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۹۸۱۴۳۰۲
تمرین سری ۸ ـ آمار احتمال مهندسی

#1

$$\begin{cases} n_1 = 25 \\ n_2 = 16 \\ \bar{x}_1 = 8_2 \\ \bar{x}_2 = 78 \\ S_1^2 = 64 \\ S_2^2 = 49 \\ 1-\alpha = 0.98 \\ \dfrac{\sigma_1^2}{\sigma_2^2} = ? \end{cases}$$

$$\frac{\sigma_1^2}{\sigma_2^2} \in \left\{ \frac{S_1^2}{S_2^2} F_{\frac{\alpha}{2}, (n_1-1, n_2-1)} , \frac{S_2^2}{S_1^2} F_{1-\frac{\alpha}{2}, (n-1, n_2-1)} \right\}$$

$$1-\alpha = 0.98 \Rightarrow \alpha = 0.02$$

$$\Rightarrow \begin{cases} F_{1-\frac{\alpha}{2}, n_1-1, n_2-1} = F_{0.99, 24, 15} = 3.29 \\[2ex] F_{\frac{\alpha}{2}, n_1-1, n_2-1} = F_{0.01, 24, 15} = \dfrac{1}{F_{0.99, 15, 24}} = \dfrac{1}{2.89} = 0.346 \end{cases}$$

$$\Rightarrow \frac{\sigma_1^2}{\sigma_2^2} \in \left\{ \frac{64}{49}(0.3460), \frac{64}{49}(3.29) \right\} = (0.45192, 4.2971) \ \blacksquare$$

#2

Group1	12	10	14	13	11		
Group2	17	15	14	16	17	17	16

الف)

$$\begin{cases} (10.92, 13.08) \\ \alpha = ? \end{cases}$$

$$\bar{x}_1 = \frac{1}{n} \sum x_{i_1} = \frac{60}{5} = 12$$

$$S_1^2 = \frac{1}{n-1} \sum_{n-1} (x_{i_1} - \bar{x}_1)^2 = \frac{(12-12)^2 + (10-12)^2 + \cdots}{4} = 2.5$$

$$\mu \in \left(\bar{x} - t_{1-\frac{\alpha}{2}, n-1} \sqrt{\frac{S^2}{n}} , \bar{x} + t_{1-\frac{\alpha}{2}, n-1} \sqrt{\frac{S^2}{n}} \right)$$

$$\bar{x} - t_{1-\frac{\alpha}{2}, n-1} \sqrt{\frac{S^2}{n}} = 12 - t_{1-\frac{\alpha}{2}, n-1} \sqrt{\frac{2.5}{5}} = 10.92 \Rightarrow 12 - 10.92 = \sqrt{0.5}\, t_{1-\frac{\alpha}{2}, n-1}$$

$$\Rightarrow t_{1-\frac{\alpha}{2}, n-1} = \frac{1.08}{\sqrt{0.5}} \Rightarrow t_{1-\frac{\alpha}{2}, n-1} = 1.5274 \xrightarrow{n=5} t_{1-\frac{\alpha}{2}, 4} = 1.5274 \simeq 1.53$$

از جدول توزیع t: $t_{0.90, 4} = 1.53 \Rightarrow 1-\frac{\alpha}{2} = 0.9 \Rightarrow \underline{\alpha = 0.2}$

دارم

ب)
$$\begin{cases} H_0 : \mu_2 > \mu_1 \\ H_1 : \mu_2 \leq \mu_1 \end{cases}$$

$1 - \alpha = 0.95 \implies \alpha = 0.05$

$\implies t_{1-\alpha, n_1+n_2-2} = t_{0.95, 10} = 1.81$

$\bar{x}_2 = \frac{1}{n_2} \sum x_{i2} = \frac{112}{7} = 16 \implies S_2^2 = \frac{1}{n-1} \sum (x_{i2} - \bar{x}_2)^2 = 1.3\bar{3}$

$\implies T = \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{(16-12) - 0}{\sqrt{\frac{2.5}{5} + \frac{1.33}{7}}} = 4.81$

$\implies T = 4.81 \not< t_{0.95, 10} = 1.81 \implies$ فرض H_0 رد می‌شود $\Leftarrow$ میانگین نمره بردن میانگین 20 ↑

#3
$$\begin{cases} \sigma = 6 \\ n = 9 \\ H_0 : \mu > 65 \\ H_1 : \mu \leq 65 \end{cases}$$

$\bar{x} = \frac{1}{n} \sum x_i = \frac{578}{9} = 65.22$

$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{65.22 - 65}{\frac{6}{\sqrt{9}}} = 0.11$

$\alpha = 0.05 \implies Z_{1-\alpha} = Z_{0.95} = 1.645$

$\implies Z = 0.11 < Z_{0.95} = 1.645 \implies \alpha = 0.05$ فرض H_0 نادرست است و ادعای مدیر دبیرستان در سطح
اشتباه است

#4
$$\begin{cases} \sigma = 6 \\ n = 9 \\ \mu > 65 \end{cases}$$

الف)
$$\begin{cases} H_0 : \mu > 65 \\ H_1 : \mu \leq 65 \end{cases}$$

$\alpha = 0.05 \quad , \quad \theta_0 = \mu$

$\implies Z_{1-\alpha} = Z_{0.95} = 1.645$

$\implies \left(\bar{x} < \theta_0 - Z_{1-\alpha} \frac{\sigma}{\sqrt{n}} \right) \Longleftrightarrow RH_0 \implies \bar{x} < 65 - (1.645)\frac{6}{\sqrt{9}} = 65 - 3.29 = 61.71$

$\bar{x} > 61.71$: ناحیه بحرانی

ب)
$$\begin{cases} \mu = 67 \\ \beta = ? \\ توان = ? \end{cases}$$

$\beta = P(RH | صدق H_1) = P(\bar{x} > 61.71 | \mu = 67) = P\left(\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} > \frac{61.71 - 67}{\frac{6}{\sqrt{9}}} \right)$

$\implies P(Z > -2.645) = 1 - P(Z \leq -2.645) = 1 - 0.0041 = 0.9959$

توان : $1 - \beta = 1 - 0.9959 = 0.0041$

ج)
$$\begin{cases} H_0 : \mu > 65 \\ H_1 : \mu < 65 \end{cases}$$

$\bar{x} = \frac{1}{n} \sum x_i = \frac{587}{9} = 65.22 \quad , \quad Z_0 = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{65.22 - 65}{\frac{6}{\sqrt{9}}} = 0.11$

$\implies \alpha = 0.05 \implies Z_{1-\alpha} = Z_{0.95} = 1.645$

$\implies Z_0 = 0.11 < Z_{0.95} = 1.645 \implies$ فرض H_0 نادرست است و ادعای اینه متوسد طول عمرها
بیشتراز $65 cm$ است رد می‌شود

#5

$$\begin{cases} H_0 : \sigma^2 = 0.9 \\ \\ H_1 : \sigma^2 \neq 0.9 \end{cases}$$

$$\bar{x} = \frac{1}{n} \sum x_i = \frac{36.23}{5} = 7.246$$

$$S^2 = \frac{1}{n-1} \sum (x_{ii} - \bar{x})^2 = 0.00083$$

$$X_0^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{4(0.00083)}{0.9} = 0.003689$$

$$\Rightarrow X_0^2 = 0.003689 \not> X_{0.975}^2 = 11.1 \qquad \Rightarrow \qquad فرض H_0 \ درست است$$

#6

نوع A	10.28	10.27	10.30	10.32	10.27	10.27	10.28	10.29
نوع B	10.31	10.31	10.26	10.30	10.27	10.31	10.29	10.26

الف) $\begin{cases} \sigma_n^2 = \sigma_B^2 \\ \sigma_n^2 \neq \sigma_B^2 \end{cases}$ $\quad \alpha = 0.02 \quad , \quad \bar{x}_A = \frac{1}{n_A} \sum x_i = \frac{82.28}{8} = 10.285$

$$\bar{x}_B = \frac{1}{n_B} \sum x_i = \frac{82.31}{8} = 10.29$$

$$S_A^2 = \frac{1}{n-1} \sum (x_{iA} - \bar{x}_A)^2 = \frac{0.0022}{7} = 0.0003143 \quad , \quad S_B^2 = \frac{1}{n-1} \sum (x_{iB} - \bar{x}_B)^2 = \frac{0.0035}{7} = 0.0005$$

$$\Rightarrow F_0 = \frac{S_A^2}{S_B^2} = \frac{0.0003143}{0.005} = 0.6286$$

$$\alpha = 0.02 \Rightarrow F_{1-\frac{\alpha}{2}, n_A-1, n_B-1} = F_{0.99, 7.7} = 6.99$$

$$F_0 = 0.6286 \not> F_{0.99, 7.7} = 6.99 \Rightarrow \qquad فرض H_0 \ ردی یعنی واریانس دو جامعه مساوی است$$

ب) $\begin{cases} H_0 : \mu_A = \mu_B \\ H_1 : \mu_A \neq \mu_B \end{cases}$ $\quad S_P^2 = \frac{(n_A-1)S_A^2 + (n_B-1)S_B^2}{n_A + n_B - 2} = \frac{7(0.0003143) + 7(0.0005)}{8+8-2} = 0.00040715$

$$\Rightarrow S_P = 0.020178 \qquad , \qquad T = \frac{(\bar{x}_A - \bar{x}_B) - (\mu_A - \mu_B)}{S_P \sqrt{\frac{1}{n_A} + \frac{1}{n_B}}} = \frac{(10.285 - 10.29) - 0}{0.020178 \sqrt{\frac{1}{8} + \frac{1}{8}}} = -0.4956$$

$$\alpha = 0.05 \Rightarrow t_{1-\frac{\alpha}{2}, n_A + n_B - 2} = t_{0.975, 14} = 2.14$$

$$\Rightarrow |T| = 0.4956 \not> t_{0.975, 14} = 2.14 \Rightarrow فرض H_0 \ ردی یعنی میانگین دو جمعیت باهم برابراست$$

#7

$$\begin{cases} \bar{x}_1 = 75 \\ \bar{x}_2 = 60 \end{cases} \quad \begin{matrix} S_1 = 6.1 & n_1 = 16 & \alpha = 0.01 \\ S_2 = 5.3 & n_2 = 14 \end{matrix}$$

$$\begin{cases} H_0 : \sigma_1^2 < \sigma_2^2 \\ H_1 : \sigma_1^2 > \sigma_2^2 \end{cases} \quad F_c = \frac{S_1^2}{S_2^2} = \frac{(6.1)^2}{(5.3)^2} = 1.3246 \quad , \quad \alpha = 0.01 \rightarrow F_{1-\alpha,\, n_1-1,\, n_2-1}$$

$$= F_{0.99,\, 15,\, 13} = 3.82$$

$$\Rightarrow F_0 = 1.3246 \nparallel F_{0.99,\, 15,\, 13} = 3.82 \quad \longrightarrow H_0 \text{ نرفض است}$$