Reglunarfræði T
501

Æfingardæmi úr kafla 6

28. September 2015

Þorgeir Pálsson

E6.2 A system has a characteristic equation $s^3 + 10s^2 + 2s + 30 = 0$. Using the Routh–Hurwitz criterion, show that the system is unstable.

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E6.2 A system has a characteristic equation $s^3 + 8s^2 + 2s + 30 = 0$. Using the Routh–Hurwitz criterion, show that the system is unstable.

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Lausn:

E6.2 The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 8 & 30 \\ s^1 & -7/4 & 0 \\ s^o & 30 & \end{array}$$

The system is unstable since the first column shows two sign changes.

E6.5 A unity feedback system has a loop transfer function

$$L(s) = \frac{K}{(s+1)(s+3)(s+6)},$$

where $K = 20$. Find the roots of the closed-loop system's characteristic equation.

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where $K = 20$. Find the roots of the closed-loop system's characteristic equation.

Lausn (fundin með tölvu sem finnur rætur margliðu):

E6.5 The closed-loop transfer function is

$$T(s) = \frac{K}{s^3 + 10s^2 + 27s + 18 + K} \ .$$

When $K = 20$, the roots of the characteristic polynomial are

$$s_{1,2} = -1.56 \pm j1.76$$

and

$$s_3 = -6.88 \ .$$

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E6.7 Designers have developed small, fast, vertical-take-off fighter aircraft that are invisible to radar (stealth aircraft). This aircraft concept uses quickly turning jet nozzles to steer the airplane [16]. The control system for the heading or direction control is shown in Figure E6.8. Determine the maximum gain of the system for stable operation.

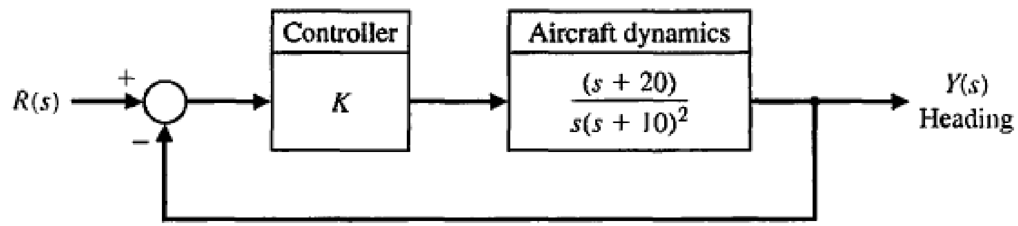

FIGURE E6.8
Aircraft heading control.

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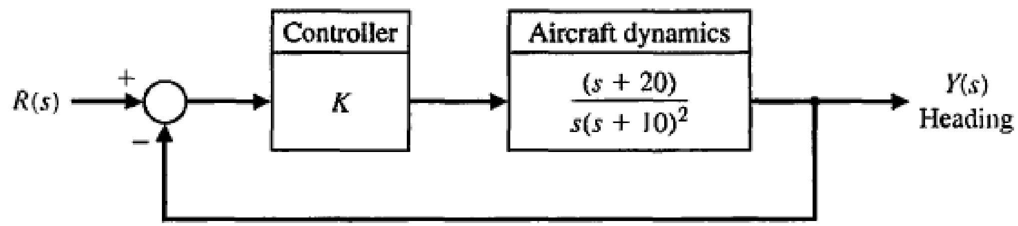

FIGURE E6.8 Aircraft heading control.

Lausn:

E6.7 The closed-loop system characteristic equation is

$$s^3 + 20s^2 + (100 + K)s + 20K = 0 \ .$$

The corresponding Routh array is

$$\begin{array}{c|cc} s^3 & 1 & (100+K) \\ s^2 & 20 & 20K \\ s^1 & b & 0 \\ s^o & 20K & \end{array}$$

where

$$b = \frac{20(100+K) - 20K}{20} = \frac{20(100)}{20} = 100 \ .$$

Therefore, the system is stable for all $K > 0$.

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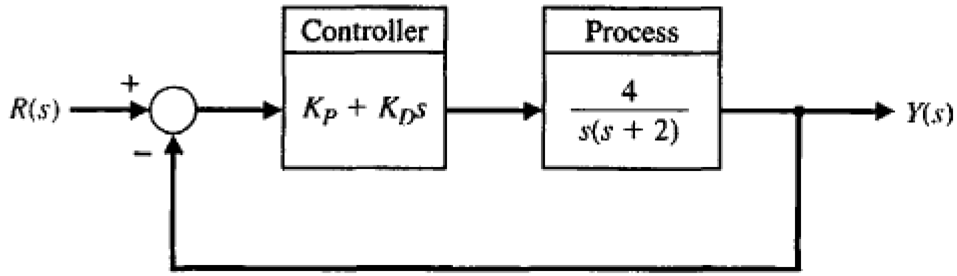
E6.13. Consider the feedback system in Figure E6.13. Determine the range of K_P and K_D for stability of the closed-loop system.

FIGURE E6.13 Closed-loop system with a proportional plus derivative controller $G_c(s) = K_P + K_D s$.



E6.13. Consider the feedback system in Figure E6.13. Determine the range of K_P and K_D for stability of the closed-loop system.

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Lausn:

E6.13 The characteristic equation is

$$s^2 + (4K_D + 2)\, s + 4K_P = 0.$$

The Routh array is

$$\begin{array}{c|cc} s^2 & 1 & 4K_P \\ s^1 & 4K_D + 2 & 0 \\ s^o & 4K_P & \end{array}$$

The system is stable if and only if $K_P > 0$ and $K_D > -1/2$

E6.16 A system has a characteristic equation

$$q(s) = s^4 + 9s^3 + 45s^2 + 87s + 50 = 0.$$

(a) Determine whether the system is stable, using the Routh–Hurwitz criterion. (b) Determine the roots of the characteristic equation.

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$$q(s) = s^4 + 9s^3 + 45s^2 + 87s + 50 = 0.$$

(a) Determine whether the system is stable, using the Routh–Hurwitz criterion. (b) Determine the roots of the characteristic equation.

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Lausn:

E6.16 The Routh array is

$$\begin{array}{c|ccc}
s^4 & 1 & 13 & 10 \\
s^3 & 5 & 19 & \\
s^2 & 9.20 & 10 & \\
s^1 & 13.57 & 0 & \\
s^o & 10 & &
\end{array}$$

The system is stable. The roots of $q(s)$ are $s_{1,2} = -1 \pm j2$, $s_3 = -2$ and $s_4 = -1$.

E6.25 A closed-loop feedback system is shown in Figure E6.25. For what range of values of the parameters K and p is the system stable?

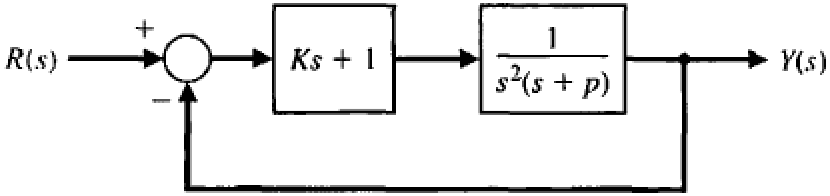

FIGURE E6.25 Closed-loop system with parameters K and p.

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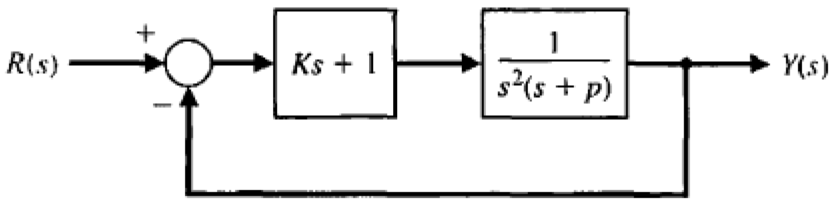

FIGURE E6.25 Closed-loop system with parameters K and p.

Lausn:

E6.25 The closed-loop transfer function is

$$T(s) = \frac{Ks+1}{s^2(s+p)+Ks+1} \ .$$

Therefore, the characteristic equation is

$$s^3 + ps^2 + Ks + 1 = 0 \ .$$

The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & \text{K} \\ s^2 & \text{p} & 1 \\ s^1 & (pK-1)/p & \\ s^o & 1 & \end{array}$$

We see that the system is stable for any value of $p > 0$ and $pK - 1 > 0$.