یا لطیف

دانشگاه صنعتی شاهرود
دانشکده مهندسی برق و رباتیک

مدار های الکتریکی ۱

نیم سال اول ۰۰-۹۹

پاسخ تمرین سری پنجم — آپ امپ

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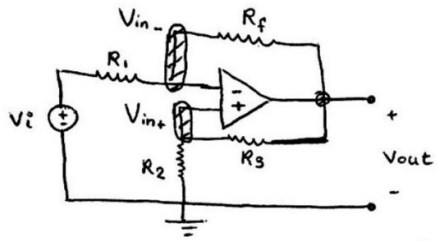

$$KCL\ in\ V_{in-}:\ \frac{V_{in-}-V_i}{R_1}+\frac{V_{in-}-V_{out}}{R_f}=0 \quad (II)$$

$$KCL\ in\ V_{in+}:\ \frac{V_{in+}-0}{R_2}+\frac{V_{in+}-V_{out}}{R_3}=0$$

$$V_{in-}=V_{in+} \rightarrow V_{in+}\left(\frac{1}{R_2}+\frac{1}{R_3}\right)=\frac{V_{out}}{R_3}$$

$$\Rightarrow V_{in+}=\frac{V_{out}}{R_3\left(\frac{1}{R_2}+\frac{1}{R_3}\right)}=\frac{V_{out}}{1+\frac{R_3}{R_2}} \quad (I)$$

$$(I)\ in\ (II) \rightarrow \frac{V_i}{R_1}=\left(\frac{1}{R_1}+\frac{1}{R_f}\right)\cdot V_{in+}-\frac{V_{out}}{R_f} \Rightarrow \frac{V_i}{R_1}=\left(\frac{1}{R_1}+\frac{1}{R_f}\right)\cdot\frac{V_{out}}{1+\frac{R_3}{R_2}}-\frac{V_{out}}{R_f}$$

$$\Rightarrow \frac{V_i}{V_{out}}=\frac{1+\frac{R_1}{R_f}}{1+\frac{R_3}{R_2}}-\frac{R_1}{R_f}=\frac{R_2R_f-R_1R_3}{R_f(R_2+R_3)} \Rightarrow \frac{V_{out}}{V_i}=\frac{R_f(R_2+R_3)}{R_2R_f-R_1R_3}$$

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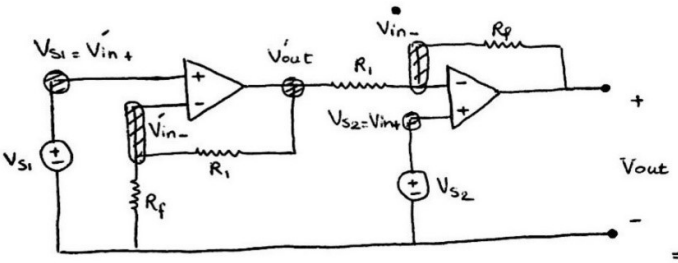

$$KCL\ in\ V'_{in-}:\ \frac{V'_{in-}-0}{R_f}+\frac{V'_{in-}-V'_{out}}{R_1}=0$$

$$\Rightarrow V'_{out}=\left(1+\frac{R_1}{R_f}\right)V_{s1} \quad (I)$$

$$KCL\ in\ V_{in-}:\ \frac{V_{in-}-V'_{out}}{R_1}+\frac{V_{in-}-V_{out}}{R_f}=0$$

$$\Rightarrow V_{out}=\left(1+\frac{R_f}{R_1}\right)V_{s2}-\frac{R_f}{R_1}V'_{out} \quad (II)$$

$$(I)\ in\ (II) \rightarrow V_{out}=\left(1+\frac{R_f}{R_1}\right)V_{s2}-\frac{R_f}{R_1}\left(1+\frac{R_1}{R_f}\right)V_{s1}$$

$$\Rightarrow V_{out}=\left(1+\frac{R_f}{R_1}\right)\cdot(V_{s2}-V_{s1})$$

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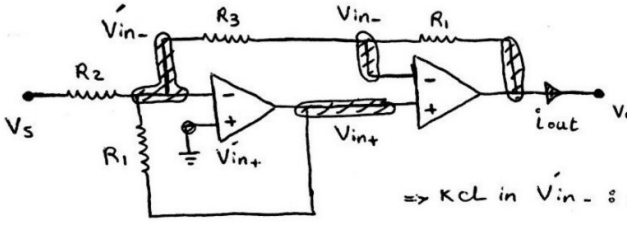

$V_{in-} = V_{in+} = 0 \Rightarrow V_{in-} = 0$

$V_{in+} = V_{in-}$

$\Rightarrow$ KCL in V_{in-} : $\frac{V_{in-} - V_s}{R_2} + \frac{V_{in-} - V_{in+}}{R_1} + \frac{V_{in-} - V_{in-}}{R_3} = 0$

$\Rightarrow V_{in-} = \frac{-R_1 R_3}{R_1 + R_3} \times \frac{1}{R_2} V_s$ (I)

KCL in V_{in-} : $\frac{V_{in-} - V_{in-}}{R_3} + \frac{V_{in-} - V_{out}}{R_1} = 0 \Rightarrow V_{out} = \left(1 + \frac{R_1}{R_3}\right) \cdot V_{in-}$ (II)

(I) in (II) $\rightarrow$ $V_{out} = \frac{R_1 + R_3}{R_3} \times \frac{R_1 R_3}{R_1 + R_3} \times \frac{V_s}{R_2} = \frac{R_1}{R_2} \cdot V_s$ **

** $\rightarrow$ KCL in V_{out} : $\frac{V_{out} - V_{in-}}{R_1} - i_{out} = 0 \Rightarrow i_{out} = \frac{V_{out} - V_{in-}}{R_1}$

$= \frac{\frac{R_1}{R_2} V_s + \frac{R_1 R_3}{R_1 + R_3} \times \frac{V_s}{R_2}}{R_1} = \frac{V_s}{R_2}\left(1 + \frac{R_3}{R_1 + R_3}\right) = i_{out}$

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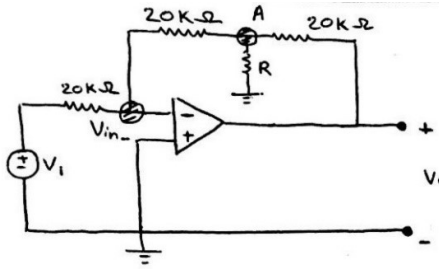

KCL in V_{in-} : $\frac{V_{in-} - V_1}{20} + \frac{V_{in-} - V_A}{20} = 0$

$V_{in-} = V_{in+} = 0 \rightarrow V_A = -V_1$ (I)

$V_{out} = -100\, V_{in}$

KCL in A : $\frac{V_A - V_{in-}}{20} + \frac{V_A - 0}{R} + \frac{V_A - V_{out}}{20}$ (II)

(I) in (II) $\Rightarrow V_{out} = -\left(2 + \frac{20}{R}\right) V_i$, $V_{out} = -100 V_i$

$\Rightarrow R = \frac{20\,K\Omega}{98} = 204\ K\Omega$

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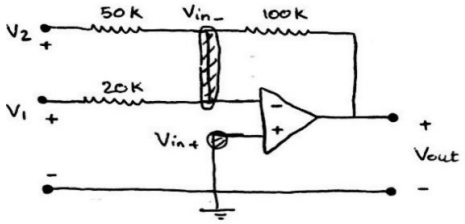

$\Rightarrow V_{in+} = V_{in-} = 0$

KCL in V_{in-} : $\frac{V_{in-} - V_2}{50} + \frac{V_{in-} - V_1}{20} + \frac{V_{in-} - V_{out}}{100} = 0$

$\Rightarrow V_{out} = -(2V_2 + 5V_1)$

if $V_1 = V_2 = 0$ $\rightarrow$ $V_{out} = V_{TH} = 0$ $\xrightarrow{V_{TH} = R_{TH} \times I}$ $R_{TH} = 0$

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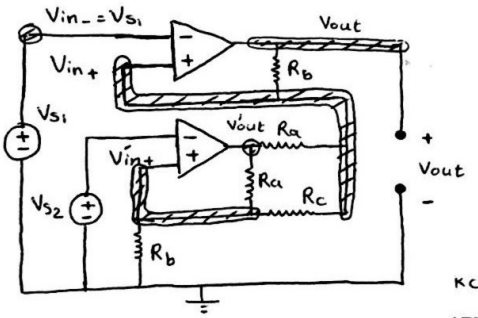

$$\begin{cases} V_{in-} = V_{in+} = V_{s1} \\ V'_{in-} = V'_{in+} = V_{s2} \end{cases}$$

KCL in V_{in+} : $\frac{V_{in+} - V_{out}}{R_b} + \frac{V_{in+} - V'_{out}}{R_a} + \frac{V_{in+} - V'_{in+}}{R_c} = 0$

$$\Rightarrow V_{s1}\left(\frac{1}{R_b} + \frac{1}{R_a} + \frac{1}{R_c}\right) - \frac{V_{s2}}{R_c} - \frac{V_c}{R_a} = \frac{V_{out}}{R_b} \quad (I)$$

KCL in V'_{in-} : $\frac{V'_{in-} - 0}{R_b} + \frac{V'_{in-} - V'_{out}}{R_a} + \frac{V'_{in-} - V_{in+}}{R_c} = 0$ (II)

$$\Rightarrow V_{s2}\left(\frac{1}{R_b} + \frac{1}{R_a} + \frac{1}{R_c}\right) - \frac{V_c}{R_a} - \frac{V_{s1}}{R_c} = 0 \xrightarrow{\text{in (I)}} \frac{V_{out}}{R_b} = V_{s1}\left(\frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_c}\right) - \frac{V_{s2}}{R_c} - V_{s2}\left(\frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_c}\right) + \frac{V_{s1}}{R_c}$$

$$\Rightarrow V_{out} = R_b\left(\left(\frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_c}\right)\cdot(V_{s1} - V_{s2}) + \frac{1}{R_c}(V_{s1} - V_{s2})\right)$$

$$\Rightarrow V_{out} = (V_{s1} - V_{s2})\cdot\left(\frac{R_b}{R_a} + \frac{2R_b}{R_c} + 1\right)$$

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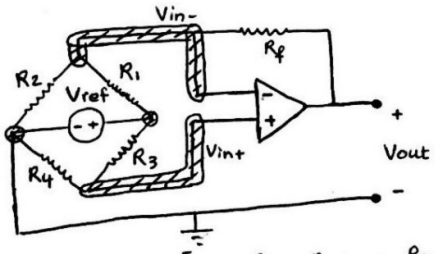

KCL in V_{in-} : $\frac{V_{in-} - 0}{R_2} + \frac{V_{in-} - V_{ref}}{R_1} + \frac{V_{in-} - V_{out}}{R_f} = 0$ (I)

KCL in V_{in+} : $\frac{V_{in+} - 0}{R_4} + \frac{V_{in+} - V_{ref}}{R_3} = 0 \Rightarrow V_{in+} = \left(\frac{R_4}{R_4 + R_3}\right)V_{ref}$

$\xrightarrow[\text{(II) in (I)}]{V_{in+} = V_{in-}}$ $\left(\frac{1}{R_f} + \frac{1}{R_2} + \frac{1}{R_1}\right)\cdot V_{in-} - \frac{V_{ref}}{R_1} = \frac{V_{out}}{R_f}$ (II)

$$\Rightarrow \left(\frac{1}{R_f} + \frac{1}{R_2} + \frac{1}{R_1}\right)\cdot\left(\frac{R_4}{R_4 + R_3}\right)\cdot V_{ref} - \frac{V_{ref}}{R_1} = \frac{V_{out}}{R_f}$$

$$\Rightarrow V_{out} = V_{ref}\cdot\left[\left(1 + \frac{R_f}{R_2} + \frac{R_f}{R_1}\right)\cdot\left(\frac{R_4}{R_4 + R_3}\right) - \frac{R_f}{R_1}\right] \xrightarrow{V_{out} = 0} \frac{R_1}{R_f} = \frac{R_3}{R_4} - \frac{R_1}{R_2} \Rightarrow \frac{1}{R_f} = \frac{R_3}{R_1 R_4} - \frac{1}{R_2}$$

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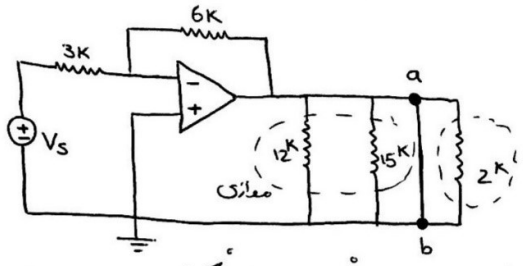
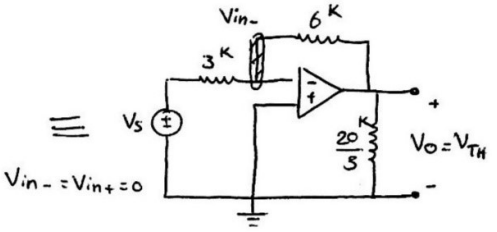
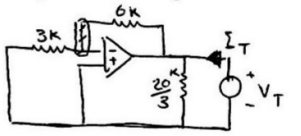


اتصال کوتاه



KCL in V_{in-} : $\frac{V_{in-} - V_S}{3} + \frac{V_{in-} - V_{TH}}{6} = 0 \Rightarrow V_{TH} = -2V_S$

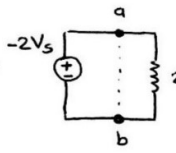
برای محاسبه R_{TH}، V_S را اتصال کوتاه می کنیم



$\Rightarrow$ KCL in V_{in-} : $\frac{V_{in-} - 0}{3} + \frac{V_{in-} - V_T}{6} = 0$

$\Rightarrow V_T = 0 \Rightarrow R_{TH} = 0$

مدار معادل برحسب آمده با در سمت چپ خروجی a و b جایگزین می کنیم



$\Rightarrow P_R = \frac{V^2}{R} = \frac{(-2V_S)^2}{2^K} = \frac{4V_S^2}{2^K} = 2V_S^2 \times 10^{-3}\,W$

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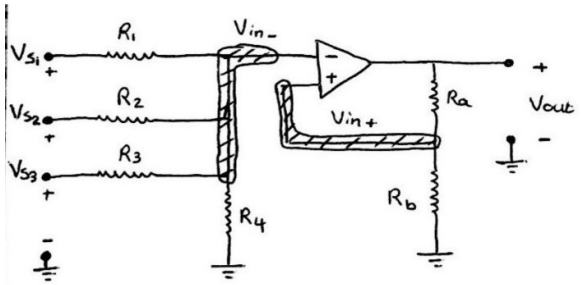

KCL in V_{in-} : $\frac{V_{in-} - V_{S1}}{R_1} + \frac{V_{in-} - V_{S2}}{R_2} + \frac{V_{in-} - V_{S3}}{R_3} + \frac{V_{in-} - 0}{R_4} = 0$ (I)

$\Rightarrow \frac{V_{S1}}{R_1} + \frac{V_{S2}}{R_2} + \frac{V_{S3}}{R_3} = V_{in-} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)$

KCL in V_{in+} : $\frac{V_{in+} - 0}{R_b} + \frac{V_{in+} - V_{out}}{R_a} = 0$

$\Rightarrow V_{in+} = V_{in-} = \frac{R_b}{R_b + R_a} \cdot V_{out}$ (II)

(II) in (I)

$\Rightarrow \frac{V_{S1}}{R_1} + \frac{V_{S2}}{R_2} + \frac{V_{S3}}{R_3} = \frac{R_b}{R_b + R_a} \cdot V_{out} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)$

$\Rightarrow V_{out} = \underbrace{\frac{R_b + R_a}{R_b\left(\frac{1}{R_{a1}} + \frac{1}{R_{b2}} + \frac{1}{R_{a3}} + \frac{1}{R_4}\right)}}_{= K'} \cdot \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$

$\Rightarrow V_{out} = \underbrace{K'\frac{1}{R_1}}_{= K_1} V_1 + \underbrace{K'\frac{1}{R_2}}_{= K_2} V_2 + \underbrace{K'\frac{1}{R_3}}_{K_3} V_3$

$\Rightarrow V_{out} = K_1 V_1 + K_2 V_2 + K_3 V_3$

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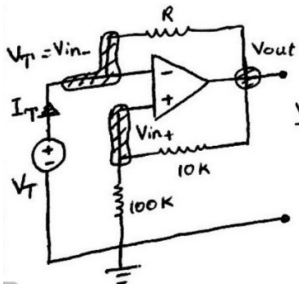

$\frac{V_T}{I_T} = R_{TH} = ?$

KCL in V_{in+} : $\frac{V_{in+} - 0}{100} + \frac{V_{in+} - V_{out}}{10} = 0 \Rightarrow V_{in+} = \frac{100}{100+10} \cdot V_{out}$

$\Rightarrow V_{out} = \frac{11}{10} V_T$ (I)

KCL in V_{in-} : $-I_T + \frac{V_T - V_{out}}{R} = 0$ (II)

(I) in (II) $\longrightarrow$ $I_T = \frac{1}{R}\left(V_T - \frac{11}{10} V_T\right) \Rightarrow \frac{V_T}{I_T} = R_{TH} = 10R$

$\Rightarrow 10R = 1\,M\Omega \Rightarrow R = \frac{1 \times 10^6\,\Omega}{10} = 10^5\,\Omega = 100\,K\Omega$