Reglunarfræði T
501

Æfingardæmi Kafli 4

Þorgeir Pálsson

E4.6 A unity feedback system has the loop transfer function

$$L(s) = G_c(s)G(s) = \frac{10K}{s(s+b)}.$$

Determine the relationship between the steady-state error to a ramp input and the gain K and system parameter b. For what values of K and b can we guarantee that the magnitude of the steady-state error to a ramp input is less than 0.1?

E4.6 A unity feedback system has the loop transfer function

$$L(s) = G_c(s)G(s) = \frac{10K}{s(s+b)}.$$

Determine the relationship between the steady-state error to a ramp input and the gain K and system parameter b. For what values of K and b can we guarantee that the magnitude of the steady-state error to a ramp input is less than 0.1?

Reykjavik University

Lausn:

E4.6 The closed-loop transfer function is

$$T(s) = \frac{10K}{s^2 + bs + 10K}.$$

The tracking error is

$$E(s) = [1 - T(s)]\, R(s) = \frac{s(s+b)}{s^2 + bs + 10K}\frac{1}{s^2},$$

where we let $R(s) = 1/s^2$. Using the final value theorem we obtain the steady-state tracking error as

$$e_{ss} = \lim_{s \to 0} sE(s) = \frac{b}{10K}\ .$$

If we require that $b < K$ then the steady-state error is less than 0.1 to the ramp input.

E4.9 Submersibles with clear plastic hulls have the potential to revolutionize underwater leisure. One small submersible vehicle has a depth-control system as illustrated in Figure E4.9.

(a) Determine the closed-loop transfer function $T(s) = Y(s)/R(s)$.

(b) Determine the sensitivity $S^T_{K_1}$ and S^T_K.

(c) Determine the steady-state error due to a disturbance $T_d(s) = 1/s$.

(d) Calculate the response $y(t)$ for a step input $R(s) = 1/s$ when $K = K_2 = 1$ and $1 < K_1 < 10$. Select K_1 for the fastest response.

Reykjavik University

Exercises

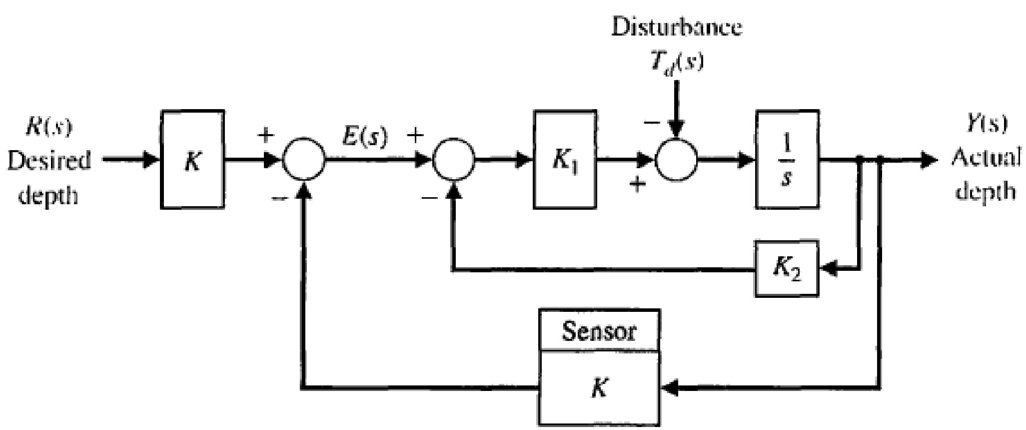


FIGURE E4.9
Depth control system.

Lausn

E4.9 (a) The closed-loop transfer function is

$$T(s) = \frac{KK_1}{s + K_1(K + K_2)} \ .$$

(b) The sensitivities are

$$S_K^T = \frac{\partial T/T}{\partial K/K} = \frac{s + K_1K_2}{s + K_1(K + K_2)}$$

and

$$S_{K_1}^T = \frac{s}{s + K_1(K + K_2)} \ .$$

(c) The transfer function from $T_d(s)$ to $Y(s)$ is

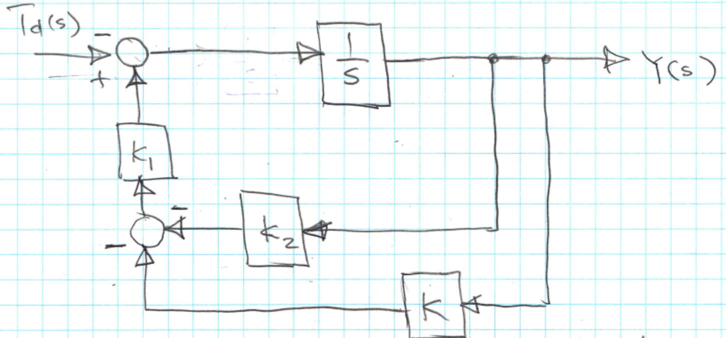
$$\frac{Y(s)}{T_d(s)} = \frac{-1}{s + K_1(K_2 + K)} \ .$$

Therefore, since $E(s) = -KY(s)$ (when $R(s) = 0$), we have

$$E(s) = \frac{K}{s + K_1(K_2 + K)} T_d(s)$$

Dæmi E 4.9

(c) Til að finna eftstöðu skekkju vegna einingar þreps, $T_d(s)$, getum við fundið yfirfærslufallið með því að endurteikna blokkritið:



takið eftir að þetta er neikvæð bakverkun:

$$\frac{Y(s)}{T_d(s)} = \frac{-1/s}{1+\frac{1}{s}K_1(K+K_2)} = \frac{-1}{s+K_1(K+K_2)}$$

Skekkjan er: $E(s) = \frac{1}{s+K_1(K+K_2)}\,\frac{1}{s}$

$$e(\infty) = \frac{1}{K_1(K+K_2)}$$

Auðveldast er þó að beita Mason á upphaflega blokkritið:

$$P_1 = \frac{1}{s} \qquad \Delta = 1 + \frac{K_1K_2}{s} + \frac{K_1K}{s}$$

$$\frac{Y(s)}{T_d(s)} = \frac{-1}{s+K_1K_2+K_1K} = \frac{-1}{s+K_1(K+K_2)}$$

DAGS.
NAFN
BLAÐ 2 AF 2

Regla Mason fyrir kerfi, þar sem allar lykkjur snertast og snerta allar tengi-rásir milli inn- og útmerkis er:

$$T(s) = \frac{\sum_k P_k \Delta_k}{\Delta}$$

þar sem P_k er yfirfærslufall tengileiðar k milli inn- og útmerkis og $\Delta_k = 1$. (co-factor)

$$\Delta = 1 - \sum_{n=1}^{N} L_n \qquad (\text{ákveða kerfisins})$$

L_n = yfirfærslufall lykkju n.

Þegar einhverjar tvær lykkjur snertast ekki er:

$$\Delta = 1 - \sum_{n=1}^{N} L_n + \sum_{l,m} L_l L_m$$

þar sem $L_l L_m$ er margfeldi yfirfærslufalla slíkra lykkja, sem ekki snertast.

Ef einhverjar lykkjur snerta ekki P_i er co-factorinn $\Delta_i = 1 - \sum L_j$,
þar sem L_j er yfirfærslufall lykkju sem ekki snertir P_i.

Lausn.....

and

$$e_{ss} = \lim_{s \to 0} sE(s) = \frac{K}{K_1(K + K_2)} \ .$$

(d) With $K = K_2 = 1$, we have

$$T(s) = \frac{K_1}{s + 2K_1} \ .$$

Then,

$$Y(s) = \frac{K_1}{s + 2K_1} \frac{1}{s}$$

and

$$y(t) = \frac{1}{2} \left[1 - e^{-2K_1 t}\right] u(t) \ ,$$

where $u(t)$ is the unit step function. Therefore, select $K_1 = 10$ for the fastest response.

E4.11 Consider the closed-loop system in Figure E4.11, where

$$G(s) = \frac{K}{s + 10} \quad \text{and} \quad H(s) = \frac{14}{s^2 + 5s + 6}.$$

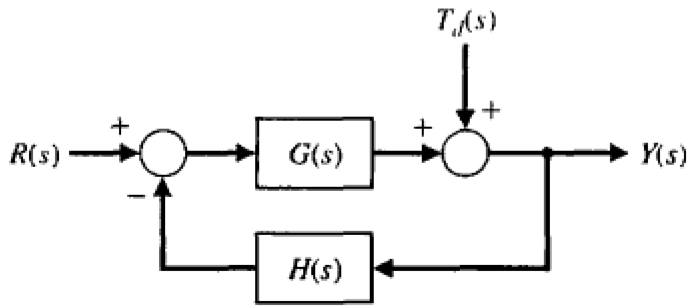


FIGURE E4.11 Closed-loop system with nonunity feedback.

(a) Compute the transfer function $T(s) = Y(s)/R(s)$.

(b) Define the tracking error to be $E(s) = R(s) - Y(s)$. Compute $E(s)$ and determine the steady-state tracking error due to a unit step input, that is, let $R(s) = 1/s$.

(c) Compute the transfer function $Y(s)/T_d(s)$ and determine the steady-state error of the output due to a unit step disturbance input, that is, let $T_d(s) = 1/s$.

(d) Compute the sensitivity S_K^T.

Reykjavik University

Lausn

Reykjavik University

E4.11 (a) The closed-loop transfer function is

$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{K(s^2+5s+6)}{s^3+15s^2+56s+60+14K}$$

(b) With $E(s) = R(s) - Y(s)$ we obtain

$$E(s) = \left[1 - \frac{G(s)}{1+G(s)H(s)}\right] R(s) = \frac{1-G(s)(1-H(s))}{1+G(s)H(s)} R(s)$$
$$= \frac{s^3+(15-K)s^2+(56-5K)s+(60+8K)}{s^3+15s^2+56s+60+14K} \cdot \frac{1}{s} \ .$$

Then, using the final value theorem we find

$$\lim_{s\to 0} sE(s) = \frac{(60+8K)}{60+14K} \ .$$

(c) The transfer function from the disturbance $T_d(s)$ to the output is

$$Y(s) = \frac{1}{1+G(s)H(s)} T_d(s) = \frac{s^3+15s^2+56s+60}{s^3+15s^2+56s+60+14K} T_d(s) \ .$$

The steady-state error to a unit step disturbance is

$$\lim_{s\to 0} sY(s) = \lim_{s\to 0} s \frac{s^3+15s^2+56s+60}{s^3+15s^2+56s+60+14K} \cdot \frac{1}{s} = \frac{60}{60+14K} \ .$$

(d) The sensitivity is

$$S_K^T = \frac{\partial T}{\partial K}\frac{K}{T} = \frac{\partial T}{\partial G}\frac{\partial G}{\partial K}\frac{K}{T}$$
$$= \frac{1}{(1+G(s)H(s))^2}\left(\frac{K}{s+10}\right)\frac{1+G(s)H(s)}{G(s)} = \frac{1}{1+G(s)H(s)} \ .$$

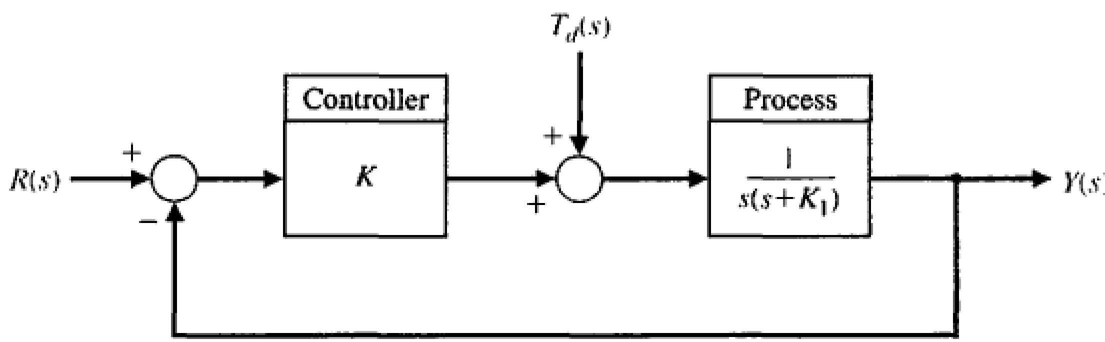
Reykjavik University

E4.14 Consider the unity feedback system shown in Figure E4.14. The system has two parameters, the controller gain K and the constant K_1 in the process.

(a) Calculate the sensitivity of the closed-loop transfer function to changes in K_1.

(b) How would you select a value for K to minimize the effects of external disturbances, $T_d(s)$?

FIGURE E4.14 Closed-loop feedback system with two parameters, K and K_1.



Lausn

E4.14 (a) The closed-loop transfer function is

$$T(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)H(s)} = \frac{K}{s^2 + K_1 s + K} .$$

The sensitivity is

$$S_{K_1}^T = \frac{\partial T/T}{\partial K_1/K_1} = -\frac{sK_1}{s^2 + K_1 s + K}.$$

(b) You would make K as large as possible to reduce the sensitivity to changes in K_1. But the design trade-off would be to keep K as small as possible to reject measurement noise.

E4.15 Reconsider the unity feedback system discussed in E4.14. This time select $K = 120$ and $K_1 = 10$. The closed-loop system is depicted in Figure E4.15.

(a) Calculate the steady-state error of the closed-loop system due to a unit step input, $R(s) = 1/s$, with $T_d(s) = 0$. Recall that the tracking error is defined as $E(s) = R(s) - Y(s)$.

(b) Calculate the steady-state response, $y_{ss} = \lim_{t\to\infty} y(t)$, when $T_d(s) = 1/s$ and $R(s) = 0$.

Reykjavik University

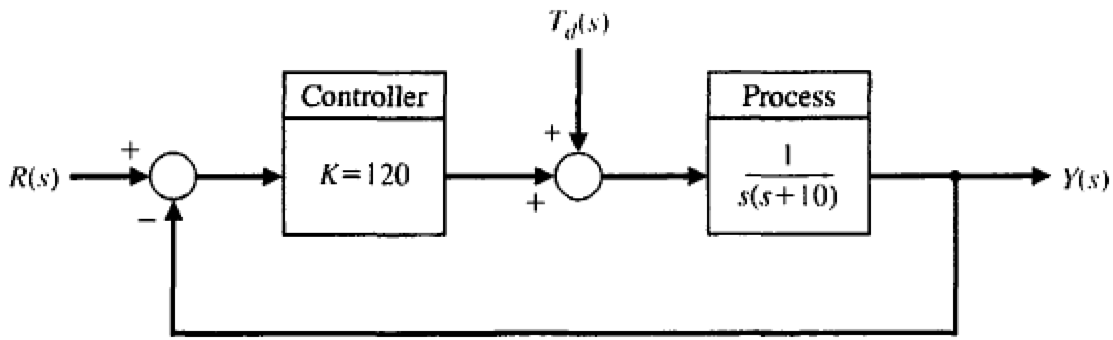


FIGURE E4.15 Closed-loop feedback system with $K = 120$ and $K_1 = 10$.

Lausn

vik University

E4.15 (a) The closed-loop transfer function is

$$T(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)H(s)} = \frac{150}{s^2 + 10s + 150} .$$

The steady-state tracking error is

$$E(s) = R(s) - Y(s) = \left[\frac{1}{1 + G_c(s)G(s)}\right] R(s)$$
$$= \frac{s^2 + 10s}{s^2 + 10s + 150} \cdot \frac{1}{s}$$

and

$$\lim_{s \to 0} sE(s) = 0 .$$

(b) The transfer function from the disturbance $T_d(s)$ to the output $Y(s)$ is

$$Y(s) = \left[\frac{1}{s^2 + 10s + 150}\right] T_d(s) .$$

The steady-state error to a unit step $T_d(s) = 1/s$ is

$$\lim_{s \to 0} sY(s) = \lim_{s \to 0} s \left[\frac{1}{s^2 + 10s + 150}\right] \cdot \frac{1}{s} = \frac{1}{150} .$$