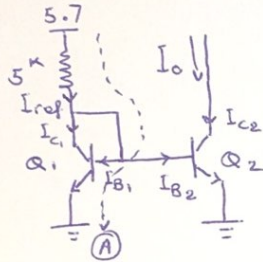


#1



KVL @ A: $-5.7 + 5 I_{ref} + 0.7 = 0$

$$I_{ref} = \frac{5.7 - 0.7}{5} = 1 \text{ mA}$$

b) $\frac{I_o}{I_{ref}} = ?$

$$I_{C1} = I_{S1} \exp\left(\frac{V_{BE1}}{V_T}\right)$$

$$I_{C2} = I_o = I_{S2} \exp\left(\frac{V_{BE2}}{V_T}\right)$$

$$I_{ref} = I_{C1} + I_{B1} + I_{B2}$$

$$\Rightarrow I_{ref} = I_{C1} + \frac{I_{C1}}{\beta_1} + \frac{I_{C2}}{\beta_2}$$

$$\Rightarrow I_{ref} = \left(1 + \frac{1}{\beta}\right) I_{C1} + \frac{I_{C2}}{\beta} \quad \xrightarrow{I_{C1} = \frac{I_{S1}}{I_{S2}} I_{out}} \quad I_{ref} = \left(1 + \frac{1}{\beta}\right) \left(\frac{I_{S1}}{I_{S2}} I_{out}\right) + \frac{I_{out}}{\beta}$$

$$\Rightarrow I_{ref} = \frac{I_{S1}}{I_{S2}} I_{out} + \frac{I_{S1}}{\beta I_{S2}} I_{out} + \frac{I_{out}}{\beta} \Rightarrow I_{ref} = I_{out} \left(\frac{I_{S1}}{I_{S2}} + \frac{I_{S1}}{\beta I_{S2}} + \frac{1}{\beta} \right)$$

$$\Rightarrow \frac{I_{out}}{I_{ref}} = \frac{1}{\left(\frac{I_{S1}}{I_{S2}} + \frac{I_{S1}}{\beta I_{S2}} + \frac{1}{\beta} \right)} = \frac{I_{S2}}{I_{S1}} \cdot \left(\frac{1}{1 + \frac{1 + \frac{I_{S2}}{I_{S1}}}{\beta}} \right)$$

$$\text{if } \beta = 50 \rightarrow \frac{I_{out}}{I_{ref}} = \frac{I_{S2}}{I_{S1}} \cdot \left(\frac{1}{1 + \frac{1 + \frac{I_{S2}}{I_{S1}}}{50}} \right)$$

$$\text{if } \beta = 200 \rightarrow \frac{I_{out}}{I_{ref}} = \frac{I_{S2}}{I_{S1}} \cdot \left(\frac{1}{1 + \frac{1 + \frac{I_{S2}}{I_{S1}}}{200}} \right)$$

$$\text{if } \beta = \infty \rightarrow \frac{I_{out}}{I_{in}} = \frac{I_{S2}}{I_{S1}}$$

[illegible]

$$I_{ref} = \frac{5.7 - 1.4}{5} = \underline{0.86 \text{ mA}}$$

$$\Rightarrow V_{BE1} = V_{BE2} = V_{BE3}$$

$$I_{C2} = I_0 = I_{S2} \exp\left(\frac{V_{BE2}}{V_T}\right)$$

$$I_{C3} = I_{S3} \exp\left(\frac{V_{BE3}}{V_T}\right)$$

$$\left\{ \begin{aligned} I_{ref} = I_{C1} + I_{B3} &= I_{C1} + \frac{I_{C3}}{\beta} \Rightarrow I_{C1} = I_{ref} - \frac{I_{C3}}{\beta} \quad (I) \end{aligned} \right.$$

$$I_{33} = I_{B1} + I_{B2} = \frac{1}{\beta} (I_{c1} + I_{c2}) = \frac{1}{\beta} (I_{c1} + I_o) \quad (II)$$

$$\xrightarrow{(I) \text{ in } (\Pi)} I_{c3} = \frac{1}{\beta} \left(I_{ref} - \frac{I_{c3}}{\beta} + I_c \right) \Rightarrow I_{c3} \left(1 + \frac{1}{\beta^2} \right) = \frac{I_{ref}}{\beta} + \frac{I_c}{\beta} \quad \underline{I_{c3} = \frac{I_{s3}}{I_{s2}} I_c} \rightarrow$$

$$\left(\frac{I_{S3} I_0}{I_{S2}} \right) \left(1 + \frac{1}{\beta^2} \right) = \frac{I_{ref}}{\beta^3} + \frac{I_0}{\beta^3} \Rightarrow \frac{I_{S3}}{I_{S2}} I_0 + \frac{I_{S3} I_0}{\beta^2 I_{S2}} = \frac{I_0}{\beta} + \frac{I_{ref}}{\beta}$$

$$\Rightarrow I_0 \left(\frac{I_{s3}}{I_{s2}} + \frac{I_{s3}}{\beta^2 I_{s2}} - \frac{1}{\beta} \right) = \frac{I_{ref}}{\beta} \Rightarrow \frac{I_0}{I_{ref}} = \frac{1}{\beta \left(\frac{I_{s3}}{I_{s2}} + \frac{I_{s3}}{\beta^2 I_{s2}} - \frac{1}{\beta} \right)} = \frac{1}{\left(\frac{\beta I_{s3}}{I_{s2}} + \frac{I_{s3}}{\beta I_{s2}} - 1 \right)}$$

$$\therefore \beta = 50 \rightarrow \frac{1}{\left(50 \frac{I_{S3}}{I_{S2}} + \frac{I_{S3}}{50 I_{S2}} - 1\right)} \approx \frac{1}{50 - \frac{I_{S2}}{I_{S1}}} = \frac{1}{\beta + \frac{1}{\beta} - \frac{I_{S2}}{I_{S1}}} \times \frac{I_{S2}}{I_{S3}}$$

[illegible]

assume $\lambda \neq 0$ and neglect β effect

$$-V_{CC} + V_{BE3} + V_{BE1} = 0 \Rightarrow V_{CC} = 2V_{BE}$$

$$I_{ref} = I_{c3} + \frac{1}{\beta} (I_{c3} + I_{c4}) = I_{c3} + \frac{1}{\beta} (I_{c3} + I_0)$$

$$\Rightarrow I_{ref} = I_{C3} + \frac{I_{C3}}{\beta} + \frac{I_o}{\beta} \Rightarrow I_{ref} = I_{C3} \left(1 + \frac{1}{\beta}\right) + \frac{I_o}{\beta}$$

$$I_{C3} = I_{S3} \exp\left(\frac{V_{BE3}}{V_T}\right)$$

$$I_{C1} = I_{S1} \exp\left(\frac{V_{BE1}}{V_T}\right)$$

$$I_0 = I_{C4} = I_{C2} = I_{S4} \exp\left(\frac{V_{BE}}{V_T}\right)$$

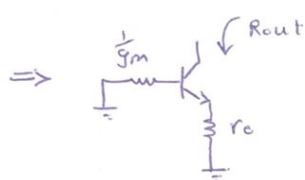
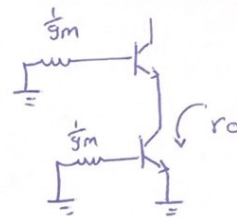
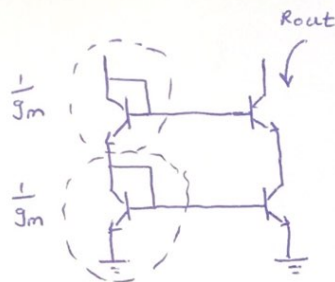
$$I_{C3} = \frac{I_{S3}}{I_{S4}} I_0$$

$$\xrightarrow{\hspace{1.5cm}} I_{ref} = \frac{I_{S3} I_0}{I_{S4}} \left(1 + \frac{1}{\beta}\right) + \frac{I_0}{\beta}$$

$$= \gamma_{ref} \frac{I_{s3}}{I_{s4}} I_0 + \frac{I_{s3} I_0}{\beta I_{s4}} + \frac{I_0}{\beta}$$

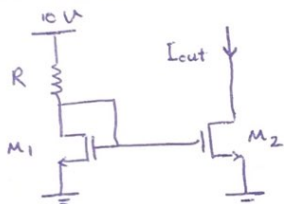
$$\Rightarrow I_{ref} = I_0 \left(\frac{I_{S3}}{I_{S4}} + \frac{I_{S3}}{\beta I_{S4}} + \frac{1}{\beta} \right) \Rightarrow I_0 \left(\frac{I_{ref}}{\frac{I_{S3}}{I_{S4}} + \frac{I_{S3}}{\beta I_{S4}} + \frac{1}{\beta}} \right)$$

$R_{out} = ?$



$$R_{out} = r_o (1 + g_m (r_o || r_{\pi})) \approx r_o (1 + \underbrace{g_m r_{\pi}}_{= \beta}) = r_o (1 + \beta)$$

#3



$$\beta_1 = 2 \frac{\text{mA}}{\text{V}^2}$$

$$\beta_2 = 10 \frac{\text{mA}}{\text{V}^2}$$

$$V_{th1} = V_{th2} = 2\text{V}$$

$$\text{if } I_{out} = 5\text{mA} \rightarrow R = ?$$

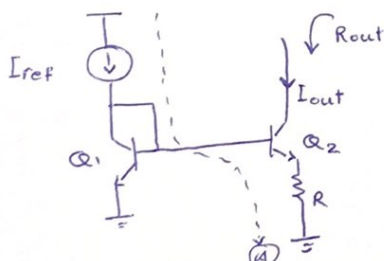
$$\frac{I_{out}}{I_{in}} = \frac{\beta_2}{\beta_1} \Rightarrow I_{in} = 1\text{mA}$$

$$-10 + R I_D + V_{GS} = 0 \Rightarrow V_{GS} = 10 - R I_D$$

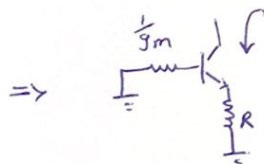
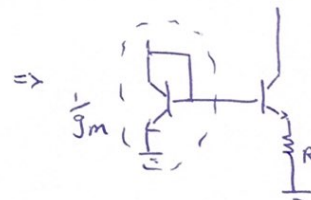
$$I_D = \frac{\beta_1}{2} (V_{GS} - V_{th})^2 = \frac{2}{2} (10 - R I_D - 2)^2$$

$$I_{in} = I_D = 1 \rightarrow 1 = (10 - R - 2)^2 \rightarrow \begin{cases} R = 7\text{k} \\ R = 9\text{k} \end{cases}$$

#4



a) $R_{out} = ?$



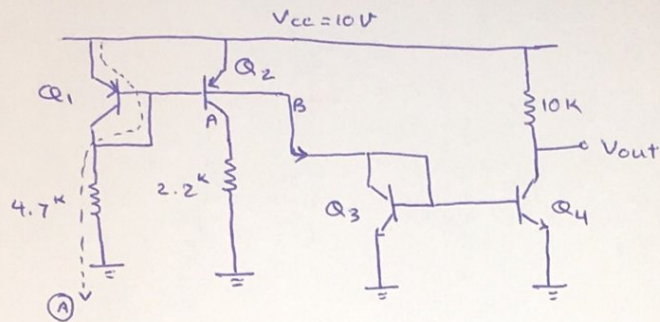
$$R_{out} = r_o (1 + g_m (R || r_{\pi})) \approx \beta r_o$$

b) if $I_{ref} = 2 I_{out} \rightarrow R = ?$
 $\beta \gg 1$

KVL @ A: $-V_{CC} + V_{BE} + R I_{out} = 0 \rightarrow I_{out} = \frac{1}{2} I_{ref}$

$$\Rightarrow \frac{R}{2} I_{ref} = V_{CC} - V_{BE} \Rightarrow R = \frac{2(V_{CC} - V_{BE})}{I_{ref}}$$

#5



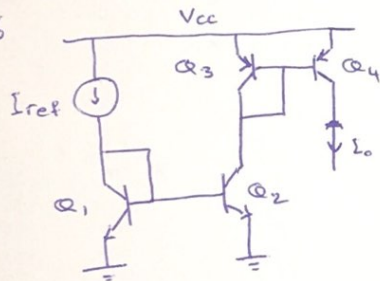
$$A = 3B$$

$$\text{KVL @ A: } -10 + 0.7 + 4.7^k(I_{C1}) = 0 \Rightarrow I_{C1} = \frac{10 - 0.7}{4.7} = 1.9 \text{ mA}$$

$$\Rightarrow I_{C2} = \frac{1}{5} I_{C1} = 0.38 \text{ mA}$$

$$\Rightarrow -10 + 10^k I_{C4} + V_{out} = 0 \Rightarrow V_{out} = 10 - 10^k I_{C4} = 10 - 10^k (0.38) = 6.2 \text{ V}$$

#6



$$\begin{cases} I_{C2} = I_{C4} = 2 I_{C1} \\ I_{C3} = 3 I_{C1} \\ \Rightarrow I_o = I_{C4} = 2 I_{ref} \end{cases}$$