بالطیف

رضا ادیب پور
۹۸۱۴۳۰۳
تمرین سری ۳ کنترل خطی

#1 سیستمی LTI است که :

additivity : $x_1(t) + x_2(t) \xrightarrow{T} y_1(t) + y_2(t)$

scaling: $a x(t) \xrightarrow{T} a y(t)$

Time Invarient : $\forall t_0 : x(t) \xrightarrow{T} y(t) \Rightarrow x(t-t_0) \xrightarrow{T} y(t-t_0)$

الف $y(t) = \dfrac{dx(t)}{dt}$

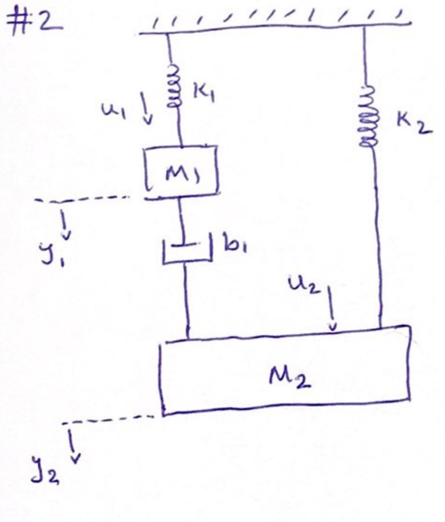
additivity : $\dfrac{dx_1(t)}{dt} + \dfrac{dx_2(t)}{dt} \xrightarrow{T} y_1(t) + y_2(t)$ ✓

scaling : $a x(t) \xrightarrow{T} \dfrac{d\,a x(t)}{dt} = a \dfrac{dx(t)}{dt} = a y(t)$ ✓

Time Invarient : $x(t-t_0) \rightarrow \dfrac{dx(t-t_0)}{dt} = \dfrac{dx(t)}{dt} - \dfrac{dx(t_0)}{dt}$
$= y(t-t_0)$

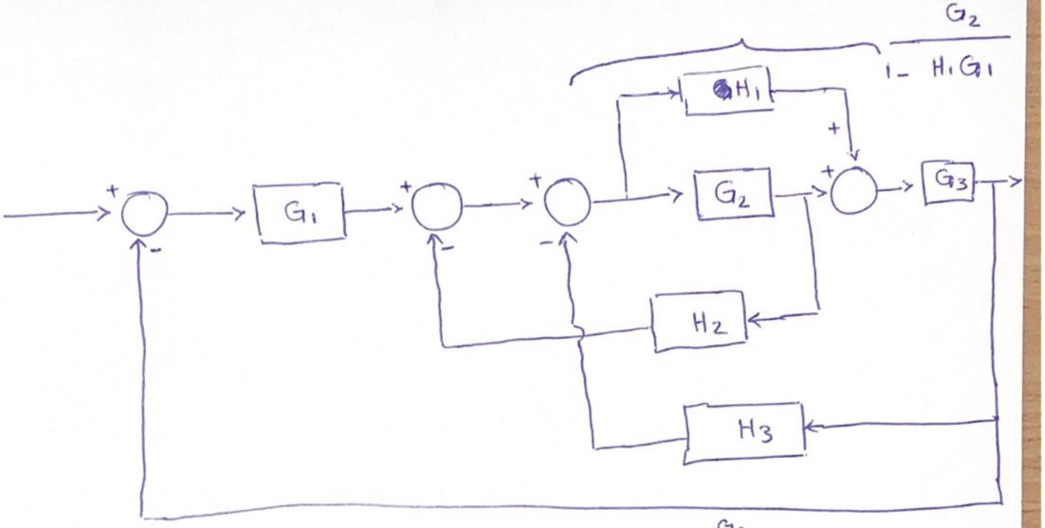
پس سیستم مشتق گیر LTI است.

#2



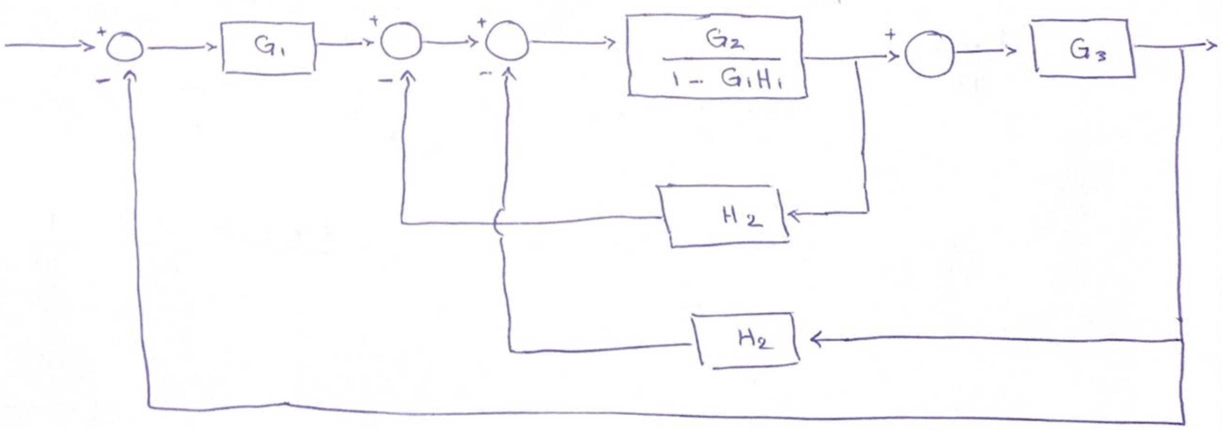
inputs : u_1, u_2
outputs : y_1, y_2

#3 $\frac{C(s)}{R(s)} = ?$

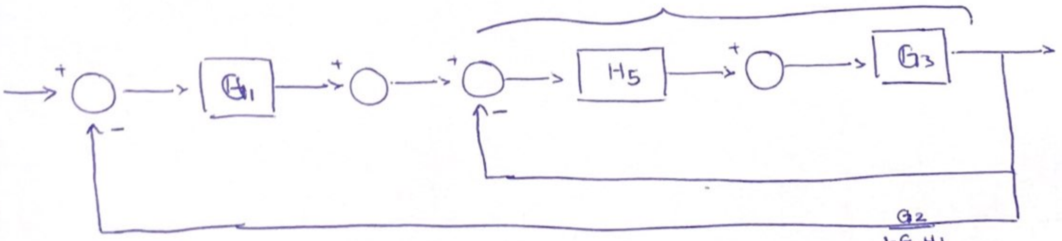


$$H_5 = \frac{\frac{G_2}{1-G_1H_1}}{1+\left(\frac{G_2}{1-G_1H_1}\right)H_2}$$

=>

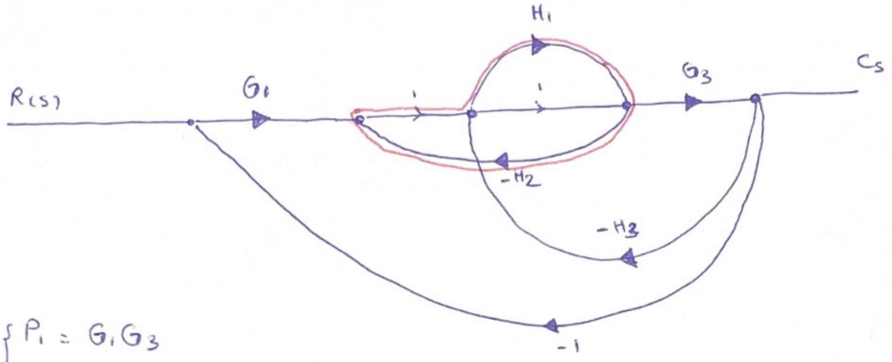


$= G_6 = H_5 + G_3 \quad \Rightarrow G_6 = \frac{G_3+H_5}{1+G_3+H_5}$



$$\Rightarrow H_T = \frac{G_1+G_6}{1+G_1+G_6} = \frac{G_1+\frac{G_3+H_5}{1+G_3+H_5}}{1+G_3+H_5} = \frac{G_1+\frac{G_3+\frac{\frac{G_2}{1-G_1H_1}}{1+\left(\frac{G_2}{1-G_1H_1}\right)H_2}}{1+G_3+\frac{\frac{G_2}{1-G_1H_1}}{1+\left(\frac{G_2}{1-G_1H_1}\right)H_2}}}{1+G_3+\frac{\frac{G_2}{1-G_1H_1}}{1+\left(\frac{G_2}{1-G_1H_1}\right)H_2}}$$

#4



مسیرها :
$P_1 = G_1 G_3$
$P_2 = G_1 H_1 G_3$

حلقه‌ها :
$L_1 = -H_1 H_2$
$L_2 = -G_1 G_3$
$L_3 = -G_3 H_3$
$L_4 = -H_1 H_3 G_3$
$L_5 = -G_1 H_1 G_3$

$$\Delta = 1 - \sum L_n + \sum L_m L_n - \sum L_m L_n L_p + \cdots$$

$$\Delta = 1 - \sum_{1}^{5} L_n + \sum (L_1 L_2 +) - 0$$

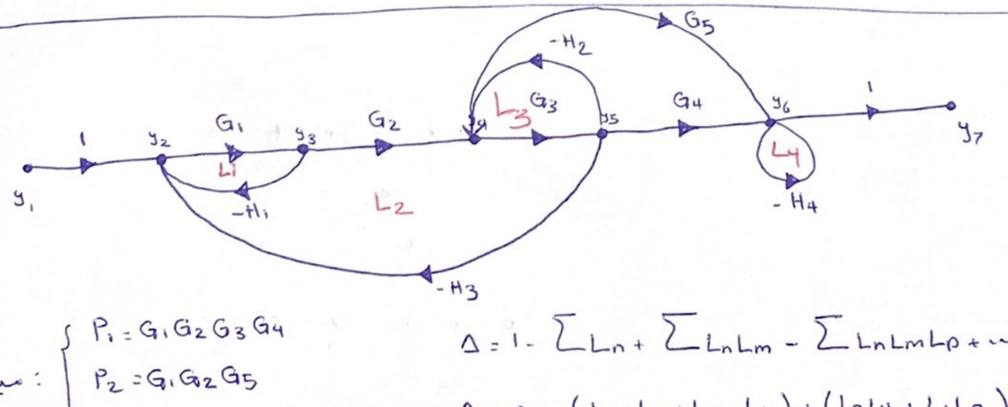
$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + L_1 L_2$$

$$\Delta = 1 - (-H_1H_2 - G_1G_3 - G_3H_3 - H_1H_3G_3 - G_1H_1G_3) + H_1H_2G_1G_3$$

$$\Delta = 1 + H_1H_2 + G_1G_3 + G_3H_3 + H_1H_3G_3 + G_1H_1G_3 + H_1H_2G_1G_3$$

(برای مسیر P_1) $\Delta_1 = 1$, (برای مسیر P_2) $\Delta_2 = 1$

$$\Rightarrow M(s) = \frac{C(s)}{R(s)} = \frac{\sum_K P_{ijK} \Delta_{ijK}}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_3 + G_1H_1G_3}{1 + H_1H_2 + G_1G_3 + H_1H_3G_3 + G_1H_1G_3 + H_1H_2G_1G_3}$$



$\frac{y_7}{y_2} = ?$

مسیرها :
$P_1 = G_1 G_2 G_3 G_4$
$P_2 = G_1 G_2 G_5$

حلقه‌ها :
$L_1 = -G_1 H_1$
$L_2 = -G_1 G_2 G_3 H_3$
$L_3 = -G_3 H_2$
$L_4 = -H_4$

$$\Delta = 1 - \sum L_n + \sum L_n L_m - \sum L_n L_m L_p + \cdots$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_3 L_4 + L_1 L_3) - L_1 L_3 L_4$$

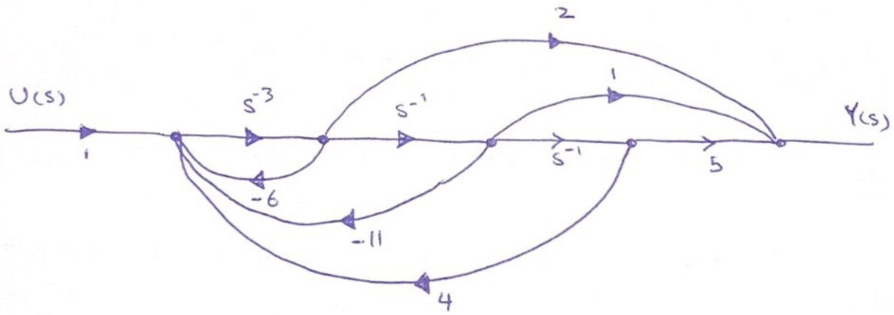
$$\Delta = 1 + G_1H_1 + G_1G_2G_3H_3 + G_3H_2 + H_4 + G_3H_2H_4 + G_1H_1G_3H_2 + G_1H_1G_3H_2H_4$$

$\Delta_1 = 1 \quad , \quad \Delta_2 = 1 \quad \Rightarrow M(s) = \frac{y_7}{y_2} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$

$$= \frac{G_1G_2G_3G_4 + G_1G_2G_5}{1 + G_1H_1 + G_1G_2G_3H_3 + G_3H_2 + H_4 + G_3H_2H_4 + G_1H_1G_3H_2 + G_1H_1G_3H_2H_4}$$

#6 ~~Y(s)~~

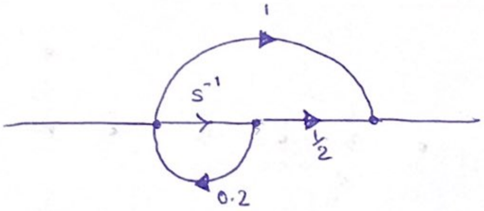
$$G(s) = \frac{2s^2 + s + 5}{s^3 + 6s^2 + 11s + 4} \times \frac{s^{-3}}{s^{-3}} = \frac{5s^{-3} + s^{-2} + 2s^{-1}}{4s^{-3} + 11s^{-2} + 6s^{-1} + 1} = \frac{Y(s)}{U(s)}$$



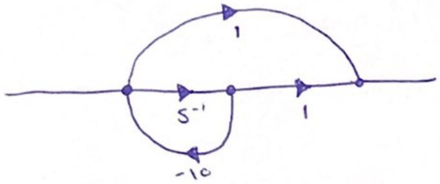
#7 $\frac{Y(s)}{\bar{U}(s)} = \frac{10s^2 + 15s + 5}{s^2 + 10.2s + 2} = \frac{10(s^2 + 1.5s + 0.5)}{s^2 + 10.2s + 2} = \frac{10(s + \frac{1}{2})(s + 1)}{(s + \frac{1}{5})(s + 10)}$

$$= 10 \times \underbrace{\frac{s + \frac{1}{2}}{s + \frac{1}{5}}}_{\bar{U}_1(s)} \times \underbrace{\frac{s + 1}{s + 10}}_{\bar{U}_2(s)}$$

$\bar{U}_1(s) = \frac{s + \frac{1}{2}}{s + \frac{1}{5}} \overset{\times s^{-1}}{=} \frac{1 + \frac{1}{2}s^{-1}}{1 + \frac{1}{5}s^{-1}} \Rightarrow$



$\bar{U}_2(s) = \frac{s + 1}{s + 10} \xrightarrow{\times s^{-1}} \frac{1 + s^{-1}}{1 + 10s^{-1}} \Rightarrow$



$\Rightarrow$