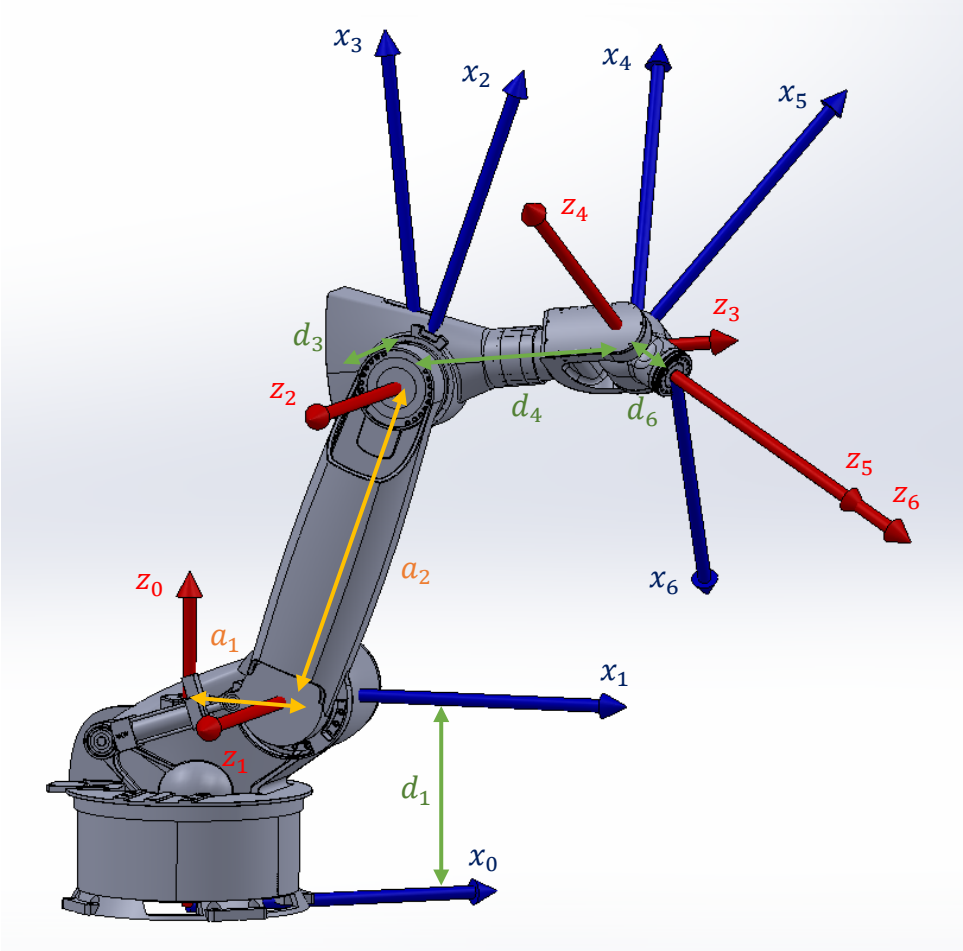
6 DOF robot




Denavit–Hartenberg Parameters

| Link | $\theta_i\ (z_{i-1})$ | $\alpha_i\ (x_i)$ | $d_i\ (z_{i-1})$ | $a_i\ (x_i)$ |
|---|---|---|---|---|
| 1 | $\theta_1$ | $\pi/2$ | $d_1$ | $a_1$ |
| 2 | $\theta_2$ | 0 | 0 | $a_2$ |
| 3 | $\theta_3$ | $\pi/2$ | $-d_3$ | 0 |
| 4 | $\theta_4$ | $-\pi/2$ | $d_4$ | 0 |
| 5 | $\theta_5$ | $\pi/2$ | 0 | 0 |
| 6 | $\theta_6$ | 0 | $d_6$ | 0 |

Transformation matrix

$$T_{i-1}^{i} = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$T_0^1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & a_1\cos\theta_1 \\ \sin\theta_1 & 0 & -\cos\theta_1 & a_1\sin\theta_1 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$T_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$T_2^3 = \begin{bmatrix} \cos\theta_3 & 0 & \sin\theta_3 & 0 \\ \sin\theta_3 & 0 & -\cos\theta_3 & 0 \\ 0 & 1 & 0 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$T_3^4 = \begin{bmatrix} \cos\theta_4 & 0 & -\sin\theta_4 & 0 \\ \sin\theta_4 & 0 & \cos\theta_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$T_4^5 = \begin{bmatrix} \cos\theta_5 & 0 & \sin\theta_5 & 0 \\ \sin\theta_5 & 0 & -\cos\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

$$T_5^6 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$T_0^6 = T_0^1 T_1^2 T_2^3 T_3^4 T_4^5 T_5^6 \quad (8)$$

End effector position

$$p_x = a1 * cos(theta1(t)) - d3 * sin(theta1(t)) + (a2 * cos(theta1(t) + theta2(t)))/2 + d4 * (sin(theta1(t) + theta2(t) + theta3(t))/2 + sin(theta2(t) - theta1(t) + theta3(t))/2) + (a2 * cos(theta1(t) - theta2(t)))/2 + d6 * (cos(theta5(t)) * (sin(theta1(t) + theta2(t) + theta3(t))/2 + sin(theta2(t) - theta1(t) + theta3(t))/2) + sin(theta5(t)) * (cos(theta1(t) - theta4(t))/2 - cos(theta1(t) + theta4(t))/2 + cos(theta4(t)) * (cos(theta1(t) + theta2(t) + theta3(t))/2 + cos(theta2(t) - theta1(t) + theta3(t))/2))) \quad (9)$$

$$p_y = d3 * cos(theta1(t)) - d6 * (cos(theta5(t)) * (cos(theta1(t) + theta2(t) + theta3(t))/2 - cos(theta2(t) - theta1(t) + theta3(t))/2) - sin(theta5(t)) * (sin(theta1(t) - theta4(t))/2 - sin(theta1(t) + theta4(t))/2 + cos(theta4(t)) * (sin(theta1(t) + theta2(t) + theta3(t))/2 - sin(theta2(t) - theta1(t) + theta3(t))/2))) + a1 * sin(theta1(t)) - d4 * (cos(theta1(t) + theta2(t) + theta3(t))/2 - cos(theta2(t) - theta1(t) + theta3(t))/2) + (a2 * sin(theta1(t) + theta2(t)))/2 + (a2 * sin(theta1(t) - theta2(t)))/2 \quad (10)$$

$$p_z = d1 - d6 * ((sin(theta4(t) - theta5(t)) * sin(theta2(t) + theta3(t)))/2 + cos(theta5(t)) * cos(theta2(t) + theta3(t)) - (sin(theta2(t) + theta3(t)) *$$

$$sin(theta4(t) + theta5(t)))/2) + a2 * sin(theta2(t)) - d4 * cos(theta2(t) + theta3(t)) \quad (11)$$

Torques of joints

$$\tau = J^T F \quad (12)$$

$$\begin{Bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{Bmatrix} = J^T \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \quad (13)$$