یا لطیف

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تمرین سری ۷ آمار و احتمال مهندسی

#1

| تعداد | شماره |
|---|---|
| 2 → $\frac{1}{3}$ | |
| 4 → $\frac{1}{3}$ | |
| 6 → $\frac{1}{3}$ | |

الف) $\mu_{(x)} = \sum x_i \cdot P_X(x) = 2\times\frac{1}{3} + 4\times\frac{1}{3} + 6\times\frac{1}{3} = \frac{11}{3}$

$\Rightarrow E(x) = 3.6$

$E(x^2) = \sum x_i^2 \cdot P_X(x) = 4\times\frac{1}{3} + 16\times\frac{1}{3} + 36\times\frac{1}{3} = \frac{56}{3}$

$\Rightarrow E(x^2) = 18.6$

$\Rightarrow Var(x) = E(x^2) - E(x)^2 = 18.6 - (3.6)^2 = \underline{5.64}$

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#2

$\begin{cases} X \sim N \\ \mu = 9000 \\ \sigma = 2500 \\ n = 225 \end{cases}$

$\bar{x} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$

$P(\bar{x} > 9100) = 1 - P(\bar{x} \le 9100)$

$P\left(\frac{\bar{x} - 9000}{\frac{2500}{\sqrt{225}}} \le \frac{9100 - 900}{\frac{2500}{\sqrt{225}}}\right) = \Phi(0.43) = 0.6$

$\Rightarrow P(\bar{X} > 9100) = \underline{0.33}$

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#3

$$\sigma^2 \in \left(\frac{(n-1)S_{n-1}^2}{\chi^2_{(1-\frac{\alpha}{2}, n-1)}}, \frac{(n-1)S_{n-1}^2}{\chi^2_{(\frac{\alpha}{2}, n-1)}}\right)$$

$$\sigma^2 \in \left(\frac{19 S_{n-1}^2}{\chi^2}, \frac{19 S_{n-1}^2}{\chi}\right) \Rightarrow \sigma^2 \in (6.1, 14.6)$$

$S_{n-1}^2 = \frac{19}{3}$

$$\begin{cases} \frac{19(\frac{19}{3})}{\chi^2_{(1-\frac{\alpha}{2}, 19)}} = 6.1 \Longrightarrow \chi^2_{1-\frac{\alpha}{2}} = 19.726 \\ \frac{19(\frac{19}{3})}{\chi^2_{(\frac{\alpha}{2}, 19)}} = 14.6 \xRightarrow{\frac{\alpha}{2} = 0.5} \chi^2 = 8.242 \end{cases}$$

با احتمال ۹۵٪ واریانس نمونه، در بازه (6.1, 14.6) است.

# #4

با توجه به جدول توزیع $t$ داریم : $df = 9$

الف) $P_r(X > a) = 0.05 = 1 - P_r(X \leq a) = 0.95$

$\Rightarrow a = 2.2622$

ب) $P_r(X < a) = 0.01 \Rightarrow a = 3.2498$

ج) $P_r(-a < X < a) = 0.95 \Rightarrow a = 1.8331$

د) $P_r(-a < X < a) = 0.99 \Rightarrow a = 2.8214$

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# #5

$$\begin{cases} n = 25 \\ X \sim N \\ \mu = 1410 \\ \sigma = 200 \end{cases}$$

$1 - \alpha = 0.95 \Rightarrow \alpha = 0.05$

$x_1, x_2, \ldots, x_{25} \sim N(\mu, \sigma^2) \Rightarrow \bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

$$\Rightarrow P_r\left(Z_{0.025} < \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} < Z_{0.025}\right) = 0.095$$

$$\Rightarrow \mu \in \left(\bar{x} \pm Z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}\right) \Rightarrow \mu \in \left(1410 \pm 1.96 \times \frac{200}{\sqrt{25}}\right)$$

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# #6

$x_1, x_2, x_3, \ldots, x_n \sim N(\mu, \sigma^2)$

I) $x_1, x_2, x_3, \ldots, x_8 \sim N(\mu, S^2)$

II) $g(x_1, x_2, \ldots, x_8) = \bar{X} \sim N(\mu, \frac{\sigma^2}{n}) \Rightarrow \bar{X} = 2.1$

III) $h(\bar{x}, \mu) = \dfrac{\bar{X} - \mu}{\frac{S_{n-1}}{\sqrt{n}}} \sim t_{df} = n - 1 = 7$

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