In the name of God

Assignment 5:

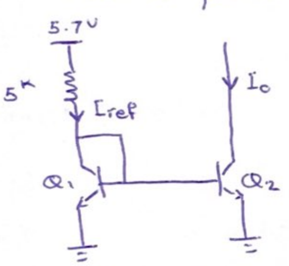
1. For the following circuits, the transistors are the same and $V_{BE} = 0.7^V$.

   a) Calculate I_{ref} in Fig. 1.a.

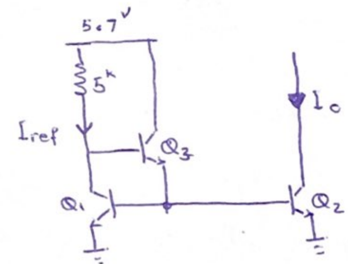
   b) Determine $\frac{I_o}{I_{ref}}$ in terms of β and compute it's value for $\beta = 50$, $\beta = 200$, $\beta = \infty$. Discuss about the results. (Fig 1.a)

   c) In order to alleviate the undesirable effect of β in BJT current mirrors, the circuit which is depicted in Fig 1.b can be used. For this circuit, calculate $\frac{I_o}{I_{ref}}$ and compare the results with those in (b).

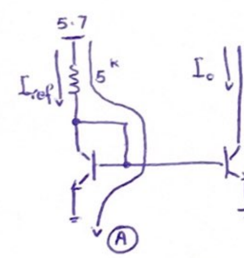
   Assume $\beta = 50$



(Fig. 1.a)

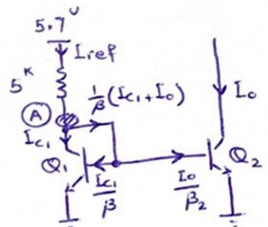


a)



KVL @ A : $-5.7 + 5^k I_{ref} + 0.7 = 0 \implies I_{ref} = \frac{5.7 - 0.7}{5} = 1^{mA}$

b) $\frac{I_o}{I_{ref}} = ?$



Transistors are the same : $\begin{cases} I_{S_1} = I_{S_2} \\ V_{T_1} = V_{T_2} \end{cases}$

$I_c = I_s e^{\frac{V_{BE}}{\eta V_T}}$

$V_{BE_1} = V_{BE_2} \implies \boxed{I_{c_1} = I_o}$ (I)

KCL @ A : $-I_{ref} + I_{c_1} + \frac{I_o}{\beta} + \frac{I_{c_1}}{\beta} = 0 \implies I_{ref} = I_{c_1} + \frac{1}{\beta}(I_{c_1} + I_o) \xrightarrow{(I)}$

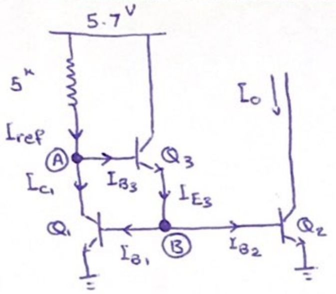
$I_{ref} = I_o + \frac{1}{\beta}(I_o + I_o) = I_o + \frac{2I_o}{\beta} = I_o\left(1 + \frac{2}{\beta}\right) \implies \boxed{\frac{I_o}{I_{ref}} = \frac{1}{1 + \frac{2}{\beta}}}$

if $\beta = 50$: $\frac{I_o}{I_{ref}} = \frac{1}{1 + \frac{2}{50}} = 0.96$

if $\beta = 200$: $\frac{I_o}{I_{ref}} = \frac{1}{1 + \frac{2}{200}} = 0.99$

if $\beta = \infty$: $\frac{I_o}{I_{ref}} \simeq 1$

c)



$V_{BE1} = V_{BE2} \Rightarrow I_{C1} = I_o$ (I)

KCL in (A): $-I_{ref} + I_{C1} + I_{B3} = 0 \Rightarrow I_{B3} = I_{ref} - I_{C1} \xrightarrow{(I)}$

$$\boxed{I_{B3} = I_{ref} - I_o}$$

$$I_{E3} = (\beta+1) I_{B3} = (\beta+1) I_{ref} - I_o \xrightarrow{\beta=50}$$

$$I_{E3} = 51(I_{ref} - I_o)$$

$$\left.\begin{array}{l} I_{B1} = \dfrac{I_{C1}}{\beta} = \dfrac{I_o}{\beta} = \dfrac{I_o}{50} \\ I_{B2} = \dfrac{I_o}{\beta} = \dfrac{I_o}{50} \end{array}\right) \longrightarrow I_{B1} = I_{B2}$$

KCL @ B: $I_{B1} + I_{B2} - I_{E3} = 0 \Rightarrow \dfrac{I_o}{50} + \dfrac{I_o}{50} - 51(I_{ref} - I_o) = 0$

$$\Rightarrow I_o\left(\frac{2}{50} + 51\right) = 51 I_{ref} \Rightarrow \frac{I_o}{I_{ref}} = \frac{51}{51 + \frac{2}{50}} = 0.9992$$

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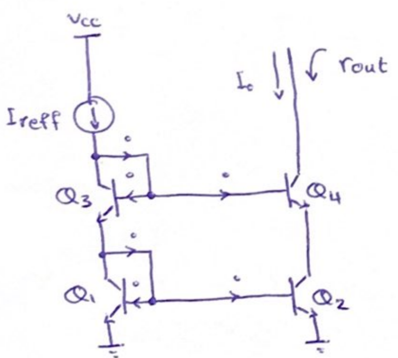
2. The following circuit is known as the "cascade" current mirror. Determine the output current and the output resistance. (Assume $\lambda \neq 0$ and neglect β effect). What are the advanteges of this configuration over the simple current-mirror scheme?



$\lambda \neq 0 \rightarrow V_A \neq \infty \longrightarrow r_o \neq \infty$

neglect β effect $\longrightarrow \beta = \infty$

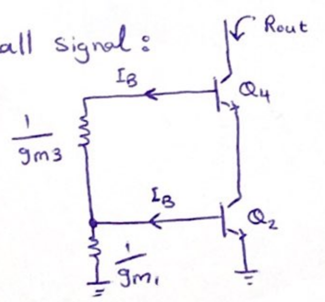
$$I_{ref} = I_c = I_{S1} \exp\left(\frac{V_{BE}}{\eta V_T}\right)\cdot\left(1 + \frac{V_{CE1}}{V_A}\right)$$

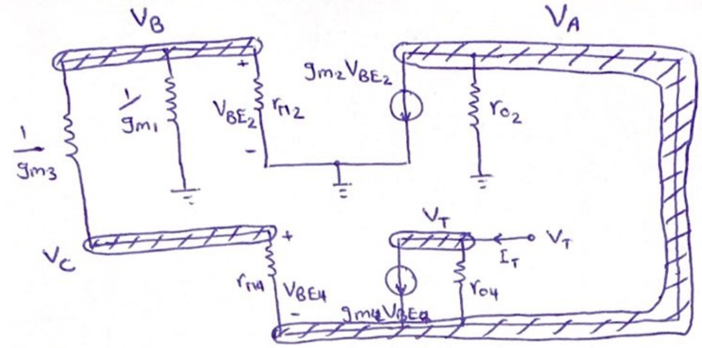
$$I_o = I_c = I_{S2} \exp\left(\frac{V_{BE}}{\eta V_T}\right)\left(1 + \frac{V_{CE2}}{V_A}\right)$$

$$\frac{I_o}{I_{ref}} = \frac{I_{S2} \cancel{\exp\left(\frac{V_{BE}}{\eta V_T}\right)}\left(1 + \frac{V_{CE2}}{V_A}\right)}{I_{S1} \cancel{\exp\left(\frac{V_{BE}}{\eta V_T}\right)}\left(1 + \frac{V_{CE1}}{V_A}\right)} = \boxed{\frac{I_{S1}}{I_{S2}} \times \frac{1 + \frac{V_{CE2}}{V_A}}{1 + \frac{V_{CE1}}{V_A}}}$$

if $V_{BE3} = V_{BE4} \Rightarrow V_{CE1} = V_{CE2}$: $\boxed{\dfrac{I_o}{I_{ref}} = \dfrac{I_{S2}}{I_{S1}}}$

small signal:





$V_B = V_{BE2}$

$V_C = V_{BE4} + V_A$

KCL @ V_B : $\frac{V_B - V_C}{\frac{1}{g_{m3}}} + \frac{V_B}{\frac{1}{g_{m1}}} + \frac{V_B}{r_{\pi 2}} = 0 \implies V_B\left[g_{m3} + g_{m1} + \frac{1}{r_{\pi 2}}\right] = g_{m3} V_C$ (I)

KCL @ V_A : $g_{m2} V_{BE2} + \frac{V_A}{r_{o2}} - g_{m4} V_{BE4} + \frac{V_A - V_T}{r_{o4}} + \frac{V_A - V_C}{r_{\pi 4}} = 0 \xrightarrow{V_{BE2} = V_B}$

$\implies g_{m2} V_B + V_A\left[g_{m4} + \frac{1}{r_{o2}} + \frac{1}{r_{o4}} + \frac{1}{r_{\pi 4}}\right] - \frac{V_T}{r_{o4}} - V_C\left[g_{m4} + \frac{1}{r_{\pi 4}}\right] = 0$ (II)

KCL @ V_C : $\frac{V_C - V_A}{r_{\pi 4}} + \frac{V_C - V_B}{\frac{1}{g_{m3}}} = 0 \implies V_C\left[g_{m3} + \frac{1}{r_{\pi 4}}\right] - g_{m3} V_B - \frac{V_A}{r_{\pi 4}} = 0$ (III)

KCL @ V_T : $-I_T + \frac{V_T}{r_{o4}} + g_{m4} V_{BE4} = 0 \implies \frac{V_T}{r_{o4}} + g_{m4}(V_C - V_A) = I_T$ (IV)

$\xrightarrow{(I),(II),(III),(IV)} \frac{V_T}{I_T} = R_{out} = 2r_o + \frac{\beta_4 r_o^2}{r_{\pi 4} + \frac{g_{m1} + g_{m4}}{g_{m1}} r_o} \approx 2r_o + \frac{\beta_4 r_o}{\frac{g_{m1} + g_{m4}}{g_{m1}} r_o} \approx 2r_o + \frac{g_{m1} \beta_4 r_o}{g_{m1} + g_{m2}}$

$\implies R_{out} = \beta_4 r_o \left(\frac{g_{m1}}{g_{m1} + g_{m2}}\right)$

if $g_{m1} = g_{m2}$: $\boxed{R_{out} = \frac{\beta r_o}{2}}$

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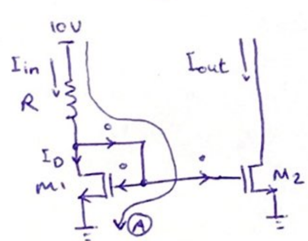
3. Specify the value of R so that $I_{out} = 5^{mA}$.



$\beta_1 = 2 \frac{mA}{V^2}$

$\beta_2 = 10 \frac{mA}{V^2}$

$V_{th1} = V_{th2} = 2^V$

$\frac{I_{out}}{I_{in}} = \frac{\beta_2}{\beta_1} \implies \frac{5^{mA}}{I_{in}} = \frac{10}{2} \implies \boxed{I_{in} = 1^{mA}}$

KVL @ A : $-10 + R I_D + V_{gs} = 0 \implies \boxed{V_{gs} = 10 - R I_D}$

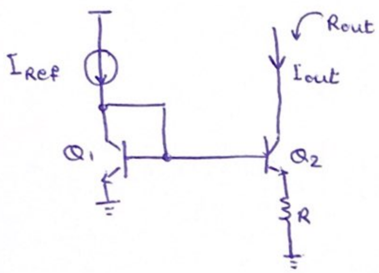
$I_D = \frac{\beta_1}{2}(V_{gs} - V_{th})^2 = \frac{2}{2}(10 - R I_D - 2)^2 \xrightarrow{I_D = I_{in}} 1 = (10 - R - 2)^2$

$\begin{cases} R = 7^k \checkmark \\ R = 9^k \end{cases}$

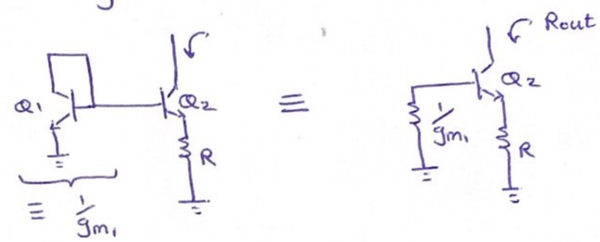
4. In the following circuit:

a) Determine the output resistance. Assume that the current source is Ideal

b) Specify R such a way that $I_{REF} = 2 I_{out}$. the transistors are the same and $\beta \gg 1$.

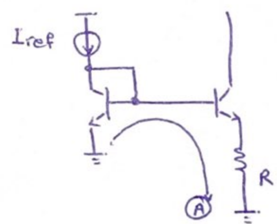


a) ac Analysis:



$$R_{out} = r_o\left[1 + g_{m2}(R \| r_{\pi_2})\right] \approx \beta r_o$$

b) if $I_{ref} = 2 I_{out} \Rightarrow R = ?$



KVL @ A: $-V_{BE_1} + V_{BE_2} + R I_o = 0$

$$-V_T \ln\left(\frac{I_{ref}}{I_s}\right) + V_T \ln\left(\frac{I_{out}}{I_s}\right) = -R I_o \Rightarrow V_T \ln\left[\frac{I_{out}}{\cancel{I_s}} \times \frac{\cancel{I_s}}{I_{ref}}\right] = -R I_{out}$$

$$V_T \ln\left(\frac{I_{out}}{I_{ref}}\right) = -R I_{out} \Rightarrow V_T \ln\left(\frac{I_{ref}}{I_{out}}\right) = R I_{out} \Rightarrow R = \frac{V_T \ln\left(\frac{I_{ref}}{I_{out}}\right)}{I_{out}}$$

$$\xrightarrow{I_{ref} = 2 I_{out}} R = \frac{V_T \ln\left(\frac{2 I_{out}}{I_{out}}\right)}{I_{out}} = \boxed{\frac{V_T \ln(2)}{I_{out}} = \frac{2 V_T \ln(2)}{I_{ref}} = \frac{V_T \ln(4)}{I_{ref}}}$$

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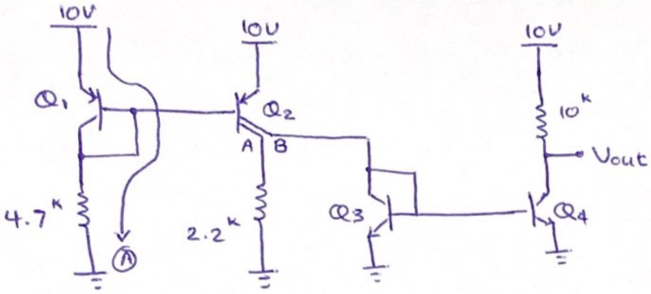
5. In the following circuit, all of the transistors are the same.
the effective area of the collector "A" of Q_2 is 3 times larger than the effective area of the collector "B" of Q_2. Calculate the output voltage.

Hint: Q_2 is a transistors with two collector terminals, which, their currents are proportional to their effective areas. the total effective collector area of Q_2 is the same as that of Q_1.



$A_{c,A} = 3A_{c,B}$

$A_{c1} = A_{c2} = A_{c3} = A_{c4}$

$V_{BE3} = V_{BE4} \Longrightarrow I_{c3} = I_{c4}$

$I_{c2,A} = 3\, I_{c2,B} \xrightarrow{I_{c2,B} = I_{c3} = I_{out}} I_{c2,A} = 3\, I_{out}$

$V_{EB1} = V_{EB2} \Longrightarrow I_{c1} = I_{c2} = 4\, I_{out}$

KVL @ A: $-10 + 0.7 + 4.7\, I_{c1} = 0 \xrightarrow{I_{c1} = 4 I_{out}} -10 + 0.7 + 4.7\,(4 I_{out}) = 0$

$\boxed{I_{out} = 0.5^{mA}} \Rightarrow V_{out} = 10 - 10^k I_{out} = 10 - 10(0.5) = \boxed{5^v}$

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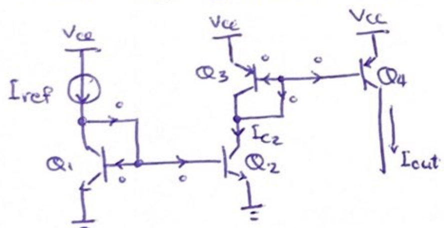
6. In the following circuit, specify a relation for I_o in terms of I_{ref} (neglect β and λ effect). The collector area of Q_2 and Q_4 is 2 times larger than Q_1 and the collector area of Q_3 is 3 times larger than Q_1:



$\begin{cases} A_{c,2,4} = 2A_{c1} \\ A_{c3} = 3A_{c1} \end{cases}$

$r_o = \infty$

$\beta \gg 1$

$V_{BE1} = V_{BE2} \Longrightarrow I_{c2} = 2 I_{c1} = 2 I_{ref} \xrightarrow{I_{c2} = I_{c3}} I_{c3} = 2 I_{ref} \quad (I)$

$V_{EB3} = V_{EB4} \Longrightarrow \dfrac{A_{c3}}{A_{c4}} = \dfrac{3A_{c1}}{2A_{c1}} = \dfrac{3}{2} \Rightarrow \boxed{I_{c4} = \frac{2}{3} I_{c3}} \xrightarrow{(I)}$

$I_{c4} = \frac{2}{3}(2 I_{ref}) = \frac{4}{3} I_{ref} = I_{out} \Longrightarrow \boxed{I_{out} = \frac{4}{3} I_{ref}}$