عباسعلی عباسی

Subject: ۹۸۲۶۳۴۳

Year: Month: Date: HW04

۱- سیستم LTI پیوسته در زمان: $H(j\omega)=\int_{-\infty}^{+\infty} h(t)e^{-j\omega t}dt = \frac{\sin(4\omega)}{\omega}$

ورودی $\Rightarrow x(t)=\begin{cases} 1 & 0<t<4 \\ -1 & 4<t<8 \end{cases}$ $\quad T=8 \quad \omega_0=\frac{2\pi}{T}=\frac{2\pi}{8}=\frac{\pi}{4}$

$a_0=0$ مقدار dc سیگنال صفر است.

خروجی a_k ?

$$a_k=\frac{1}{T}\int_{-\infty}^{+\infty}x(t)e^{-jk\omega_0 t}dt=\frac{1}{8}\int_0^8 x(t)e^{-j\frac{\pi}{4}kt}dt=\frac{1}{8}\int_0^4 \overset{1}{x(t)}e^{-jk\frac{\pi}{4}t}dt$$

$$+\frac{1}{8}\int_4^8 \overset{-1}{x(t)}e^{-jk\frac{\pi}{4}t}dt=\frac{1}{jk\pi}(1-e^{-jk\pi})$$

$$a_k=\begin{cases} \frac{1}{jk\pi}(1-e^{-jk\pi})=0 & k=2n \quad \text{زوج} \\ \frac{1}{jk\pi}(1-e^{-jk\pi})=\frac{2}{jk\pi} & k=2n-1 \quad \text{فرد} \end{cases}$$

$y(t)=\sum_{k=-\infty}^{+\infty} a_k H(j\omega_0 k)e^{jk\omega_0 t}$ سری را باید برای $k=2n-1$ حساب کرد:

$H(j\omega)=(jk(\frac{\pi}{4}))=\frac{\sin(k\pi)}{\sin(\frac{\pi}{4})}$ $\quad y(t)=0 \quad k=2n-1$ ✓

۲- قطار ضربه: $x[n]=\sum_{k=-\infty}^{+\infty}\delta[n-4k] \rightarrow \boxed{LTI}_{H(e^{j\omega})} \rightarrow y[n]=\cos(\frac{5}{2}n\pi+\frac{\pi}{4})$

$H(e^{jk\frac{\pi}{2}})$ $\quad k=0,1,2,3,\dots$?

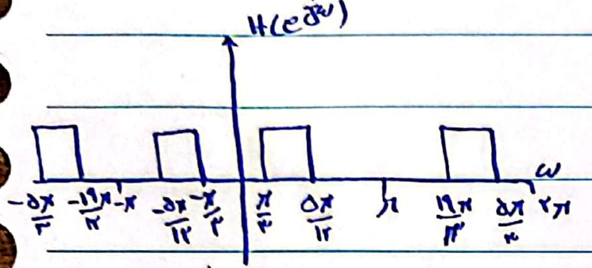
$x[n+N]=x[n] \quad N=4 \qquad a_k=\frac{1}{4}\sum_{n=0}^{4}x[n]e^{-j\frac{\pi}{2}}=\frac{1}{4}$

$$y[n]=\sum_{k=0}^{3}a_k H(e^{jk(\frac{\pi}{2})})e^{jk(\frac{\pi}{2})n}=\frac{1}{4}H(e^{j0})e^{j0}+\frac{1}{4}H(e^{j\frac{\pi}{2}})e^{j\frac{\pi}{2}}+\frac{1}{4}H(e^{j\frac{3\pi}{2}})$$

$$e^{j\frac{3\pi}{2}}+\frac{1}{4}H(e^{j\pi})e^{j\pi}=y[n]=\cos(\frac{5\pi}{2}n+\frac{\pi}{4})=\cos(\frac{\pi}{2}n+\frac{\pi}{4})$$

$= \frac{1}{2} e^{j(\frac{\pi}{2}n+\frac{\pi}{4})} + \frac{1}{2} e^{-j(\frac{\pi}{2}n+\frac{\pi}{4})} - \frac{1}{2} e^{j(\frac{\pi}{2}n+\frac{\pi}{4})} + \frac{1}{2} e^{j(\frac{3\pi}{2}n-\frac{\pi}{4})}$

$H(e^{j0}) = H(e^{j\pi}) = 0 \qquad H(e^{j\frac{\pi}{2}}) = e^{2j\frac{\pi}{4}} \qquad H(e^{3j\frac{\pi}{2}}) = 2e^{-j\frac{\pi}{4}}$



۳_ خروجی ؟

الف) $x[n] = (-1)^n$

ب) $x[n] = 1 + \sin(\frac{3\pi}{8}n + \frac{\pi}{4})$

ج) $x[n] = \sum_{k=-\infty}^{+\infty} (\frac{1}{2})^{n-4k} u(n-4k)$

$x[n] = e^{j2(\frac{\pi}{2})n} \rightarrow N = 2$ ✓

$a_k \rightarrow 0 \leq k \leq 1 \qquad a_0 = 0 \qquad a_1 = 1$

مقدار dc سیگنال صفر است

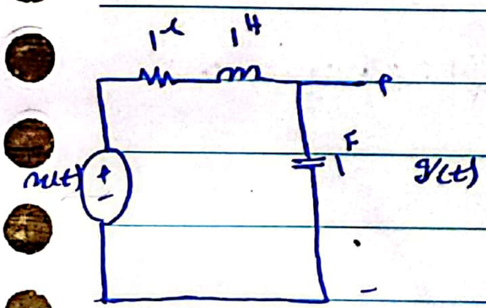
$y[n] = \sum_{k=0}^{1} a_k H(e^{j\frac{2k\pi}{2}}) e^{\frac{2k\pi}{2}} = 0 + a_1 H(e^{j\pi}) e^{j\pi} = 0$

$x[n] \longrightarrow N = 16 \Rightarrow e^{j(\frac{2\pi}{16})0(n)} - \frac{1}{2}je^{j\frac{\pi}{4}} e^{j\frac{\pi}{8}3n} + \frac{1}{2}je^{-j\frac{\pi}{4}} e^{-j\frac{\pi}{8}3n}$

$e^{j\frac{\pi}{8}n} - \frac{1}{2}je^{j\frac{\pi}{8}3n} + \frac{1}{2}je^{-j\frac{\pi}{4}} e^{j\frac{\pi}{8}3n} \qquad a_0 = 1$

$a_3 = -\frac{1}{2}je^{j\frac{\pi}{4}} \qquad a_{13} = \frac{1}{2}je^{-j\frac{\pi}{4}}$

$y[n] = \sum_{k=0}^{15} a_k H(e^{\frac{jk\pi}{16}}) e^{\frac{k\pi}{8}} = \sin \frac{3\pi}{8}n + \frac{\pi}{4}$



۴_ LTI علی به صورت RLC ورودی $x(t)$ خروجی $y(t)$

الف) معادله‌ی دیفرانسیل $x(t)$ و $y(t)$ را بنویسید.

ب) $H(j\omega)$ ؟ $x(t) = e^{j\omega t}$

ج) $y(t)$ ؟ $x(t) = \sin t$

$x(t) = v_R + v_L + v_C \rightarrow v_R = RC\frac{dy(t)}{dt}$

$I = I_C = C\frac{dv_C}{dt} = C\frac{dy(t)}{dt} \Rightarrow RC\frac{dy(t)}{dt} + LC\frac{d^2}{dt^2}y(t) + y(t) = x(t)$

$v_L = L\frac{dI_L}{dt} = LC\frac{d^2y(t)}{dt^2}$

$\frac{d^2}{dt^2}y(t) + \frac{d}{dt}y(t) + y(t) = x(t)$

$v_C = \frac{1}{C}\int I_C dt \rightarrow \frac{1}{C}\int C \frac{d}{dt}y(t) dt = y(t)$

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$e^{j\omega t} \to \boxed{LTI} \to Ae^{j\omega t}$ $\quad A = H(j\omega)$ $\quad$ گین

$x(t) = e^{j\omega t} = \frac{d^2}{dt^2}y(t) + \frac{d}{dt}y(t) + y(t) \to H(j\omega) = \frac{1}{-\omega^2 + j\omega + 1}$

ج)

$x(t) = x(t+T) \to T = 2\pi$

$x(t) = \frac{1}{2j}e^{j(\frac{2\pi}{2\pi})t} - \frac{1}{2j}e^{-j(\frac{2\pi}{2\pi})t} \Rightarrow a_1 = a_{-1}^* = \frac{1}{2j}$

$y(t) = a_1 H(j)e^{jt} - a_{-1}H(-j)e^{-jt} = \frac{1}{j}e^{jt} - \frac{1}{-j}e^{-jt} = (-\frac{1}{2})(e^{jt} + e^{-jt})$

$= -\cos(t)$

$x[n] \to \boxed{LTI} \to y[n]$ $\quad a_k \quad y[n] = ?$ $\quad$ ۵-

$y[n] - \frac{1}{4}y[n-1] = x[n]$ $\quad$ الف) $x[n] = \sin(\frac{3\pi}{4}n)$

ب) $x[n] = \cos(\frac{\pi}{4}n) + 2\cos(\frac{\pi}{2}n)$

$e^{j\omega n} \to \boxed{LTI} \to H(e^{j\omega})e^{j\omega n}$

$H(e^{j\omega})e^{j\omega n} - \frac{1}{4}e^{-j\omega}e^{j\omega n}H(e^{j\omega}) = e^{j\omega n} \to H(j\omega) = \frac{1}{1 - \frac{1}{4}e^{-j\omega}}$

$y[n] = \sum_{k=\langle N\rangle} a_k H\{e^{j\frac{2k\pi}{N}}\}e^{jk(\frac{2\pi}{N})n}$ $\quad \omega_0 = \frac{2\pi}{N}$

$a_k \Rightarrow a_k H(e^{j\frac{2k\pi}{N}})$

الف) $\to N = 4$ $\quad a_3 = a_{-3}^* = \frac{1}{2j}$ $\quad b_3 = a_1 H(e^{\frac{3j\pi}{4}}) = \frac{1}{2j} \times \frac{1}{(1 - \frac{1}{4}e^{-\frac{3j\pi}{4}})^*}$

$b_{-3} = a_{-1}H(e^{\frac{3j\pi}{4}}) = \frac{1}{2j} \times \frac{1}{(1 - \frac{1}{4}e^{\frac{3j\pi}{4}})^*}$

ب) $N = 8$ $\quad a_1 = a_{-1}^* = \frac{1}{2}$ $\quad a_2 = a_{-2}^* = 1$

$b_1 = a_1 H(e^{\frac{j\pi}{4}}) = \frac{1}{2} \times \frac{1}{(1 - \frac{1}{4}e^{-\frac{j\pi}{4}})^*}$ $\quad b_{-1} = a_{-1}H(e^{-\frac{j\pi}{4}}) = \frac{1}{2} \times \frac{1}{(1 - \frac{1}{4}e^{\frac{j\pi}{4}})^*}$

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$b_2 = a_2 H(e^{j\frac{\pi}{2}}) = \frac{1}{(1-\frac{1}{4}e^{-j\frac{\pi}{2}})^*}$ $\quad$ $b_{-2} = a_{-2} H(e^{-j\frac{\pi}{2}}) = \frac{1}{(1-\frac{1}{4}e^{+j\frac{\pi}{2}})^*}$

$h[n] = (\frac{1}{2})^{|n|}$ $\quad$ $y[n] = ?$ -1

$x[n] = \sum_{k=-\infty}^{+\infty} \delta[n-4k]$ $\quad$ $H(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} - \frac{1}{1-2e^{-j\omega}}$

$a_k \to x[n] = \frac{1}{4}$ $\quad$ $N=4$ $\quad$ $a_k \to y[n] =$

$b_k = a_k H(e^{j\frac{2k\pi}{N}}) = \frac{1}{4}\left[\frac{1}{1-\frac{1}{2}e^{-j\frac{k\pi}{2}}} - \frac{1}{1-2e^{-j\frac{k\pi}{2}}}\right]$