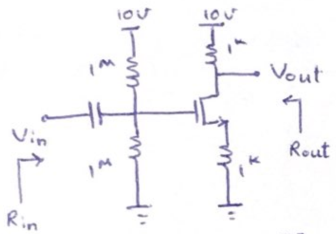
In the name of God

ASSignment 3 :

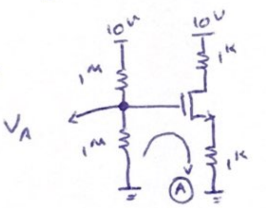
1. In the following circuits, determine the Voltage gain, input resistance and output resistance.

a)



$V_{TH} = 2V$, $\mu_n C_{ox} \frac{W}{L} = 0.5 \frac{mA}{V^2}$, $\lambda = 0\, V^{-1} \longrightarrow r_{ds} = \infty$

dc Analysis :



$V_A = \frac{1^M \times 10}{1+1} = 5^V$

KVL @ A : $-5 + \cancel{(1^M \| 1^M)} I_G + V_{gs} + 1^k I_D = 0$

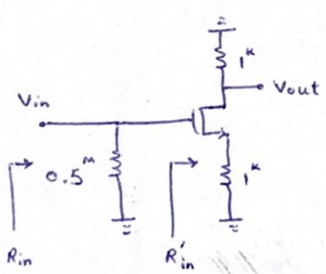
$\boxed{V_{gs} = 5 - 1^k I_D}$ (I)

$I_D = \frac{k'}{2}\frac{W}{L}(V_{gs} - V_{th})^2 = \frac{0.5}{2}(5 - 1^k I_D - 2)^2 \longrightarrow$ $I_D = 1^{mA}$ ✓, $I_D = 9^{mA}$

$g_m = \sqrt{2 \frac{k'W}{L} I_D} = \sqrt{2(0.5) \times 1} = 1^{mmho}$

<u>hint</u> : $g_m = \begin{cases} 1)\ k'\frac{W}{L}(V_{gs} - V_{th}) \\ 2)\ \sqrt{2k'\frac{W}{L} I_D} \\ 3)\ \frac{2 I_D}{V_{gs} - V_{th}} \end{cases}$

ac Analysis :

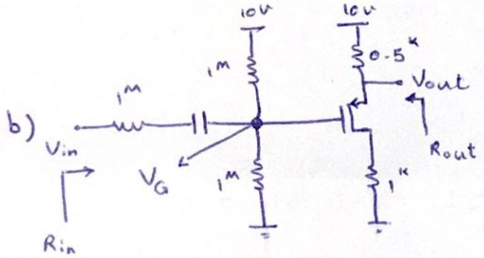


$A_v = \frac{V_{out}}{V_{in}} = \frac{-R_D}{R_S + \frac{1}{g_m}} = \frac{-1^k}{1^k + \frac{1}{1m}} = \boxed{-0.5}$

$R_{in} = 0.5^M \| R'_{in} \xrightarrow{R'_{in} = \infty} R_{in} = 0.5^M = 500^k$

$R_{out} = 1^k \| r_{ds}(1 + g_m R_s) = 1^k$

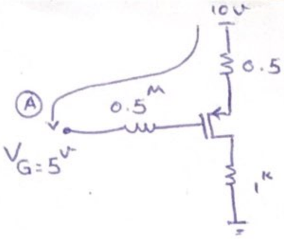
b)



$V_G = \frac{1^M \times 10}{1+1} = 5^V$

$V_{TH} = -1^V$, $\mu_p C_{ox} \frac{W}{L} = 2 \frac{mA}{V}$, $\lambda = 0\, V^{-1} \longrightarrow r_{ds} = \infty$

dc Analysis :



KVL @ A : $-10 + 0.5^k I_D + V_{SG} + \cancel{0.5^M I_G} + 5 = 0$

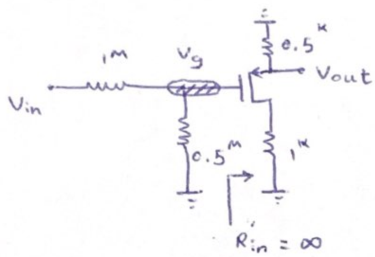
$V_{SG} = 5 - 0.5^k I_D$

$I_D = \frac{k'}{2}\frac{W}{L}(V_{SG} - |V_{th}|)^2 = (5 - 0.5^k I_D - 1)$

$\begin{cases} I_D = 4^{mA} \checkmark \\ I_D = 16^{mA} \end{cases}$

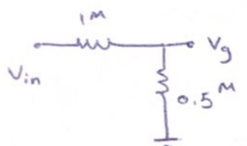
$g_m = \sqrt{2k'\frac{W}{L} I_D} = \sqrt{2 \times 2 \times 4} = 4^{mmho}$

ac Analysis :



$A_v = \frac{V_{out}}{V_g} \times \frac{V_g}{V_{in}}$

$\frac{V_{out}}{V_g} = \frac{R_S}{R_S + \frac{1}{g_m}} = \frac{0.5^k}{0.5^k + \frac{1}{4}} = 0.6$



$\frac{V_g}{V_{in}} = \frac{0.5^M}{1.5^M} = 0.3 \Rightarrow A_v = \frac{V_{out}}{V_{in}} = 0.6 \times 0.3 = 0.18$

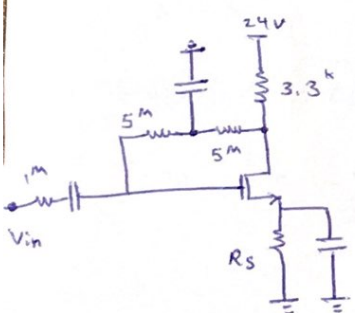
$R_{in} = 1^M + [0.5^M \| R'_{in}] = 1^M + [0.5^M \| \infty] = \boxed{1.5^{M\Omega}}$

$R_{out} = 0.5 \| (\frac{1}{g_m} \| \infty) = 0.5 \| 0.25 = \boxed{0.16^k}$

#2. In the following circuit :

a) Specify the source resistance so that $I_D = 2.5^{mA}$

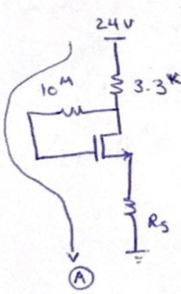
b) Calculate the voltage gain and the input and the output resistance.



$\beta = 0.25 \frac{mA}{V^2}$, $|V_{th}| = 3^v$

$\lambda = 0 \rightarrow r_{ds} = \infty$

dc Analysis :



KVL @ A : $-24 + 3.3^k I_D + \cancel{10^M I_G} + V_{gs} + R_S I_D = 0$

$V_{gs} = 24 - 3.3(2.5) R_S = 15.75 - 2.5^m R_S$

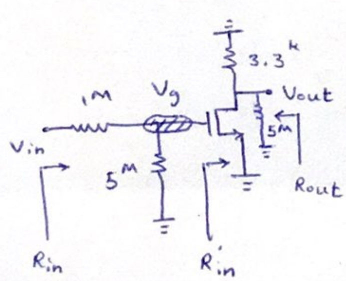
$I_D = \frac{k'}{2}\frac{W}{L}(V_{gs} - V_{th})^2 \Rightarrow 2.5 = \frac{0.25}{2}(15.75 - 2.5^m R_S - 3)^2$

$R_S = 3.3^k \checkmark$

$R_S = 6.8^k$

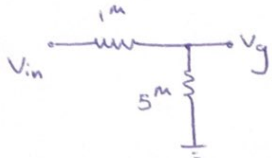
$g_m = \sqrt{2k'\frac{W}{L} I_D} = \sqrt{2(0.25) \times 2.5} = 1.1^{mmho}$

AC Analysis :



$$\frac{V_{out}}{V_{in}} = A_v = \frac{V_{out}}{V_g} \times \frac{V_g}{V_{in}} \Rightarrow \frac{V_{out}}{V_g} = -g_m\,(R_o \| r_{ds}) = -1.1\,(5^M \| 3.3^k \| \infty) \simeq -3.63$$

$\frac{V_g}{V_{in}}$:



$$\frac{V_g}{V_{in}} = \frac{5}{6} \simeq 0.8 \Rightarrow A_v = \frac{V_{out}}{V_{in}} = -3.63 \times 0.8 \simeq \boxed{-2.9}$$

$$R_{in} = 1^M + \underbrace{(5^M \| \infty)}_{R'_{in}} = 6^M \quad , \quad R_{out} = 5^M \| 3.3^k \| r_{ds}(1+g_m R_s) = 3.3^k$$

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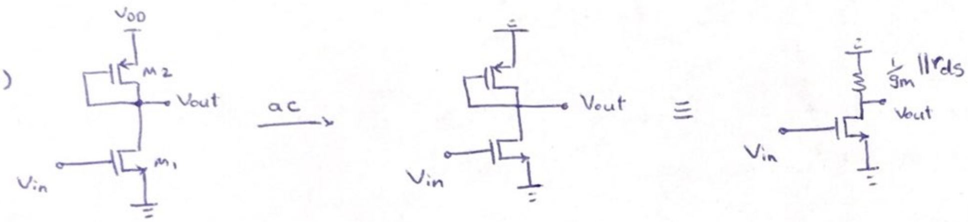
3. Determine a relation the voltage gain ($A_v = \frac{V_{out}}{V_{in}}$) of the following circuits. Assume that the transistors operate in saturation and $\lambda \neq 0$

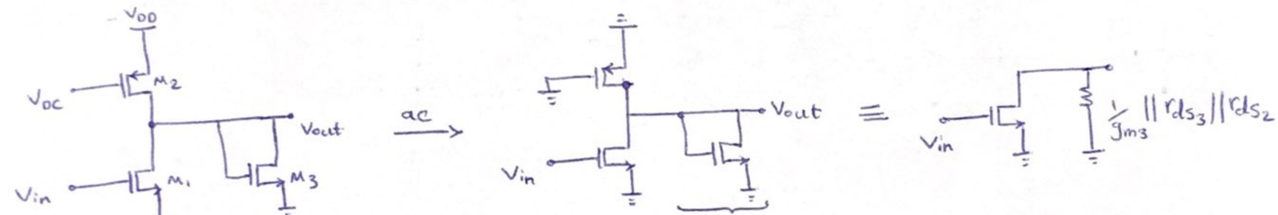
a)



$\frac{1}{g_{m2}} \| r_{ds_2}$

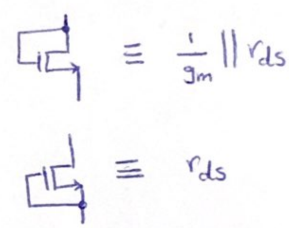
$$A_v = -g_{m_1}(R_o \| r_{ds_1})$$

b)

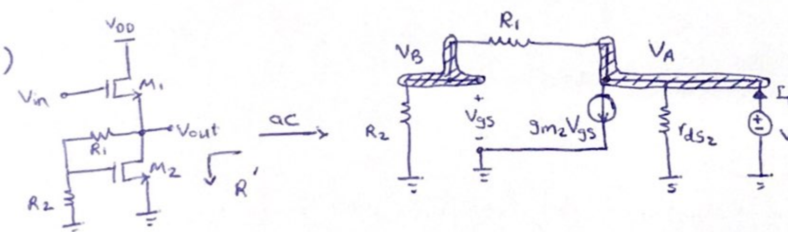


$$A_v = -g_{m_1}(R_o \| r_{ds_1}) = -g_{m_1}\left(\frac{1}{g_{m3}} \| r_{ds_3} \| r_{ds_2} \| r_{ds_1}\right)$$

hint :



c)



$$\text{KCL @ } V_A : \frac{V_A}{r_{ds2}} + g_{m2}V_{gs} - I_T + \frac{V_A - V_B}{R_1} = 0 \xrightarrow{V_{gs} = V_B} \frac{V_A}{r_{ds2}} + g_{m2}V_{gs} - I_T + \frac{V_A}{R_1} - \frac{V_{gs}}{R_1} = 0$$

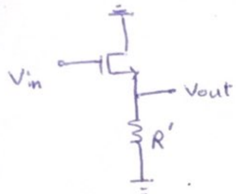
$$\Rightarrow V_{gs}\left(g_{m2} - \frac{1}{R_1}\right) + V_A\left(\frac{1}{r_{ds2}} + \frac{1}{R_1}\right) = I_T \quad (\mathrm{I})$$

$$\text{KCL @ } V_B : \frac{V_{gs}}{R_2} + \frac{V_{gs} - V_A}{R_1} = 0 \Rightarrow V_A = V_{gs}\left(\frac{R_1}{R_2} + 1\right) \quad (\mathrm{II})$$

$$\xrightarrow{(\mathrm{II}) \text{ in } (\mathrm{I})} V_{gs}\left(g_{m2} - \frac{1}{R_1}\right) + V_{gs}\left(\frac{R_1}{R_2} + 1\right)\left(\frac{1}{r_{ds2}} + \frac{1}{R_1}\right) = I_T \Rightarrow V_{gs}\left[\left(g_{m2} - \frac{1}{R_1}\right) + \left(\frac{R_1}{R_2} + 1\right)\left(\frac{1}{r_{ds2}} + \frac{1}{R_1}\right)\right] = I_T$$

$$\xrightarrow{V_{gs} = \frac{V_A}{1 + R_1/R_2}} \frac{V_A}{1 + \frac{R_1}{R_2}}\left[g_{m2} - \frac{1}{R_1} + \left(\frac{R_1}{R_2} + 1\right)\left(\frac{1}{r_{ds2}} + \frac{1}{R_1}\right)\right] = I_T \xrightarrow{V_A = V_T}$$

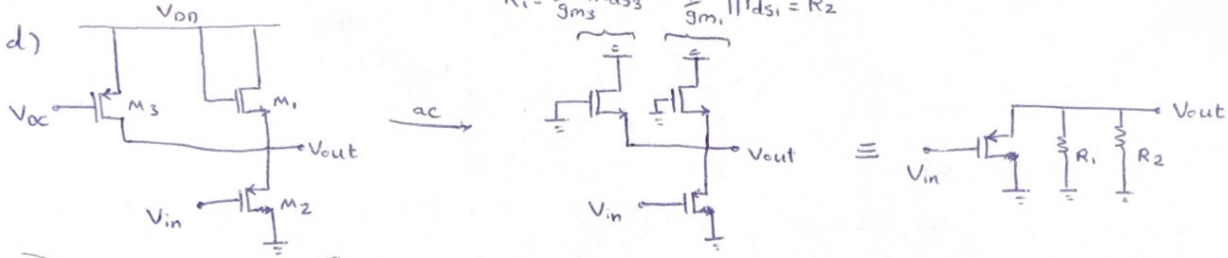
$$\frac{V_T}{I_T} = \frac{1}{\frac{R_2}{R_1 + R_2}\left(g_{m2} - \frac{1}{R_1} + \left(\frac{R_1}{R_2} + 1\right)\left(\frac{1}{r_{ds2}} + \frac{1}{R_1}\right)\right)} = R'$$



$$A_v = \frac{R_S}{R_S + \frac{1}{g_m}} = \frac{R'}{R' + \frac{1}{g_m}}$$

d)

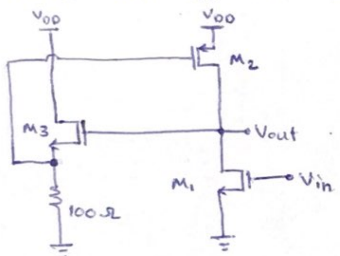
$R_1 = \frac{1}{g_{m3}} \| r_{ds3}$ $\quad \frac{1}{g_{m1}} \| r_{ds1} = R_2$

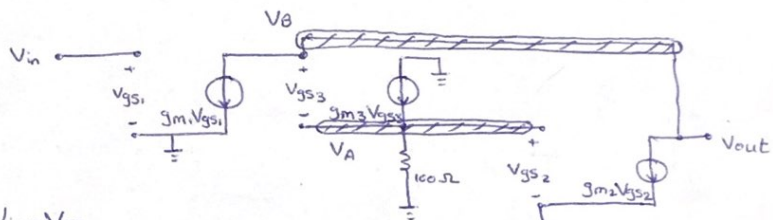


$$A_u = -g_{m2}\left(\cancel{r_{ds2} \| R_1 \| R_2}\right)$$

$$A_v = \frac{R_S}{R_S + \frac{1}{g_{m2}}} = \frac{R_1 \| R_2}{(R_1 \| R_2) + \frac{1}{g_{m2}}}$$

4. Draw the small signal model of the following circuit and calculate the voltage gain. Assume that all of the transistors are in saturation.





$g_m = 10 \ \frac{mA}{V}$

$\lambda = 0 \rightarrow r_{ds} = \infty$

$V_{in} = V_{gs1}$

KCL @ V_B : $g_{m1} V_{gs1} + g_{m2} V_{gs2} = 0 \xrightarrow{V_{gs1} = V_{in}} V_{gs2} = \frac{-g_{m1}}{g_{m2}} V_{in}$

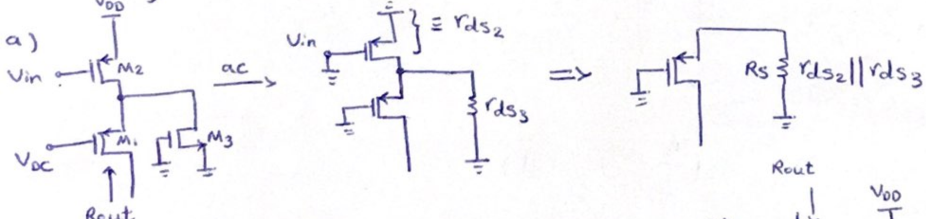
KCL @ V_A : $-g_{m3} V_{gs3} + \frac{V_A}{100} = 0 \xrightarrow{V_A = V_{gs2}} V_{gs3} = \frac{V_{gs2}}{100 g_{m3}}$

$$V_o = V_{gs2} + V_{gs3} = \frac{V_{gs2}}{100 g_{m3}} - \frac{g_{m1}}{g_{m2}} V_{in} = \frac{1}{100 g_{m3}}\left[\frac{-g_{m1}}{g_{m2}} V_{in}\right] - \frac{g_{m1}}{g_{m2}} V_{in}$$

$$A_V = \frac{V_o}{V_{in}} = \frac{-g_{m1}}{g_{m2}}\left[\frac{1}{100 g_{m3}} - 1\right]$$

5. Specify a relation for the output resistance of the following circuits. Assume $\lambda \neq 0$

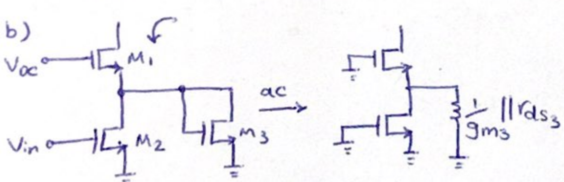
a)

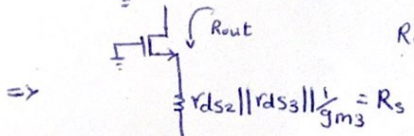


$r_{ds2} \| r_{ds3}$

$\Rightarrow R_{out} = r_{ds1}(1 + g_{m1} R_S)$

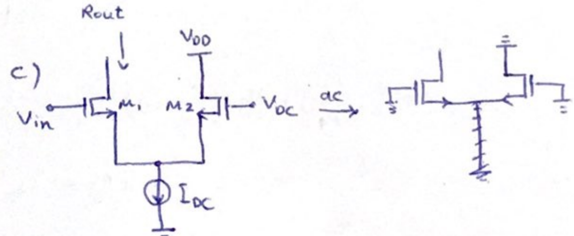
b)

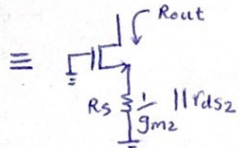




$R_{out} = r_{ds2}(1 + g_{m2} R_S)$

c)





$R_{out} = r_{ds1}(1 + g_{m1} R_S)$