یا لطیف

تمرین سری 3 سیگنال سیستم :

#1

$$x[n] = \left(\tfrac{1}{2}\right)^{n-2} u[n-2]$$

$$h[n] = u[n+2]$$

$$y[n] = x[n] * h[n] = ?$$

hint :
$$\begin{cases} y[n] = x[n]*h[n] = \sum_{k=-\infty}^{+\infty} h[k]\,x[n-k] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k] \\ y(t) = x(t)*h(t) = \int_{-\infty}^{+\infty} h(\tau)\,x(t-\tau)\,d\tau = \int_{-\infty}^{+\infty} x(\tau)\,h(t-\tau)\,d\tau \end{cases}$$

فرض می کنیم :
$$\begin{cases} x_1[n] = \left(\tfrac{1}{2}\right)^{n} u[n] \\ h_1[n] = u[n] \end{cases} \xrightarrow{\text{می دانیم}} \begin{cases} x[n] = x_1[n-2] \\ h[n] = h_1[n+2] \end{cases}$$

$$\Rightarrow y[n] = x[n]*h[n] = x_1[n-2]*h_1[n+2] = \sum_{k=-\infty}^{+\infty} x_1[k-2]\cdot h_1[n-k+2]$$

$$y[n] = \sum_{k=-\infty}^{+\infty} \underbrace{\left(\tfrac{1}{2}\right)^{k} u[k]}_{x_1[n]} \cdot \underbrace{u[n-k]}_{h_1[n]} = \sum_{k=0}^{n} \left(\tfrac{1}{2}\right)^{n} = \frac{\left(\tfrac{1}{2}\right)^{0} - \left(\tfrac{1}{2}\right)^{n+1}}{1-\tfrac{1}{2}} = \frac{1-\left(\tfrac{1}{2}\right)^{n+1}}{\tfrac{1}{2}}$$

$$= 2\left(1-\left(\tfrac{1}{2}\right)^{n+1}\right) \longrightarrow \sum_{k=-\infty}^{+\infty} x_1[k-2]\,h_1[n-k+2] = \sum_{k=-\infty}^{+\infty} \left(\tfrac{1}{2}\right)^{k-2} u[k-2]\,u[n-k+2]$$

$$= \sum_{k=2}^{n+2} \left(\tfrac{1}{2}\right)^{k-2} = 2\left(\tfrac{1}{4}-\left(\tfrac{1}{2}\right)^{n+3}\right)$$

hint :
$$\sum_{n=n_1}^{n_2} a^n = \frac{a^{n_1} - a^{n_2+1}}{1-a}$$

#2

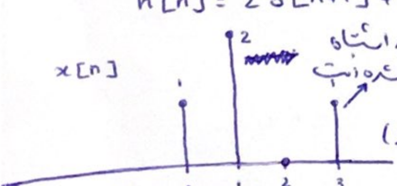
$$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$$

$$h[n] = 2\delta[n+1] + 2\delta[n-1]$$

a) $x[n] * h[n] = ?$

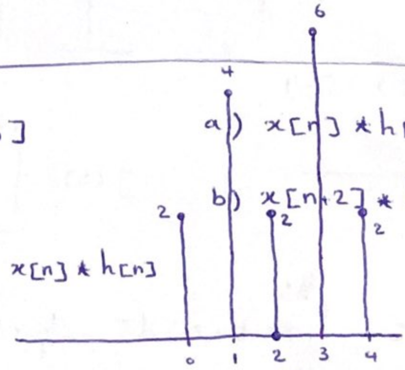
b) $x[n+2] * h[n] = ?$

$x[n]$

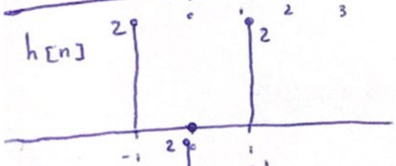


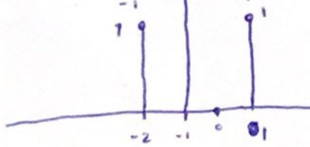
دامنه این نمونه به اشتباه 1+ در نظر گرفته شده است باید 1- باشد (اصلاح شود) $\Longrightarrow$

$x[n] * h[n]$



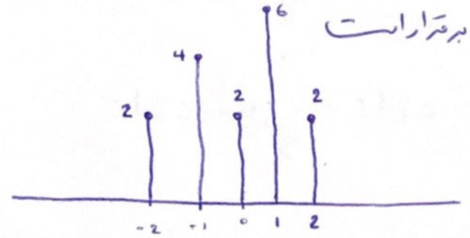
$h[n]$





$\Rightarrow$

خاصیت جابه جایی برقرار است



#3 $\begin{cases} h(t) = e^{2t} u(1-t) \\ \text{سیستم LTI} \\ x(t) = u(t) - 2u(t-2) + u(t-5) \end{cases}$ $\quad y(t) = ?$

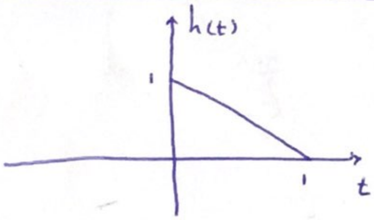
$$y(t) = \int_{-\infty}^{+\infty} x(\tau) h(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} x(t-\tau) h(\tau)\, d\tau$$

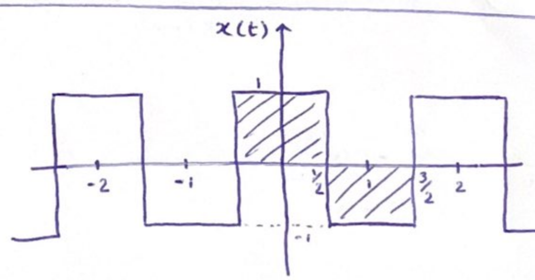
$$= \int_{-\infty}^{+\infty} \big(u(\tau) - 2u(\tau-2) + u(\tau-5)\big) \cdot e^{2(t-\tau)} u(1-(t-\tau))\, d\tau$$

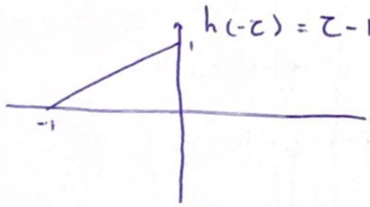
$$= \int_0^2 h(t-\tau)\, d\tau - \int_2^5 h(t-\tau)\, d\tau = \begin{cases} \int_0^2 e^{2(t-\tau)} d\tau - \int_2^5 e^{2(t-\tau)} d\tau & ; \quad t \le 1 \\ \int_{t-1}^2 e^{2(t-\tau)} d\tau - \int_2^5 e^{2(t-\tau)} d\tau & ; \quad 1 \le t \le 3 \\ -\int_{t-1}^5 e^{2(t-\tau)} d\tau & ; \quad 3 \le t \le 6 \\ 0 & ; \quad 6 < t \end{cases}$$

$$= \begin{cases} 0.5\left(e^{2t} - 2e^{2(t-2)} + e^{2(t-5)}\right) & ; \quad t \le 1 \\ 0.5\left(e^{2} + e^{2(t-5)} - 2e^{2(t-2)}\right) & ; \quad 1 \le t \le 3 \\ 0.5\left(e^{2(t-5)} - e^{2}\right) & ; \quad 3 \le t \le 6 \\ 0 & ; \quad 6 < t \end{cases}$$

#4







$$y(t) = \int_{-\infty}^{+\infty} x(\tau) h(t-\tau)\, d\tau$$

$$= \begin{cases} \int_{t-1}^{-1/2} (t-\tau-1)\, d\tau + \int_{-1/2}^{t} (1-t+\tau)\, d\tau = \frac{1}{4} + t - t^2 & ; \quad -\frac{1}{2} \le t \le \frac{1}{2} \\ \int_{t-1}^{1/2} (1-t+\tau)\, d\tau + \int_{1/2}^{t} (t-1-\tau)\, d\tau = t^2 - 3t + \frac{7}{4} & ; \quad \frac{1}{2} \le t \le \frac{3}{2} \end{cases}$$