یا لطیف

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#2

$$\begin{cases} y' = x^2 + y^2 \ , \ y(0) = 1 \\ \text{الف) } y(0.2) \rightarrow \text{روش اویلر} \ , \ (h = 0.1) \\ \text{ب) } y(0.2) \rightarrow \text{روش رانگ-کوتا مرتبه ۲} \ , \ (h = 0.1) \\ \text{ج) } y(0.2), y(0.3) \rightarrow \text{روش آدامس-بشفورث دو گامی} \ (h = 0.1) \end{cases}$$

الف) $x_0 = 0 \Rightarrow \begin{cases} x_2 = 0.2 \\ x_1 = 0.1 \end{cases} \Rightarrow y(0.1) \simeq y(0) + h\,y'(0) = 1 + (0.1) \times 1 = 1.1$

$$y(0.2) \simeq y(0.1) + h\,y'(0.1) = 1.1 + 0.1\,(1.22) = 1.222$$

ب) $f(x,y) = x^2 + y^2$:

(I) $\quad k_1 = h f(x_0, y_0) = 0.1 f(0.1) = 0.1$

$\quad k_2 = h f(x_0 + h, y_0 + k_1) = 0.1 f(0.1, 1.1) = 0.122$

$$\Rightarrow y_1 = y_0 + \frac{1}{2}(k_1 + k_2) = 1 + 0.5\,(0.1 + 0.122) = 1.111$$

(II) :

$$\begin{cases} k_1 = h f(x_1, y_1) = 0.1 f(0.1, 1.111) = 0.12443 \\ k_2 = h f(x_1 + h, y_1 + k_1) = 0.1 f(0.2, 1.23543) = 0.15663 \end{cases}$$

$$\Rightarrow y_2 = y_1 + 0.5\,(k_1 + k_2) = 1.111 + 0.5\,(0.12443 + 0.15663) = 1.25153 \Rightarrow y_2(0.2) = 1.25153$$

ج) $y_{i+1} = y_i + \frac{h}{2}\left(3f(x_i, y_i) - f(x_{i-1}, y_{i-1})\right) \quad i = 1, 2 \xrightarrow{\text{تمرین}} y_1 = 1.111$

اگر $i = 1$: $y_2 = y_1 + \frac{h}{2}\left(3f(x_1, y_1) - f(x_0, y_0)\right) = 1.111 + \frac{0.1}{2}\left(3f(0.1, 1.111) - f(0,1)\right)$

$= 1.24765$

اگر $i = 2$: $y_3 = y_2 + \frac{h}{2}\left(3f(x_2, y_2) - f(x_1, y_1)\right) = 1.24765 + \frac{0.1}{2}\Big(3f(0.2, 1.24765)$

$- f(0.1, 1.111)\Big) = 1.32493$

#7 $\begin{cases} y' = -y\ln y \ , \ y(0) = \frac{1}{2} \\ y(0.375) \to (AB2) \ , \ h = \frac{1}{8} \end{cases}$

$f(x,y) = -y\ln y \xrightarrow{x_0 = 0} x_3 = 0.375 \Rightarrow y_{i+1} = y_i + \frac{h}{2}\left(3f(x_i, y_i) - f(x_{i-1}, y_{i-1})\right)$

$\begin{cases} k_1 = hf(x_0, y_0) = \frac{1}{8} f(0, \frac{1}{2}) = 0.04332 \\ k_2 = hf(x_0 + h, y_0 + k_1) = \frac{1}{8} f(0.125, \frac{1}{2} + 0.04332) = 0.04143 \end{cases}$

$\Rightarrow y_1 = y_0 + \frac{1}{2}(k_1 + k_2) = \frac{1}{2} + \frac{1}{2}(0.08475) = 0.54238$

$y_2 = y_1 + \frac{h}{2}\left(3f(x_1, y_1) - f(x_0, y_0)\right) = 0.58293$

$y_3 = y_2 + \frac{h}{2}\left(3f(x_2, y_2) - f(x_1, y_1)\right) = \boxed{0.62118}$ $\Big) \to y' = -y\ln y \Rightarrow \frac{dy}{dx} = -y\ln y$

$\Rightarrow \frac{dy}{y\ln y} = -dx \to \int \frac{dy}{y\ln y} = -\int dx \Rightarrow \ln(\ln y) = -x + c \quad (y(0) = \frac{1}{2})$

$\to c = \ln(\ln(\frac{1}{2})) \Rightarrow \ln y = e^{-x+c} = e^{-x+\ln(\ln\frac{1}{2})} = e^{-x}\ln(\frac{1}{2}) = -\ln 2 e^{-x} \Rightarrow y = e^{-\ln 2 e^{-x}}$

$\xrightarrow{x = 0.375} y(0.375) = \boxed{0.62102}$ ← باتقریب مناسب متناظریان هستند

#12 $\begin{cases} AM4\ ? \\ E \overset{?}{=} -\frac{19h^5}{720} y^{(5)}(\eta) \end{cases}$

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$$\begin{cases} y' = \sin x + \sin y \quad , \ y(0) = 1 \\ h = 0.1 \\ x = 0 \ \text{to} \ x = 0.2 \\ \varepsilon = 0.001 \end{cases}$$

$$f_1^{(0)} = y_0 + h f(x_0, y_0) = 1 + 0.1 f(0.1) = 1.08415$$

$$y_1^{(1)} = y_0 + \frac{h}{2}\left(f(x_0, y_0) + f(x_1, y_1^{(0)})\right) = 1 + \frac{0.1}{2}\left(f(0,1) + f(0.1, 1.08415)\right) = 1.09126$$

$|y_1^{(1)} - y_1^{(0)}| \not< 0.001 \longrightarrow$ $y_1^{(2)} = y_0 + \frac{h}{2}\left(f(x_0, y_0) + f(x_1, y_1^{(1)})\right) = 1.09143$

$|y_1^{(2)} - y_1^{(1)}| < 0.001 \longrightarrow$ $y(0.1) \simeq 1.09143 = y_1$

$$y_2^{(0)} = y_1 + h f(x_1, y_1) = 1.09143 + 0.1 f(0.1, 1.09143) = 1.19014$$

$$y_2^{(1)} = y_1 + \frac{h}{2}\left(f(x_1, y_1) + f(x_2, y_2^{(0)})\right) = 1.09143 + \frac{0.1}{2}\left(f(0.1, 1.09143) + f(0.2, 1.19014)\right)$$

$$= 1.19714 \longrightarrow |y_2^{(1)} - y_2^{(0)}| \not< 0.001$$

$$\Rightarrow y_2^{(2)} = y_1 + \frac{h}{2}\left(f(x_1, y_1) + f(x_2, y_2^{(1)})\right) = 1.19726 \longrightarrow |y_2^{(2)} - y_2^{(1)}| < 0.001$$

$$\Rightarrow \underline{y(0.2) = 1.19726}$$

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$$\begin{cases} y' = -y + e^t \ , \ y(0) = 0 \ , \ 0 \le t \le 1 \\ h = 0.1 \end{cases}$$

$$y_{i+1} = y_i + \frac{h}{24}\left(55 f(t_i, y_i) - 59 f(t_{i-1}, y_{i-1}) + 37 f(t_{i-2}, y_{i-2}) - 9 f(t_{i-3}, y_{i-3})\right)$$

$$\begin{cases} y_1 = y(0.1) = 0.0905 \\ y_2 = y(0.2) = 0.1637 \\ y_3 = y(0.3) = 0.2222 \end{cases} \Rightarrow$$

t_i	y_i	y
0.4	0.2681	0.2681
0.5	0.3032	0.3033
0.6	0.3292	0.3293
0.7	0.3475	0.3476
0.8	0.3594	0.3595
0.9	0.3658	0.3659
1	0.3678	0.3679