یا لطیف

رضا ادیب پور
۹۸۱۴۳۰۳

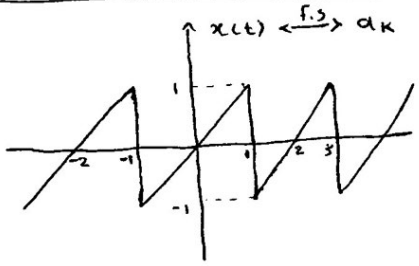
تمرین سری ۴- سیگنال و سیستم

#1 $\begin{cases} x_1(t) \xleftrightarrow{F.S} a_k \longrightarrow T_1 \text{ پریودیک با پریود} \\ x_2(t) \xleftrightarrow{F.S} b_k \longrightarrow \text{پریود} = ? \end{cases}$

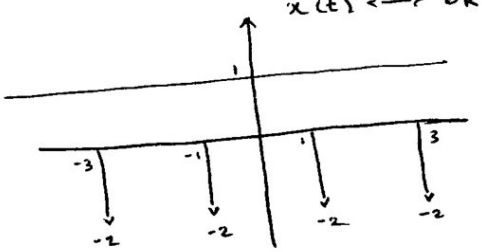
$x_2(t) = 2x_1(1-t) - \frac{1}{3}x_1(2t-1) \Rightarrow \begin{cases} x_1(t-1) \xleftrightarrow{F.S} a_k e^{-jk\frac{2\pi}{T_1}\times 1} \longrightarrow x_1(t-1) \xleftrightarrow{F.S} a_k e^{-jk\frac{4\pi}{T_1}} \\ x_1(1+t) \xleftrightarrow{F.S} a_k e^{jk\frac{2\pi}{T_1}\times 1} \longrightarrow x_1(-t+1) \xleftrightarrow{F.S} a_{-k} e^{+jk\frac{2\pi}{T_1}} \end{cases}$

$\Rightarrow x_2(t) = 2x_1(1-t) - \frac{1}{3}x_1(2t-1) \xleftrightarrow{F.S} 2a_k e^{-jk\frac{4\pi}{T_1}} - \frac{1}{3}a_{-k} e^{+jk\frac{2\pi}{T_1}} = b_k \longrightarrow T = T_1$

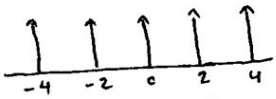
#2



مشتق می‌گیریم ⟶



ضربه‌ی دیراک:



$s(t) \xleftrightarrow{F.S} d_k = \frac{1}{T} = \frac{1}{2}$

$\Rightarrow x'(t) = -2\,s(t-1) + 1$

$(jkw_0)a_k = -2 \times e^{-jkw_0 \times 1} \times \frac{1}{2}$

$\Rightarrow a_k = \frac{-e^{-jkw_0}}{jkw_0}$

$w_0 = \frac{2\pi}{2} = \pi$

$a_k = \frac{-e^{-jk\pi}}{jk\pi} = \begin{cases} \frac{j}{k\pi}(-1)^k & ; k \neq 0 \\ 0 & , a_0 = 0 \end{cases}$

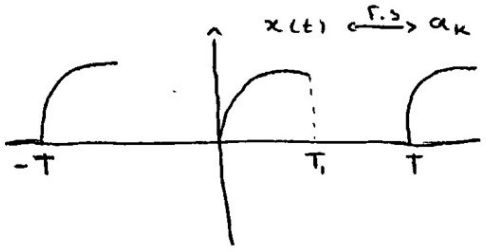
#4 $\begin{cases} x(t) \xleftrightarrow{F.S} a_k \longrightarrow T = 6 \text{ پریودیک با} \\ z(t) = 2x\left(\frac{1}{2}t - 1\right) + 3x(2t) \xleftrightarrow{F.S} b_k = ? \end{cases}$

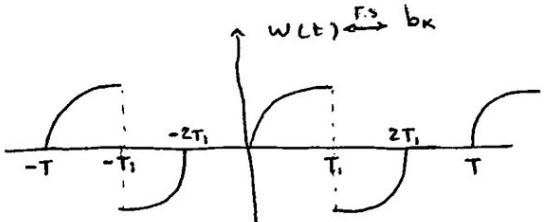
$2x\left(\frac{1}{2}t - 1\right) \xleftrightarrow{F.S} 2a_k \times e^{-jkw_0 \times 1} = 2a_k e^{-jkw_0}$

$\begin{cases} 2x\left(\frac{1}{2}t\right) \xleftrightarrow{F.S} 2a_k \\ \quad \hookrightarrow T = 12 \\ 3x(2t) \xleftrightarrow{F.S} 3a_k \\ \quad \hookrightarrow T = 3 \end{cases}$

$\Rightarrow z(t) \xleftrightarrow{F.S.} 2a_k e^{-jkw_0} + 3a_k = [illegible]$

#4





$\Rightarrow \quad w(t) = x(t) - x(-t-2T_1) \quad \Rightarrow \quad b_k = a_k - a_{-k} e^{-jk\omega_0 \times 2T_1} = a_k - a_{-k} e^{-jk4\pi\frac{T_1}{T}}$

#5

$$\begin{cases} x(t) \\ T = 3 \\ a_k = a_{k+2} \\ a_k = a_{-k} \\ \int_{-0.5}^{0.5} x(t)\,dt = 1, \quad \int_{0.5}^{1.5} x(t)\,dt = 2 \end{cases}$$