BUFFON`S NEEDLE

Name: Seyed Mohammad Amin Sadat

Student number: 401436077

Course: Advanced Mathematics

For this problem the needle length is l,x is the minimum distance between lines and middle of the needles, θ is the angle between lines and needle and d is the distance between two parallel lines.

For this problem we have:

$$0 \leq x \leq \frac{d}{2}$$

$$0 \leq \theta \leq \frac{\pi}{2}$$

The distribution is normal so we have:

$$p(x) = \frac{1}{b-a} \quad for \quad a \leq x \leq b$$

So we have:

$$p_x(x) = \begin{cases} \frac{2}{d} & : 0 \leq x \leq \frac{2}{d} \\ 0 & : elsewhere \end{cases}$$
$$p_\theta(x) = \begin{cases} \frac{2}{\pi} & : 0 \leq \theta \leq \frac{\pi}{2} \\ 0 & : elsewhere \end{cases}$$

The random variables x and θ are independent so the joint probability density function is the product of two functions:

$$p_{x,\theta}(x) = \begin{cases} \frac{4}{\pi d} & : 0 \leq \theta \leq \frac{\pi}{2}, 0 \leq x \leq \frac{2}{d} \\ 0 & : elsewhere \end{cases}$$

The needles we intersect the line if:

$$x \leq \frac{l}{2} \sin(\theta)$$

CASE 1: SHORT NEEDLES $(L \leq D)$

By integrating the joint probability density function over the specified limits we have:

$$Pr = \int_{\theta=0}^{\frac{pi}{2}} \int_{x=0}^{\frac{d}{2}\sin(\theta)} \frac{4}{\pi d} dx d\theta = \frac{2l}{d\pi}$$

CASE 2 : LONG NEEDLES $(L > D)$

Let f(x) be:

$$f(x) = min(\frac{l}{2}\sin(\theta), \frac{d}{2})$$

Then we have:

$$Pr = \int_{\theta=0}^{\frac{pi}{2}} \int_{x=0}^{f(x)} \frac{4}{\pi d} dx d\theta$$

By integrating this equation we have the probability as:

$$Pr = \frac{2}{\pi} \left\{ \cos^{-1}(\frac{d}{l}) + \frac{l}{d} \left\{ 1 - \sqrt{1 - \frac{d^2}{l}} \right. \right.$$

Estimating π

1. Count the intersections and calculate the probability
2. find the estimate for π with the formulas for the two cases