Subject: 9824063 ۱ کیا کرمانی

Year. Month. Date. ( )

if $k=15$, $k=14.9$ find $Ax=b$ $\Rightarrow \begin{bmatrix} 8 & 5 & 2 \\ 21 & 19 & 16 \\ 39 & 48 & 53 \end{bmatrix} x = \begin{bmatrix} k \\ 56 \\ 140 \end{bmatrix}$ -1

$|A| = 8\times 239 - 5\times 489 + 2\times 267 = 1$, $A^{-1} = \frac{1}{|A|} \times adj(A)$

$$adj(A) = \begin{bmatrix} \begin{vmatrix} 19 & 16 \\ 48 & 53 \end{vmatrix} & -\begin{vmatrix} 5 & 2 \\ 48 & 53 \end{vmatrix} & \begin{vmatrix} 5 & 2 \\ 19 & 16 \end{vmatrix} \\ -\begin{vmatrix} 21 & 16 \\ 89 & 53 \end{vmatrix} & \begin{vmatrix} 8 & 2 \\ 89 & 53 \end{vmatrix} & -\begin{vmatrix} 8 & 2 \\ 21 & 16 \end{vmatrix} \\ \begin{vmatrix} 21 & 19 \\ 89 & 48 \end{vmatrix} & -\begin{vmatrix} 8 & 5 \\ 89 & 48 \end{vmatrix} & \begin{vmatrix} 8 & 5 \\ 21 & 19 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 239 & -169 & 42 \\ -311 & 246 & -86 \\ 683 & -67 & 47 \end{bmatrix}$$

$\Rightarrow Ax = b \Rightarrow A^{-1}Ax = A^{-1}b \Rightarrow x = A^{-1}b$

if $k=15 \Rightarrow A^{-1}b = x = \begin{bmatrix} 1 \\ -2929 \\ 13409 \end{bmatrix}$

cond. $= \|A\| . \|A^{-1}\| \Rightarrow$

if $k=14.9 \Rightarrow A^{-1}b = x = \begin{bmatrix} -22.9 \\ -2897.9 \\ 13340.7 \end{bmatrix}$

$\|A\| = \sqrt{\lambda_{max}} \Rightarrow A^TA - \lambda I = \begin{bmatrix} 8 & 21 & 39 \\ 5 & 19 & 48 \\ 2 & 16 & 53 \end{bmatrix} \begin{bmatrix} 8 & 5 & 2 \\ 21 & 19 & 16 \\ 39 & 48 & 53 \end{bmatrix}$

$$-\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2026-\lambda & 2311 & 2419 \\ 2311 & 2690-\lambda & 2858 \\ 2419 & \cancel{2858}\ 2858 & 3069-\lambda \end{bmatrix} \Rightarrow |A^TA - \lambda I| = 0$$

$\Rightarrow \lambda =$

Rashen-s

Scanned with CamScanner

Subject: 9824063 ② کیا گرمابی
Year. Month. Date. ( )

1) $\forall A, B \in R \Rightarrow A+B \in R$ $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ -2

$A+B=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}+\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ $B=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

$=\begin{bmatrix} \underbrace{a_{11}+b_{11}}_{c_{11}} & \underbrace{a_{12}+b_{12}}_{c_{12}} \\ \underbrace{a_{21}+b_{21}}_{c_{21}} & \underbrace{a_{22}+b_{22}}_{c_{22}} \end{bmatrix}$ ✓

2) $\forall A \in R, \forall c \in F \Rightarrow cA \in R$

$cA = c\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} ca_{11} & ca_{12} \\ ca_{21} & ca_{22} \end{bmatrix} = cA$ ✓

3) $\forall A, B \in R \Rightarrow A+B = B+A$

$A+B=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}+\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}=\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix}=\begin{bmatrix} b_{11}+a_{11} & b_{12}+a_{12} \\ b_{21}+a_{21} & b_{22}+a_{22} \end{bmatrix}$

$=B+A$ ✓

4) $\forall A, B, C \in R \Rightarrow A+(B+C)=(A+B)+C$

$A+(B+C)=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}+\begin{bmatrix} b_{11}+c_{11} & b_{12}+c_{12} \\ b_{21}+c_{21} & b_{22}+c_{22} \end{bmatrix}=\begin{bmatrix} a_{11}+b_{11}+c_{11} & a_{12}+b_{12}+c_{12} \\ a_{21}+b_{21}+c_{21} & a_{22}+b_{22}+c_{22} \end{bmatrix}$

$=\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix}+\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}=(A+B)+C$ ✓

5) $\forall A \in R ; \exists 0 \in R \Rightarrow A+0=0+A$

Rashen-s

Subject: 9824063 (3) کیا کرمانی

Year. Month. Date. ( )

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a_{11}+0 & a_{12}+0 \\ a_{21}+0 & a_{22}+0 \end{bmatrix} = \begin{bmatrix} 0+a_{11} & 0+a_{12} \\ 0+a_{21} & 0+a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = 0 + A \checkmark$$

6) $\forall A \in R, \exists -A \in R \Rightarrow A + (-A) = (-A) + A = 0$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} -a_{11} & -a_{12} \\ -a_{21} & -a_{22} \end{bmatrix} = \begin{bmatrix} a_{11}+(-a_{11}) & a_{12}+(-a_{12}) \\ a_{21}+(-a_{21}) & a_{22}+(-a_{22}) \end{bmatrix}$$

$$= \begin{bmatrix} -a_{11}+a_{11} & -a_{12}+a_{12} \\ -a_{21}+a_{21} & -a_{22}+a_{22} \end{bmatrix} = \begin{bmatrix} -a_{11} & -a_{12} \\ -a_{21} & -a_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= (-A) + A = 0 \checkmark$$

7) $\forall A, B \in R, \forall a, b \in F \Rightarrow \begin{cases} (a+b)A = aA + bA \\ a(A+B) = aA + aB \end{cases}$

$$(a+b)\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11}a+a_{11}b & a_{12}a+a_{12}b \\ a_{21}a+a_{21}b & a_{22}a+a_{22}b \end{bmatrix} = \begin{bmatrix} aa_{11} & +aa_{12} \\ aa_{21} & aa_{22} \end{bmatrix}$$

$$+ \begin{bmatrix} ba_{11} & ba_{12} \\ ba_{21} & ba_{22} \end{bmatrix} = aA + bA \checkmark$$

$$a\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix} = \begin{bmatrix} aa_{11}+ab_{11} & aa_{12}+ab_{12} \\ aa_{21}+ab_{21} & aa_{22}+ab_{22} \end{bmatrix} = \begin{bmatrix} aa_{11} & aa_{12} \\ aa_{21} & aa_{22} \end{bmatrix}$$

Scanned with CamScanner

Subject: 9824063
Year. Month. Date. ( )

(4)

کیانا کرمانی

$$+\begin{bmatrix} ab_{11} & ab_{12} \\ ab_{21} & ab_{22} \end{bmatrix} = aA + aB \checkmark$$

$$8)\ \forall A \in R,\ \forall a,b \in F \Rightarrow a(bA) = (ab)A$$

$$a(bA) = a\begin{bmatrix} ba_{11} & ba_{12} \\ ba_{21} & ba_{22} \end{bmatrix} = \begin{bmatrix} aba_{11} & aba_{12} \\ aba_{21} & aba_{22} \end{bmatrix} =$$

$$ab\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = (ab)A \checkmark$$

$$9)\ \forall A \in R,\ \exists 1 \in F \Rightarrow 1A = A$$

$$1 \times \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1a_{11} & 1a_{12} \\ 1a_{21} & 1a_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = A \checkmark$$

-3

$$u = \begin{bmatrix} 1-\lambda \\ 2+\lambda \end{bmatrix},\quad v = \begin{bmatrix} 2+\lambda \\ 1-\lambda \end{bmatrix}$$

$$c_1 u + c_2 v = 0 \Rightarrow c_1\begin{bmatrix} 1-\lambda \\ 2+\lambda \end{bmatrix} + c_2\begin{bmatrix} 2+\lambda \\ 1-\lambda \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} c_1 - c_1\lambda + 2c_2 + c_2\lambda \\ 2c_1 + c_1\lambda + c_2 - c_2\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} c_1 - c_1\lambda + 2c_2 + c_2\lambda = 0 \overset{\times 2}{\Rightarrow} \\ 2c_1 + c_1\lambda + c_2 - c_2\lambda = 0 \end{cases}$$

Scanned with CamScanner

Subject: 9824063 ⑤ کیا گرمابی

Year. Month. Date. ( )

1) Span:

-4

$$c_1\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} + c_2\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix} + c_3\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix} + c_4\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}c_1 & 0\\ 0 & 0\end{bmatrix} + \begin{bmatrix}c_2 & c_2\\ 0 & 0\end{bmatrix} + \begin{bmatrix}c_3 & c_3\\ c_3 & 0\end{bmatrix} + \begin{bmatrix}c_4 & c_4\\ c_4 & c_4\end{bmatrix}$$

$$= \begin{bmatrix}r_{11} & r_{12}\\ r_{21} & r_{22}\end{bmatrix} \Rightarrow \begin{bmatrix}c_1+c_2+c_3+c_4 & c_2+c_3+c_4\\ c_3+c_4 & c_4\end{bmatrix} = r_{2\times 2}$$

$$\begin{bmatrix}1 & 0 & 1 & 1 & 1 & 1 & 1 & 1\\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1\end{bmatrix}_{1\times 8} \begin{bmatrix} \\ \end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 1 & 1\\ 0 & 0 & 0 & 0\\ 1 & 1 & 1 & 1\\ 1 & 0 & 1 & 1\end{bmatrix}_{4\times 4} \begin{bmatrix} \\ \end{bmatrix}$$

$A_{4\times 4}$

2) Linear Independent:

Rashen-s

Subject: 9824063

Year. Month. Date. ( )

6

کیا کرمانی

-5

$$A=\begin{bmatrix} 3 & 12 & -1 & -6 \\ 6 & 24 & -2 & -12 \\ -3 & -12 & 1 & 6 \end{bmatrix} \qquad R(A) \leqslant \min(3,4)=3$$

کهاد 1×1 ✓ کهاد 2×2 : $\begin{bmatrix} 3 & -12 \\ -1 & -6 \end{bmatrix}$ ✓ کهاد 3×3 : ✗

همهٔ کهادهای 3×3 برابر صفر خواهند شد. $\boxed{Rank=2}$

$$A=\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad R(A) \leqslant \min(3,4)=3$$

کهاد 1×1 ✓ کهاد 2×2 : $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ ✓ کهاد 3×3 : ✗

همهٔ کهادهای 3×3 برابر صفر خواهند شد. $\boxed{Rank=2}$

$$A=\begin{bmatrix} 1 & 1 & a \\ -a & -1 & 1 \end{bmatrix} \qquad R(A) \leqslant \min(2,3)=2$$

کهاد 1×1 ✓ کهاد 2×2 : $\begin{bmatrix} 1 & 1 \\ -a & -1 \end{bmatrix}$ ✓ $\boxed{Rank=2}$