باللطیف

رضا ادیب پور
٩٨١٤٣٠٣
تمرین سری ۸ سیگنال (تبدیل Z)

#1

$$x[n]=\begin{cases}(\frac13)^n\cos(\frac{\pi}{4}n) & n\le 0\\ 0 & n>0\end{cases} = (\frac13)^n\cos(\frac{\pi}{4}n)\,u[-n]$$

$$=(\frac13)^n\left(\frac12 e^{j\frac{\pi}{4}n}+\frac12 e^{-j\frac{\pi}{4}n}\right)u[-n] \Rightarrow X(Z)=\frac12\sum_{n=-\infty}^{0}(\frac13)^n e^{j\frac{\pi}{4}n}Z^{-n}+\frac12\sum_{n=-\infty}^{0}(\frac13)^n e^{-j\frac{\pi}{4}n}Z^{-n}$$

$$=\frac12\sum_{n=-\infty}^{0}(\frac13 e^{j\frac{\pi}{4}}Z^{-1})^n+\frac12\sum_{n=-\infty}^{0}(\frac13 e^{-j\frac{\pi}{4}}Z^{-1})^n=\frac{(\frac13 e^{j\frac{\pi}{4}}Z^{-1})^{-\infty}-(\frac13 e^{j\frac{\pi}{4}}Z^{-1})^{1}}{1-\frac13 e^{j\frac{\pi}{4}}Z^{-1}}\times\frac12$$

$$+\frac12\times\frac{(\frac13 e^{-j\frac{\pi}{4}}Z^{-1})^{-\infty}+(\frac13 e^{-j\frac{\pi}{4}}Z^{-1})^{1}}{1-\frac13 e^{-j\frac{\pi}{4}}Z^{-1}}=\left(\frac{-\frac13 e^{j\frac{\pi}{4}}Z^{-1}}{1-\frac13 e^{j\frac{\pi}{4}}Z^{-1}}+\frac{\frac13 e^{-j\frac{\pi}{4}}Z^{-1}}{1-\frac13 e^{-j\frac{\pi}{4}}Z^{-1}}\right)\times\frac12 \qquad |Z|<\frac13$$

قطب ها : $\frac13 e^{j\frac{\pi}{4}}$ ، $\frac13 e^{-j\frac{\pi}{4}}$

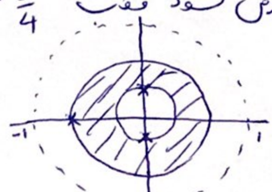
#2 $$X(Z)=\frac{1-\frac14 Z^{-2}}{(1+\frac14 Z^{-2})(1+\frac54 Z^{-1}+\frac38 Z^{-2})} \qquad Roc: |Z|>\frac12$$

$$=\frac{(1-\frac12 Z^{-1})\cancel{(1+\frac12 Z^{-1})}}{(1+\frac12 e^{j\frac{\pi}{2}}Z^{-1})(1-\frac12 e^{j\frac{\pi}{2}}Z^{-1})\cancel{(1+\frac12 Z^{-1})}(1+\frac34 Z^{-1})}=\frac{(1-\frac12 Z^{-1})}{(1+\frac12 e^{j\frac{\pi}{2}}Z^{-1})(1-\frac12 e^{j\frac{\pi}{2}}Z^{-1})(1+\frac34 Z^{-1})}$$

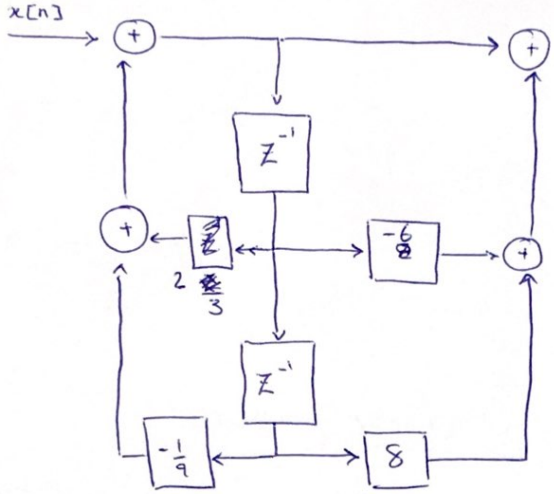
$$=\frac{A}{(1+\frac12 e^{j\frac{\pi}{2}}Z^{-1})}+\frac{B}{(1-\frac12 e^{j\frac{\pi}{2}}Z^{-1})}+\frac{C}{(1+\frac34 Z^{-1})} \Rightarrow \begin{cases}A=\frac{-1}{13}-j\frac{5}{13}\\ B=\frac{-1}{13}+j\frac{5}{13}\\ C=\frac{15}{13}\end{cases}$$

$$\Rightarrow x[n]=(-\frac{1}{13}-j\frac{5}{13})\cdot(-\frac12 e^{j\frac{\pi}{2}})^n u[n]+(\frac{-1}{13}+j\frac{5}{13})(\frac12 e^{j\frac{\pi}{2}})^n u[n]\underbrace{-\frac{15}{13}(-\frac34)^n u[-n-1]}$$

با فرض $Roc: \frac12<|Z|<\frac34$ ، اگر $Roc: |Z|>\frac12$ فرض شود قطب $Z=-\frac34$ درون Roc می افتد که اشتباه است.



#3



$$H(Z) = \frac{c + dZ^{-1} + eZ^{-2}}{1 + aZ^{-1} + bZ^{-2}}$$

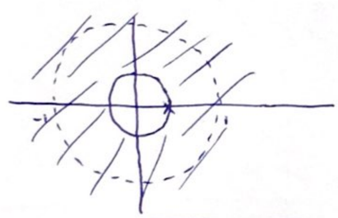
$$H(Z) = \frac{1 - 6Z^{-1} + 8Z^{-2}}{1 - \frac{2}{3}Z^{-1} + \frac{1}{9}Z^{-2}}$$

$$H(Z) = \frac{Y(Z)}{X(Z)} = \frac{1 - 6Z^{-1} + 8Z^{-2}}{1 - \frac{2}{3}Z^{-1} + \frac{1}{9}Z^{-2}}$$

$$\Rightarrow Y(Z) - \frac{2}{3}Z^{-1}Y(Z) + \frac{1}{9}Z^{-2}Y(Z) = X(Z) - 6Z^{-1}X(Z) + 8Z^{-2}X(Z) \xrightarrow{x[n-n_0] \overset{Z}{\longleftrightarrow} Z^{-n_0}X(Z)}$$

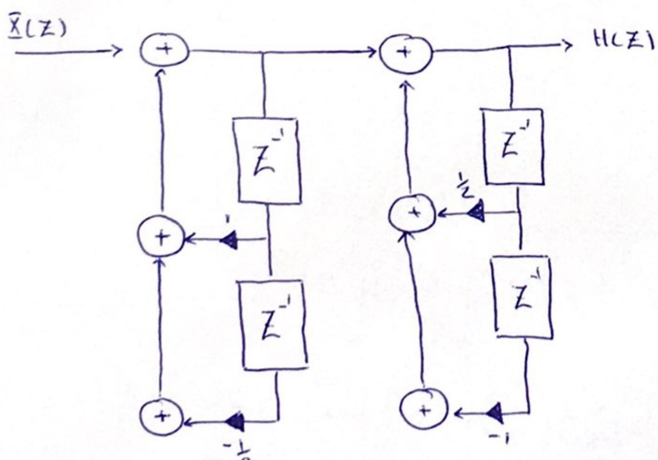
$$\Rightarrow y[n] - \frac{2}{3}y[n-1] + \frac{1}{9}y[n-2] = x[n] - 6x[n-1] + 8x[n-2]$$

ب) ~~هم می‌تواند علّی باشد هم می‌تواند نباشد~~ چون سیستم علّی است (ROC دست راستی) و چون دو قطب در $Z = \frac{1}{3}$ داریم ، سیستم پایدار است ← $|Z|=1$ را پوشش می‌دهد.



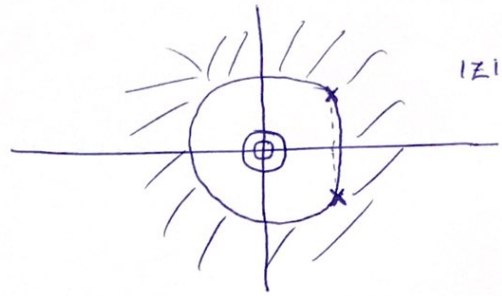
#4

$$H(Z) = \frac{1}{(1 - Z^{-1} + \frac{1}{2}Z^{-2})(1 - \frac{1}{2}Z^{-1} + Z^{-2})} = \frac{1}{(1 - Z^{-1} + \frac{1}{2}Z^{-2})} \times \frac{1}{(1 - \frac{1}{2}Z^{-1} + Z^{-2})}$$



#5

$$\begin{cases} \text{الف)} \quad x[n] \to \text{Real \& Causal} \\ \text{ب)} \quad X(z) \to \text{دو قطب دارد} \\ \text{پ)} \quad X(z) \to \text{دو صفر در مبدأ دارد} \\ \text{ت)} \quad Z_1 = \frac{1}{2}e^{j\frac{\pi}{3}} \quad \text{قطب} \\ \text{ث)} \quad X(1) = \frac{8}{3} \end{cases}$$



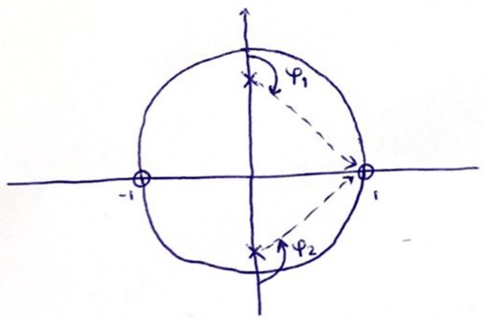
$|Z| > \frac{1}{2}$

$$\Rightarrow X(Z) = \frac{A Z^2}{(1-\frac{1}{2}e^{j\frac{\pi}{3}}Z^{-1})(1-\frac{1}{2}e^{-j\frac{\pi}{3}}Z^{-1})} \xrightarrow{(\text{ث})} X(1) = \frac{A}{(1-\frac{1}{2}e^{j\frac{\pi}{3}})(1-\frac{1}{2}e^{-j\frac{\pi}{3}})} = \frac{8}{3}$$

$$\Rightarrow \frac{A}{1-\frac{1}{2}e^{-j\frac{\pi}{3}}-\frac{1}{2}e^{j\frac{\pi}{3}}+\frac{1}{4}} = \frac{A}{\frac{5}{4}-(\cos(\frac{\pi}{3}))} = \frac{8}{3} \Rightarrow \frac{4}{3}A = \frac{8}{3} \Rightarrow \boxed{A=2}$$

$$\Rightarrow X(Z) = \frac{2Z^2}{(1-\frac{1}{2}e^{j\frac{\pi}{3}}Z^{-1})(1-\frac{1}{2}e^{-j\frac{\pi}{3}}Z^{-1})} \qquad \text{Roc}: |Z| > \frac{1}{2}$$

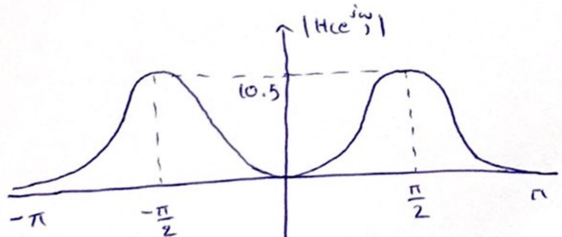
#6



$$|H(e^{j\omega})| = \frac{\prod_{i=1}^{M} d_i \to \text{فاصله صفرها تا دایره واحد}}{\prod_{i=1}^{N} l_i \to \text{فاصله قطب ها تا دایره واحد}}$$

$$\begin{cases} |H(e^{j0})| = \frac{0\times 2}{1+0.81} = 0 \\ |H(e^{j\frac{\pi}{2}})| = \frac{\sqrt{1+1}\times\sqrt{1+1}}{0.1\times 0.9} = 10.5 \\ |H(e^{j\pi})| = \frac{0\times 2}{1+0.81} = 0 \end{cases}$$

$\Rightarrow$

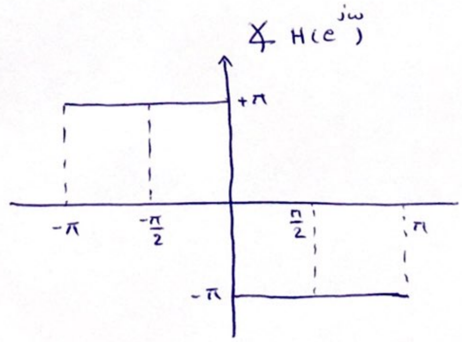


$$\measuredangle H(e^{j\omega}) = \cancel{\sum_{i=1}^{N} \psi_i}^{\,0} - \sum_{i=1}^{M} \varphi_i$$

$$\measuredangle H(e^{j0}) = -\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = -\pi$$

$$\measuredangle H(e^{j\frac{\pi}{2}}) = -(0+\pi) = -\pi$$

$\Rightarrow$



در بین 0 تا $\frac{\pi}{2}$، هر چقدر که φ_1 کم می شود به همان میزان φ_2 زیاد می شود

اگر سیستم علی باشد ($|z|>0.9$) سیستم پایدار نیز هست چون دایره واحد را پوشش می دهد.