یالطیف

پاسخ تمرین سری 3 (تکمیلی سیستم‌های LTI) سیگنال سیستم

#1

$$\begin{cases} x_1(t) = u(t+1) - u(t-1) \longrightarrow y_1(t) \\ x_2(t) = 5t\big(u(t) - u(t-2)\big) \longrightarrow y_2(t) \implies x_2(t) = 5t\big(x_1(t-1)\big) \\ h(t) = ? \end{cases}$$

$$y_1(t) = x_1(t) * h_1(t) = \int_{-\infty}^{+\infty} x_1(\tau) h_1(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} \big(u(t+1) - u(t-1)\big) h_1(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{+\infty} u(t+1) h_1(t-\tau)\, d\tau - \int_{-\infty}^{+\infty} u(t-1) h_1(t-\tau)\, d\tau = \int_{-1}^{t} d\tau - \int_{+1}^{t} d\tau = \tau\Big|_{-1}^{t} - \tau\Big|_{+1}^{t}$$

$$= t+1 - (t-1) = \boxed{2}$$

$$y_2(t) = x_2(t) * h_2(t) = \int_{-\infty}^{+\infty} 5\tau\big(u(\tau) - u(\tau-2)\big) h_2(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} 5\tau u(\tau) h_2(t-\tau)\, d\tau$$

$$- \int_{-\infty}^{+\infty} 5\tau u(\tau-2) h_2(t-\tau)\, d\tau = 5\int_{0}^{t} \tau\, d\tau - 5\int_{2}^{t} \tau\, d\tau = 5\left[\frac{\tau^2}{2}\Big|_{0}^{t} - \frac{\tau^2}{2}\Big|_{2}^{t}\right] =$$

$$\frac{5}{2}\left[(t^2 - 0) - (t^2 - 4)\right] = \boxed{10}$$

$$\boxed{h(t) = 2y_1(t)}$$

#2

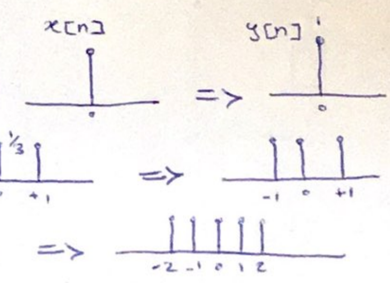
$$y[n] = \frac{1}{2N+1} \cdot \sum_{m=-N}^{N} x[m+n] \qquad \text{(moving average system)}$$

$h[n] = ?$

If $N=0 \longrightarrow x[m+n] \longrightarrow y[N] = \frac{1}{1} x[m+n] \xrightarrow{\text{if } x[n] = \delta[n]}$

If $N=1 \longrightarrow \sum_{m=-1}^{+1} x[m+n] \longrightarrow y[N] = \frac{1}{3} \sum_{m=-1}^{+1} x[m+n] \longrightarrow$

If $N=2 \longrightarrow y[N] = \frac{1}{5} \sum_{m=-2}^{2} x[m+n] \longrightarrow$



If $N=3 \longrightarrow y[N] = \frac{1}{7} \sum_{m=-3}^{3} x[m+n]$

$N=0: \quad x[n] * h[n] = y[n] \quad \Rightarrow \quad h[n]=1$

$N=1: \quad h[n] = \frac{1}{3} \quad \Rightarrow \quad \boxed{h[n] = \frac{1}{2N+1}}$

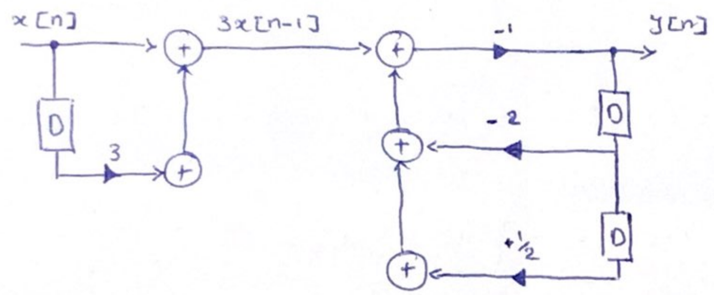
#3

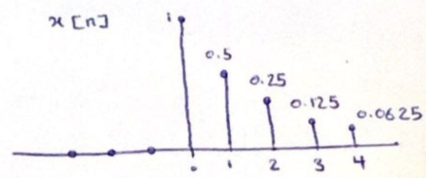
$$y[n] + 2y[n-1] - \frac{1}{2}y[n-2] = 3x[n-1]$$

a) رسم بلوک دیاگرام

b) if $x[n] = (\frac{1}{2})^n u[n]$, سیستم در حالت سکون باشد $\Rightarrow$ $y[0]$ تا $y[4]=?$, فرم کلی $y[n]=?$

$\hookrightarrow y[0]=0$





$$y[n] = 3x[n-1] - 2y[n-1] + \frac{1}{2}y[n-2]$$

if $n=0 \longrightarrow \cancel{y[0]} = 3x[-1] - 2y[-1] + \frac{1}{2}y[-2] \longrightarrow y[-1] = \underbrace{\frac{3}{2}x[-1]}_{=0} + \frac{1}{4}y[-2]$

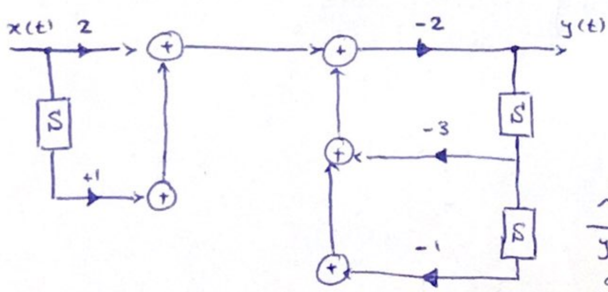
if $n=+1 \longrightarrow y[1] = \underbrace{3x[0]}_{=3} - 2\cancel{y[0]} + \frac{1}{2}y[-1] \longrightarrow y[1] = 3 + \frac{1}{2}\left(\frac{1}{4}y[-2]\right)$

if $n=2 \longrightarrow y[2] = 3x[1] - 2y[1] + \cancel{\frac{1}{2}y[0]} \longrightarrow y[2] = \frac{3}{2} - 2\left(3 + \frac{1}{8}y[-2]\right)$

if $n=3 \longrightarrow y[3] = 3x[2] - 2y[2] + \frac{1}{2}y[1] \longrightarrow y[3] = \frac{3}{4} - 2\left(-2\left(3+\frac{1}{8}y[-2]\right)\right) + \frac{1}{2}\left(3 + \frac{1}{2}\left(\frac{1}{4}y[-2]\right)\right)$

#4

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y(t) = \frac{dx}{dt} + 2x(t) \quad \Rightarrow \quad \frac{dx}{dt} + 2x(t) - \frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 2y(t) = 0$$



if $x(t) = e^{-3t}u(t) \longrightarrow y(t)=?$

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y(t) = \frac{d}{dt}\left(e^{-3t}u(t)\right) + 2e^{-3t}u(t)$$

$$\xrightarrow[\substack{y(0)=0 \\ \frac{dy(0)}{dt}=0,\ \frac{d^2y(0)}{dt^2}=0}]{\text{شرایط اولیه صفر}} \frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y(t) = -3e^{-3t}u(t) + \delta(t)e^{-3t} + 2e^{-3t}u(t)$$

$$\Rightarrow \frac{d^2y}{dt^2} + \overset{=2\alpha}{3}\frac{dy}{dt} + \overset{=\omega_0^2}{2}y(t) = 1 - e^{-3t}u(t) \longrightarrow y(t) = y_g(t) + y_p(t)$$

$$s^2 + 3s + 2 = 0 \longrightarrow \begin{cases} s = -2 \\ s = -1 \end{cases}$$

$$2\alpha = 3 \rightarrow \alpha = 1.5$$
$$\omega_0^2 = 2 \rightarrow \omega_0 = \sqrt{2} \simeq 1.4$$
$$\Rightarrow \alpha > \omega_0$$

میرای شدید

$$y_g(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t} = k_1 e^{-2t} + k_2 e^{-t}$$

$$y(0) = 0 \quad : \quad k_1 + k_2 = 0 \Rightarrow k_1 = -k_2 \quad (I)$$

$$\frac{dy(0)}{dt} = 0 \quad : \quad -2k_1 e^{-2t} - k_2 e^{-t} = 0 \xrightarrow{t=0} -2k_1 - k_2 = 0 \xrightarrow{(I)}$$