1) Find the response of the following equation for $k = 15$ and $k = 14.9$. Is the system ill-condition? Find the condition number.

$$A\mathbf{x} = \mathbf{b} \quad \rightarrow \quad \begin{bmatrix} 8 & 5 & 2 \\ 21 & 19 & 16 \\ 39 & 48 & 53 \end{bmatrix} \mathbf{x} = \begin{bmatrix} k \\ 56 \\ 140 \end{bmatrix}$$

2) Consider $M_{2\times2}(R)$ is a set of all 2×2 matrix with real elements. Show this set form a vector space on R (real number).

3) Find the linear independent condition for the following vectors.

$$\mathbf{u} = \begin{bmatrix} 1-\lambda \\ 2+\lambda \end{bmatrix}, \qquad \mathbf{v} = \begin{bmatrix} 2+\lambda \\ 1-\lambda \end{bmatrix}$$

4) Check the "**basis**" conditions for the following set. (For the vector space $M_{2\times2}$)

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$$

5) Find the rank of the following matrices.

$$A = \begin{bmatrix} 3 & 12 & -1 & -6 \\ 6 & 24 & -2 & -12 \\ -3 & -12 & 1 & 6 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & a \\ -a & -1 & 1 \end{pmatrix}, \ where \ a \in \mathbb{R}.$$