یالطیف

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تمرین سری ۶ آمار و احتمال مهندسی

#1

$$2 \to \tfrac{1}{3} \quad 4 \to \tfrac{1}{3} \quad 6 \to \tfrac{1}{3}$$

$$\mu_{(x)} = \sum x_i \cdot P_X(x) = \frac{1}{3}(2) + \frac{1}{3}(4) + \frac{1}{3} \times 6 = \frac{11}{3}$$

$$E(x) = 3.6$$

$$E(x^2) = \sum x_i^2 \cdot P_X(x) = \frac{1}{3} \times 4 + \frac{1}{3} \times 16 + \frac{1}{3} \times 36 = 18.6$$

$$\Rightarrow Var(x) = E(x^2) - E(x)^2 = 18.6 - \left(\frac{11}{3}\right)^2 = \underline{5.64}$$

#2

$$\begin{cases} n = 70 \\ X \sim (\mu, \frac{10}{6}) \end{cases}$$

$$P(x < 45.6) = 0.33 \Rightarrow \frac{45.6 - \mu}{10} = 0.44$$

$$\Rightarrow \mu = 41.2$$

$$P(x > 75) = 1 - P(x \leq 75) = 1 - \left(P\left(\frac{X-\mu}{\varphi} \leq \frac{75 - 41.2}{10}\right)\right) = 0.0087 = 0.87\%$$

$$\begin{cases} X : \text{نمره درس} \\ y : \text{نمره پرسشده} \end{cases}$$

$$y \sim N(15, 2), \quad P(x+y > 70) = 1 - P(x+y \leq 70) = 1 - P\left(\frac{x+y-(15+41.2)}{10+2} \leq \frac{70 - 56.2}{\frac{12}{1.15}}\right)$$

$$\Rightarrow 1 - \varphi(1.15) = 1 - 0.8749 = 0.1251 = 12.51\%$$

که قبول می شوند

#3 $f(t) = \begin{cases} 2e^{-2t} & ; \ t \geqslant 0 \\ 0 & ; \ \text{other} \end{cases}$

$$\frac{1}{\lambda} = E(t) = \int_0^\infty t\, f(t)\, dt = \int_0^\infty t \times 2e^{-2t}\, dt$$

$$= \frac{-(2t+1)e^{-2t}}{2} \Big|_0^\infty = \frac{1}{2} \qquad t \sim \exp(\lambda = 2)$$

$\lambda = 4$, $f(x) = 4e^{-4x}$; $x \geqslant 0$

$$P(X \leq 8) = \int_0^8 e^{-4x}\, dx = -e^{-4t} \Big|_0^8$$
$$= e^{-32}(e^{32} - 1) = 0.9$$

#4 $X \sim$ Passion (λ) , $\lambda = 1$, $P(x > 10) = 1 - P(x < 10)$

$$= 1 - \sum_{x=5}^{10} e^{-7} \cdot \frac{7^x}{x!}$$

$$P(x > 2) = 1 - P(x \leq 2) = 1 - P(0) + P(1) + P(2)$$

$$\Rightarrow 1 - \left[\left(\frac{e^{-1} \times 1^0}{0!} \right) + \left(\frac{e^{-1} \times 1^1}{1!} \right) + \left(\frac{e^{-1} \cdot 1^2}{2!} \right) \right]$$

#5 $\begin{cases} \text{کمتر از 25 دقیقه جواب می دهد} : 0.7 \\ \text{در 25 دقیقه جواب می دهد} : 0.3 \end{cases}$

$x \sim bin(12, 0.3)$

$$P(x=2) = \binom{12}{2} \cdot (0.3)^2 \cdot (0.7)^{10} = 0.1677$$

#6 $\begin{cases} P(\text{برخورد}) = 0.8 \\ P(\text{عدم برخورد}) = 0.2 \end{cases}$

$$P(x < 5) = P(x < 4) = \sum_{x=0}^{4} 0.8(0.2)^x$$
$$= 0.8(1 + 0.2 + 0.04 + 0.008 + 0.0016)$$

$X \sim Geo(0.8) \Rightarrow f(x) = 0.8(0.2)^x$; $x = \{0, 1, 2, 3, \dots\}$

$P(x < 5) = 0.99328$, $P(x \geqslant 5) = 1 - P(X < 5) = 1 - 0.2048 = 0.7954$

$\begin{cases} n = 5 \\ k = 3 \end{cases} \Rightarrow x \sim bin(5, 0.8)$, $P(x) = \binom{5}{3} \times (0.8)^3 \cdot (0.2)^2 = 0.2048$