Reglunarfræði T
501

Sýnidæmi – kafli 9

Þorgeir Pálsson

E9.6 A system has a loop transfer function

$$L(s) = G_c(s)G(s) = \frac{K(s + 100)}{s(s + 10)(s + 40)}.$$

When $K = 500$, the system is unstable. Show that if we reduce the gain to 50, the resonant peak is 3.5 dB. Find the phase margin of the system with $K = 50$.

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E9.6 The Bode plot of the closed-loop transfer function is shown in Figure E9.6. The value of $M_{p_\omega} = 3$ dB. The phase margin is $P.M. = 40^o$ when $K = 50$.

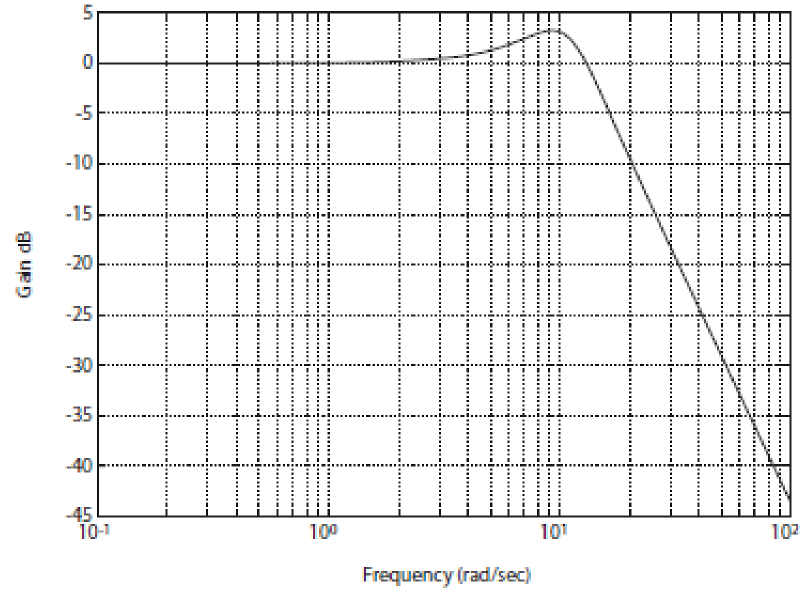

FIGURE E9.6
Closed-loop Bode Diagram for $T(s) = \frac{50(s+100)}{s^3+50s^2+450s+5000}$.

E9.8 Consider a unity feedback system with the loop transfer function

$$L(s) = G_c(s)G(s) = \frac{K}{s(s+1)(s+2)}.$$

(a) For $K = 4$, show that the gain margin is 3.5 dB.

(b) If we wish to achieve a gain margin equal to 16 dB, determine the value of the gain K.

Answer: (b) $K = 0.98$

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Answer: (b) $K = 0.98$

E9.8 (a) When $K = 4$, the $G.M. = 3.5$ dB. This is illustrated in Figure E9.8.

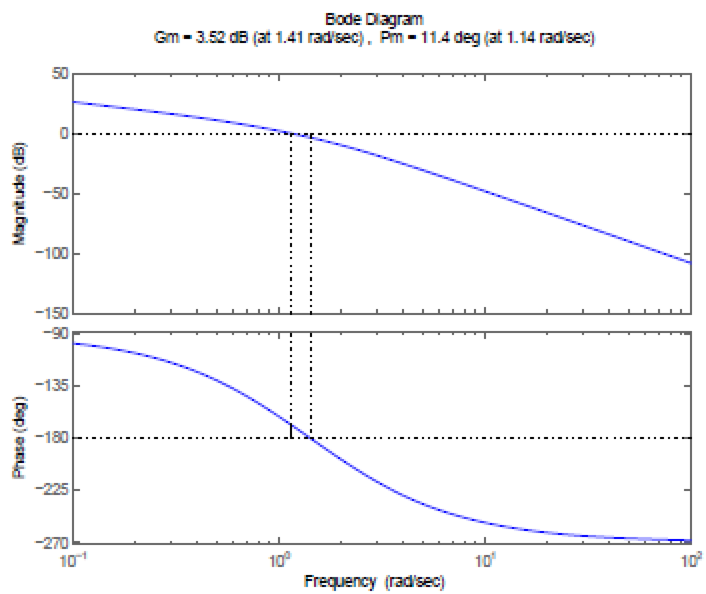

FIGURE E9.8
Bode Diagram for $G_c(s)G(s) = \frac{K}{s(s+1)(s+2)}$, where $K = 4$.

(b) The new gain should be $K = 1$ for a gain margin $G.M. = 16$ dB.

E9.13 A Nichols chart is given in Figure E9.14 for a system with $G_c(j\omega)G(j\omega)$. Using the following table, find (a) the peak resonance $M_{p\omega}$ in dB; (b) the resonant frequency ω_r; (c) the 3-dB bandwidth; and (d) the phase margin of the system.

	ω_1	ω_2	ω_3	ω_4
rad/s	1	3	6	10

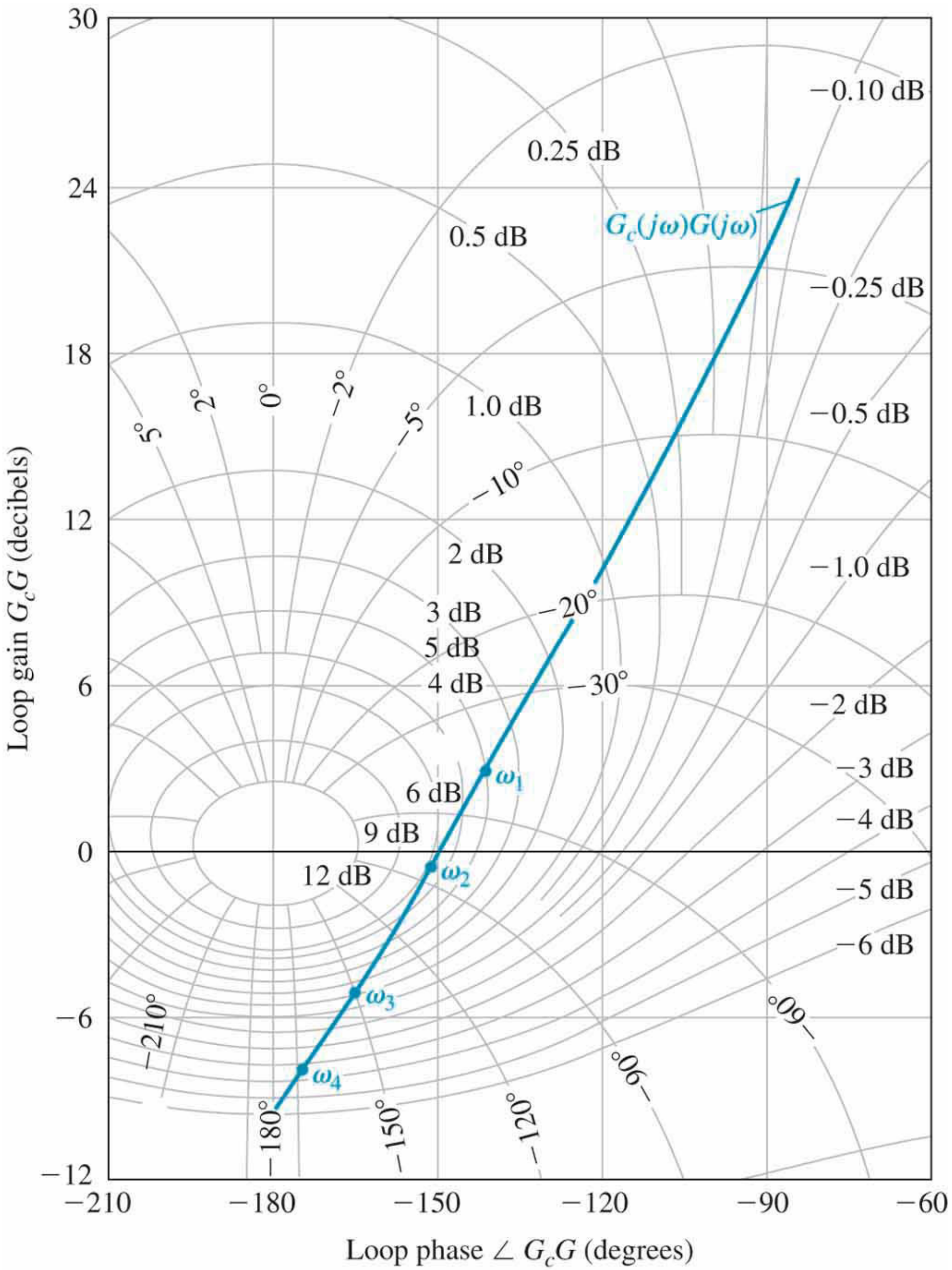

Fig E9.13. Nichols Chart for G_cG

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rad/s	1	3	6	10

Svörin má lesa út úr Nichols grafinu:

E9.13 (a) The peak resonance $M_{p_\omega} = 6$ dB.

(b) The resonant frequency is $\omega_r = \omega_2 = 3$ rad/sec.

(c) The bandwidth is $\omega_B = \omega_4 = 10$ rad/sec.

(d) The phase margin is $P.M. = 30^o$.

E9.15 Consider a unity feedback system with the loop transfer function

$$L(s) = G_c(s)G(s) = \frac{100}{s(s + 20)}.$$

Find the bandwidth of the closed-loop system.

Answers: $\omega_B = 6.4$ rad/sec

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Find the bandwidth of the closed-loop system.

Answers: $\omega_B = 6.4$ rad/sec

E9.15 The loop transfer function is

$$G_c(s)G(s) = \frac{100}{s(s+20)} \, ,$$

and the closed-loop transfer function is

$$T(s) = \frac{100}{s^2 + 20s + 100} \, .$$

The magnitude plot for the closed-loop system is shown in Figure E9.15. With bandwidth defined as frequency at which the magnitude is reduced

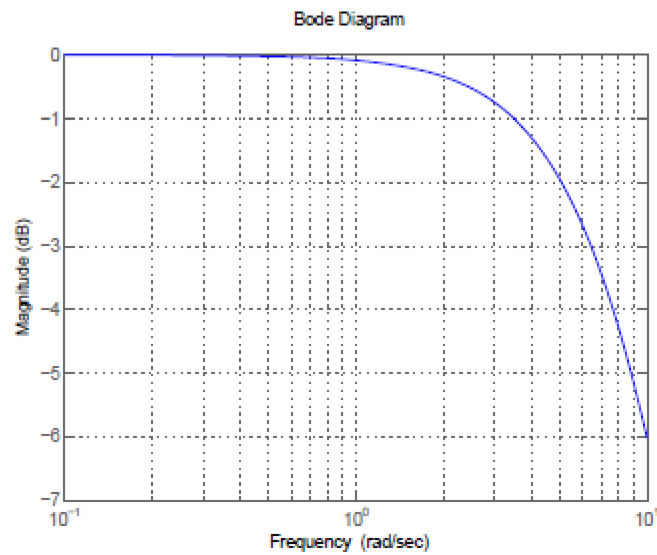

FIGURE E9.15
Magnitude plot for the closed-loop $T(s) = \frac{100}{s^2+20s+100}$.

-3 dB from the dc value, we determine that $\omega_B = 6.4$ rad/sec.

E9.16 The pure time delay e^{-sT} may be approximated by a transfer function as

$$e^{-sT} \approx \frac{1 - Ts/2}{1 + Ts/2}$$

for $0 < \omega < 2/T$. Obtain the Bode diagram for the actual transfer function and the approximation for $T = 0.2$ for $0 < \omega < 10$.

E9.16 The transfer function of the approximation is

$$G(j\omega) = \frac{1 - j\omega/10}{1 + j\omega/10} ,$$

and the magnitude is

$$|G(j\omega)| = \left|\frac{1 - j\omega/10}{1 + j\omega/10}\right| = 1 ,$$

which is equivalent to the actual time delay magnitude. The phase approximation is

$$\phi = -\tan^{-1}\omega/10 + \tan^{-1}(-\omega/10) = -2\tan^{-1}\omega/10$$

and the actual phase is

$$\phi = -0.2\omega .$$

The phase plots are shown in Figure E9.16. The approximation is accurate for $\omega < 3$ rad/sec.

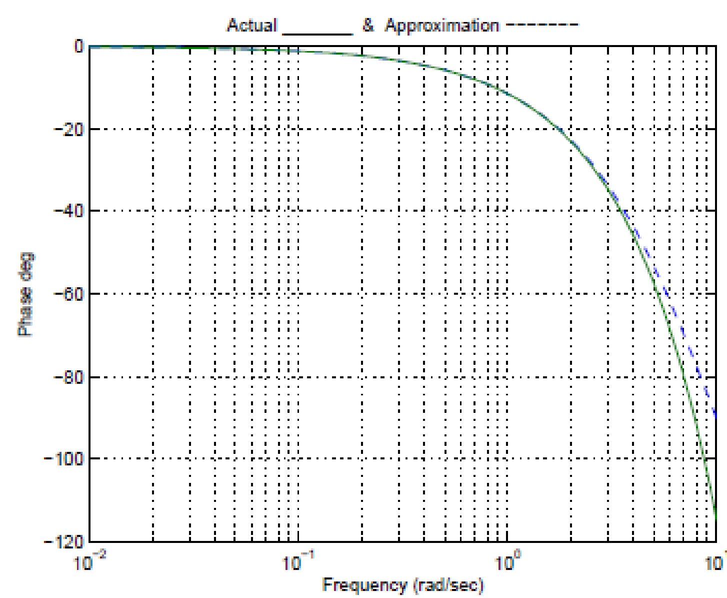

FIGURE E9.16
Phase plots for time delay actual vs approximation.

E9.21 A unity feedback control system has a loop transfer function

$$L(s) = G_c(s)G(s) = \frac{K}{s(s+2)(s+50)}.$$

Determine the phase margin, the crossover frequency, and the gain margin when $K = 1300$.

Answers: $PM = 16.6°$, $\omega_c = 4.9$, $GM = 4$ or 12 dB

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E9.21 The Bode plot is shown in Figure E9.21.

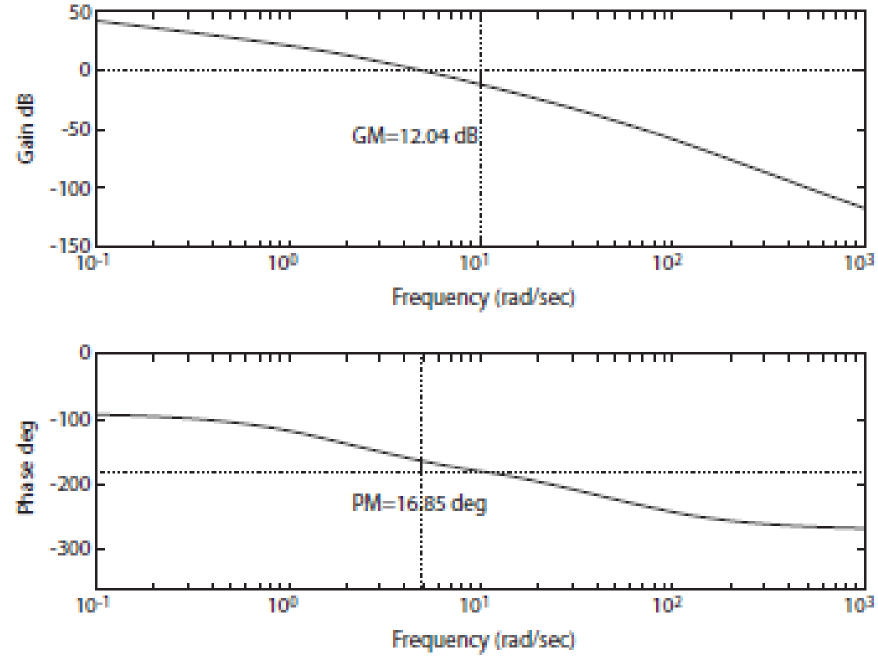

FIGURE E9.21
Bode Diagram for $G_c(s)G(s) = \frac{1300}{s(s+2)(s+50)}$.

E9.23 Consider again the system of E9.21 when $K = 438$. Determine the closed-loop system bandwidth, resonant frequency, and $M_{p\omega}$ using the Nichols chart.

Answers: $\omega_B = 4.25$ rad/s, $\omega_r = 2.7$, $M_{p\omega} = 1.7$

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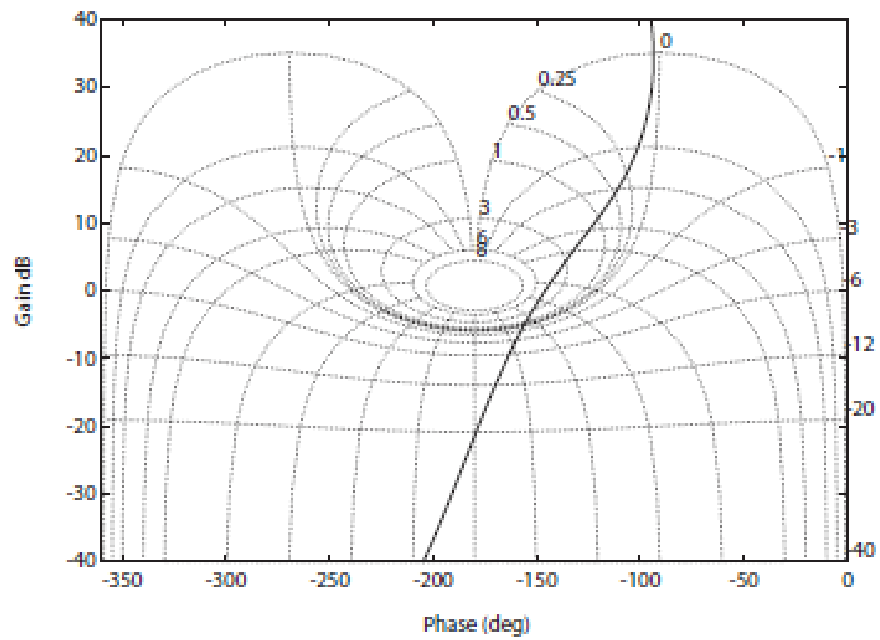

FIGURE E9.23
Nichols chart for $G_c(s)G(s) = \frac{438}{s(s+2)(s+50)}$.

The actual values are

$$M_{p\omega} = 1.6598 \text{ (4.4 dB)} \quad \omega_r = 2.4228 \text{ rad/sec} \quad \omega_B = 4.5834 \text{ rad/sec} .$$