تمرینات سری هفتم:

۱.

a) $\sum_{k=0}^{\infty} a^k \delta(t-kT)$

$a^0 e^{-0T} + a^1 e^{-T} + a^2 e^{-2T} \Rightarrow \sum_{k=0}^{\infty} a^k e^{-kT}$

تمام صفحه

غیر از بی نهایت

c) $\cos(\omega_0 t+\varphi)u(t)$ $\quad Re\{s\}>0$

$$\frac{s\cos(\varphi)-\omega_0\sin(\varphi)}{s^2+\omega_0^2}$$

صفر $= \omega_0\tan(\varphi)$

$\rightarrow s^2=-\omega_0^2 \Rightarrow s=\pm j\omega_0$

b) $te^{-at}u(t) \quad a>0$

از مشتق $\left(\frac{1}{s+a}\right)^2$ $\quad Re\{s\}>-a$ $\quad s+a=0 \Rightarrow s=-a$ مضاعف

d) $\underbrace{e^{-at}}_{\text{جابجایی}}\sin(\omega_0 t)u(t) \quad a>0 \Rightarrow \frac{\omega_0}{(s+a)^2+\omega_0^2} \rightarrow s=-a\pm j\omega_0$

$Re\{s\}>-a$

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۲.

a) $\frac{(s-2)(s-(-2))}{1} = (s-2)(s+2) = s^2-4 \xrightarrow{\mathcal{L}^{-1}} \frac{d^2\delta(t)}{dt^2}-4\delta(t)$

b) $\frac{1}{(s+2)(s-2)} = \frac{1}{s^2+4} = \frac{-1/4}{s+2}+\frac{-1/4}{s-2} \xrightarrow{\mathcal{L}^{-1}} 1/4(e^{2t}-e^{-2t})u(t)$

c) $\frac{1}{s(s-(2j))(s-(-2j))} = \frac{1}{s\,(s-2j)(s+2j)} = \frac{1/4}{s}+\frac{1/8}{s-2j}+\frac{-3/8}{s+2j}$

$\xrightarrow{\mathcal{L}^{-1}} 1/4\,u(t)+\frac{1}{8}e^{+2jt}u(t)-\frac{3}{8}e^{-2jt}u(t)$

d) $\frac{(s-2)}{(s+2)} = 1+\frac{-4}{s+2} \xrightarrow{\mathcal{L}^{-1}} \delta(t)-4(e^{-2t})u(t)$

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۳.

a) $X(s) = \frac{s^2-s+1}{(s+1)^2} \quad Re\{s\}>-1$

$\frac{s^2-s+1+2s-2s}{s^2+2s+1} = 1+\frac{-3s}{s^2+2s+1} = 1+\frac{A}{(s+1)^2}+\frac{B}{(s+1)} = 1+\frac{3}{(s+1)^2}+\frac{-3}{(s+1)}$

$\xrightarrow{\mathcal{L}^{-1}} \delta(t)+3te^{-t}u(t)-3e^{-t}u(t)$

b) $X(s) = \frac{s+1}{(s+1)^2+4}$ $\quad Re\{s\} > -1$

$$\frac{s+a}{(s+a)^2+w_0^2} \overset{s}{\longleftrightarrow} (e^{-at} \cos w_0 t)\, u(t)$$

$$\overset{s^{-}}{\Rightarrow} (e^{-t} \cos 2t)\, u(t)$$

۴-

الف

$Re\{s\} < -1$ غیرعلی و ناپایدار (ب و ج

$-1 < Re\{s\} < 1$ غیرعلی و پایدار

$Re\{s\} > 1$ علی و ناپایدار

$$H(s)=\frac{s+1}{s^2+2s+2}$$

۵-

$$x(t)=e^{-|t|}=e^{-t}u(t)+e^{t}u(-t)\xrightarrow{s}\frac{1}{s+1}-\frac{1}{s-1}=\frac{-2}{s^2-1}$$

$H(s)$ $\rightarrow s^2+2s+2=0 \Rightarrow s=-1\pm j$ ، حل $-1<Re<1$

$$\Rightarrow Re\{s\}_H>-1$$

$$\Rightarrow Y(s)=H(s)X(s)=\frac{-2}{(s^2+2s+2)(s-1)}$$

$$Re\{s\}_y=Re\{s\}_x\cap Re\{s\}_H$$

$$=-1<Re\{s\}_y<1$$

$$\rightarrow Y(s)=\frac{-0.4}{s-1}+\frac{0.4(s+1)}{(s+1)^2+1}+\frac{0.8}{(s+1)^2+1}$$

$$\xrightarrow{s^{-1}} y(t)=0.4e^{t}u(-t)+0.4e^{-t}\cos t\,u(t)+0.8e^{-t}\sin t\,u(t)$$

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۴- حل:

الف) $x(t)=e^{2t}$ $\qquad y(t)=\frac{1}{6}e^{2t}$

$$\Rightarrow H(2)=\frac{1}{6}$$

ب) $\frac{dh}{dt}+2h=e^{-4t}u(t)+bu(t) \Rightarrow H(s)=\frac{b(s+4)+s}{s(s+4)(s+2)}$

$$H(2)=\frac{1}{6}\Rightarrow b=1\Rightarrow H(s)=\frac{2}{s(s+4)}$$

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