(1

a) $\delta(t+1)+\delta(t-3)$

$$\rightarrow X(j\omega)=\int_{-\infty}^{+\infty} x(t)\,e^{-j\omega t}\,dt \longrightarrow \int_{-\infty}^{+\infty}\delta(t+1)dt+\int_{-\infty}^{+\infty}\delta(t-3)dt=e^{j\omega}+e^{-3j\omega}$$

b) $1+\cos(7\pi t+\frac{\pi}{8})$

$$\rightarrow 1+\frac{1}{2}\left(e^{j(7\pi t+\frac{\pi}{8})}+e^{-j(7\pi t+\frac{\pi}{8})}\right)\rightarrow X(j\omega)=\int_{-\infty}^{+\infty}x(t)\,e^{-j\omega t}\,dt$$

$$\rightarrow 2\pi\delta(\omega)+\frac{1}{2}\left(2\pi\delta(\omega-7\pi)e^{\frac{j\pi}{7}}+2\pi\delta(\omega+7\pi)e^{\frac{-j\pi}{8}}\right.$$

c) $e^{-2|t-1|}$

$$X(j\omega)=\int_{-\infty}^{+\infty}x(t)\,e^{-j\omega t}\,dt=\int_{-\infty}^{+\infty}e^{-2|t-1|}\,e^{-j\omega t}\,dt\xrightarrow{t-1=\tau}\int_{-\infty}^{+\infty}e^{-2|\tau|}\,e^{-j\omega(\tau+1)}\,dt$$

$$=\int_{-\infty}^{0}e^{2\tau}e^{-j\omega t}e^{-j\omega}dt+\int_{0}^{+\infty}e^{-2\tau}e^{-j\omega\tau}e^{-j\omega}d\tau=e^{-j\omega}\left[\frac{e^{(2-j\omega)\tau}}{2-j\omega}\Bigg|_{-\infty}^{0}+\frac{e^{-(2+j\omega)\tau}}{-(2+j\omega)}\Bigg|_{0}^{\infty}\right]$$

d) $$X(j\omega)=\int_{-\infty}^{+\infty}x(t)\,e^{-j\omega t}\,dt=\int_{-1}^{1}(1)\,e^{-j\omega t}\,dt=\frac{e^{-j\omega t}}{-j\omega}\Bigg|_{-1}^{1}=\frac{e^{-j\omega}-e^{j\omega}}{-j\omega}$$

(2

$$x(t)=\frac{\sin\omega_c t}{\pi t}\xrightarrow{F^{-1}}X(j\omega)=\Pi\left(\frac{\omega}{2T}\right)\rightarrow T=\omega_c$$

$$x(t)=\frac{\sin\omega_c t}{\pi t}=\frac{1}{2\pi}\int_{-\omega_c}^{\omega_c}\Pi\left(\frac{\omega}{2T}\right)e^{j\omega t}\,d\omega\xrightarrow{t\to -t}\frac{2\pi\sin\omega_c t}{\pi t}=\int_{-\omega_c}^{\omega_c}\Pi\left(\frac{\omega}{2T}\right)e^{j\omega t}\,dt$$

$$\longrightarrow\frac{2\pi\sin\omega_c(-t)}{\pi(t)}=\int_{-\omega_c}^{\omega_c}\Pi\left(\frac{\omega}{2T}\right)e^{-j\omega t}\,d\omega$$

$$\longrightarrow\frac{2\sin\omega_c\omega}{\omega}=\int_{-\omega_c}^{\omega_c}\Pi\left(\frac{t}{2T}\right)e^{-j\omega t}\,dt\longrightarrow F\left\{\Pi\left(\frac{t}{2T}\right)\right\}=\left\{\frac{2\sin\omega\omega_c}{\omega}\right\}$$

۳-

$$x(t) = t\left(\frac{\sin t}{\pi t}\right)^2$$

$$x_1(t) = \frac{\sin t}{\pi t} \xrightarrow{F} X_1(j\omega) = \Pi\left(\frac{\omega}{2T}\right)$$

$$x_2(t) = \left(\frac{\sin t}{\pi t}\right)^2 \xrightarrow{F} X_2(j\omega) = \frac{1}{2\pi} X_1(j\omega) * X_1(j\omega) \longrightarrow X_2(j\omega) \begin{cases} \omega+2 & -2\leq\omega<0 \\ -\omega+2 & 0\leq\omega\leq 2 \end{cases}$$

$$X(j\omega) = t\left(\frac{\sin t}{\pi t}\right)^2 = \begin{cases} \frac{j}{2\pi} \times \frac{d(\omega+2)}{d\omega} = \frac{j}{2\pi} & 2\leq\omega<0 \\ \frac{j}{2\pi} \times \frac{d(-\omega+2)}{d\omega} = \frac{-j}{2\pi} & 0\leq\omega\leq 2 \end{cases}$$

۴-

$$x(t) = \begin{cases} 1+\cos\pi t & |t|\leq 1 \\ 0 & |t|>1 \end{cases}$$

$$x(t) = (1+\cos\pi t)\,\Pi\left(\frac{t}{2T}\right) \xrightarrow{F} X(j\omega) = \frac{1}{2\pi} F\left\{1+\frac{e^{j\pi t}+e^{-j\pi t}}{2}\right\} * F\left\{\Pi\left(\frac{t}{2T}\right)\right\}$$

$$= \frac{1}{2\pi}\left\{2\pi\delta(\omega)+\frac{1}{2}\left(2\pi\delta(\omega-\pi)+2\pi\delta(\omega+\pi)\right)\right\} * \frac{2\sin\omega}{\omega}$$

۵-

الف) $x(t) = te^{-|t|} \xrightarrow{F} X(j\omega) = j\frac{d}{d\omega} F\{e^{-|t|}\}$

$$\to \int_{-\infty}^{\infty} e^{-|t|} e^{-j\omega t}\,dt = \int_{-\infty}^{0} e^{t} e^{-j\omega t}\,d\omega + \int_{0}^{+\infty} e^{-t} e^{-j\omega t}\,d\omega = \frac{e^{(1-j\omega)t}}{1-j\omega}\Big|_{-\infty}^{0} + \frac{e^{-(1+j\omega)t}}{1+j\omega}\Big|_{0}^{+\infty}$$

$$= \frac{1}{1-j\omega}+\frac{1}{1+j\omega} = \frac{2}{1+\omega^2} \to X(j\omega) = j\frac{d}{\omega}\left(\frac{2}{1+\omega^2}\right) = \frac{-4j\omega}{(1+\omega^2)^2}$$

ب) $x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}\,d\omega \longrightarrow te^{-|t|} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{-4\omega j}{(1+\omega^2)^2} e^{j\omega t}\,d\omega$

$$\xrightarrow{\times 2\pi j} 2\pi j t e^{-|t|} = \int_{-\infty}^{+\infty} \frac{4\omega}{(1+\omega^2)^2} e^{j\omega t}\,d\omega \to -2\pi j t e^{-|t|} = \int_{-\infty}^{+\infty} \frac{4\omega}{(1+\omega^2)^2} e^{-j\omega t}\,d\omega$$

$$-2\pi j\omega e^{-|\omega|} = \int_{-\infty}^{\infty} \frac{4t}{(1+t^2)^2} e^{-j\omega t}\,dt \to F\left\{\frac{4t}{(1+t^2)^2}\right\} = -2\pi j\omega e^{-|\omega|}$$

-۶

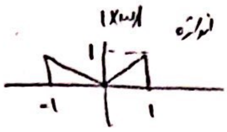
a) $X(\omega)=\frac{2\sin(3(\omega-2\pi))}{\omega-2\pi}$ → تأخیر $= X(j(\omega-\omega_0)) \xrightarrow{F^{-1}} e^{j\omega_0 t}x(t)$

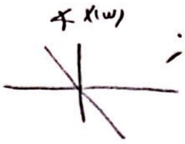
$x(t)=e^{j2\pi t}\,\Pi\left(\frac{t}{6}\right)$ → $\frac{2\sin\omega T}{\omega} \xrightarrow{F} \Pi\left(\frac{t}{2T}\right)$

b) $2[\delta(\omega-1)-\delta(\omega+1)]+3[\delta(\omega-2\pi)+\delta(\omega+2\pi)]$ $\xrightarrow{\text{تأخیر}}$ $2\pi\delta(\omega)\xrightarrow{F^{-1}}1$

$\rightarrow 2\left[\frac{e^{jt}}{2\pi}-\frac{e^{-jt}}{2\pi}\right]+3\left[\frac{e^{j2\pi t}}{2\pi}+\frac{e^{-j2\pi t}}{2\pi}\right]$

c)





$\rightarrow$ تقارن فرد = فاز

تقارن زوج = اندازه

$X(j\omega)=|X(\omega)|\,e^{j\measuredangle X(\omega)}=\begin{cases}|-\omega|\,e^{-2j\omega} & -1<\omega<0\\ |\omega|\,e^{-2j\omega} & 0<\omega<1\end{cases}$

$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}d\omega=\frac{1}{2\pi}\int_{-1}^{1}|\omega|e^{-2j\omega}e^{j\omega t}d\omega=\frac{1}{2\pi}\left[\int_{-1}^{0}-\omega e^{j(t-2)\omega}d\omega+\int_{0}^{1}\omega e^{j(t-2)\omega}d\omega\right]$

-۷

الف) $x(t)=x_1(t-1)\xrightarrow{F}X(j\omega)=e^{-j\omega}X_1(j\omega)\Rightarrow \measuredangle X(j\omega)=-\omega$

ب) $X(0)=\int_{-\infty}^{+\infty}x(t)e^{-j\omega t}d\omega\xrightarrow{\omega=0}\int_{-\infty}^{+\infty}x(t)\,dt=$ مساحت شکل

ج) $\int_{-\infty}^{+\infty}X(\omega)\,d\omega=2\pi x(t)$

د) $\int_{-\infty}^{+\infty}X(\omega)\frac{2\sin\omega}{\omega}e^{2j\omega}d\omega=\int_{-\infty}^{+\infty}X(\omega)X_2(\omega)e^{j\omega_0}d\omega=2\pi F^{-1}\overbrace{[X(\omega)X_2(\omega)]}^{x_1(t)*x_2(t)}\Big|_{t=0}$

$\rightarrow 2\pi\int_{-\infty}^{\infty}x(\tau)x_2(-\tau)\,d\tau$; $\frac{2\sin\omega}{\omega}e^{2j\omega}\xrightarrow{F^{-1}}\Pi\left(\frac{t+2}{2}\right)$

ه) $Re\{X(\omega)\}=\frac{x(t)+x(-t)}{2}=\frac{1}{2}\times\begin{cases}2 & -1\le t<0\\ -t+2 & 0<t<1\\ t & 1\le t<2\\ 2 & 2\le t\le 3\end{cases}+\frac{1}{2}\times\begin{cases}2 & 0\le t<1\\ t+2 & -1\le t<0\\ -t & -2\le t<-1\\ 2 & -3\le t<-2\end{cases}$

و) $\int_{-\infty}^{+\infty}|X(\omega)|^2dt=2\pi\int_{-\infty}^{+\infty}|x(t)|^2dt$

$\int_{-1}^{0}4\,dt+\int_{0}^{1}(-t+2)^2dt+\int_{1}^{2}t^2dt+\int_{2}^{3}4\,dt=$

8-

$$g(t)=x(t)*h(t)\xrightarrow{F}Y(j\omega)=X(j\omega)\cdot H(j\omega)$$

$$g(t)=x(3t)*h(3t)\xrightarrow{F}G(j\omega)=\underbrace{\frac{1}{3}X\left[\frac{j\omega}{3}\right]}_{x}\cdot\underbrace{\frac{1}{3}H\left[\frac{j\omega}{3}\right]}_{h}=\frac{1}{9}X\left[\frac{j\omega}{3}\right]H\left[\frac{j\omega}{3}\right]$$

$$\xrightarrow{F^{-1}}\frac{1}{3}y(3t)=g(t)\qquad g(t)=Ay(Bt)=\frac{1}{3}y(3t)\qquad A=\frac{1}{3},\ B=3$$

9-

$$\frac{d^2y(t)}{dt^2}+6\frac{dy(t)}{dt}+8y(t)=2x(t)$$

الف) $\frac{d^2h(t)}{dt^2}+6\frac{dh(t)}{dt}+8h(t)=2\delta(t)\xrightarrow{F}(j\omega)^2H(j\omega)+6(j\omega)+H(j\omega)+8(H(j\omega))2$

$$H(j\omega)=\frac{2}{(j\omega)^2+6j\omega+8}=\frac{2}{(j\omega+4)(j\omega+2)}=\frac{-1}{j\omega+4}+\frac{1}{j\omega+2},\ h(t)=-e^{-4t}u(t)+e^{-2t}u(t)$$

ب) $x(t)=te^{-2t}\xrightarrow{F}X(j\omega)=\frac{1}{(2+j\omega)^2}\longrightarrow\left[\frac{1}{(2+j\omega)^2}\cdot\frac{2}{(j\omega+4)(j\omega+2)}\right]\Rightarrow$

$$=\frac{A}{(2+j\omega)}+\frac{B}{(2+j\omega)^2}+\frac{C}{(2+j\omega)^3}+\frac{D}{j\omega+4}$$

10-

الف) $x(t)=[e^{-t}+e^{-3t}]u(t)\xrightarrow{F}X(j\omega)=\frac{1}{1+j\omega}+\frac{1}{3+j\omega}=\frac{2(j\omega+2)}{(j\omega+1)(j\omega+3)}$

$$y(t)=[2e^{-t}-2e^{-4t}]u(t)\xrightarrow{F}Y(j\omega)=\frac{2}{1+j\omega}-\frac{2}{4+j\omega}=\frac{6}{(1+j\omega)(4+j\omega)}$$

$$H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}=\frac{6(j\omega+1)(j\omega+3)}{2(j\omega+2)(1+j\omega)(4+j\omega)}=\frac{3(j\omega+3)}{(j\omega+2)(4+j\omega)}=\frac{A}{j\omega+2}+\frac{B}{4+j\omega}$$

ب) $H(j\omega)\xrightarrow{F^{-1}}h(t)=\frac{3}{2}(e^{-2t}+e^{-4t})u(t)$