(۱)

A) $\delta(t+1)+\delta(t-2)$

$$x(jw)=\int_{-\infty}^{+\infty} x(t)e^{-jwt}dt=\int_{-\infty}^{+\infty}(\delta(t+1)+\delta(t-2))e^{-jwt}dt$$

$$=e^{jw}+e^{-2jw} \longrightarrow |x_1(jw)|=e^{jw}+e^{-2jw}$$

B) $1+\cos(2\pi t+\frac{\pi}{4})$

$$X(jw)=\mathcal{F}\{x(t)\}=\mathcal{F}\{1+\cos(2\pi t+\tfrac{\pi}{4})\}=\mathcal{F}\{1+\tfrac{1}{2}e^{j(2\pi t+\frac{\pi}{4})}+e^{-j(2\pi t+\frac{\pi}{4})}\}$$

$$=\mathcal{F}\{1\}+\frac{e^{j\frac{\pi}{4}}}{2}\mathcal{F}\{e^{j2\pi t}\}+\frac{e^{j\frac{\pi}{4}}}{2}\mathcal{F}\{e^{-j2\pi t}\}=2\pi\delta(w)$$

$$+\frac{e^{j\frac{\pi}{4}}}{2}(2\pi\delta(w-2\pi))+\frac{e^{-j\frac{\pi}{4}}}{2}(2\pi\delta(w+2\pi)=$$

$$=2\pi\delta(w)+\pi(e^{j\frac{\pi}{4}}\delta(w-2\pi)+e^{-j\frac{\pi}{4}}=e^{-j\frac{\pi}{4}}\delta(w+2\pi))$$

C) $e^{-2|t-1|}$

$$X(jw)=\int_{-\infty}^{+\infty}e^{-2|t-1|}e^{-jwt}dt=\int_{-\infty}^{+\infty}e^{-2|t-1|-jwt-1}dt$$

$$=e^{-jw}\int_{-\infty}^{0}e^{(2-jw)t}dt+e^{-jw}\int_{0}^{+\infty}e^{-(2+jw)t}dt$$

$$=e^{-jw}\left(\frac{e^{(2-jw)t}}{2-jw}\Big|_{-\infty}^{0}-\frac{e^{-(2+jw)t}}{2+jw}\Big|_{0}^{+\infty}\right)=$$

$$e^{-j\omega}\left(\frac{1}{2-j\omega}+\frac{1}{2+j\omega}\right)=\frac{4e^{-j\omega}}{4+\omega^2}$$

$$X(j\omega)=\left|\frac{4e^{-j\omega}}{2-j\omega}\right|=\frac{4}{4+\omega^2}$$

D) $$X(j\omega)=\int_{-1}^{1} x e^{-j\omega t}\,dt=\frac{-1}{j\omega}e^{-j\omega t}\Big|_{-1}^{1}=\frac{-1}{j\omega}$$

$$\left(e^{-j\omega}-e^{j\omega}\right)=\frac{2\sin\omega}{\omega}$$

سه‌شنبه ۷ شهریور

TUE. 28 AUG ۱۰ شوال

۲) $\frac{\sin w_c t}{\pi t}$

$$x(I) \leftrightarrow 2\pi x(-w) \rightarrow \frac{\sin wt}{\pi t} \leftrightarrow 2\pi\left(\pi\left(\frac{w}{2\pi}\right)\right)$$

۳)

$$\mathcal{F}\left\{\frac{\sin t}{\pi t}\right\} \begin{cases} 1 & |w|<1 \\ . & |w|>1 \end{cases} \Rightarrow \mathcal{F}\left\{\left(\frac{\sin t}{\pi t}\right)^2\right\} = \left(\frac{\sin t}{\pi t}\right) * \left(\frac{\sin t}{\pi t}\right)$$

$$= \frac{1}{2\pi}\mathcal{F}\left\{\left(\frac{\sin t}{\pi t}\right)\right\} * \mathcal{F}\left\{\frac{\sin t}{\pi t}\right\} \Rightarrow \mathcal{F}\left\{\left(\frac{\sin t}{\pi t}\right)^2\right\}$$

$$= \frac{1}{2\pi}\begin{cases} w+2 & -2 \le w < 0 \\ -w+2 & 0 \le w < 2 \\ 0 & else \end{cases}$$

$$x(jw) = \mathcal{F}\left\{t\left(\frac{\sin t}{\pi t}\right)^2\right\} = j\frac{d}{dw}\mathcal{F}\left\{\left(\frac{\sin t}{\pi t}\right)^2\right\} =$$

$$\frac{j}{2\pi}\cdot\frac{d}{dw}\begin{cases} w+1 & -2 < w < 0 \\ -w+2 & 0 < w < 2 \\ 0 & else \end{cases}$$

۴) $X(jw) = \int_{-1}^{1} (1+\cos \pi t) e^{-jwt}\, dt = \int_{-1}^{1} e^{-jwt}\, dt$

$$+ \int_{-1}^{1} \cos \pi t\, e^{-jwt}\, dt$$

$$= \frac{-1}{jw} e^{-jwt} \Big|_{-1}^{1} + \frac{1}{2j(\pi - w)} \left(e^{j(\pi - w)} - e^{-j(\pi - w)}\right)$$

$$+ \frac{-1}{2j(\pi + w)} \left(e^{-j(\pi + w)} - e^{j(\pi + w)}\right)$$

$$= \frac{2 \sin w}{w} + \frac{\sin(\pi - w)}{\pi - w} + \frac{\sin(\pi + w)}{\pi + w}$$

جمعه ۱۰ شهریور
FRI 31 AUG ۱۳ شوال

a)

$$\mathcal{F}\{te^{-|t|}\} = j\frac{d}{dw}\mathcal{F}\{e^{-|t|}\} = j\frac{d}{dw}\left(\frac{2}{1+w^2}\right) = \frac{-4jw}{(1+w^2)^2}$$

b)

$$e^{-|t|} = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(\frac{-4jw}{(1+w^2)^2}\right)e^{jwt}\,dw \rightarrow -2\pi j\,te^{-|t|}$$

$$= \int_{-\infty}^{+\infty}\left(\frac{4w}{(1+w^2)^2}\right)e^{-jwt}\,dt$$

$$-2\pi jw\,e^{-|w|} = \int_{-\infty}^{+\infty}\left(\frac{4t}{(1+t^2)^2}\right)e^{-jwt}\,dt$$

$$\mathcal{F}\left(\frac{4t}{(1+t^2)^2}\right) = -2\pi jw\,e^{-|w|}$$

شهادت آیت الله قدوسی و سرتیپ وحید دستجردی(۱۳۶۰هـ ش)

۶)

a) $X(w) = \dfrac{2\sin(3(w-2\pi))}{w-2\pi}$

$$\mathcal{F}^{-1}\{X(jw)\} = \mathcal{F}^{-1}\left\{\frac{2\sin(3(w-2\pi))}{w-2\pi}\right\} = e^{j2\pi t}\,\mathcal{F}^{-1}\left\{\frac{2\sin 3w}{w}\right\}$$

$$= e^{j2\pi t}\,sp(t)\ ,\quad T=3\ ,$$

پالس مستطیل
به ارتفاع ۱

b) $2[\delta(w-1)-\delta(w+1)] + 3[\delta(w-2\pi)+\delta(w+2\pi)]$

$$\mathcal{F}^{-1}\{X(jw)\} = \mathcal{F}^{-1}\{2(\delta(w-1)-\delta(w+1)) + 3(\delta(w-2\pi)+\delta(w+2\pi)$$

$$= 2\mathcal{F}^{-1}\{\delta(w-1)\} - 2\mathcal{F}^{-1}\{\delta(w+1)\} + 3\mathcal{F}^{-1}\{\delta(w-2\pi)\}$$

$$+ 3\mathcal{F}^{-1}\{\delta(w+2\pi)\}$$

$$= \frac{2}{2\pi}e^{jt} - \frac{2}{2\pi}e^{-jt} + \frac{3}{2\pi}e^{j2\pi t} + \frac{3}{2\pi}e^{j2\pi t} = \frac{2}{\pi}\sin t + \frac{3}{\pi}\cos 2\pi t$$

پنج‌شنبه ۱۶ شهریور

THU. 6 SEP ۱۹ شوال

2012 سپتامبر

c) $X(jw) = |x(jw)|e^{j\angle X(jw)} = \begin{cases} |w|e^{-jcw} & |w|<1 \\ 0 & else \end{cases}$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(jw)e^{jwt}dw = \frac{1}{2\pi}\int_{-1}^{1}|w|e^{-jcw}e^{jcw}dw$$

$$= \frac{-1}{2\pi}\int_{-1}^{0} we^{jw(t-c)}dw + \frac{1}{2\pi}\int_{0}^{1} we^{jw(t-c)}dw$$

$$= \frac{1}{2\pi}\int_{0}^{1} we^{-jw(t-c)}dw + \frac{1}{2\pi}\int_{0}^{1} we^{jw(t-c)}dw$$

جمعه ۱۷ شهریور

FRI. 7 SEP ۲۰ شوال

قیام ۱۷ شهریور و کشتار جمعی از مردم به دست مأموران ستم شاهی پهلوی(۱۳۵۷هـ.ش)

$$= \frac{1}{\pi}\int_{0}^{1} w\cos w(t-c)dw$$

$$= \frac{1}{\pi}\left\{\frac{w\sin w(t-c)}{t-c}\Big|_0^1 - \int_0^1 \frac{\sin w(t-c)}{t-c}dw\right\} = \frac{1}{\pi}$$

$$= \frac{1}{\pi}\left\{\frac{\sin(t-c)}{t-c} + \frac{\cos(t-c)-1}{(t-c)^2}\right\}$$

۷) $x(t) = z(t-1) \rightarrow X(j\omega) = e^{-j\omega} Z(j\omega)$

الف) $|X(j\omega)| = |e^{-j\omega}| \, |Z(j\omega)| = |Z(j\omega)| \quad \measuredangle X(j\omega) = -\omega$

ب) $X(0) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \Big|_{\omega=0} = \int_{-\infty}^{+\infty} x(t)\, dt = \pi$

ج) $\int_{-\infty}^{+\infty} X(j\omega)\, d\omega = \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega(0)} d\omega = 2\pi x(0) = 4\pi$

د) $Y(j\omega) = \frac{2\sin\omega}{\omega} e^{j2\omega} \rightarrow y(t) = \mathcal{F}^{-1}\left\{\frac{2\sin\omega}{\omega} e^{j2\omega}\right\} = \mathrm{sp}(t+2)$

پالس مستطیل ارتفاع ۱ و $t=1$

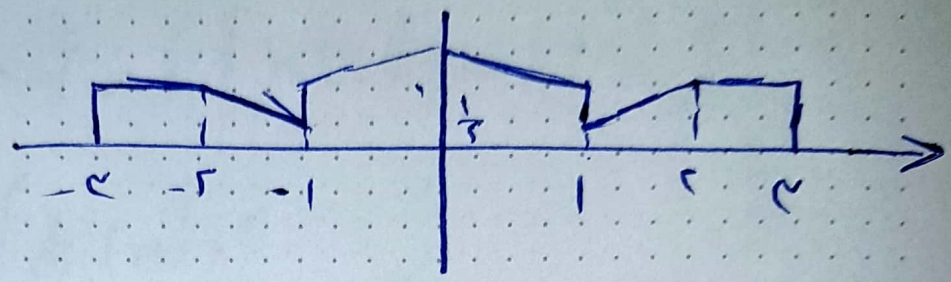
$$\int_{-\infty}^{+\infty} X(j\omega) \frac{2\sin\omega}{\omega} e^{j2\omega} d\omega = \int_{-\infty}^{+\infty} X(j\omega) Y(j\omega)\, d\omega$$

$$= \int_{-\infty}^{+\infty} X(j\omega) Y(j\omega) e^{j\omega(0)} d\omega$$

$$= 2\pi \mathcal{F}^{-1}\{X(j\omega) Y(j\omega)\}_{t=0} = 2\pi \{x(t) * y(t)\}_{t=0}$$

$$= 2\pi \int_{-\infty}^{+\infty} x(z)\, y(0-z)\, dz = 7\pi$$

۵) $Re\{X(j\omega)\} = x(t)$



۶) $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi \int_{-\infty}^{+\infty} |x(t)|^2 dt$

$= 2\pi \left(\frac{38}{3}\right) = 268 - \frac{2}{3}\pi$

1) $Y(j\omega) = \mathcal{F}\{y(t)\} = \mathcal{F}\{x(t) * h(t)\} = X(j\omega)H(j\omega)$

$G(j\omega) = \mathcal{F}\{y(j\omega)\} = \mathcal{F}\{x(at) * h(at)\}$

$\mathcal{F}\{x(at)\} \cdot \mathcal{F}\{h(at)\}$

$= \left[\frac{1}{a} \times \frac{j\omega}{a}\right]\left[\frac{1}{a} H\left(\frac{j\omega}{a}\right)\right] = \frac{1}{a^2}\left(\frac{j\omega}{a}\right) H\left(\frac{j\omega}{a}\right)$

$G(j\omega) = \frac{1}{a^2} Y\left(\frac{j\omega}{a}\right)$

۹)

الف) $x(t)=\delta(t) \quad y(t)=h(t)$

$$\frac{d^2h(t)}{dt^2}+4\frac{dh(t)}{dt}+8h(t)=2\delta(t)$$

$$(j\omega)^2H(j\omega)+4(j\omega)H(j\omega)+8H(j\omega)=2$$

$$H(j\omega)=\frac{2}{(j\omega)^2+4j\omega+8}=\frac{2}{(j\omega+2)(j\omega+4)}=\frac{-1}{j\omega+2}+\frac{1}{j\omega+4}$$

$$h(t)=\mathcal{F}^{-1}\{H(j\omega)\}=e^{-4t}u(t)+e^{-2t}u(t)=(e^{-2t}-e^{-4t})u(t)$$

ب) $y(t)=x(t)*h(t)\rightarrow Y(j\omega)=X(j\omega)H(j\omega)=\mathcal{F}\{te^{-2t}\}H(j\omega)$

$$=\left[\frac{d}{d\omega}\left(\frac{1}{j\omega+2}\right)\right]\left(\frac{1}{(j\omega+2)^2}\right)\left(\frac{2}{(j\omega+2)(j\omega+4)}\right)$$

$$=\frac{3/4}{j\omega+4}-\frac{1/2}{(j\omega+2)^2}+\frac{1}{(j\omega+2)^3}+\frac{1/4}{(j\omega+2)^2}-\frac{3/4}{j\omega+2}$$

$$=\frac{1}{4}e^{-2t}+te^{-2t}+\frac{1}{2}t^2e^{-2t}-\frac{3}{4}e^{-2t}-\frac{1}{4}e^{-4t}$$

۱۰)

الف) $X(j\omega) = \mathcal{F}\{x(t)\} = \mathcal{F}\{e^{-t}u(t) + e^{-3t}u(t)\}$

$$= \frac{1}{j\omega+1} + \frac{1}{j\omega+3} = \frac{2(j\omega+2)}{(j\omega+1)(j\omega+3)}$$

$$y(j\omega) = \mathcal{F}\{y(t)\} = \mathcal{F}\{2e^{-t}u(t) - 2e^{-4t}u(t)$$

$$= \frac{2}{j\omega+1} - \frac{2}{j\omega+4} = \frac{6}{(j\omega+1)(j\omega+4)}$$

ب) $h(t) = \mathcal{F}^{-1}\{H(j\omega)\} = \mathcal{F}^{-1}\left\{\frac{3(3+j\omega)}{(2+j\omega)(4+j\omega)}\right\}$

$$\mathcal{F}^{-1}\left\{\frac{3/2}{2+j\omega} + \frac{3/2}{4+j\omega}\right\} = \frac{3}{2}e^{-2t}u(t) + \frac{3}{2}e^{-4t}u(t)$$

$$\frac{3}{2}(e^{-2t} + e^{-4t})u(t)$$

سعید رفیع
۹۸۱۹۰۵۳