BUFFON'S NEEDLE

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Course: Advanced Mathematics

For this problem the needle length is I,x is the minimum distance between lines and middle of the needles, θ is the angle between lines and needle and d is the distance between two parallel lines.

For this problem we have:

$$0 \le x \le rac{d}{2}$$

$$0 \le \theta \le \frac{\pi}{2}$$

The distribution is normal so we have:

$$p(x) = \frac{1}{b-a}$$
 for $a \le x \le b$

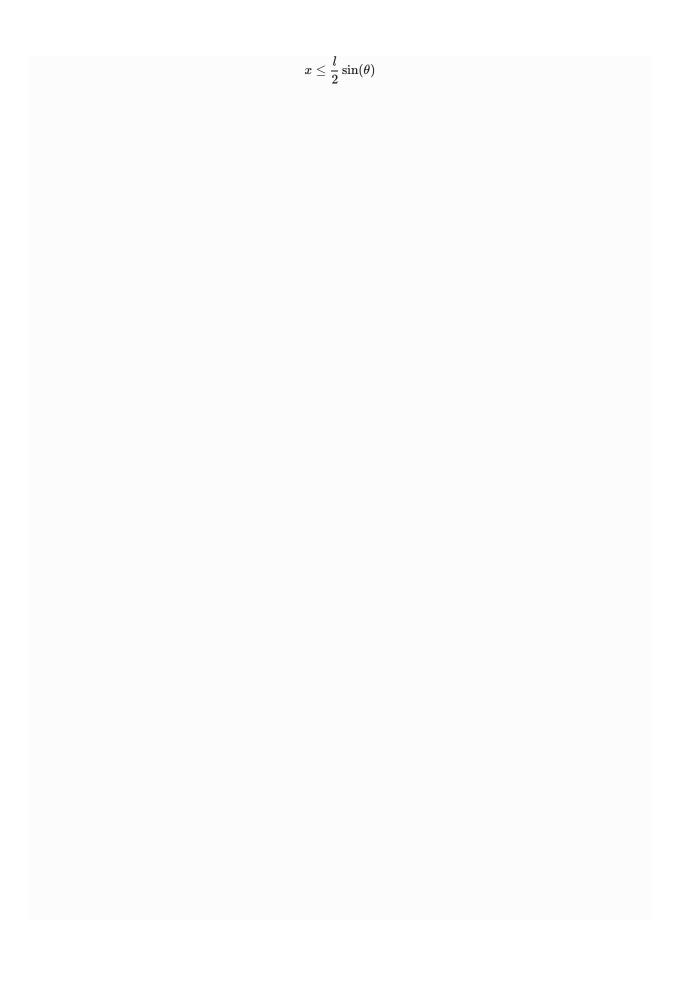
So we have:

$$p_x(x) = egin{cases} rac{2}{d} & :0 \leq x \leq rac{2}{d} \ 0 & :elsewhere \ p_ heta(x) = egin{cases} rac{2}{\pi} & :0 \leq heta \leq rac{\pi}{2} \ 0 & :elsewhere \end{cases}$$

The random variables x and θ are independent so the joint probability density function is the product of two functions:

$$p_{x, heta}(x) = egin{cases} rac{4}{\pi d} &: 0 \leq heta \leq rac{\pi}{2}, 0 \leq x \leq rac{2}{d} \ 0 &: elsewhere \end{cases}$$

The needles we intersect the line if:



CASE 1: SHORT NEEDLES $(L \leq D)$

By integrating the joint probability density function over the specified limits we have:

$$Pr=\int_{ heta=0}^{rac{pi}{2}}\int_{x=0}^{rac{d}{2}\sin(heta)}rac{4}{\pi d}dxd heta=rac{2l}{d\pi}$$

CASE 2 : LONG NEEDLES (L>D)

Let f(x) be:

$$f(x) = min(rac{l}{2}\sin(heta),rac{d}{2})$$

Then we have:

$$Pr = \int_{\theta=0}^{\frac{pi}{2}} \int_{x=0}^{f(x)} \frac{4}{\pi d} dx d\theta$$

By integrating this equation we have the probability as:

$$Pr = rac{2}{\pi} \left\{ \cos^{-1}(rac{d}{l}) + rac{l}{d} \left\{ 1 - \sqrt{1 - rac{d^2}{l}}
ight.
ight.$$

Estimating π

- 1. Count the intersections and calculate the probability
- 2. find the estimate for π with the formulas for the two cases