سوال ۱-

$$y[n] = \sum_{m=-\infty}^{+\infty} x[m]\, h_m[n]$$

$$x[n] = (-\tfrac{1}{2})^n u[n-4]$$

$$h[n] = 4^n u[2-n]$$

$$\Rightarrow y[n] = \sum_{m=-\infty}^{+\infty} (-\tfrac{1}{2})^m u[m-4] \times 4^{(n-m)} u[2-n+m]$$

$m \geq 4$ $\qquad$ $2-n+m \geq 0 \Rightarrow m \geq n-2$

$n < m+2$

$$y[n] = 4^n \sum_{m=4}^{\infty} (-\tfrac{1}{2} \times \tfrac{1}{4})^m = 4^n \sum_{m=4}^{\infty} (-\tfrac{1}{8})^m$$

$n < 6$ /

$u[m-4]$ تعیین کننده بازه است.

$n > 6$ /

$u[2-n+m]$ تعیین کننده بازه است.

$$y[n] = 4^n \sum_{m=n-2}^{\infty} (-\tfrac{1}{8})^m$$

سوال ۲-

$$y(t) = \int_{-\infty}^{+\infty} x(\tau) h(t-\tau)\, d\tau \qquad h(t) = -t+1$$

$$-\tfrac{1}{2} < t < \tfrac{1}{2} \Rightarrow \int_{-1/2}^{t} (-t+\tau+1)\, d\tau + \int_{t-1}^{-1/2} (t-\tau-1)\, d\tau$$

$$(1-t)\tau + \tfrac{\tau^2}{2} \Big]_{-1/2}^{t} + (t-1)\tau - \tfrac{\tau^2}{2} \Big]_{t-1}^{-1/2}$$

$$\left(t - t^2 + \tfrac{t^2}{2} + \tfrac{1}{2} - \tfrac{t}{2} - \tfrac{1}{8}\right) + \left(\tfrac{1}{2} - \tfrac{t}{2} - \tfrac{1}{8} - (t-1)^2 + \tfrac{(t-1)^2}{2}\right)$$

$$= -t^2 + t + \tfrac{1}{4}$$

$$\tfrac{1}{2} < t < \tfrac{3}{2} \Rightarrow \int_{t-1}^{1/2} (-t+\tau+1)\, d\tau + \int_{1/2}^{t} (t-\tau-1)\, d\tau$$

$$(1-t)\tau + \tfrac{\tau^2}{2} \Big]_{t-1}^{1/2} + (t-1)\tau - \tfrac{\tau^2}{2} \Big]_{1/2}^{t}$$

$$\left(\tfrac{1}{2} - \tfrac{t}{2} + \tfrac{1}{8} + (1-t)^2 - \tfrac{(t-1)^2}{2}\right) + \left(t^2 - t - \tfrac{t^2}{2} - \tfrac{1}{2}t + \tfrac{1}{2} + \tfrac{1}{8}\right)$$

$$= t^2 - 3t + \tfrac{7}{4}$$

سوال ۳ -

a) $h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[1-n]$

$n<0 \Rightarrow (1.01)^n u[1-n] \neq 0$ وقتی $1-n>0 \Rightarrow n<1$

$\Rightarrow h[n] \neq 0$ علی نیست

$$\sum_{n=0}^{\infty} (-\tfrac{1}{2})^n = \frac{(-\frac{1}{2})^0 - (-\frac{1}{2})^{\infty+1}}{1-(-\frac{1}{2})} = \frac{1-0}{3/2} = 2/3 < \infty$$

$\Rightarrow$ پایدار

$$\sum_{n=-\infty}^{1} (1.01) = \frac{(1.01)^{-\infty} - (1.01)^2}{1-(1.01)} = \frac{0-(1.01)^2}{-0.01} = 102.01 < \infty$$

b) $h(t) = (2e^{-t} - e^{-\frac{t-100}{100}}) u(t)$

$t<0 \Rightarrow h(t) = 0 \Rightarrow$ علی است

$$\int_{-\infty}^{+\infty} |h(t)|\, dt = \int_0^{\infty} 2e^{-t} - e^{-(\frac{t-100}{100})}\, dt = \left(-2e^{-t} + \frac{100}{t-100} e^{-(\frac{t-100}{100})}\right)\Big]_0^{\infty}$$

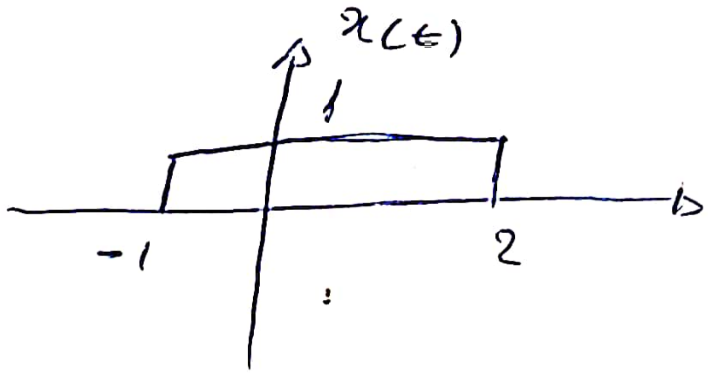
$$\Rightarrow \left(\cancel{-2e^{-\infty}}^{\,0} + \frac{100}{\infty-100} \cancel{e^{-(\frac{\infty-100}{100})}}^{\,0}\right) - \left(-2e^{0} + \frac{100}{0-100} e^{1}\right) = 2+e < \infty$$

پایدار است

سوال ۱۵ -

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}\, x(\underbrace{\tau-2}_{\tau' = \tau - 2 \;\Rightarrow\; d\tau' = d\tau})\, d\tau$$

$$\tau = \tau' + 2$$




الف)

$$\Rightarrow \int_{-\infty}^{t-2} e^{-(t-(\tau'+2))}\, x(\tau')\, d\tau'$$

$$\Rightarrow e^{-(t-\tau'-2)}$$


$$\Rightarrow h(t) = e^{-(t-2)}\, u(t-2)$$

$$x(t) = u(t+1) - u(t-2)$$

ب)

$$y(t) = \int_{-\infty}^{+\infty} h(\lambda)\, x(t-\lambda)\, d\lambda = \int_{-\infty}^{+\infty} e^{-(\lambda-2)}\, \underbrace{u(\lambda-2)}_{\lambda > 2}\, [\underbrace{u(t-\lambda+1)}_{\lambda < t+1} - \underbrace{u(t-\lambda-2)}_{\lambda < t-2}]\, d\lambda$$

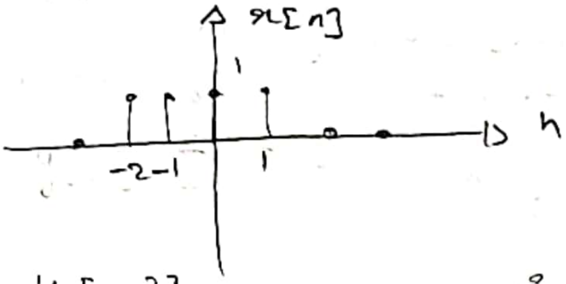
$$y(t) = \begin{cases} 0 & t < 1 \\ \int_{2}^{t+1} e^{-(t-\lambda-2)}\, d\lambda & 1 < t < 4 \\ \int_{t-2}^{t+1} e^{-(t-\lambda-2)}\, d\lambda & t > 4 \end{cases} = \begin{cases} 0 & t < 1 \\ 1 - e^{-(t-1)} & 1 < t < 4 \\ e^{-(t-4)}(1 - e^{-3}) & t > 4 \end{cases}$$

Scanned with CamScanner

سؤال ۵ -

$$y[n] + 2y[n-1] = x[n] + 2x[n-2]$$

$$y[n] = x[n] + 2x[n-2] - 2y[n-1]$$




$$y[-3] = 0$$

$$y[-2] = x[-2] + 2\underbrace{x[-4]}_{0} - 2\underbrace{y[-3]}_{} = 1$$

$$y[-1] = \underbrace{x[-1]}_{1} + 2\underbrace{x[-3]}_{0} - 2\underbrace{y[-2]}_{1} = 0$$

$$y[0] = \underbrace{x[0]}_{1} + \underbrace{2x[-2]}_{2} - \underbrace{2y[-1]}_{0} = 3$$

$$y[1] = \underbrace{x[1]}_{1} + \underbrace{2x[-1]}_{2} - \underbrace{2y[0]}_{6} = -3$$

$$y[2] = \underbrace{x[2]}_{0} + \underbrace{2x[0]}_{2} - \underbrace{2y[1]}_{-6} = 8$$

$$y[3] = \underbrace{x[3]}_{0} + \underbrace{2x[1]}_{2} - \underbrace{2y[2]}_{2\times 8} = 2 - 2\times 8 = -14$$

$$y[4] = \underbrace{x[4]}_{0} + \underbrace{2x[2]}_{0} - 2y[3] = -2\times -14$$

$$-2\times -2\times(-14)$$

$$y[n] = (-1)^n \times 2^{n-3} \times 14$$

---

سؤال ۴ -

$$2y[n] - y[n-1] + y[n-3] = x[n] - 5x[n-4]$$

$$S_1 : 2y_1[n] = x_1[n] - 5x_1[n-4]$$

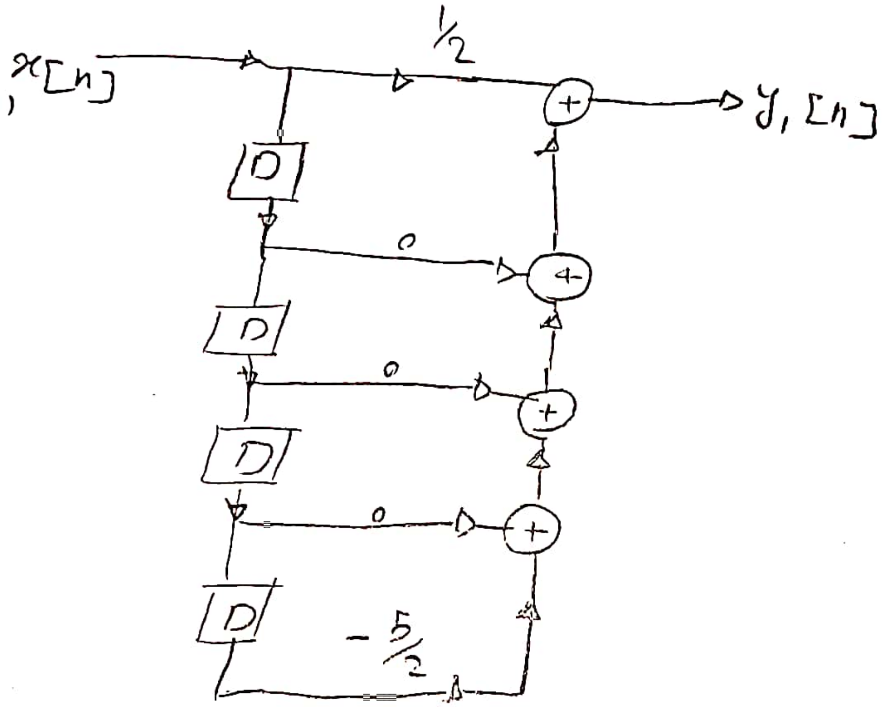
$$S_2 : y_2[n] - \tfrac{1}{2}y_2[n-1] + \tfrac{1}{2}y_2[n-3] = x_2[n]$$

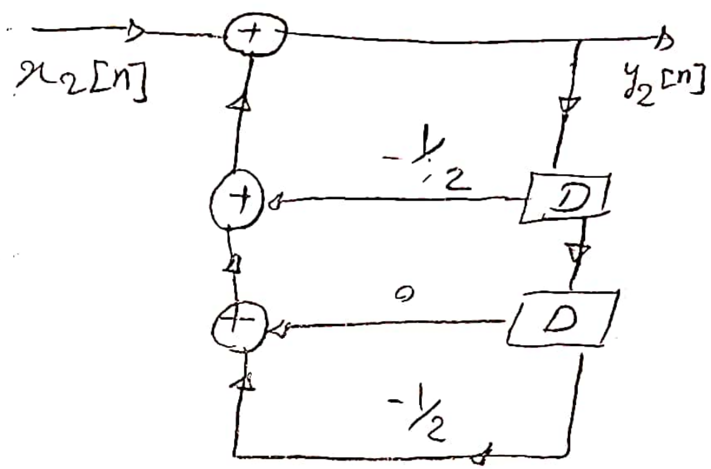
الف) با فرض : $y_1[n] = x_2[n]$

$$\Rightarrow S_1 : 2y_2[n] - y_2[n-1] + y_2[n-3] = x_1[n] - 5x_1[n-4]$$

که مشابه معادله دیفرانسیل است

ب)



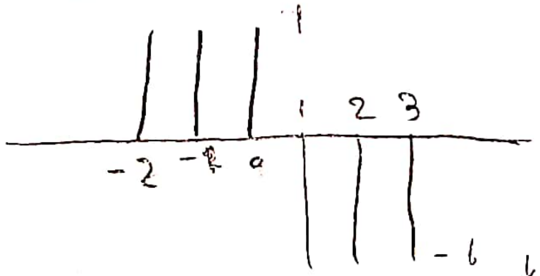




سؤال ۷-

$x[n] \longrightarrow y[n]$

$z[n] \longrightarrow y'[n] = ?$

$\frac{1}{3}x[n+2]$




$$z[n] = \sum_{n=\langle N\rangle} \frac{1}{3} x[n+2]$$

$$\rightarrow y'[n] = \frac{1}{3} \sum_{n=\langle N\rangle} y[n+2]$$

$\frac{1}{3}\sum x[n+2]$

$\rightarrow$