یا لطیف

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تمرین سری ۵ سیگنال سیستم

#1

$$\begin{cases} x[n] : \text{Real \& odd} \\ N = 10 \\ P = 8 \\ a_3, a_4 \neq 0 \\ 0 < \measuredangle a_3, a_4 < \pi \\ 2|a_3| = |a_4| \end{cases}$$

$$P = \frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$$

$$= \sum_{k=0}^{10} |a_k|^2 = |a_3|^2 + |a_4|^2 = |a_3|^2 + 4|a_3|^2 = 8$$

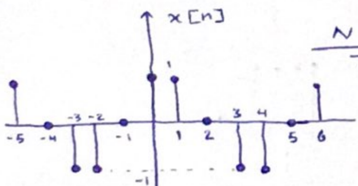
$$\Rightarrow 5|a_3|^2 = 8 \Rightarrow |a_3|^2 = \frac{8}{5} \Rightarrow |a_3| = 2\sqrt{\frac{2}{5}}$$

$$|a_4| = 4\sqrt{\frac{2}{5}}$$

$$\xrightarrow{a_k = -a_{-k}} \begin{cases} a_3 = -a_{-3} = 2j\sqrt{\frac{2}{5}} \\ a_4 = -a_{-4} = 4j\sqrt{\frac{2}{5}} \end{cases} \Rightarrow x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\frac{\pi}{5}n} = \sum_{k=0}^{10} a_k e^{jk\frac{\pi}{5}n}$$

$$= 2j\sqrt{\frac{2}{5}}\, e^{j\frac{3\pi}{5}n} + 4j\sqrt{\frac{2}{5}}\, e^{j\frac{4\pi}{5}n} = x[n]$$

#2



$\underline{N = 6}$

$$a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]\, e^{-jk\left(\frac{2\pi}{N}\right)n}$$

$$a_k = \frac{1}{6}\sum_{n=0}^{6} x[n]\, e^{-jk\frac{\pi}{3}n}$$

$$= \frac{1}{6}\left(x[0]e^{-jk\frac{\pi}{3}\times 0} + x[1]e^{-jk\frac{\pi}{3}\times 1} + \cancel{x[2]e^{-jk\frac{\pi}{3}\times 2}} + x[3]e^{-jk\frac{\pi}{3}\times 3} + x[4]e^{-jk\frac{\pi}{3}\times 4} + \cancel{x[5]e^{-jk\frac{\pi}{3}\times 5}} + x[6]e^{-jk\frac{\pi}{3}\times 6}\right)$$

$$= \frac{1}{6}\left(1 + e^{-jk\frac{\pi}{3}} \underbrace{- e^{-jk\pi}}_{=(-1)^k} - e^{-jk\frac{4\pi}{3}} + \underbrace{e^{-jk2\pi}}_{=(-1)^{2k}}\right) = \frac{1}{6}\left(1 + e^{-jk\frac{\pi}{3}} - e^{-jk\frac{4\pi}{3}} + (-1)^k + (-1)^{2k}\right)$$

#3

$$\begin{cases} \xrightarrow{x[n]} \boxed{\text{علی}} \longrightarrow y[n] \\ y[n] - \frac{1}{4}y[n-1] = x[n] \\ x[n] = \cos\left(\frac{\pi}{4}n\right) + 3\sin\left(\frac{2\pi}{3}n\right) \end{cases}$$

در صورت سوال گفته نشده سیستم LTI است، اما برای حل ما فرض کردیم که سیستم $\underline{LTI}$ است.

$$e^{j\omega n} \longrightarrow \boxed{LTI} \longrightarrow H(j\omega)e^{j\omega n} \Rightarrow H(j\omega)e^{j\omega n} - \frac{1}{4}H(j\omega)\cdot e^{j\omega(n-1)} = e^{j\omega n} \Rightarrow H(j\omega)\cancel{e^{j\omega n}} - \frac{1}{4}H(j\omega)\cancel{e^{j\omega n}}e^{-j\omega} = \cancel{e^{j\omega n}}$$

$$\Rightarrow H(j\omega)\left(1 - \frac{1}{4}e^{-j\omega}\right) = 1 \Rightarrow \boxed{H(j\omega) = \frac{1}{1 - \frac{1}{4}e^{-j\omega}}}$$

$$x(t) = \sum a_k e^{jk\omega_0 n} \longrightarrow \boxed{H(j\omega)} \longrightarrow y[n] = \sum_{k=\langle N\rangle} \underbrace{a_k H(j\omega)}_{b_k} e^{jk\omega_0 n}$$

#3 ادامه :

$$x[n] = \underbrace{\cos\left(\frac{\pi}{4}n\right)}_{N_1 = 8} + \underbrace{3\sin\left(\frac{2\pi}{3}n\right)}_{N_2 = 3} \Rightarrow \text{lcm}\{3, 8\} = 24$$

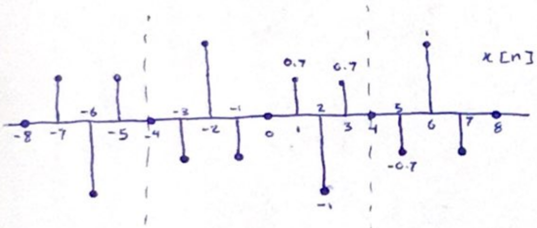
$$\Rightarrow x[n] = \underset{a_3}{\frac{1}{2}} e^{j\frac{\pi}{4}n} + \underset{a_{-3}}{\frac{1}{2}} e^{-j\frac{\pi}{4}n} + \underset{a_8}{\frac{3}{2j}} e^{j\frac{2\pi}{3}n} - \underset{a_{-8}}{\frac{3}{2j}} e^{-j\frac{2\pi}{3}n} \Rightarrow \begin{cases} a_3 = a_{-3} = \frac{1}{2} \\ a_{-8} = -\frac{3}{2j} \\ a_8 = \frac{3}{2j} \end{cases}$$

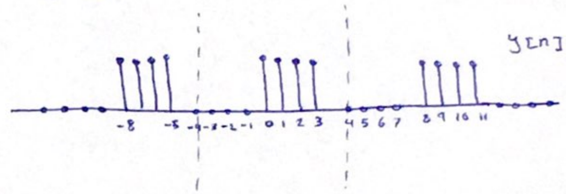
$$b_3 = a_3 H(e^{j\frac{\pi}{4}}) = \frac{1}{2} \times \frac{1}{1 - \frac{1}{4}e^{-j\frac{\pi}{4}}}, \quad b_{-3} = a_{-3} H(e^{-j\frac{\pi}{4}}) = \frac{1}{2} \times \frac{1}{1 - \frac{1}{4}e^{j\frac{\pi}{4}}}$$

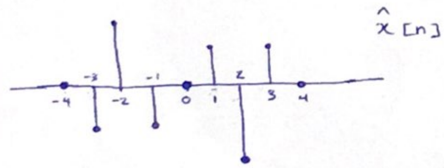
$$b_8 = a_8 H(e^{j\frac{2\pi}{3}}) = \frac{3}{2j} \times \frac{1}{1 - \frac{1}{4}e^{-j\frac{2\pi}{3}}}, \quad b_{-8} = a_{-8} H(e^{-j\frac{2\pi}{3}}) = \frac{-3}{2j} \times \frac{1}{1 - \frac{1}{4}e^{j\frac{2\pi}{3}}}$$

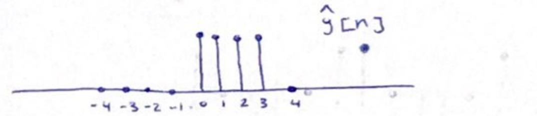
#4

$$x[n] = \sin\left(\frac{3\pi}{4}n\right), \quad y[n] = \begin{cases} 1 & ; 0 \le n \le 3 \\ 0 & ; 4 \le n \le 7 \end{cases} \qquad N = 8$$

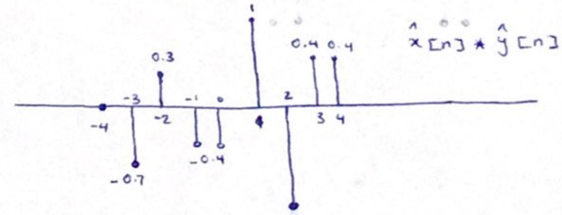




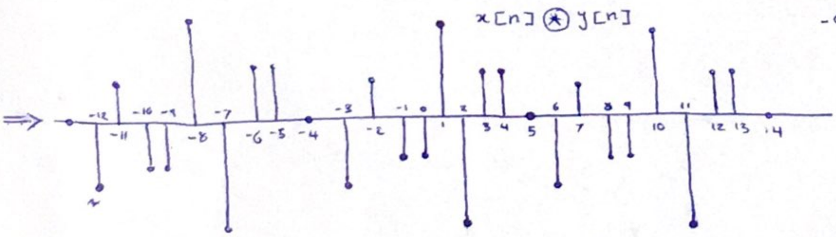




$$\Rightarrow \hat{x}[n] * \hat{y}[n] = \sum_{k=-\infty}^{+\infty} \hat{x}[k]\, \hat{y}[n-k] \Rightarrow$$



$\Rightarrow$



$$a_k = \frac{1}{8}\sum_{k=-4}^{3} x[n] e^{-jk\frac{\pi}{4}n} = \frac{1}{8}\left(\cancel{x[-4] e^{-jk\frac{\pi}{4}\times -4}} + x[-3] e^{-jk\frac{\pi}{4}\times(-3)} + x[-2] e^{-jk\frac{\pi}{4}\times -2} + x[-1] e^{-jk\frac{\pi}{4}(-1)} \right.$$

$$\left. + x[0] e^{-jk\frac{\pi}{4}(0)} + x[1] e^{-jk\frac{\pi}{4}\times 1} + x[2] e^{-jk\frac{\pi}{4}(+2)} + x[3] e^{-jk\frac{\pi}{4}\times 3} \right)$$

$$= \frac{1}{8}\left(-0.7 e^{-jk\frac{3\pi}{4}} + 0.3 e^{+jk\frac{\pi}{2}} - 0.4 e^{jk\frac{\pi}{4}} - 1 + e^{-jk\frac{\pi}{4}} - e^{-jk\frac{\pi}{2}} + 0.4 e^{-jk\frac{3\pi}{4}} \right)$$