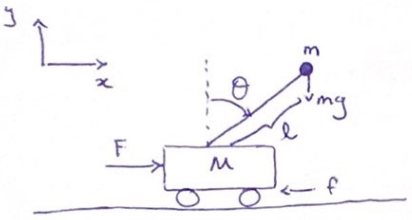
یا علی

رضا ادیبی پور
۹۸۱۴۳۰۴
تمرین سری ۴ کنترل خطی

#1

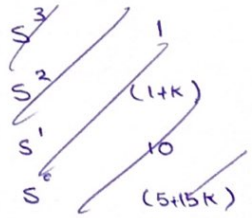


a) $\begin{cases} x_g = x + l\sin\theta \\ y_g = l\cos\theta \end{cases} \Rightarrow M\ddot{x} + m\ddot{x}_g = f \Rightarrow M\ddot{x} + m\ddot{x} + m(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) = f$

$\Rightarrow (F_x \cos\theta)l - (F_y \sin\theta)l = (mg\sin\theta)l \Rightarrow \begin{cases} F_x = m\ddot{x}_g = m(\ddot{x} - l\dot{\theta}^2\sin\theta + l\ddot{\theta}\cos\theta) \\ F_y = m\ddot{y}_g = -m(l\dot{\theta}^2\cos\theta + l\ddot{\theta}\sin\theta) \end{cases}$

$\Rightarrow m\ddot{x}\cos\theta + ml\ddot{\theta} = mg\sin\theta$

#2 $\quad s^3 + (1+K)s^2 + 10s + (5+15K) = 0$



s^3	1	10
s^2	$(1+K)$	$(5+15K)$
s^1	$\frac{10(1+K)-5-15K}{1+K}$	0
s^0	$5+15K$	

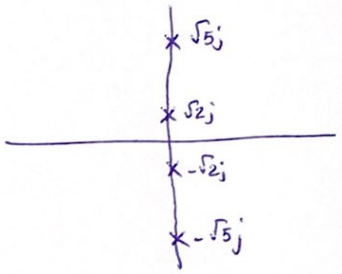
$10(1+K) - 5 + 15K > 0 \rightarrow K > -\frac{1}{5}$ OR $K > -0.2$

#3

$$\begin{cases} G(s) = \dfrac{10s^2 + 0.25s + 2}{s^4 + 7s^2 + 10} \\ u = 2\sin 10t \end{cases}$$

قطب‌ها: $s^4 + 7s^2 + 10 = 0$

$$\begin{cases} s_1 = \sqrt{2}j \\ s_2 = -\sqrt{2}j \\ s_3 = \sqrt{5}j \\ s_4 = -\sqrt{5}j \end{cases}$$



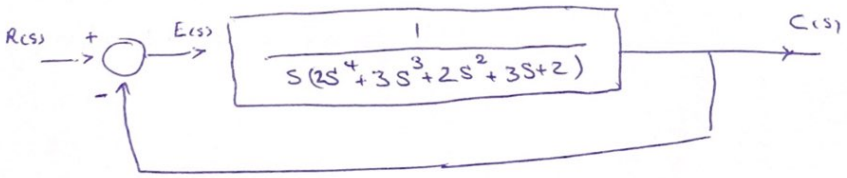
$$u(t) = 2\sin 10t \longleftrightarrow U(s) = \frac{20}{s^2+100}$$

$$G(s) = \frac{Y(s)}{U(s)} \Rightarrow Y(s) = U(s)G(s) = \frac{20}{s^2+100} \times \frac{10s^2 + 0.25s + 2}{s^4 + 7s^2 + 10}$$

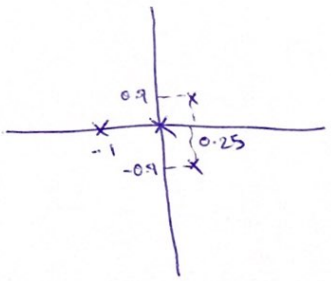
$$= \frac{20(10s^2 + 0.25s + 2)}{(s^2+100)(s^4+7s^2+10)} \Rightarrow \text{قطب‌ها:} \begin{cases} s_{1,2} = \pm\sqrt{2}j \\ s_{3,4} = \pm\sqrt{5}j \\ s_{5,6} = \pm 10j \end{cases}$$

تمام قطب‌ها روی محور $j\omega$ قرار دارند ← مرزی مستقر هم پایدار است.

#4



$$s(2s^4 + 3s^3 + 2s^2 + 3s + 2) = 0 \longrightarrow \begin{cases} s_1 = 0 \\ s_2 = -1 \\ s_3 = \frac{1}{4} + j0.9 \\ s_4 = \frac{1}{4} - j0.9 \end{cases}$$



یک قطب در مبدأ، یک قطب در سمت چپ محور $j\omega$ و دو قطب مختلط در سمت راست محور $j\omega$ دارد

#5 $s^4 + 3s^3 + 30s^2 + 30s + 200 = 0$

s^4	1	30	0
s^3	3	200	0
s^2	$\frac{-110}{3}$	0	0
s^1	200	0	0
s^0	0		