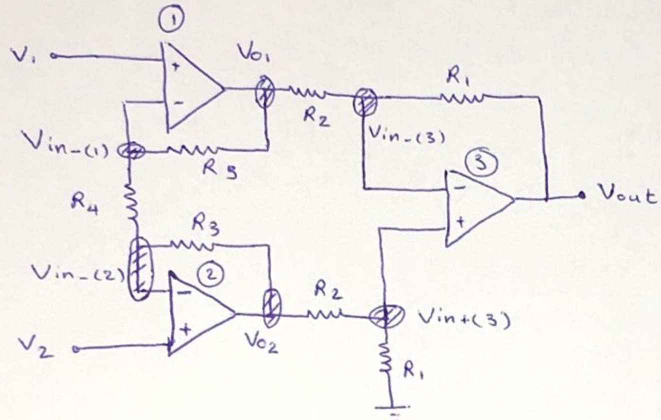
یا لطیف

تمرین سری ۹ الک 2

رضا دینی پور

۹۸۱۴۳۰۳

1



$V_1 = V_{in-(1)}$

$V_2 = V_{in-(2)}$

KCL @ $V_{in-(1)}$: $\frac{V_{in-(1)} - V_{in-(2)}}{R_4} + \frac{V_{in-(1)} - V_{o1}}{R_3} = 0 \xrightarrow[V_{in-(2)}=V_2]{V_{in-(1)}=V_1}$

$$\frac{V_1 - V_2}{R_4} + \frac{V_1 - V_{o1}}{R_3} = 0 \Rightarrow \boxed{V_{o1} = V_1\left(\frac{R_3}{R_4}+1\right) - \frac{R_3}{R_4} V_2} \quad (I)$$

KCL @ $V_{in-(2)}$: $\frac{V_{in-(2)} - V_{in-(1)}}{R_4} + \frac{V_{in-(2)} - V_{o2}}{R_3} = 0 \Rightarrow \frac{V_2 - V_1}{R_4} + \frac{V_2 - V_{o2}}{R_3} = 0$

$$\Rightarrow \boxed{V_{o2} = V_2\left(\frac{R_3}{R_4}+1\right) - \frac{R_3}{R_4} V_1} \quad (II)$$

KCL @ $V_{in-(3)}$: $\frac{V_{in-(3)} - V_{o1}}{R_2} + \frac{V_{in-(3)} - V_{out}}{R_1} = 0 \xrightarrow{(I)}$

$$\frac{V_{in-(3)}}{R_2} - \frac{1}{R_2}\left[V_1\left(\frac{R_3}{R_4}+1\right) - \frac{R_3}{R_4}V_2\right] + \frac{V_{in-(3)} - V_{out}}{R_1} = 0$$

$$\Rightarrow \frac{V_{in-(3)}}{R_2} - V_1\left(\frac{R_3}{R_2R_4} + \frac{1}{R_2}\right) + \frac{R_3}{R_2R_4}V_2 + \frac{V_{in-(3)}}{R_1} = \frac{V_{out}}{R_1}$$

KCL @ $V_{in+(3)}$: $\frac{V_{in+(3)}}{R_1} + \frac{V_{in+(3)} - V_{o2}}{R_2} = 0 \xrightarrow{(II)} \frac{V_{in+(3)}}{R_1} + \frac{V_{in+(3)}}{R_2} = \frac{1}{R_2}\left[V_2\left(\frac{R_3}{R_4}+1\right) - \frac{R_3}{R_4}V_1\right]$

$$\Rightarrow V_{in+(3)}\underbrace{\left(\frac{1}{R_2}+\frac{1}{R_1}\right)}_{2} = \underbrace{V_2\left(\frac{R_3}{R_2R_4}+\frac{1}{R_2}\right) - \frac{R_3}{R_2R_4}V_1}_{1} \Rightarrow V_{in+(3)} = \frac{1}{2}$$

$$V_{in+(3)} = V_{in-(3)} \Rightarrow \frac{V_2\left(\frac{R_3}{R_2R_4}+\frac{1}{R_2}\right) - \frac{R_3}{R_2R_4}V_1}{\frac{1}{R_2}+\frac{1}{R_1}} = \frac{\frac{V_{out}}{R_1} + V_1\left(\frac{R_3}{R_2R_4}+\frac{1}{R_2}\right) - \frac{R_3}{R_2R_4}V_2}{\frac{1}{R_2}+\frac{1}{R_1}}$$

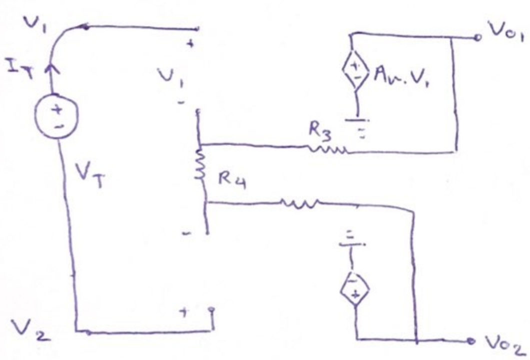
$$\Rightarrow V_2\left(\frac{R_3}{R_2R_4}+\frac{1}{R_2}\right)-\frac{R_3}{R_2R_4}V_1 = V_1\left(\frac{R_3}{R_2R_4}+\frac{1}{R_2}\right)-\frac{R_3}{R_2R_4}V_2+\frac{V_{out}}{R_1}$$

$$\Rightarrow V_2\left[\frac{R_3}{R_2R_4}+\frac{1}{R_2}+\frac{R_3}{R_2R_4}\right]-V_1\left[\frac{R_3}{R_2R_4}+\frac{R_3}{R_2R_4}+\frac{1}{R_2}\right]=\frac{V_{out}}{R_1}$$

$$\Rightarrow V_2\left(\frac{1}{R_2}+\frac{2R_3+R_4}{R_2R_4}\right)-V_1\left(\frac{1}{R_2}+\frac{2R_3+R_4}{R_2R_4}\right)=\frac{V_{out}}{R_1}$$

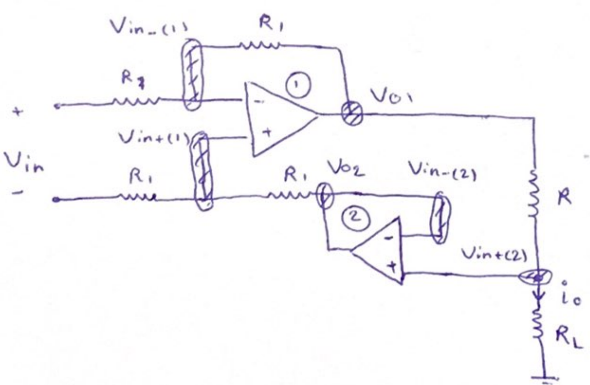
$$\Rightarrow V_{out}=V_2\left(\frac{R_1}{R_2}+\frac{2R_1R_3+R_1R_4}{R_2R_4}\right)-V_1\left(\frac{R_1}{R_2}+\frac{2R_1R_3+R_1R_4}{R_2R_4}\right)$$

$$\Rightarrow V_{out}=(V_2-V_1)\cdot\left[\frac{R_1}{R_2}+\frac{2R_1R_3+R_1R_4}{R_2R_4}\right]$$



$$\frac{V_T}{I_T}=R_{in}=\infty$$

#2



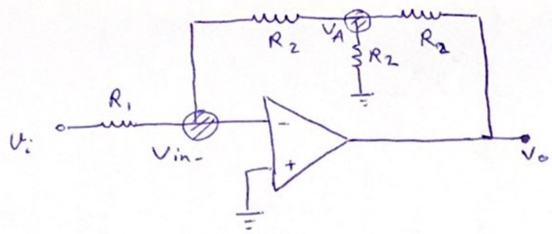
$$\text{kcl @ } V_{in-(1)}: \frac{V_{i-(1)}-V_{in}}{R_{\cancel{2}1}}+\frac{V_{i-(1)}-V_{o1}}{R_1}=0 \xrightarrow{V_{i-(1)}=V_{i+(1)}} \boxed{V_{i-(1)}=\tfrac{1}{2}(V_{in}+V_{o1})} \quad (\mathrm{I})$$

$$\text{kcl @ } V_{i+(1)}: \frac{V_{i+(1)}+V_{in}}{R_1}+\frac{V_{i+(1)}-V_{o2}}{R_1}=0 \xrightarrow{(\mathrm{I})} \frac{1}{R_1}\left(\tfrac{1}{2}(V_{in}+V_{o1})\right)+\frac{V_{in}}{R_1}+\frac{1}{R_1}\left(\tfrac{1}{2}(V_{in}+V_{o1})\right)$$

$$-\frac{V_{o2}}{R_1}=0 \Rightarrow \underbrace{\boxed{\frac{1}{R_1}(V_{in}+V_{o1})+\frac{V_{in}}{R_1}-\frac{V_{o2}}{R_1}=0}}_{(\mathrm{II})} \xrightarrow[=V_{in+(2)}]{V_{o2}=V_{in-(2)}} \underline{V_{o2}=2V_{in}+V_{o1}} \quad (\mathrm{III})$$

$$\text{kcl @ } V_{in+(2)}: \frac{V_{i+(2)}-V_{o1}}{R}+i_o=0 \Rightarrow \frac{1}{R}(2V_{in}+V_{o1})-\frac{V_{o1}}{R}+i_o=0 \Rightarrow i_o=\frac{-2V_{in}}{R}$$

3

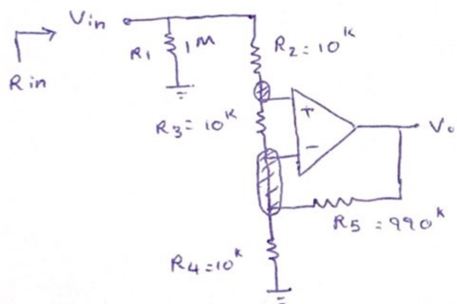


KCL in V_{in-}: $\dfrac{\cancel{V_{in-}}^{\,0} - V_i}{R_1} + \dfrac{\cancel{V_{in-}}^{\,0} - V_A}{R_2} = 0 \Rightarrow \dfrac{V_i}{R_1} = \dfrac{-V_A}{R_2} \Rightarrow \boxed{V_A = -\dfrac{R_2}{R_1} V_i}$ (I)

KCL @ V_A: $\dfrac{V_A - 0}{R_2} + \dfrac{V_A - \cancel{V_{in-}}^{\,0}}{R_2} + \dfrac{V_A - V_o}{R_2} = 0 \overset{(I)}{\Rightarrow} -\dfrac{\cancel{R_2}}{R_1}V_i \left(\dfrac{1}{\cancel{R_2}}\right) + \dfrac{1}{\cancel{R_2}}\left(-\dfrac{\cancel{R_2}}{R_1}V_i\right) + \dfrac{1}{\cancel{R_2}}\left(-\dfrac{\cancel{R_2}}{R_1}V_i\right)$

$= \dfrac{V_o}{R_2} \Rightarrow \dfrac{-3}{R_1}V_i = \dfrac{V_o}{R_2} \Rightarrow \boxed{\dfrac{V_o}{V_i} = -3\dfrac{R_2}{R_1}}$

4



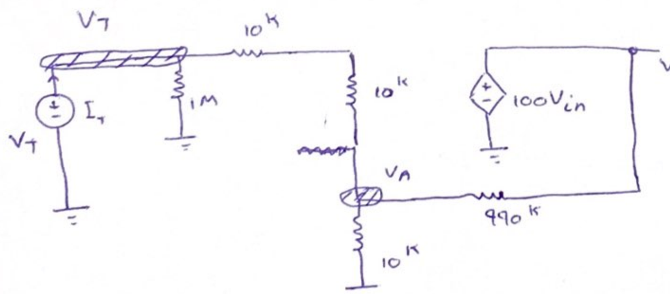
KCL in V_{in+}: $\dfrac{V_{in+} - V_{in-}}{10^k} + \dfrac{V_{in+} - V_{in}}{10^k} = 0$

$\Rightarrow V_{i+} = V_{i-} = V_{in}$

KCL @ V_{i-}: $\dfrac{V_{i-} - 0}{10^k} + \dfrac{V_{i-} - V_o}{990^k} = 0$

$\Rightarrow \dfrac{V_{in}}{10^k} + \dfrac{V_{in}}{990^k} = \dfrac{V_o}{990^k}$

$\Rightarrow \dfrac{V_o}{V_{in}} = 100$



KCL @ V_T: $-I_T + \dfrac{V_T}{1^M} + \dfrac{V_T - V_A}{20^k} = 0$

~~KCL @ V_A~~

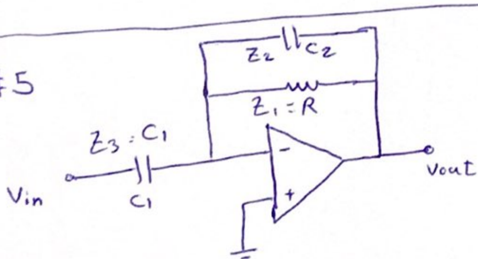
$V_A = \dfrac{10}{10+990} \times V_{out}$

$= \dfrac{10}{1000} \times 100 V_{in} \Rightarrow V_A = V_{in}$

$\Rightarrow V_{in} = V_{i+} = V_{i-} = 0$

$\Rightarrow \dfrac{V_T}{1^M} + \dfrac{V_T}{20^k} = I_T \Rightarrow \dfrac{V_T}{I_T} = R_{in} = \dfrac{1000}{51} \simeq \boxed{19.6^k}$

5



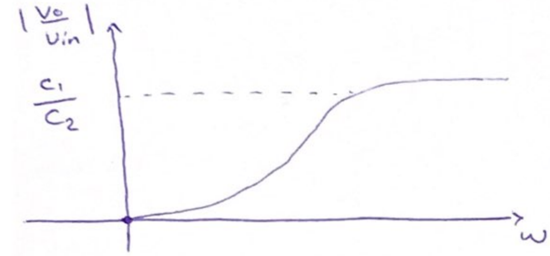
$V_{i+} = V_{i-} = 0$

KCL @ V_{i-}: $\dfrac{\cancel{V_{i-}}^{\,0} - V_{in}}{Z_3} + \dfrac{\cancel{V_{i-}}^{\,0} - V_{out}}{Z_1} + \dfrac{\cancel{V_{i-}}^{\,0} - V_{out}}{Z_2} = 0$

$\Rightarrow -\dfrac{V_{in}}{Z_3} = V_{out}\left(\dfrac{1}{Z_1} + \dfrac{1}{Z_2}\right)$

$\Rightarrow \dfrac{-V_{in}}{\frac{1}{C_1 S}} = V_{out}\left(\dfrac{1}{R} + \dfrac{1}{\frac{1}{C_2 S}}\right) \Rightarrow \dfrac{V_{out}}{V_{in}} = \dfrac{-C_1 S}{\frac{1}{R_1} + C_2 S} \xrightarrow{S = j\omega}$

$$\frac{V_{out}}{V_{in}} = \frac{-jC_1\omega}{\frac{1}{R_1} + jC_2\omega} \longrightarrow \begin{cases} \left|\frac{V_{out}}{V_{in}}\right| = \dfrac{C_1\omega}{\sqrt{(\frac{1}{R_1})^2 + C_2^2\omega^2}} \\ \measuredangle \frac{V_{out}}{V_{in}} = -\frac{\pi}{2} - \tan^{-1}\left(\dfrac{C_2\omega}{\frac{1}{R_1}}\right) = -\frac{\pi}{2} - \tan^{-1}(R_1C_2\omega) \end{cases}$$

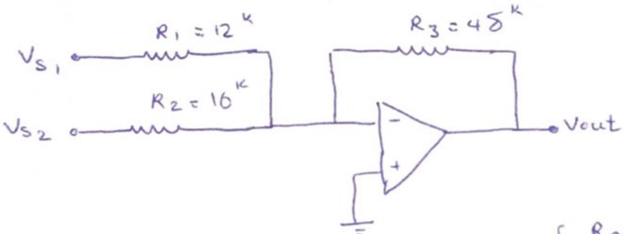


High Pass Filter

#6

$$V_{out} = 4V_1 + 3V_2$$

می‌دانیم:



$$V_{out} = -\left(\frac{R_3}{R_1}V_{s1} + \frac{R_3}{R_2}V_{s2}\right) \longrightarrow \begin{cases} R_3 = 4R_1 \Rightarrow \frac{R_3}{R_1} = 4 \\ R_3 = 3R_2 \Rightarrow \frac{R_3}{R_2} = 3 \end{cases}$$

$$\Rightarrow \begin{cases} R_1 = \frac{3}{4}R_2 \\ R_2 = \frac{4}{3}R_1 \end{cases} \Rightarrow R_3 = 48^k,\ R_2 = 16^k,\ R_1 = 12^k$$

$$\Rightarrow V_{out} = -\left(\frac{48^k}{12^k}V_1 + \frac{48^k}{16^k}V_2\right) \Rightarrow V_{out} = +(4V_1 + 3V_2) + \underbrace{\pi}_{\text{برای جبران (-)}}$$

#7

$$\frac{d^2v}{dt^2} + \frac{20dv}{dt} + 100v = 25 \Rightarrow \frac{d^2v}{dt^2} = -20\frac{dv}{dt} - 100v + 25$$