سوال سوم

۱) فرض کنید $x[n]=\delta[n]+2\delta[n+1]$ و $h[n]=2\delta[n+1]-5[n-2]$ است. کانولوشن $x[n+1]*h[n-1]$ را به دو روش محاسبه کنید.

$$x[n+1] \Rightarrow x[n+1]=\delta[n+1]+2\delta[n+2]$$
$$h[n-1] \Rightarrow h[n-1]=2\delta[n]-5[n-3]$$

$$\underbrace{x[n+1]}_{N} * \underbrace{h[n-1]}_{N'} = x[N]*h[N'] \rightarrow \int_{-\infty}^{+\infty} x(\lambda)\,h(t-\lambda)\,d\lambda$$

$$\left.\begin{array}{l} t+1=\lambda \longrightarrow x(\lambda) \\ t-1=t-\lambda \quad h(t-\lambda) \end{array}\right\} \int_{-\infty}^{+\infty} \Big(\delta[\lambda+1]+2\delta[\lambda+2]\Big)\Big(2\delta[t-\lambda]-5[t-\lambda-3]\Big)\,d\lambda$$

$$\int_{-\infty}^{+\infty} \Big(\delta[\lambda+1]2\delta[t-\lambda]-\delta[\lambda+1]5[t-\lambda-3]+2\delta[\lambda+2]2\delta[t-\lambda]$$
$$-2\delta[\lambda+2]5[t-\lambda-3]\Big)\,d\lambda$$

② پاسخ یک سیستم LTI با پاسخ ضربه $h[n]=(3)^n u[-n+2]$ به ورودی $x[n]=(\frac{1}{2})^{n-1}(u(n)-u(n-10))$ چیست؟

$$y[n]=\sum_{k=-\infty}^{+\infty} x[n]\,h_k[n]$$

$$=\sum_{-\infty}^{+\infty}(\tfrac{1}{2})^{n-1}\big(u[n]-u[n-10]\big)\times(3)^n u[-n+2]$$

$$=\sum_{-\infty}^{+\infty}\Big((\tfrac{1}{2})^{n-1}u[n]-(\tfrac{1}{2})^{n-1}u[n-10]\Big)\cdot 3^n u[-n+2]$$

$$=\sum_{-\infty}^{+\infty}(3)^n\times(\tfrac{1}{2})^{n-1}u[n]u[-n+2]-(3)^n(\tfrac{1}{2})^{n-1}u[n-10]u[-n+2]$$

($n\geq 10$)

$$\sum_{0}^{+\infty}(3)^n\times(\tfrac{1}{2})^{n-1}u[-n+2]-\sum_{10}^{+\infty}(3)^n(\tfrac{1}{2})^{n-1}u[-n+2]$$

$$\sum_{k=0}^{2}(3)^n\times(\tfrac{1}{2})^{n-1}$$

④ پاسخ یک سیستم LTI سببی با پاسخ ضربه $h(t)=e^{t+1}(u(t)-u(t-\delta))$ به ورودی $x(t)=2e^{t-1}u(-t)$ را بیابید.

$u(t-\lambda)=e^{t-\lambda+1}(u(t-\lambda)-u(t-\lambda-\delta))$ سببی

$x(\lambda)=2e^{\lambda-1}u(-\lambda)$

$$y(t)=\int_{-\infty}^{+\infty}x(\lambda)h(t-\lambda)d\lambda$$

$$=\int_{-\infty}^{+\infty}2e^{\lambda-1}u(-\lambda)\times e^{t-\lambda+1}(u(t-\lambda)-u(t-\lambda-\delta))$$

$$=\int_{-\infty}^{\infty}2e^{\lambda-1}u(-\lambda)\times\left(e^{t-\lambda+1}u(t-\lambda)-e^{t-\lambda+1}u(t-\lambda-\delta)\right)$$

$$=\int_{-\infty}^{\infty}2e^{\lambda-1}.u(-\lambda).e^{t-\lambda+1}.u(t-\lambda)-\int_{-\infty}^{\infty}2e^{\lambda-1}.u(-\lambda).e^{t-\lambda+1}.u(t-\lambda-\delta)$$

$$2\int_{-\infty}^{\infty}e^{\lambda-1+t-\lambda+1}.u(-\lambda).u(t-\lambda)-2\int_{-\infty}^{\infty}e^{\lambda-1+t-\lambda+1}.u(-\lambda).u(t-\lambda-\delta)$$

$$2e^t\int_0^t 1\,d\lambda-2e^t\int_0^{t-\delta}1\,d\lambda=0\rightarrow 2e^t\times\lambda\Big]_0^t-2e^t\lambda\Big]_0^{t-\delta}$$

$$2e^t(t-0)-2e^t(t-\delta-0)\rightarrow 2e^tt-2e^t(t-\delta)$$

$$\rightarrow 2e^tt\,u(-t)-2e^t(t-\delta)u(t-\lambda-\delta)$$