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کیانا ارمانی

1-

$$I = A^{-1}A \Rightarrow \det(I) = \det(A^{-1}A) = \det(A^{-1})\det(A)$$

$$\Rightarrow 1 = \det(A^{-1})\det(A) \Rightarrow \det(A^{-1}) = 1/\det(A).$$

2-

$$\alpha A = (\alpha I)A \Rightarrow \det(\alpha A) = \det(\alpha I)\det(A) = \alpha^n \det(A).$$

3-

فرض می‌کنیم:

$$X = \begin{bmatrix} b & c \\ d & e \end{bmatrix} \Rightarrow X^2 = \begin{bmatrix} b & c \\ d & e \end{bmatrix}\begin{bmatrix} b & c \\ d & e \end{bmatrix}$$

$$\Rightarrow X^2 = \begin{bmatrix} b^2+cd & c(b+e) \\ d(b+e) & cd+e^2 \end{bmatrix} \Rightarrow X^2 = \begin{bmatrix} b^2+cd & c(b+e) \\ d(b+e) & cd+e^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{cases} b^2+cd = 1 \\ d(b+e) = 0 \Rightarrow \begin{cases} d = 0 \Rightarrow \begin{cases} b^2 = 1 \Rightarrow b = \pm 1 \\ e^2 = 1 \Rightarrow e = \pm 1 \end{cases} \\ b = -e \Rightarrow \begin{cases} (-e)^2 + cd = 1 \\ cd + e^2 = 1 \end{cases} \\ c = 0 \end{cases} \\ cd + e^2 = 1 \\ c(b+e) = a \; \star \end{cases}$$

$$\Rightarrow b = 1, \quad e = -1, \quad d = 0, \quad c = 0$$

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$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 2 & 7 \\ 2 & 4 & -3 & 2 \\ 3 & 0 & 15 & 3 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}\begin{vmatrix} -3 & 2 \\ 15 & 3 \end{vmatrix}$$

$$- \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}\begin{vmatrix} 2 & 7 \\ 15 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix}\begin{vmatrix} 1 & 7 \\ -3 & 2 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 2 & 4 \end{vmatrix}\begin{vmatrix} 3 & -1 \\ 15 & 3 \end{vmatrix}$$

$$-\begin{vmatrix}0&1\\3&0\end{vmatrix}\begin{vmatrix}3&-1\\-3&2\end{vmatrix}+\begin{vmatrix}2&4\\3&0\end{vmatrix}\begin{vmatrix}3&-1\\2&7\end{vmatrix}\Rightarrow$$

$$|A|=\left((1)\times(\cancel{-3}-9)\right)-\underbrace{\left((0)\times(-99)\right)}_{0}+\left((-6)\times(23)\right)+$$

$$\left((-2)\times(24)\right)-\left((-3)\times(3)\right)+\left((-12)\times(23)\right)=(-39)-(0)$$

$$+(-138)+(-48)-(-9)+(-276)=-492$$

-5

$$A=\begin{bmatrix}2&0&3\\0&7&0\\4&0&5\end{bmatrix}\qquad A^{-1}=\frac{1}{|A|}\operatorname{adj}(A)$$

$$\Rightarrow A^{-1}=\frac{1}{-14}\times\begin{bmatrix}+\begin{vmatrix}7&0\\0&5\end{vmatrix}&-\begin{vmatrix}0&3\\0&5\end{vmatrix}&+\begin{vmatrix}0&3\\7&0\end{vmatrix}\\-\begin{vmatrix}0&0\\4&5\end{vmatrix}&+\begin{vmatrix}2&3\\4&5\end{vmatrix}&-\begin{vmatrix}2&3\\0&0\end{vmatrix}\\+\begin{vmatrix}0&7\\4&0\end{vmatrix}&-\begin{vmatrix}2&0\\4&0\end{vmatrix}&+\begin{vmatrix}2&0\\0&7\end{vmatrix}\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}-2{,}5&0&1{,}5\\0&0{,}142857&0\\2&0&-1\end{bmatrix},\quad A^{T}=\begin{bmatrix}2&0&4\\0&7&0\\3&0&5\end{bmatrix}\Rightarrow$$

$$A^{-1}A^{T}A=\begin{bmatrix}-0{,}5&0&-2{,}5\\0&0{,}999999&0\\1&0&3\end{bmatrix}\begin{bmatrix}2&0&3\\0&7&0\\4&0&5\end{bmatrix}$$

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~~[illegible]~~ (3) کلا 0.4

$$= \begin{bmatrix} -11 & 0 & -14 \\ 0 & 6.999993 & 0 \\ 14 & 0 & 18 \end{bmatrix} \Rightarrow \det(A^{-1}A^{T}A) = (-11 \times 125.999874)$$

$$-(0 \times 0) + (-14 \times -97.999902) = \boxed{-13.999986}$$

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(4)

کیاگرمابی

$x+y+z=0$, $v=(x,y,z)$, $w=(z,x,y)$. -6

می دانیم نرم v و w با هم برابر است: $\|v\| = \|w\| = \sqrt{x^2+y^2+z^2}$,

$$\|v\|\cdot\|w\| = \sqrt{x^2+y^2+z^2}\cdot\sqrt{x^2+y^2+z^2} = x^2+y^2+z^2.$$

ضرب داخلی v و w نیز به صورت زیر است:

$$v\cdot w = (x,y,z)\cdot(z,x,y) = xz + yx + zy.$$

می دانیم: $(a+b+c)^2 = a^2+b^2+c^2+2(ab+ac+bc)$

حال فرمول بالا را برای x و y و z می نویسیم:

$$\underbrace{(x+y+z)^2}_{0} = \underbrace{x^2+y^2+z^2}_{\|v\|\cdot\|w\|} + 2\underbrace{(xz+yx+zy)}_{v\cdot w}$$

$$0^2 = \|v\|\cdot\|w\| + 2v\cdot w \Rightarrow -2v\cdot w = \|v\|\cdot\|w\|$$

$$v\cdot w = -\frac{1}{2}\|v\|\cdot\|w\|$$

می دانیم: $\cos\theta = \frac{v\cdot w}{\|v\|\|w\|} \Rightarrow \cos\theta = \frac{-\frac{1}{2}\|v\|\|w\|}{\|v\|\|w\|} = -\frac{1}{2}$

$$\Rightarrow \frac{v\cdot w}{\|v\|\|w\|} = -\frac{1}{2}$$