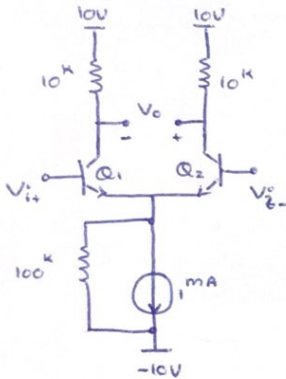


In the name of God

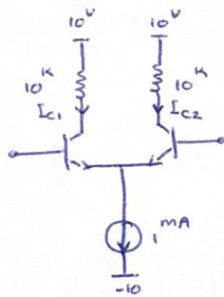
## Assignment 6:

1. Determine  $V_{o+}$ ,  $V_{o-}$ , and  $V_{od} = V_{o+} - V_{o-}$  for the following circuit.



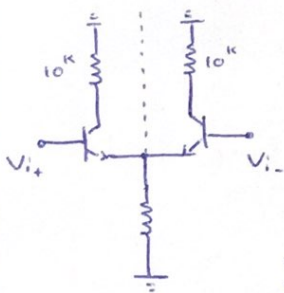
$$\begin{aligned} V_A &= \infty \\ V_T &= 25 \text{ mV} \\ \beta &= 100 \\ V_{i+} &= 0.5 V_{id} \\ V_{i-} &= -0.5 V_{id} \\ V_{id} &= 10 \sin 10t \text{ mV} \end{aligned}$$

dc Analysis:



$$\begin{aligned} I_{C1} &= I_{C2} = 0.5 \text{ mA} \\ g_{m1} &= g_{m2} = 40(0.5) = 20 \text{ mmho} \\ r_{\pi 1} &= r_{\pi 2} = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k} \\ r_{o1} &= r_{o2} = \infty \end{aligned}$$

ac Analysis: inputs are differential  $\Rightarrow$  draw half circuit  $\Rightarrow \bar{V}$  in symmetry line is equal to 0



$$\begin{aligned} A_{vd} &= \frac{V_o}{V_{id}} = \frac{1}{2} g_m R_c \\ &= \frac{1}{2} (20) 10 = 100 \end{aligned}$$

$$\frac{V_{od}}{V_{id}} = A_{vd} = \frac{V_{o+} - V_{o-}}{V_{id}} = -g_m R_c = -200$$

$$V_{o+} = A_{vd} V_{id} = 100 (10 \sin 10t) = 1000 \sin 10t \text{ mV} = 1 \sin 10t \text{ V}$$

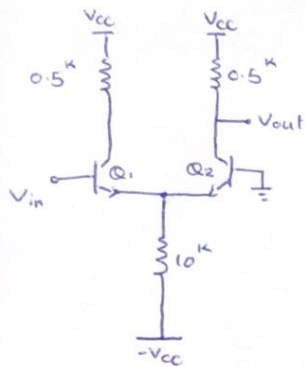
$$V_{o-} = A_{vd} V_{id} = -100 (10 \sin 10t) = -1000 \sin 10t \text{ mV} = -1 \sin 10t \text{ V}$$

$$V_{o+} - V_{o-} = 2000 \sin 10t \text{ mV} = 2 \sin 10t \text{ V}$$

2. Calculate the voltage gain ( $A_v = \frac{V_{out}}{V_{in}}$ ) of the circuit shown below using the following methods:

a) Direct analysis:

b) Half circuit analysis (break the input voltage to a differential term and a common-mode one, compute the differential and common-mode output voltage and add the result together)



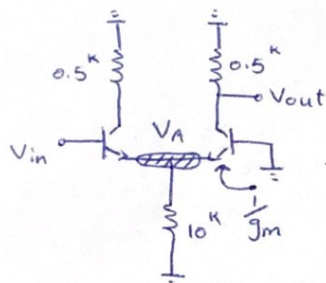
$$I_{C1} = I_{C2} = 1 \text{ mA} \Rightarrow g_{m1} = g_{m2} = 40 \text{ mho}$$

$$\beta = 100 \Rightarrow r_{\pi 1} = r_{\pi 2} = \frac{\beta}{g_m} = 2.5 \text{ k}\Omega$$

$$V_A = \infty \Rightarrow r_{o1} = r_{o2} = \infty$$

$$V_T = 25 \text{ mV}$$

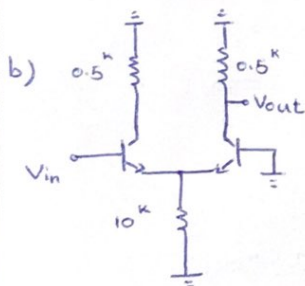
ac Analysis:



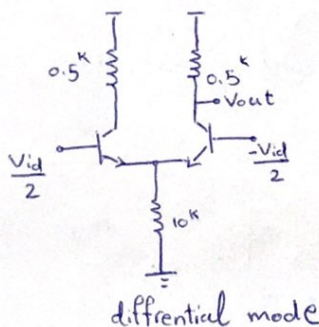
$$A_v = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_A} \times \frac{V_A}{V_{in}}$$

$$= \left[ +g_m R_C \right] \times \left[ \frac{R_E}{R_E + \frac{1}{g_m}} \right]$$

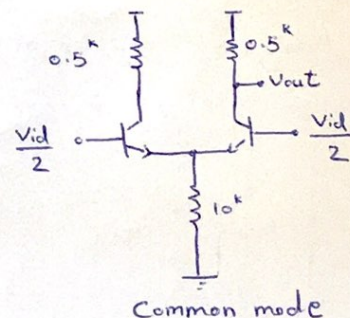
$$= (40 \times 0.5) \times \left( \frac{10 \text{ k} \parallel \frac{1}{40}}{(10 \text{ k} \parallel \frac{1}{40}) + \frac{1}{40}} \right) \approx \boxed{9.98}$$



$\equiv$

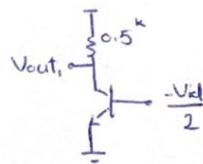


(+)



$$A_v = A_v \Big|_{\text{differential}} + A_v \Big|_{\text{common mode}}$$

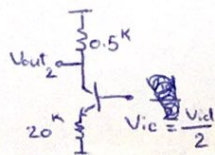
half circuit (diff):



$$\frac{V_{out1}}{-\frac{V_{id}}{2}} = -g_m R_C \Rightarrow V_{out1} = \frac{1}{2} g_m R_C V_{in} \quad (1)$$

$$V_{out1} = 10 V_{in}$$

half circuit (com):



$$\frac{V_{out2}}{V_{ic}} = \frac{-R_C}{2R_{EE} + \frac{1}{g_m}} = \frac{-0.5 \text{ k}}{20 + \frac{1}{40}} \Rightarrow \frac{V_{out2}}{V_{id}} = \frac{1}{2} \left( \frac{-0.5}{20 + \frac{1}{40}} \right) = -0.012$$



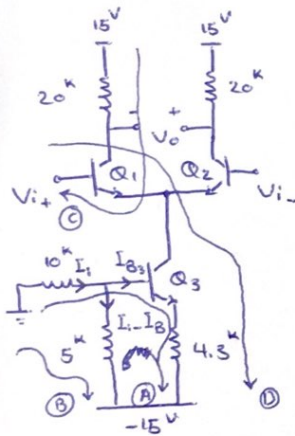
$$\Rightarrow V_{out2} = -0.012 V_{id}$$

$$V_{out} = V_{out1} + V_{out2} = 10 V_{in} - 0.012 V_{in} = 9.98 V_{in} \Rightarrow A_v = \frac{V_{out}}{V_{in}} = \boxed{9.98}$$

3. In the circuit shown below, the transistor are the same. neglect  $\beta$  effect in dc Analysis.

a) calculate the input dc common mode voltage range and the output swing?

b) Determine CMMR?  $V_{BE} = 0.6$ ,  $V_{CE,sat} = 0$ ,  $V_A = 10$ ,  $\beta = 100$



$$\text{KVL @ A: } 10 I_1 + 0.6 + 4.3 I_{C3} - 15 = 0 \quad \xrightarrow{I_{C3} = 100 I_{B3}} \quad \boxed{10 I_1 + 430 I_B = 14.4} \quad (I)$$

$$\text{KVL @ B: } 10 I_1 + 5 (I_1 - I_B) - 15 = 0 \Rightarrow \boxed{15 I_1 - 5 I_B = 15} \quad (II)$$

$$(I), (II) \rightarrow \begin{cases} I_1 = 1 \text{ mA} \\ I_B = 0.01 \text{ mA} \end{cases} \rightarrow I_{C3} = 100 I_B = \boxed{1 \text{ mA}}$$

$$\text{KVL @ C: } -15 + 20 (0.5) + V_{CE} - 0.6 + V_{in,dc} = 0 \Rightarrow V_{in,dc} = 15 - 10 - V_{CE}$$

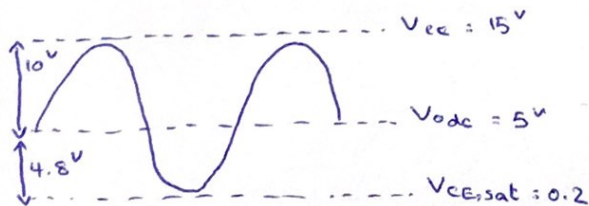
$$V_{idc} = 5 - V_{CE} \rightarrow \text{Max } \{V_{in,dc}\} = 5 \text{ V}$$

$$\text{KVL @ D: } -V_{in,dc} + 0.6 + V_{CE3} + 4.3 (1 \text{ mA}) - 15 = 0 \Rightarrow V_{in,dc} = -10.1 + V_{CE3}$$

$$\text{Min } \{V_{in,dc}\} = -10.1$$

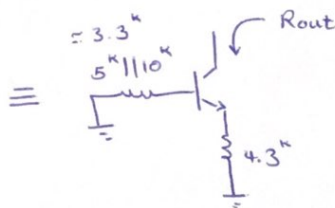
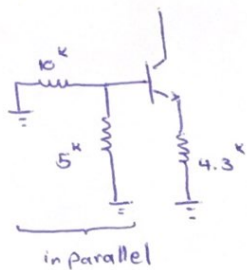
$$\boxed{-10.1 < V_{in,dc} < 5 \text{ V}}$$

calculate Swing:  $V_{CC} - V_{odc} = 20 \frac{I_{EE}}{2} \Rightarrow V_{odc} = V_{CC} - 20 \frac{I_{EE}}{2} = 15 - 20 \left(\frac{1}{2}\right) = 5$



$$\Rightarrow \text{Swing } \{V_o\} = 10 + 4.8 = \boxed{14.8 \text{ V}}$$

b)



$$CMMR = \frac{A_{v,d}}{A_{v,cm}}$$

$$R_{out} = r_o \left( 1 + \frac{\beta R_E}{R_E + r_{\pi} + R_B} \right)$$

$$= 10^k \left( 1 + \frac{100(4.3^k)}{4.3^k + 2.5^k + (5^k || 10^k)} \right) \approx 436^k$$

$$I_{C1} = I_{C2} = 0.5 \text{ mA}$$

$$g_{m1} = g_{m2} = 20 \text{ mmho}$$

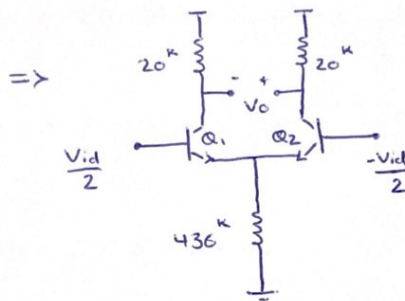
$$r_{\pi 1} = r_{\pi 2} = \frac{\beta}{g_m} = \frac{100}{20} = 5^k$$

$$r_{o1} = r_{o2} = \frac{V_A}{I_C} = \frac{10}{0.5} = 20^k$$

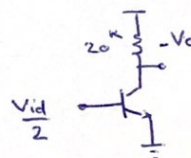
$$r_{o3} = \frac{10}{1} = 10^k$$

$$g_{m3} = 40 \text{ mmho}$$

$$r_{\pi 3} = \frac{100}{40} = 2.5^k$$



I) differential mode :

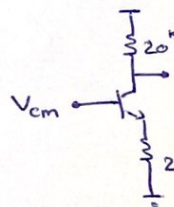


$$A_v = \frac{-V_o}{\frac{V_{id}}{2}} = -g_m (R_c || r_o)$$

$$= 20(20^k || 20^k) = 200$$

$$\Rightarrow \frac{V_o}{V_{id}} = \frac{1}{2}(200) = 100$$

II) common mode :



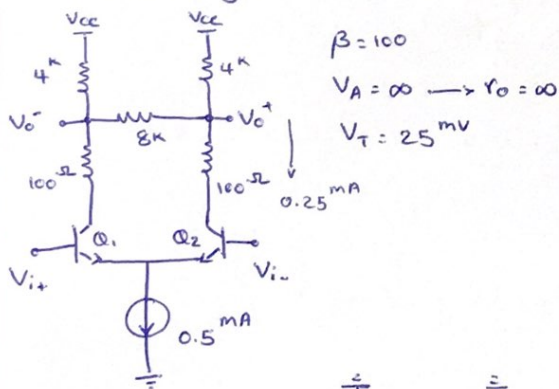
$$A_v = \frac{-R_c}{R_E + \frac{1}{g_m} + \frac{1}{20}}$$

$$= \frac{-20}{2(436) + \frac{1}{40} + \frac{1}{20}}$$

$$= -0.023$$

$$\Rightarrow CMMR = \frac{A_{v,d}}{A_{v,cm}} = \frac{2(100)}{0.023} = \boxed{8695.65}$$

4. In the following circuit,  $Q_1$  and  $Q_2$  are the same and are biased in the active region. Calculate the differential voltage gain ( $A_{v,d} = \frac{V_o}{V_{id}} = \frac{V_{o+} - V_{o-}}{V_{i+} - V_{i-}}$ )



$$\beta = 100$$

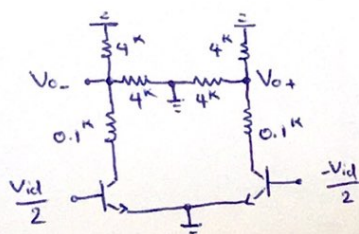
$$V_A = \infty \rightarrow r_o = \infty$$

$$V_T = 25 \text{ mV}$$

$$0.25 \text{ mA}$$

$$\left\{ \begin{array}{l} g_{m1} = g_{m2} = 10 \text{ mmho} \\ r_{\pi 1} = r_{\pi 2} = \frac{100}{10} = 10^k \\ r_{o1} = r_{o2} = \infty \end{array} \right.$$

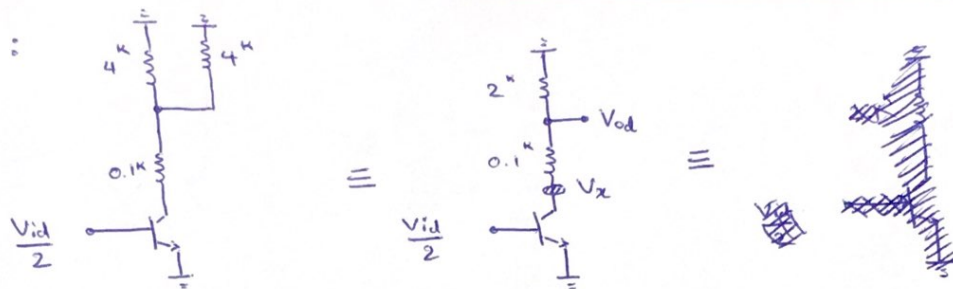
ac Analysis :



$$A_{v,d} = \frac{V_{o,d}}{V_{id}} = \frac{V_{o+} - V_{o-}}{V_{id}}$$

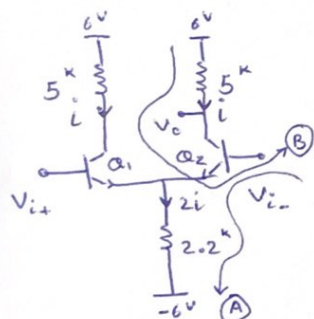


half circuit:



$$A_{vd} = \frac{V_{od}}{\frac{V_{id}}{2}} = \frac{V_{od}}{V_x} \times \frac{V_x}{\frac{V_{id}}{2}} = [-g_m R_c] \times \left[ \frac{2}{2+0.1} \right] = [-10(0.1)] \times \left[ \frac{2}{2.1} \right] = \boxed{-0.9}$$

5. In the following differential Amplifier Circuit, determine the input dc common mode range. ( $V_{BE} = 0.6V$ ,  $V_{CE,sat} = 0.2$ )



$$\text{KVL @ A: } -V_{in,dc} + 0.6 + 2.2^k(2i) - 6 = 0$$

$$\Rightarrow i = \frac{5.4 + V_{in,dc}}{4.4} \quad (I)$$

$$\text{KVL @ B: } -6 + 5^k i + V_{CE2} - 0.6 + V_{in,dc} = 0$$

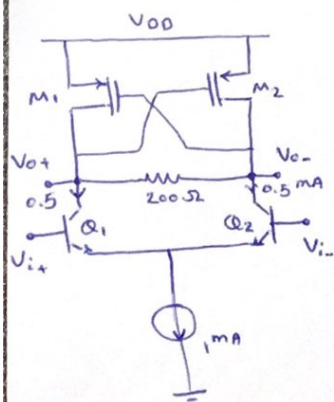
$$\Rightarrow V_{CE2} = 6 - 5i + 0.6 - V_{in,dc} \quad (II)$$

$$V_{CE2} = 6.6 - 5 \left( \frac{5.4 + V_{in,dc}}{4.4} \right) - V_{in,dc}$$

$$V_{CE2} = -2.13 V_{in,dc} + 0.46 \geq 0.2$$

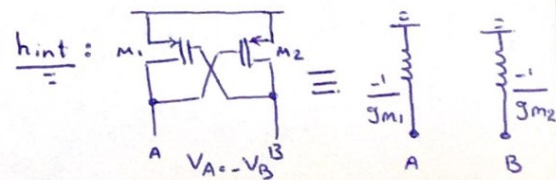
$$\Rightarrow \boxed{V_{in,dc} \leq 0.122V}$$

6. Calculate the single-ended voltage gain as well as the fully differential voltage gain ( $\frac{V_{o+} - V_{o-}}{V_{i+} - V_{i-}}$ ) for the following circuit.



$$Q_{1,2}: \begin{cases} \beta = 100 \\ V_T = 25mV \\ |V_A| = 10V \end{cases}$$

$$M_{1,2}: \begin{cases} g_m = 10 \frac{mA}{V} \\ |A| = 0V^{-1} \end{cases}$$



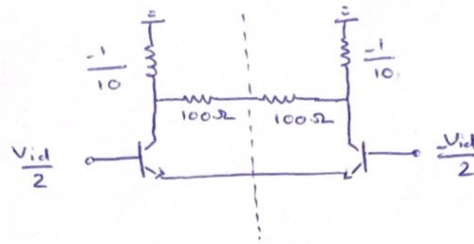
\* cross-coupled configuration

$$I_{C1} = I_{C2} = 0.5 \text{ mA}$$

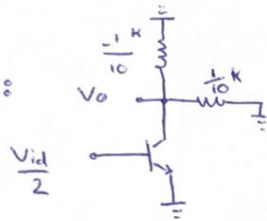
$$g_{m1} = g_{m2} = 20 \text{ mmho}$$

$$r_{n1} = r_{n2} = \frac{\beta}{g_m} = 5^k$$

$$r_{o1} = r_{o2} = \frac{V_A}{I_C} = 20^k$$



half circuit :



$$A_{v_d} = \frac{V_o}{\frac{V_{id}}{2}} = -g_m (R_C \parallel r_o) = -20 \left( -0.1 \parallel 0.1 \parallel 20^k \right)$$

$$= -400$$

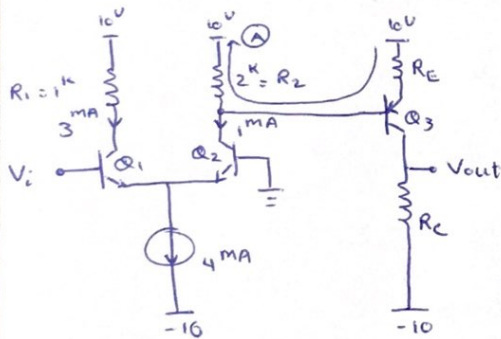
$$\frac{V_o}{V_{id}} = \frac{1}{2} \times 400 = 200 \rightarrow \frac{V_{od}}{V_{id}} = 2(200) = 400$$

7. In the following circuit, the dc output voltage is equal to zero.

Calculate the voltage gain:

$$(V_{i(DC)} = 0^V, V_{o(DC)} = 0^V, |V_{BE}| = 0.6^V, V_T = 25^{\text{mV}},$$

$$V_A = \infty, \beta \gg 1, R_E \gg \frac{1}{g_m})$$



$$\text{DC Analysis : } I_{C1} = 3 \text{ mA}, I_{C2} = 1 \text{ mA}$$

~~Derive~~

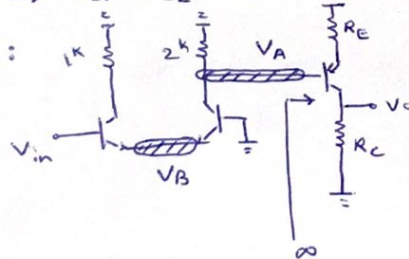
$$g_{m1} = 120 \text{ mmho}, g_{m2} = 40 \text{ mmho}$$

$$r_n = \infty, r_o = \infty$$

$$\text{KVL @ A : } -10 + R_E I_{C3} + 0.6 - 2^k(1^{\text{mA}}) + 10 = 0$$

$$\text{if } R_1 = R_2 \Rightarrow I_{C1} = I_{C2} = 2 \text{ mA}$$

ac Analysis :



$$A_v = \frac{V_o}{V_i} = \frac{V_o}{V_A} \times \frac{V_A}{V_B} \times \frac{V_B}{V_i} = \left[ \frac{-R_C}{R_E + \frac{1}{g_{m3}}} \right] \times \left[ g_{m2} (R_C \parallel r_o) \right] \times \left[ \frac{R_E}{R_E + \frac{1}{g_m}} \right]$$

$$R_E \gg \frac{1}{g_{m3}} \Rightarrow \frac{1}{g_{m3}} \approx 0$$

$$= \frac{-R_C}{R_E} \times (g_{m2} R_C) = -80 \frac{R_C}{R_E}$$