یا لطیف

دانشگاه صنعتی شاهرود
دانشکده مهندسی برق و رباتیک

مدار های الکتریکی ۱

نیم سال اول ۰۰-۹۹

پاسخ تمرین سری سوم

KVL و KCL و شکل موج ها

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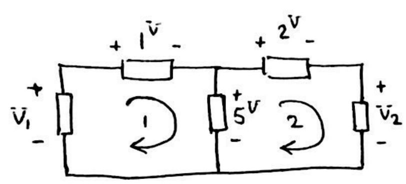
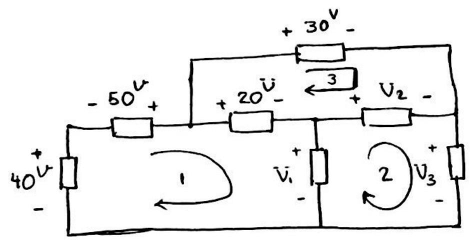
a)



$$\text{KVL in } 1 : +1 + 5 - V_1 = 0 \Rightarrow V_1 = 6^v$$

$$\text{KVL in } 2 : +2 + V_2 - 5 = 0 \Rightarrow V_2 = 3^v$$

b)

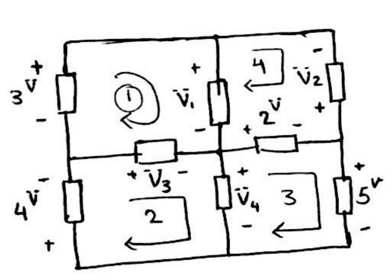


$$\text{KVL in } 1 : -50 + 20 + V_1 - 40 = 0 \Rightarrow V_1 = 70^v$$

$$\text{KVL in } 3 : +30 - V_2 - 20 = 0 \Rightarrow V_2 = 10^v$$

$$\text{KVL in } 2 : +V_2 + V_3 - V_1 = 0 \Rightarrow V_3 = 60^v$$

c)



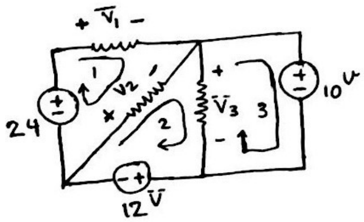
$$\text{KVL in } 3 : +2 + 5 - V_4 = 0 \Rightarrow V_4 = 7^v$$

$$\text{KVL in } 2 : +V_3 + V_4 + 4 = 0 \xrightarrow{V_4 = 7} V_3 = -11^v$$

$$\text{KVL in } 1 : +V_1 - V_3 - 3 = 0 \xrightarrow{V_3 = -11} V_1 = -8^v$$

$$\text{KVL in } 4 : -V_2 - 2 - V_1 = 0 \xrightarrow{V_1 = -8} V_2 = 6^v$$

d)



KVL in ③ : $+10 - V_3 = 0 \Rightarrow V_3 = 10^V$

KVL in ② : $+V_2 + V_3 + 12 = 0 \xrightarrow{V_3=10} V_2 = -22^V$

KVL in ① : $+V_1 - V_2 - 24 = 0 \xrightarrow{V_2=-22} V_1 = 2^V$

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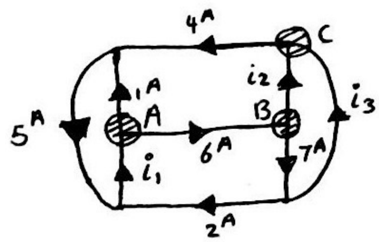
a)

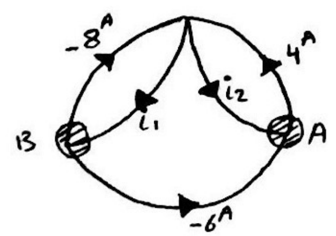


KCL in Ⓐ : $+1 - i_1 + 6 = 0 \Rightarrow i_1 = 7^A$

KCL in Ⓑ : $+i_2 + 7 - 6 = 0 \Rightarrow i_2 = -1^A$

KCL in Ⓒ : $-i_2 + 4 - i_3 = 0 \Rightarrow i_3 = 5^A$

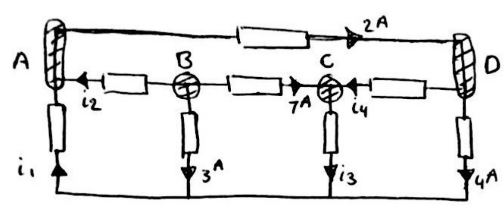
b)



KCL in Ⓐ : $+4 - i_2 - (-6) = 0 \Rightarrow i_2 = 10^A$

KCL in Ⓑ : $-i_1 - 8 - 6 = 0 \Rightarrow i_1 = -14^A$

c)



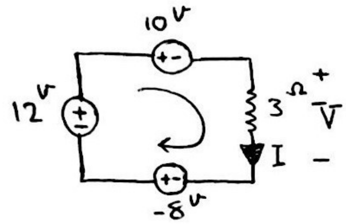
KCL in Ⓑ : $+7 + 3 + i_2 = 0 \Rightarrow i_2 = -10^A$

KCL in Ⓐ : $-i_2 - i_1 + 2 = 0 \Rightarrow i_1 = 12^A$

KCL in Ⓓ : $-2 + 4 + i_4 = 0 \Rightarrow i_4 = -2^A$

KCL in Ⓒ : $-7 - i_4 + i_3 = 0 \Rightarrow i_3 = 5^A$

d)



$$KVL: +10 + \bar{V} - (-8) - 12 = 0 \Rightarrow \bar{V} = -6^{\bar{V}}$$

$$\xrightarrow{\bar{V} = iR} i = \frac{-6}{3} = -2^A$$

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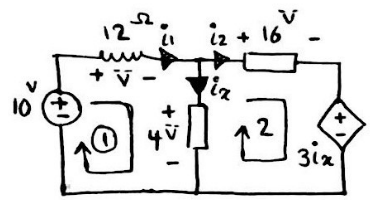

$$KVL \text{ in } ①: +\bar{V} + 4 - 10 = 0 \Rightarrow \bar{V} = 6^{\bar{V}}$$

$$KVL \text{ in } ②: +16 + 3i_x - 4^V = 0 \Rightarrow i_x = -4^A$$

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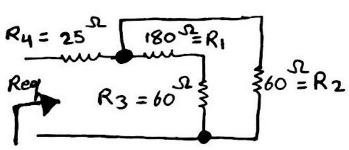
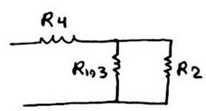


$$R_1 \xlongequal{S} R_3 : R_{1,3} = R_1 + R_3 = 180 + 60 = 240$$

$\Rightarrow$



$$R_{1,3} \| R_2 : \frac{R_{1,3} R_2}{R_{1,3} + R_2} = \frac{240 \times 60}{240 + 60} = 48^\Omega$$

$\Rightarrow$

$$\Rightarrow Req = R_4 + R_{1,3,2} = 25 + 48 = 73^\Omega$$

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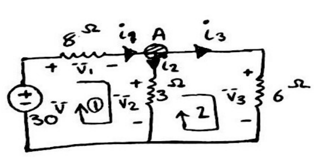

$$\bar{V} = iR \Rightarrow \begin{cases} \bar{V}_1 = 8i_1 \\ \bar{V}_2 = 3i_2 \\ \bar{V}_3 = 6i_3 \end{cases} \qquad KCL \text{ in } A: -i_1 + i_3 + i_2 = 0$$

$$KVL \text{ in } ①: +8i_1 + 3(\underbrace{i_1 - i_3}_{= i_2}) - 30 = 0$$

$$\Rightarrow 11i_1 - 3i_3 = 30 \text{ (I)}$$

$$KVL \text{ in } 2: +6i_3 + 3(i_3 - i_1) = 0 \Rightarrow 9i_3 = 3i_1 \Rightarrow i_1 = 3i_3 \text{ (II)}$$

$$\Rightarrow \text{(II) in (I)}: 11(3i_3) - 3i_3 = 30 \Rightarrow \boxed{i_3 = 1^A}, \boxed{i_1 = 3^A}, \boxed{i_2 = 2^A}$$

$$\Rightarrow \begin{cases} \bar{V}_1 = i_1 R_1 = 8 \times 3 = 24^V \\ \bar{V}_2 = i_2 R_2 = 2 \times 3 = 6^V \\ \bar{V}_3 = i_3 R_3 = 1 \times 6 = 6^V \end{cases}$$

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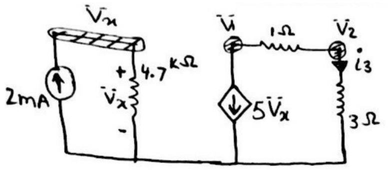

KCL in $\bar{V}_x$: $-2 + \frac{\bar{V}_x}{4.7} = 0 \Rightarrow \bar{V}_x = 9.4^{V}$

KCL in $\bar{V}_1$: $+5\bar{V}_x + \frac{\bar{V}_1 - \bar{V}_2}{1} = 0$ (II)

KCL in $\bar{V}_2$: $+i_3 + \frac{\bar{V}_2 - \bar{V}_1}{1} = 0 \Rightarrow i_3 = \bar{V}_1 - \bar{V}_2$ (I)

$\Rightarrow$ (I) in (II) : $5V_x + i_3 = 0 \Rightarrow i_3 = -5\bar{V}_x$

$\xrightarrow{\bar{V}_x = 9.4} i_3 = -47^{A}$

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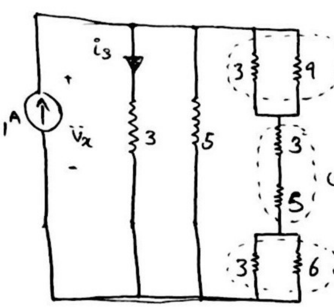


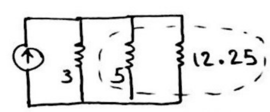
موازی : $\frac{9\times 3}{9+3} = 2.25^{\Omega}$

سری : $5+3 = 8$

موازی : $\frac{6\times 3}{6+3} = 2^{\Omega}$

سری : $2.25+8+2 = 12.25^{\Omega}$

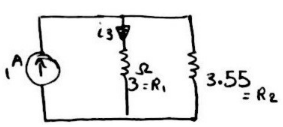
$\Rightarrow$



موازی

$\Rightarrow \frac{5\times 12.25}{5+12.25} = \frac{61.25}{17.25} = 3.55$

$\Rightarrow$



تقسیم جریان : $i_3 = \frac{G_1}{G_1+G_2} = \frac{0.3}{0.3+0.28} = 0.49^{A}$

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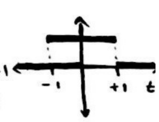
a)

$f_a(t) = u(1-t^2)$: $\quad 1-t^2 > 0 \Rightarrow -1 < t < +1 \Rightarrow f_a(t) = \begin{cases} 0 & : t < -1 \\ 1 & : -1 < t < +1 \\ 0 & : t > +1 \end{cases}$

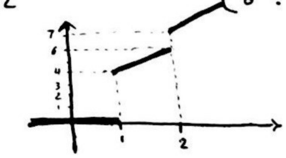


b)

$f_b(t) = u(t-2) + 2(t+1)u(t-1)$:

$u(t-2) = \begin{cases} 1 & : t > 2 \\ 0 & : t < 2 \end{cases}$, $u(t-1) = \begin{cases} 1 & : t > 1 \\ 0 & : t < 1 \end{cases}$

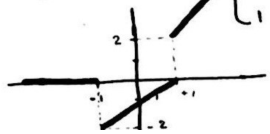
$\Rightarrow f_b(t) = \begin{cases} 0+2(t+1)(0) & : t < 1 \\ 0+2(t+1)(1) & : 1 < t < 2 \\ 1+2(t+1)(1) & : t > 2 \end{cases} = \begin{cases} 0 & : t < 1 \\ 2t+2 & : 1 < t < 2 \\ 2t+3 & : t > 2 \end{cases}$



c)

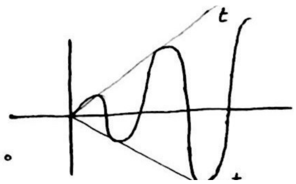
$$f_c(t) = (t-1)u(t+1) + (t+1)u(t-1) \; : \; u(t+1) = \begin{cases} 0 : t<-1 \\ 1 : t>+1 \end{cases}, \quad u(t-1) = \begin{cases} 0 : t<1 \\ 1 : t>1 \end{cases}$$

$$\Rightarrow f_c(t) = \begin{cases} 0 : t<-1 \\ t-1 : -1<t<+1 \\ 2t : t>1 \end{cases}$$



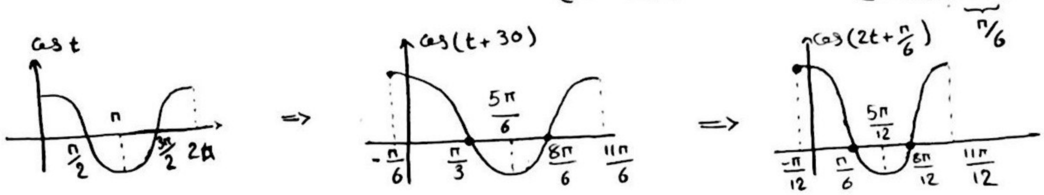
d)

$$f_d(t) = r(t)\sin(t) \; : \; r(t) = \begin{cases} 0 : t<0 \\ t : t>0 \end{cases} \Rightarrow f_d(t) = \begin{cases} 0 : t<0 \\ t\sin t : t>0 \end{cases}$$



e)

$$f_e(t) = u(t)\cos(2t+30) \; : \; u(t) = \begin{cases} 0 : t<0 \\ 1 : t>1 \end{cases} \Rightarrow f_e(t) = \begin{cases} 0 : t<0 \\ \cos(\underbrace{2t+30}_{\pi/6}) : t>0 \end{cases}$$



f)

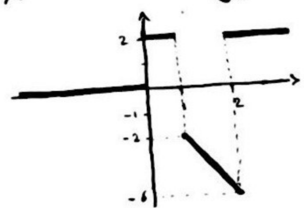
$$f_f(t) = u(1-t) + u(t-1) \; : \; 1-t>0 \Rightarrow t<1 \Rightarrow u(1-t) = \begin{cases} 1 : t<1 \\ 0 : t>1 \end{cases}$$

$$, \; u(t-1) = \begin{cases} 0 : t<1 \\ 1 : t>1 \end{cases} \Rightarrow f_f(t) = \begin{cases} 1 : t<1 \\ 1 : t>1 \end{cases}$$

g)

$f_g(t) = 2u(t) - 4r(t-1) + 4r(t-2)$: $u(t) = \begin{cases} 0 & : t<0 \\ 1 & : t>0 \end{cases}$, $r(t-1) = \begin{cases} 0 & : t<1 \\ t & : t>1 \end{cases}$

, $r(t-2) = \begin{cases} 0 & : t<2 \\ t & : t>2 \end{cases}$ => $f_g(t) = \begin{cases} 0 & : t<0 \\ 2 & : 0<t<1 \\ 2-4t & : 1<t<2 \\ 2 & : t>2 \end{cases}$



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a) $\int_{-5}^{2}(t+4)\left[\delta(t)-\delta(t+4)+\delta(t-3)\right]dt = \int_{-5}^{2}(t+4)\underbrace{\delta(t)}_{t_0=0}dt - \int_{-5}^{2}(t+4)\underbrace{\delta(t+4)}_{t_0=-4}dt + \cancel{\int_{-5}^{2}(t+4)\underbrace{\delta(t-3)}_{t_0=3}}^{0}$

$\int_{t_1}^{t_2} f(t)\delta(t-t_0) = f(t_0) \longrightarrow$ $\int_{-5}^{2}(0+4)\delta(t)dt - \int_{-5}^{2}(-4+4)\delta(t+4)dt = 4\underbrace{\int_{-5}^{2}\delta(t)dt}_{=1} - 0 = \boxed{4}$

b) $\int_{-2}^{4}(t^3+4)\left[\delta(t)+4\delta(t-2)\right]dt = \underbrace{\int_{-2}^{4}(t^3+4)\delta(t)dt}_{=4} + \int_{-2}^{4}(t^3+4)\times 4\underbrace{\delta(t-2)}_{t=2}dt$

$\Rightarrow 4 + 48 = 52$

c) $\int_{-3}^{4} t^2\left[\delta(t)+\delta(t+2.5)+\delta(t-5)\right]dt = \int_{-3}^{4}t^2\delta(t)dt + \int_{-3}^{4}t^2\underbrace{\delta(t+2.5)}_{t=-2.5}dt + \cancel{\int_{-3}^{4}t^2\underbrace{\delta(t-5)}_{t=-5}dt}^{0}$

$= 0 + 6.25 + 0 = 6.25$

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a) $(1-te^{-t})u(t)$: $(-e^{-t}+te^{-t})u(t) + \cancel{(1-te^{-t})}\delta(t) = (-e^{-t}+te^{-t})u(t) + \delta(t)$

b) $\cos 2t\, u(t)$: $-2\sin 2t\, u(t) + \cancel{\cos 2t\, \delta(t)} = -2\sin 2t\, u(t) + \delta(t)$

c) $e^{-t}u(t)$: $-e^{-t}u(t) + e^{-t}\delta(t) = -e^{-t}u(t) + \delta(t)$

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مبدأ زمانی بازه

$$v_c(t) = v_c(t_0) + \frac{1}{C}\int_{t_0}^{t} i_c(\tau)\,d\tau$$

$0<t<1 \Rightarrow i_s = -1^A$

$1<t<2 \Rightarrow i_s = 4^A$

$t>2 \Rightarrow i_s = 0^A$

$t<0 \rightarrow i_s = 0 \Rightarrow v_c(t) = v_c(t_0=0) + \frac{1}{C}\int^{t} \cancel{i_c(\tau)}^{=0}\,d\tau$

$v_c(t) = -2 \Rightarrow v_c(t=0) = -2^V$

I) $0<t<1 \Rightarrow i_s = -1^A$

$$v_c(t) = v_c(t=0) + \frac{1}{C}\int_0^t i_c(\tau)\,d\tau = -2^V + \frac{1}{2}\int_0^t -1\,d\tau$$

$$= -2 - \frac{1}{2}\tau\Big|_0^t = -2 - \frac{t}{2} \Rightarrow \boxed{v_c(t) = -2 - \frac{t}{2}} \; : 0<t<1$$

$\Rightarrow$ if $t=1 \Rightarrow v_c(1) = -2 - \frac{1}{2} = \frac{-5}{2}$ (**)

II) $1<t<2 \Rightarrow i_s = 4^A$

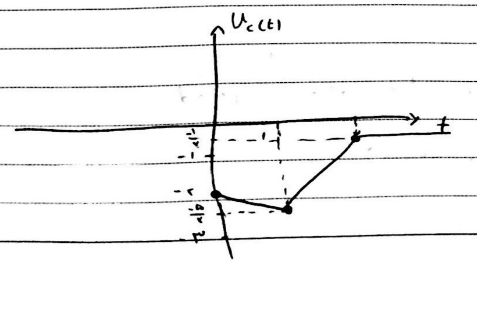
$$v_c(t) = v_c(t_0=1) + \frac{1}{C}\int_1^t i_c(\tau)\,d\tau \overset{(**)}{=} -\frac{5}{2} + \frac{1}{2}\int_1^t 4\,d\tau$$

$$= \frac{-5}{2} + 2\int_1^t d\tau \Rightarrow \frac{-5}{2} + 2\tau\Big|_1^t = \frac{-5}{2} + (2t-2) = 2t - \frac{9}{2}$$

$\Rightarrow v_c(t) = 2t - \frac{9}{2} \Longrightarrow$ if $t=2 \Rightarrow v_c(2) = 2(2) - \frac{9}{2} = \frac{-1}{2}$ (***)

III) $t>2 \Rightarrow i_s = 0^A$

$$v_c(t) = v_c(t_0=2) + \frac{1}{C}\int_2^t \cancel{i_c(\tau)}^{=0}\,d\tau \overset{(***)}{=} v_c(t) = v_c(t_0=2) = \frac{-1}{2}$$



SHAFAGH