سوال (5)

$x_1 \xleftrightarrow{F.S} a_k$ periodic with $T_1 \left(\frac{2\pi}{\omega_1}\right)$

$$x_2(t) = \underbrace{2x_1(1-t)}_{(I)} - \frac{1}{3}x_1(2t-1) \longrightarrow \begin{cases} T = ? \\ b_k = ? \end{cases}$$

$$\text{If } x_1(t) \xleftrightarrow{F.S} a_k \Rightarrow x_1(t+1) \xleftrightarrow{F.S} a_k e^{+jk\omega_1 \times 1}$$

$$\text{If } x(-t) \xleftrightarrow{F.S} a_{-k} \Rightarrow x_1(-t+1) = x(1-t) \xleftrightarrow{F.S} a_{-k} e^{+jk\omega_1}$$

$$x(t-1) \xleftrightarrow{F.S} a_k e^{-jk\omega_1}, \quad x(2t-1) \xleftrightarrow{F.S} a_k e^{-jk2\omega_1}$$

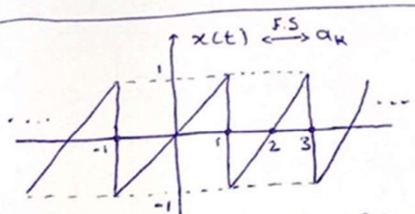
ضرایب تغییری نمی‌کند

فقط ω_1، 2 برابر می‌شود ← (T نصف می‌شود)

$$\boxed{\Rightarrow a_k = 2a_{-k}e^{jk\omega_1} - \frac{1}{3}a_k e^{-jk2\omega_1}}$$

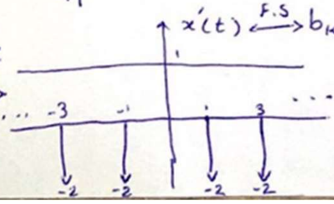
$$, \quad \begin{cases} 2x_1(1-t) \longrightarrow T_1 \\ -\frac{1}{3}x_1(2t-1) \longrightarrow \frac{T_1}{2} \end{cases} \Rightarrow \text{kmm}\left\{T_1, \frac{T_1}{2}\right\} = T_1$$

سوال (6)



روش اول ← استفاده از خواص

مشتق می‌گیریم



می‌دانیم :

$$s(t) \xleftrightarrow{S.S} d_k = \frac{1}{T} = \frac{1}{2}$$

$$\left\{\begin{array}{l} x'(t) = -2\delta(t-1) + 1 \\ \to (jk\omega_0)\, a_k = -2\, e^{-jk\omega_0 \times 1} \times \underbrace{\tfrac{1}{2}}_{= d_k} \end{array}\right\} \to a_k = \frac{-1}{jk\omega_0} e^{-jk\omega_0} \xrightarrow{\omega_0 = \frac{2\pi}{2} = \pi}$$

$$\boxed{a_k = \frac{-1}{jk\pi} e^{-jk\pi} = \frac{j}{k\pi}(-1)^k \quad \text{for } k \neq 0}$$

روش دوم $\longrightarrow$ فرمول آنالیز : $a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt \xrightarrow{T=2} a_k = \frac{1}{2}\int_{-1}^{+1} t\, \underbrace{e^{-jk\omega_0 t}\, dt}$

مشتق	انتگرال
t	$e^{-jk\omega_0 t}$
1	$\frac{-1}{jk\omega_0} e^{-jk\omega_0 t}$
0	$\frac{-1}{(jk\omega_0)^2} e^{-jk\omega_0 t}$

$$= \frac{1}{2}\left[\frac{-t}{jk\omega_0} e^{-jk\omega_0 t} \Big|_{-1}^{+1} + \frac{1}{(jk\omega_0)^2} e^{-jk\omega_0 t} \Big|_{-1}^{+1} \right]$$

$$= \frac{1}{2}\left[e^{-jk\omega_0}\left(\frac{1}{(k\omega_0)^2} - \frac{1}{jk\omega_0} \right) - e^{jk\omega_0}\left(\frac{1}{(k\omega_0)^2} + \frac{1}{jk\omega_0} \right) \right]$$

$$= -\frac{1}{2}\left[\frac{1}{(k\omega_0)^2}\left(e^{jk\omega_0} - e^{-jk\omega_0} \right) + \left(e^{jk\omega_0} + e^{-jk\omega_0} \right) \frac{1}{jk\omega_0} \right]$$

a) $a_k = a_{k+2}$	سوال 7
b) $a_k = a_{-k}$	$T=3$
c) $\int_{-0.5}^{0.5} x(t)\,dt = 1$	
d) $\int_{0.5}^{1.5} x(t)\,dt = 2$	
$x(t) = ?$	

طبق خواص سری فوریه می‌دانیم $x(-t) \xleftrightarrow{F.S} a_{-k}$

پس از خاصیت b می‌توان فهمید که: $\boxed{x(t)=x(-t)}$

طبق خاصیت جابه‌جایی در فرکانس: $e^{j\omega_0 M t}x(t) \xleftrightarrow{F.S} a_{k-M}$

از خاصیت a می‌توان فهمید که: ⟵

$$\underbrace{x(t)=x(t)e^{-j\frac{4\pi}{3}t}}$$

فقط در نقاط $\Rightarrow t=0, \pm 1.5, \pm 3, \pm 4.5, \pm 6, \ldots$

$t = \pm 1.5t$ مقدار غیرصفر دارد به طوری که شرط a را ارضا کند

باید $e^{-j\frac{4\pi}{3}t}=1 \Leftarrow$ شود تا a صحیح باشد

از خاصیت c می‌توان فهمید که: $x(t)=\delta(t) \;;\; -\frac{1}{2}<t<\frac{1}{2}$

همچنین از خاصیت d می‌توان فهمید: $x(t)=2\delta(t-1.5) \;;\; 0.5<t<1.5$

بنابراین می‌توان نوشت: $x(t) = \sum_{k=-\infty}^{+\infty} \delta(t-3k) + 2\sum_{k=-\infty}^{+\infty} \delta(t-1.5-3k)$

if $x(t) \xleftrightarrow{F.S} a_k$ سؤال (8)

$$z(t) = \underbrace{\cos(M\omega_0 t)}_{(III)} \cdot \underbrace{\frac{d}{dt}\left(\underbrace{x(t-t_0)}_{(I)}\right)}_{(II)} \xleftrightarrow{F.S} ?$$

I) $x(t-t_0) \xleftrightarrow{F.S} a_k e^{-jk\omega_0 t_0}$

II) $\frac{d}{dt}\left(x(t-t_0)\right) \xleftrightarrow{F.S} jk\omega_0 \left(a_k e^{-jk\omega_0 t_0}\right)$

III) $\cos(M\omega_0 t) : \Rightarrow \frac{1}{2}\left(e^{jM\omega_0 t} + e^{-jM\omega_0 t}\right)$

یادآوری : $\cos x = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)$

$$\cos(\omega_0 t) = \frac{1}{2} e^{j\omega_0 t} + e^{-j\omega_0 t} \Rightarrow b_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t} dt = \frac{\omega_0}{2\pi} \times \frac{1}{2}\int_0^{\frac{2\pi}{\omega_0}} \left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) e^{-jk\omega_0 t} dt$$

$$= \frac{\omega_0}{4\pi} \int_0^{\frac{2\pi}{\omega_0}} e^{j\omega_0 t} e^{-jk\omega_0 t} dt + \int_0^{\frac{2\pi}{\omega_0}} e^{-j\omega_0 t} \cdot e^{-jk\omega_0 t} dt$$

$$= \frac{\omega_0}{4\pi} \int_0^{\frac{2\pi}{\omega_0}} e^{j(\omega_0 - k\omega_0)t} dt + \int_0^{\frac{2\pi}{\omega_0}} e^{-j(\omega_0 + k\omega_0)t} dt$$

$$= \frac{\omega_0}{4\pi}\left(\frac{2\pi}{\omega_0} e^{j(\omega_0 - k\omega_0)} + \frac{2\pi}{\omega_0} e^{-j(\omega_0 + k\omega_0)}\right) = \underbrace{\frac{1}{2}\left(e^{j(\omega_0 - k\omega_0)} + e^{-j(\omega_0 + k\omega_0)}\right)}$$

ضرایب سری فوریه $\cos(\omega_0 t)$

$$\Rightarrow \cos(M\omega_0 t) \xleftrightarrow{F.S} \frac{1}{2}\left(e^{j(M\omega_0 - k\omega_0 M)} + e^{-j(M\omega_0 + kM\omega_0)}\right)$$

$$\Rightarrow z(t) \xleftrightarrow{F.S} c_k = \frac{jk\omega_0}{2}\left(a_k e^{-jk\omega_0 t_0}\right) \cdot \left(e^{jM(\omega_0 - k\omega_0)} + e^{-jM(\omega_0 + k\omega_0)}\right)$$

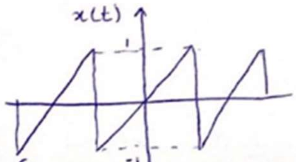
سوال ۹

$$w(t) = \sum_{k=-\infty}^{+\infty} \left(\delta(t-0.5-2k) - \delta(t+0.5-2k) \right)$$

یادآوری :

$$x(t) \circledast y(t) = \int_T x(\tau)\, y(t-\tau)\, d\tau$$

$$x(t) \circledast y(t) \xleftrightarrow{F.S} T a_k b_k$$

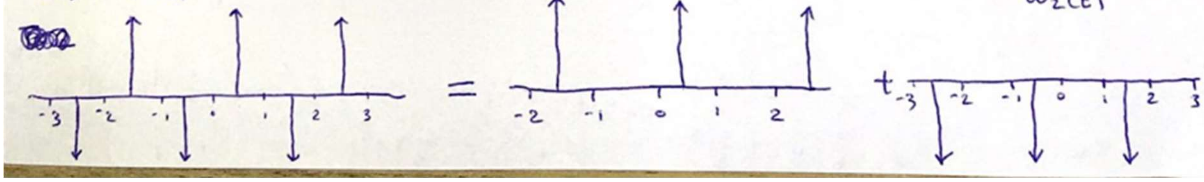


$\xleftrightarrow{a_k}$

ضرایب سری فوریه این سیگنال در مثال 6 بدست آورده شد

$w(t) \xleftrightarrow{F.S} b_k$

$w_1(t)$ $w_2(t)$



$$\Rightarrow w(t) = w_1(t) + w_2(t) \xleftrightarrow{F.S} b_k = c_k + d_k$$

$$w_1(t) \xleftrightarrow{F.S} \frac{1}{2} e^{-jk\frac{\pi}{2}}$$

$$w_2(t) \xleftrightarrow{F.S} -\frac{1}{2} e^{jk\frac{\pi}{2}} \quad \Rightarrow b_k = \frac{1}{2}\left(e^{-jk\frac{\pi}{2}} - e^{jk\frac{\pi}{2}} \right)$$

$$x(t) \xleftrightarrow{F.S} a_k = \frac{-1}{jk\pi} e^{-jk\pi} \Rightarrow T a_k b_k = 2 \times \frac{1}{2}\left(e^{-jk\frac{\pi}{2}} - e^{jk\frac{\pi}{2}} \right) \cdot \frac{-1}{jk\pi} e^{-jk\pi}$$

$$= \underbrace{\frac{-1}{jk\pi} e^{-jk\pi} \left(e^{-jk\frac{\pi}{2}} - e^{jk\frac{\pi}{2}} \right)}_{(I)} \Longrightarrow x(t) \circledast y(t) = FS^{-1}\{I\}$$

روش دوم : $x(t) \circledast y(t) = \int_T x(\tau)\, y(t-\tau)$

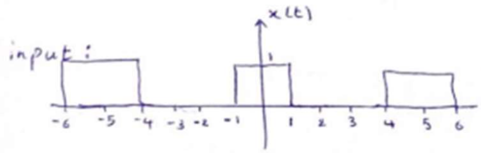
تفاوت کانولوشن پریودیک با کانولوشن معمولی تنها در بازه انتگرال گیری است .

سوال (10)

[illegible]

$$\frac{d^2y(t)}{dt^2} - 5\frac{dy(t)}{dt} + 6y(t) = 3\frac{dx(t)}{dt}$$

input :



$$x(t) = e^{j\omega t} \longrightarrow \boxed{LTI} \longrightarrow H(j\omega)e^{j\omega t} = y(t)$$

$$\Rightarrow \frac{d^2}{dt^2}\left(H(j\omega)e^{j\omega t}\right) - 5\left(H(j\omega)e^{j\omega t}\right) + 6H(j\omega)e^{j\omega t} = 3e^{j\omega t}$$

$$= (j\omega)^2 H(j\omega)e^{j\omega t} - 5(j\omega)H(j\omega)e^{j\omega t} + 6H(j\omega)e^{j\omega t} = 3e^{j\omega t}$$

$$= -\omega^2 H(j\omega)e^{j\omega t} - 5j\omega H(j\omega)e^{j\omega t} + 6H(j\omega)e^{j\omega t} = 3e^{j\omega t}$$

$$\Rightarrow H(j\omega)\left(-\omega^2 e^{j\omega t} - 5j\omega e^{j\omega t} + 6e^{j\omega t}\right) = 3e^{j\omega t}$$

$$\Rightarrow H(j\omega) = \frac{3e^{j\omega t}}{6e^{j\omega t} - 5j\omega e^{j\omega t} - \omega^2 e^{j\omega t}} = \frac{3\cancel{e^{j\omega t}}}{\cancel{e^{j\omega t}}(6 - 5j\omega - \omega^2)} = \frac{3}{-\omega^2 - 5j\omega + 6}$$

$$\Rightarrow y(t) = \sum_{k=-\infty}^{+\infty} \underbrace{a_k H(jk\omega_0)}_{= b_k} e^{jk\omega_0 t} \quad , \quad x(t) \xleftrightarrow{F.S} a_k = \underset{\text{duty cycle}}{d} \cdot \mathrm{Sinc}(kd) = \frac{2}{5}\mathrm{Sinc}\left(\frac{2k}{5}\right)$$

$$\boxed{[illegible]: \quad y(t) = x(t) * h(t) \xleftrightarrow{F.T} Y(j\omega) = X(j\omega)H(j\omega)}$$

$$a_0 = \frac{2}{5} \qquad a_2 = \frac{2}{5}\mathrm{Sinc}\left(\frac{4}{5}\right)$$

$$a_1 = \frac{2}{5}\mathrm{Sinc}\left(\frac{2}{5}\right) \qquad a_3 = \frac{2}{5}\mathrm{Sinc}\left(\frac{6}{5}\right)$$

$$\Rightarrow y(t) = \frac{2}{5}H(0)e^{j0} + \frac{2}{5}\left(\mathrm{Sinc}\left(\frac{2}{5}\right)H(j\omega_0)e^{j\omega_0 t} + \mathrm{Sinc}\left(\frac{4}{5}\right)H(2j\omega_0)e^{j\omega_0 t} + \dots\right)$$