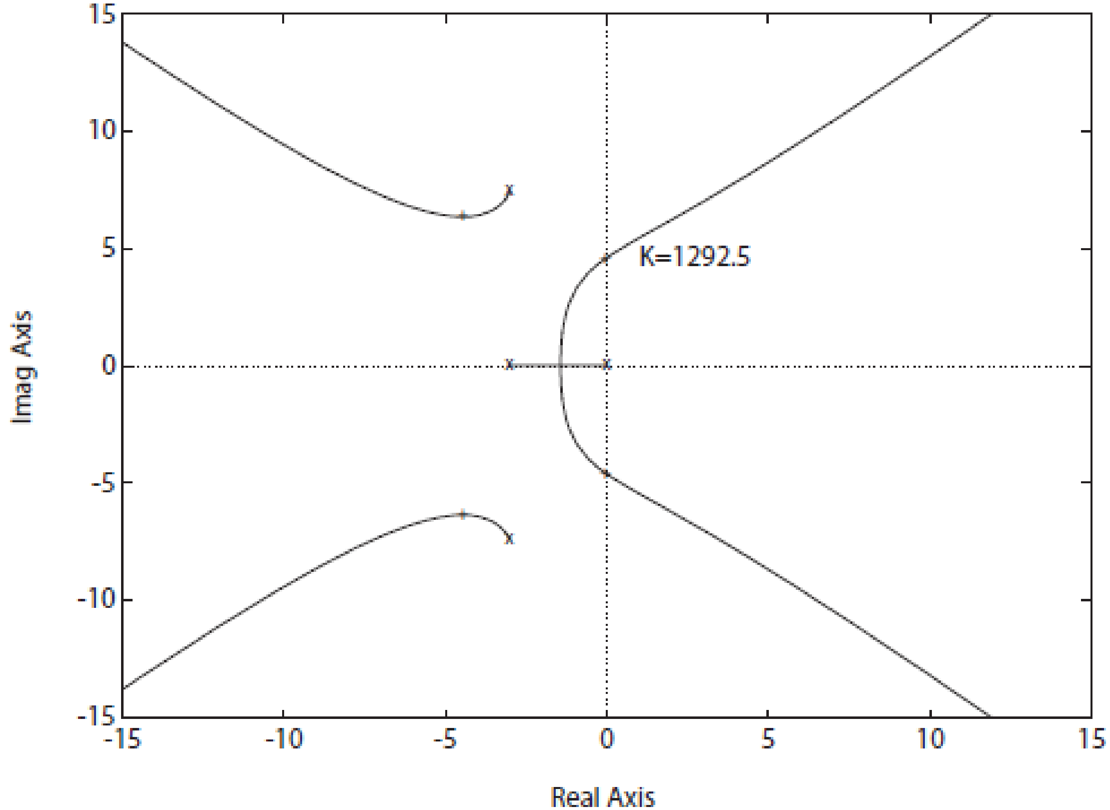

FIGURE E7.19
Root locus for $1 + \frac{K}{s(s+3)(s^2+6s+64)} = 0$.

E7.25 A closed-loop feedback system is shown in Figure E7.25. For what range of values of the parameters K is the system stable? Sketch the root locus as $0 < K < \infty$.

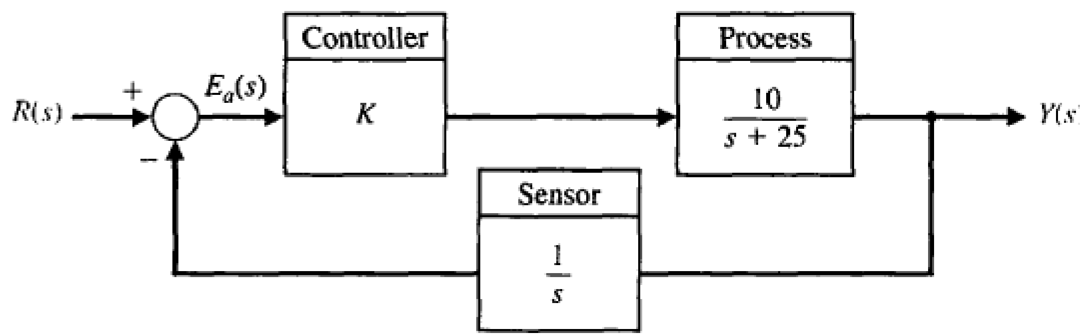

FIGURE E7.25
Nonunity feedback system with parameter K.

E7.25 The characteristic equation is

$$1 + K\frac{10}{s(s+25)} = 0 \ .$$

The root locus shown in Figure E7.25 is stable for all $0 < K < \infty$.

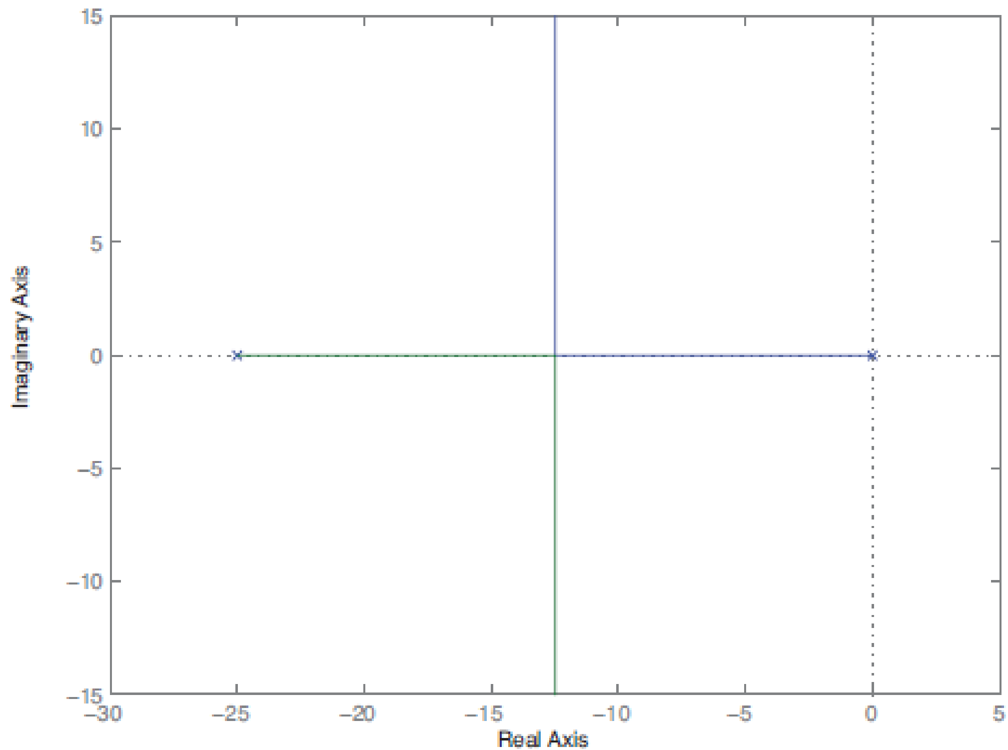

FIGURE E7.25
Root locus for $1 + K\frac{10}{s(s+25)} = 0$.

E7.27 Consider the unity feedback system in Figure E7.27. Sketch the root locus as $0 \leq p < \infty$.

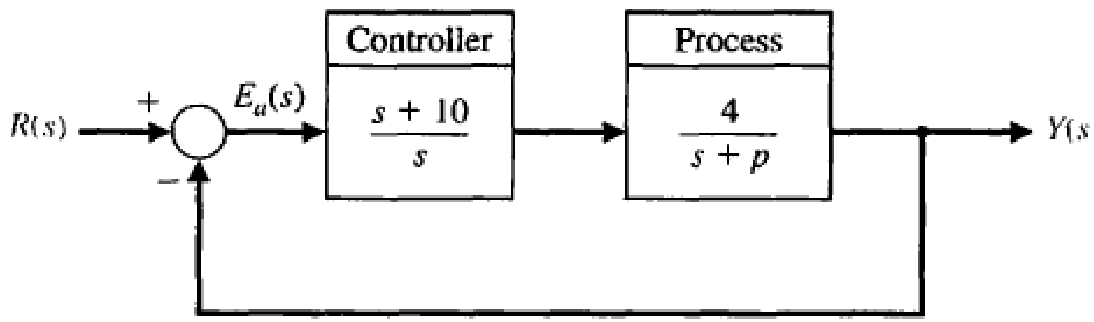

FIGURE E7.27
Unity feedback system with parameter p.

E7.27 The characteristic equation is

$$1 + p\frac{s}{s^2 + 4s + 40} = 0 \ .$$

The root locus shown in Figure E7.27 is stable for all $0 < p < \infty$.

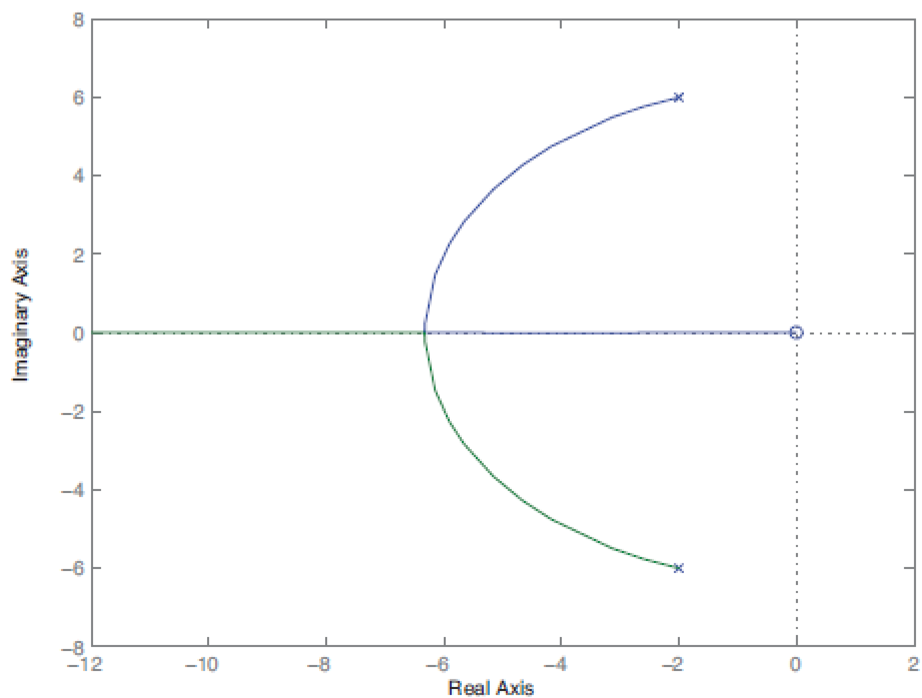

FIGURE E7.27
Root locus for $1 + p\frac{s}{s^2+4s+40} = 0$.