Reglunarfræði T
501

Æfingardæmi – kafli 8

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E8.1 Increased track densities for computer disk drives necessitate careful design of the head positioning control [1]. The loop transfer function is

$$L(s) = G_c(s)G(s) = \frac{K}{(s+1)^2}$$

Plot the frequency response for this system when $K = 4$. Calculate the phase and magnitude at $\omega = 0.5, 1, 2, 4,$ and ∞.

Answer: $|L(j0.5)| = 0.94$ and $\angle L(j0.5) = -28.1°$.

E8.2 A tendon-operated robotic hand can be implemented using a pneumatic actuator [8]. The actuator can be represented by

$$G(s) = \frac{5000}{(s + 70)(s + 500)}.$$

Plot the frequency response of $G(j\omega)$. Show that the magnitude of $G(j\omega)$ is -17 dB at $\omega = 10$ and -27.1 dB at $\omega = 200$. Show also that the phase is $-138.7°$ at $\omega = 700$.

E8.2 The transfer function is

$$G(s) = \frac{5000}{(s+70)(s+500)} .$$

The frequency response plot is shown in Figure E8.2. The phase angle is computed from

$$\phi = -\tan^{-1}\frac{\omega}{70} - \tan^{-1}\frac{\omega}{500} .$$

The phase angles for $\omega = 10, 100$ and 700 are summarized in Table E8.2.

ω	10	200	700
$\lvert G(j\omega)\rvert$	-16.99	-27.17	-41.66
ϕ (deg)	-9.28	-92.51	-138.75

TABLE E8.2 **Magnitude and phase for** $G(s) = \frac{5000}{(s+70)(s+500)}$.

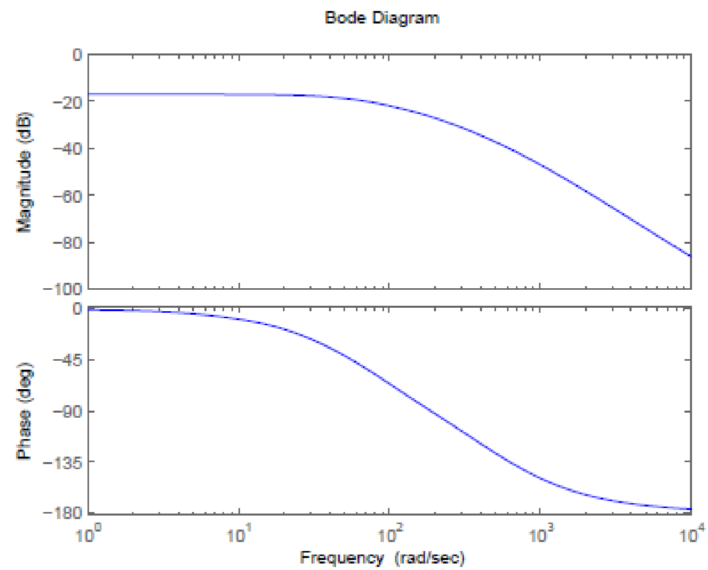

FIGURE E8.2
Frequency response for $G(s) = \frac{5000}{(s+70)(s+500)}$.

E8.3 The frequency response for a process of the form

$$G(s) = \frac{Ks}{(s + a)(s^2 + 20s + 100)}$$

is shown in Figure E8.4. Determine K and a by examining the frequency response curves.

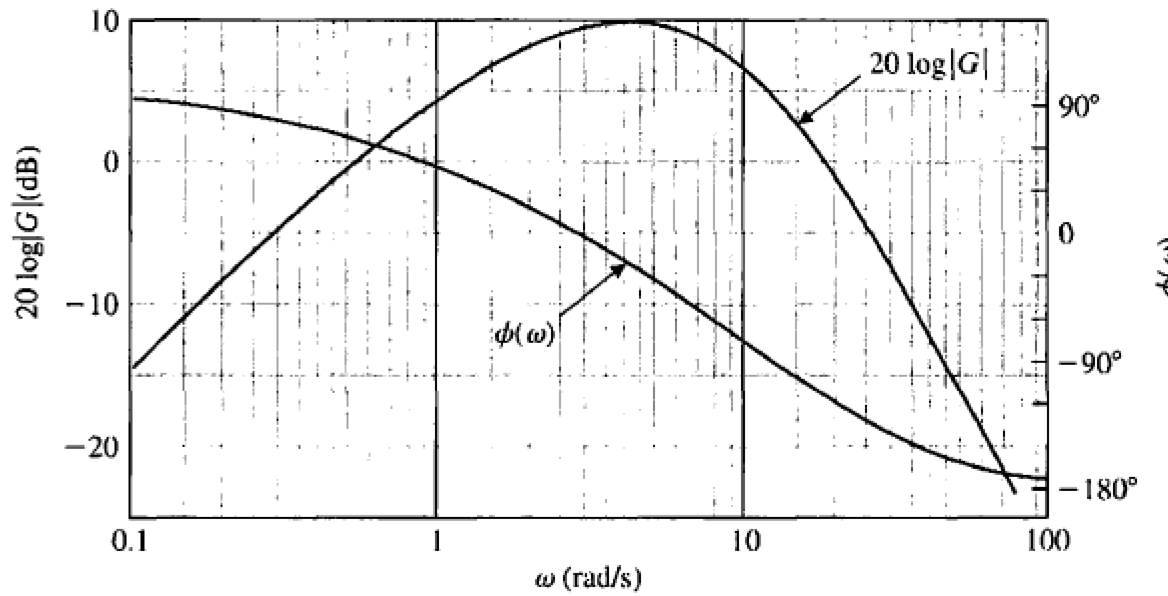

FIGURE E8.3
Bode diag

E8.3 The transfer function is

$$G(s) = \frac{Ks}{(s+a)(s+10)^2} \ .$$

Note that $\phi = 0^o$ at $\omega = 3$, and that

$$\phi = +90^o - \tan^{-1}\frac{\omega}{a} - 2\tan^{-1}\frac{\omega}{10} \ .$$

Substituting $\omega = 3$ and solving for a yields

$$a = 2 \ .$$

Similarly, from the magnitude relationship we determine that

$$K = 400 \ .$$

E8.4 A robotic arm has a joint-control loop transfer function

$$L(s) = G_c(s)G(s) = \frac{300(s + 100)}{s(s + 10)(s + 40)}.$$

Show that the frequency equals 28.3 rad/s when the phase angle of $L(j\omega)$ is $-180°$. Find the magnitude of $L(j\omega)$ at that frequency.

Answer: $|L(j28.3)| = -2.5$ dB

E8.4 The loop transfer function is

$$L(s) = \frac{300(s+100)}{s(s+10)(s+40)} \ .$$

The phase angle is computed via

$$\phi(\omega) = -90^o - \tan^{-1}\frac{\omega}{10} - \tan^{-1}\frac{\omega}{40} + \tan^{-1}\frac{\omega}{100} \ .$$

At $\omega = 28.3$, we determine that

$$\phi = -90^o - 70.5^o - 35.3^o + 15.8^o = 180^o \ .$$

Computing the magnitude yields

$$|L(j\omega)| = \frac{300(100)(1+(\frac{\omega}{100})^2)^{\frac{1}{2}}}{\omega 10(1+(\frac{\omega}{10})^2)^{\frac{1}{2}} 40(1+(\frac{\omega}{40})^2)^{\frac{1}{2}}} = 0.75 \ ,$$

when $\omega = 28.3$. We can also rewrite $L(s)$ as

$$L(s) = \frac{75(\frac{s}{100}+1)}{s(\frac{s}{10}+1)(\frac{s}{40}+1)} \ .$$

Then, the magnitude in dB is

$$\begin{aligned} 20\log_{10}|L| = 20\log_{10}(75) + 10\log_{10}(1+(\frac{\omega}{100})^2) - 10\log_{10}(1+(\frac{\omega}{10})^2) \\ - 10\log_{10}(1+(\frac{\omega}{40})^2) - 20\log_{10}\omega = -2.5 \text{ dB} \ , \end{aligned}$$

at $\omega = 28.3$.

E8.5 The magnitude plot of a transfer function

$$G(s) = \frac{K(1 + 0.5s)(1 + as)}{s(1 + s/8)(1 + bs)(1 + s/36)}$$

is shown in Figure E8.5. Determine K, a, and b from the plot.

Answer: $K = 8, a = 1/4, b = 1/24$

E8.7 The lower portion for $\omega < 2$ is

$$20 \log \frac{K}{\omega} = 0 \text{ dB} ,$$

at $\omega = 8$. Therefore,

$$20 \log \frac{K}{8} = 0 \text{ dB}$$

which occurs when

$$K = 8 .$$

We have a zero at $\omega = 2$ and another zero at $\omega = 4$. The zero at $\omega = 4$ yields

$$a = 0.25 .$$

We also have a pole at $\omega = 8$, and a second pole at $\omega = 24$. The pole at $\omega = 24$ yields

$$b = 1/24 .$$

Therefore,

$$G(s) = \frac{8(1 + s/2)(1 + s/4)}{s(1 + s/8)(1 + s/24)(1 + s/36)} .$$

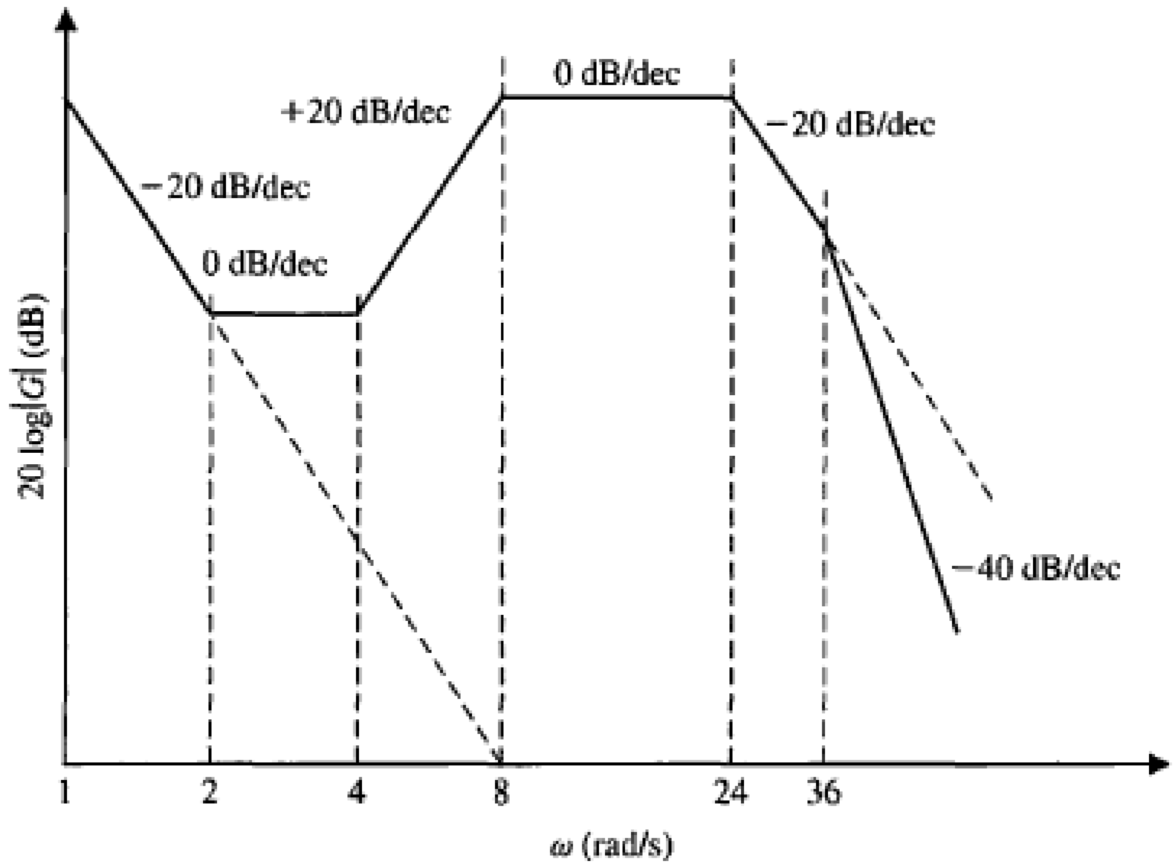

FIGURE E8.5
Bode diagram.

E8.7 Consider a system with a closed-loop transfer function

$$T(s) = \frac{Y(s)}{R(s)} = \frac{4}{(s^2 + s + 1)(s^2 + 0.4s + 4)}.$$

This system will have no steady-state error for a step input. (a) Plot the frequency response, noting the two peaks in the magnitude response. (b) Predict the time response to a step input, noting that the system has four poles and cannot be represented as a dominant second-order system. (c) Plot the step response.

E8.5 The transfer function is

$$T(s) = \frac{4}{(s^2 + s + 1)(s^2 + 0.4s + 4)} .$$

(a) The frequency response magnitude is shown in Figure E8.5. The frequency response has two resonant peaks at

$$\omega_{r_1} = 0.8 \text{ rad/sec} \quad \text{and} \quad \omega_{r_2} = 1.9 \text{ rad/sec} .$$

(b) The percent overshoot is

$$P.O. = 35\% ,$$

and the settling time is

$$T_s \approx 16 \text{ sec} .$$

(c) The step response is shown in Figure E8.5.

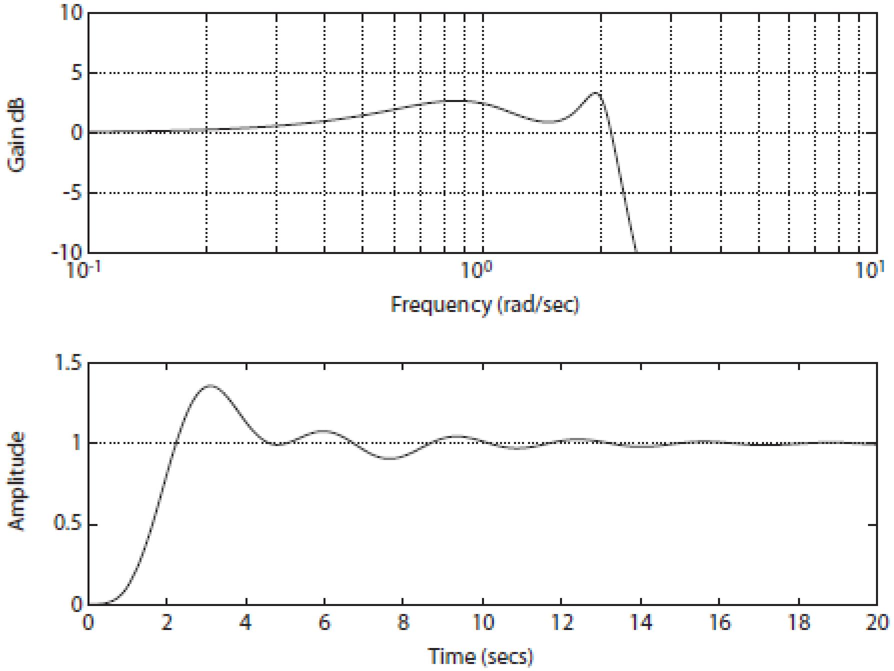

FIGURE E8.5
(a) Bode Diagram for $T(s) = \frac{4}{(s^2+s+1)(s^2+0.4s+4)}$. (b) Unit step response.

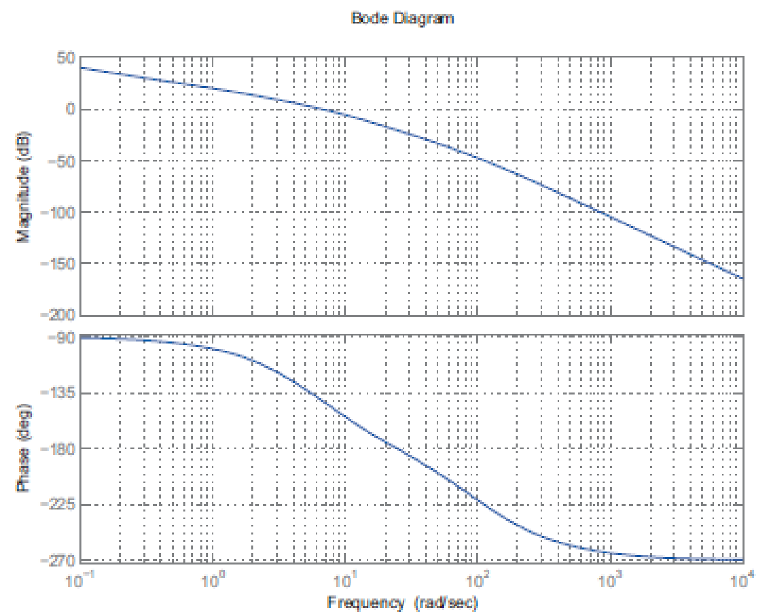

FIGURE E8.6
Bode Diagram for $L(s) = \frac{10}{s(s/6+1)(s/100+1)}$.

E8.9 The Bode diagram of a system is shown in Figure E8.9. Determine the transfer function $G(s)$.

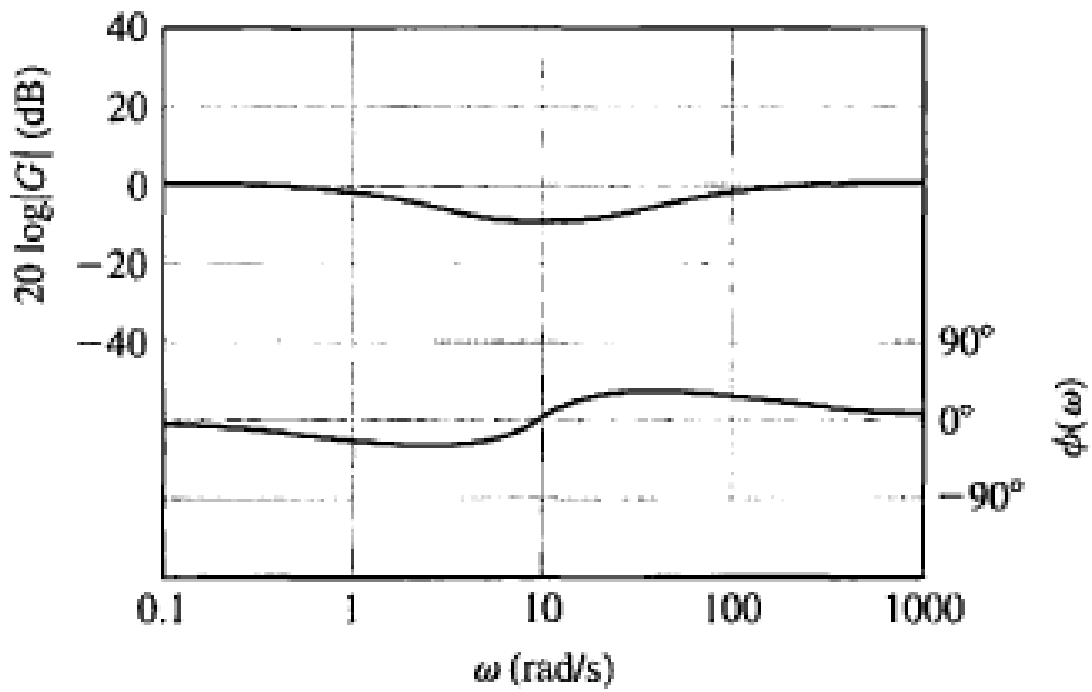

FIGURE E8.9
Bode diagram.

E8.9 The Bode diagram for $G(s)$ is shown in Figure E8.9.

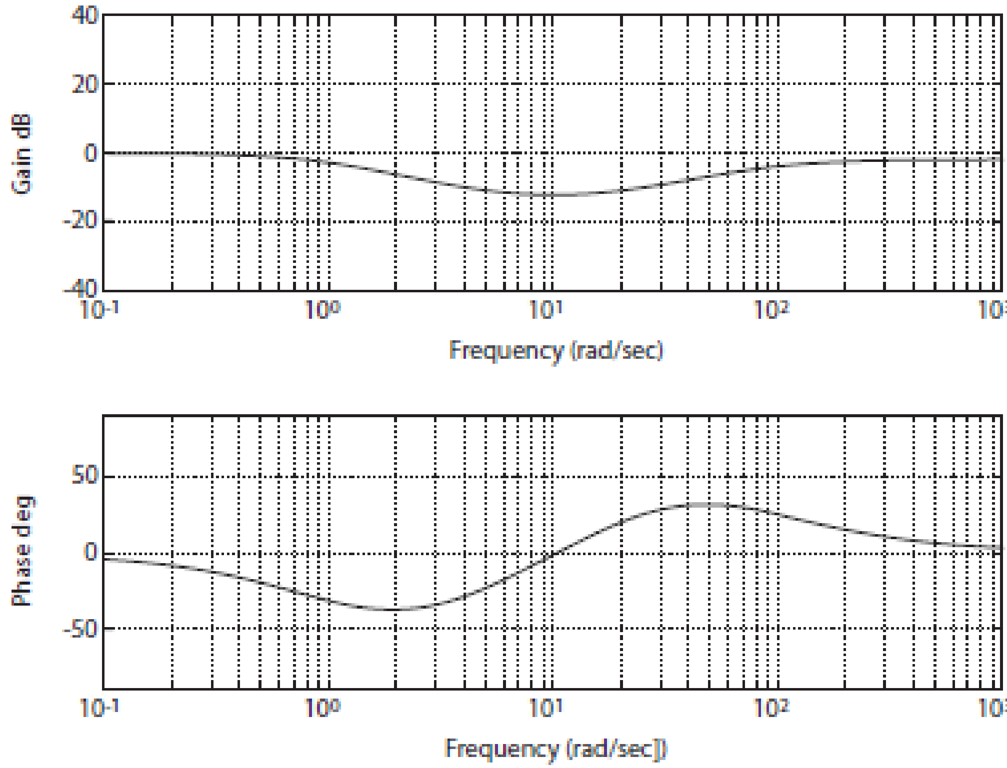

FIGURE E8.9
Bode Diagram for $G(s) = \frac{(s/5+1)(s/20+1)}{(s+1)(s/80+1)}$.

E8.14 Consider the nonunity feedback system in Figure E8.14, where the controller gain is $K = 2$. Sketch the Bode plot of the loop transfer function. Determine the phase of the loop transfer function when the magnitude $20 \log|L(j\omega)| = 0$ dB. Recall that the loop transfer function is $L(s) = G_c(s)G(s)H(s)$.

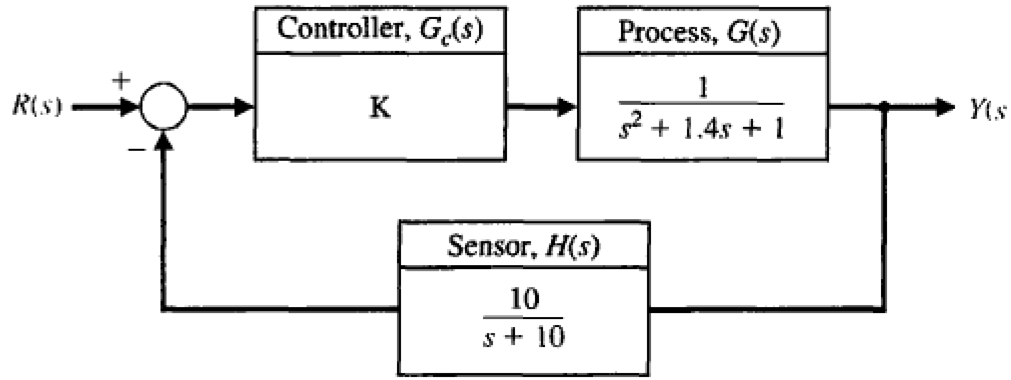

FIGURE E8.14 Nonunity feedback system with controller gain K.

E8.14 Consider the nonunity feedback system in Figure E8.14, where the controller gain is $K = 2$. Sketch the Bode plot of the loop transfer function. Determine the phase of the loop transfer function when the magnitude $20 \log|L(j\omega)| = 0$ dB. Recall that the loop transfer function is $L(s) = G_c(s)G(s)H(s)$.

Lausn:

E8.14 The loop transfer function is

$$L(s) = \frac{20}{(s^2 + 1.4s + 1)(s + 10)} .$$

The Bode plot of $L(s)$ is shown in Figure E8.14. The frequency when $20 \log_{10} |L(\jmath\omega)| = 0$ is $\omega = 1.32$ rad/sec.

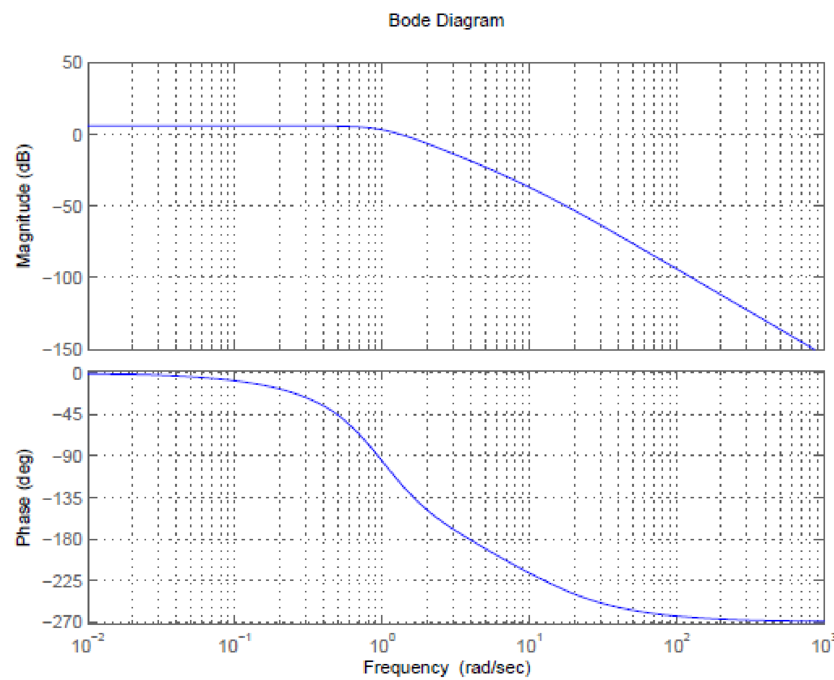

FIGURE E8.14
Bode Diagram for $L(s) = \frac{20}{(s^2+1.4s+1)(s+10)}$.

E8.14 The loop transfer function is

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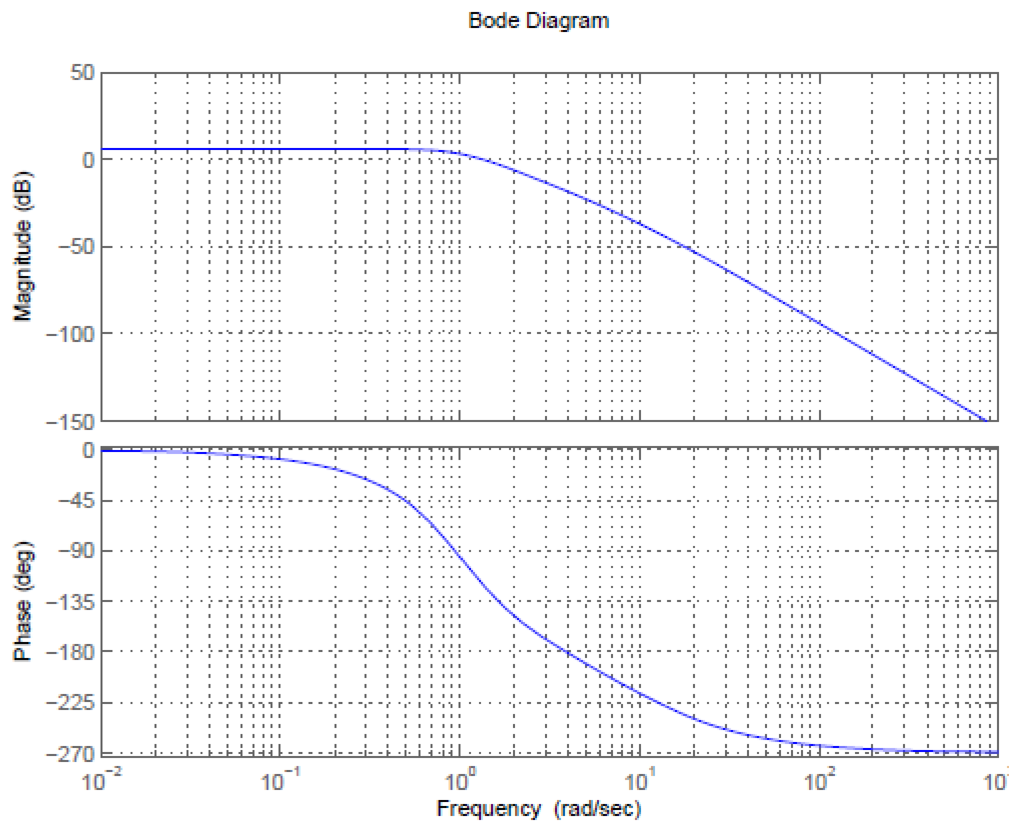

FIGURE E8.14
Bode Diagram for $L(s) = \frac{20}{(s^2+1.4s+1)(s+10)}$.