الف) $x[n]=\alpha^n \sin w_0 n$ ۱_

$x_1[n]=\sin w_0 n\, u[n] \xrightarrow{\text{تبدیل } Z} \dfrac{(\sin w_0) z^{-1}}{1-2(\cos w_0)z^{-1}+z^{-2}} \qquad |z|>1$

$$X(z)=\frac{(\sin w_0)(\frac{z}{\alpha})^{-1}}{1-2(\cos w_0)(\frac{z}{\alpha})^{-1}+(\frac{z}{\alpha})^{-2}} \qquad |\tfrac{z}{\alpha}|>1 \to |z|>\alpha$$

ب) $x[n]=\alpha^{|n|}$

$$x[n]=\alpha^n u[n]+\alpha^{-n}u[-n-1]=\alpha^n u[n]+(\alpha^{-1})^n u[-n-1] \xrightarrow{Z} X(z)=\underbrace{\frac{1}{1-\alpha z^{-1}}}_{|z|>\alpha}+\underbrace{\frac{-1}{1-\alpha^{-1}z^{-1}}}_{|z|<\alpha^{-1}}$$

$\to \alpha<|z|<\alpha^{-1}$

الف) $$X(z)=\frac{1-\frac{1}{2}z^{-1}}{1-\frac{7}{12}z^{-1}+\frac{1}{12}z^{-2}}=\frac{1-\frac{1}{2}z^{-1}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z^{-1})}=\frac{\overset{3}{A}}{1-\frac{1}{4}z^{-1}}+\frac{\overset{-2}{B}}{1-\frac{1}{3}z^{-1}}$$

$$\begin{cases} x[n]=3(\frac{1}{4})^n u[n]-2(\frac{1}{3})^n u[n] & |z|>\frac{1}{4},\ |z|>\frac{1}{3} \\ x[n]=3(\frac{1}{4})^n u[n]+2(\frac{1}{3})^n u[-n-1] & |z|>\frac{1}{4},\ |z|<\frac{1}{3} \\ x[n]=-3(\frac{1}{4})^n u[n-1]+2(\frac{1}{3})^n u[-n-1] & |z|<\frac{1}{4},\ |z|<\frac{1}{3} \end{cases}$$

ب) $X(z)=\dfrac{1}{1024}\left[\dfrac{1024-z^{-10}}{1-\frac{1}{2}z^{-1}}\right] \quad , |z|>0$

$$X(z)=\frac{1-\frac{z^{-10}}{1024}}{1-\frac{1}{2}z^{-1}} \xrightarrow{\times z^{10}} X(z)=\frac{z^{10}-\frac{1}{1024}}{z^{10}-\frac{1}{2}z^{9}}=\frac{z^{10}-\frac{1}{1024}}{z^{9}(z-\frac{1}{2})}=\frac{(z-\frac{1}{2})(z^{9}+\frac{1}{2}z^{8}+(\frac{1}{2})^{2}z^{7}+\dots(\frac{1}{2})^{9}z^{0})}{z^{9}(z-\frac{1}{2})}$$

$$X(z)=z^{0}+\frac{1}{2}z^{-1}+(\frac{1}{2})^{2}z^{-2}+\dots(\frac{1}{2})^{9}z^{-9} \xrightarrow{z^{-1}} x[n]=(\frac{1}{2})^n \quad 0\le n\le 9 \quad , X(z)=\sum_{n=-\infty}^{\infty}x[n]z^{-n}$$

ج) $X(z)=\log(1-2z) \quad , |z|<\frac{1}{2}$

$$X(z)=\sum_{i=1}^{\infty}\frac{(2z)^i}{i}=-(2z+\frac{2^2z^2}{2}+\frac{2^3z^3}{3}+\dots)$$

$$X(z)=\sum_{n=-\infty}^{\infty}x[n]z^{-n} \Rightarrow x[n]=\frac{2^{-n}}{n}u[-n-1]$$

$$2)\ X(z) = \frac{z^{-1} - \frac{1}{2}}{(1-\frac{1}{2}z^{-1})^2}, \quad |z| < \frac{1}{2}$$

$$\frac{\overset{\nearrow -2}{A}}{(1-\frac{1}{2}z^{-1})^2} + \frac{B \rightarrow \frac{3}{2}}{(1-\frac{1}{2}z^{-1})^2} = \frac{2}{1-\frac{1}{2}z^{-1}} + 3z\frac{\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})^2}$$

$$x[n] = 2\left(\frac{1}{2}\right)^n u[-n-1] + 3(n+1)\left(\frac{1}{2}\right)^{n+1} u[-(n+1)-1]$$

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$$Y(z) - \frac{1}{2}z^{-1}Y(z) + \frac{1}{4}z^{-2}Y(z) = X(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}$$

$$z^2 - \frac{1}{2}z + \frac{1}{4} = 0 \longrightarrow z_{1,2} = \frac{1}{4} \pm j\frac{\sqrt{3}}{4} \quad |z| > \frac{1}{4}$$ یا

$$H(z) = \frac{1}{(1-(\frac{1}{4}+j\frac{\sqrt{3}}{4})z^{-1})(1-(\frac{1}{4}-j\frac{\sqrt{3}}{4})z^{-1})} \qquad X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \quad |z| > \frac{1}{2}$$

$$Y(z) = X(z)H(z) = \frac{1}{(1-\frac{1}{2}z^{-1})(1-(\frac{1}{4}+j\frac{\sqrt{3}}{4})z^{-1})(1-(\frac{1}{4}-j\frac{\sqrt{3}}{4})z^{-1})} = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1-(\frac{1}{4}+j\frac{\sqrt{3}}{4})z^{-1}}$$

$$+ \frac{C}{1-(\frac{1}{4}-j\frac{\sqrt{3}}{4})z^{-1}} \rightarrow \begin{aligned} A &= 1 \\ B &= \frac{-j}{\sqrt{3}} \\ C &= \frac{j}{\sqrt{3}} \end{aligned}$$

$$y[n] = \left(\frac{1}{2}\right)^n u[n] - \frac{j}{\sqrt{3}}\left(\frac{1}{4}+j\frac{\sqrt{3}}{4}\right)^n u[n] + \frac{j}{\sqrt{3}}\left(\frac{1}{4}-j\frac{\sqrt{3}}{4}\right)^n u[n]$$

$$= \left(\frac{1}{2}\right)^n u[n] - \frac{j}{\sqrt{3}}\left(\frac{1}{2}\right)^n\left(\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)^n u[n] + \frac{j}{\sqrt{3}}\left(\frac{1}{2}\right)^n\left(\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)^n u[n]$$

$$= \left(\frac{1}{2}\right)^n\left[1 - \frac{j}{\sqrt{3}}\left(\left(\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)^n - \left(\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)^n\right)\right]u[n] = \left(\frac{1}{2}\right)^n\left[1 - \frac{j\,2j}{\sqrt{3}}\left(\frac{e^{j\frac{\pi}{3}n} - e^{-j\frac{\pi}{3}n}}{2j}\right)\right]u[n]$$

$$= \left(\frac{1}{2}\right)^n\left[\frac{2}{\sqrt{3}}\sin\frac{n\pi}{3} + 1\right]u[n]$$

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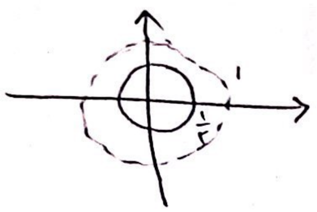
$X(z) = \frac{1}{1-z^{-1}} \quad |z|>1$

$Y(z) = \sum_{n=-\infty}^{\infty} y[n] z^{-n} = \sum_{n=-\infty}^{\infty} (\frac{1}{2})^{n-1} u[n+1] z^{-n} = \sum_{n=-1}^{\infty} (\frac{1}{2})^{n-1} z^{-n} = \sum_{0}^{\infty} (\frac{1}{2})^{(n-1)-1} z^{-(n-1)}$

$= 4z \sum_{n=0}^{\infty} (\frac{1}{2} z^{-1})^{n} = 4z \frac{1-0}{1-\frac{1}{2}z^{-1}} = \frac{4z}{1-\frac{1}{2}z^{-1}} \quad |z|>\frac{1}{2}$

$H(z) = \frac{Y(z)}{X(z)} = (1-z^{-1})(\frac{4z}{1-\frac{1}{2}z^{-1}}) = \frac{4(z-1)}{1-\frac{1}{2}z^{-1}}$ $\quad z=1$ صفر $\quad z=\frac{1}{2}$ قطب $\Rightarrow |z|>\frac{1}{2}$



ROC شامل دایره واحد است

می همگرا است.

علی

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$x[n] = (-1)^n = z^n \Rightarrow y[n] = H(z)z^n \xrightarrow{z=-1} y[n] = H(-1)(-1)^n = 0 \rightarrow H(-1)=0$

$H(z) = \frac{1}{1-\frac{1}{2}z^{-1}}, \quad Y(z) = 1 + \frac{a}{1-\frac{1}{2}z^{-1}} \quad |z|>\frac{1}{2}$

$H(z) = \frac{Y(z)}{X(z)} = (1-\frac{1}{2}z^{-1})(1+\frac{a}{1-\frac{1}{2}z^{-1}}) \quad |z|>\frac{1}{2} \xrightarrow{H(-1)=0} H(-1) = (1-\frac{1}{2}(-1)^{-1})(1+\frac{a}{1-\frac{1}{2}(-1)^{-1}})$

$= 0 \rightarrow \frac{3}{2}(1+\frac{2}{3}a) = 0 \quad a = \frac{-3}{2}$

$H(z) = (1-\frac{1}{2}z^{-1})(\frac{1-\frac{1}{2}z^{-1}+a}{1-\frac{1}{2}z^{-1}}), \quad |z|>\frac{1}{2}$

$\rightarrow 1-\frac{1}{2}z^{-1}-\frac{3}{2} = \frac{1}{2}z^{-1}-\frac{1}{2}$

$x[n] = 1 = z^n, \quad y[n] = H(z).z^n \xrightarrow{z=1} y[n] = H(1)(1)^n = 0 \quad H(1)=0$