یا لطیف

رضا آذین پور
۹۸۱۴۳۰۳
تمرین سری 5 کنترل خطی

ζ : ضریب میرایی
ω_n : فرکانس طبیعی

#1

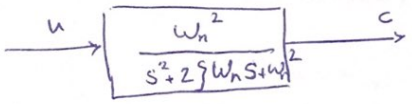
الف) $0.6 \le \zeta \le 0.8$, $\omega_n \le 10$

ب) $0.5 \le \zeta \le 0.707$, $\omega_n \ge 10$

ج) $0.5 \le \zeta \le 1$, $5 \le \omega_n \le 10$

د) $0 \le \zeta \le 0.707$, $5 \le \omega_n \le 10$

ه) $0.6 \le \zeta \le 1$, $\omega_n \le 6$

الف) $\begin{cases} \omega_n = 10 \\ \zeta = 0.7 \end{cases} \Rightarrow$ 

$P_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -7 \pm 7.141 \Rightarrow G_s = \frac{100}{s^2+14s+100}$

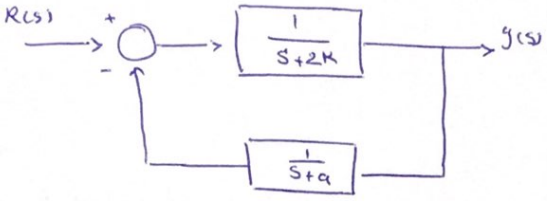
ب) $\begin{cases} \omega_n = 12 \\ \zeta = 0.6 \end{cases}$

$P_{1,2} = -3.6 \pm 11.44$

$G_s = \frac{144}{s^2+7.2s+144}$

ج) $\begin{cases} \omega_n = 10 \\ \zeta = 0.6 \end{cases}$

#5



الف) خطای حالت دائم پله ورودی $= 0 \Rightarrow$ $W_e(s) = \frac{y(s)}{R(s)} = \frac{1}{1+GH(s)}$

$$W_e(s) = \frac{1}{1+\left(\frac{1}{s+2K}\right)\left(\frac{1}{s+a}\right)} = \frac{1}{1+\frac{s+a+s+2K}{s^2+(2K+a)s+2Ka}} = \frac{s^2+(2K+a)s+1Ka}{s^2+(2K+2+a)s+2Ka+a+2K}$$

$$\Rightarrow E(s) = \frac{s^2+(2K+a)s+2Ka}{s^2+(2K+2+a)+(2Ka+a+2K)} \times \frac{1}{S} \quad \Rightarrow C_\circ = \lim_{S\to 0} W_e(s) = \lim_{S\to 0} \frac{s^2+(2K+a)s+2Ka}{s^2+(2K+2+a)s+2Ka+a+2K}$$

$$= \frac{2Ka}{2Ka+a+2K} = 0 \Rightarrow \begin{cases} K=0 \\ a=0 \end{cases}$$

#6 $\frac{y(s)}{U(s)} = \frac{108(s+3)}{(s+9)(s^2+8s+36)} \Rightarrow (s+9)(s^2+8s+36)=0$ $\begin{cases} s=-9 \\ s=-4+2\sqrt{5}j \\ s=-4-2\sqrt{5}j \end{cases}$

سیستم پایدار است $\Leftarrow$ No Root on RHP

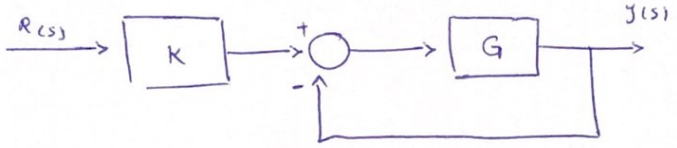
$$\frac{108(3+s)}{9\left(1+\frac{1}{9}\right)\left(2.5+5.4j\left(1+\frac{1}{2.5+j5.4}s\right)+2.5-5.4j\left(1+\frac{1}{2.5-5.4j}\right)\right.}$$

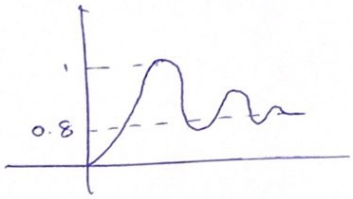
$$= \frac{1.57\left(1+\frac{1}{3}s\right)}{\left(1+\frac{1}{9}s\right)\left(1+\frac{1}{2.5+5.4j}s\right)\left(1+\frac{1}{2.5-5.4j}\right)}$$

$R_{ss} = \frac{R}{S} \Rightarrow e_{ss} = \frac{R}{1+K_p}$

$K_p = \lim_{S\to 0} G(s)K(s) = 1.57 \Rightarrow e_{ss} = \frac{R}{1+K_p} = \frac{R}{1+1.57} = \frac{R}{2.57} = \frac{1}{2.57}$

#7





$E(s) = R(s) - C(s)$

$\therefore K=1 \Rightarrow y_{ss} = 0.8 = \lim_{s\to 0} s\, y(s) = \lim_{s\to 0} s\, R(s) \frac{G(s)}{1+G(s)} = \lim \frac{s}{s} \left(\frac{G(s)}{1+G(s)}\right)$

$\Rightarrow \quad 0.8 = \frac{G(0)}{1+G(0)} \Rightarrow G(0) = 4$

$E(s) = R(s) - G(s) = R(s) - \frac{K\,G(s)}{1+G(s)} R(s) \quad \Rightarrow \quad E(s) = R(s)\left(1 - \frac{k\,G(s)}{1+G(s)}\right) = \frac{1}{s}\left(1 - \frac{k\,G(s)}{1+G(s)}\right)$

$e_{ss} = \lim_{s\to 0} s\, R(s) = 0$

$\lim s \frac{1}{s}\left(1 - \frac{k\,G(s)}{1+G(s)}\right) = 0 \Rightarrow \frac{1 - k\,G(0)}{1+G(0)} = 0 \Rightarrow G_0 = 4 \Rightarrow K = 1.25$