یالطیف

رضا آدینه پور
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$$\begin{cases} f \in C^2[x_0, x_1] \\ |f(x) - P_1(x)| \le \frac{h^2}{8} M \end{cases} \qquad M = \max_{x_0 < x < x_1} |f''(x)| \qquad h = x_1 - x_0$$

$$E(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_n)}{(n+1)!} f^{(n+1)}(c(x)) \Rightarrow |f(x) - P_1(x)| = \frac{\overbrace{(x-x_0)(x-x_1)}^{g(x)}}{2!} f''(\xi)$$

$$\Rightarrow g'(x) = 2x - (x_1 + x_0) = 0 \Rightarrow x = \frac{x_0 + x_1}{2} \xrightarrow[g(x_0) = g(x_1)]{g''(x) = 2} \left| \frac{(x-x_0)(x-x_1)}{2!} f''(\xi) \right|$$

$$\le \frac{h^2}{n} |f''(\xi)| \le \frac{h^2}{n} M \Rightarrow M = \max_{x_0 < x < x_1} |f''(x)|$$

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$$L'_j(x_j) \overset{?}{=} \sum_{i=0, i \neq j}^{n} \frac{1}{x_j - x_i}$$

$$L_j(x) = \frac{(x-x_0)\cdots(x-x_{j-1})(x-x_{j+1})\cdots(x-x_n)}{\underbrace{(x_j-x_0)\cdots(x_j-x_{j-1})(x_j-x_{j+1})\cdots(x_j-x_n)}_{=A}} \Rightarrow L'_j(x) = \frac{(x-x_1)\cdots(x-x_{j-1})(x-x_{j+1})}{A}$$

$$+ \frac{(x-x_0)(x-x_2)\cdots(x-x_{j-1})(x-x_{j+1}) + \cdots (x-x_n)}{A} + \cdots + \frac{(x-x_0)\cdots(x-x_{j-1})(x-x_{j+1})\cdots(x-x_{n-1})}{A}$$

$$\xrightarrow{x = x_j} L'_j(x_i) = \frac{1}{x_j - x_0} + \frac{1}{x_j - x_1} + \cdots + \frac{1}{x_j - x_{j-1}} + \frac{1}{x_j - x_{j+1}} + \cdots + \frac{1}{x_j - x_n}$$

$$= \sum_{i=0, i \neq j}^{n} \frac{1}{x_j - x_i}$$

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$$\begin{cases} f(x) = \log_2 x \\ \text{گره ها} : 1, \frac{1}{2}, \frac{1}{4} \end{cases}$$

x	$\frac{1}{4}$	$\frac{1}{2}$	1
$f(x)$	-2	-1	0

$\xrightarrow{\text{در فواصل}} \left(\frac{1}{2}, 1\right), \left(\frac{1}{4}, \frac{1}{2}\right)$

$$S(x) = \begin{cases} \frac{x-0.5}{-0.25} f\left(\frac{1}{4}\right) + \frac{x-0.25}{0.25} f\left(\frac{1}{2}\right) \\ \frac{x-1}{-0.5} f\left(\frac{1}{2}\right) + \frac{x-0.5}{0.5} f(1) \end{cases} = \begin{cases} 4x-3 \ , & \frac{1}{4} \le x \le \frac{1}{2} \\ 2(x-1) \ , & \frac{1}{2} \le x \le 1 \end{cases}$$

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$$\begin{cases} S(x) \to \text{اسپلاین مکعبی طبیعی} \\ a = x_0 < x_1 < \cdots < x_n = b \\ \int_a^b (S''(x))^2 dx \overset{?}{\le} \int_a^b (f''(x))^2 dx \end{cases}$$

داریم : $S''(a) = S''(b) = 0 \quad \Rightarrow f(x) - S(x) = g(x) \Rightarrow g(x_i) = 0 \qquad i = 0, 1, \ldots, n$

$$\int_a^b (f''(x))^2 dx = \int_a^b (S''(x))^2 dx + \int_a^b (g''(x))^2 dx + 2\int_a^b S''(x) g''(x) dx \qquad *$$

$$\int_a^b S''(x) g''(x)\, dx = \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} S''(x) g''(x) dx = \sum_{i=1}^{n} \left((S''g')(x_i) - (S''g')(x_{i-1}) - \int_{x_{i-1}}^{x_i} S'''(x) g'(x)\, dx \right) \xrightarrow{\sum_{i=1}^{n} (S''g')(x_i) - (S''g')(x_{i-1}) = (S''g')(b) - (S''g')(a)} \int_a^b S''(x) g''(x)\, dx$$

$$= - \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} S'''(x)\, g'(x) dx = - \sum_{i=1}^{\infty} c_i \int_{x_{i-1}}^{x_i} g'(x)\, dx \xrightarrow{-\sum_{i=1}^{n} c_i (g(x_i) - g(x_{i-1})) = 0} \int_a^b S''(x) g''(x) dx = 0$$

$$\xrightarrow{*} \int_a^b (f''(x))^2 dx = \int_a^b (S''(x))^2 dx + \int_a^b (g''(x))^2 dx \Rightarrow \int_a^b (f''(x))^2 dx \ge \int_a^b (S''(x))^2 dx$$

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x	-2	-1	0	1	3
y	1	2	3	3	4

فرض می کنیم جواب به صورت مقابل است : $Y = a + bx \qquad (n = 5)$

$$* \begin{cases} 5a + b = 13 \\ a + 15b = 11 \end{cases} \Rightarrow a = \frac{92}{37} \ , \ b = \frac{21}{37} \quad \Rightarrow Y = a + bx = \frac{92}{37} + \frac{21}{37} x$$

$$* \begin{cases} na + b \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i \\ a \sum_{i=1}^{n} x_i + b \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i \end{cases}$$