باللطیف

رضا ادیب پور
۹۸۱۴۳۳۰۳

امتحان دوم محاسبات عددی

گروه ۳

#1

$$\begin{cases} f(x) = \sqrt[3]{x^3+5} \\ f''(0.1) = ? \end{cases}$$

+0.2

x	0.1	0.3	0.5	0.7	0.9	1.1
f(x)	1.710	1.713	1.724	1.748	1.789	1.849

$\Delta f_0 = f_1 - f_0 = 1.713 - 1.710 = 0.0030$

$\Delta f_1 = f_2 - f_1 = 0.0110$

... الی آخر

x	f(x)	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$	$\Delta^5 f$
0.1	1.710	0.0030	0.0080	0.0050	-0.0010	-0.0010
0.3	1.713	0.0110	0.0130	0.0040	-0.0020	
0.5	1.724	0.0240	0.0170	0.0020		
0.7	1.748	0.0410	0.0190			
0.9	1.789	0.0600				
1.1	1.849					

$$f''(x_0) = \frac{1}{h^2}\left(\Delta^2 f_0 - \Delta^3 f_0 + \Delta^4 f_0 - \Delta^5 f_0\right) = \frac{1}{(0.2)^2} \times \left(0.0080 - 0.0050 - 0.0010 + 0.0010\right)$$

$$\approx 0.075 = f''(0.1)$$

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#2 $I = \int_0^1 e^{-x^2} dx$

$h = 1 \rightarrow T_1 = \frac{1}{2}\left(f(0) + f(1)\right) = \frac{1}{2}\left(1 + 0.376\right) = 0.6839$

$h = \frac{1}{2} \rightarrow T_2 = \frac{1}{4}\left(f(0) + 2f(\frac{1}{2}) + f(1)\right) = 0.7313$

$h = \frac{1}{4} \rightarrow T_4 = \frac{1}{8}\left(f(0) + 2f(\frac{1}{4}) + 2f(\frac{1}{2}) + 2f(\frac{3}{4}) + f(1)\right) = 0.7429$

$h = \frac{1}{8} \rightarrow$ [illegible] $\rightarrow T_8 = 0.7458$

$T_2^{(1)} = \frac{4T_2 - T_1}{3} = 0.7471$, $T_4^{(1)} = \frac{4T_4 - T_2}{3} = 0.7468$, $T_8^{(1)} = \frac{4T_8 - T_4}{3} = 0.7468$

$T_4^{(2)} = \frac{16T_4^{(1)} - T_2^{(1)}}{15} = T_8^{(2)} = T_8^{(3)} = 0.7468$

0.6839			
0.7313	0.7471		
0.7429	0.7468	0.7468	
0.7458	0.7468	0.7468	0.7468

$\Rightarrow I \approx 0.7468$

یا لطیف

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#3

$$\begin{cases} 2x - \ln(x) - 3.5 = 0 \\ \text{Aitken} \\ \varepsilon = 0.0001 \end{cases}$$

$f(x) = 2x - \ln(x) - 3.5 \Rightarrow f(2) = -0.19 \,,\ f(3) = 1.40 \Rightarrow f(2)f(3) < 0$

لم دربازه $[2,3]$ یک ریشه داریم.

$2x = \ln(x) + 3.5 \Rightarrow x = \frac{1}{2}\ln(x) + 1.75 = g(x)$

$\Rightarrow g'(x) = \frac{0.5}{x} \Rightarrow |g'(x)| = \frac{0.5}{x} \le \frac{1}{2} < 1 \quad ; \quad \forall x \in [2,3]$

$\Rightarrow$ بنابراین شرایط قضیه نقطه ثابت برقرار است.

$\boxed{x_0 = 0.5} \Rightarrow x_{n+1} = g(x_n) \Rightarrow$

n	x_n	
1	1.40343	
2	1.91946	
3	2.07602	
4	2.11523	
5	2.12458	
6	2.12679	
7	2.12731	
8	2.12743	✓

$$\hat{x}_{n+2} = x_n - \frac{(x_{n+1} - x_n)^2}{x_{n+2} - 2x_{n+1} + x_n}$$

$$\hat{x}_2 = 0.5 - \frac{(1.40343 - 0.5)^2}{1.91946 - 2(1.40343) + 0.5}$$

$= +2.606$

$x_3 = g(\hat{x}_2) = \frac{0.5}{+2.106} = +0.237 \,,\quad x_4 = g(x_3) = \frac{0.5}{-0.237} = +2.109$

$x_5 = g(x_4) = \frac{0.5}{+2.109} = +0.237$

$$\hat{x}_5 = +0.237 - \frac{(+2.109 - 0.237)^2}{+0.237 - 2(+2.109) + 0.237} = 1.173$$

$$\begin{cases} x_6 = g(\hat{x}_5) = 0.426 \\ x_7 = g(x_6) = 1.173 \ \checkmark \\ x_8 = g(x_7) = 0.426 \end{cases}$$