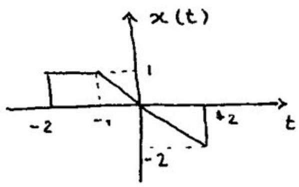
یالطیف

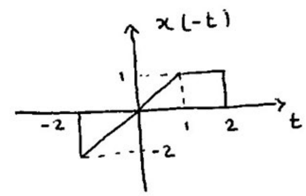
رضا ادیب‌پور
۹۸۱۴۳۰۳
تمرین سری ۱، سیگنال و سیستم

#1 الف)



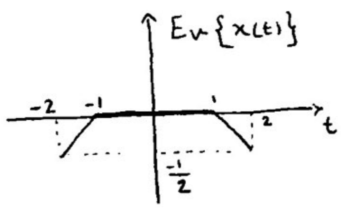
$$\begin{cases} Ev\{x(t)\} = ? \\ odd\{x(t)\} = ? \end{cases}$$

$$x(t) = \begin{cases} -x ; & -1<x<2 \\ 1 ; & -2<x<-1 \end{cases} \quad , \quad x(-t) = \begin{cases} x ; & -2<x<+1 \\ 1 ; & 1<x<2 \end{cases} , \Rightarrow$$



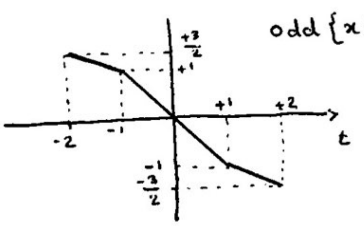
$$\Rightarrow x(t)+x(-t) = \begin{cases} 0 ; & 0<x<1 \\ 1-x ; & 1<x<2 \\ 0 ; & -1<x<0 \\ 1+x ; & -2<x<-1 \end{cases} \Rightarrow Ev\{x(t)\} = \frac{x(t)+x(-t)}{2} = \begin{cases} 0 ; & 0<x<1 \\ \frac{1-x}{2} ; & 1<x<2 \\ 0 ; & -1<x<0 \\ \frac{1+x}{2} ; & -2<x<-1 \end{cases}$$

$\Rightarrow$

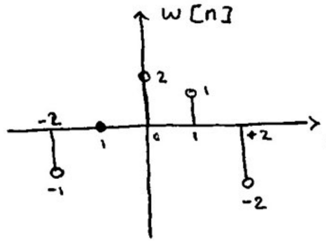


$$\Rightarrow x(t)-x(-t) = \begin{cases} -2x ; & 0<x<1 \\ -x-1 ; & 1<x<2 \\ -2x ; & -1<x<0 \\ 1-x ; & -2<x<-1 \end{cases} \Rightarrow odd\{x(t)\} = \frac{x(t)-x(-t)}{2} = \begin{cases} -x ; & 0<x<1 \\ \frac{-x-1}{2} ; & 1<x<2 \\ -x ; & -1<x<0 \\ \frac{1-x}{2} ; & -2<x<-1 \end{cases}$$

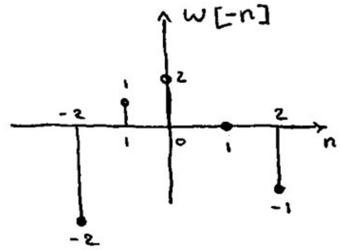
$\Rightarrow$



ب)

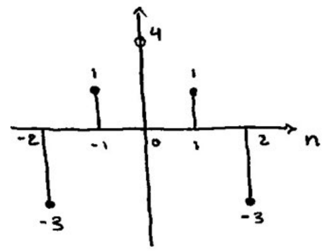


$\Rightarrow$

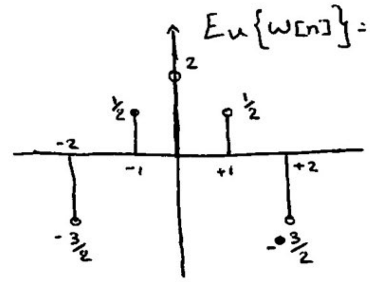


$$Ev\{w[n]\} = \frac{w[n]+w[-n]}{2}$$

$\Rightarrow W[n]+W[-n]$:

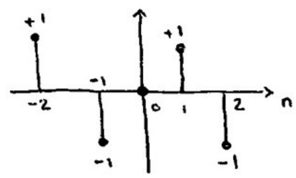


$\Rightarrow$

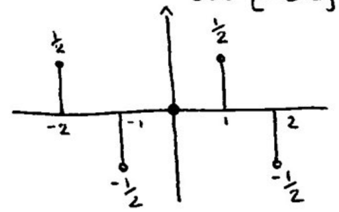


$$odd\{w[n]\} = \frac{w[n]-w[-n]}{2}$$

$\Rightarrow W[n]-W[-n]$:



$\Rightarrow$



#2 الف) $x_1(t) = j\, e^{(-2+j100)t} \Rightarrow \begin{cases} Re\{x_1(t)\} = Ae^{-at}\cos(\omega t+\varphi) \\ Im\{x_1(t)\} = Ae^{-at}\sin(\omega t+\varphi) \end{cases}$

$$\Rightarrow x_1(t) = e^{j\frac{\pi}{2}} \cdot e^{(-2+j100)t} = \underbrace{e^{j(100t+\frac{\pi}{2})}} \cdot e^{-2t}$$

$$= \cos(100t+\tfrac{\pi}{2}) + j\sin(100t+\tfrac{\pi}{2})$$

$$\Rightarrow \begin{cases} Re\{x_1(t)\} = 1e^{-2t}\cos(100t+\frac{\pi}{2}) \\ Im\{x_1(t)\} = 1e^{-2t}\sin(100t+\frac{\pi}{2}) \end{cases}$$

ب) $x_2(t) = \underbrace{\sqrt{2}\, e^{j\frac{\pi}{2}}}\cos(3t+\frac{\pi}{3}) \Rightarrow x_2(t) = \underbrace{j\sqrt{2}}_{A}\cos(3t+\frac{\pi}{3})$

$$= \sqrt{2}\left(\cancel{\cos(\tfrac{\pi}{2})} + j\sin(\tfrac{\pi}{2})\right) = j\sqrt{2}$$

$$\Rightarrow A\cos(3t+\tfrac{\pi}{3}) = A\,Re\left\{e^{j(3t+\frac{\pi}{3})}\right\} = A\,Re\left\{\cos(3t+\tfrac{\pi}{3}) + j\sin(3t+\tfrac{\pi}{3})\right\}$$

$\underline{A = j\sqrt{2} = j\sqrt{2}e^{-0t}} \rightarrow \begin{cases} Re\{x_2(t)\} = j\sqrt{2}\,e^{-0t}\cos(3t+\frac{\pi}{3}) \\ Im\{x_2(t)\} = j\sqrt{2}\,e^{-0t}\sin(3t+\frac{\pi}{3}) \end{cases}$

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الف) $x[n] = 3e^{j3\pi(n+\frac{1}{2})/5} = 3e^{j\frac{3\pi}{5}(n+\frac{1}{2})}$

$\Rightarrow \omega_0 = 2\pi\left(\frac{m}{N_0}\right) \Rightarrow$ اگر بتوان $\left(\omega_0 = \frac{3\pi}{5}\right)$ را به فرم روبه رو نوشت به طوری که ω_0 مضرب گویایی از π باشد، N پریود سیگنال است

$\Rightarrow \omega_0 = 2\pi\left(\frac{3}{10}\right) \longrightarrow N_0 = 10$ پریود اصلی $\Rightarrow \underline{N = 10m}$

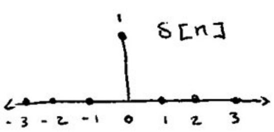
ب) $2\cos(10t+1) - \sin(4t-1)$:

$$\begin{cases} \cos(10t+1) = \cos(10(t+T_1)+1) \\ \Rightarrow 10T_1 = 2K_1\pi \Rightarrow T_1 = \frac{K_1\pi}{5} \\ \sin(4t-1) = \sin(4(t+T_2)-1) \\ \Rightarrow 4T_2 = 2K_2\pi \Rightarrow T_2 = \frac{K_2\pi}{2} \end{cases}$$

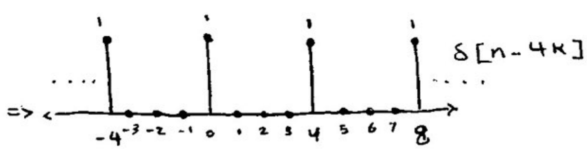
$\Rightarrow T = \text{Kmm}\left[\frac{\pi}{5}, \frac{\pi}{2}\right] = 100\pi$

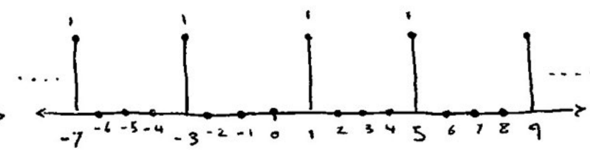
$\Rightarrow T = 100K\pi \Rightarrow \underline{T_0 = 100\pi}$

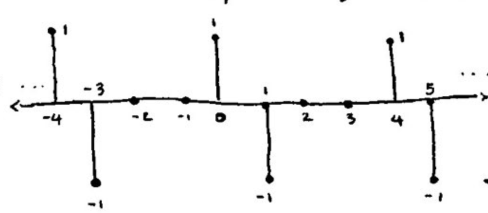
ج) $Z[n] = \sum_{K=-\infty}^{+\infty} \left\{ \delta[n-4K] - \delta[n-1-4K] \right\}$ $\qquad \delta[n] = \begin{cases} 1 \ ; & n = 0 \\ 0 \ ; & n \neq 0 \end{cases}$



$\Rightarrow \delta[n-4K] = \begin{cases} \text{if } K=0 \Rightarrow \delta[n] \\ \text{if } K=1 \Rightarrow \delta[n-4] \\ \text{if } K=2 \Rightarrow \delta[n-8] \\ \text{if } K=-1 \Rightarrow \delta[n+4] \\ \text{if } K=-2 \Rightarrow \delta[n+8] \end{cases}$

$\Rightarrow$ 

$\Rightarrow \delta[n-1-4K] = \delta[(n-4K)-1) \Rightarrow$ 

$\Rightarrow \delta[n-4K] - \delta[n-1-4K]$:  سیگنال $Z[n]$ پریودیک است

$\underline{T_0 = 4}$

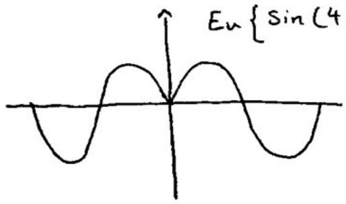
د) $S[n] = \cos\left[\frac{\pi}{2}n\right] \cdot \cos\left[\frac{\pi}{4}n\right] = \frac{1}{2}\left(\underbrace{\cos\left(\frac{3\pi}{4}n\right)}_{*} + \underbrace{\cos\left(\frac{\pi}{4}n\right)}_{**}\right)$

$$\begin{cases} *)\ \omega_0 = 2\pi\left(\frac{3}{8}\right) \xrightarrow[\pi \text{ است}]{\omega_0 \text{ مضرب گویایی از}} N_0 = 8 \\ **)\ \omega_0 = 2\pi\left(\frac{1}{8}\right) \xrightarrow[\text{از } \pi \text{ است}]{\omega_0 \text{ مضرب گویایی}} N_0 = 8 \end{cases} \longrightarrow N_0 = \text{kmm}[8,8] = \underline{8}$$

ج) $Q(t) = E_v\left\{\sin(4\pi t)\,u(t)\right\} = \dfrac{\sin(4\pi t)u(t) + \sin(-4\pi t)u(-t)}{2} \xrightarrow{\sin(-\alpha) = -\sin\alpha}$

$= \frac{1}{2}\Big(\underbrace{\sin(4\pi t)\,u(t)}_{(I)} - \underbrace{\sin(4\pi t)\,u(-t)}_{(II)}\Big)$ $\qquad u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}, \quad u(-t) = \begin{cases} 1 & t \leq 0 \\ 0 & t \geq 0 \end{cases}$

$(I):$ $\begin{cases} \text{برای } t > 0 \Rightarrow \sin(4\pi t) \\ \text{برای } t < 0 \Rightarrow 0 \end{cases}$, $(II):$ $\begin{cases} \text{برای } t < 0 \Rightarrow -\sin(4\pi t) \\ \text{برای } t > 0 \Rightarrow 0 \end{cases}$

$\Rightarrow$ 

$\Rightarrow$ پریودیک نیست

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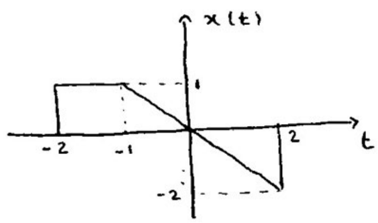
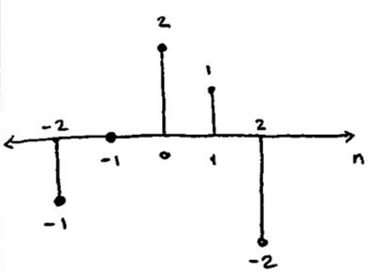


$x(t) = \begin{cases} -x & ; -1 < x < 2 \\ 1 & ; -2 < x < -1 \end{cases}$ $\qquad T_0 = 5$

$E_\infty = E_{[-2,-1]} + E_{[-1,0]} + E_{[0,2]}$

$= \int_{-2}^{-1} dt + \int_{-1}^{0} (-t)^2\,dt + \int_{0}^{2} (-t)^2 = t\Big|_{-2}^{-1} + \frac{t^3}{3}\Big|_{-1}^{2} = (-1+2) + \frac{1}{3}(8+1) = 4$

$\Rightarrow E_{[-2,+2]} = 4 \Rightarrow$ توان یک سیگنال پریودیک $= \dfrac{E_{[-T,T]}}{\text{پریود}} = \dfrac{4}{5}$



$E_\infty = \sum_{n=-\infty}^{+\infty} |x[n]|^2$