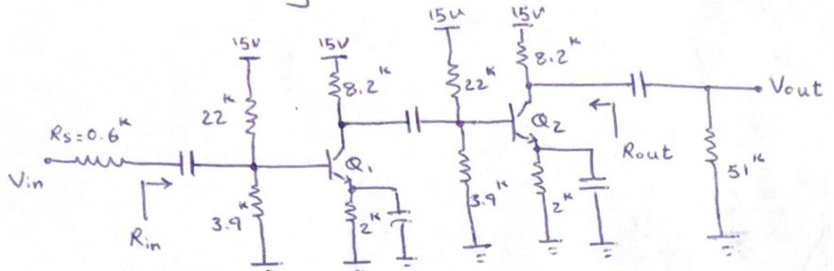
In the name of God

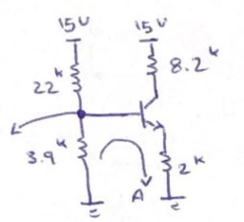
Assignment 4:

1. Calculate the voltage gain, input resistance and output resistance of the following circuit.



$\beta = 100$

DC analysis:



$V_A = \dfrac{3.9^k \times 15}{3.9 + 22}$

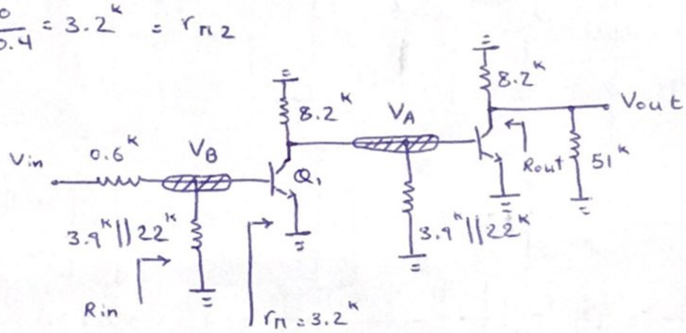
$= 2.25\,V$

KVL @ A : $-2.25 + (22^k \| 3.9^k)\dfrac{I_C}{100} + 0.7 + 2^k I_C = 0$

$I_c\left(\dfrac{3.3^k}{100} + 2^k\right) = 2.25 - 0.7 \Rightarrow I_{C_1} = 0.76^{mA} = I_{C_2}$

$\begin{cases} g_{m_1} = 40 I_{C_1} = 30.4^{mmho} = g_{m2} \\ r_{\pi_1} = \dfrac{\beta}{g_{m_1}} = \dfrac{100}{30.4} = 3.2^k = r_{\pi 2} \\ r_{o_1} = \infty = r_{o2} \end{cases}$

ac Analysis:



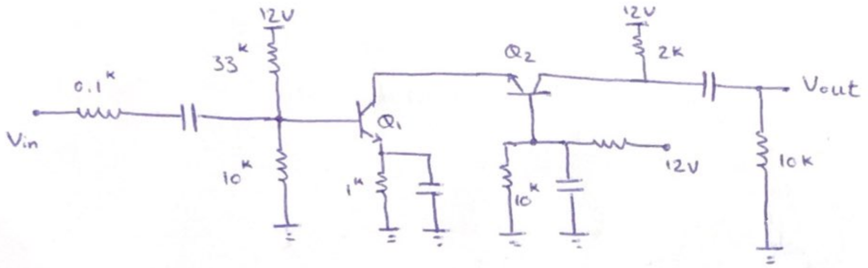
$A_v = \dfrac{V_{out}}{V_{in}} = \dfrac{V_{out}}{V_A} \times \dfrac{V_A}{V_B} \times \dfrac{V_B}{V_{in}} = \left[g_{m_2}\left(R_{C_2} \| r_{o_2}\right)\right] \cdot \left[-g_{m_1}\left(R_{C_1} \| r_{o_1}\right)\right] \cdot \left[\dfrac{3.9^k \| 22^k \| r_{\pi_1}}{0.6^k + 3.9^k \| 22^k \| r_{\pi_1}}\right]$

$= \left[-30.4\left(8.2^k \| 51^k \| \infty\right)\right] \times \left[-30.4\left(8.2^k \| 3.9^k \| 22^k \| 3.2^k \| \infty\right)\right] \times \left[\dfrac{3.9^k \| 22^k \| 3.2^k}{0.6^k + 3.9^k \| 22^k \| 3.2^k}\right]$

$\simeq 11007$

$R_{in} = 3.9^k \| 22^k \| r_\pi \simeq \boxed{1.62^k}$, $R_{out} = 8.2^k \| \underbrace{r_o}_{=\infty} = \boxed{8.2^k}$

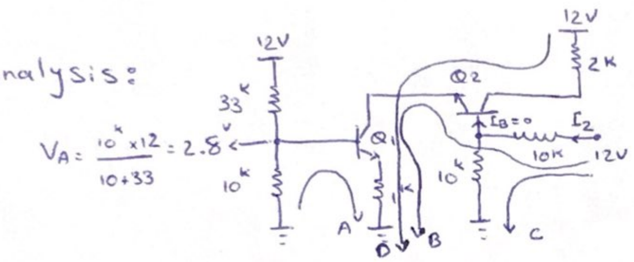
2. In the Circuit shown below, determine the Voltage gain.



$$\begin{cases} Q_1 : \beta_1 = 100 \\ Q_2 : \alpha_2 = 1 \longrightarrow \beta_2 = \infty \end{cases}$$

hint : $\begin{cases} \alpha = \dfrac{\beta}{\beta+1} \\ \beta = \dfrac{\alpha}{1-\alpha} \end{cases}$

dc analysis:



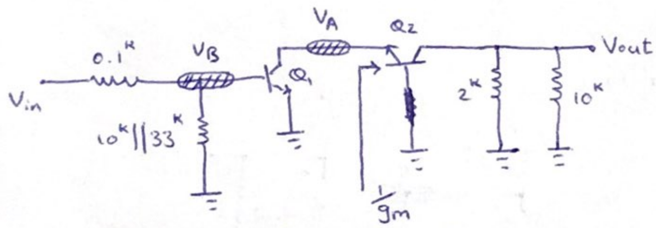
KVL @ A : $-2.8^V + (10^K \| 33^K) \cdot \frac{I_{C_1}}{100} + 0.7 + 1^K I_{C_1} = 0 \Rightarrow \boxed{I_{C_1} = 1.96^{mA}} = I_{C_2} \longrightarrow \begin{cases} g_m = 78.4^{mmho} \\ r_\pi = 1.27^K \\ r_o = \infty \end{cases}$

KVL @ B : $-12 + 10^K (I_2) + 0.7 + V_{CE_1} + 1^K I_{C_1} = 0 \xrightarrow{I_{C_1} = 1.96^{mA}} \boxed{V_{CE_1} = 3.34^V} > V_{CE,sat}$

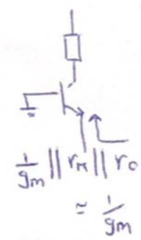
KVL @ C : $-12 + 10^K I_2 + 10^K I_2 = 0 \Rightarrow \boxed{I_2 = 0.6^{mA}}$

KVL @ D : $-12 + 2^K I_C + V_{CE_2} + V_{CE_1} + 1^K I_C = 0 \xrightarrow[V_{CE_1} = 3.34]{I_C = 1.96^{mA}} V_{CE_2} = 2.78 > V_{CE,sat}$

ac analysis:



hint:



$$A_v = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_A} \times \frac{V_A}{V_B} \times \frac{V_B}{V_{in}} = \left[+g_m (R_c \| r_o)\right] \times \left[-g_m (R_c \| r_o)\right] \times \left[\frac{10^K \| 33^K}{(10^K \| 33^K) + 0.1^K}\right]$$

$$= \left[78.4 (2^K \| 10^K \| \infty)\right] \times \left[-78.4 \left(\frac{1}{78.4}\right)\right] \times \left[\frac{10^K \| 33^K}{(10^K \| 33^K) + 0.1^K}\right] = 130 \times (-1) \times 0.9 = \boxed{-117}$$

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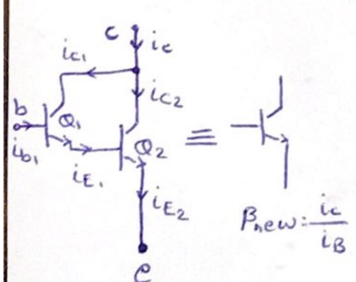
3. The following structure is known to ~~the~~ be the "Darlington pair" Configuration and is used in order to increase the β of single transistor. This Configuration can be modeled as a single NPN transistor. Determine it's equivalent r_π, g_m and β.



× $i_{C_1} = \beta_1 i_{B_1}$

× $i_{E_1} = (\beta_1 + 1) i_{B_1}$

× $i_{C_2} = \beta_2 i_{E_1} = \beta_2 i_{B_2}$

× $i_{B_2} = i_{E_1}$

× $i_{C_2} = \beta_2 (\beta_1 + 1) i_{B_1}$

× $i_C = i_{C_1} + i_{C_2} = \beta_1 i_{B_1} + \beta_2 (\beta_1 + 1) i_{B_1} = i_{B_1} (\beta_1 + \beta_2 (\beta_1 + 1))$

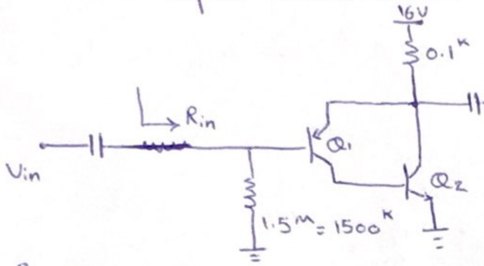
$\Rightarrow \beta_{new} = \frac{i_C}{i_{B_1}} = \beta_1 + \beta_2 (\beta_1 + 1)$

$= \beta_1 + \beta_2 + \beta_1 \beta_2$

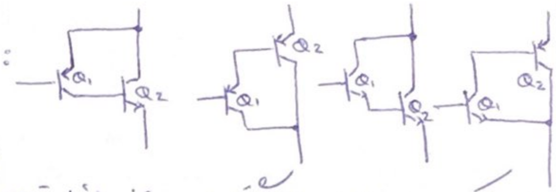
$g_{m,new} = 40 I_C$

$r_{\pi,new} = \frac{\beta_{new}}{g_{m,new}} = \frac{\beta_1 + \beta_2 + \beta_1 \beta_2}{40 I_C}$

4. The following Configuration is called the "Sziklai Pair" or the complementary Darlington pair. This Configuration Can be used to Construct a PnP transistor with a larg β. For the following Circuit, determine the Voltag gain and input resistance.



hint :

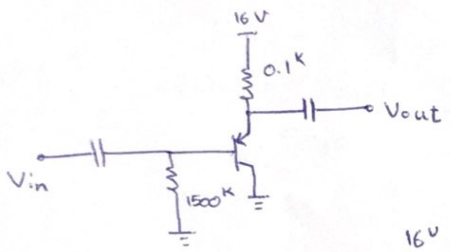


تمامی ترکیب های بالا زوج دارلینگتون هستند. اثبات می شود

β معادل در ترکیب های بالا به صورت زیر است : $\beta_{new} \simeq \beta_1 \beta_2$

$\beta_1 = 160$

$\beta_2 = 200$

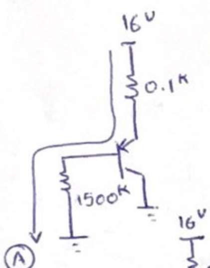


$\beta_{new} \simeq \beta_1 \beta_2 = 160 \times 200 = 32000$

$g_m = 40\, I_c$

$r_\pi = \dfrac{\beta_{new}}{g_m}$

dc analysis :

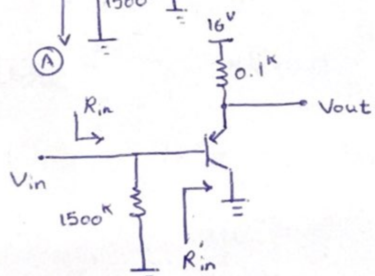


$KVL @ A : -16^V + 0.1^k I_c + 0.7 + 1500^k \times \dfrac{I_c}{32000} = 0$

$\Rightarrow I_c = 109^{mA} \longrightarrow g_m = 40(109) = 4360$ mmho

$r_\pi = \dfrac{32000}{4360} = 7.3^k$

ac analysis :



$A_v = \dfrac{V_{out}}{V_{in}} = \dfrac{R_E}{R_E + \frac{1}{g_m}} = \dfrac{0.1^k}{0.1^k + \frac{1}{4360}} \simeq \boxed{1}$

$R_{in} = R'_{in} \parallel 1500^k = 3207.3^k \parallel 1500^k \simeq \underline{1022^k}$

$R'_{in} = r_\pi + (\beta+1) R_E = 7.3^k + (32000) 0.1^k$
$= 3207.3^{k\Omega}$

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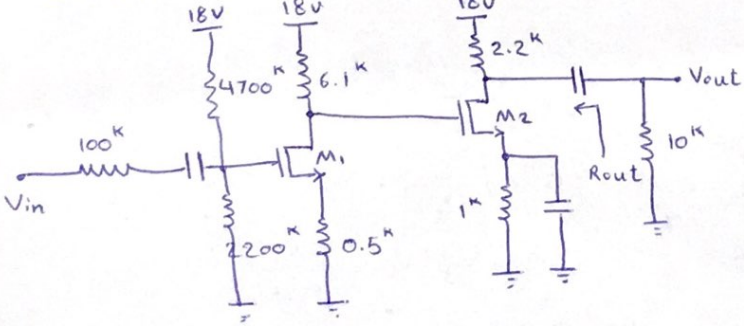
5. Determine the Voltage gain and the output resistance of the following Circuit.

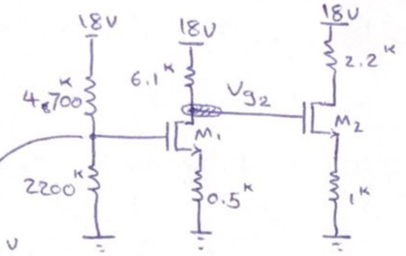


$\beta_1 = 0.5 \ \frac{mA}{V^2}$, $\beta_2 = 0.3 \ \frac{mA}{V^2}$

$V_{th_1} = 2^V$, $V_{th_2} = 2.6^V$

$r_{ds_1} = 75^k$, $r_{ds_2} = 100^k$

dc analysis:



$$V_{g_1} = \frac{2200 \times 18}{4700+2200} = 5.7^V$$

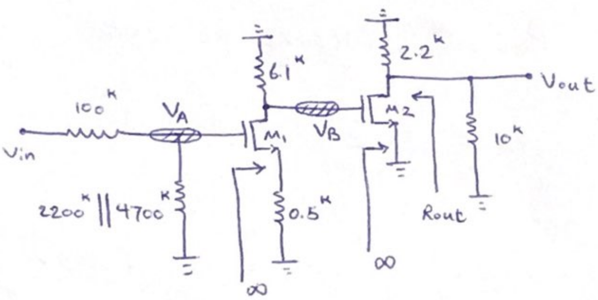
$$I_{D_1} = \frac{\beta_1}{2}\left(V_{gs_1} - V_{th_1}\right)^2 = \frac{0.5}{2}\left(5.7 - 0.5^k I_D - 2\right)^2$$

$$\begin{cases} I_{D_1} = 28.9^{mA} \\ I_{D_1} = 1.8^{mA} \checkmark \longrightarrow g_{m_1} = \sqrt{2\beta I_D} = 1.3^{mmho} \end{cases}$$

$$18 - V_{g2} = 6.1^k I_{D_1} \xrightarrow{I_{D_1}=1.8^{mA}} \boxed{V_{g2} = 7.02^V}$$

$$I_{D2} = \frac{\beta_2}{2}\left(V_{gs2} - V_{th2}\right)^2 = \frac{0.3}{2}\left(7.02 - 1^k I_D - 2.6^V\right)^2 = \begin{cases} I_{D2} = 14.12^{mA} \\ I_{D2} = 1.38^{mA} \checkmark \longrightarrow g_{m2} = 0.9^{mmho} \end{cases}$$

ac Analysis:



$$R_{out} = 2.2^k || r_{ds2} = 2.2^k || 100^k \simeq 2.2^k$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_B} \times \frac{V_B}{V_A} \times \frac{V_A}{V_{in}} = \left[-g_{m_2}(R_{D_2} || r_{ds_2})\right] \times \left[\frac{-R_{D_1}}{\frac{1}{g_{m_1}} + R_{S_1}}\right] \times \left[\frac{2200^k || 4700^k}{(2200^k || 4700^k) + 100^k}\right]$$

$$= \left[-0.9(2.2^k || 10^k || 100^k)\right] \times \left[\frac{6.1^k}{\frac{1}{1.3} + 0.5^k}\right] \times \left[\frac{2200^k || 4700^k}{(2200^k || 4700^k) + 100^k}\right] \simeq \boxed{-7.18}$$

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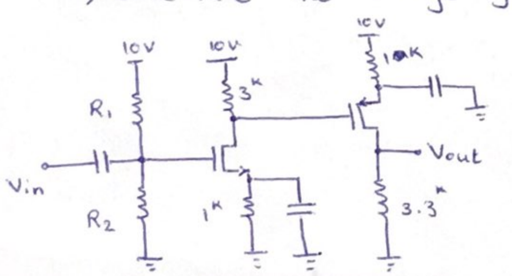
6. a) Specify R_1 and R_2 so that the bias current of M_1 will be equal to 1^{mA}.

b) Calculate the Voltage gain and the output resistance.

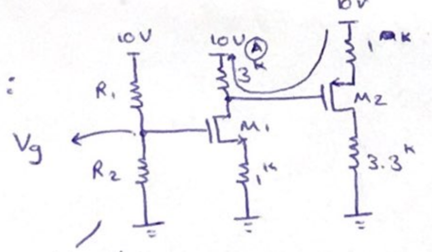


$\beta = 0.25 \ \frac{mA}{V^2}$

$|V_{th}| = 0.5^V$

$\lambda = 0 \longrightarrow r_{ds} = \infty$

dc analysis:



$$V_g = \frac{R_2 \times 10}{R_1 + R_2}$$

$$V_{gs_1} = \sqrt{\frac{I_{D_1}}{\frac{K'}{2}\frac{W}{L}}} + V_{th_1} = \sqrt{\frac{1}{\frac{1}{8}}} + 0.5 = \boxed{3.3}$$

$$V_{gs_1} = V_g - V_s = \frac{10R_2}{R_1+R_2} - 1^k(1^m) = 3.3^V$$

$$\frac{10R_2}{R_1+R_2} = 4.3^V \xrightarrow[R_2=1000^k]{\text{فرض می کنیم}} \frac{10(1000)}{R_1+1000} = 4.3 \Rightarrow \boxed{R_1 = 1325.5^k}$$

b) KVL @ A: $-10 + 1^K I_{D_2} + V_{sg_2} - 3^K(1^m) + 10 = 0 \Rightarrow V_{sg_2} = 3 - 1^K I_{D_2}$

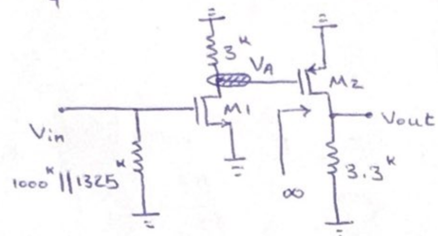
$I_{D_2} = \frac{\beta}{2}\left(V_{sg} - |V_{th}|\right)^2 = \frac{0.25}{2}\left(3 - 1^K I_{D_2} - 0.5\right)^2 = \begin{cases} I_{D_2} = 0.5^{mA} \checkmark \\ I_{D_2} = 12.5^{mA} \end{cases}$

$g_{m_2} = \sqrt{2\beta I_{D_2}} = \sqrt{2 \times \frac{1}{4} \times \frac{1}{2}} = 0.5^{mmho}$

$g_{m_1} = \sqrt{2\beta I_{D_1}} = \sqrt{2 \times \frac{1}{4} \times 1} = 0.7^{mmho}$

$R_{out} = 3.3^K \| r_{ds} = \boxed{3.3^K}$

ac analysis:



$A_v = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_A} \times \frac{V_A}{V_{in}} = \left[-g_{m_2}(R_o \| r_{ds})\right] \cdot \left[-g_{m_1}(R_D \| r_{ds})\right] = \left[-0.5(3.3^K \| \infty)\right] \times \left[-0.7(3^K \| \infty)\right]$

$= \boxed{3.46}$

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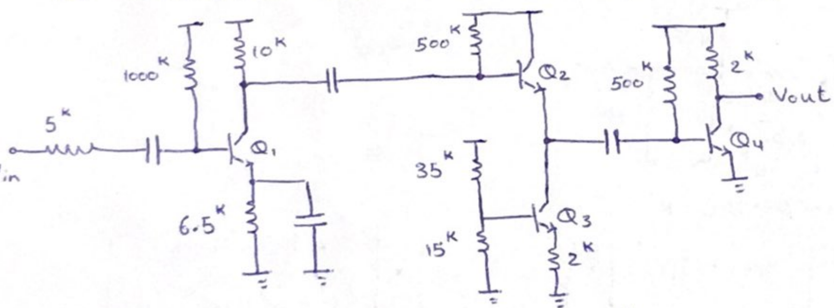
7. Calculate the Voltage gain of the structure depicted in the following figure. The transistors are in the saturation regin.



$V_A = 100^v$

$V_T = 25^{mv}$

$\beta = 100$

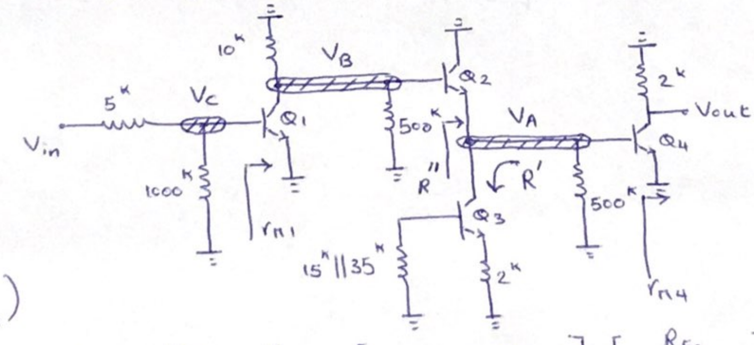
$I_{c_1} = I_{c_4} = 1^{mA}$

$I_{c_2} = 0.5^{mA}$

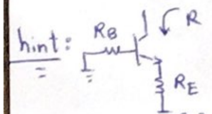
$I_{c_1} = I_{c_4} = 1^{mA} \longrightarrow g_{m_1} = g_{m_4} = 40^{mmho} \longrightarrow r_{\pi_1} = r_{\pi_4} = \frac{100}{40} = 2.5^K \longrightarrow r_{o_1} = r_{o_4} = \frac{V_A}{I_c} = 100^K$

$I_{c_2} = I_{c_3} = 0.5^{mA} \longrightarrow g_{m_2} = g_{m_3} = 20^{mmho} \longrightarrow r_{\pi_2} = r_{\pi_3} = \frac{100}{20} = 5^K \longrightarrow r_{o_2} = r_{o_3} = 200^K$

ac analysis:



hint:



$R = r_o\left(1 + \frac{\beta R_E}{R_E + r_\pi + R_B}\right)$

$R' = 200^K\left(1 + \frac{100(2^K)}{2^K + 5^K + 15^K \| 35^K}\right)$

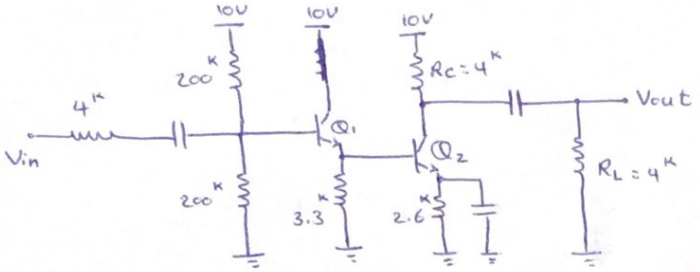
$R'' = r_{\pi_2} + (\beta + 1) R_E$
$= 5^K + 101\,(R' \| 500^K \| 2.5^K)$

$A_v = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_A} \times \frac{V_A}{V_B} \times \frac{V_B}{V_C} \times \frac{V_C}{V_{in}} = \left[-g_{m_4}(R_{C_4} \| r_{o_4})\right] \times \left[\frac{R_{E_2}}{R_{E_2} + \frac{1}{g_{m_2}}}\right] \times \left[-g_{m_1}(R_{C_1} \| r_{o_1})\right]$

$\times \left[\frac{(r_{\pi_1} \| 1000^K)}{(r_{\pi_1} \| 1000^K) + 5^K}\right] = \left[-40(2^K \| 100^K)\right] \times \left[\frac{500^K \| 2.5^K \| R'}{(500^K \| 2.5^K \| R') + \frac{1}{20}}\right] \times \left[-40(10^K \| 500^K \| R'' \| 100^K)\right]$

$\times \left[\frac{100^K \| 1000^K}{(100^K \| 1000^K) + 5^K}\right] \simeq \boxed{9390}$

8. Calculate the Voltage gain in the following Circuit:



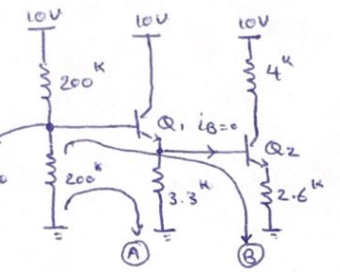
$\beta_1 = \beta_2 = 100$

$V_{ce,sat} = 0.2$

$V_{be,on} = 0.7$

dc analysis:

$V_A = \frac{200(10)}{200+200} = 5^V$



KVL @ A: $-5^V + (200^K \| 200^K)\frac{I_{C1}}{100} + 0.7 + 3.3^K I_{C1} = 0$

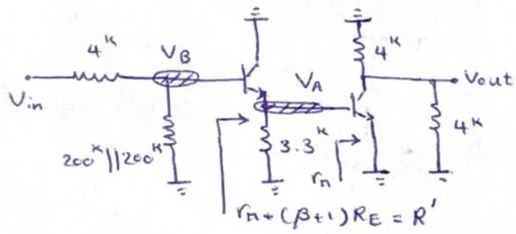
$\Rightarrow I_{C1} = 1^{mA}$

KVL @ B: $-5^V + (200^K \| 200^K)\frac{I_{C1}}{100} + 0.7 + 0.7 + 2.6^K I_{C2} = 0$

$\Rightarrow I_{C2} = 1^{mA}$

$I_{C1} = I_{C2} = 1^{mA} \longrightarrow \begin{cases} g_{m1} = g_{m2} = 40^{mmho} \\ r_\pi = 2.5^K \\ r_o = \infty \end{cases}$

ac analysis:



$$A_v = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_A} \times \frac{V_A}{V_B} \times \frac{V_B}{V_{in}} = \left[-g_m (R_C \| r_o)\right] \times \left[\frac{R_E}{R_E + \frac{1}{g_m}}\right] \times \left[\frac{R' \| 200^K \| 200^K}{R' \| 200^K \| 200^K + 4^K}\right]$$

$$= \left[-40(4^K \| 4^K \| \infty)\right] \times \left[\frac{3.3^K \| 2.5^K}{3.3^K \| 2.5^K + \frac{1}{40}}\right] \times \left[\frac{(2.5 + 101(3.3^K)) \| 100^K}{(2.5+101(3.3^K)) \| 100^K + 4^K}\right] \cong \boxed{-78.5}$$