$$H(j\omega) = \int_{-\infty}^{+\infty} h(t) e^{-j\omega t} dt = \frac{\sin(4\omega)}{\omega} \qquad x(t) = \begin{cases} 1 & 0 \le t < 4 \\ -1 & 4 \le t < 8 \end{cases} \qquad T = 8$$ (1

$$\omega_0 = \frac{2\pi}{T} \rightarrow \omega_0 = \frac{\pi}{4} \qquad x(t) = \text{فرد} \longrightarrow a_0 = 0$$

$$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t} dt = \frac{1}{8}\left[\int_0^4 e^{-jk\frac{\pi}{4}t} dt + \int_4^8 -e^{-jk\frac{\pi}{4}t} dt\right] \rightarrow a_k = \begin{cases} 0 & k = \text{even} \\ \frac{2}{jk\pi} & k = \text{odd} \end{cases}$$

$$y(t) = \sum_{k=-\infty}^{+\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}, \quad H(j\omega) = \frac{\sin(\cancel{4}k\frac{\pi}{\cancel{4}})}{k\frac{\pi}{4}}$$

$$\longrightarrow \text{فرد} \quad y(t) = \sum_{k=-\infty}^{+\infty} \frac{2}{jk\pi} \frac{\sin(k\pi)}{\frac{k\pi}{4}} e^{jk\frac{\pi}{4}t} \longrightarrow b_k = a_k H(jk\omega_0) = \frac{2}{jk\pi} \frac{4\sin(k\pi)}{k\pi}$$

$$\text{زوج} \quad y(t) = 0$$

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k], \quad y[n] = \cos\left(\frac{5\pi}{2} n + \frac{\pi}{4}\right), \quad \omega_0 = \frac{\pi}{2}, \quad \frac{N}{m} = \frac{2\pi}{\frac{5\pi}{2}} \rightarrow \boxed{N = 4}$$ -2

$$a_0 = \frac{1}{4}\sum_{k=0}^{3} x[n] = \frac{1}{4}$$

$$a_k = \frac{1}{N}\sum_{k=0}^{3} x[n] e^{-jk\omega_0 n} = \frac{1}{4}\sum_{k=0}^{3}\sum_{k=-\infty}^{\infty} \delta[n-4k] e^{-jnk\frac{\pi}{2}} = \frac{1}{4}$$

$$e^{-jk\frac{\pi}{2}} = \cos\left(k\frac{\pi}{2}\right) - j\sin\left(k\frac{\pi}{2}\right) = \begin{cases} k=0 \longrightarrow 1 \\ k=1 \longrightarrow -j \\ k=2 \longrightarrow -1 \\ k=3 \longrightarrow j \end{cases}$$

$$y[n] = \sum_{-\infty}^{+\infty} a_k H(e^{jk\omega_0}) e^{jk\omega_0 n} = \sum_{k=0}^{3} \frac{1}{4} H(e^{jk\frac{\pi}{2}}) e^{jk(\frac{\pi}{2})n} = \frac{1}{4}H(e^0)e^0 + \frac{1}{4}H(e^{j\frac{\pi}{2}})e^{j\frac{\pi}{2}n}$$

$$+ \frac{1}{4}H(e^{j\pi})e^{j\pi n} + \frac{1}{4}H(e^{j\frac{3\pi}{2}})e^{j\frac{3\pi}{2}n}$$

$$\longrightarrow \begin{cases} H(e^{j\frac{\pi}{2}}) = 2e^{j\frac{\pi}{4}} \\ H(e^{j\frac{3\pi}{2}}) = 2e^{-j\frac{\pi}{4}} \\ H(e^0) = 0 \\ H(e^{j\pi}) = 0 \end{cases}$$

الف) $x[n]=(-1)^n \quad \omega_0=\pi \quad N=2$

$$a_0=\frac{1}{N}\sum_{K=0}^{1}x[n]=\frac{1}{2}\sum_{K=0}^{1}(-1)^n=0$$

$$a_K=\frac{1}{N}\sum_{K=0}^{1}x[n]e^{-jK\omega_0 n}=\frac{1}{2}\sum_{K=1}^{2}(-1)^n e^{-jK\pi n}=\frac{1}{2}\left[e^{j\pi n}\left(e^{-j\pi n}+e^{-j2\pi n}\right)\right]$$

$$y[n]=\sum_{K=0}^{1}a_K H(e^{jK\pi})e^{jK\pi n}=0$$

ب) $x[n]=1+\sin\left(\frac{3\pi}{8}n+\frac{\pi}{4}\right) \qquad \frac{N}{m}=\frac{2\pi}{\frac{3\pi}{8}}\rightarrow N=16 \qquad \omega_0=\frac{2\pi}{N}=\frac{\pi}{8}$

$$x[n]=e^0+\frac{1}{2j}\left[e^{j(\frac{3\pi}{8}n+\frac{\pi}{4})}-e^{-j(\frac{3\pi}{8}n+\frac{\pi}{4})}\right]=e^0+\frac{1}{2j}e^{j\frac{3\pi}{8}n}e^{j\frac{\pi}{4}}-\frac{1}{2}e^{-j\frac{3\pi}{8}n}e^{-j\frac{\pi}{4}}$$

$$\left[a_0=1\ ,\ a_3=\frac{1}{2j}e^{j\frac{\pi}{4}}\ ,\ a_{-3}=\frac{-1}{2j}e^{-j\frac{\pi}{4}}\right]$$

$$y[n]=\sum_{K=0}^{15}a_K H(e^{jK\frac{\pi}{8}})e^{jK\frac{\pi}{8}n}=H(e^0)e^0+\frac{1}{2j}e^{j\frac{\pi}{4}}H(e^{j\frac{3\pi}{8}})e^{j\frac{3\pi}{8}n}-\frac{1}{2j}e^{-j\frac{\pi}{4}}H(e^{-j\frac{3\pi}{8}})e^{-j\frac{3\pi}{8}n}$$

ج) $x[n]=\left[\left(\frac{1}{2}\right)^n u[n]\right]*\sum_{K=-\infty}^{\infty}\delta[n-4K]=g[n]*r[n]$

$$x[n]=\sum_{K=-\infty}^{\infty}\left(\frac{1}{2}\right)^{n-4K}u[n-4K]$$

$$g[n]=\left(\frac{1}{2}\right)^n u[n]\ ,\ r[n]=\sum_{K=-\infty}^{\infty}\delta[n-4K]$$

$$\rightarrow\sum_{K=0}^{3}a_K H(e^{j\frac{2\pi K}{4}})e^{K(\frac{2\pi}{4})}=\frac{1}{4}\left(H(e^{j0})e^{j0}+H(e^{j(\frac{\pi}{2})})e^{j(\frac{\pi}{2})}+H(e^{j\pi})e^{j\pi}+H(e^{j3(\frac{\pi}{2})})e^{j3(\frac{\pi}{2})}\right)=0$$

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الف)

$$V=RC\frac{dy(t)}{dt} \qquad V=LC\frac{d^2y(t)}{dt^2}$$

$$x(t)=LC\frac{d^2y(t)}{dt^2}+RC\frac{dy}{dt}+y$$

$$\rightarrow\frac{d^2y}{dt^2}+\frac{dy}{dt}+y=x(t)$$

ب) $H(j\omega) = \dfrac{1}{-\omega^2 + j\omega + 1} \longrightarrow 1^{j\omega t}$ جزئی مبهمی بعلرم است.

ج) $x(t) = \dfrac{1}{2j} e^{j(\frac{2\pi}{2\pi})t} - \dfrac{1}{2j} e^{-j(\frac{2\pi}{2\pi} t)} \rightarrow a_1 = a_{-1}^* = \dfrac{1}{2j}$

$y(t) = (\frac{1}{j} e^{jt} - \frac{1}{-j} e^{-jt}) = (-\frac{1}{2})(e^{jt} + e^{-jt})$

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الف) $N=4 \rightarrow a_3 = a_{-3}^* = \dfrac{1}{2j} \rightarrow b_3 = a_1 H(e^{3j\frac{\pi}{2}}) = \dfrac{1}{2j(1 - \frac{1}{4} e^{-\frac{3\pi j}{2}})^*}$

$b_{-3} = a_{-1} H(e^{-3j\frac{\pi}{2}}) = \dfrac{1}{2j(1 - \frac{1}{4} e^{\frac{j3\pi}{2}})^*}$

ب) $N=8 \rightarrow a_1 = a_{-1} = \dfrac{1}{2} \qquad a_{-2} = 1 \qquad b_1 = a_1 H(e^{j\frac{\pi}{4}}) = \dfrac{1}{2(1 - \frac{1}{4} e^{-j\frac{\pi}{4}})^*}$

$b_2 = a_2 H(e^{j\frac{\pi}{2}}) = \dfrac{1}{(1 - \frac{1}{4} e^{-j\frac{\pi}{2}})} \qquad b_{-2} = \dfrac{1}{(1 - \frac{1}{4} e^{j\frac{\pi}{2}})^*}$

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$H(e^{j\omega}) = \dfrac{1}{1 - \frac{1}{2} e^{-j\omega}} - \dfrac{1}{1 - 2 e^{-j\omega}}$

$b_k = a_k H(e^{j\frac{2k\pi}{N}}) = \dfrac{1}{4}\left[\dfrac{1}{1 - \frac{1}{2} e^{-jk\frac{\pi}{2}}} - \dfrac{1}{1 - 2e^{-jk\frac{\pi}{2}}}\right]$