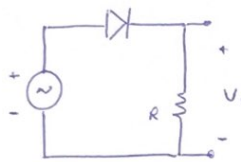
یالطیف

رضا دینه پرور

۹۸۱۴۲۳۰۴

تمرین سری ۳ الک صنعتی

3-3



a) $PF \overset{?}{=} \frac{1}{2} \Rightarrow PF = \frac{P_R}{S} = \frac{P_R}{V_{s,rms} I_{rms}}$

$$\begin{cases} P_R = \frac{V_{R,rms}^2}{R} \\ V_{R,rms} = \frac{V_m}{2} \\ I_{rms} = \frac{V_m}{2R} \end{cases} \Rightarrow PF = \frac{\frac{V_{R,rms}^2}{R}}{\frac{V_m}{\sqrt{2}} \times \frac{V_m}{2R}} = \frac{\frac{1}{R}\left(\frac{V_m^2}{4}\right)}{\frac{V_m^2}{2\sqrt{2}R}}$$

$$\Rightarrow PF = \frac{2\sqrt{2}R}{4R} = \frac{\sqrt{2}}{2} = \boxed{\frac{1}{\sqrt{2}}}$$

b) $\cos(\theta_1 - \varphi) = \cos(0) = 1 \qquad I_1 = \frac{V_1}{R} = \frac{1}{R} \cdot \frac{V_m}{2} \angle 0 \quad \Rightarrow PF = \cos(\theta_1 - \varphi_1) \cdot DF$

$\Rightarrow DF = \frac{1}{\sqrt{2}}$

3-5

$$\begin{cases} V_s = 120\,V\ (rms) \\ f = 60\ Hz \\ RL\ Load \begin{matrix} \nearrow R = 10\,\Omega \\ \searrow L = 15\ mH \end{matrix} \\ a)\ i(\omega t) = ? \end{cases}$$

$$i(\omega t) = \begin{cases} \frac{V_m}{Z}\left(\sin(\omega t - \theta) + \sin(\theta) e^{\frac{-\omega t}{\omega \tau}}\right) & ; 0 \le \omega t \le \beta \\ 0 & ; \text{other} \end{cases}$$

$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + (15 \times 10^{-3} \times 2\pi \times 60)^2} = 11.48 \quad , \omega\tau = \frac{\omega L}{R} = \frac{2\pi \times 60 \times 15 \times 10^{-3}}{10} = 0.56$

$V_m = 120\sqrt{2} = 169.7$

$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{2\pi \times 60 \times 15 \times 10^{-3}}{10}\right) = 0.51$

$i(\omega t) = \frac{169.7}{11.48}\left(\sin(\omega t - 0.51) + \sin(0.51) e^{\frac{-\omega t}{0.56}}\right) = 14.77\left[\sin(\omega t - 0.51) + 0.49 e^{\frac{-\omega t}{0.56}}\right]$

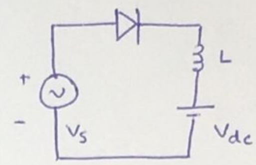
$\xRightarrow{\omega t = \beta} \sin(\beta - 0.51) + \sin(0.51) e^{\frac{-\beta}{0.56}} = 0 \Rightarrow \beta = 3.65 = 209.5^\circ$

b) $I_{avg} = \frac{1}{2\pi}\int_0^\beta i(\omega t)\, d\omega t = \frac{1}{2\pi}\int_0^{3.65}\left(14.77\left(\sin(\omega t - 0.51) + 0.49 e^{\frac{-\omega t}{0.56}}\right)\right) d\omega t = 4.98\ A$

c) $I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^\beta i^2(\omega t)\, d\omega t} = \sqrt{\frac{1}{2\pi}\int_0^{3.65} i^2(\omega t)\, d\omega t} = 3.97\ A \qquad$ d) $PF = \frac{P}{S} = \frac{584}{120\,(7.65)} = 63.7\%$

#3_10

$\begin{cases} V_s = 120\,V \text{ (rms)} \\ f = 60\text{ HZ} \\ \text{RL Source load} \begin{cases} L = 100\,mH \\ V_{dc} = 48\,V \end{cases} \\ P = ? \end{cases}$



$$i(\omega t) = \begin{cases} \frac{V_m}{\omega L}\left[\cos\alpha - \cos(\omega t)\right] + \frac{V_{dc}}{\omega L}(\alpha - \omega t) & ; \quad \alpha \leq \omega t \leq \beta \\ 0 & ; \quad \text{other} \end{cases}$$

$$\alpha = \sin^{-1}\left(\frac{V_{dc}}{V_m}\right) = \frac{48}{120\sqrt{2}} = 0.28$$

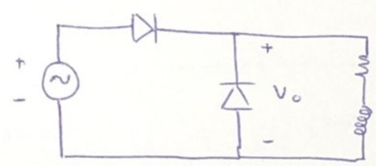
$$i(\omega t) = 4.5\left(\cos(0.28) - \cos(\omega t)\right) + 1.27(0.28 - \omega t) = 4.5\left(0.95 - \cos(\omega t)\right) + 1.27(0.28 - \omega t)$$

$\xrightarrow{\omega t = \beta}$ $i(\beta) = 0 \Rightarrow \beta = 4.48 = 256°$

$$I_o = \frac{1}{2\pi}\int_\alpha^\beta i(\omega t)\, d\omega t = \frac{1}{2\pi}\int_{0.28}^{4.48} i(\omega t)\, d\omega t = 1.97\,A \Rightarrow P_{dc} = V_{dc} I_o = 48(1.97) = 94$$

#3_13

$\begin{cases} R = 12\,\Omega \\ L = 60\,mH \\ V_s = 120\,V \text{ (rms)} \\ f = 60\text{ HZ} \\ a) \text{ dc component?} \end{cases}$



$$V_o(t) = \underbrace{\frac{V_m}{\pi}}_{= \text{dc term}} + \frac{V_m}{2}\sin(\omega_o t) - \sum_{n=2,4,\dots 2k}^{\infty} \frac{2V_m}{(n^2-1)\pi} \cdot \cos(n\omega_o t)$$

$$\Rightarrow V_{o(dc)} = \frac{V_m}{\pi} = \frac{120\sqrt{2}}{\pi} = 54\,V \quad , \quad I_o = \frac{V_{dc}}{R} = \frac{54}{12} = \boxed{4.5\,A}$$

b) $Z_o = R = 12\,\Omega \Rightarrow \begin{cases} Z_1 = |R + j\omega_o L| = 25.60 \\ V_1 = \frac{120\sqrt{2}}{2} = 84.85 \end{cases}$

$I_1 = \frac{V_1}{Z_1} = 3.31$

n	V_n	I_n	Z_n
0	54.01	4.50	12
1	84.85	3.31	25.60
2	36.01	4.50	46.80
4	7.20	0.76	91.27

#3_22

$\begin{cases} V_s = 120\,V \text{ (rms)} \\ f = 60\text{ HZ} \\ P_L = 50\,W \\ a)\ C = ? \end{cases}$

$$P \approx \frac{V_o^2}{R} \approx \frac{V_m^2}{R} \Rightarrow R = \frac{V_m^2}{P} = \frac{(120\sqrt{2})^2}{50} = 576\,\Omega$$

$$C = \frac{V_m}{fR\Delta V_o} = \frac{120\sqrt{2}}{60(576)(1.5)} = 3.270\,\mu F$$

$$\alpha = \sin^{-1}\left(1 - \frac{\Delta V_o}{V_m}\right) = \sin^{-1}\left(1 - \frac{1.5}{120\sqrt{2}}\right) = 1.438 = 82.5°$$

$$I_{D,peak} = V_m\left(\omega C\cos\alpha + \frac{\sin\alpha}{R}\right) = 28.1\,A \qquad I_{D,Avg} = \frac{V_m}{R} = 0.295\,A$$

#3-27

$$\begin{cases} V_s = 120\ V\ (rms) \\ f = 60\ HZ \\ RL\ Load \begin{cases} R = 25 \\ L = 50\ mH \end{cases} \\ \alpha = 30^\circ \end{cases}$$

a) $i(\omega t)$

$$i(\omega t) = \begin{cases} \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)\right] e^{\frac{\alpha - \omega t}{\omega \tau}} & ;\ \alpha \le \omega t \le \beta \\ 0 & ;\ other \end{cases}$$

$V_m = 120\sqrt{2}$

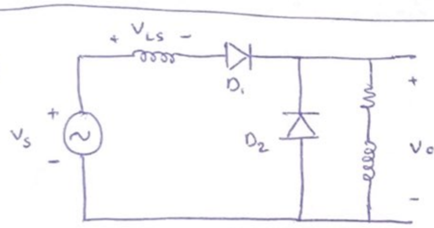
$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{25^2 + (2\pi \times 60 \times 50 \times 10^{-3})^2} = 31.30$ $\qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = 0.64$

$\Rightarrow i(\omega t) = 3.46 \sin(\omega t - 0.615) - 6.38 e^{\frac{-\omega t}{0.07}} \xrightarrow{\omega t = \beta} i(\beta) = 0 \Rightarrow \beta = 3.74 = 215^\circ$

b) $I_o = \frac{1}{2\pi}\int_\alpha^\beta i(\omega t)\, d\omega t = 0.893$

c) $I_{rms} = \sqrt{\frac{1}{2\pi}\cdot\left[\int_\alpha^\beta i^2(\omega t)\, d\omega t\right]} = 1.5^A \Rightarrow P_o = P_R = I_{rms}^2 R = (1.5)^2 \times 40 = 90.3\ W$

#3-40



in $\omega t = \pi$ $\begin{cases} D_1: ON \\ D_2: \text{turns ON} \end{cases}$

$V_{LS} = V_m \sin \omega t = L_s \frac{di_{D_1}}{dt} = \omega L_s \frac{di_{D_1}}{d\omega t}$

$\Rightarrow \frac{di_{D_1}}{d\omega t} = \frac{V_m}{\omega L_s}\int_\pi^{\omega t} \sin(\omega t)\, d\omega t + i_{D_1}(\pi) \xrightarrow{\omega t = \pi + u} i_{D_1} = 0 = \frac{V_m}{\omega L_s}\cdot\left[-1 - \cos(\pi + u)\right] + I_L$

$\cos(\pi + u) = -\cos u \quad = 0 = \frac{V_m}{\omega L_s}(-1 + \cos u) + I_L = \frac{-V_m}{\omega L_s}\cos u + I_L$

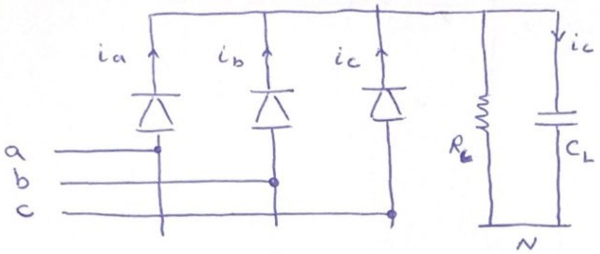
$u = \cos^{-1}\left[1 - \frac{I_L X_s}{V_m}\right]$

3-42

Design with RL source Load:

سؤالات پیوست :

#1

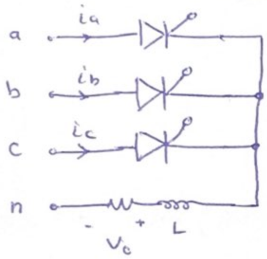


$$i_c(t) = C\,\frac{dV_o}{dt}$$

$$\begin{cases} i_c = \dfrac{V_o}{R} \\ \Delta t = \dfrac{T}{3} = \dfrac{1}{3f} \end{cases}$$

$$\Rightarrow \frac{V_o}{R} = \frac{1}{\frac{1}{3f}} \cdot \frac{C\,\Delta V_m}{1} \Rightarrow C \simeq \frac{V_m}{3Rf\Delta V_o}$$

#2



I.) Assume $L = 0$:

$$V_{o(dc)_a} = \frac{1}{\frac{2\pi}{3}} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin\omega t \, d\omega t = \frac{V_m \cdot 3\sqrt{3}}{2\pi} \cos\alpha$$

$$V_{o(dc)_b} = \frac{1}{\frac{2\pi}{3}} \int_{\frac{5\pi}{6}+\alpha}^{\frac{3\pi}{2}+\alpha} V_m \sin(\omega t - 120^\circ)\, d\omega t$$

$$= \frac{3}{2\pi} \left[V_m \cos\left(\frac{5\pi}{6} + \alpha - 120\right) - V_m \cos\left(\frac{3\pi}{2} + \alpha - 120\right) \right]$$

$$V_{o(dc)_c} = \frac{1}{2\pi/3} \int_{\frac{3\pi}{2}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin(\omega t + 120)\, d\omega t = 3\left(V_m \cos\left(\frac{3\pi}{2} + \alpha + 120^\circ\right) - V_m \cos(126.45 + \alpha) \right)$$

$$\Rightarrow V_{rms} = \sqrt{\frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^2 \sin^2(\omega t)\, d\omega t} = \sqrt{3} \cdot V_m \left[\frac{1}{6} + \frac{\sqrt{2}}{8\pi} \cos 2\alpha \right]^{\frac{1}{2}}$$

$$PF = \frac{P}{S} = \frac{V_{rms}^2 / R}{3 V_{s,rms} \cdot \frac{V_{rms}}{R}}$$

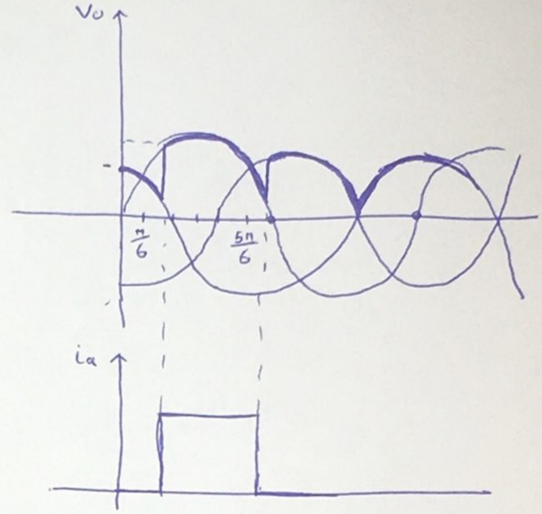
II) Assume $L \neq 0$:

$$V_{o(dc)} = \frac{1}{2\pi/3} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin(\omega t)\, d\omega t = \frac{3\sqrt{3}\, V_m}{2\pi} \cos\alpha$$

$$V_{rms} = \left[\frac{1}{2\pi/3} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^2 \sin^2\omega \, d\omega t \right] = \sqrt{3}\, V_m \left[\frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha \right]^{\frac{1}{2}}$$

$$PF = \frac{P}{S} = \frac{V_{o(dc)}\, I_d}{\sqrt{3} \cdot V_{L,rms}\, I_d}$$

#3 $\begin{cases} V_{s,rms} = 380\text{ V} \\ f = 50\text{ Hz} \end{cases}$



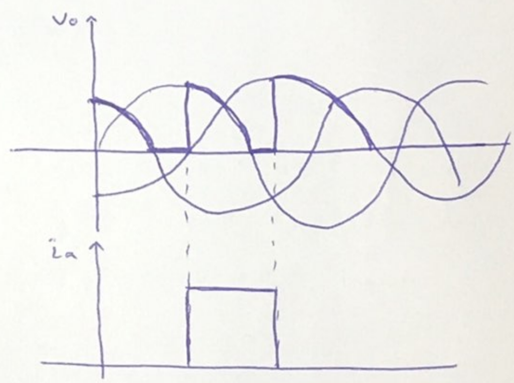
iii) $V_{s,rms} = \frac{380}{\sqrt{3}}$, $V_m = \frac{380\sqrt{2}}{\sqrt{3}} = 310.28$

if $R = 10$: $V_{o\,dc} = \frac{380\sqrt{3} \times 3\sqrt{3}}{2\pi} \cos(5) = 247.86$

$V_{rms} = \sqrt{3} \cdot \frac{\sqrt{2} \times 380}{\sqrt{3}} \cdot \left[\frac{1}{6} + \frac{\sqrt{3}}{8\pi}\cos(30)\right]^{\frac{1}{2}} = 255.69$

$P = \frac{(255.69)^2}{10} = 6537.73$ $S = 3 \times \frac{380}{\sqrt{3}} \times \frac{255.69}{10} = 16829$

$PF = \frac{P}{S} = 38.8\%$ $Q = \sqrt{S^2 - P^2} = 15507.19$



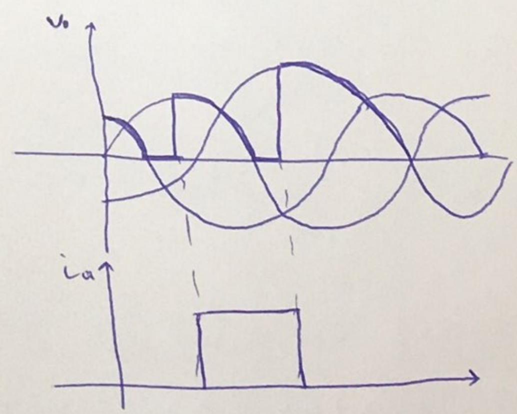
if $\alpha = 45^\circ$

$V_{o,dc} = \frac{1}{\frac{2\pi}{3}} \int_{\frac{\pi}{6}+45}^{\pi} V_m \sin(wt)\, dwt = 186.5$

$V_{rms} = \left[\frac{1}{2\pi/3} \int_{\frac{\pi}{6}+\frac{\pi}{4}}^{\pi} V_m^2 \sin^2(wt)\, dwt\right]^{\frac{1}{2}} = 218.79$

$P = \frac{(218.79)^2}{10} = 4786.97$ $S = 3 \times \frac{380}{\sqrt{3}} \times \frac{218.79}{10} = 14397$

$PF = \frac{P}{S} = 33.2\%$ $Q = \sqrt{S^2 - P^2} = 13582.1$



if $\alpha = 75^\circ$

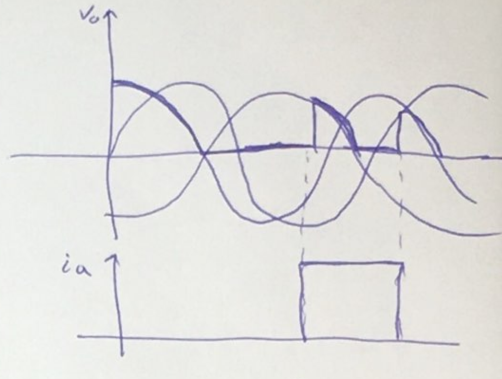
$V_{o(dc)} = \frac{1}{2\pi/3} \int_{\frac{\pi}{6}+75^\circ}^{\pi} V_m \sin(wt)\, dwt = 109.81$

$V_{rms} = \frac{\sqrt{2} \times 380}{\sqrt{3}} : \left[\frac{1}{2\pi/3} \int_{\frac{\pi}{6}+75}^{\pi} V_m^2 \sin^2(wt)\, dwt\right]^{\frac{1}{2}} = 156.024$

$P = \frac{(156)^2}{10} = 2434.37$

$S = 3 \times \frac{380}{\sqrt{3}} \times \frac{156}{10} = 10268$ $PF = \frac{P}{S} = 0.237$ $Q = \sqrt{S^2 - P^2} = 9975.25$

$\therefore \alpha = 105$



$V_{o(dc)} = 310.28$

$V_{o,rms} = \frac{38}{\sqrt{3}}$

$V_o = \frac{1}{\frac{2\pi}{3}} \int_{105^\circ}^{\pi} V_m \sin\omega t \, d\omega t = 43.4$

$V_{rms} = \left[\frac{3\pi}{2} \int_{135}^{\pi} (310.28)^2 \cdot \sin(\omega t) \, d\omega t\right]^{\frac{1}{2}} = 81$

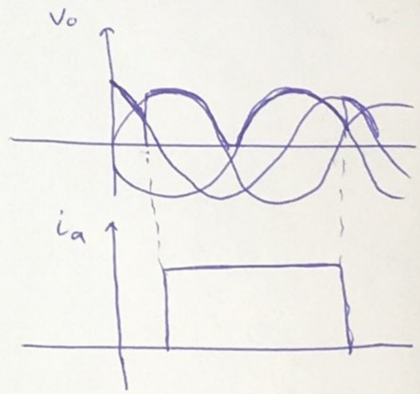
$P = \frac{V_{rms}^2}{R} = \frac{81^2}{10} = 656.16$

$S = 3 \times \frac{380}{\sqrt{3}} \times \frac{81}{10} = 5331.4$

$PF = \frac{P}{S} = 12.3\%$

$Q = \sqrt{S^2 - P^2} = 5290.86$

ب) كامل اندوكتيو



$\therefore \alpha = 15^\circ$, $i_d = 310.28$, $V_{L,rms} = 380$

$V_{odc} = 247.86$

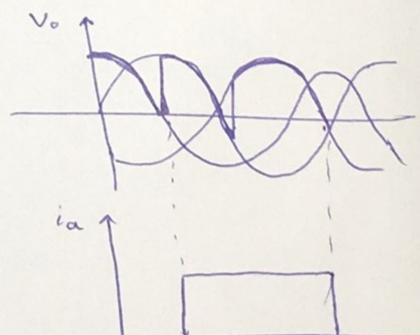
$V_{rms} = 255.64$

$P = V_{odc} \, i_d = 247.86 \times 20$

$S = \sqrt{3} \times 380 \times 20 = 13164$

$Q = 12194.96$

$P.F = \frac{P}{S} = 37.65\%$



$\therefore \alpha = 45$

$V_{L,rms} = 380$

$V_{s,rms} = \frac{380}{\sqrt{3}}$

$V_{an} = \frac{380}{\sqrt{3}} \times \sqrt{2} = 310.28$

$V_{odc} = \frac{3\pi}{2} \int_{\frac{\pi}{6}+45^\circ}^{\frac{5\pi}{6}+45^\circ} 310.28 \sin\omega t \, d\omega t = \frac{3\sqrt{3}}{2\pi} \times 310.28 \cdot \cos 45 = 181.44$

$V_{rms} = \sqrt{3} \times 310.28 \left(\frac{1}{6}\right)^{\frac{1}{2}} = 219.41$

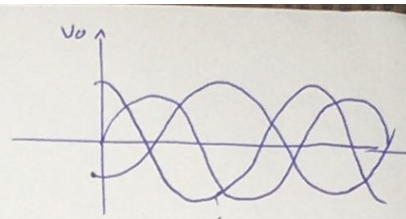
$P = 181.44 \times 20$

$S = \sqrt{3} \times \cancel{310.28}\ 380 \times 20 = 13164$

$PF = 27.56\%$

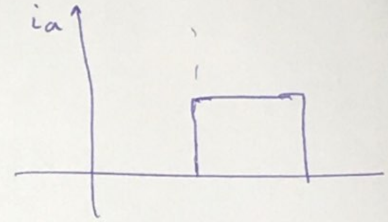
$Q = \sqrt{S^2 - P^2} = 12653.94$

$\alpha = 75$



$V_{o\,dc} = \frac{3\sqrt{3}}{2\pi} \times 310.28 \cos(75) = 66.41$

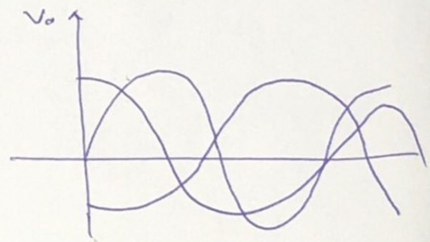
$V_{rms} = 175.78$



$P = 66.41 \times 20 = 1328.2$ $\qquad S = 13164$

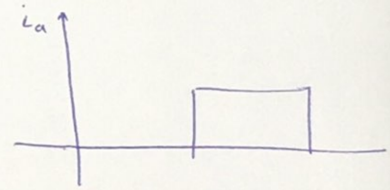
$PF = 10\%$ $\qquad Q = 13096.82$

if $\alpha = 105$



$V_{o(dc)} = -66.41$

$V_{rms} = 175.79$



$P = -66.41 \times 20 = -1328.2$

$S = 13164$ $\qquad PF = \frac{-1328.2}{13164}$

#4

$$\begin{cases} L_s = 2.5 \times 10^{-3}\,H \\ L_s \frac{di_s}{dt} = \sqrt{2}\, V_s \sin(\omega t) \end{cases} \qquad \alpha < \omega t < \alpha + u$$

$$\Rightarrow \int_{\alpha}^{\alpha+u} V_{an} \sin(\omega t)\, d\omega t = \omega L_s \int_{-i_d}^{+i_d} di_s \quad \Rightarrow u = \cos^{-1}\left[\frac{-2\omega L_s i_d + V_m \cos\alpha}{V_m}\right] - \alpha$$

$$V_{odc} = \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin(\omega t)\, d\omega t - \int_{\alpha}^{\alpha+u} V_m \sin(\omega t) = \frac{3\sqrt{3}}{2\pi} V_m \cos\alpha - V_m \cos(\alpha) + V_m \cos(\alpha+u)$$

if $\alpha = 15$:

$$u = \cos^{-1}\left[\frac{-2 \times 100\pi \times 2.5 \times 10^{-3} \times 20 + \frac{380\sqrt{2}}{\sqrt{3}} \cos(15)}{\frac{380\sqrt{2}}{\sqrt{3}}}\right] - 0.261 = 0.268$$

$V_{L,rms} = 380$

$V_{s,rms} = \frac{380}{\sqrt{3}}$

$V_{odc} = 216.62$

$\alpha = 45^\circ = 0.785 \Rightarrow u = 0.125 \Rightarrow V_{odc} = 133.04$

$\alpha = 75^\circ = 1.308 \Rightarrow u = 0.103 \Rightarrow V_{odc} = 35.24$

$\alpha = 105^\circ = 1.832 \Rightarrow u = 0.106 \Rightarrow V_{odc} = -97.66$