Statistical Process Control

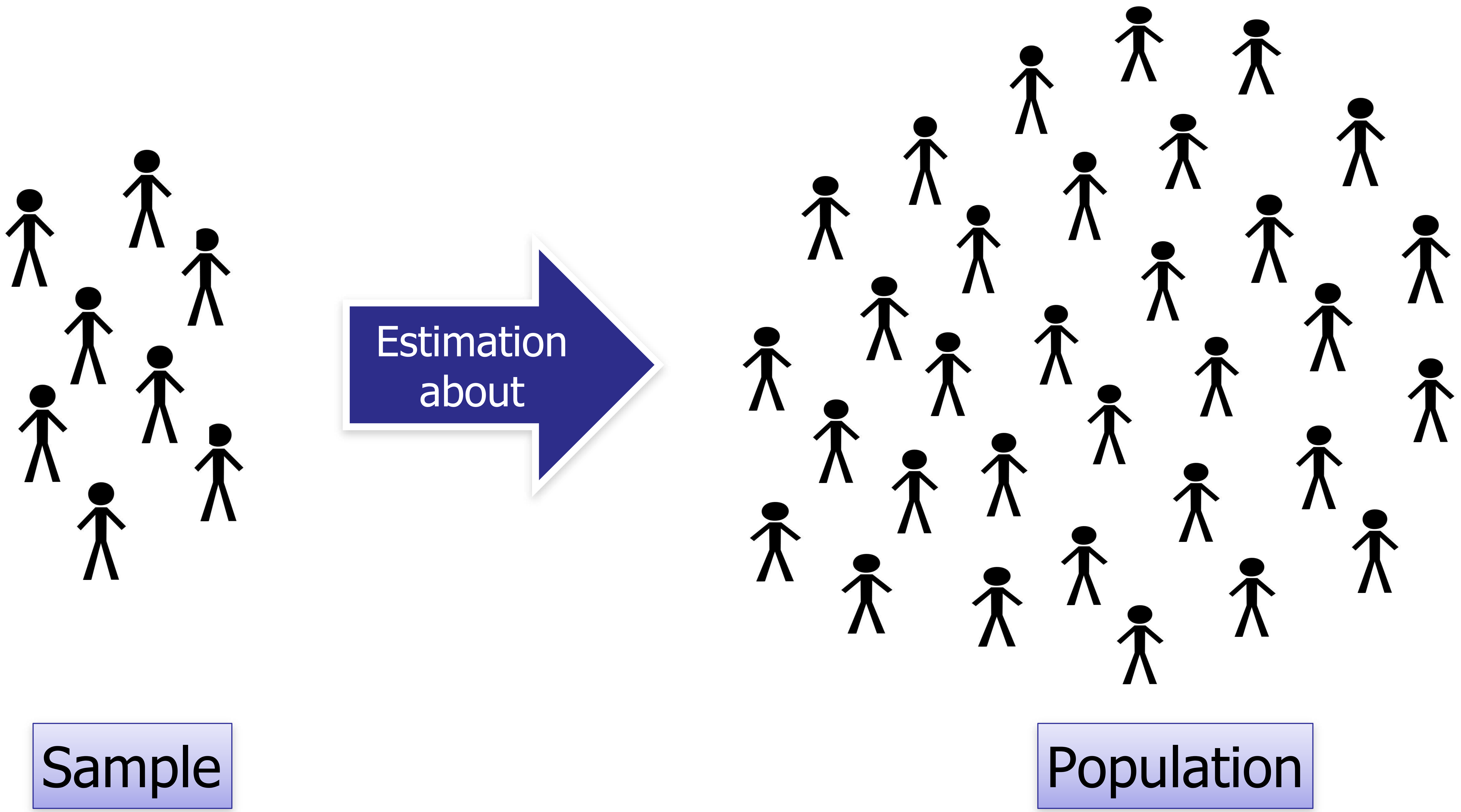
Trainer: Amit Ballal

Reference: AIAG SPC 2^{nd} Edition manual

At the end of this training, you will learn:

- Understand basics of statistics
- Understand variation, state of statistical control Vs process control
- Identify correct control chart to be used
- Identify causes of variation and take action to reduce variation
- Apply SPC techniques in practical and real life

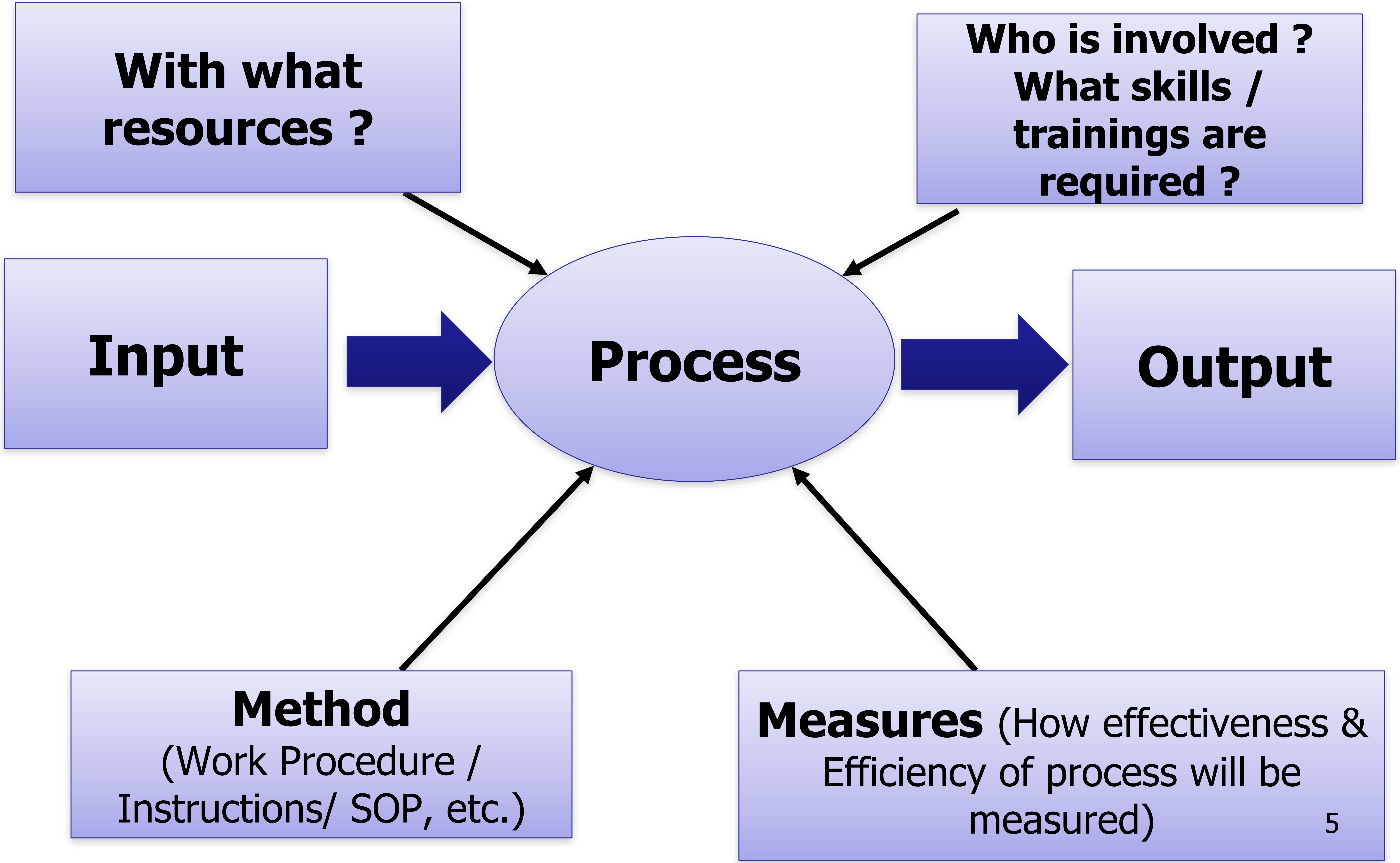
Statistics



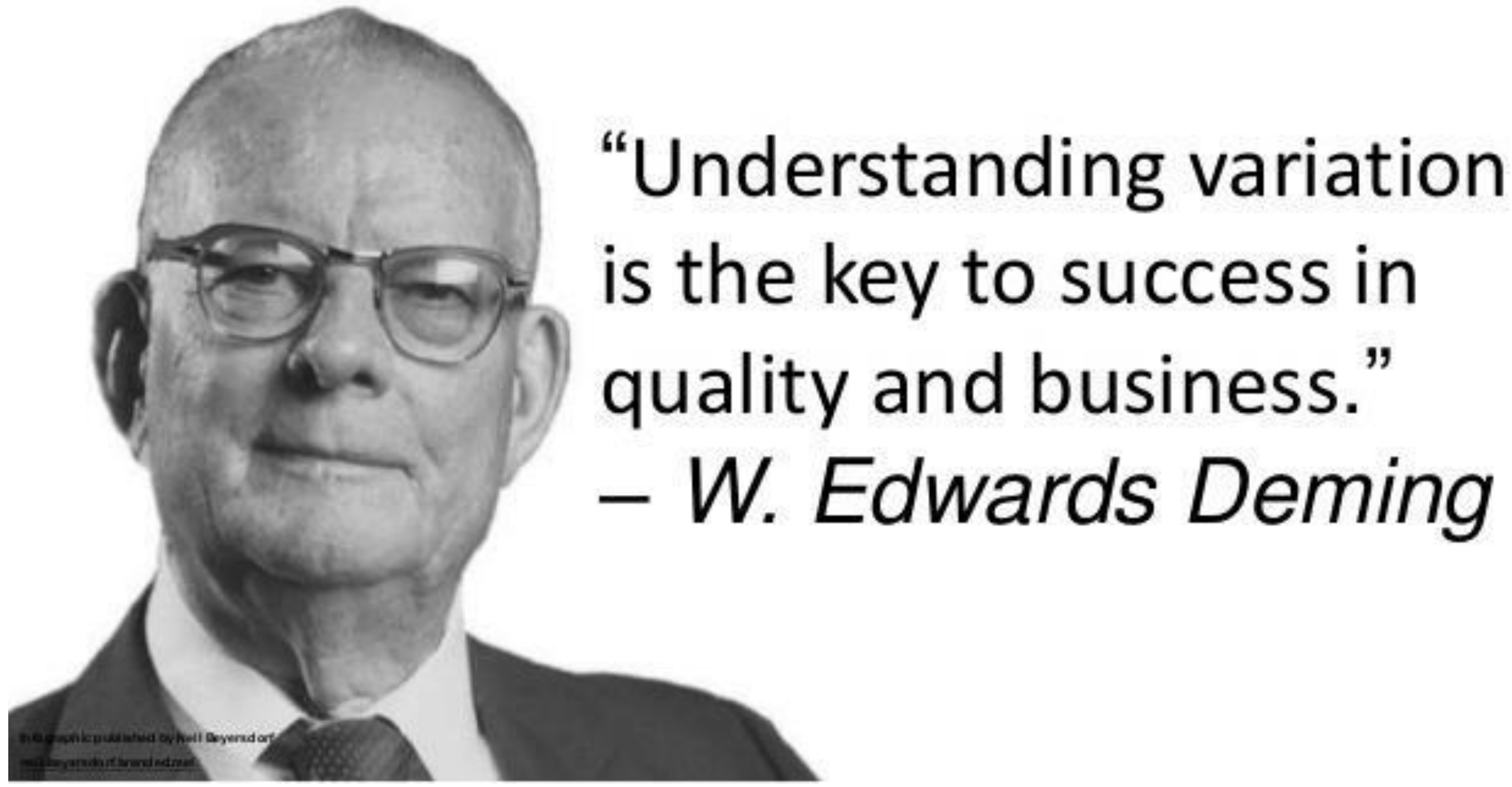
Symbols used

Parameter	Sample	Population
Sample size	n	N
Mean (Average)	$\bar{X}$	μ
Standard Deviation (Sigma)	s	σ
Variance	s^2	σ^2
Range	R	R
Mode	-------	-------
Median	M or m or $\tilde{x}$	-------

Process



With what resources ?
Who is involved ? What skills / trainings are required ?
Input
Process
Output
Method (Work Procedure / Instructions/ SOP, etc.)
Measures (How effectiveness & Efficiency of process will be measured)



"Variation is Everywhere"

Control

Variation is only due to common causes

(No special causes)

Dr. Walter A. Shewhart

Dr. Walter A. Shewhart

Born:
18 March 1891

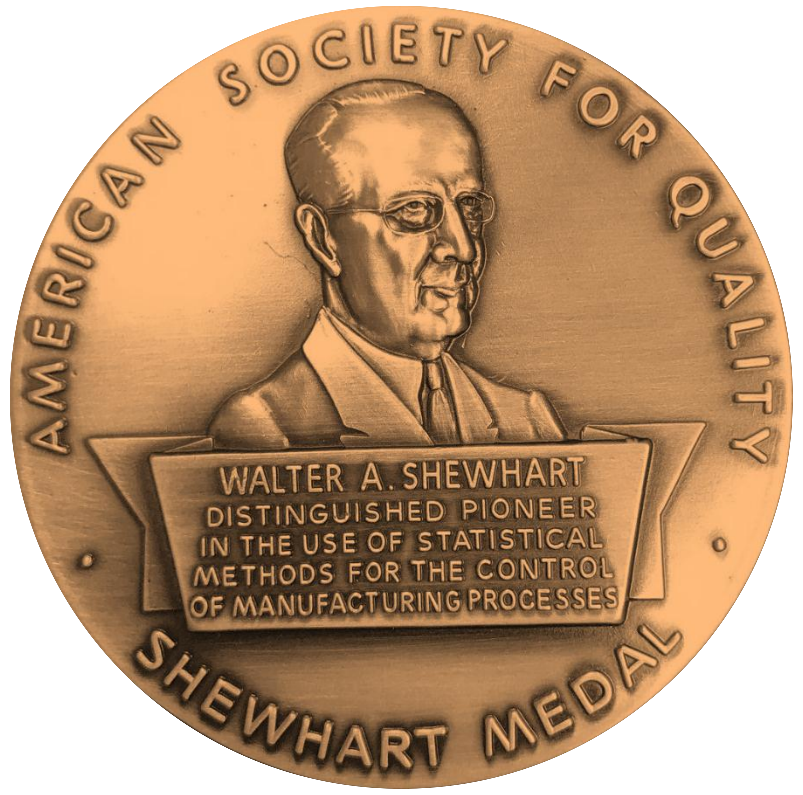
Died:
11 March 1967

Summary:
American physicist, engineer and statistician who worked on statistical quality control

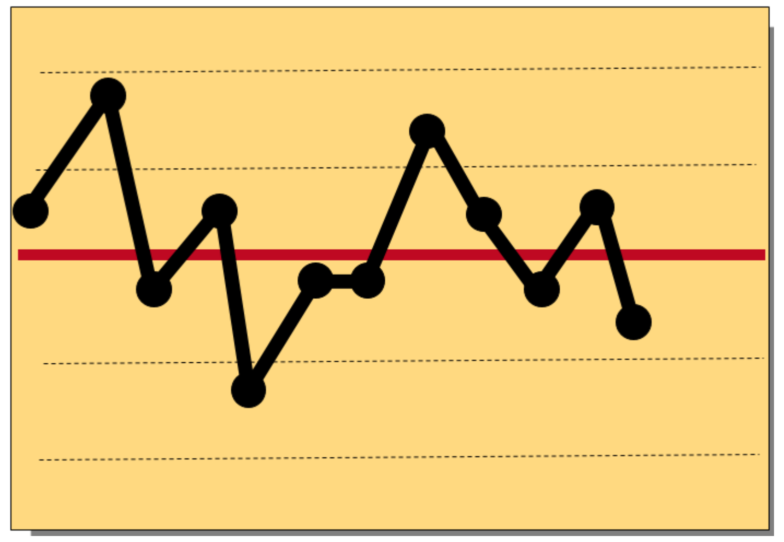
Achievements:

"Shewhart medal" was named after him which was started in 1948 by ASQ

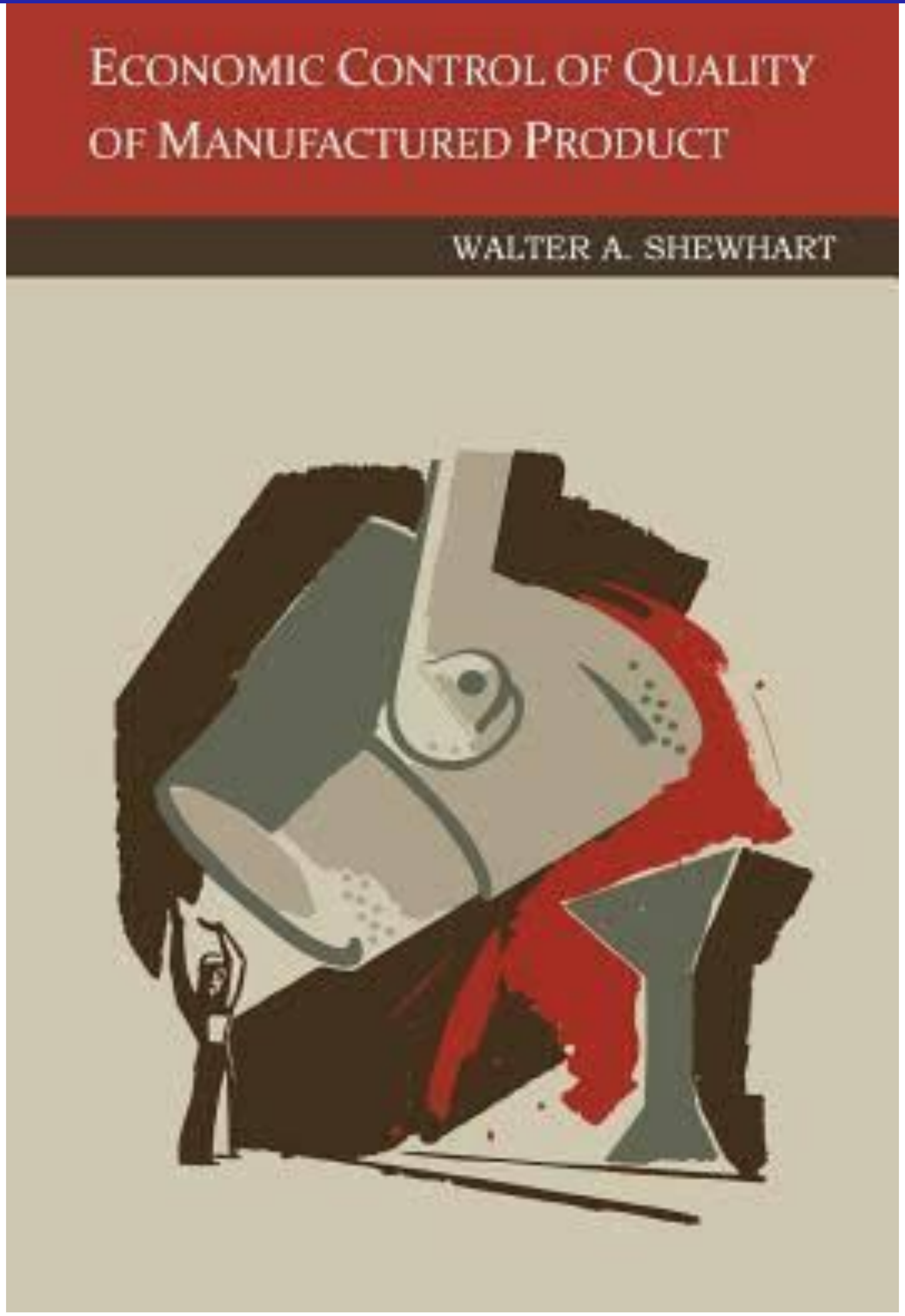
- Awarded to Deming in 1955
- Awarded to Kaoru Ishikawa in 1982
- Awarded to Dorian Shainin in 1989

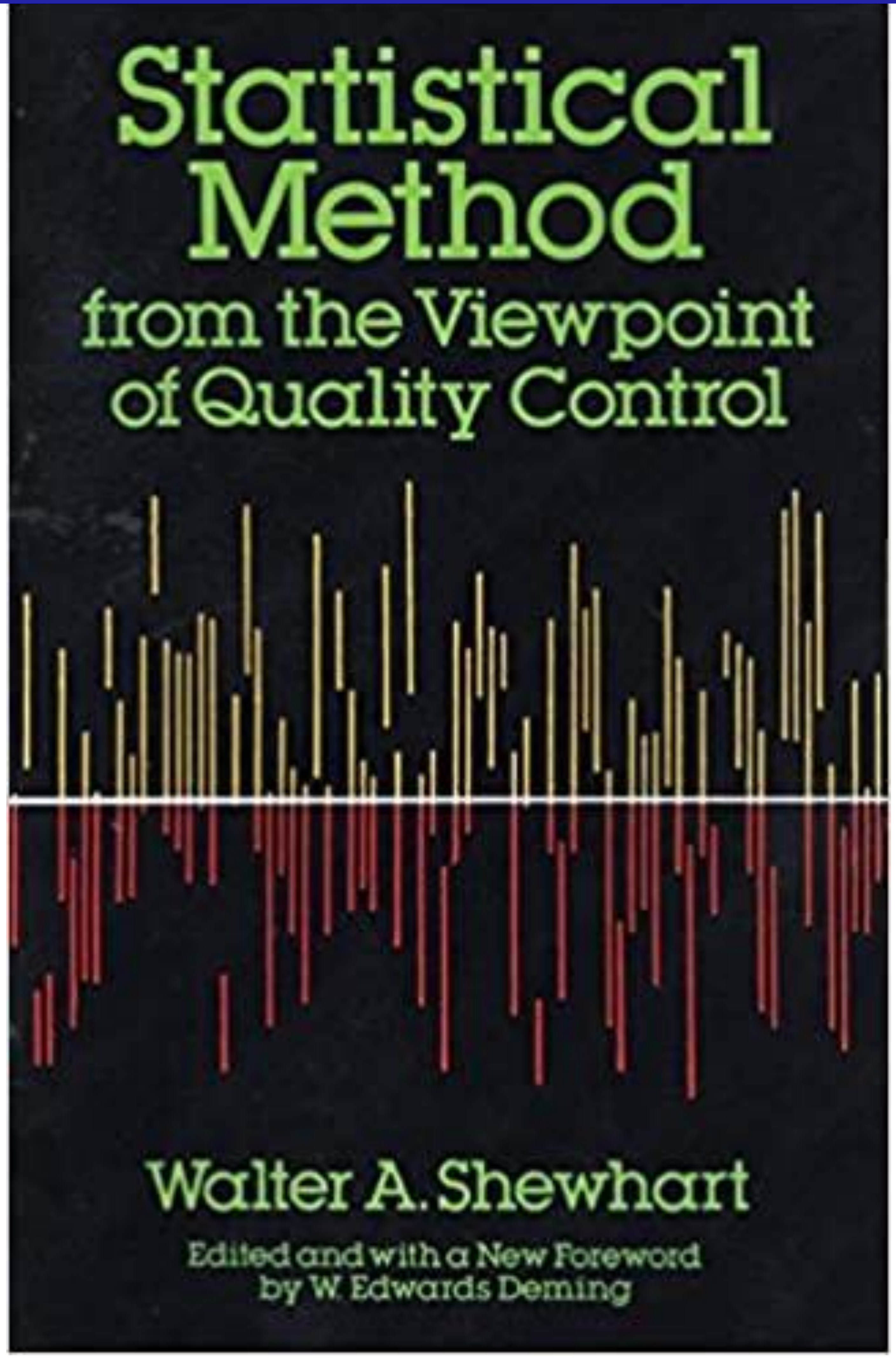


Invented control charts at Bell telephones



Books by Dr. Walter A. Shewhart





A study of the accelerated motion of small drops through a viscous medium

Correction of Data for Errors of Averages: A Discussion of a Method for Correcting Data for Errors of Averages Obtained from Small Samples

Stability

Common Vs Special causes

Common cause	Special cause
➢ Occurs regularly / part of the process	➢ Doesn’t occur regularly / Not part of the process
➢ Difficult to identify	➢ Easy to identify
➢ Can be reduced at some extent (that too with lots of effort / process re-design), cannot be eliminated	➢ Can be eliminated completely

I-MR (X-MR) Chart

- I = Individual
- MR = Moving Range

Where to use ?

- **Sample (Subgroup) size = 1**

Sr. No.	Reading	MR (Moving Range)
1	290	
2	288	2
3	285	3
4	290	5
5	291	1
6	287	4
7	284	3
8	290	6
9	290	0
10	288	2
	$\bar{X} = 288.3$	$\overline{MR} = 2.89$

Setting Control Limits

I Chart

$\mathbf{UCL_x} = \bar{x} + E_2\overline{MR}$

$\mathbf{LCL_x} = \bar{x} - E_2\overline{MR}$

MR Chart

$\mathbf{UCL_{MR}} = D4.\overline{MR}$

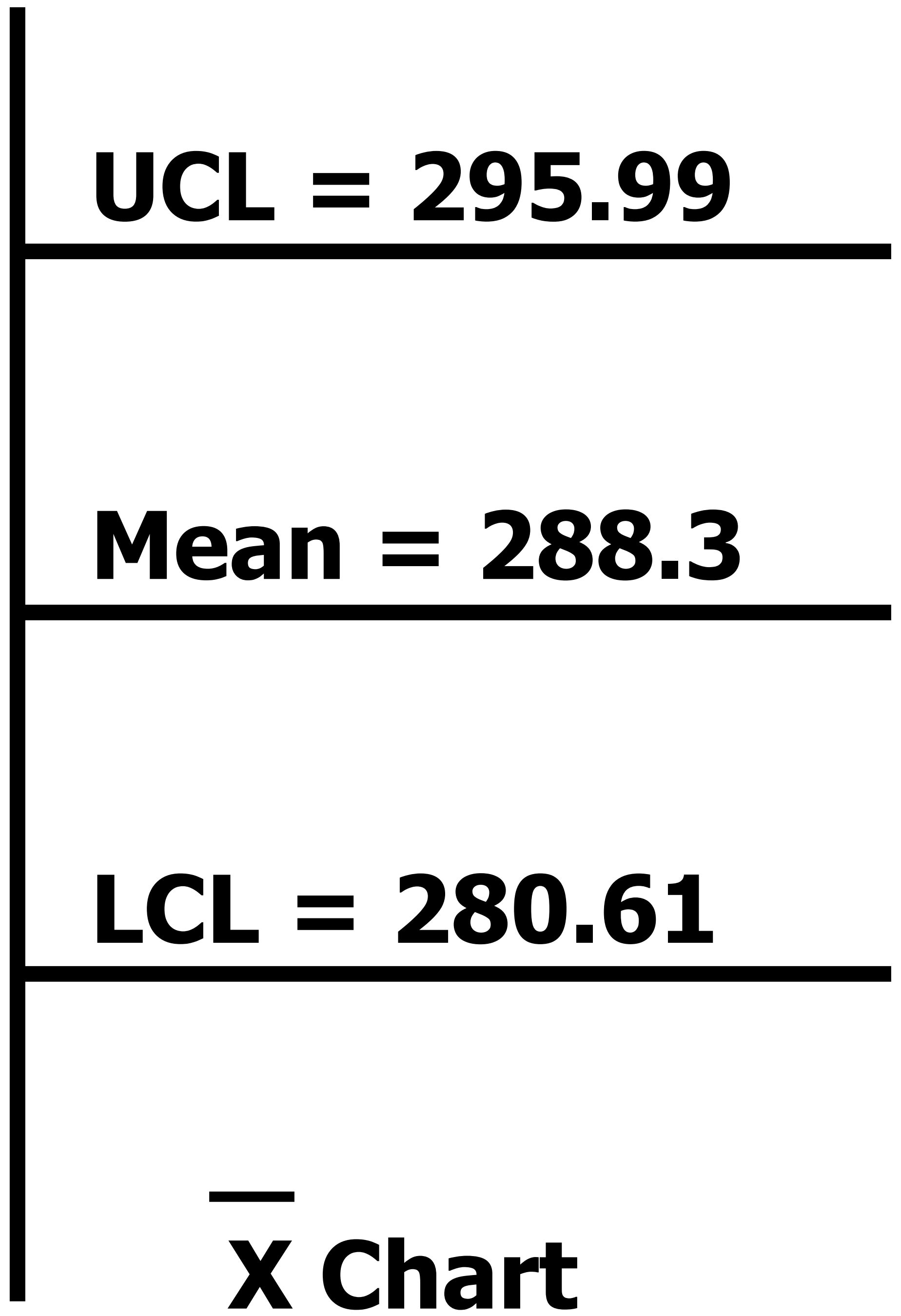
$\mathbf{LCL_{MR}} = D3.\overline{MR}$

Setting Control Limits

Process average $\bar{x}$ = 288.3 mm
Average range $\overline{MR}$ = 2.89 mm
Sample(subgroup) size n = 1

$$
\begin{aligned}
UCL_x &= \bar{x} + E_2 \overline{MR} \\
&= 288.3 + (2.66)(2.89) \\
&= 288.3 + 7.69 \\
&= 295.99 \text{ mm}
\end{aligned}
$$

$$
\begin{aligned}
LCL_x &= \bar{x} - E_2 \overline{MR} \\
&= 288.3 - 7.69 \\
&= 280.61 \text{ mm}
\end{aligned}
$$



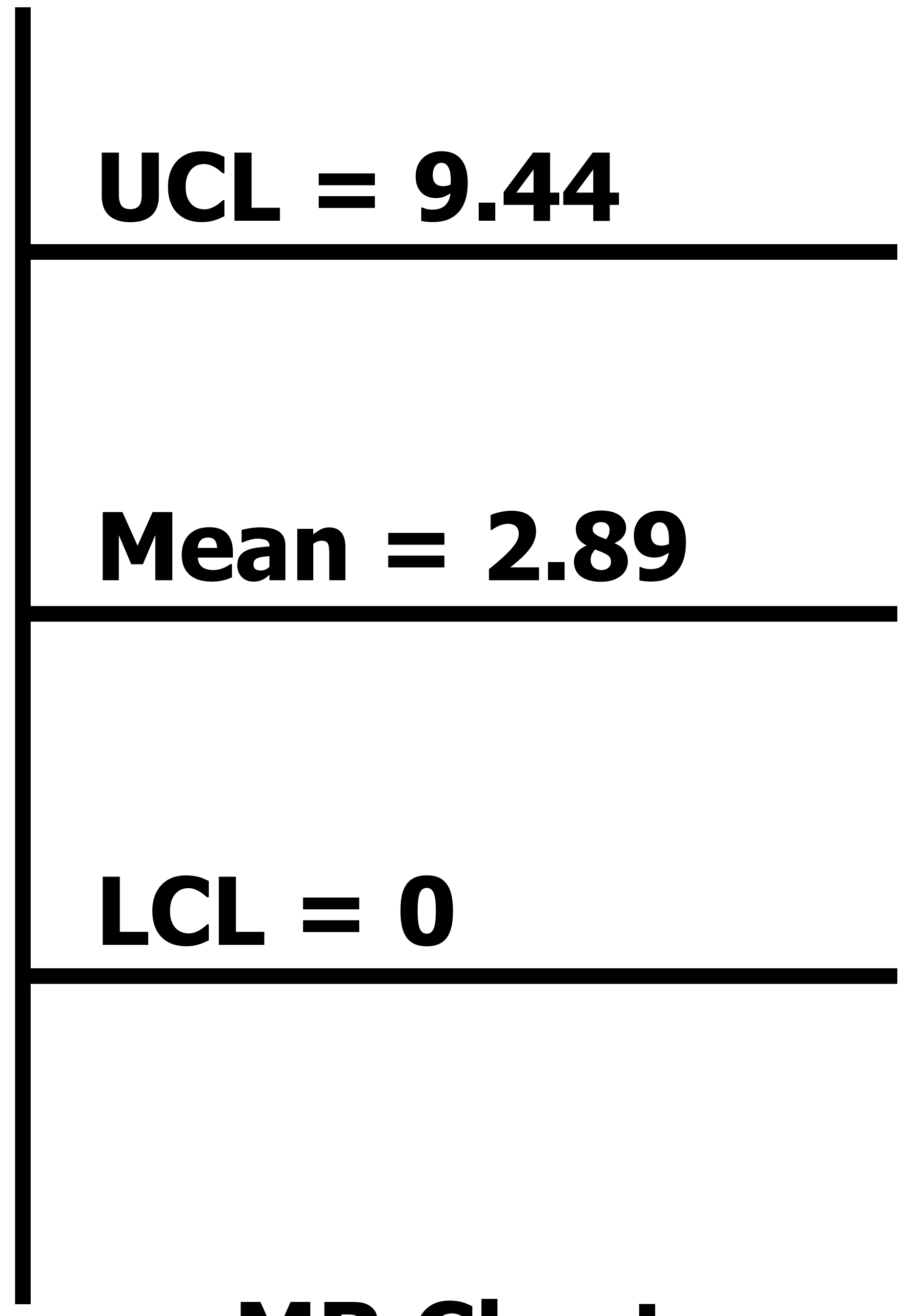
$\overline{X}$ Chart

Setting Control Limits

Process average $\overline{x}$ = 288.3 mm
Average range $\overline{MR}$ = 2.89 mm
Sample(subgroup) size n = 1

$$UCL_{MR} = D_4 . \overline{MR}$$
$$= (3.267)(2.89)$$
$$= 0.429 \text{ mm}$$

$$LCL_{MR} = D_3 . \overline{MR}$$
$$= (0)(2.89)$$
$$= 0 \text{ mm}$$

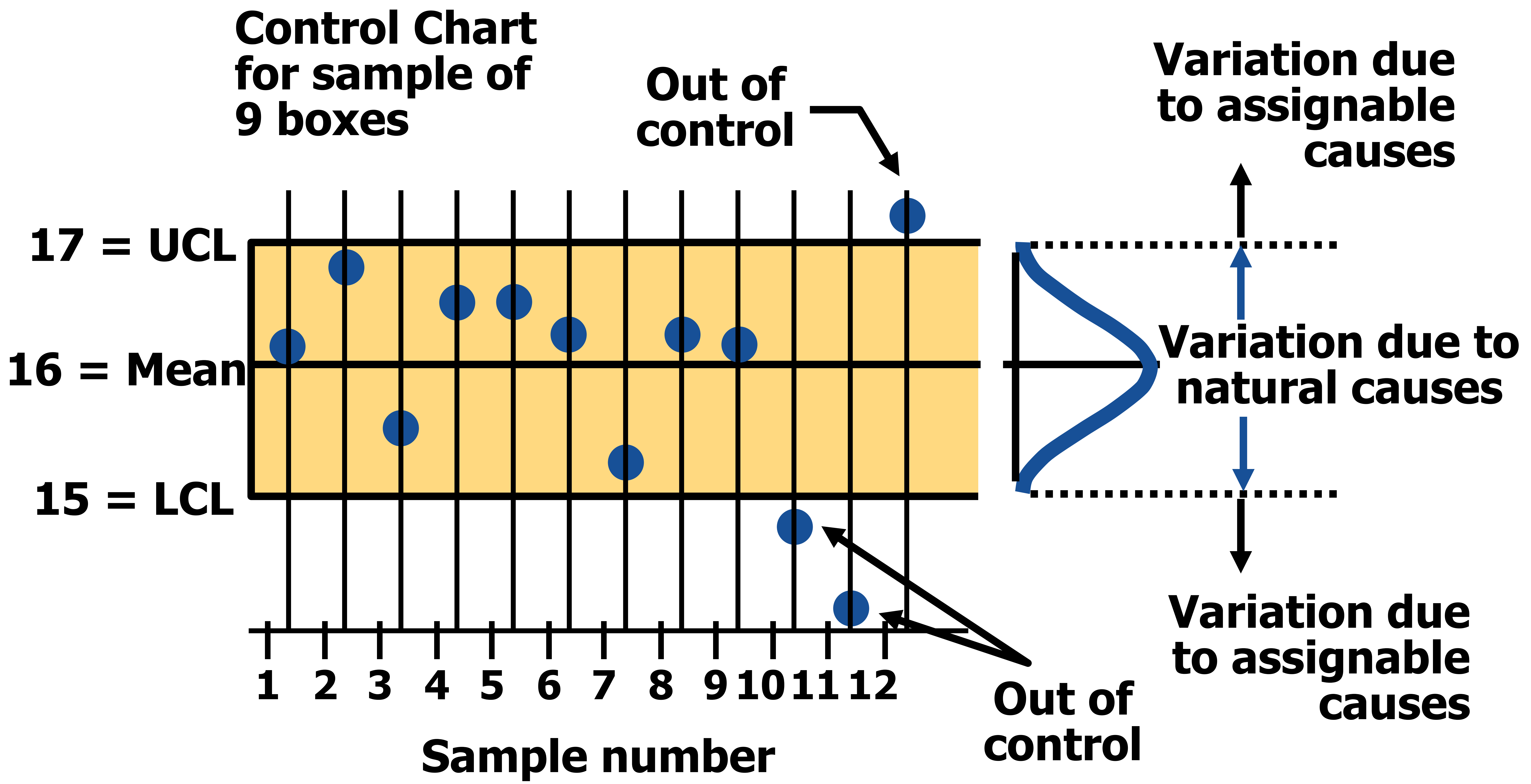


MR Chart

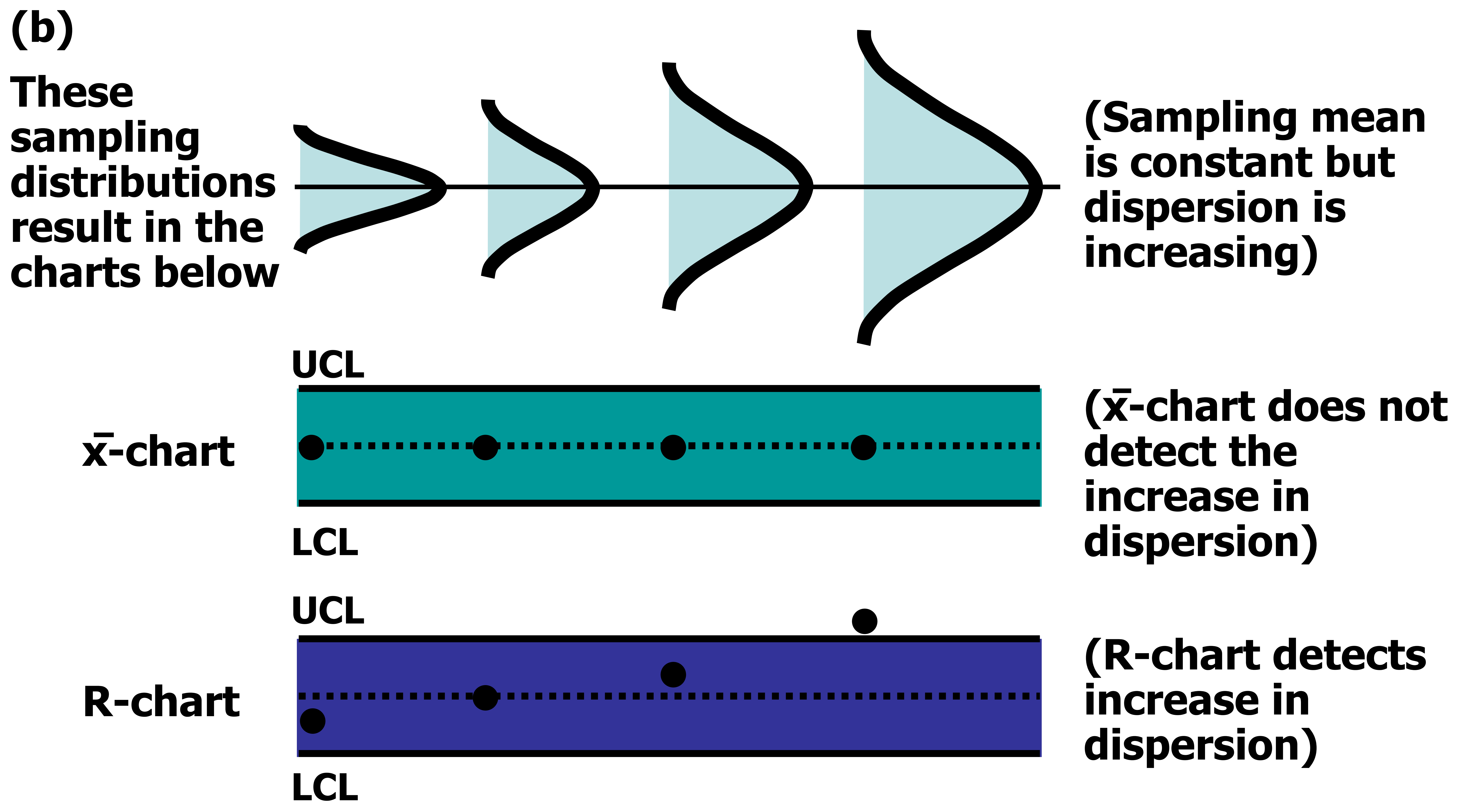
Change event log

OCS	Subgroup No.	Event log
	1	New setup, insert IC27, material lot #1984
	4	New setup, insert IC27, material lot #1991
	6	New setup, insert IC27, material lot #1997
*	7	material lot #2002, bad insert replaced
	8	material lot #2102
	11	material lot #2112
	14	material lot #2120

Setting control limits



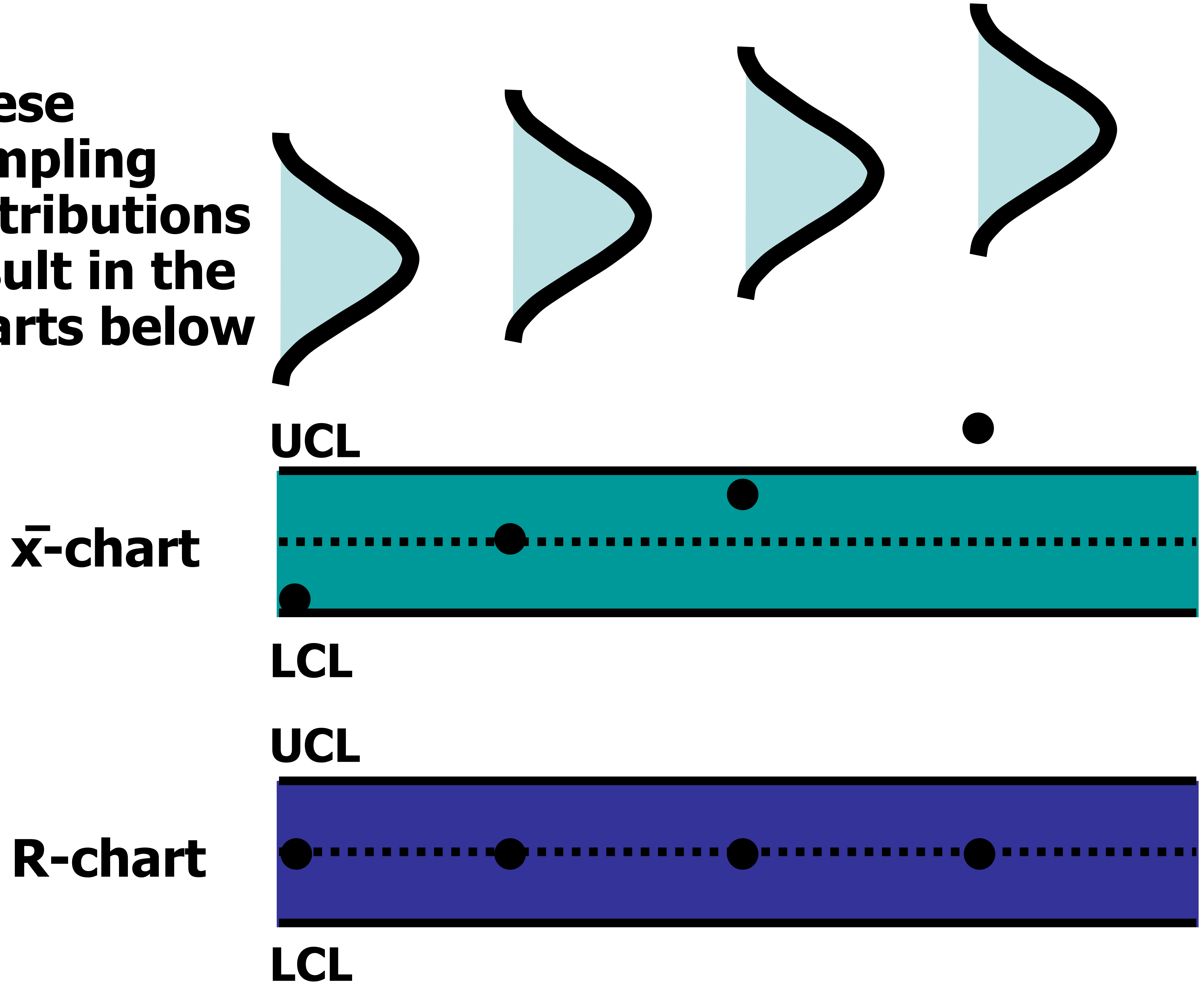
Range / variation chart



Average chart

(a)

These sampling distributions result in the charts below



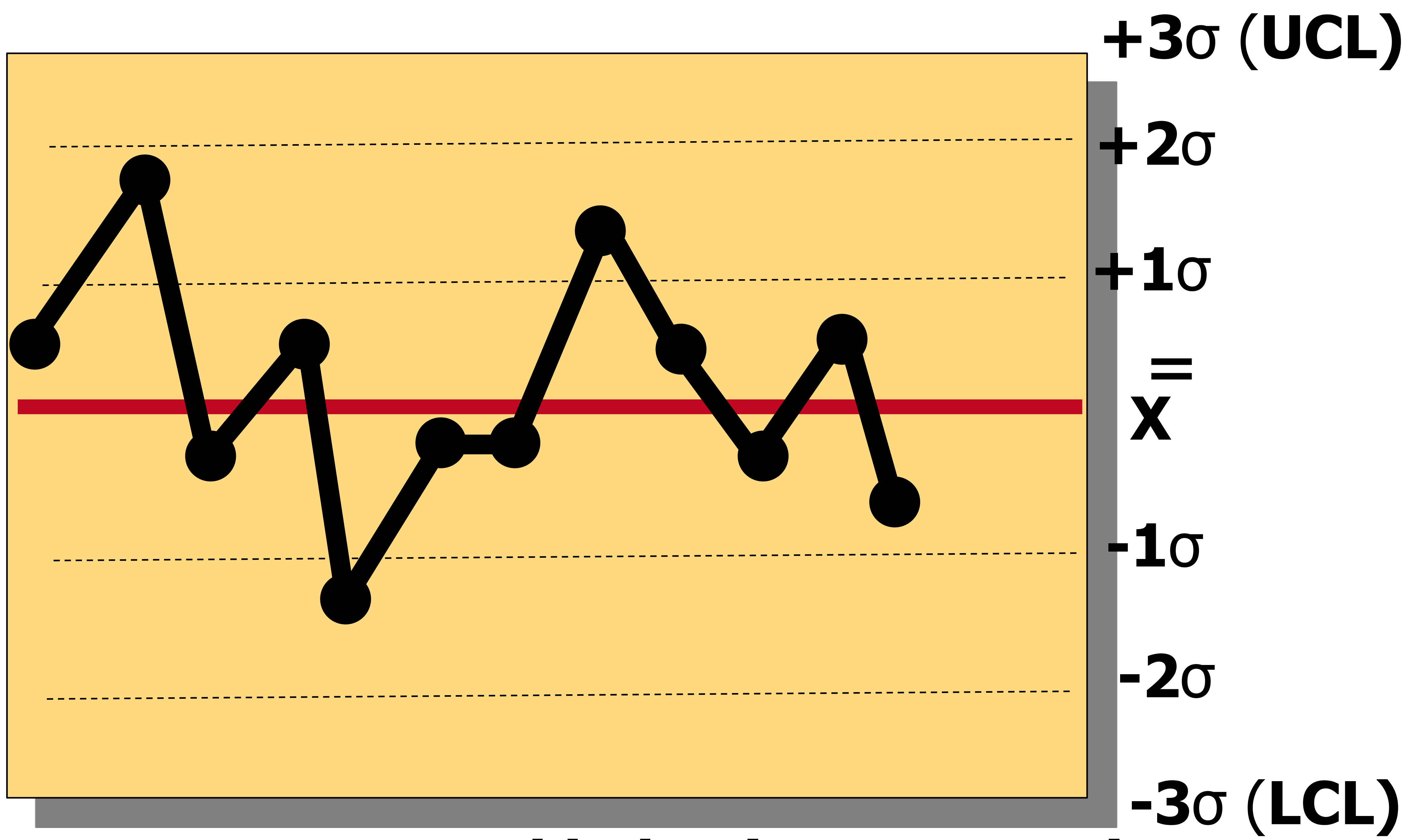
(Sampling mean is shifting upward but range is consistent)

(x̄-chart detects shift in central tendency)

(R-chart does not detect change in mean)

Patterns in control charts

UCL (Upper control limit)



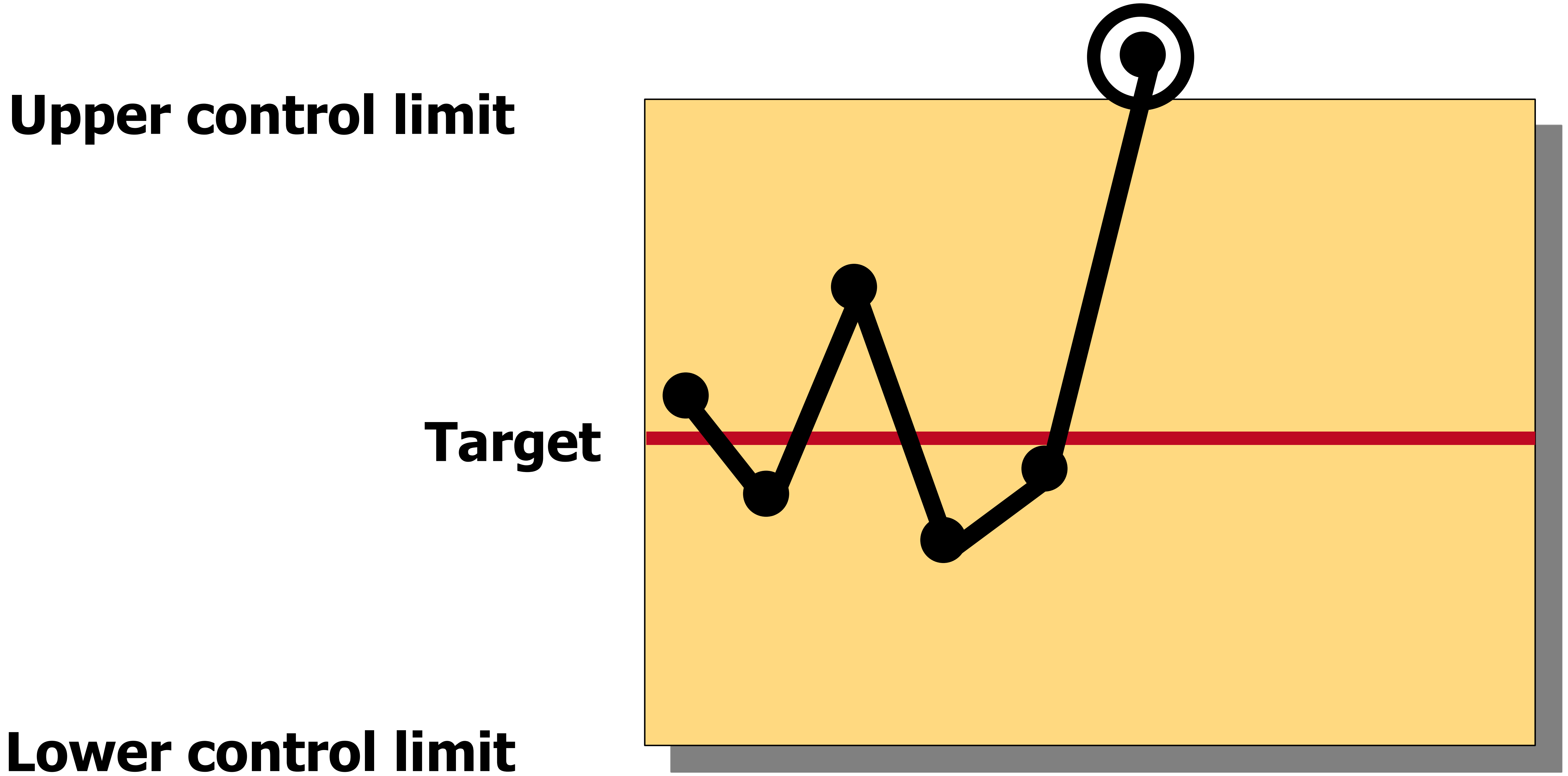
LCL (Lower control limit)

Normal behavior. Process is "in control."

How to detect special cause?

Patterns in Control Charts

Rule-1

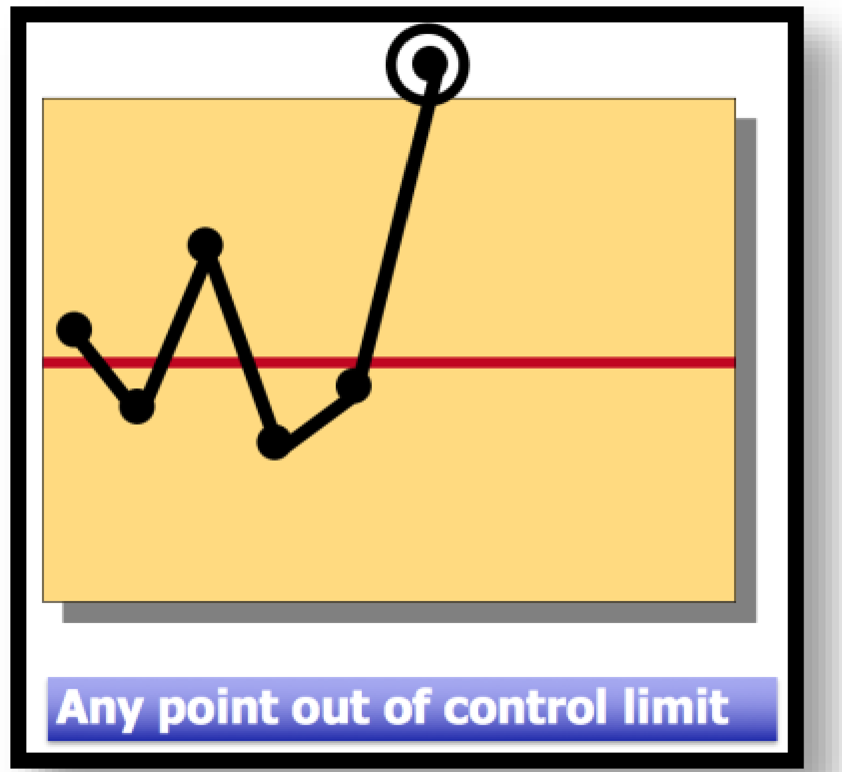


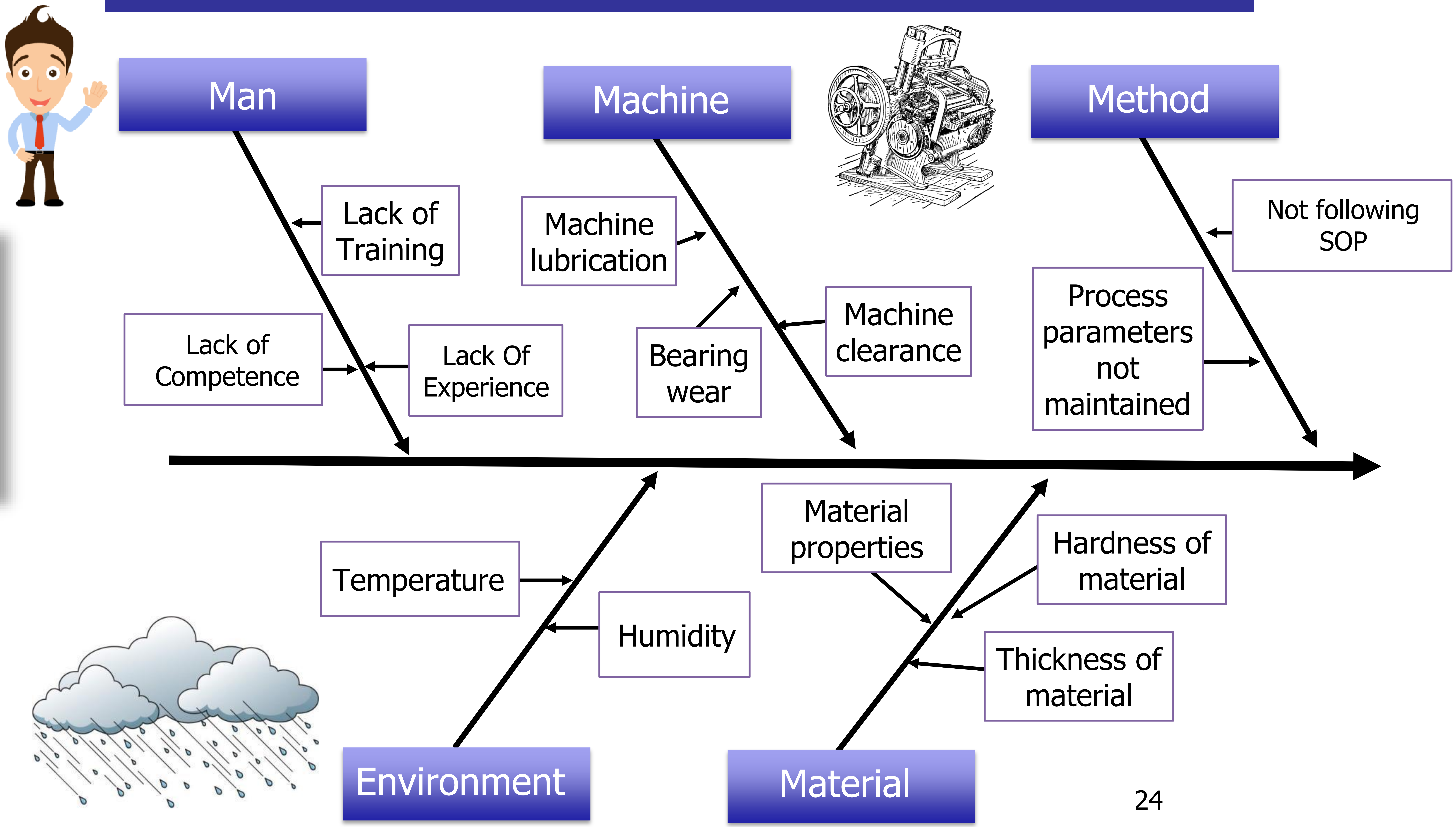
Any point out of control limit

Reference: AIAG SPC 2nd edition manual

Rule-1

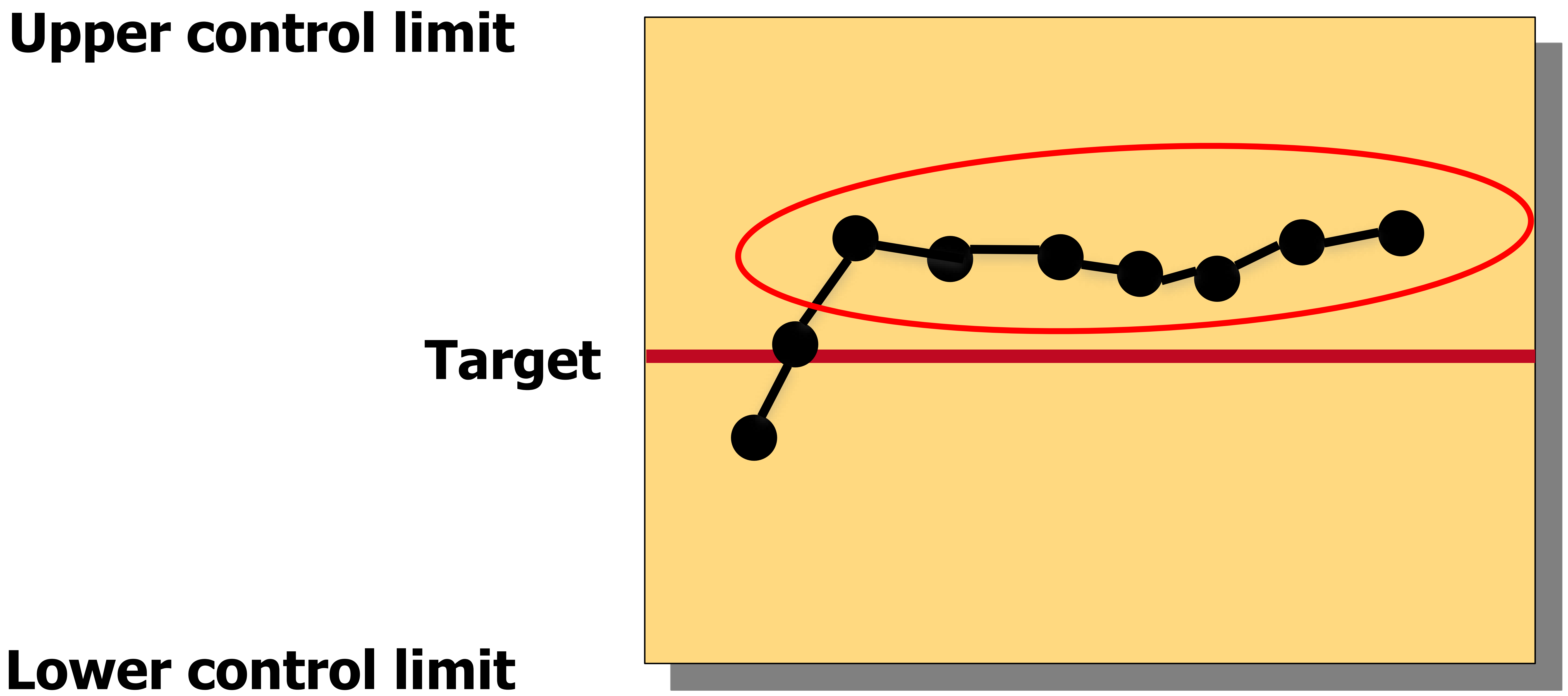
Potential causes





Rule-2

Patterns in Control Charts

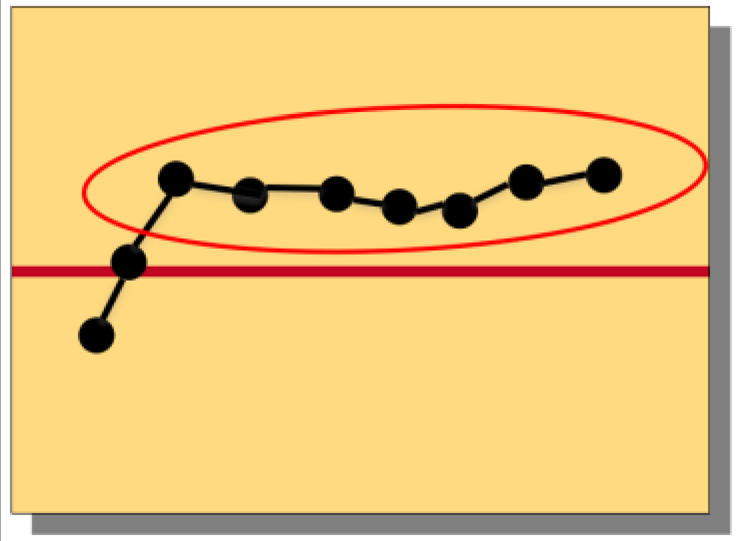


7 continuous points on one side of centre line

Reference: AIAG SPC 2nd edition manual

Rule-2

Potential causes



7 continuous points on one side of centre line

Change in tool- Such as

- Shifting of stopper
- Change in pin location

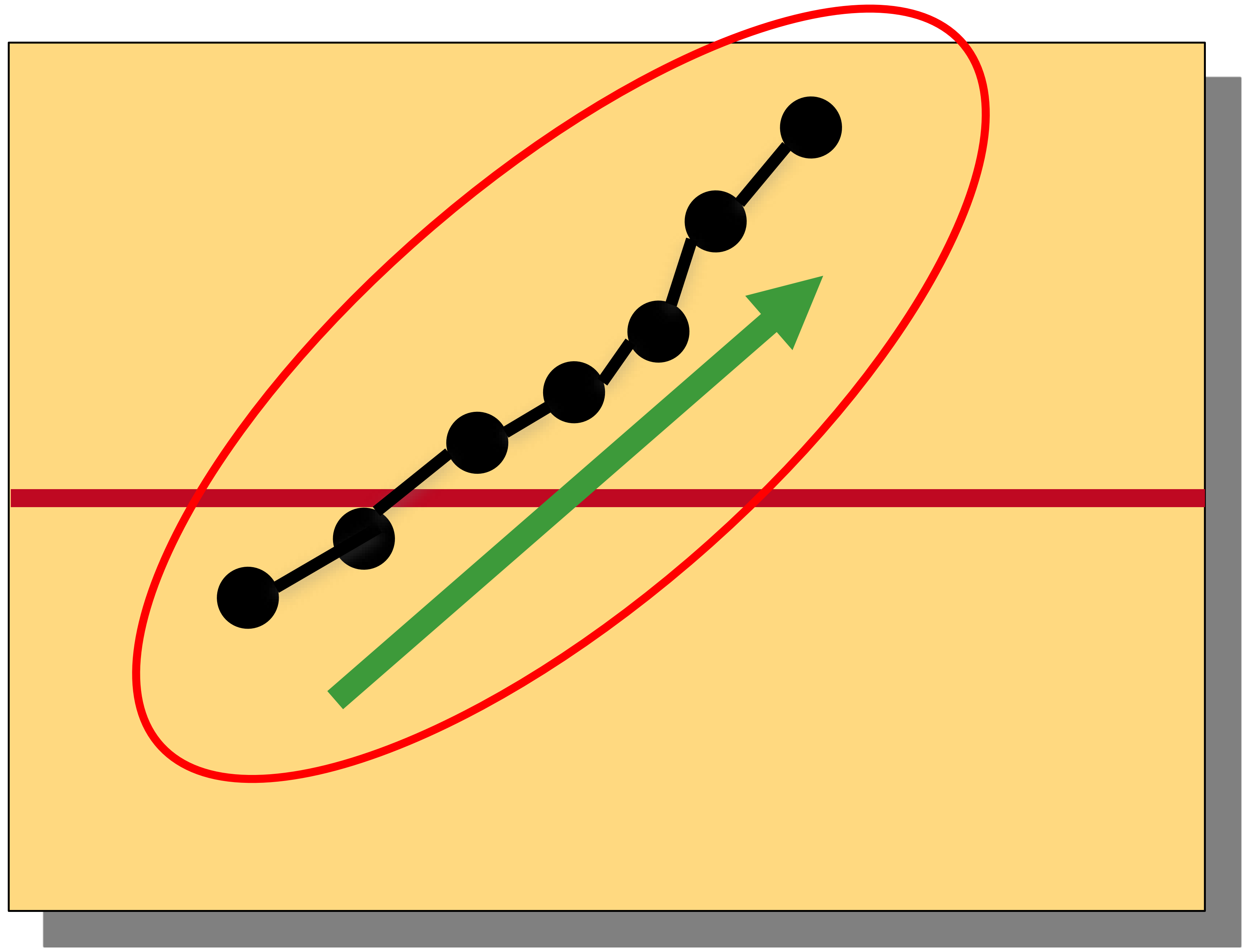
Rule-3

Patterns in Control Charts

Upper control limit

Target

Lower control limit



6 continuous points in increasing order

Reference: AIAG SPC 2nd edition manual

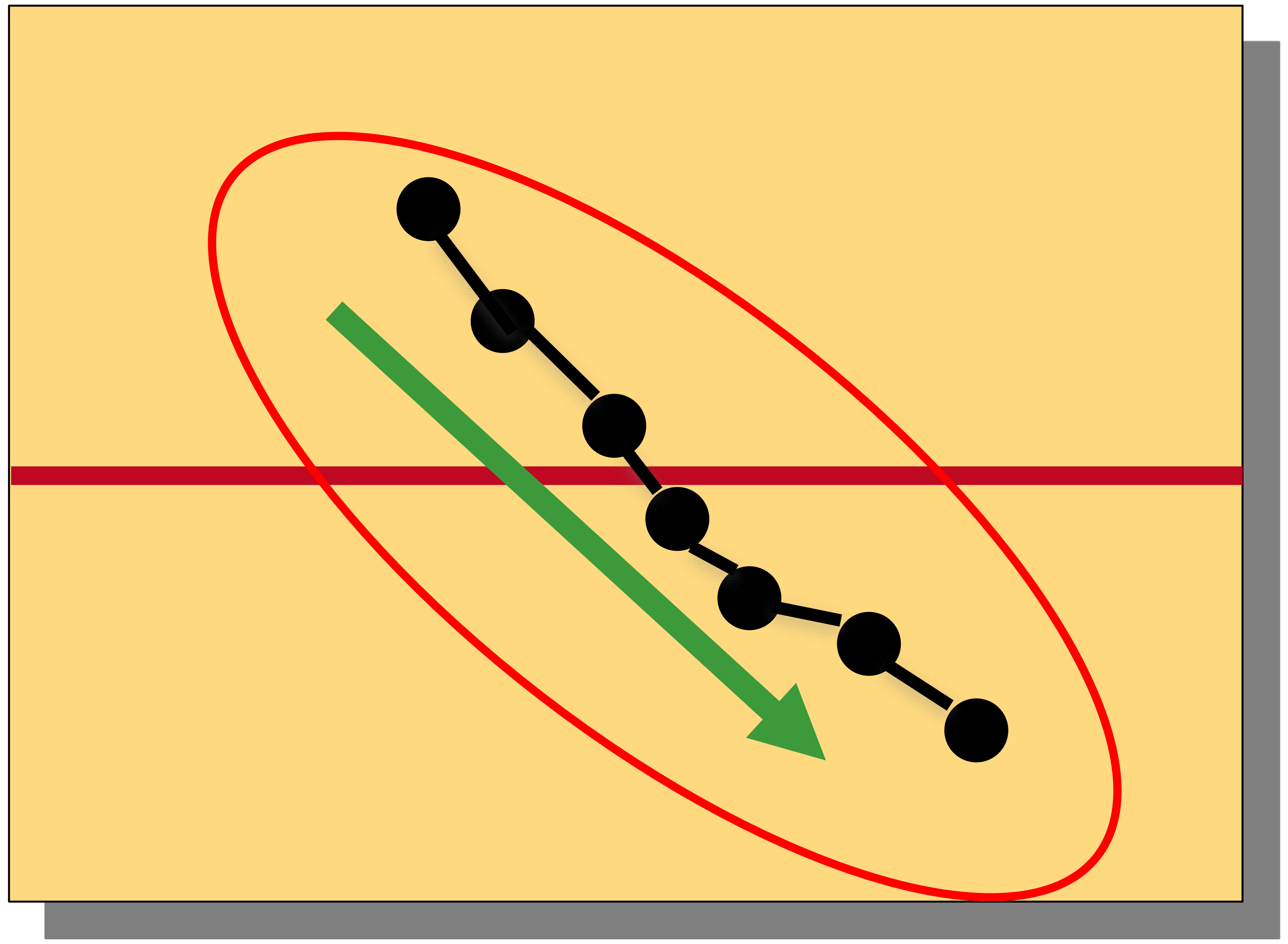
Rule-3 (Continued….)

Patterns in Control Charts

Upper control limit

Target

Lower control limit

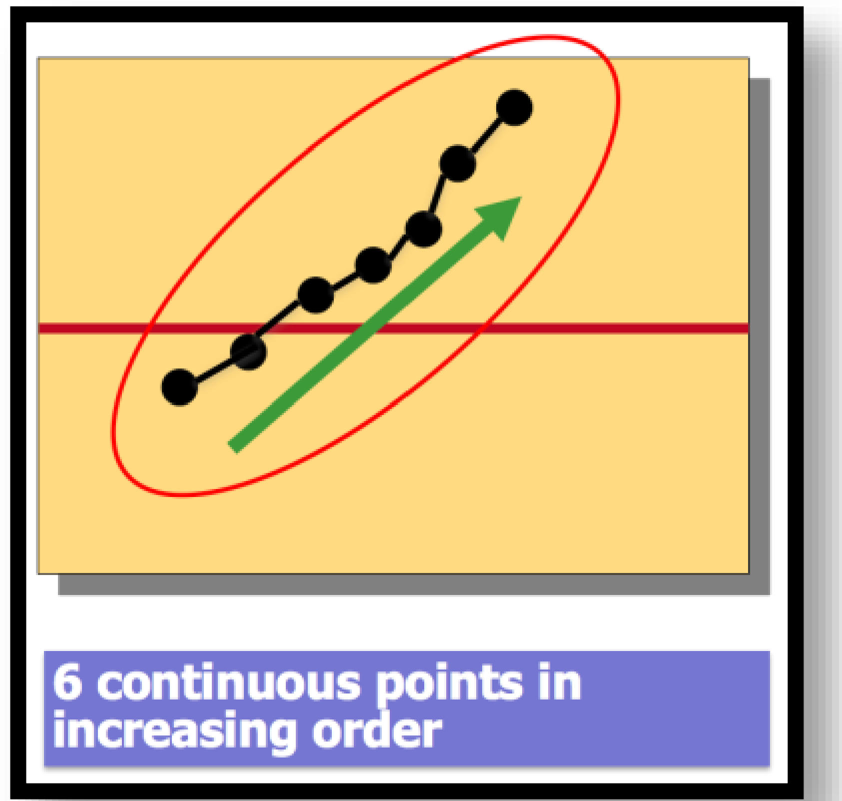


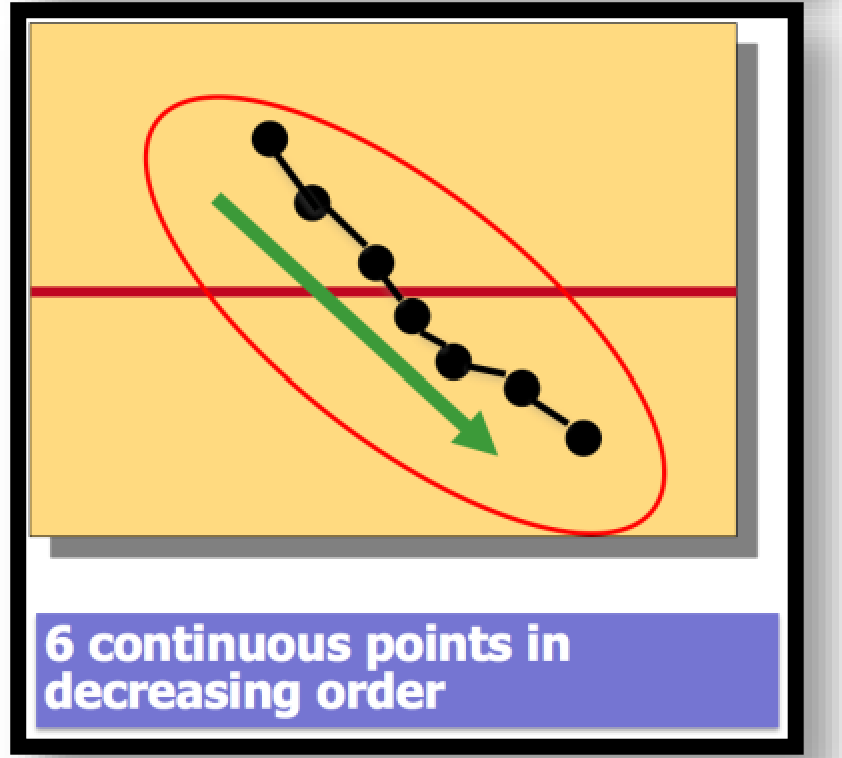
6 continuous points in decreasing order

Reference: AIAG SPC 2nd edition manual

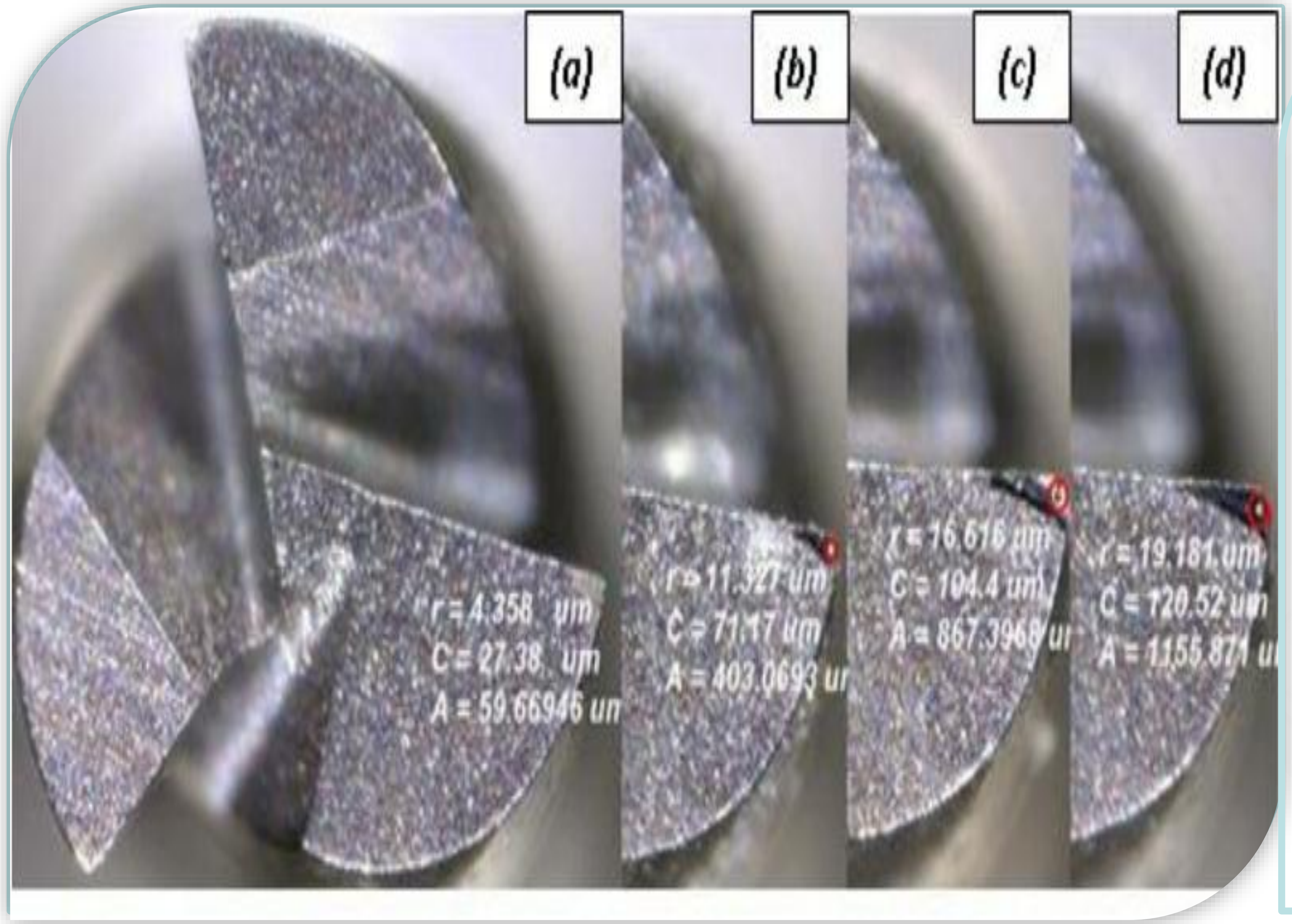
Rule-3

Potential causes



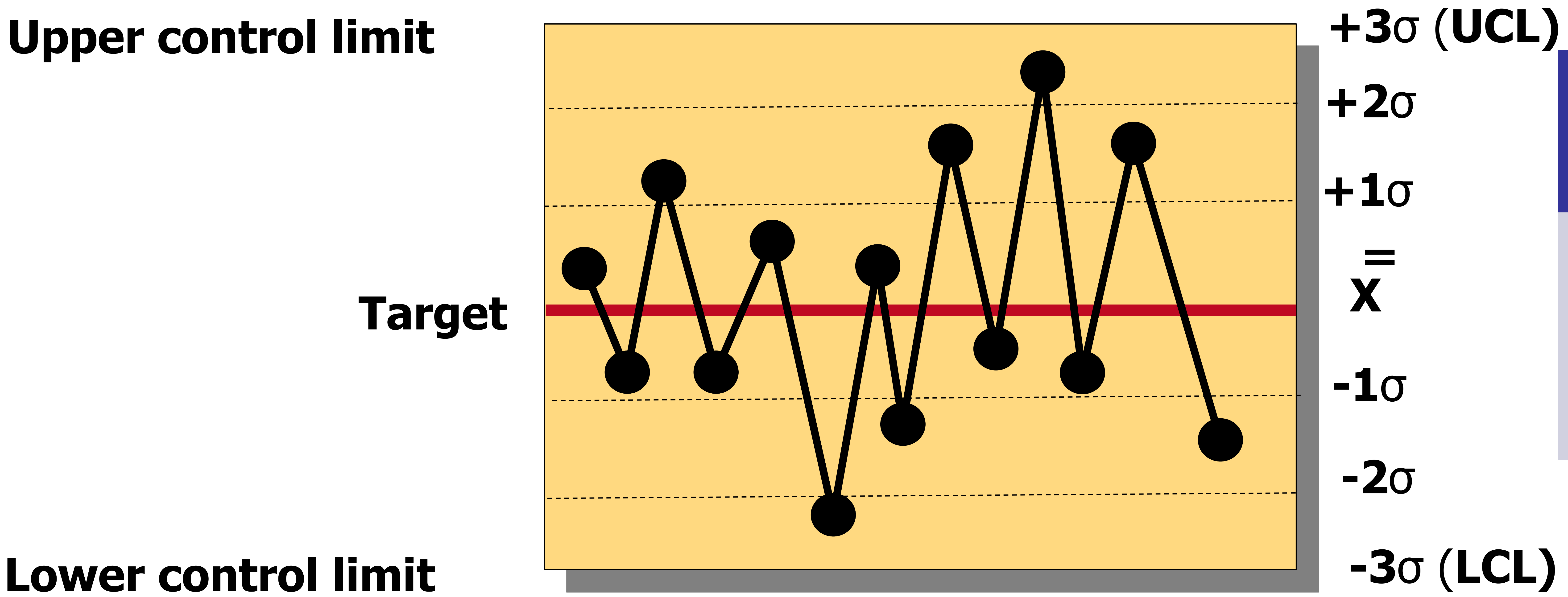


Tool wear



Rule-4

Patterns in Control Charts



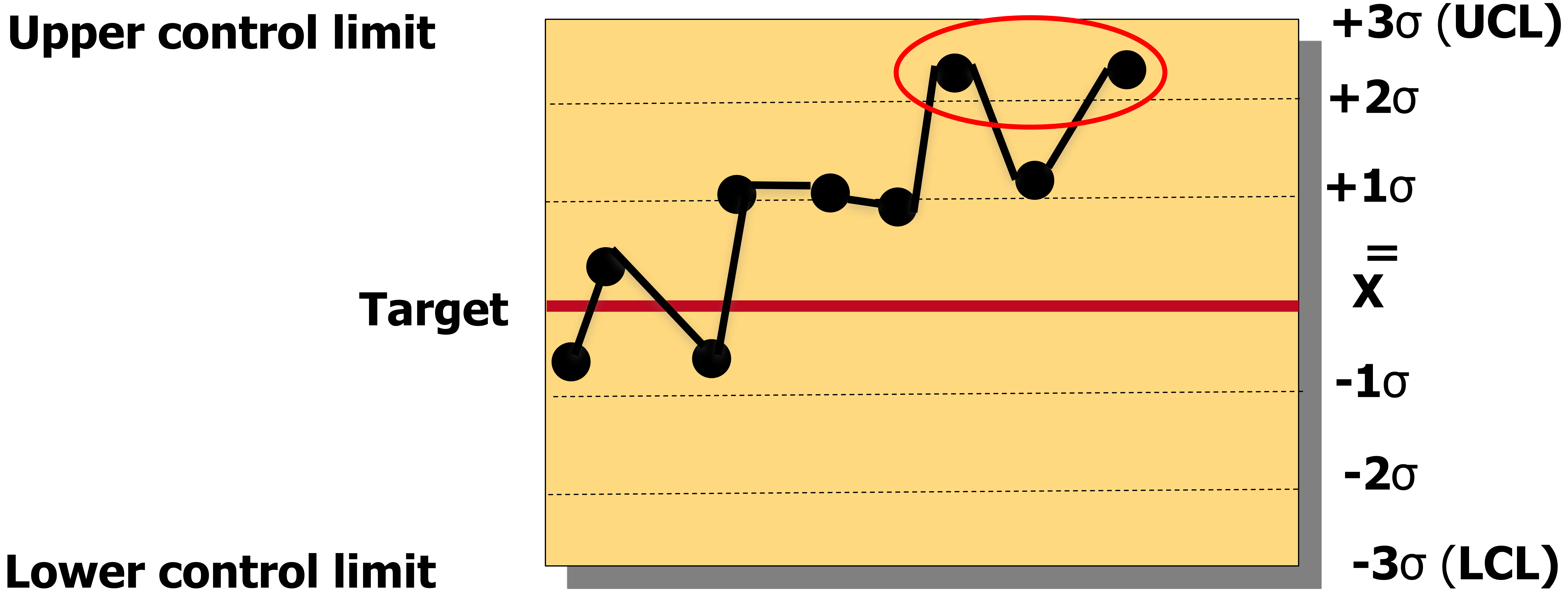
14 points in row alternating up and down

Potential causes

Reference: AIAG SPC 2nd edition manual

Rule-5

Patterns in Control Charts

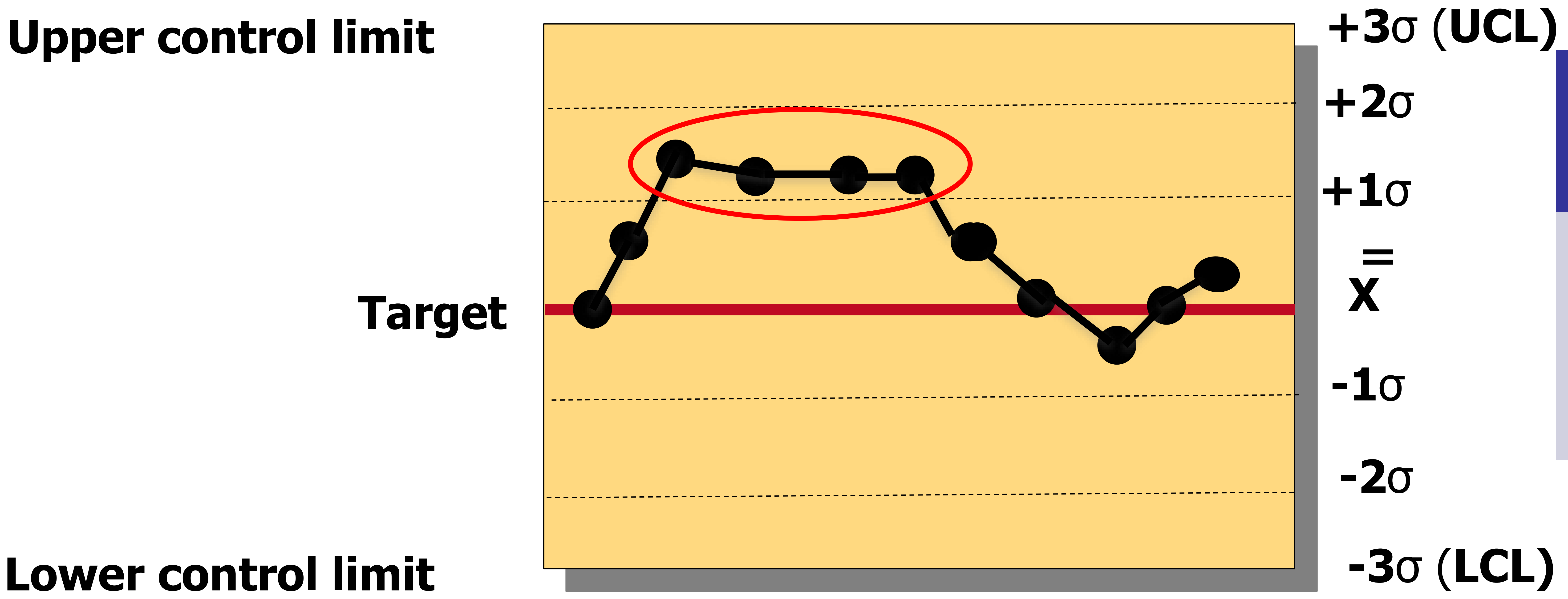


2 out of 3 points >2 std dev from centreline (Same side)

Reference: AIAG SPC 2nd edition manual

Rule-6

Patterns in Control Charts



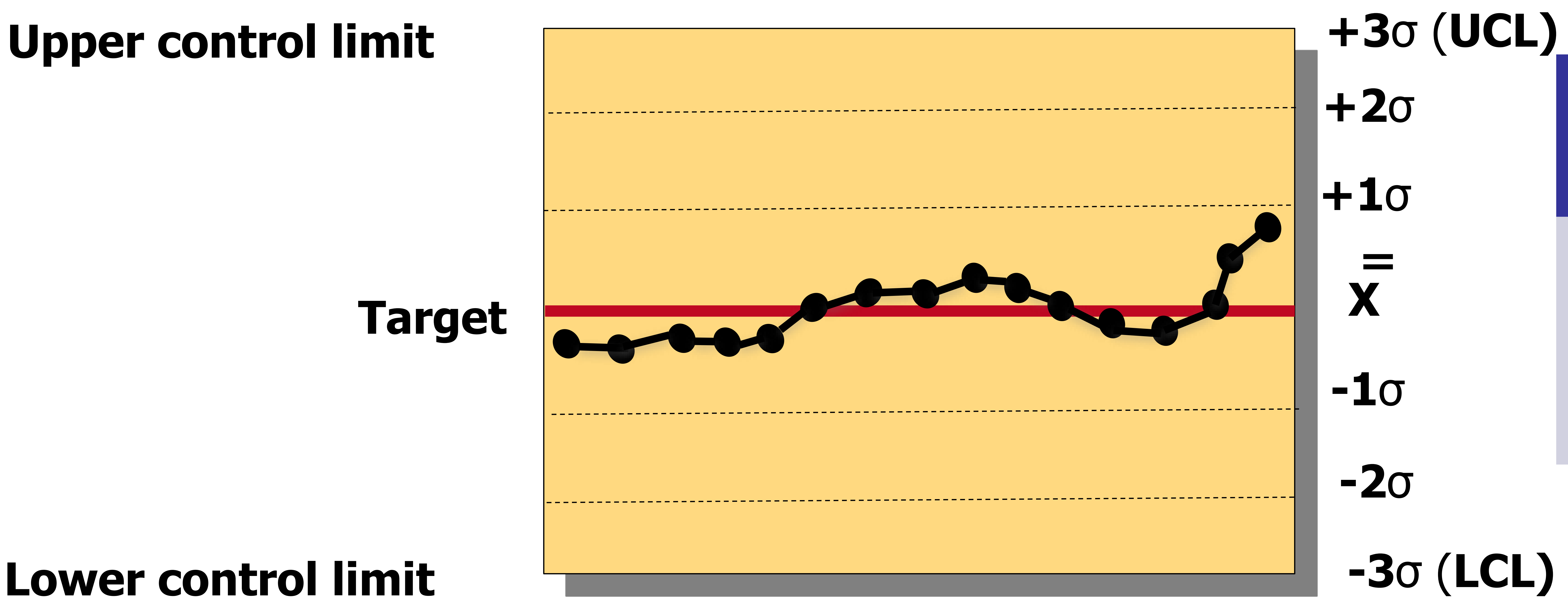
4 out of 5 points beyond 1σ limit on same side of centreline

Potential causes

Reference: AIAG SPC 2nd edition manual

Rule-7

Patterns in Control Charts



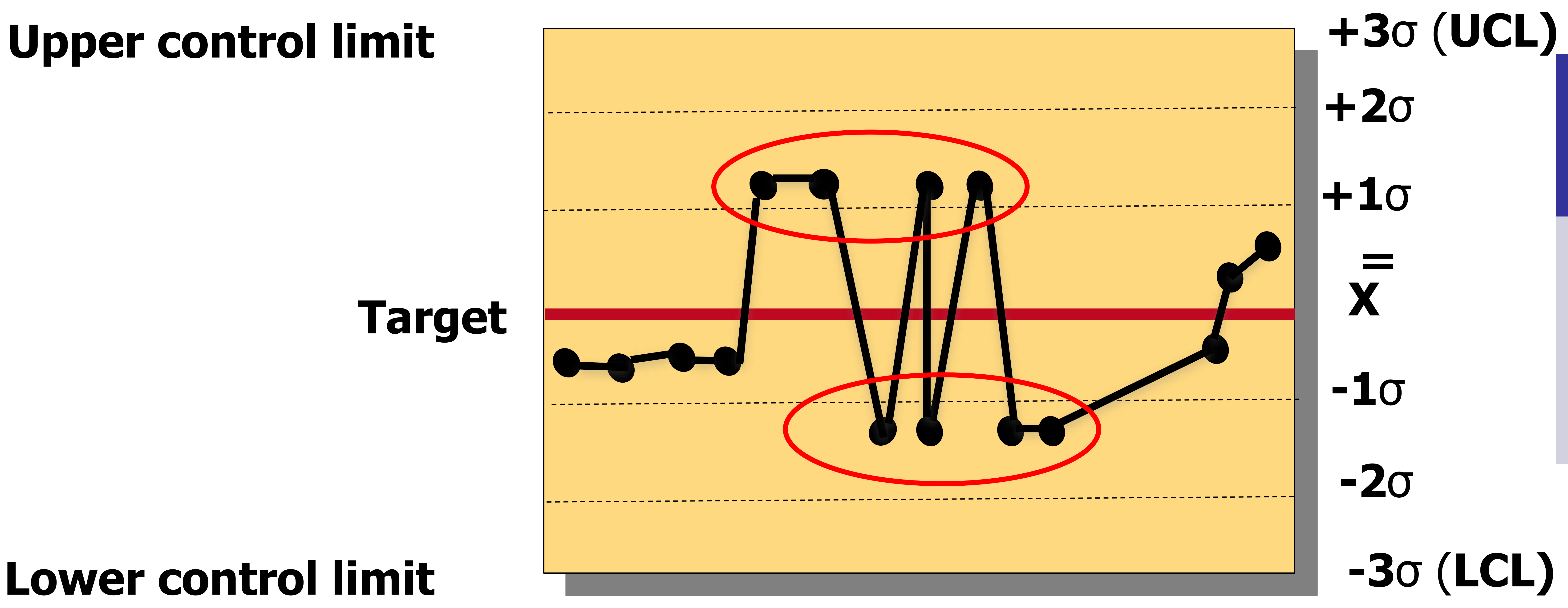
15 points in row within 1 standard deviation of Centre line

Potential causes

Reference: AIAG SPC 2nd edition manual

Rule-8

Patterns in Control Charts



8 points in row > 1 std. dev from centreline (either side)

Potential causes

Reference: AIAG SPC 2nd edition manual

Summary of “Control chart” rules

1. 1 point more than 3s limits
2. 7 points in row in same side of centreline
3. 6 points in row all increasing or decreasing
4. 14 points in row alternating up and down
5. 2 out of 3 points >2 std dev from centreline (Same side)
6. 4 out of 5 points >1 std dev from centreline (Same side)
7. 15 points in row within 1 std. dev of centreline (Either side)
8. 8 points in row > 1 std. dev from centreline (Either side)

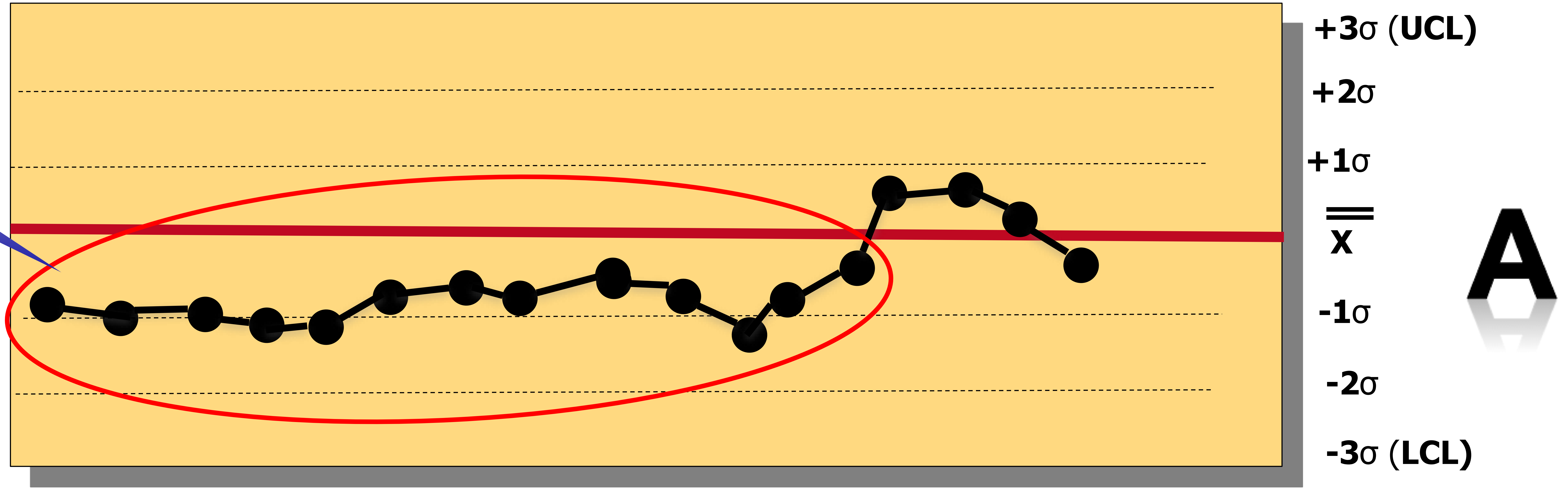
How to detect special cause?

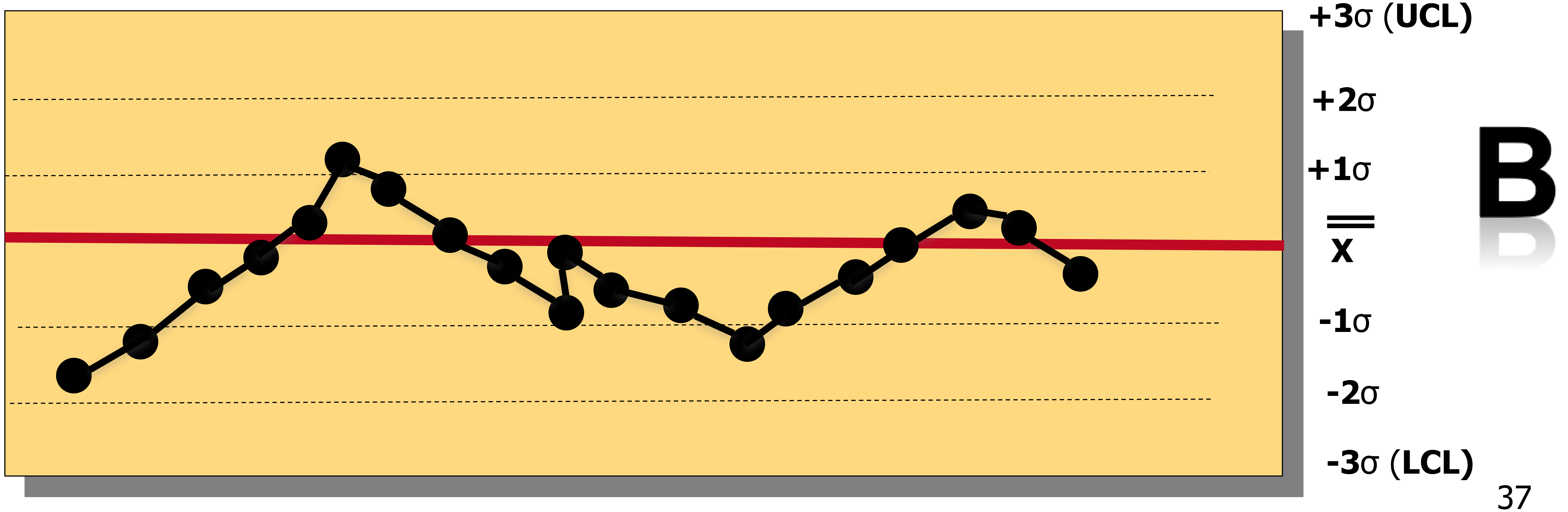
(For Practice)

Which control chart shows special cause? (1/3)

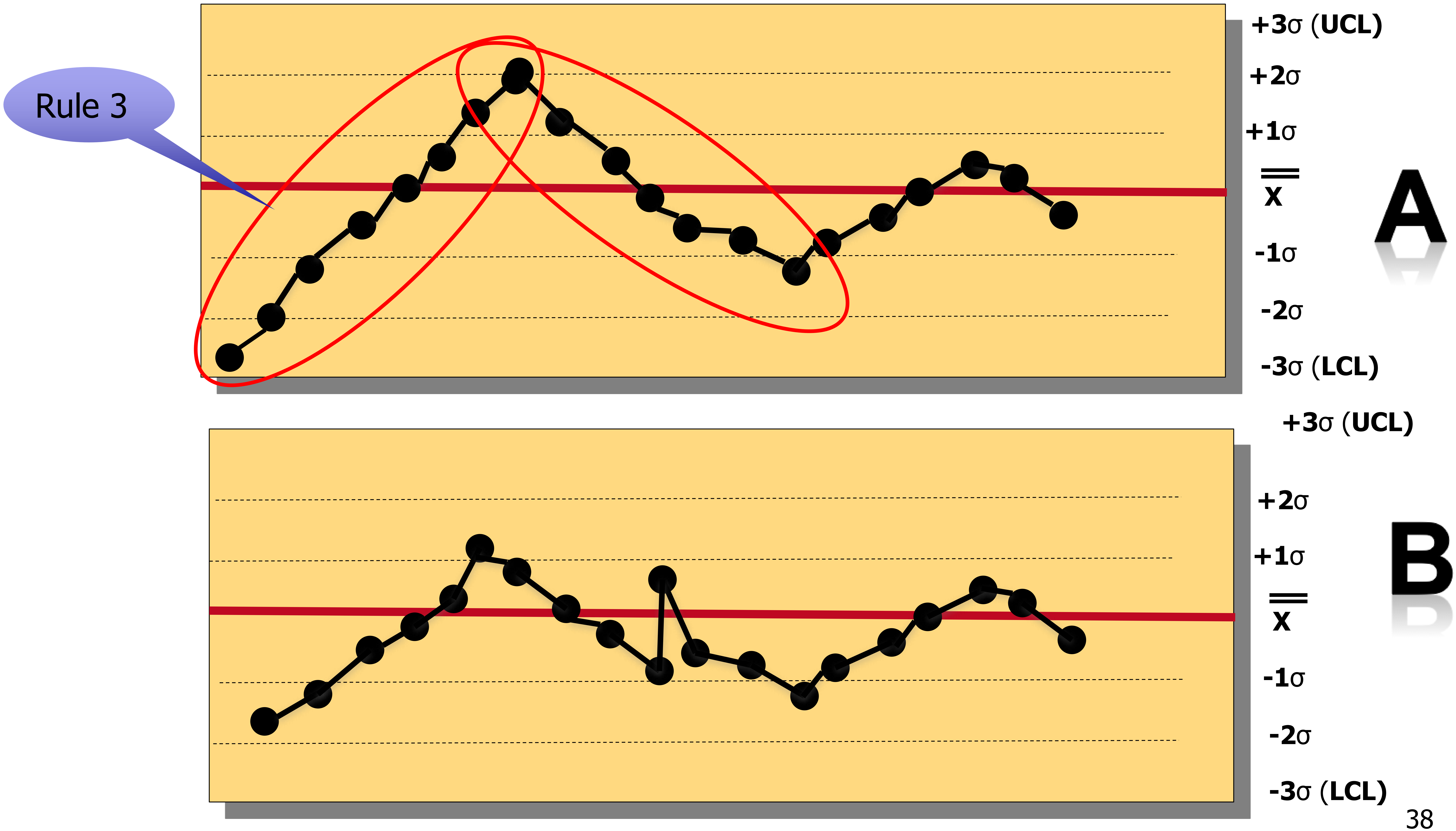
Rule 2

7 continuous points on one side of center line

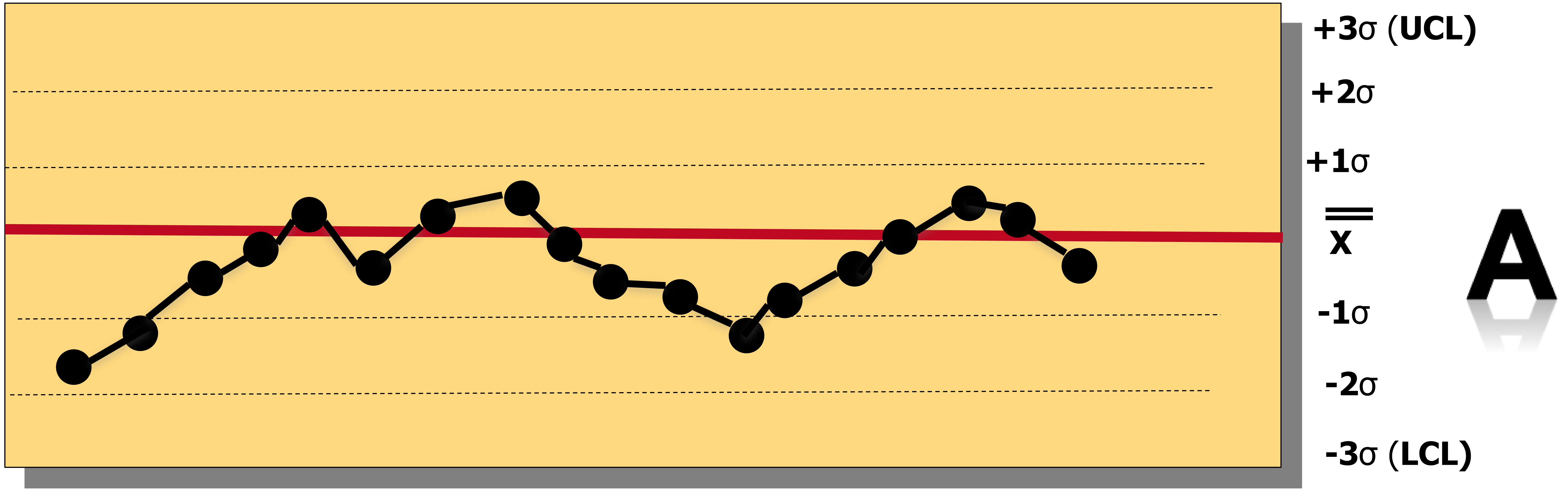


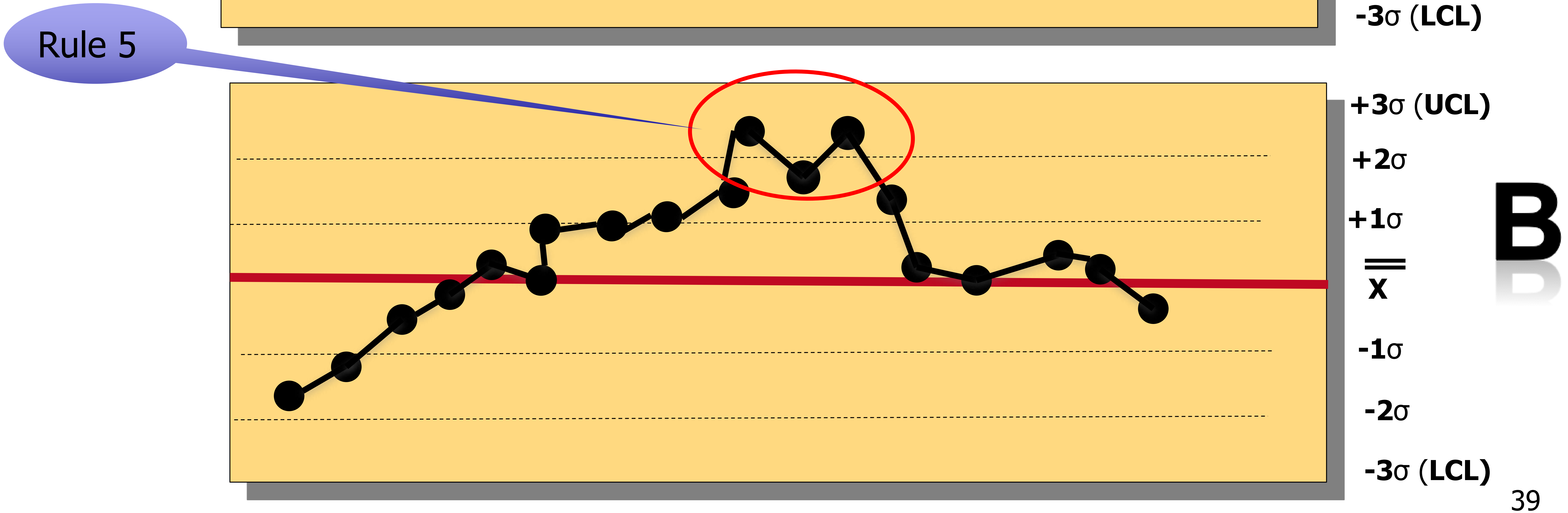


Which control chart shows special cause? (2/3)



Which control chart shows special cause? (3/3)



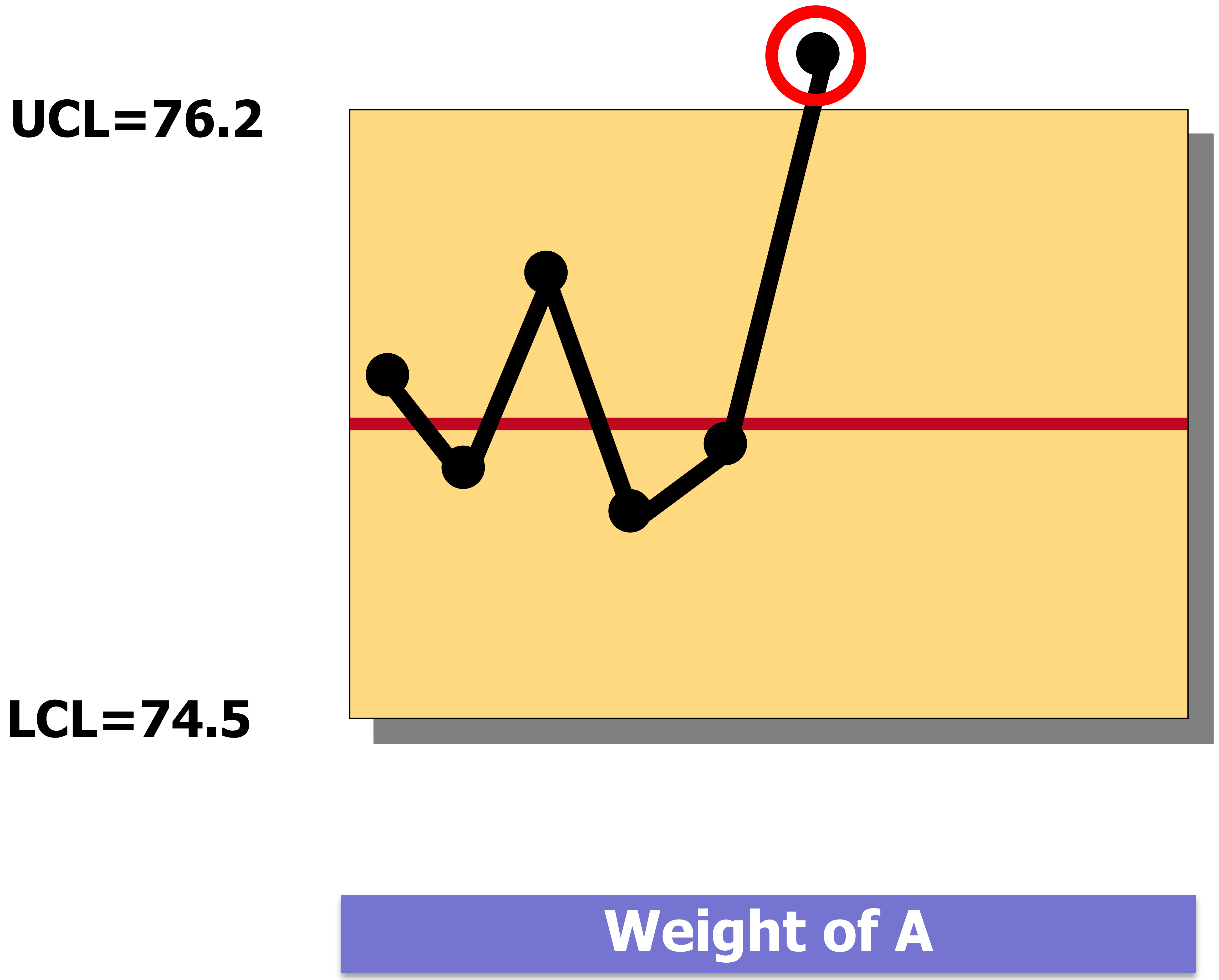


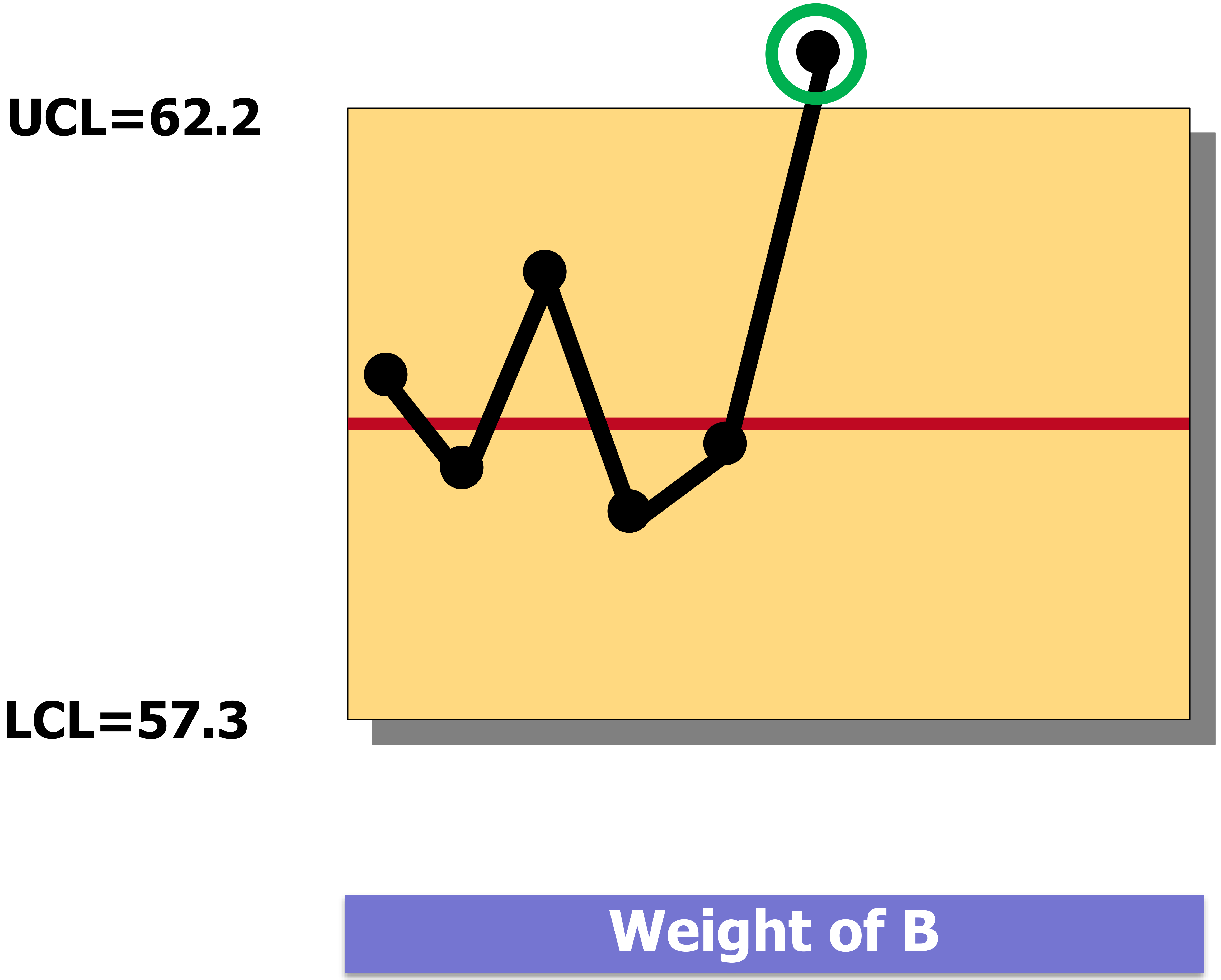
Tampering (Over adjustment)

Data has no meaning apart from its context

- Dr.Walter Shewhart

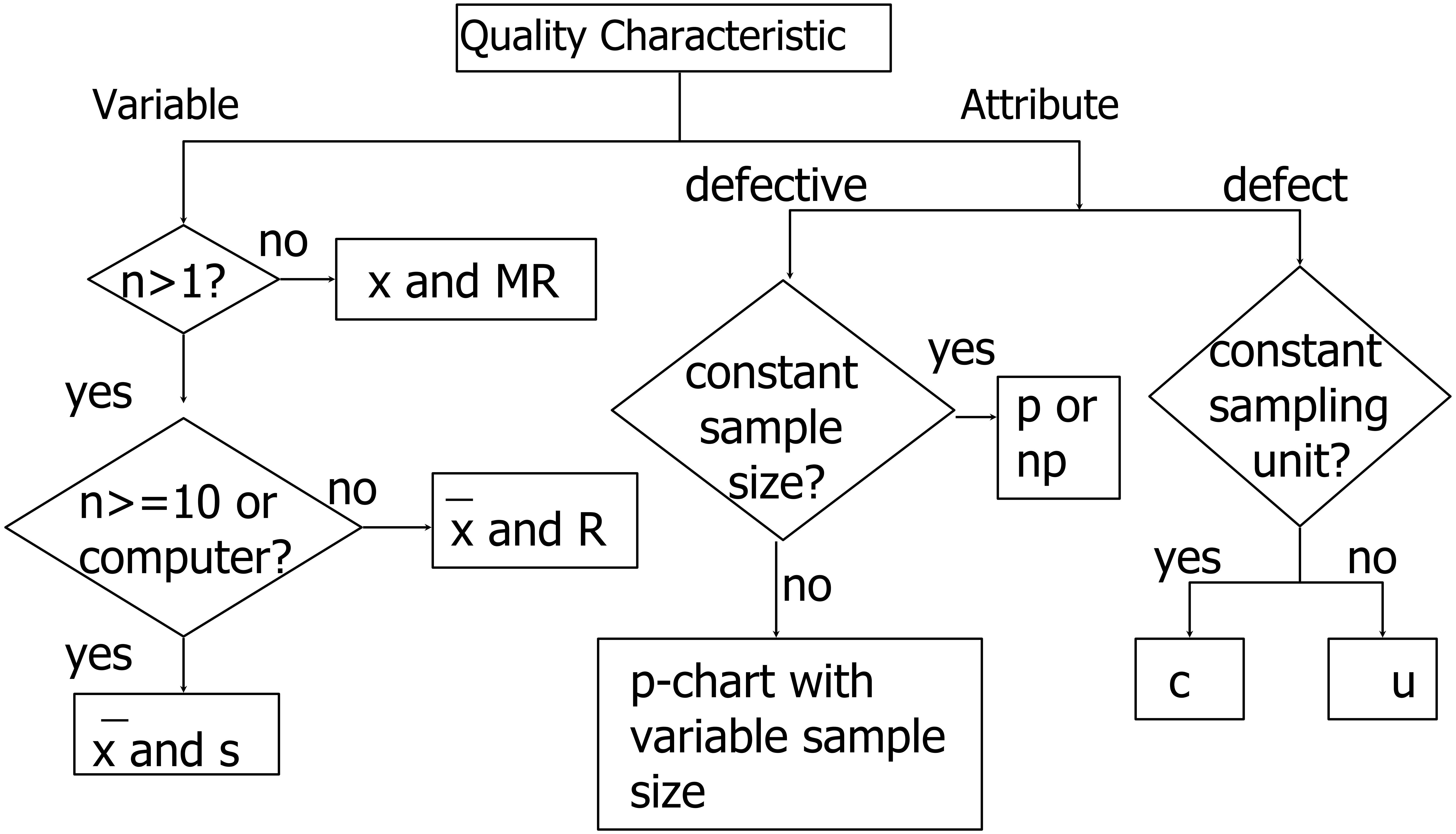
Statistical control Vs Process Control





Difference between control charts

Control Chart Selection



Control chart differences & Application

	Type of Control chart							
	Variable				Attribute			
					Defectives		Defects	
Parameter	Xbar-R chart	Xbar- s chart	Median Chart (With R chart)	I-MR (X-MR) charts	P	np chart	C chart	U chart
Use	Subgroup size $\leq$ 9	Subgroup size > 9	Any subgroup size	Subgroup size = 1	Sample size is varying	Sample size is constant	Sample size is constant	Sample size is varying
Charting	Xbar chart & R chart both to be used together	Xbar chart & s chart both to be used together	Median chart & R chart both to be used together	Individual chart & Moving range chart to be used together	Only p chart is used	Only np chart is used	Only c chart is used	Only u chart is used
Charting (Comparison with Xbar-R chart)	-------	s (Of subgroup) is plotted in place of R	Median is plotted in place of Xbar	1. Individual readings are plotted in place of Xbar 2. MR is plotted in place of R	-------	-------	-------	-------
Charting (Centre line)	1. X dbar (For Xbar chart) 2. Rbar (For R chart)	1. X dbar (For Xbar chart) 2. s bar (For s chart)	1. Median of medians (For median chart) 2. Median of R (For R chart)	1. Xbar (For individual Or X chart) 2. MR bar (For MR chart)	p bar	np bar	c bar	u bar

Xbar-R chart

Part No:	1234	**Part name**	XYZ	**Process / Operation**	ABC	**Characteristics**	8± 2 mm	**Machine no:**	M1TR-01
Subgroup #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Date, Time	**12-8-122**	**13-8-22**	**14-8-22**	**16-8-22**	**17-8-22**	**18-8-22**	**19-8-22**	**20-8-22**	**21-8-22**
1	9	11	10	10	8	10	8	8	10
2	8	10	9	9	10	9	7	6	8
3	8	11	8	8	11	8	8	9	9
4	9	8	8	10	8	10	10	10	8
5	10	10	9	8	9	8	9	9	9
X bar	**8.8**	**10**	**8.8**	**9**	**9.2**	**9**	**8.4**	**8.4**	**8.8**
R	**2**	**3**	**2**	**2**	**3**	**2**	**3**	**4**	**2**

Xbar-R chart

$\overline{X}$ Chart

UCL: $\overline{\overline{X}} + A2.\overline{R}$

LCL: $\overline{\overline{X}} - A2.\overline{R}$

Where,

$\overline{R}$ =average range of the samples

A_2 =control chart factor found in Table 1.1

$\overline{\overline{X}}$ =mean of the sample means

Range Chart

$$LCL = D_3\overline{R}$$

$$UCL = D_4\overline{R}$$

Where,

$\overline{R}$ =average range of the samples

D_3 = Control chart factor found in Table 1.1

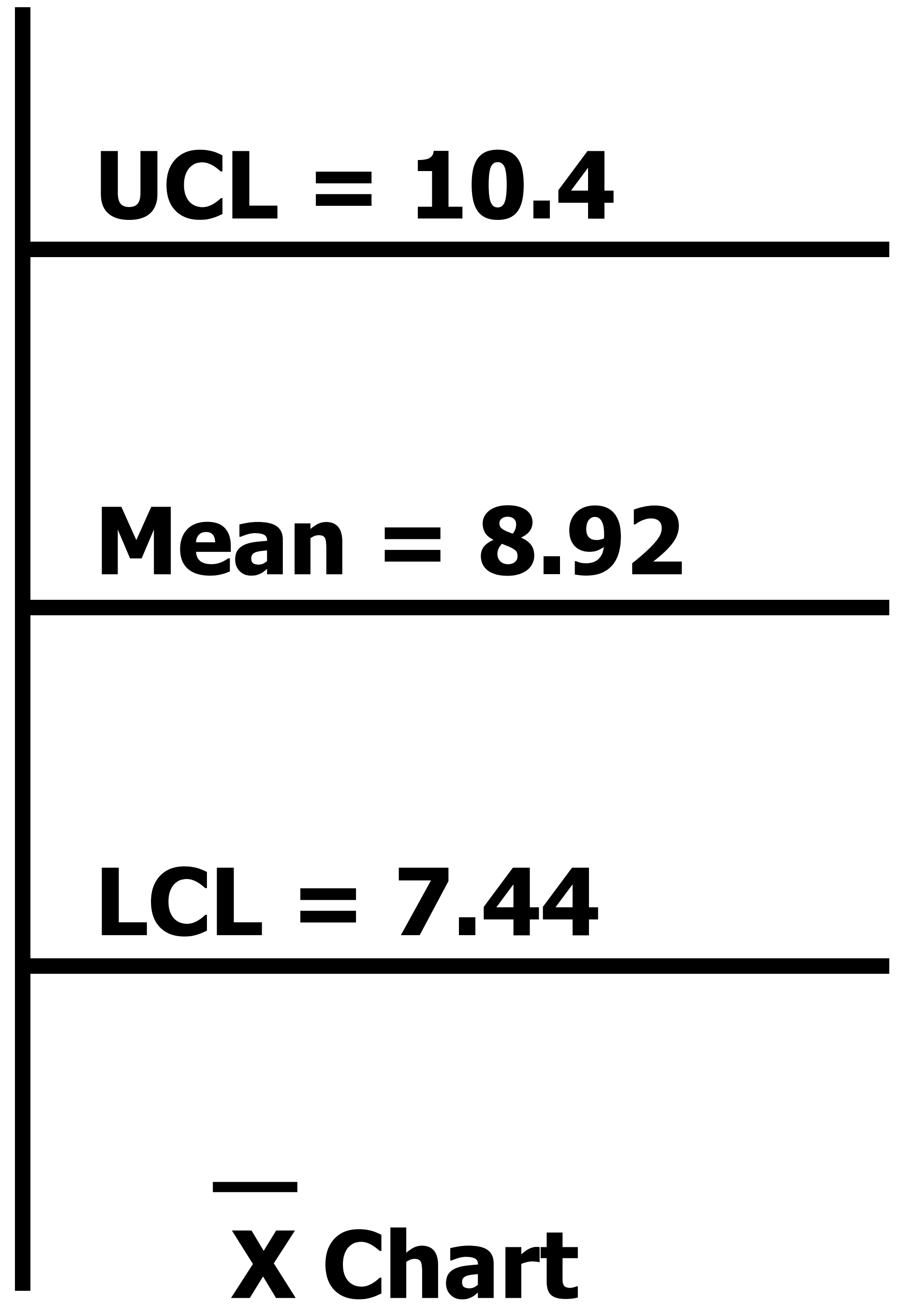
D_4 = Control chart factor found in Table 1.1

Setting Control Limits

Process average $\bar{\bar{x}}$ = 8.92 mm
Average range $\bar{R}$ = 2.56 mm
Sample(subgroup) size n = 5

$$
\begin{aligned}
UCL_{\bar{x}} &= \bar{\bar{x}} + A_2\bar{R} \\
&= 8.92 + (0.577)(2.56) \\
&= 8.92 + 1.48 \\
&= 10.4 \text{ mm}
\end{aligned}
$$

$$
\begin{aligned}
LCL_{\bar{x}} &= \bar{\bar{x}} - A_2\bar{R} \\
&= 8.92 - (0.577)(2.56) \\
&= 8.92 - 1.48 \\
&= 7.44 \text{ mm}
\end{aligned}
$$

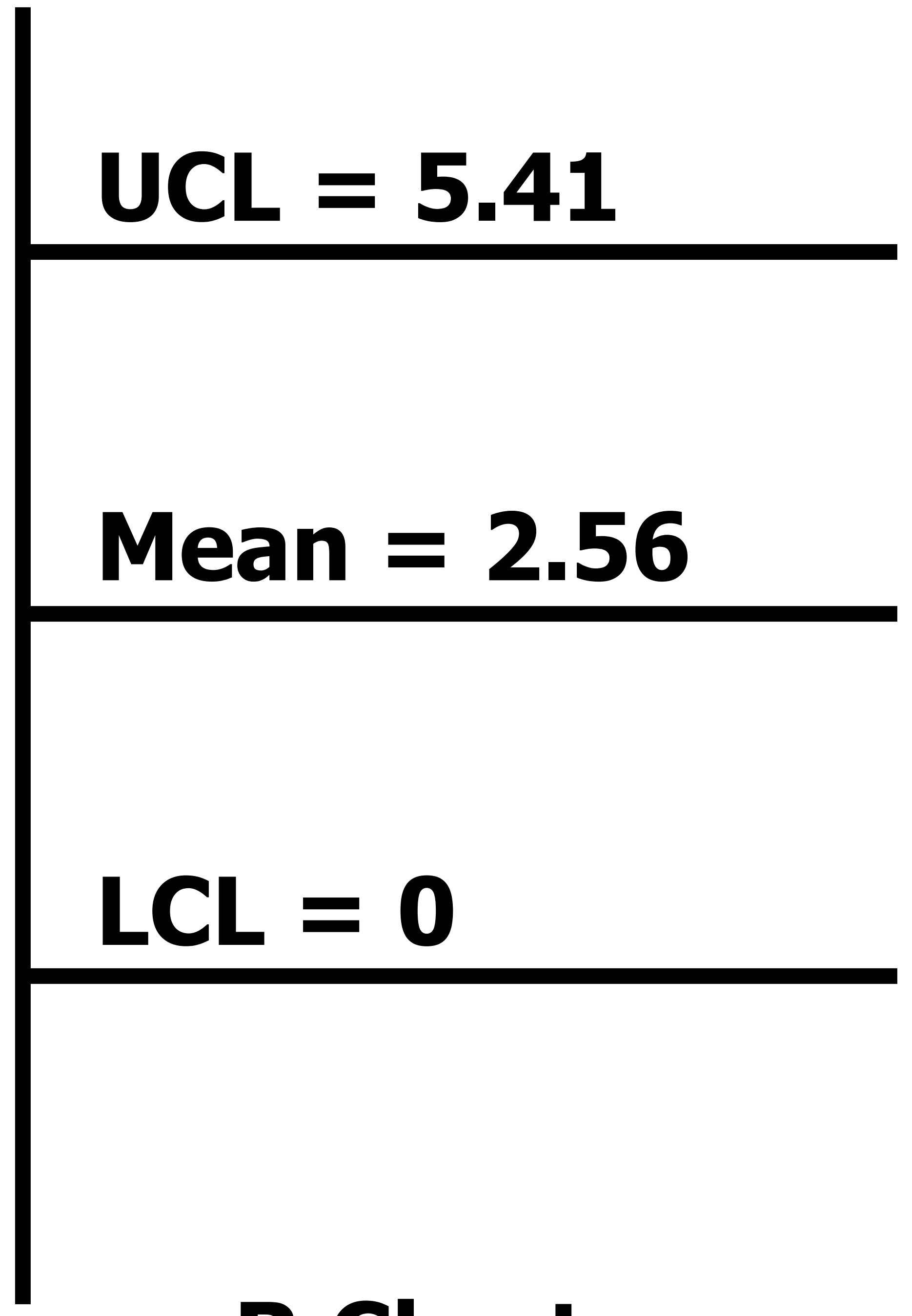


Setting Control Limits

Process average $\bar{\bar{x}}$ = 8.92 mm
Average range $\bar{R}$ = 2.56 mm
Sample(subgroup) size n = 5

$$UCL_R = D_4\bar{R} = (2.114)(2.56) = 5.41 \text{ mm}$$

$$LCL_R = D_3\bar{R} = (0)(2.56) = 0 \text{ mm}$$

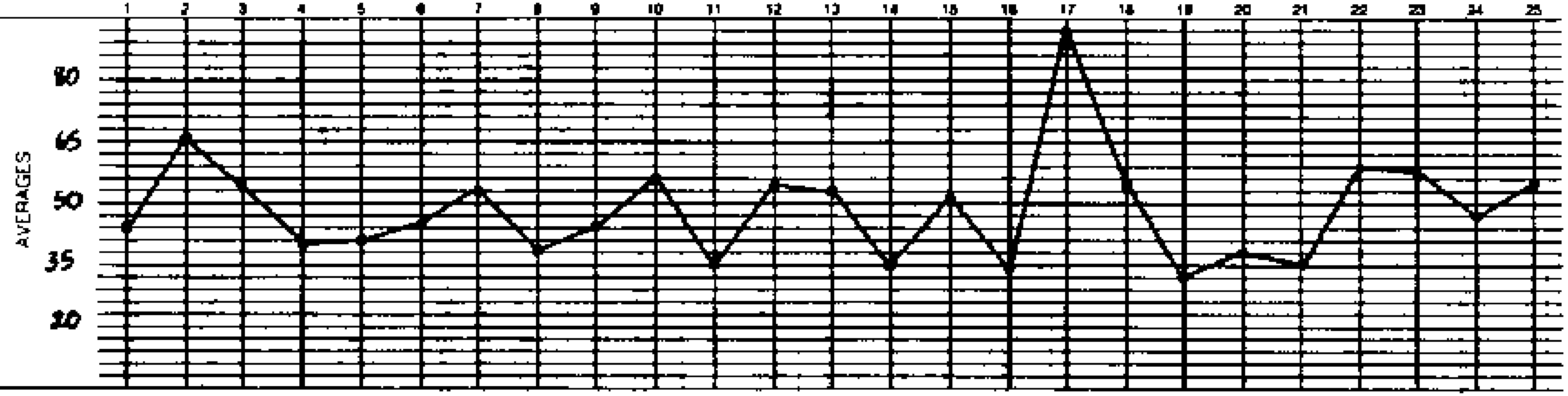


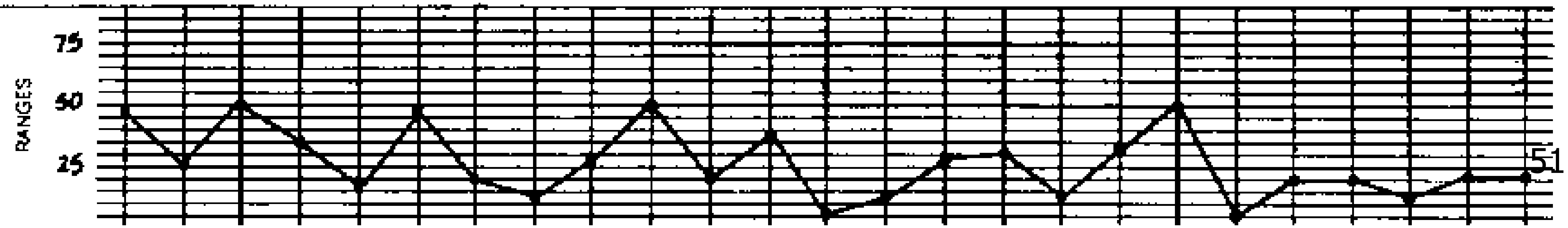
R Chart

VARIABLES CONTROL CHART ($\overline{X}$ & R)

PART NO	
CHART NO	
PART NAME (PRODUCT)	SILICON WAFER
OPERATION (PROCESS)	
SPECIFICATION LIMITS	0-100
OPERATOR	
MACHINE	
GAGE	
UNIT OF MEASURE	.0000
ZERO EQUALS	

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
DATE																									
TIME																									
SAMPLE MEASUREMENTS 1	41	78	84	60	46	64	43	37	50	57	24	78	51	41	56	46	99	71	41	41	22	62	64	44	41
2	70	53	34	36	47	16	53	43	29	83	42	48	57	29	64	41	86	54	2	39	40	70	52	38	63
3	22	68	48	25	29	56	64	30	57	32	39	39	50	35	36	16	98	39	53	36	46	46	57	60	62
4																									
5																									
SUM	133	199	166	121	122	136	160	110	136	172	105	165	158	105	156	103	283	164	96	116	108	178	173	142	166
AVERAGE, $\overline{X}$	44	66	55	40	41	45	53	37	45	57	35	55	53	35	52	34	94	54	32	39	36	59	58	47	55
RANGE, R	48	25	50	35	18	48	21	13	28	51	18	39	7	12	28	30	13	32	51	5	24	24	12	22	22
NOTES																									





Normality

Mean

(Average)

Mean

1, 46, 11, 46, 8

$\Downarrow$

1+46+11+46+8=112

$\Downarrow$

Sum / n = (112/5) = 22.4

10, 42, 16, 46, 42, 46

$\Downarrow$

10+42+16+46+42+46 = 202

$\Downarrow$

Sum / n = (112/5) = 33.67

Mean (For practice)

10, 9, 11, 12, 11

10, 9, 11, 13, 11,12

Median

(Middle Value)

Median

1, 46, 11, 46, 8

1, 8, 11, 46, 46

11

10, 42, 16, 46, 42, 46

10, 16, 42, 42, 46, 46

$$\frac{42+42}{2} = 42$$

Median (For practice)

10, 9, 11, 12, 11

10, 9, 11, 13, 11,12

Mode

(Maximum occurring Value)

Mode

1, 82, 11, 46, 82

↓

1,11, 46, 82, 82

↓

82

10, 42, 16, 46, 42, 46

↓

10, 16, 42, 42, 46, 46

↓

42, 46

Mode (For practice)

10, 9, 11, 13, 12

9, 9, 11, 12, 11,12

Standard Deviation

$$s = \sqrt{\frac{\sum(x-\overline{x})^2}{N-1}}$$

$where$

$s = the\ standard\ deviation$

$x = each\ value\ in\ the\ sample$

$\overline{x} = the\ mean\ of\ the\ values$

$N = the\ number\ of\ values\ (the\ sample\ size)$

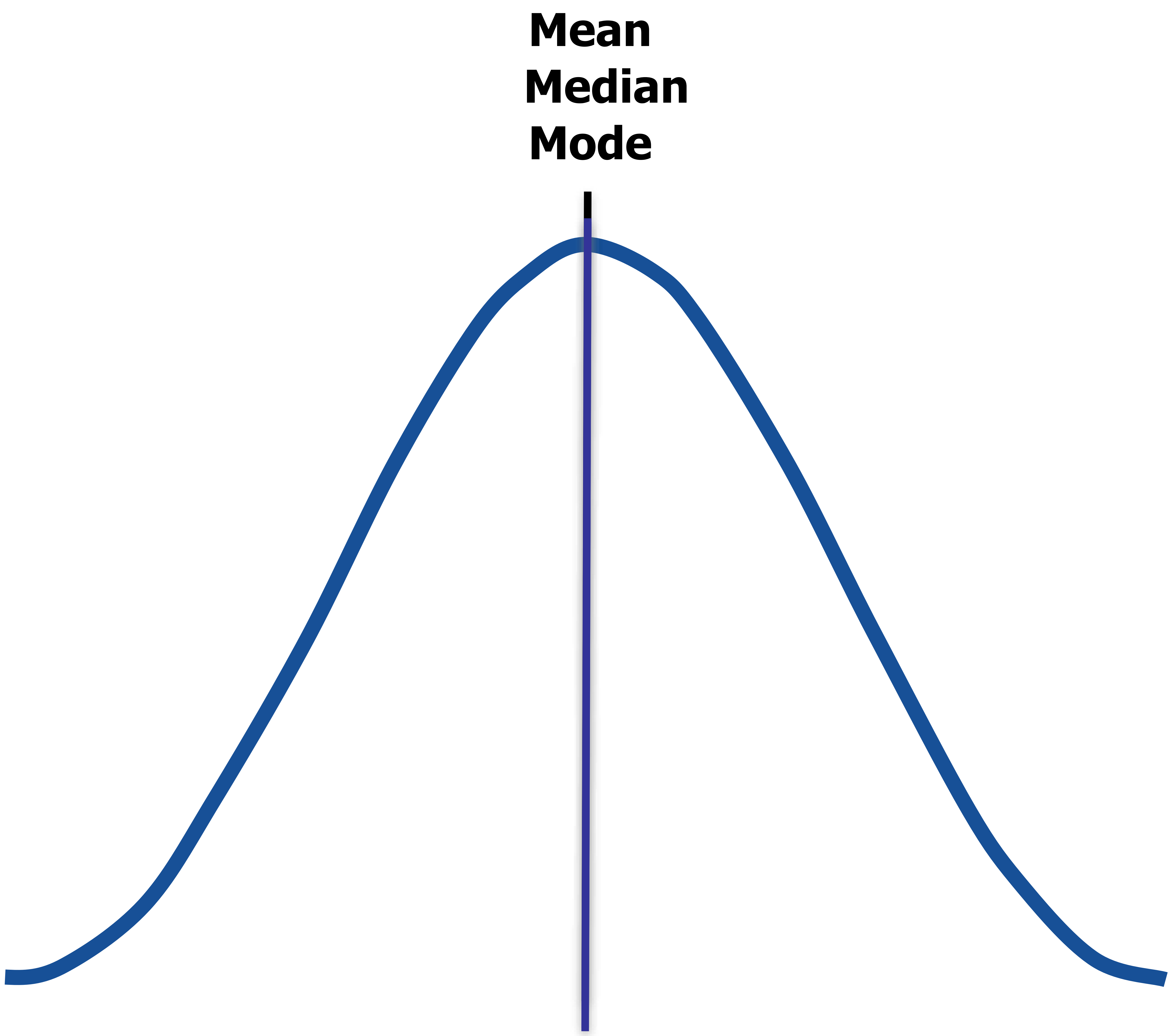
Standard Deviation

Sr. No.	Readings	$X - \bar{X}$	$(X - \bar{X})^2$	$\Sigma(X - \bar{X})^2$
1	10	10-10 = **0**	$(0)^2$ = **0**	0+4+1+1+4 = **10**
2	12	12-10 = **2**	$(2)^2$ = **4**	
3	11	11-10 = **1**	$(1)^2$ = **1**	
4	9	9-10 = **-1**	$(-1)^2$ = **1**	
5	8	8-10 = **-2**	$(-2)^2$ = **4**	
	Mean (Average): 10		$s^2 = \dfrac{\Sigma(X - \bar{X})^2}{n - 1}$	= 10 / 4 = 2.5
	n = 5	**n – 1** = 4	$s = \sqrt{\dfrac{\Sigma(X - \bar{X})^2}{n - 1}}$	= √ 2.5 = 1.58

Standard Deviation (For practice)

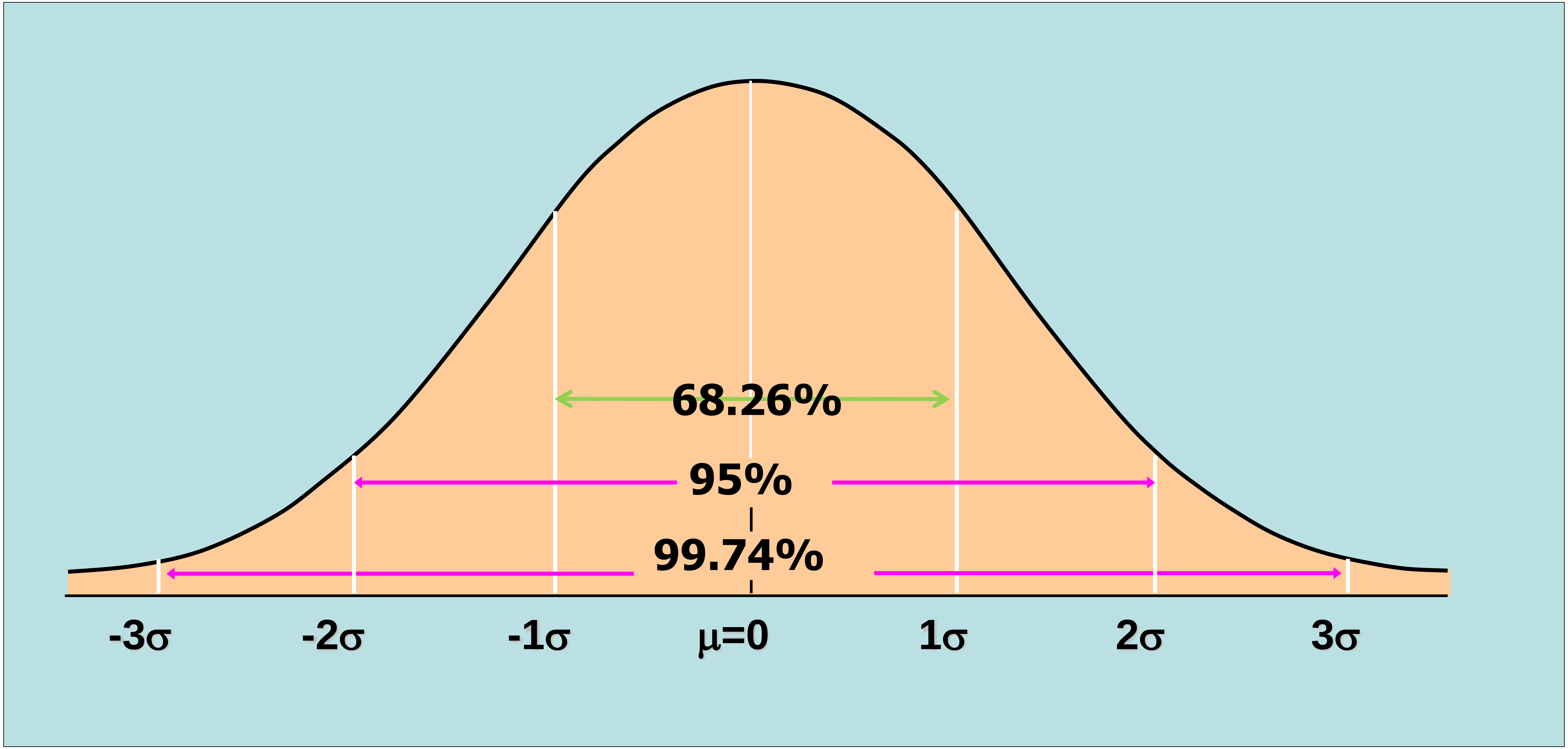
Sr. No.	Readings	$X - \overline{X}$	$(X - \overline{X})^2$	$\Sigma(X - \overline{X})^2$
1	21			
2	18			
3	17			
4	23			
5	26			
	Mean (Average):		$s^2 = \dfrac{\Sigma(X - \overline{X})^2}{n - 1}$	=
	n =	**n – 1** =	$s = \sqrt{\dfrac{\Sigma(X - \overline{X})^2}{n - 1}}$	=

Normal Distribution



Normal Distribution

Normal Distribution

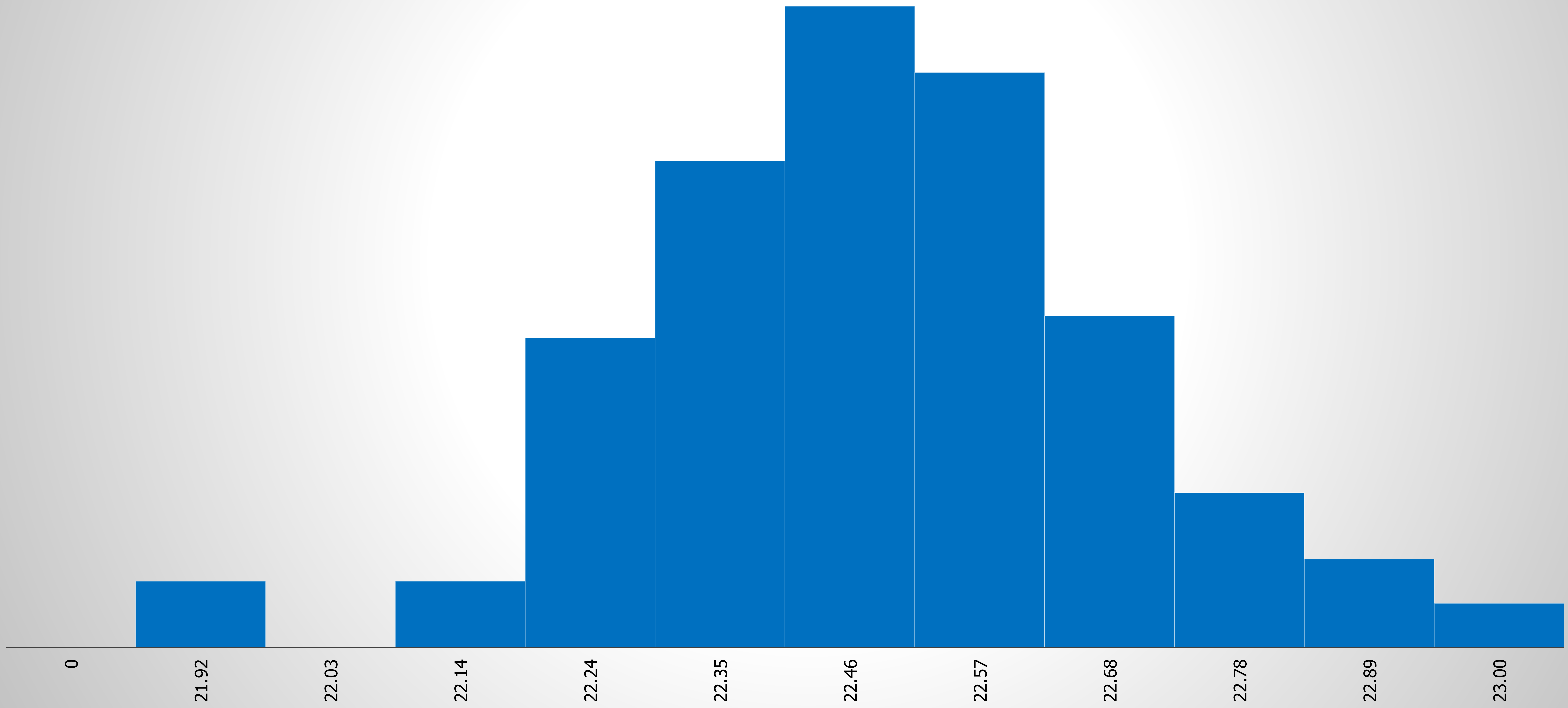


Example

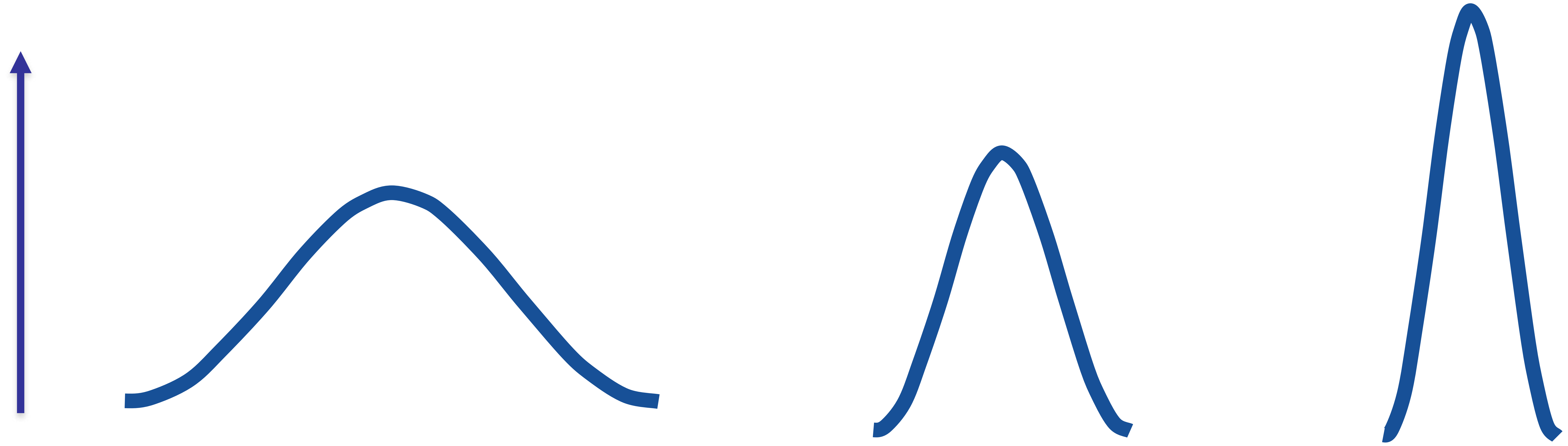
- India Vs Pakistan cricket match is going on, India's score is 192/8
- India needs 52 runs to win
- 2 players are remaining, run rate of both players is same
- You are captain of Indian cricket team. Who'll you choose as next batsman.

	Player-A	Player-B
Mean	106	62
Median	28	65
Mode	17	67

Histogram



Kurtosis

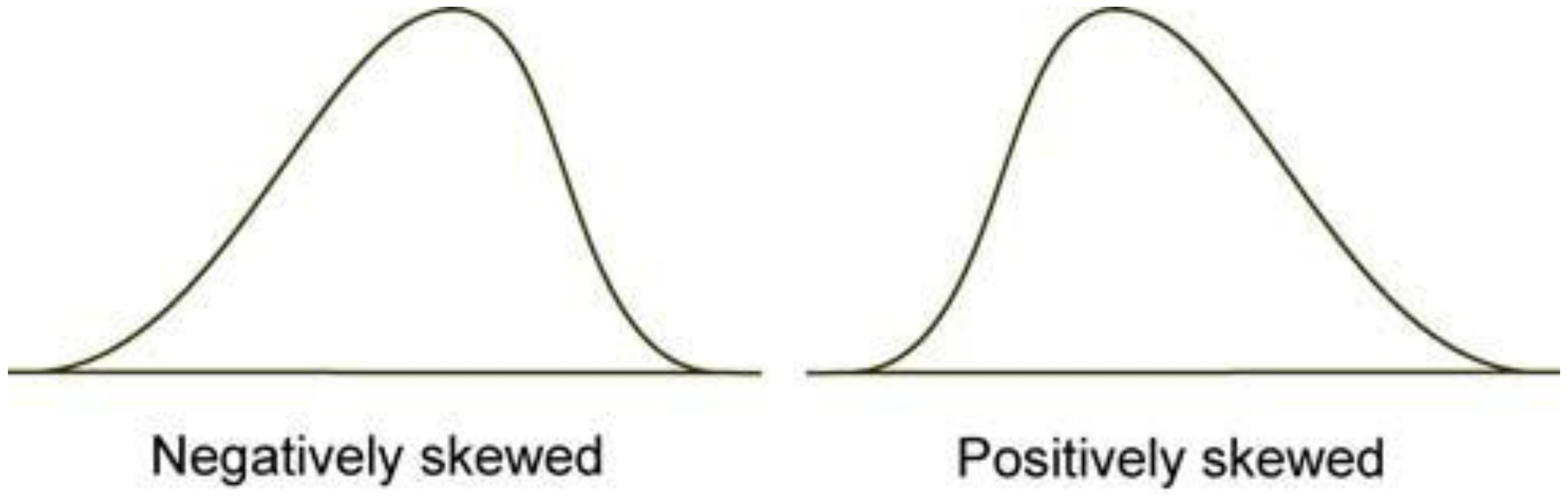


The following formula is used to determine the degree of kurtosis:

$$\frac{\Sigma\,(y - \mu)^3}{\sigma^4} - 3$$

- Where y = the deviation, μ = the mean, and σ = the standard deviation.

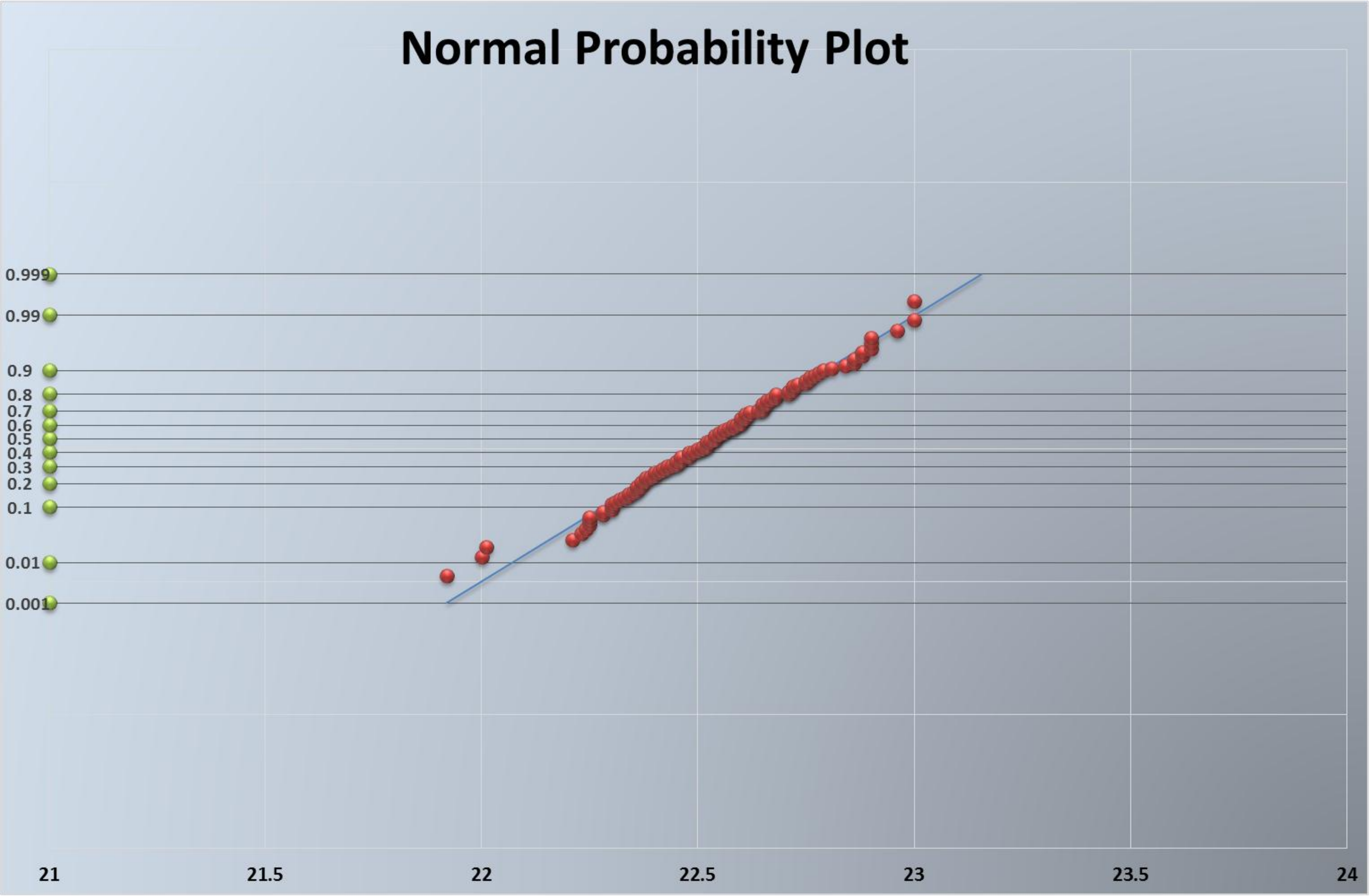
Skewness



The formula for determining the skew factor is:

$$\frac{\Sigma\,(y - \mu)^3}{\sigma^4}$$

- If the skew factor is zero, there is no skew.
- Along with letting you compare histograms, the skew factor can tell you if the process has a tendency to lean toward upper or lower specification limits.



Normal Probability Plot
0.999
0.99
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.01
0.001
21
21.5
22
22.5
23
23.5
24

Anderson-Darling test

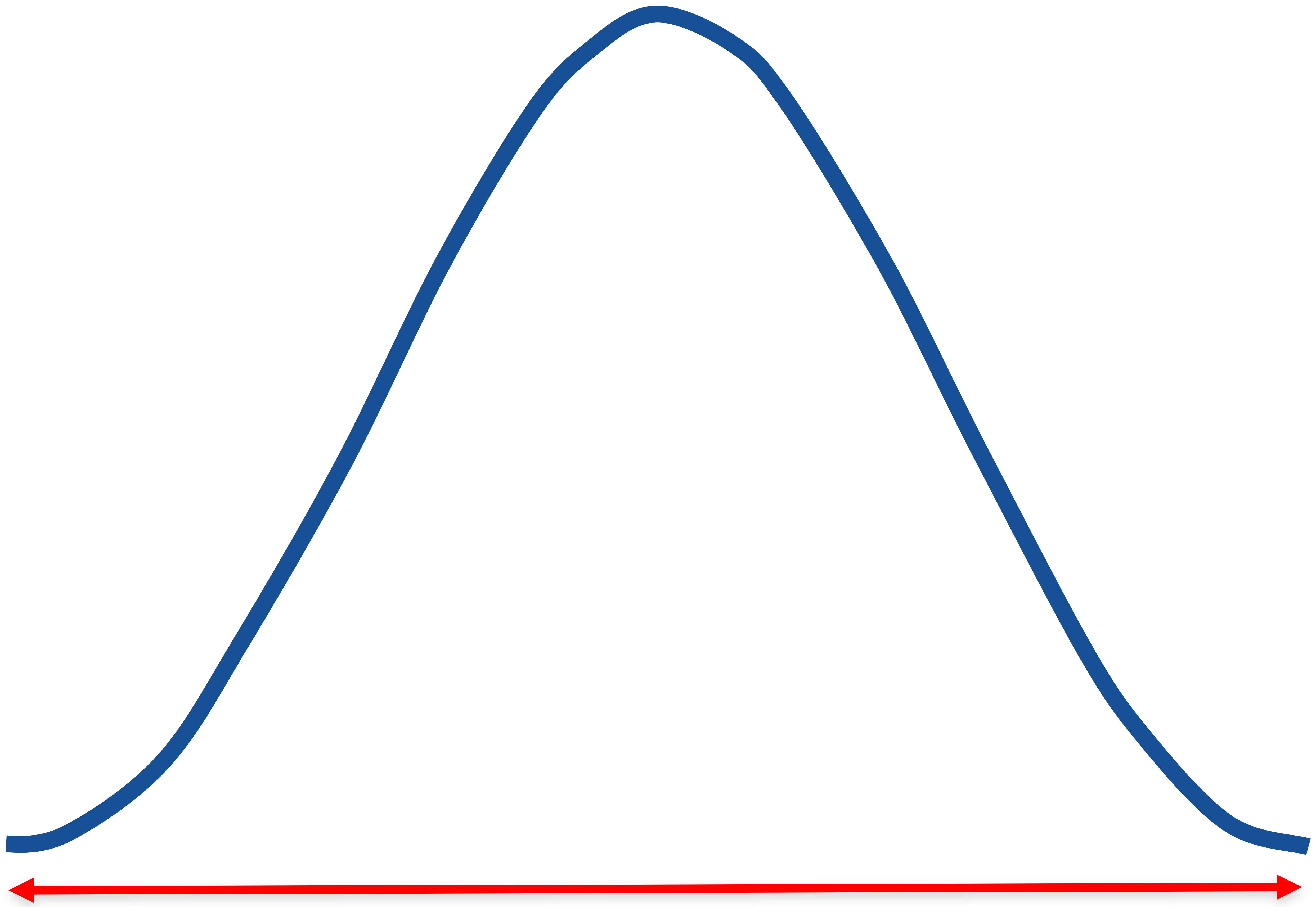
Capability

Cp, Cpk

Cp: $$\frac{USL - LSL}{6s}$$

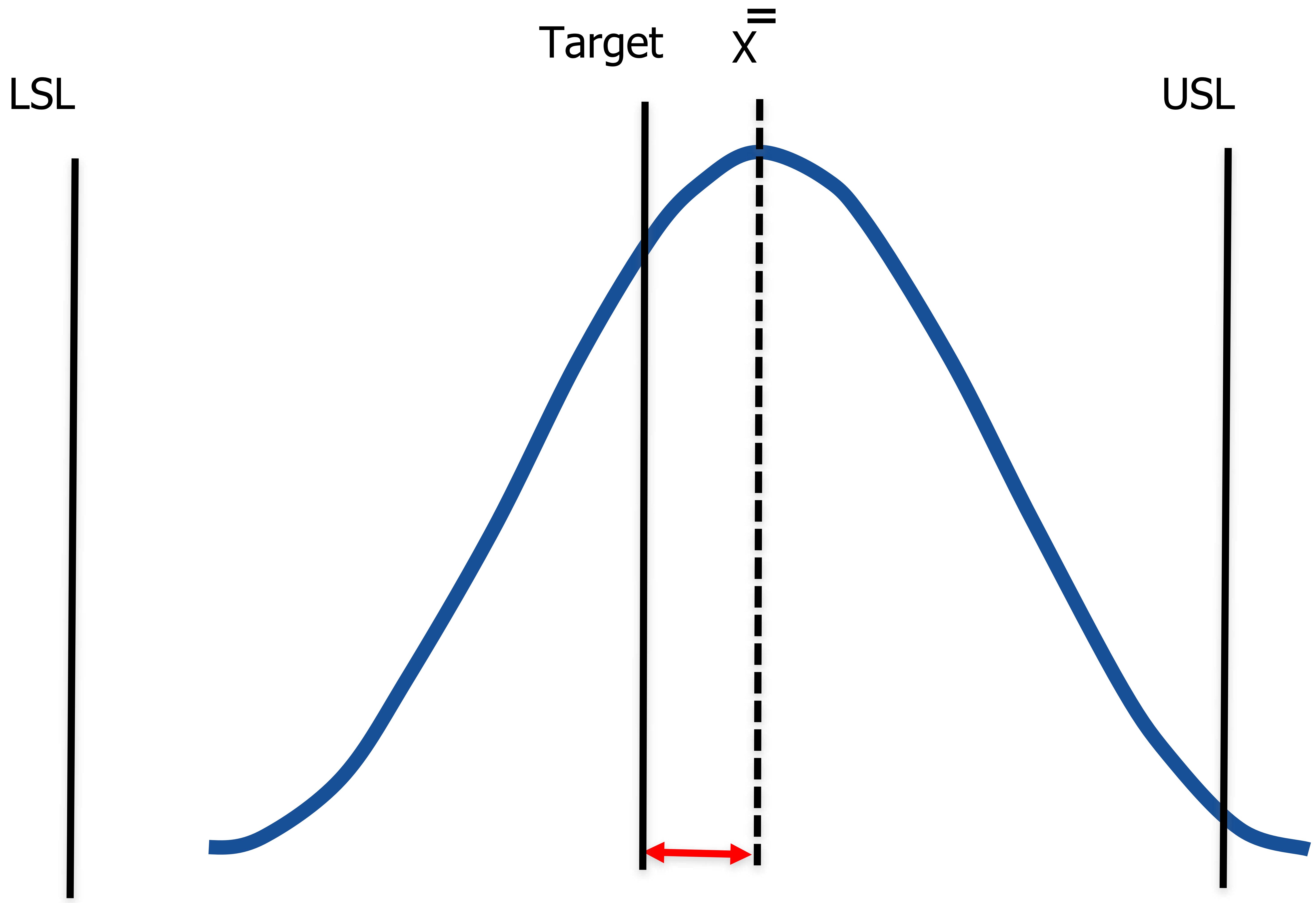
Cpk: $$\frac{USL - \bar{\bar{X}}}{3s} \quad \text{Or} \quad \frac{\bar{\bar{X}} - LSL}{3s}$$

Cp (Process Capability) / Pp(Process Performance?



Process Variation

Cpk (Process Capability Index) / Ppk (Process Performance Index)?



Position of Process (Shift)

Can Cpk be Greater than Cp ?

No

Acceptance Criteria of Cp/Pp & Cpk/Ppk

- Cp / Pp: 1.67 minimum
- Cpk / Ppk: 1.67 Minimum

Six sigma converter

Without shift		
Sigma(σ) level to PPM/DPMO & Cp		
Sigma(σ) level	PPM/DPMO	Cp
6	3	2.00

PPM/DPMO to Sigma(σ) level & Cp		
PPM	Sigma(σ) level	Cp
233	5.000	1.67

Cp to Sigma(σ) level & PPM/DPMO		
Cp	Sigma(σ) level	PPM
1.6667	5.000	233

With 1.5σ shift		
PPM/DPMO to Sigma(σ) level & Pp		
Sigma(σ) level	PPM/DPMO	Pp
5.5	0.0190	1.83

PPM/DPMO to Sigma(σ) level & Pp		
PPM/DPMO	Sigma level	Pp
3.4	4.500	1.50

Pp to Sigma(σ) level & PPM/DPMO		
Pp	Sigma(σ) level	PPM/DPMO
1.67	5.010	0.272

Enter in these cells only

Cp is 2.00 & Cpk is 0.67

What does it indicate & what should be done?

1

- Cp = 2 means process variation is less
- Therefore process can achieve desired results

2

- Cpk=0.67 which means process is shifted
- This indicates that it is shifted from target

3

- So, conclusion is process can achieve desired results since variation is less.
- But process needs to shifted towards target, so that required results can be achieved (Cpk can be improved)

What does Cp (Process Capability) / Pp(Process Performance) Represent ?

- Lots of Process Variation is present

How to improve Cp/Pp ?

- **By reducing variation in the process**
- **Brainstorm what can be the probable causes for variation**
- **Validate actual causes & eliminate the cause**

Relation between Process capability and Rejection rate

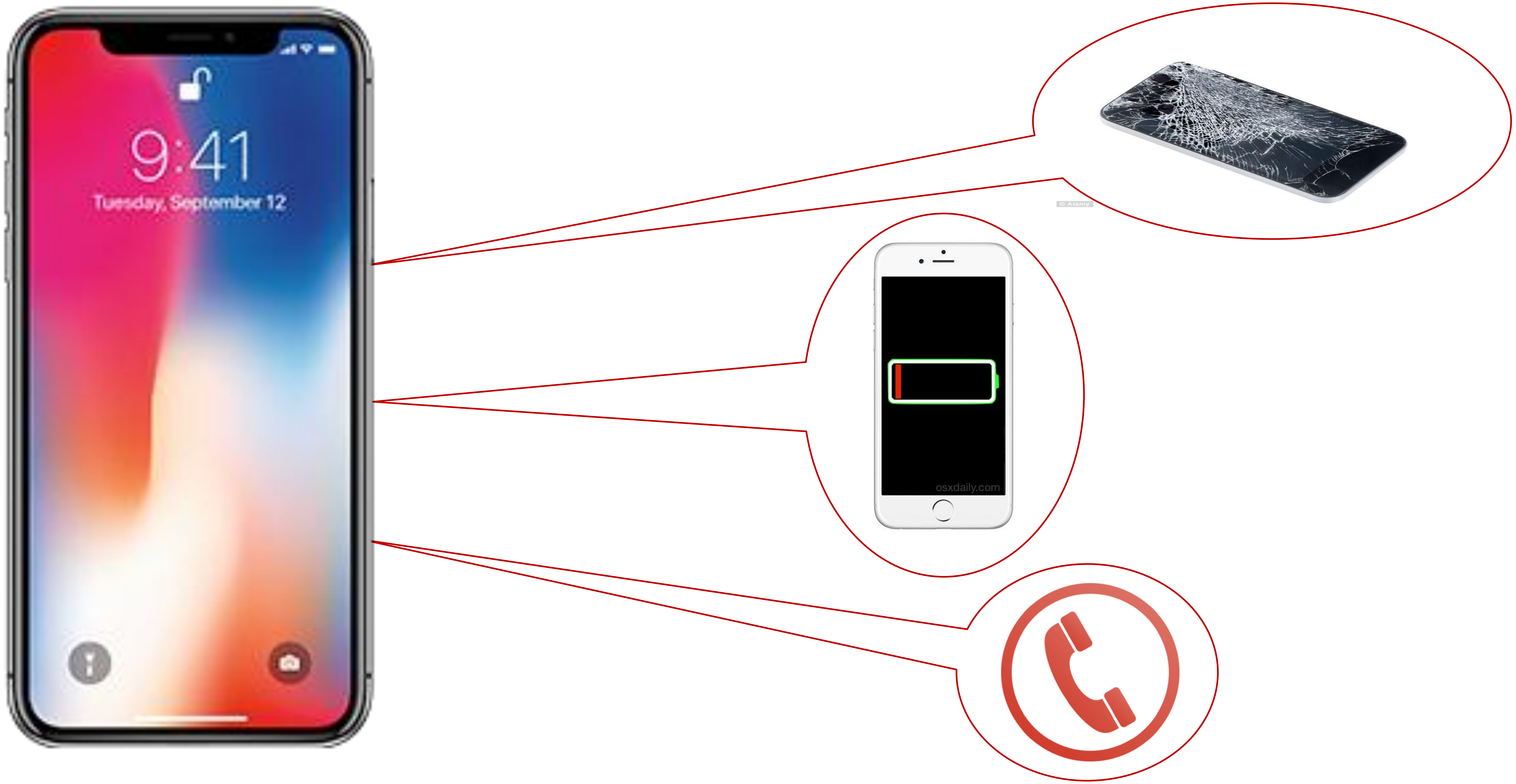
Cp	Rejection rate
1.00	0.270%
1.33	0.007%
1.50	6.8 PPM
2.00	2.0 PPB

Attribute control charts

Defect Vs Defective

Defective

Defect



Use of Attribute Control Charts

	Chart for varying sample size	*Chart for constant sample size*
Chart for no. of non-conforming	p	np
Chart for no. of non-conformities	u	c

For Defectives

P Chart

(When subgroup size varies)

- Subgroup size > 50
- Most sensitive attribute control chart

Formulae

- **p = Defectives found in subgroup / total nos. inspected in subgroup**
- **P bar = Total defectives found / total nos inspected**

$$\bar{p} = \frac{\text{Total non-conforming units}}{\text{Total units inspected}}$$

P Chart

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$LCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Population will be a binomial distribution, but applying the Central Limit Theorem allows us to assume a normal distribution for the sample statistics

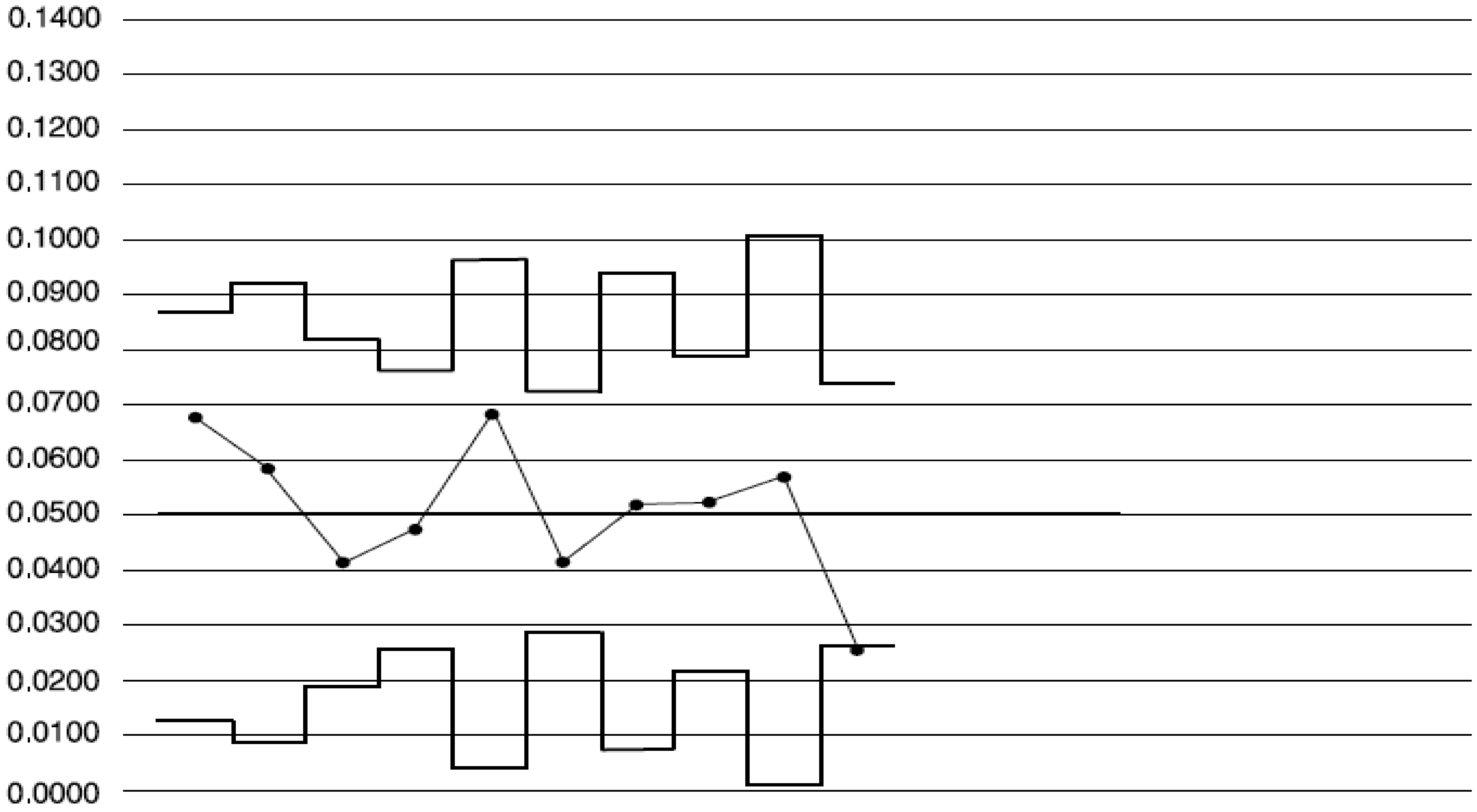
p Chart calculations

Sr. No.	Inspected / lot Qty.	Rejection Qty.	Proportion
1	100	2	0.02
2	82	3	0.0365
3	56	5	0.089
4	75	2	0.027
5	87	1	0.011
n (Average)	80	**p bar =**	0.0325

Sample	UCL	LCL
1	0.08742	0.01396
2	0.09198	0.00940
3	0.08224	0.01914
4	0.07602	0.02536
5	0.09699	0.00439
6	0.07333	0.02805
7	0.09400	0.00739
8	0.07973	0.02165
9	0.10019	0.00155
10	0.07418	0.02720

Part: *AR2456* Inspected by: *KLG* Date: *03-30-92*

# Checked	321	254	435	675	202	845	231	513	785									
# Defective	22	15	18	32	14	35	12	27	20									



np Chart

(When subgroup size is same)

Formulae

$$\overline{np} = \frac{\text{Total non-conforming units}}{\text{Total units inspected}}$$

Step 2: Calculate the center line value, $n\bar{p}$.
The formula is $(n * \bar{p})$.

For our example, $n = 75$ and $\bar{p} = 0.32267$

$$n\bar{p} = 24.20$$

Setting Control Limits

np Chart

$$UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}\,(1 - \bar{p})} \qquad LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}\,(1 - \bar{p})}$$

np Chart calculations

Sr. No.	Inspected / lot Qty.	Rejection Qty.	Proportion
1	100	2	0.02
2	100	3	0.03
3	100	5	0.05
4	100	2	0.02
5	100	1	0.01
n (Constant)	100	**p bar =**	0.026
		n pbar = pbar * n	2.6

For Defects

c Chart

(When subgroup size is same)

C chart

$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$

c Chart calculations

Sr. No.	Inspected / lot Qty.	Total defects found	c
1	100	2	0.02
2	100	3	0.03
3	100	5	0.05
4	100	2	0.02
5	100	1	0.01
n (Constant)	100	**c bar =**	0.026

u chart

$$UCL_u = \bar{u} + 3\sqrt{\bar{u} / n}$$

$$LCL_u = \bar{u} - 3\sqrt{\bar{u} / n}$$

$$\bar{u} = \frac{\text{Total number of defects}}{\text{Total inspection units}}$$

u Chart calculations

Sr. No.	Inspected / lot Qty.	No. of defects	u
1	100	2	0.02
2	82	3	0.0365
3	56	5	0.089
4	75	2	0.027
5	87	1	0.011
n (Average)	80	**u bar =**	0.037

Sample	UCL	LCL
1	3.10	1.49
2	3.43	1.16
3	3.20	1.39
4	3.34	1.25
5	3.56	1.03
6	3.02	1.57
7	3.15	1.44
8	3.11	1.48
9	3.43	1.16
10	3.29	1.30
11	3.05	1.54
12	3.24	1.35
13	3.20	1.39
14	3.07	1.52
15	3.37	1.22

Other type of Control Charts

Probability Based Charts

- Stoplight Control (Also known as traffic light control
- Pre-Control

Short Run Control Charts

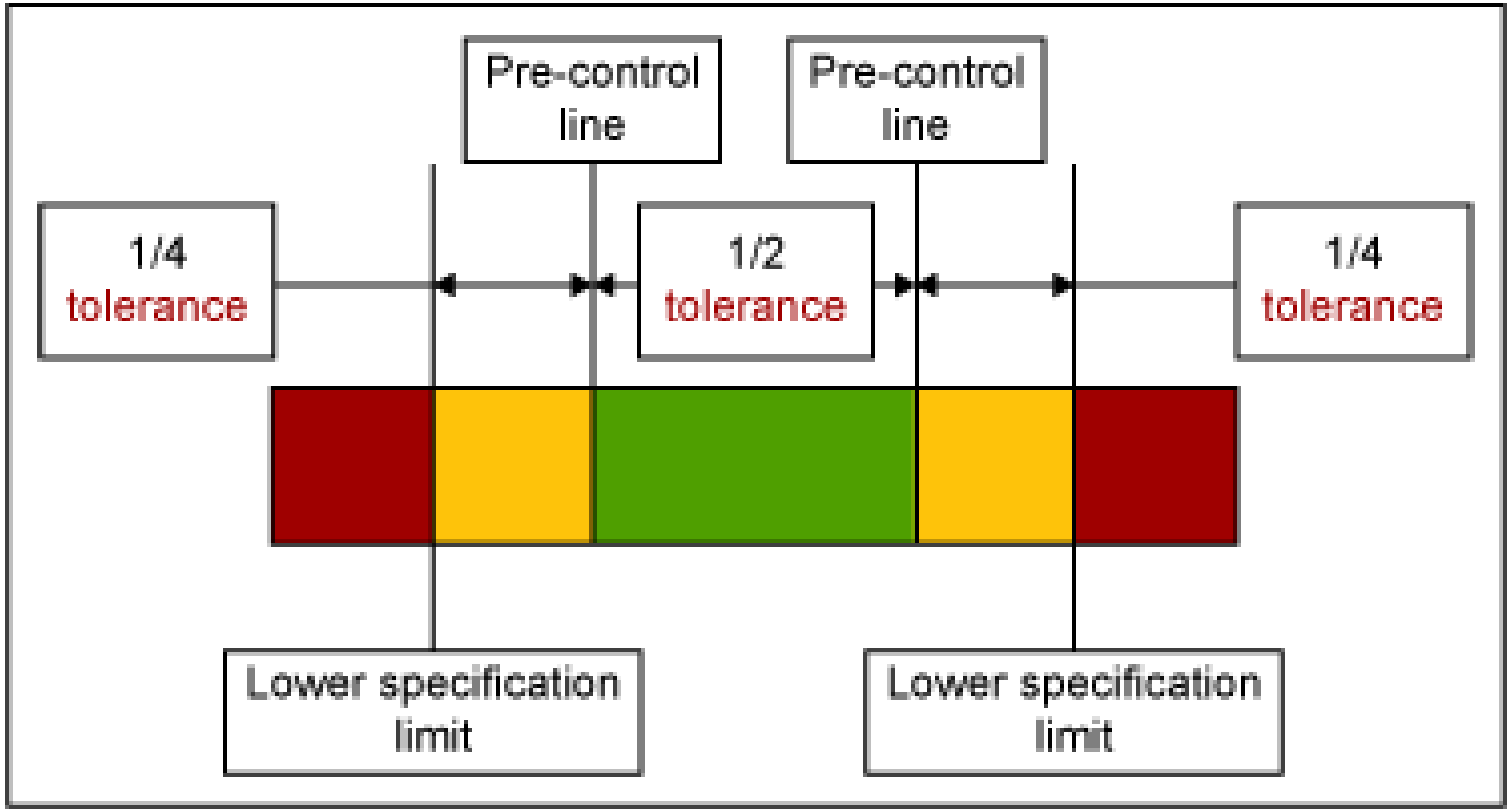
- Deviation from Nominal
- Standardized X-bar and R Charts
- Standardized Attributes Control Charts

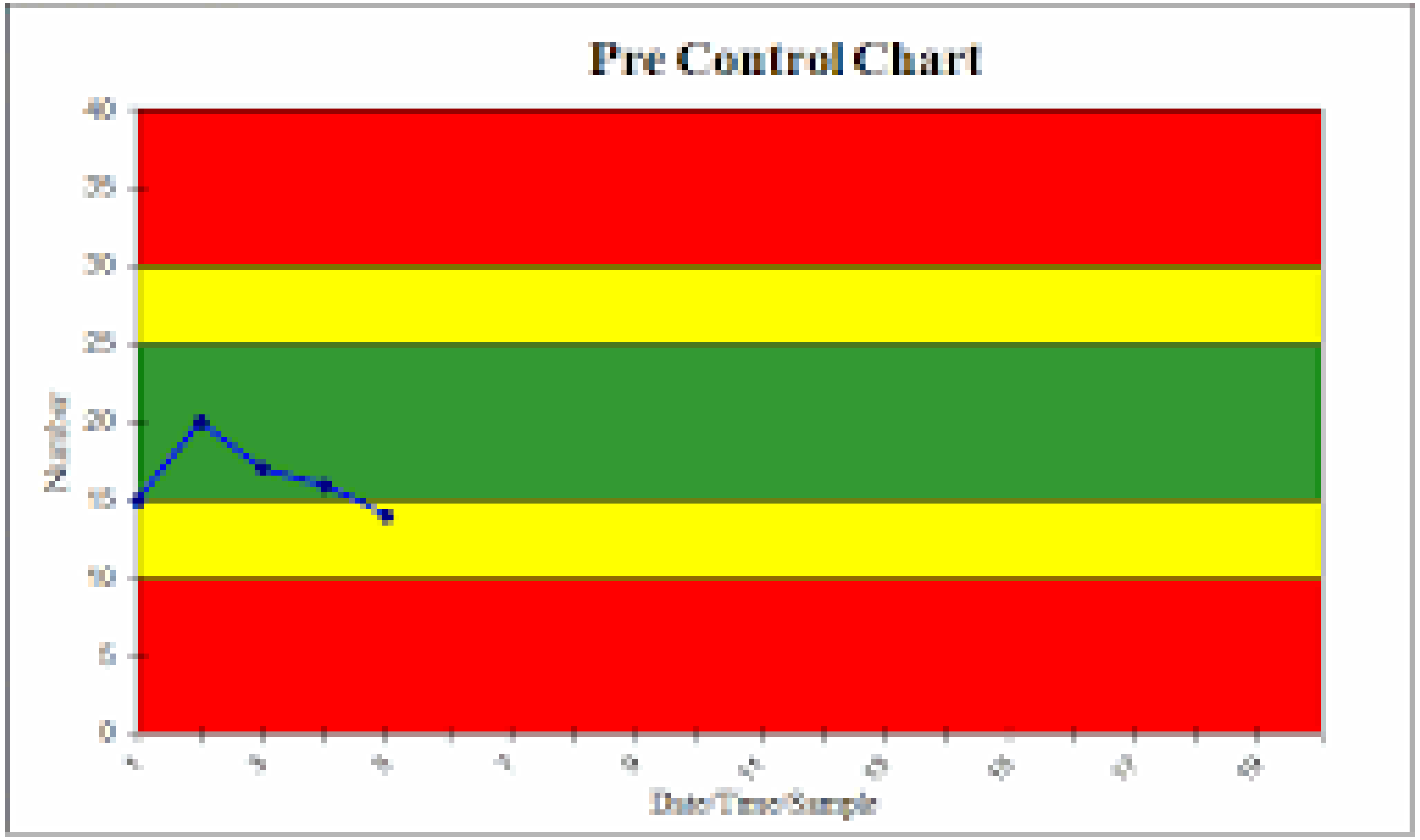
Charts for Detecting Small Changes

- CUSUM Chart (Cumulative Sum)
- EWMA Chart (Exponentially Weighted Moving Average)

Pre-Control

- Pre-Control is a technique that sets the limits of the red zones at the upper and lower specifications.




Pre Control Chart
40
35
30
25
20
15
10
5
0
Number
Date/Time/Sample

Control Charts Formulas

Type of Chart	LCL	CL	UCL
$\bar{x}$ (with R)	$\bar{\bar{x}} - A_2\bar{R}$	$\bar{\bar{x}}$	$\bar{\bar{x}} + A_2\bar{R}$
R	$D_3\bar{R}$	$\bar{R}$	$D_4\bar{R}$
p	$\bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n}$	$\bar{p}$	$\bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n}$
$\bar{x}$ (with s)	$\bar{\bar{x}} - A_3\bar{s}$	$\bar{\bar{x}}$	$\bar{\bar{x}} + A_3\bar{s}$
s	$B_3\bar{s}$	$\bar{s}$	$B_4\bar{s}$
x	$\bar{x} - 3\bar{R}/d_2$	$\bar{x}$	$\bar{x} + 3\bar{R}/d_2$
np	$n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$	$n\bar{p}$	$n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$
c	$\bar{c} - 3\sqrt{\bar{c}}$	$\bar{c}$	$\bar{c} + 3\sqrt{\bar{c}}$
u	$\bar{u} - 3\sqrt{\bar{u}/n}$	$\bar{u}$	$\bar{u} + 3\sqrt{\bar{u}/n}$

SPC Formulae

p chart

$$\bar{p} = \frac{\text{Total number of non-conforming units}}{\text{Total number of units inspected}}$$

$$n = \text{number of units inspected per sample}$$

$$UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}\ (1-\bar{p}\ )}{n}} \qquad LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}\ (1-\bar{p}\ )}{n}}$$

Use the p chart with either constant or changing sample sizes. When n changes, UCL_p and LCL_p must be recalculated. LCL_p must be equal to or greater than 0; LCL_p can never be less than 0.

np chart

$$n\bar{p} = \frac{\text{Total number of non-conforming units}}{\text{Total number of units inspected}}$$

$$n\bar{p} = \text{number of units inspected per sample}$$

$$UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}\ (1-\bar{p}\ )} \qquad LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}\ (1-\bar{p}\ )}$$

Use the np chart with constant sample sizes. When n changes, UCL_{np} and LCL_{np} must be recalculated. LCL_{np} must be equal to or greater than 0; LCL_{np} can never be less than 0.

c chart

$$\bar{c} = \frac{\text{Total number of defects}}{\text{Number of samples}}$$

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}} \qquad LCL_c = \bar{c} - 3\sqrt{\bar{c}}$$

Use the c chart when the number of units inspected per sample is constant. LCL_c must be equal to or greater than 0; LCL_c can never be less than 0.

u chart

$$\bar{u} = \frac{\text{Total number of defects}}{\text{Total number of units inspected}}$$

n = number of units inspected per sample

$$UCL_u = \bar{u} + 3\sqrt{\bar{u}/n} \qquad LCL_u = \bar{u} - 3\sqrt{\bar{u}/n}$$

Use the u chart when the number of units inspected per sample is not constant. When n changes, UCL_u and LCL_u must be recalculated. LCL_u must be equal to or greater than 0; LCL_u can never be less than 0.

X-bar chart for sample averages

$\overline{X}$ = average for each sample

$$UCL_x = \overline{\overline{X}} + A_2 * \overline{R}$$

$$LCL_x = \overline{\overline{X}} - A_2 * \overline{R}$$

$$\overline{\overline{X}} = \frac{\text{Sum of } \overline{X} \text{ values for all subgroups}}{\text{Number of subgroups}}$$

$$UCL_x = \overline{\overline{X}} + A_3 * \bar{s}$$

$$LCL_x = \overline{\overline{X}} - A_3 * \bar{s}$$

R chart for sample ranges

R = range for each subgroup

$$\overline{R} = \frac{\text{Sum of ranges for all subgroups}}{\text{Number of subgroups}}$$

$$UCL_R = D_4 * \overline{R} \qquad LCL_R = D_3 * \overline{R}$$

S chart for sample sigmas

S = Sigma for each subgroup

$$\bar{s} = \frac{\text{Sum of sigmas for all subgroups}}{\text{Number of subgroups}}$$

$$UCL_s = B_4 * \bar{s} \qquad LCL_s = B_3 * \bar{s}$$

Capability & performance indices

Cp/Pp

$$Cp/Pp = \frac{\text{Specification width}}{\text{Natural tolerance}}$$

$$Cp = \frac{USL - LSL}{6\sigma_{est}}$$

$$Pp = \frac{USL - LSL}{6\sigma_{calc}}$$

$$\sigma_{est} = \bar{R}/d_2$$

Pc/Pr

$$Pc/Pr = \frac{\text{Natural tolerance}}{\text{Specification width}}$$

$$Pc = \frac{6\sigma_{est}}{USL - LSL}$$

$$Pr = \frac{6\sigma_{calc}}{USL - LSL}$$

$$\sigma_{calc} = \sqrt{\frac{\Sigma(X - \bar{\bar{X}})^2}{n}}$$

Cpm

$$\sigma_{Cpm} = \sqrt{\frac{\Sigma(X - Nom)}{n - 1}}$$

$$Cpm = \frac{USL - LSL}{6*\sigma_{Cpm}}$$

Cpk

$$Z_U = \frac{USL - \bar{\bar{X}}}{\sigma_{est}} \qquad Z_L = \frac{\bar{\bar{X}} - LSL}{\sigma_{est}}$$

$$Cpk = \text{Minimum of } Z_U / 3 \text{ or } Z_L / 3$$

Ppk

$$Z_U = \frac{USL - \bar{\bar{X}}}{\sigma_{calc}} \qquad Z_L = \frac{\bar{\bar{X}} - LSL}{\sigma_{calc}}$$

$$Ppk = \text{Minimum of } Z_U / 3 \text{ or } Z_L / 3$$

Median & R chart formulae

$\tilde{X}$ = median for each sample

$\tilde{\tilde{X}}$ = median of $\tilde{X}$ values for all subgroups

$UCL_x = \tilde{X} + A_5\tilde{R}$

$LCL_x = \tilde{X} - A_5\tilde{R}$

R = range for each subgroup

$\tilde{R}$ = median of ranges for subgroups

$UCL_R = D_4\tilde{R}$

$LCL_R = D_3\tilde{R}$

3 chart formulae

Subgroup-to-subgroup value of X-bar

$$UCL = \bar{\bar{X}} + E_2\overline{MR}$$

$$LCL \quad \bar{\bar{X}} - E_2\overline{MR}$$

Subgroup-to-subgroup value of MR

$$UCL = D_4\overline{MR}$$

$$LCL = D_3\overline{MR}$$

Within subgroup values of R

$$UCL = D_4\bar{R}$$

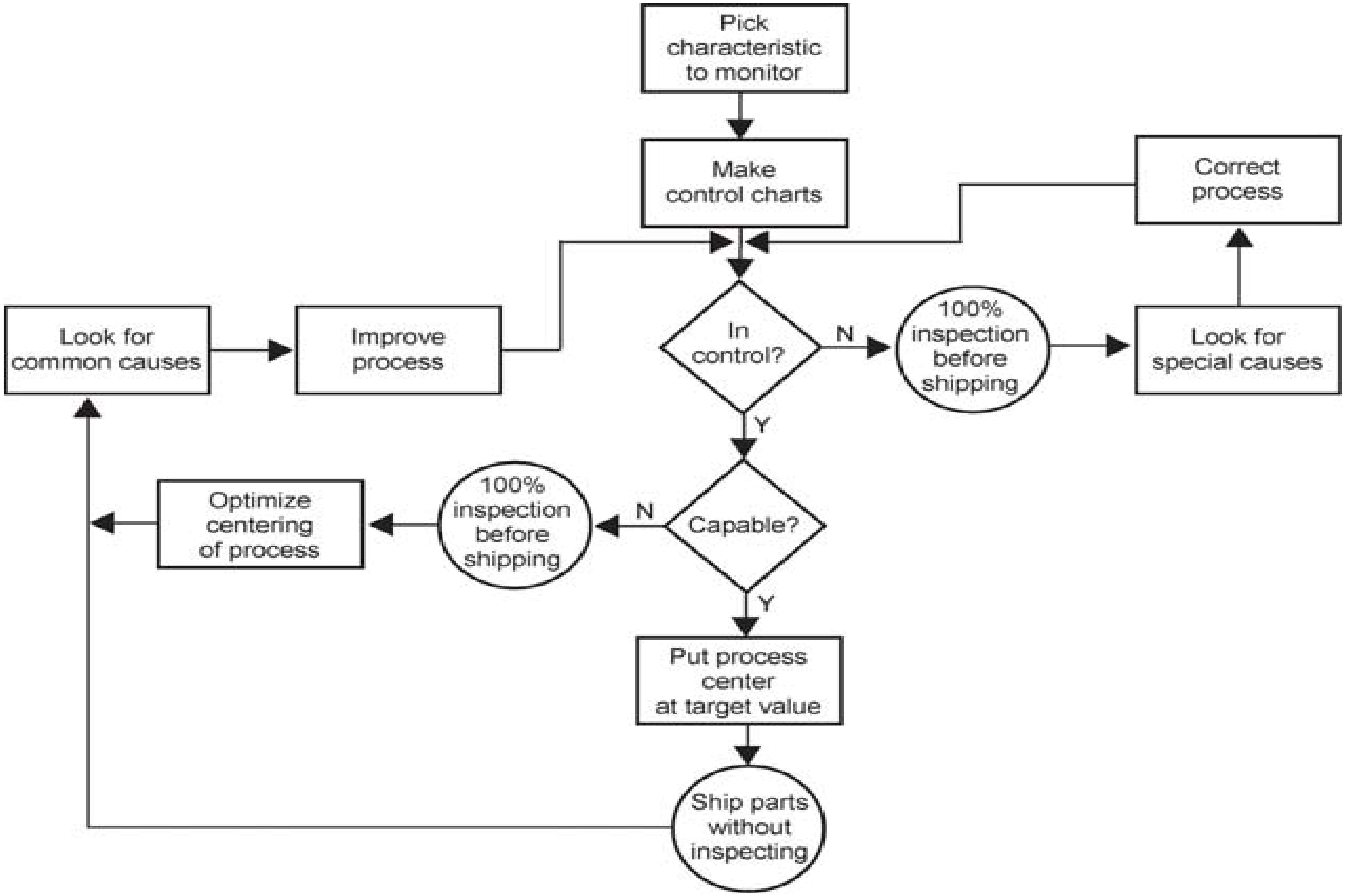
$$LCL = D_3\bar{R}$$

Factors for 3 charts

Where k refers to the size of the moving range and n refers to the subgroup size of individuals:

$\bar{\bar{X}}$	
k	E_2
2	2.66
3	1.77
4	1.46
5	1.29

MR & R			
n	D_4	D_3	d_2
2	3.267	0	1.128
3	2.575	0	1.693
4	2.284	0	2.059
5	2.114	0	2.326


Pick characteristic to monitor
Make control charts
Correct process
Look for common causes
Improve process
In control?
N
100% inspection before shipping
Look for special causes
Y
Optimize centering of process
100% inspection before shipping
N
Capable?
Y
Put process center at target value
Ship parts without inspecting

Thank you