-۱

a) $\int_0^{+\infty} e^{-at} e^{-(\sigma+j\omega)t}\,dt$ مطلقا انتگرال پذیر

$$\int_0^{\infty} |e^{-at} e^{-(\sigma+j\omega)t}|\,dt<\infty \to \int_0^{+\infty} e^{-at} e^{-\sigma t}\,dt = \int_0^{\infty} e^{-(a+\sigma)t}\,dt = \frac{e^{-(a+\sigma)t}}{-a-\sigma}\Big|_0^{+\infty}$$

$$\to \frac{1}{a+\sigma}<\infty \Rightarrow a+\sigma>0 \quad \sigma>-a$$

b) $\int_{-\infty}^{+\infty} e^{-a|t|} e^{-(\sigma+j\omega)t}\,dt$ مطلقا انتگرال پذیر

$$\int_{-\infty}^{+\infty} |e^{-a|t|} e^{-(\sigma+j\omega)t}|\,dt<\infty \to \int_{-\infty}^{+\infty} e^{-a|t|} e^{-\sigma t}\,dt = \int_{-\infty}^{0} e^{(a-\sigma)t}\,dt + \int_0^{\infty} e^{-(a+\sigma)t}\,dt$$

$$\to \frac{e^{(a-\sigma)t}}{a-\sigma}\Big|_{-\infty}^{0} + \frac{e^{-(a-\sigma)t}}{-a-\sigma}\Big|_0^{\infty} = \frac{1}{a-\sigma} - \frac{1}{-a-\sigma} <\infty \longrightarrow -a<\sigma<a$$

-۲

a) $\frac{s+1}{s^2-1} \to$ $s=-1$ صفر

$s^2-1=0 \to s=\pm 1$ قطب ها

b) $\frac{s^2-1}{s^2+s+1} \to$ $s^2-1=0 \quad s=\pm 1$ قطب ها

$s^2+s+1=0 \quad s=\frac{-1\pm\sqrt{-3}}{2}$ صفرها

-۳

a) $\frac{1}{s^2+9} \xRightarrow{L^{-1}} x(t)=\frac{1}{3}\sin(3t)$ ($\frac{1}{3}\times 3$)

b) $\frac{s+2}{s^2+7s+12} = \frac{A}{s+3} + \frac{B}{s+4} \xrightarrow{L^{-1}} -e^{-3t}+2e^{-4t}$

c) $\frac{s^2-s+1}{(s+1)^2} = \frac{s^2-s+1}{s^2+2s+1} = 1-\frac{3s}{s^2+2s+1} = 1+\frac{A}{s+1}+\frac{B}{(s+1)^2} \xrightarrow{L^{-1}} x(t) =$

$\delta(t) - 3e^{-t} + 3te^{-t}$

$$X(s) = \frac{2(s+2)}{(s+3)(s+4)} = \frac{\overset{\to -2}{A}}{s+3} + \frac{\overset{\to 4}{B}}{s+4} \qquad Re\{s\} > -3$$ -4

$$\xrightarrow{L^{-1}} x(t) = \{-2e^{-3t} + 4e^{-4t}\}u(t)$$

$$g(t) = be^{-t} + abe^{t} \xrightarrow{L} G(s) = \frac{b}{s+1} + \frac{ab}{s-1} = \frac{bs - b + abs + ab}{s^2-1} \to b = \frac{1}{2},\ a = 1$$ -5

-6

$$X(s) = \frac{M}{(s-\alpha_1)(s-\alpha_2)(s-\alpha_3)(s-\alpha_4)} = \frac{M}{(s-\frac{1}{2}e^{j\frac{\pi}{4}})(s-\frac{1}{2}e^{-j\frac{\pi}{4}})(s+\frac{1}{2}e^{j\frac{\pi}{4}})(s+\frac{1}{2}e^{-j\frac{\pi}{4}})}$$

$$= \frac{M}{(s^2-\frac{s}{2}e^{-j\frac{\pi}{4}}-\frac{s}{2}e^{j\frac{\pi}{4}}+\frac{1}{4})(s^2+\frac{s}{2}e^{-j\frac{\pi}{4}}+\frac{s}{2}e^{j\frac{\pi}{4}}+\frac{1}{4})} = \frac{M}{\left(\frac{-s\cos\frac{\pi}{4}}{\frac{\sqrt{2}}{2}}\right)\times\left(\frac{s}{\sqrt{2}}\right)}$$

$$\int_{-\infty}^{\infty} x(t)\,dt = 4 \to X(0) = \int_{-\infty}^{\infty} x(t)e^{-st}dt\Big|_{s=0} = 4 = \frac{M}{\frac{1}{4}\cdot\frac{1}{4}} = 4 \qquad M = \frac{1}{4}$$

$$X(s) = \frac{\frac{1}{4}}{(s^2-\frac{s}{\sqrt{2}}+\frac{1}{4})(s^2+\frac{s}{\sqrt{2}}+\frac{1}{4})} = \begin{cases} \overbrace{s_1 = \frac{\sqrt{2}}{4} \pm j\frac{\sqrt{2}}{4}}^{s+\frac{\sqrt{2}}{4}>0} \\ \underbrace{s_2 = -\frac{\sqrt{2}}{4} \pm j\frac{\sqrt{2}}{4}}_{s-\frac{\sqrt{2}}{4}<0} \end{cases} \to -\frac{\sqrt{2}}{4} < Re\{s\} < \frac{\sqrt{2}}{4}$$

$$\frac{d^3y(t)}{dt^3} + (a+1)\frac{d^2y(t)}{dt^2} + a(a+1)\frac{dy(t)}{dt} + a^2y(t) = x(t)$$ -7

$$\xrightarrow{h(t)} \frac{d^3h(t)}{dt^3} + (a+1)\frac{d^2h(t)}{dt^2} + a(a+1)\frac{dh(t)}{dt} + a^2h(t) = \delta(t)$$

$$\xrightarrow{L} s^3H(s) + (a+1)s^2H(s) + a(a+1)sH(s) + a^2H(s) = 1$$

$$H(s) = \frac{1}{s^3+(a+1)s^2+a(a+1)s+a^2} = \frac{1}{(s+1)(s^2+as+a^2)}$$

$$g(t) = \frac{dh(t)}{dt} + h(t) \xrightarrow{L} G(s) = sH(s) + H(s) = H(s)(s+1)$$

$$G(s) = \frac{s+1}{(s+1)(s^2+as+a^2)} = \frac{1}{s^2+as+a^2}$$

قطب‌ها :

$$\begin{cases} s_1 = -a + \dfrac{\sqrt{a^2+4a^2}}{2} \\ s_2 = -a - \dfrac{\sqrt{a^2+4a^2}}{2} \end{cases}$$

۸-

الف)

$$X(s) = \frac{1}{s+1} \qquad Re\{s+1\} > 0 \rightarrow Re\{s\} > -1$$

$$H(s) = \frac{1}{s+2} \qquad Re\{s+2\} > 0 \rightarrow Re\{s\} > -2$$

ب)

$$Y(s) = X(s)H(s) = \frac{1}{(s+1)(s+1)} = \frac{A}{s+1} + \frac{B}{s+2} \qquad A=1,\ B=-1 \qquad Re\{s\} > -1$$

$$\xrightarrow{L^{-1}} y(t) = \left[e^{-t} - e^{-2t}\right] u(t)$$

۹-

$$\frac{d^2h(t)}{dt^2} - \frac{dh(t)}{dt} - 2h(t) = \delta(t) \xrightarrow{L} s^2H(s) - sH(s) - 2H(s) = 1$$

$$H(s) = \frac{1}{s^2+s-2} = \frac{1}{(s+1)(s-2)}$$

۱) پایداری $\rightarrow$

$$Re\{s+1\} > 0 \rightarrow Re\{s\} > -1$$

$$Re\{s-2\} < 0 \rightarrow Re\{s\} < 2$$

$$\xrightarrow{\text{اشتراک}} -1 < Re\{s\} < 2$$

$$h(t) = \frac{1}{3} e^{-t} u(t) + \frac{1}{3} e^{2t} u(t)$$

۲) علی $\rightarrow$

$$Re\{s+1\} > 0 \rightarrow Re\{s\} > -1$$

$$Re\{s-2\} > 0 \rightarrow Re\{s\} > 2$$

$$\xrightarrow{\text{اشتراک}} Re\{s\} < -1$$

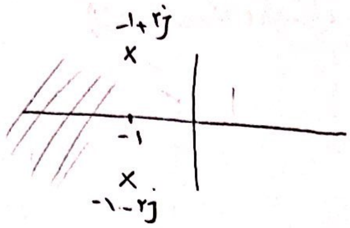
$$h(t) = \frac{-1}{3} e^{-t} u(-t) + \frac{1}{3} e^{2t} u(t)$$

۳) $Re\{s+1\}<0 \rightarrow Re\{s\}>-1$

$Re\{s-2\}<0 \rightarrow Re\{s\}>2 \xrightarrow{\text{اشتراک}} Re\{s\}>2 \quad h(t) = \frac{-1}{3}e^{-t}u(-t) + \frac{1}{3}e^{2t}u(-t)$

۱۰) $X(s) = X^*(s^*) \rightarrow s_2 = -1-2j \rightarrow$ قطب‌ها مزدوج هستن

اتفاقی یکی منفی هست قطب مثبت اون هم هست



$$X(s) = \frac{M}{(s+1-2j)(s+1+2j)} = \frac{M}{s^2+2s+5}$$

$$X(0) = \frac{M}{5} = 8 \rightarrow M = 40 \rightarrow X(s) = \frac{40}{s^2+2s+5} \rightarrow \begin{cases} s_1 = \frac{-2 \pm \sqrt{-16}}{2} \\ s_2 = \frac{-2+\sqrt{-16}}{2} \end{cases}$$

۱۱) $x(t) = e^{st} \quad y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} h(\tau)x(t-\tau) = \int_{-\infty}^{+\infty} h(\tau)e^{-s\tau}e^{st}d\tau = H(s)e^{st}$

$$y(t) = \frac{1}{6}e^{2t} \xrightarrow{s=2} H(s)e^{st} \rightarrow H(2) = \frac{1}{6}$$

$$\xrightarrow{L} sH(s) + 2H(s) = \frac{1}{s+4} + \frac{b}{s} = \frac{s+b(s+4)}{s(s+4)}, \quad H(s) = \frac{s+b(s+4)}{s(s+4)(s+2)}$$

$$H(2) = \frac{2+b(6)}{2(6)(4)} = \frac{1}{6} \Rightarrow b = 1 \rightarrow H(s) = \frac{2s+4}{2(s+4)(s+2)} = \frac{2}{s(s+4)}$$

Scanned with CamScanner