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1)

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt = \frac{\sin(4\omega)}{\omega}$$

$a_0 = 0$

$$a_k = \frac{1}{8}\int_0^8 x(t) e^{-j(\frac{2\pi}{8})} dt = \frac{1}{8}\int_0^4 e^{-j(\frac{2\pi}{8})tk} dt - \frac{1}{8}\int_4^8 e^{-j(\frac{2\pi}{8})tk} dt$$

$$= \frac{1}{j\pi k}\left[1 - e^{-j\pi k}\right]$$

$$a_k = \begin{cases} \frac{2}{j\pi k} & k \text{ فرد} \\ 0 & k \text{ زوج} \end{cases}$$

$$\omega_0 = \frac{\pi}{4}$$

$$y(t) = \sum_{-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}$$

$$H(j\omega) = (jk(\tfrac{\pi}{4})) = \frac{\sin(k\pi)}{\sin(\frac{\pi}{4})}$$

برای مقادیر فرد برابر صفر است $y(t) = 0$

۲)

$x[n] = \sum_{-\infty}^{\infty} \delta[n-4k]$ $\qquad H(e^{j\omega})$

$y[n] = \cos \frac{5\pi}{2} n + \frac{\pi}{4}$ $\qquad H(e^{jk\frac{\pi}{2}})$ $\qquad k = 0, 1, 2, 3$

$N = 4$

$$a_k = \frac{1}{4} \sum_{n=0}^{3} x[n] e^{-j\frac{2\pi}{4}} = \frac{1}{4}$$

$$y[n] = \sum_{k=0}^{3} a_k H(e^{j(\frac{2\pi}{4})k}) e^{jk(\frac{2\pi}{4})n}$$

$$= \frac{1}{4} H(e^{j0}) e^{j0} + \frac{1}{4} H(e^{j(\frac{\pi}{2})}) e^{j(\pi/2)} + \frac{1}{4} H(e^{j(3\pi/2)}) e^{j3\pi/2}$$
$$+ \frac{1}{4} H(e^{j\pi}) e^{j\pi}$$

$$y[n] = \cos\left(\frac{5\pi}{2} n + \pi/4\right) = \frac{1}{2} e^{j(\pi/2\, n + \pi/4)} + \frac{1}{2} e^{j(\frac{3\pi}{2} n - \pi/4)}$$

$H(e^{j0}) = H(e^{j\pi}) = 0$

$H(e^{j(\pi/2)}) = e^{2j\pi/4}$

$H(e^{3j\pi/2}) = 2e^{-j\pi/4}$

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۳) الف) $x[n] = (-1)^n$

$x_1[n] = e^{j\text{۲}(\frac{\pi}{\text{۲}})n}$

دوره تناوب ۲ دارد متناوب است
ضریب سری فوریه $0 \leq k \leq 1$

$a_0 = 0 \qquad a_1 = 1$

$$y_1[n] = \sum_{k=0}^{1} a_k H(e^{\text{۲}kj\pi/\text{۲}}) e^{k\pi} = 0 + a_1 H(e^{j\pi}) e^{j\pi} = 0$$

ب) $x[n] = 1 + \sin(\frac{\text{۳}\pi}{\text{۸}} n + \frac{\pi}{\text{۴}})$

با دوره تناوب ۱۶ دارد متناوب است

$$x_2[n] = e^{j(\text{۲}\pi/\text{۱۶})n(0)} - (j/\text{۲}) e^{-j(\pi/\text{۴})} e^{-j\frac{\pi}{\text{۸}}(\text{۳})n}$$

$$= e^{j(\pi/\text{۸})n} - (j/\text{۲}) e^{j(\pi/\text{۸})(\text{۳})n} + (j/\text{۲}) e^{-j(\pi/\text{۴})} e^{j(\pi/\text{۸})(\text{۱۳})n}$$

ضریب فوریه $0 \leq k \leq 15$

$a_0 = 1 \qquad a_3 = -(j/\text{۲}) e^{j\pi/\text{۴}}$

$a_{13} = (j/\text{۲}) e^{-j\pi/\text{۴}}$

$$y_2[n] = \sum_{k=0}^{15} a_k H(e^{j\pi k/\text{۱۶}}) e^{k(\pi/\text{۱۶})}$$

$$= 0 - (j/\text{۲})(e^{j\pi/\text{۴}}) e^{j(\pi/\text{۸})\text{۳}n} + (j/\text{۲}) e^{-j(\pi/\text{۴})} e^{j(\pi/\text{۸})n(\text{۱۳})}$$

$$= \sin(\frac{\text{۳}\pi}{\text{۸}})n + \frac{\pi}{\text{۴}}$$

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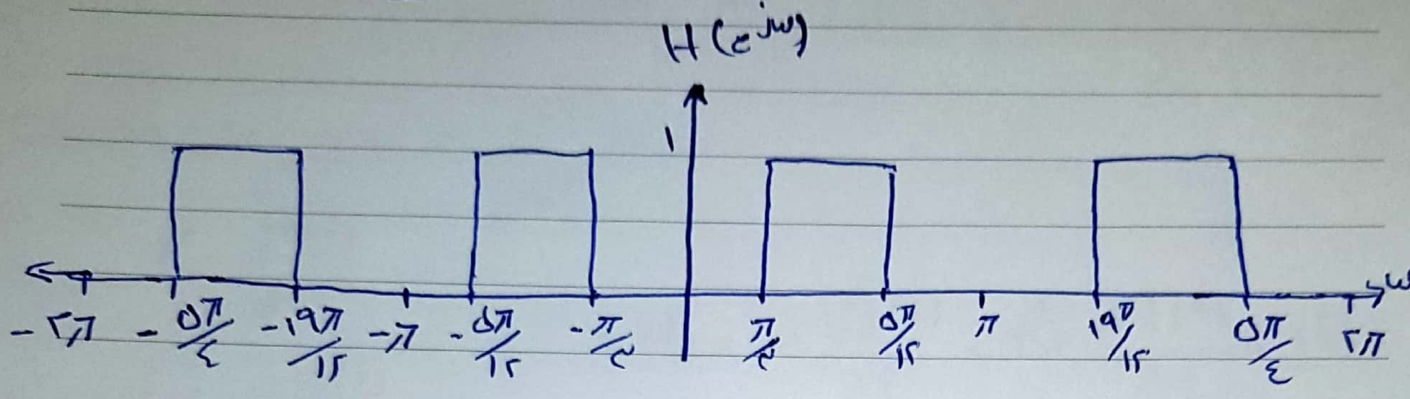
ادامه ۳:

ج) $x[n] = \sum_{-\infty}^{\infty} \left(\frac{1}{2}\right)^{n-4k} u[n-4k]$



$r[n] * g[n]$

$r[n] = \sum_{-\infty}^{\infty} \delta[n-4k]$

$g[n] = \left(\frac{1}{2}\right)^{n} u[n]$

سیگنال $r[n]$ دارای دوره تناوب ۴

$a_k = \frac{1}{4} \quad , \text{all } k$

$q[n] = \sum_{k=0}^{3} a_k H(e^{j2k\pi/4}) e^{k(2\pi/4)}$

$= \left(\frac{1}{4}\right)\left[H(e^{j0})e^{j0} + H(e^{j\pi/2})e^{j(\pi/2)}\right.$

$\left. + H(e^{j\pi})e^{j\pi} + H((e^{3j(\pi/2)})e^{j3\pi/2}\right) = 0$

۴)

الف) معادله دیفرانسیل مرتبط کننده $y(t)$, $x(t)$ را بیابید

$C\frac{dy(t)}{dt}$: جریان جاری در خازن

$RC\frac{dy(t)}{dt}$

$LC\frac{d^2y(t)}{dt^2}$

$$x(t)=LC\frac{d^2y(t)}{dt^2}+RC\frac{dy(t)}{dt}+y(t)$$

$R=C=L=1$

$$\frac{d^2y(t)}{dt^2}+\frac{dy(t)}{dt}+y(t)=x(t)$$

ب)

$$H(j\omega)=\frac{1}{-\omega^2+j\omega+1}$$

ج)

$$x(t)=\frac{1}{2j}e^{j(2\pi/2\pi)t}-\frac{1}{2j}e^{-j(2\pi/2\pi)t}$$

$$a_0=a_{-1}=\frac{1}{2j}$$

$$y(t)=a_1H(j)e^{jt}-a_{-1}H(-j)e^{-jt}=\left(\frac{1}{j}e^{jt}-\frac{1}{-j}e^{-jt}\right)$$

$$=-\frac{1}{2}\left(e^{jt}+e^{-jt}\right)=\cos(t)$$

$$y[n] - \frac{1}{۴} y[n-1] = x[n]$$ ۱د

$$\omega = \frac{۲\pi}{N}$$

الف) $x[n] = \sin(\frac{۲\pi}{۴} n)$

$$y[n] = \sum a_k H(e^{j۲k\pi/N}) e^{jk(۲\pi/N)n}$$

$N = ۴$ $\quad a_۲ = a_۲^* = \frac{1}{۲j}$

$$b_۲ = a_1 H(e^{۲j\pi/۴}) = \frac{1}{۲j(1-\frac{1}{۴}e^{-۲\pi/۴})}$$

$$b_{-۲} = a_{-1} H(e^{-۲j\pi/۴}) = \frac{1}{۲j(1-\frac{1}{۴}e^{j۲\pi/۴})}$$

ب) $x[n] = \cos(\frac{\pi}{۴} n) + ۲\cos(\frac{\pi}{۲} n)$ $\quad N = ۸$ $\quad a_1 = a_{-1} = \frac{1}{۲}$

$a_۲ = a_{-۲} = 1$

$$b_1 = a_1 H(e^{j\pi/۴}) = \frac{1}{۲(1-\frac{1}{۴}e^{-j\pi/۴})}$$

$$b_{-1} = a_{-1} H(e^{-j\pi/۴}) = \frac{1}{۲(1-\frac{1}{۴}e^{j\pi/۴})}$$

$$b_۲ = a_۲ H(e^{j\pi/۲}) = \frac{1}{(1-\frac{1}{۴}e^{-j\pi/۲})^*}$$

$$b_{-۲} = a_{-۲} H(e^{-j\pi/۲}) = \frac{1}{(1-\frac{1}{۴}e^{j\pi/۲})^*}$$

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$$h[n] = \left(\frac{1}{۲}\right)^{|n|}$$

$$x[n] = \sum_{-\infty}^{\infty} \delta(n - ۴k)$$

$$H(e^{jw}) = \frac{1}{1 - \frac{1}{۲}e^{-jw}} - \frac{1}{1 - ۲e^{-jw}}$$

$$a_k = \frac{1}{۴} \quad \text{all } k \quad N = ۴$$

$$b_k = a_k \, H(e^{j۲k\pi/N}) = \frac{1}{۴}\left[\frac{1}{1 - \frac{1}{۲}e^{-jk\pi/۲}} - \frac{1}{1 - ۲e^{jk\pi/۲}}\right]$$