نکته: حوزه‌ی پیوسته $y(t) = x(t) * h(t) \Rightarrow \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{+\infty} h(\tau)x(t-\tau)d\tau$

حوزه‌ی گسسته $y[n] = x[n] * h[n] \Rightarrow \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = \sum_{-\infty}^{+\infty} x[n-k]\,h[k]$

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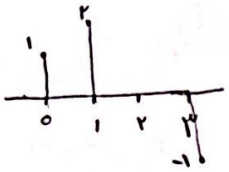
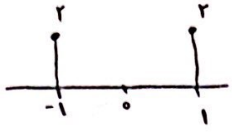
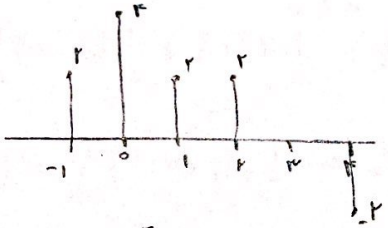
a) $y[n] = x[n] * h[n]$

$x[n]$: 

$h[n]$: 

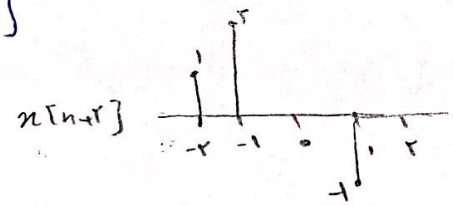
$$y[n] = x[n] * h[n] = \sum_{-\infty}^{+\infty} x(k) * h[n-k] = \sum_{-\infty}^{\infty} x[n-k]\,h[k]$$

$$y[n] = x[0] * h[n] + x[1] * h[n-1] + x[2] * h[n-2] \xrightarrow{\text{ساده‌سازی}} 2\delta[n+1] + 2\delta[n-1] + 4\delta[n] + 4\delta[n-2] + 2\delta[n-2] + 2\delta[n-4] = 4\delta[n] + 2\delta[n+1] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$$



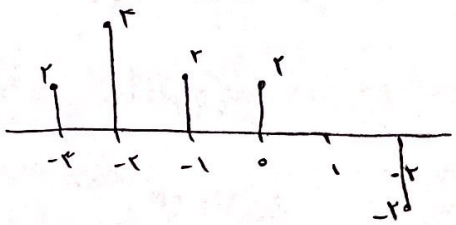
b) $y[n] = x[n+2] * h[n]$

$$= \sum_{-\infty}^{+\infty} h[k] * x[n+2-k]$$

$x[n+2]$: 

$$y[n] = h[-1] * x[n+2+1] + h[1] * x[n+2-1]$$

$$y[n] = 2\delta[n+3] + 4\delta[n+1] + 2\delta[n] - 2\delta[n-2] = 2\delta[n+3] + 4\delta[n+2] + 2\delta[n+1] + 2\delta[n] - 2\delta[n-2]$$



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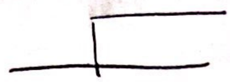
$h[n] = (\frac{1}{2})^{n-1} [u[n+3] - u[n-10]] \longrightarrow$ $u[k+3]$, $k \geq -3$

$u[k-10]$, $k \geq 10$



$$h[n-k] = \begin{cases} (\frac{1}{2})^{n-k-1} & A \leq k \leq B \\ 0 & o.w \end{cases}$$

$h[k] \longrightarrow -3 \leq k \leq 9$

$h[-k] \longrightarrow -9 \leq k \leq 3$ $\rightsquigarrow$ $-9+n \leq k \leq 3+n$

$A = -9+n$

$B = 3+n$

-۳

ورودی $x[n] = (\frac{1}{2})^{n-1} u[n-2]$

پاسخ ضربه $h[n] = u[n+2]$

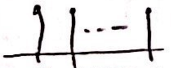
$$(\frac{1}{2})^{n-1} u[n-2] * u[n+2] = (\frac{1}{2})^{n-1} [u[n] * \delta[n-2]] * [u[n] * \delta[n+2]]$$

$$= (\frac{1}{2})^{n-1} [u[n] * u[n]] \underbrace{[\delta[n-2] * \delta[n+2]]}_{\delta[n]} = (\frac{1}{2})^{n-1} u[n] * u[n]$$

$$y[n] = \sum_{-\infty}^{+\infty} (\frac{1}{2})^{k-1} u[k] * u[n-k] = \sum_{k=0}^{n} (\frac{1}{2})^{k-1} = 2 \sum_{k=0}^{n} (\frac{1}{2})^{k} \Rightarrow 2[2-(\frac{1}{2})^{n}]$$

-۴

$$x[n] = \begin{cases} 1 & 3 \leq n \leq 8 \\ 0 & ow \end{cases}$$ ورودی



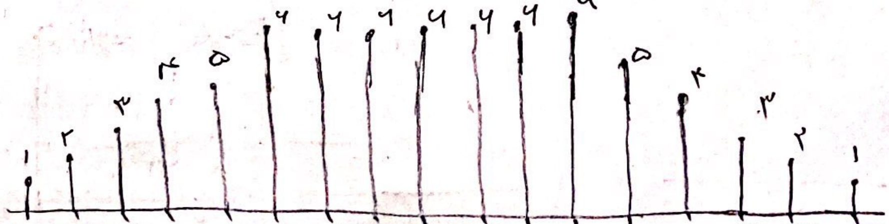
$$h[n] = \begin{cases} 1 & 4 \leq n \leq 15 \\ 0 & ow \end{cases}$$ پاسخ ضربه

$$Y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

باتوجه به اول و آخر بازه را مشخص می‌کنیم:

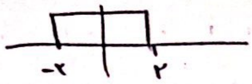
$$\hat{y}[n] = \sum_{k=4}^{15} h[k] x[n-k] =$$

(۲)



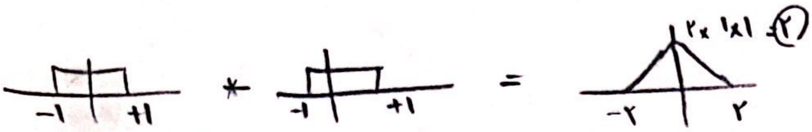
۵-

$y(t) = \text{rect}\left(\frac{t}{4}\right)$

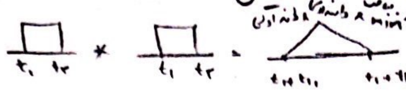


$x(t) = \text{rect}\left(\frac{t}{2}\right)$

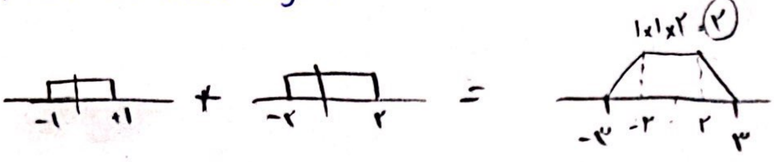
a) $h(t) = x(t) * x(t)$



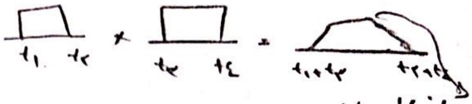
نکته: اگر دو سیگنال rect با هم کانوالو شوند و بازه‌های زمانی یکسان باشد حاصل به صورت سیگنال مثلث می‌شود.



b) $z(t) = x(t) * y(t)$



نکته: اگر دو سیگنال rect با بازه‌های زمانی متفاوت با هم کانوالو شوند حاصل به صورت ذوزنقه می‌شود.



دامنه اولی × دامنه دومی × مساحت min

۶-

$z(t) = x(t) * h(t)$

$h(t) = \sum_{k=-\infty}^{+\infty} \delta(t-3k) \longrightarrow x(t) * \sum_{k=-\infty}^{+\infty} \delta(t-3k) = \sum_{k=-\infty}^{+\infty} x(t) * \delta(t-3k) = \sum_{k=-\infty}^{+\infty} 2\Lambda\left(\frac{t-3k}{2}\right)$

$x(t) = 2\,\text{tri}\left(\frac{t}{2}\right) = 2\Lambda\left(\frac{t}{2}\right)$

$= 2 \sum_{k=-\infty}^{+\infty} \Lambda\left(\frac{t-3k}{2}\right)$

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$x(t) = \begin{cases} t+1 & 0 \leq t \leq 1 \\ 2-t & 1 < t \leq 2 \\ 0 & \text{ow} \end{cases}$

$h(t) = \delta(t+2) + 2\delta(t+1)$

$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$

$\rightarrow x(t) * h(t) = x(t+2) + 2x(t+1)$

$\rightarrow y(t) = \begin{cases} t+2 & -2 \leq t \leq -1 \\ t+4 & -1 \leq t \leq 0 \\ -2t+2 & 0 \leq t \leq 1 \\ 0 & \text{ow} \end{cases}$