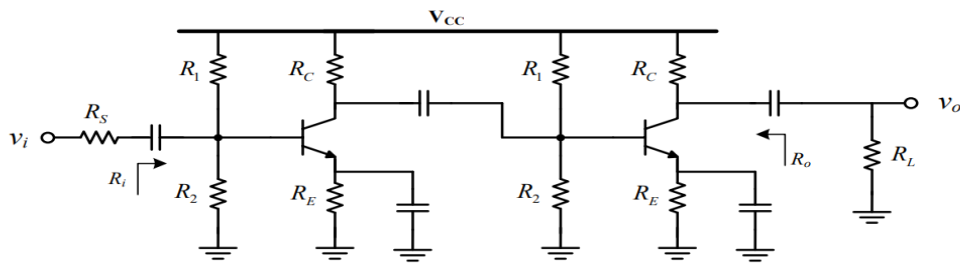
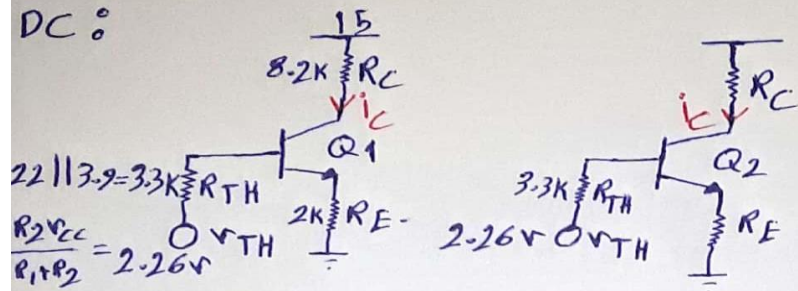


1- Calculate the voltage gain, input resistance and output resistance of the following circuit.



$$\begin{cases} R_1 = 22k\Omega \\ R_2 = 3.9k\Omega \\ R_C = 8.2k\Omega \\ R_E = 2k\Omega \\ R_S = 600\Omega \\ R_L = 51k\Omega \\ V_{CC} = 15V \\ \beta = 100 \end{cases}$$

DC :



$$V_{BE} = 0.7V$$

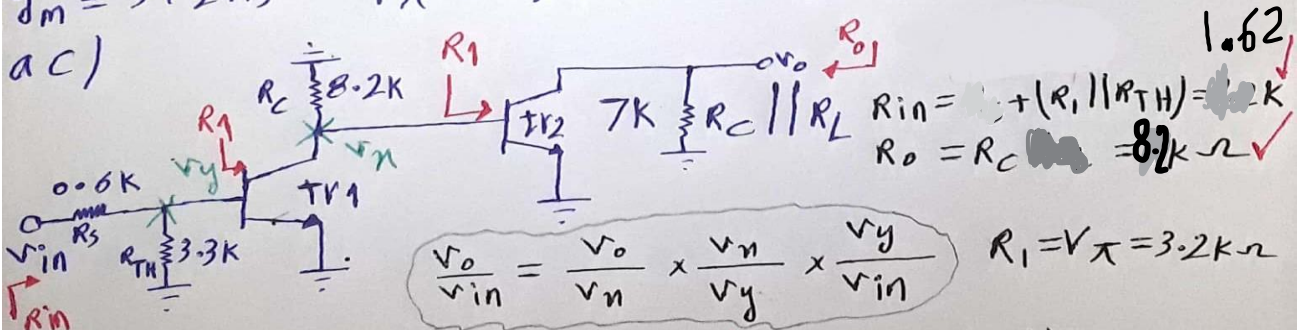
$$R_E \gg \frac{R_{TH}}{\beta} \rightarrow \text{نظير } i_B ; 2k\Omega \gg \frac{3.3k}{100} \checkmark$$

$$i_C = 100 \times \frac{2.26 - 0.7}{200} = 0.78 \text{ mA}$$

$$V_{CE} = 15 - (R_C + R_E) i_C = 15 - 10.2 \times 0.78 = 7V > 0.2V \checkmark \text{ نظير } V_{CE}$$

$$g_m = 31.2 \text{ ms} \quad V_{\pi} = 3.2k\Omega \quad V_o = \infty$$

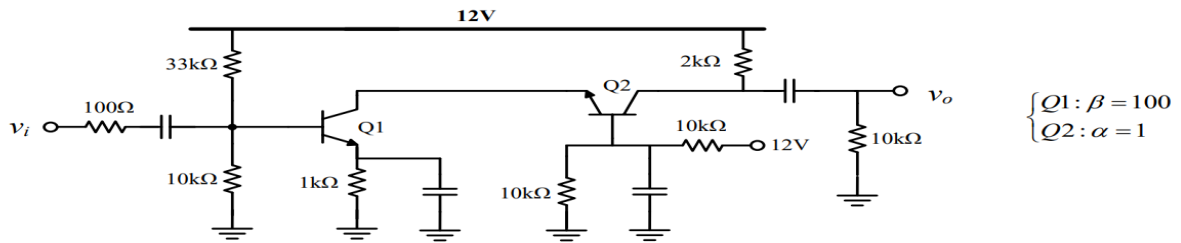
ac)



$$\frac{v_o}{v_n} = -g_m R_C = -31.2 \times 7 = -218.4 \checkmark \quad \frac{v_n}{v_y} = -g_m (R_C || R_1) = -72 \checkmark$$

$$\frac{v_y}{v_{in}} = \frac{R_1 || R_{TH}}{(R_1 || R_{TH}) + R_S} = \frac{1.62}{1.62 + 0.6} = 0.7 \checkmark \quad \frac{v_o}{v_{in}} = +11007$$

2- In the circuit shown below, determine the voltage gain.



$A_v = ?$

$\beta_1 = 100$

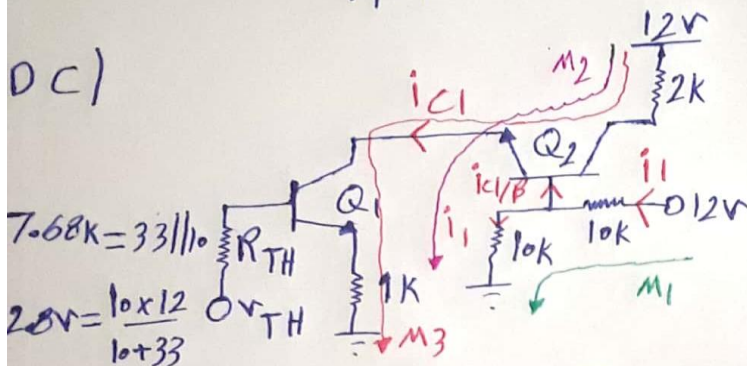
$\alpha = \frac{i_c}{i_E} = \frac{\beta}{\beta + 1}$

- 2

$\alpha = 1 \rightarrow \beta_2 = \beta + 1 \rightarrow i_c = i_E$

$\beta_2 \gg 20$

DC)



$i_{C1} = \beta_1 i_B = 100 \times \frac{2.8 - 0.7}{7.68 + 100} = 1.95 \text{ mA}$

$\beta_2 \gg 20 \rightarrow i_{C2} \approx i_{E2} \rightarrow i_{C2} = i_{C1} = 1.95 \text{ mA}$

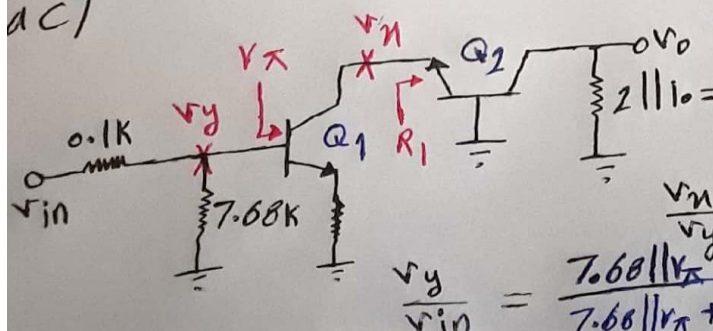
KVL @ M_1 : $-12 + 10i_1 + 10i_1 = 0 \rightarrow i_1 = 0.6 \text{ mA}$

KVL @ M_2 : $-12 + 2i_{C1} + v_{CE2} - 0.7 + 10i_1 = 0 \rightarrow v_{CE2} = 2.8 \text{ V} > 0.2$

KVL @ M_3 : $-12 + v_{CE2} + v_{CE1} + i_{C1} = 0 \rightarrow v_{CE1} = 3.35 \text{ V} > 0.2$

$g_{m1} = g_{m2} = 78 \frac{\text{mA}}{\text{V}}$ $V_{\pi 1} = 1.28 \text{ k}\Omega$

AC)



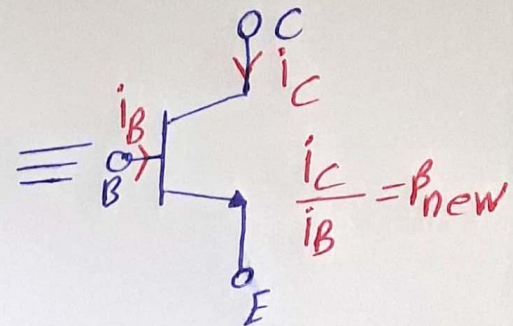
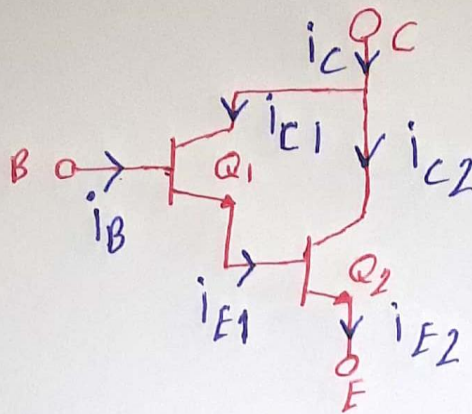
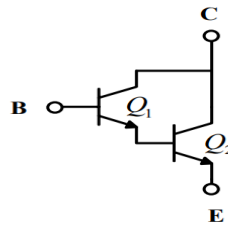
$A_v = \frac{v_o}{v_{in}} \times \frac{v_n}{v_y} \times \frac{v_y}{v_{in}}$

$\frac{v_o}{v_n} = +g_m R_C = 130$

$\frac{v_n}{v_y} = -R_1 g_m = -1$

$\frac{v_y}{v_{in}} = \frac{7.68 \parallel 1.28}{7.68 \parallel 1.28 + 0.1} = \frac{1}{1.1} = 0.9 \rightarrow \frac{v_o}{v_{in}} = -117$

- 3- The following structure is known to be the "Darlington pair" configuration and is used in order to increase the β of a single transistor. This configuration can be modeled as a single NPN transistor. Determine its equivalent r_{π} , g_m and β .



$$i_{C1} = \beta_1 i_B \quad i_{E1} = (\beta_1 + 1) i_B \quad i_{C2} = \beta_2 i_{E1}$$

$$i_{C2} = \beta_2 (\beta_1 + 1) i_B$$

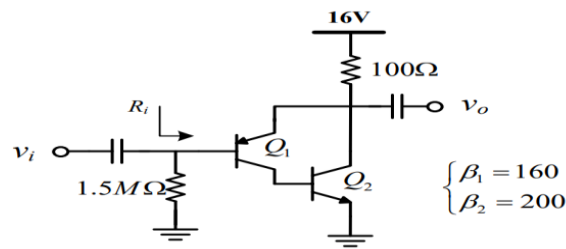
$$i_C = i_{C1} + i_{C2} = \beta_1 i_B + \beta_2 (\beta_1 + 1) i_B$$

$$\frac{i_C}{i_B} = \beta_{new} = \frac{(\beta_1 + \beta_1 \beta_2 + \beta_2) i_B}{i_B} \rightarrow \beta_{new} = \beta_1 + \beta_2 + \beta_1 \beta_2$$

$$\beta_{new} \approx \beta_1 \beta_2$$

$$g_{m, new} = 40 i_C \quad r_{\pi, new} = \frac{\beta_{new}}{g_{m, new}} = \frac{\beta_1 + \beta_2 + \beta_1 \beta_2}{40 i_C}$$

- 4- The following configuration is called the "Sziklai Pair" or the "Complementary Darlington pair". This configuration can be used to construct a PNP transistor with a large β . For the following circuit, determine the voltage gain and the input resistance.



DC)

$\beta_1 = 160$
 $\beta_2 = 200$

$A_v = ?$
 $R_{in} = ?$

$i_{C1} = 160 i_B$ $i_{C2} = 200 i_{C1} = 32 \times 10^3 i_B$

KVL @ m_1 : $-16 + \frac{1}{10} (i_{C2}) + 0.7 + 1500 i_B = 0$

$i_B = 3.25 \times 10^{-3} \rightarrow i_{C1} + i_{C2} \approx i_{C2} = 1.04 \text{ mA}$

$\beta_{new} = \frac{1.04}{3.25 \times 10^{-3}} = 32000$ $\beta_{new} \approx \beta_1 \beta_2$

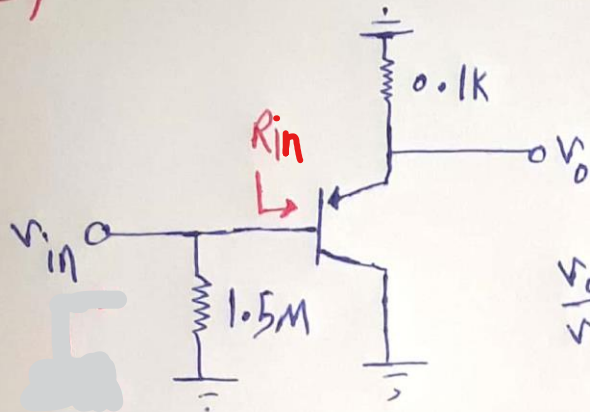
$V_{CE2} = V_{EC1} + 0.7 \rightarrow$ (Note: This is a typo in the original image, it should be $V_{CE2} = V_{CE1} + 0.7$)

KVL @ m_2 : $-16 + \frac{1.04}{10} + V_{EC1} + 0.7 = 0 \rightarrow V_{EC1} = 4.9 > 0.2$

$\beta_{new} = 32000$ $g_{m, new} = i_{C2} \times 40 = 1040 \frac{\text{mA}}{\text{V}}$

$r_{\pi, new} = \frac{\beta_{new}}{g_{m, new}} = 30.7 \text{ k}\Omega$

ac)

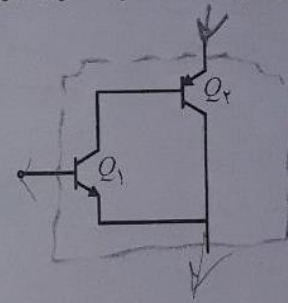
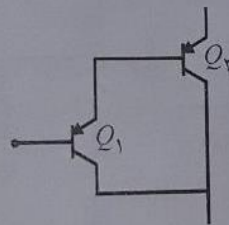
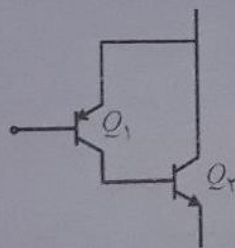


$$\frac{v_o}{v_{in}} = \frac{0.1}{0.1 + \frac{1}{g_{m,new}}} \approx 1$$

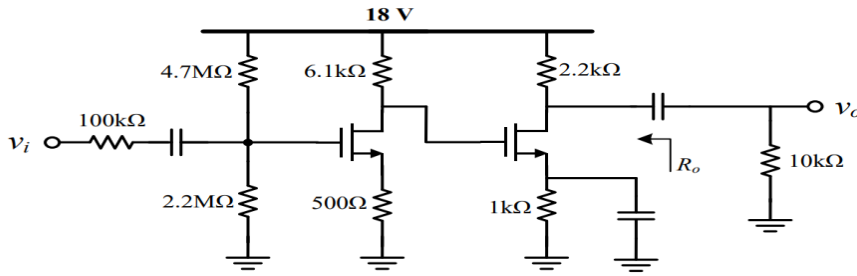
$$R_{in} = r_{\pi,new} + \beta_{new} R_E = 30.7 + 32000 \times 0.1 \approx 3200k\Omega$$

~~$$R_{in} = 1.5M \parallel 32M = 1M\Omega$$~~

β کلی یک ترکیب دارلینگتون تقریباً برابر $\beta_1 \beta_2$ است. ترکیب‌های دیگر دارلینگتون به صورت زیر است.



5- Determine to voltage gain and the output resistance of the following structure.



$$\begin{cases} \beta_1 = 0.5 \text{ mA/V} \\ \beta_2 = 0.3 \text{ mA/V} \\ V_{TH1} = 2 \text{ V} \\ V_{TH2} = 2.6 \text{ V} \\ r_{ds1} = 75 \text{ k}\Omega \\ r_{ds2} = 100 \text{ k}\Omega \end{cases}$$

2c)

$\beta_1 = 0.5$ $V_{DS1} = 75 \text{ k}\Omega$ 15
 $\beta_2 = 0.3$ $V_{DS2} = 100 \text{ k}\Omega$
 $V_{TH1} = 2$
 $V_{TH2} = 2.6 \text{ V}$

$$V_{GS1} = 5.74 - \frac{i_1}{2} \quad i_1 = \frac{\beta_1}{2} (V_{GS1} - V_{TH1})^2$$

$$V_{GS1} = 4.74 \quad i_1 = \frac{1}{4} \left(5.74 - \frac{i_1}{2} - 2 \right)^2$$

$$i_1 = \frac{1}{4} \left(3.74 - \frac{i_1}{2} \right)^2 \rightarrow 16i_1 = \left(7.48 - i_1 \right)^2$$

$$i_1^2 - 31i_1 + 56 = 0 \rightarrow i_1 = 2 \text{ mA}$$

$$\text{KVL @ } M_1: -18 + 6.1i_1 + V_{GS2} + i_2 = 0 \rightarrow V_{GS2} = 5.8 - i_2$$

$$i_2 = \frac{3}{20} \left(5.8 - i_2 - 2.6 \right)^2 \rightarrow 20i_2 = 3 \left(3.2 - i_2 \right)^2$$

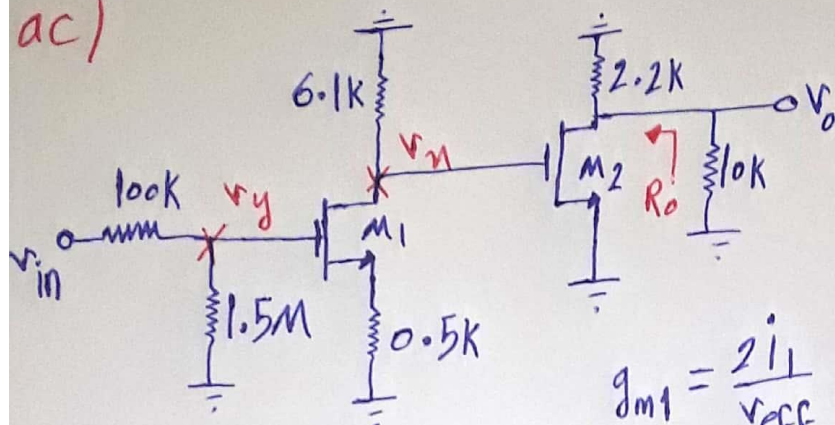
$$i_2^2 - 39.2i_2 + 30.72 = 0 \rightarrow i_2 = 0.8 \text{ mA}$$

$$\text{KVL @ } M_2: -18 + 6.1i_1 + V_{DS1} + \frac{i_1}{2} = 0 \rightarrow V_{DS1} = 4.8 > 2.74 \quad \checkmark$$

$$\text{KVL @ } M_3: -18 + 2.2i_2 + V_{DS2} + i_2 = 0 \rightarrow V_{DS2} = 15.44 \text{ V}$$

$$V_{GS2} = 5 \text{ V} \rightarrow V_{eff2} = 2.4 \text{ V} \quad V_{DS2} > V_{eff2} \quad \checkmark$$

ac)



$$g_{m1} = \frac{2I_1}{V_{eff}} = \sqrt{2\beta_1 I_1} = 1.41 \text{ mS}$$

$$g_{m2} = \sqrt{2\beta_2 I_2} = 0.7 \text{ mS}$$

$$V_{DS1} = 75 \text{ K} \quad V_{DS2} = 100 \text{ K}$$

$$\frac{v_o}{v_n} \times \frac{v_n}{v_g} \times \frac{v_g}{v_{in}} = \frac{v_o}{v_{in}}$$

$$\frac{v_o}{v_n} = -g_{m2}(R_D || V_{DS2}) = -0.7(1.8 || 100)$$

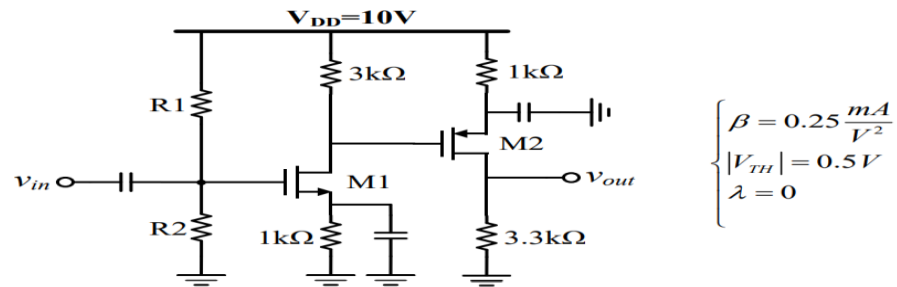
$$\frac{v_o}{v_n} = -0.7(1.8) = -1.26 \checkmark$$

$$\frac{v_n}{v_g} = \frac{-R_D}{R_S + \frac{1}{g_{m1}}} = \frac{-6.1}{\frac{1}{2} + \frac{1}{41}} = -11.63 \checkmark$$

$$\frac{v_g}{v_{in}} = \frac{1.5 \text{ M}}{1.5 \text{ M} + 0.1 \text{ M}} = 0.93 \checkmark \rightarrow \frac{v_o}{v_{in}} = +13.6$$

$$R_o = 2.2 \text{ K} || V_{DS2} = 2.1 \text{ K} \Omega$$

- 6- a) Specify R_1 and R_2 so that the bias current of M1 will be equal to 1 mA.
b) Calculate the voltage gain and the output resistance.



DC)

$\frac{10R_2}{R_1+R_2} = V_{TH}$

$\beta = \frac{1}{4}$

$|V_{TH}| = 0.5V$

$\lambda = 0$

$V_{GS1} = V_{TH1} + \sqrt{\frac{2I_{D1}}{\beta}} = \frac{1}{2} + \sqrt{\frac{2}{\frac{1}{4}}} = 3.3V$

$V_{GS1} = V_{TH} - 1 \Rightarrow \frac{10R_2}{R_1+R_2} - 1 = 3.3 \rightarrow \frac{10R_2}{R_1+R_2} = 4.3$

ضرب: $R_2 = 1K$

$10 = 4.3R_1 + 4.3 \rightarrow R_1 = 1.32K$

KVL @ M1: $-10 + I_2 + V_{SG2} - 3 + 10 = 0 \rightarrow V_{SG2} = 3 - I_2$

$I_2 = \frac{1}{8} (3 - I_2 - 0.5)^2 \rightarrow 8I_2 = (-I_2 + 2.5)^2$

$I_2^2 - 13I_2 + 6.25 = 0 \rightarrow I_D = 0.5mA = I_2$

KVL @ M2: $-10 + 3 + V_{DS1} + 1 = 0 \rightarrow V_{DS1} = 6V > 3.3 - 0.5V$

KVL @ M3: $-10 + 0.5 + V_{SD2} + \frac{3.3}{2} = 0 \rightarrow V_{SD2} = 7.8 > 2.5 - 0.5V$

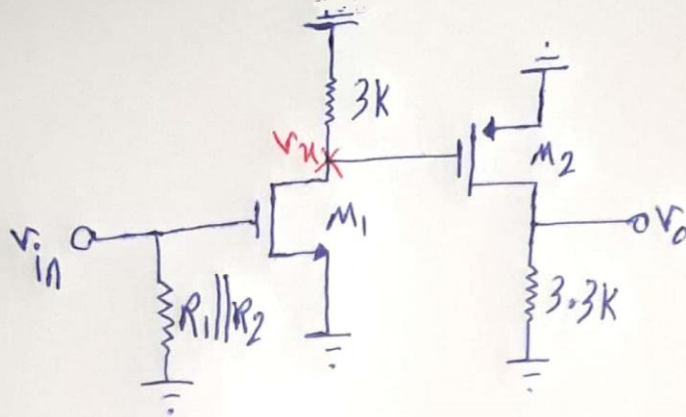
$V_{DS} = \infty$

$g_{m1} = \sqrt{2\beta I_1} = 0.7mS$

$g_{m2} = \sqrt{2\beta I_2} = 0.5mS$

1a 16

$$R_o = ? \quad A_v = ? \quad |b$$

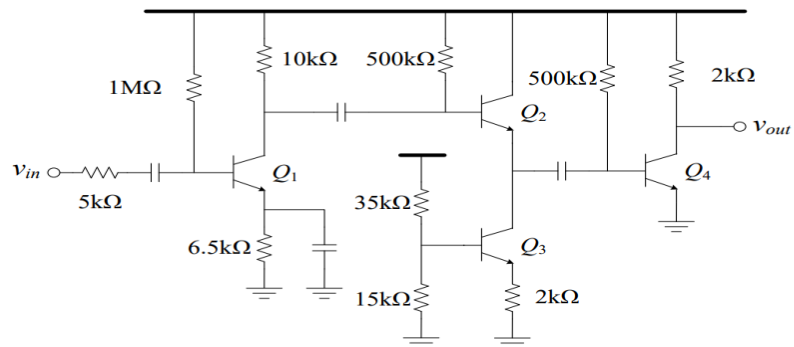


$$\frac{v_o}{v_{in}} = \frac{v_o}{v_n} \times \frac{v_n}{v_{in}} = -g_{m2} R_{D2} \times -g_{m1} R_{D1}$$

$$\frac{v_o}{v_{in}} = -\frac{1}{2} \times 3.3 \times -0.7 \times 3 = +3.46$$

$$R_{out} = 3.3k\Omega$$

- 7- Calculate the voltage gain of the structure depicted in the following figure. The transistors are in the saturation region.



$$\begin{cases} V_A = 100V \\ V_T = 25mV \\ \beta = 100 \\ I_{C1} = I_{C4} = 1mA \\ I_{C2} = 0.5mA \end{cases}$$

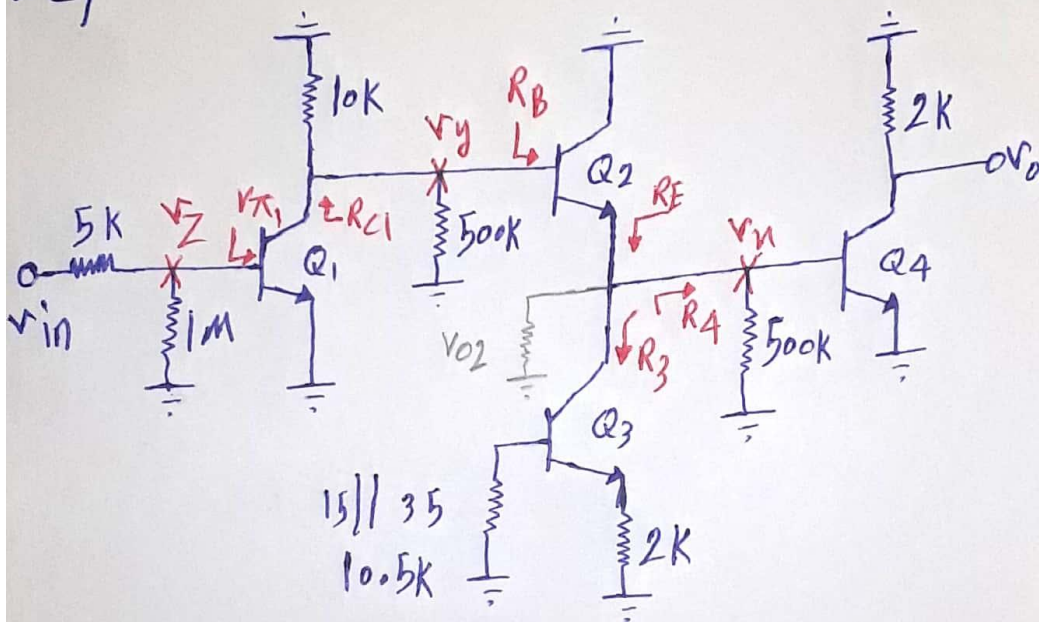
DC)

$$i_{C1} = i_{C4} = 1\text{mA} \rightarrow g_{m1,4} = 40\text{mS} \rightarrow r_{\pi 1,4} = 2.5\text{k} \rightarrow r_{o1,4} = 100\text{k}$$

$$i_{C2} = i_{C3} = 0.5\text{mA} \rightarrow g_{m2,3} = 20 \rightarrow r_{\pi 2,3} = 5\text{k} \rightarrow r_{o2,3} = 200\text{k}$$

$A_v = ?$

ac)



$$\frac{v_o}{v_{in}} = \frac{v_o}{v_n} \times \frac{v_n}{v_y} \times \frac{v_y}{v_x} \times \frac{v_x}{v_{in}}$$

$$\frac{v_o}{v_n} = -g_{m4}(r_{\pi 4} \parallel r_{o4}) = -40(2 \parallel 100) = -80$$

$$\frac{v_n}{v_y} = \frac{R_E}{R_E + \frac{1}{g_{m2}}}$$

$$R_E = r_{o2} \parallel R_3 \parallel R_4$$

$$R_3 = r_{o3}(1 + g_{m3}(R_E \parallel r_{\pi 3}))$$

$$R_3 = 200(1 + 20(2 \parallel 5)) = 5914\text{k}\Omega$$

$$r_{\pi 4} = 500\text{k} \parallel r_{\pi 4} = 2.5\text{k}$$

$$R_E = 200\text{k} \parallel 5914\text{k} \parallel 2.5\text{k} = 2.5\text{k}\Omega \rightarrow \frac{v_n}{v_y} = \frac{2.5}{2.5 + \frac{1}{20}} = 0.98$$

$$\frac{v_y}{v_z} = -g_{m_1}(R_{C1} \parallel R_o)$$

$$R_{C1} = 10 \parallel 500k \parallel R_B$$

$$R_B = r_{\pi 2} + \beta_2 R_E$$

$$R_B = 5 + 100 \times 2.5 = 250k\Omega$$

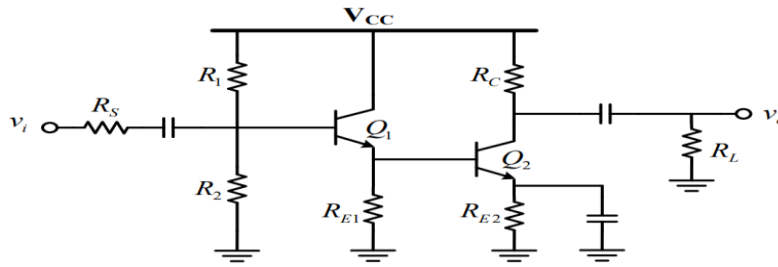
$$R_{C1} = 10 \parallel 250 \parallel 500 = 10k\Omega$$

$$\frac{v_y}{v_z} = -40(10 \parallel 100) = -363$$

$$\frac{v_z}{v_{in}} = \frac{1M \parallel 2.5k}{1M \parallel 2.5k + 5k} = \frac{2.5}{2.5 + 5} = 0.33$$

$$\frac{v_o}{v_{in}} = 9391$$

8- Calculate the voltage gain in the following circuit.



$$\begin{cases} V_{CC} = 10V \\ R_C = R_S = R_L = 4k\Omega \\ R_1 = R_2 = 200k\Omega \\ R_{E1} = 3.3k\Omega \\ R_{E2} = 2.6k\Omega \\ V_{CE,sat} = 0.2V \\ V_{BE,on} = 0.7V \\ \beta_1 = \beta_2 = 100 \end{cases}$$

DC)

-8

Av=?

$$-5 + \frac{100 i_{C1}}{100} + 0.7 + 3.3 i_{C1} = 0 \rightarrow i_{C1} = 1mA$$

$$KVL @ M_1: -3.3 i_{C1} + 0.7 + 2.6 i_{C2} = 0 \rightarrow i_{C2} = 1mA$$

$$i_{C1} \gg i_{B2} \rightarrow 1 \gg \frac{1}{100} \checkmark \quad g_m = 40mS \quad r_\pi = 2.5k$$

ac)

$$\frac{v_o}{v_{in}} = \frac{v_o}{v_n} \times \frac{v_n}{v_y} \times \frac{v_y}{v_{in}}$$

$$\frac{v_o}{v_n} = -g_m R_C = -40 \times 2 = -80$$

$$\frac{v_n}{v_y} = \frac{R_E}{R_E + \frac{1}{g_m}} = \frac{1.4}{1.4 + \frac{1}{40}} = 0.98$$

$$R_E = 3.3 \parallel r_\pi = 1.4k$$

$$\frac{v_y}{v_{in}} = \frac{R_B \parallel 100}{R_B \parallel 100 + 4} \quad R_B = r_\pi + \beta R_E = 142k \rightarrow \frac{v_y}{v_{in}} = 0.93 \rightarrow \frac{v_o}{v_{in}} = -73$$