مدار های الکتریکی ۱

نیم سال اول ۹۹-۰۰

تحلیل مش و گره ، مدار معادل تونن و نرتن   پاسخ تمرین سری چهارم

۱.
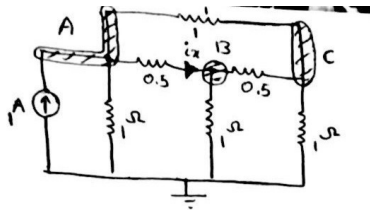
$$KCL \ in \ A : -1 + \frac{V_A - 0}{1} + \frac{V_A - V_B}{0.5} + \frac{V_A - V_C}{1} = 0$$

$$\Rightarrow 4V_A - 2V_B - V_C = 1 \qquad (I) \qquad\qquad (II)$$

$$KCL \ in \ B : \frac{V_B - V_A}{0.5} + \frac{V_B - 0}{1} + \frac{V_B - V_C}{0.5} = 0 \Rightarrow 5V_B - 2V_A - 2V_C = 0$$
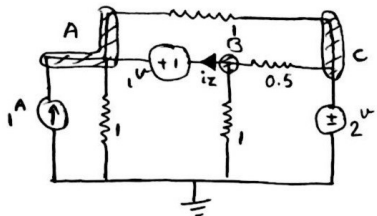
$$KCL \ in \ C : \frac{V_C - 0}{1} + \frac{V_C - V_B}{0.5} + \frac{V_C - V_A}{1} = 0 \Rightarrow 4V_C - 2V_B - V_A = 0 \quad (III)$$

$$\Rightarrow \begin{cases} 4V_A - 2V_B - V_C = 0 \\ -2V_A + 5V_B - 2V_C = 0 \\ -V_A - 2V_B + 4V_C = 0 \end{cases} \Rightarrow V_A = \frac{16}{35} , \ V_B = \frac{2}{7} , \ V_C = \frac{9}{35}$$

$$\Rightarrow \overset{\circ}{i}_x = \frac{V_A - V_B}{0.5} = 2V_A - 2V_B = \frac{32}{35} - \frac{4}{7} = \frac{12}{35} \ A$$
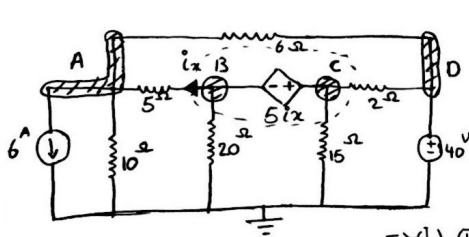
۲.

※ اگر بین یک گره در غیرمرجع ، منبع ولتاژ ، وجود داشته باشد درگره مرکب حاصل از آنها KCL می نویسیم ※

$$KCL \ in \ C : \Rightarrow V_C = 2^{\overset{v}{}}$$

$$KCL \ in \ A, B : -1 + \frac{V_A - 0}{1} + \frac{V_A - V_C}{1} + \frac{V_B - V_C}{0.5} + \frac{V_B - 0}{1} = 0$$

$$\overset{V_C = 2}{\Longrightarrow} 3V_B + 2V_A = 7 \quad , \quad V_A - V_B = 1 \Rightarrow \begin{cases} 3V_B + 2V_A = 7 \\ -V_B + V_A = 1 \end{cases} \Rightarrow V_A = 2^v , \ V_2 = 1^v$$

۳.

KCL in A: $+6 + \frac{V_A-0}{10} + \frac{V_A-V_B}{5} + \frac{V_A-40}{6} = 0 \Rightarrow \frac{7}{15}V_A - \frac{V_B}{5} = \frac{2}{3}$ (I)
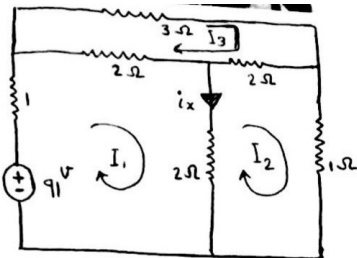
KCL in B,C: $\frac{V_B-V_A}{5} + \frac{V_B-0}{20} + \frac{V_C-0}{15} + \frac{V_C-40}{2} = 0$

$\Rightarrow \frac{V_B}{4} - \frac{V_A}{5} + \frac{17}{30}V_C = 20$ (II)

$\Rightarrow V_C - V_B = 5i_x = 5\left(\frac{V_B-V_A}{5}\right) \Rightarrow V_C - 2V_B + V_A = 0$ (III)

$\Rightarrow$ (I),(II),(III) $\Rightarrow \underline{V_A = 10\,V}$, $\underline{V_B = 20\,V}$, $\underline{V_C = 30\,V}$, $i_x = \frac{V_B-V_A}{5} = 2$

۴.

KVL in M_1: $1\times I_1 + 2(I_1-I_3) + 2(I_1-I_2) - 91 = 0$

$\Rightarrow 5I_1 - 2I_3 - 2I_2 = 91$ (I)

KVL in M_2: $2(I_2-I_3) + 1\times I_2 + 2(I_2-I_1) = 0$
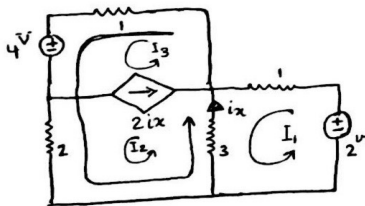
$\Rightarrow 5I_2 - 2I_3 - 2I_1 = 0$ (II)

KVL in M_3: $3\times I_3 + 2(I_3-I_2) + 2(I_3-I_1) = 0$

$\Rightarrow 7I_3 - 2I_2 - 2I_1 = 0$ (III) $\xrightarrow{(I),(II),(III)}$ $\begin{cases} 5I_1 - 2I_3 - 2I_2 = 91 \\ 4I_2 - I_3 - 2I_1 = 0 \\ 7I_3 - 2I_2 - 2I_1 = 0 \end{cases} \Rightarrow I_1 = 31^A$, $I_2 = 18^A$

$\Rightarrow i_x = I_1 - I_2 = 31 - 18 = 13^A$, $I_3 = 14^A$

۵.

* اگر منبع جریان بین دو مش مشترک بود، در مش مرکب حاصل از آنها KVL می نویسیم *

KVL in M_1: $1\times I_1 + 3\times(I_1-I_2) - 2 = 0$

$\Rightarrow 4I_1 - 3I_2 = 2^V$ (I)

KVL in M_2, M_3: $1\times I_3 + 4 + 2\times I_2 + 3(I_2-I_1) = 0$
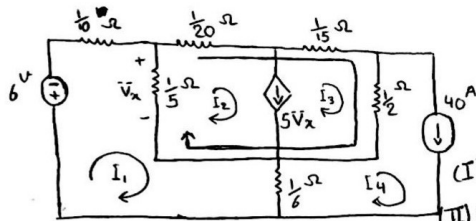
$\Rightarrow I_3 + 5I_2 - 3I_1 = -4$ (II)

$\Rightarrow I_3 - I_2 = 2i_x = 2(I_2-I_1) \Rightarrow I_3 = 3I_2 - 2I_1$ (III)

$\Rightarrow$ (III),(II),(I) : $\begin{cases} 4I_1 - 3I_2 = 2 \\ I_3 + 5I_2 - 3I_1 = -4 \\ 3I_2 - 2I_1 - I_3 = 0 \end{cases} \Rightarrow \underline{I_1 = \frac{4}{17}}^A$, $\underline{I_2 = \frac{-6}{17}}^A$, $\underline{I_3 = \frac{-26}{17}}^A$

$\Rightarrow i_x = I_2 - I_1 = \frac{-6}{17} - \frac{4}{17} = \frac{-10}{17}^A$

۶.

* هرگاه یک منبع جریان تنها متعلق به یک مش بود ، در آن مش KVL

نمی‌نویسیم زیرا جریان آن مش مشخص است ← * $I_4 = 40^A$

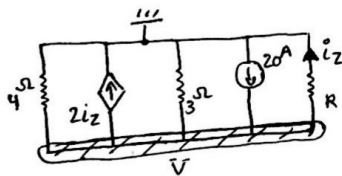(I) KVL in M_1 : $\frac{1}{10} I_1 + \frac{1}{5} (I_1 - I_2) + \frac{1}{6} (I_1 - I_4) + 6 = 0$

(II) KVL in M_1, M_2 : $\frac{1}{20} I_2 + \frac{1}{15} I_3 + \frac{1}{2} (I_3 - I_4) + \frac{1}{5} (I_2 - I_1) = 0$

$\Rightarrow I_2 - I_3 = 5V_x = 5 \times \frac{1}{5} (I_1 - I_2) \Rightarrow I_1 + I_3 - 2I_2 = 0$ (III)

$\Rightarrow$ (I) , (II) , (III) $\Rightarrow I_1 = 10^A$, $I_2 = 20^A$, $I_3 = 30^A$, $I_4 = 40^A$

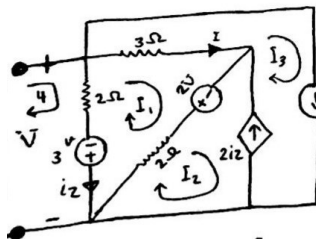$\Rightarrow V_x = \frac{1}{5} (I_1 - I_2) = -2^V$

۷.

KCL in $\bar{V}$: $\frac{\bar{V} - 0}{4} + 2i_z + \frac{\bar{V} - 0}{3} - 20 + \frac{\bar{V} - 0}{R} = 0$

$\left.\begin{array}{l} \bar{V} = 24^V \\ i_z = \frac{\bar{V}}{R} \end{array}\right\} \rightarrow 6 + 2(\frac{24}{R}) + 8 - 20 + \frac{24}{R} = 0 \Rightarrow R = 12^\Omega$

۸.

* روش تحلیل مش * KVL in M_1 : $+3 + 2I_1 + 3(I_1 - I_3) - 2 + 2(I_1 - I_2) = 0$

$\underline{I_3 = 3}$ $7I_1 - 2I_2 = 8$ (I)

* منبع جریان وابسته بین مش ۲ و ۳ مشترک است بنابراین باید در مش مرکب حاصل از آنها KVL بنویسیم اما از آنجایی که منبع جریان و کمیت تنها متعلق به مش ۲ و ۳ است بنابراین می‌توان ... مش مرکب حاصل از آنها KVL نوشت *

$\Rightarrow I_3 - I_2 = 2i_z = 2(-I_1) \Rightarrow I_2 - 2I_1 = 3$ (II) $\Rightarrow \begin{cases} 7I_1 - 2I_2 = 8 \\ -2I_1 + I_2 = 3 \end{cases}$
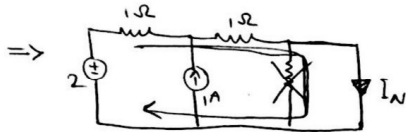
$\Rightarrow I_1 = I = \frac{14}{3}$, $I_2 = \frac{37}{3}$ $\Rightarrow$ KVL in M_4 : $-2I_1 - 3 = \frac{-28}{3} - 3 = \frac{-37}{3} = \bar{V}$
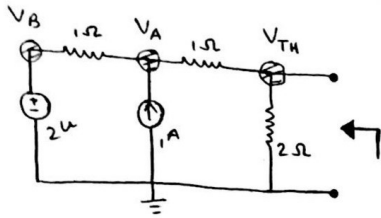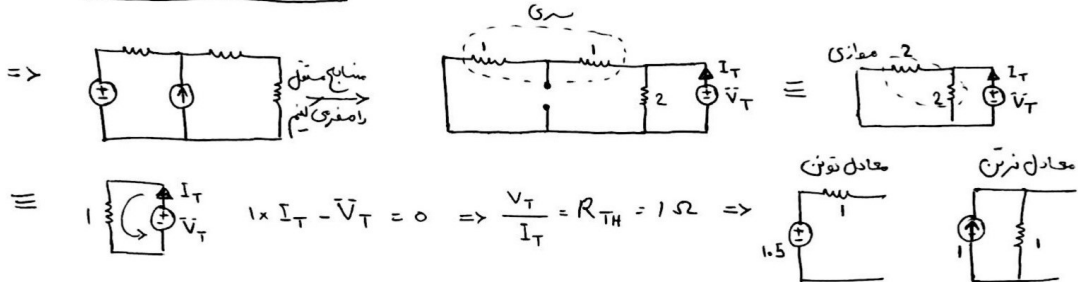
$$\text{kcl in } V_{TH} : \frac{V_{TH}-0}{2} + \frac{V_{TH}-V_A}{1} = 0 \Rightarrow 1.5 V_{TH} = V_A \quad (I)$$

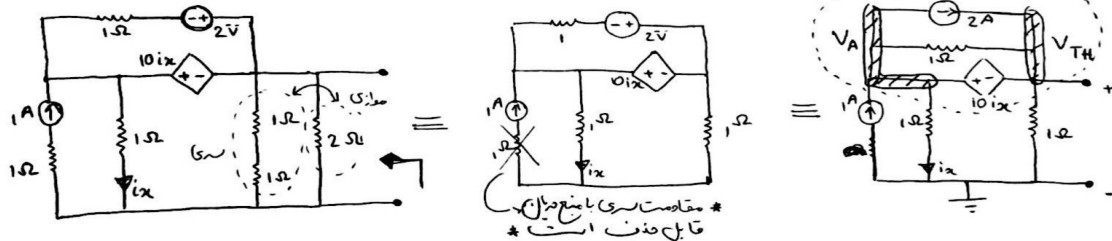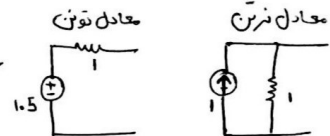$$\text{kcl in } V_A : -1 + \frac{V_A - V_{TH}}{1} + \frac{V_A - 2}{1} = 0 \Rightarrow 2V_A - V_{TH} = 3 \quad (II)$$

$$(I) \text{ in } (II) : 2(1.5 V_{TH}) - V_{TH} = 3 \Rightarrow \boxed{V_{TH} = \frac{3}{2} = 1.5^v}$$

$$KVL : 1 \times I_N + 1 \times I_N = 2 \Rightarrow \boxed{I_N = 1^A}$$
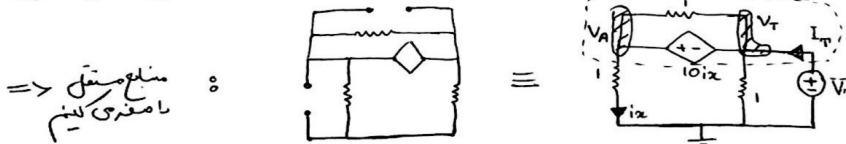

$$1 \times I_T - \bar{V}_T = 0 \Rightarrow \frac{V_T}{I_T} = R_{TH} = 1\Omega \Rightarrow$$

$$\text{kcl in } V_A , V_{TH} : -1 + \frac{V_A - 0}{1} + \frac{V_{TH} - 0}{1} = 0 \Rightarrow V_{TH} + V_A = 1 \quad (I)$$
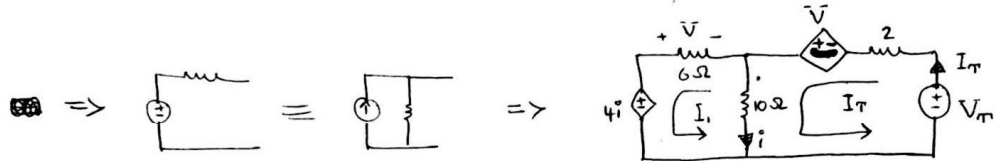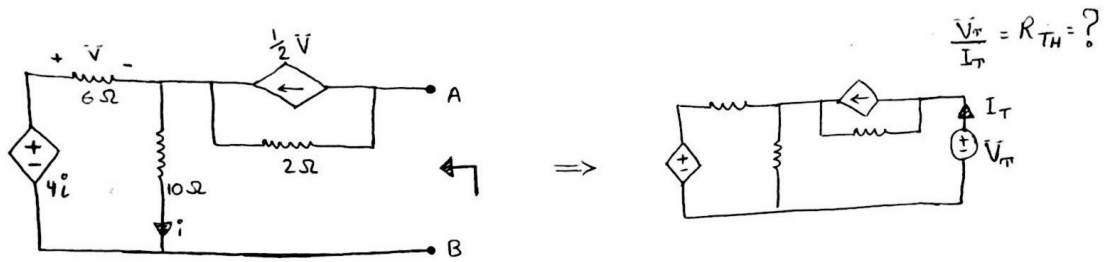
$$\Rightarrow V_A - V_{TH} = 10 ix = 10(V_A) \Rightarrow V_{TH} = -9 V_A \quad (II) \xrightarrow{(II) \text{ in } (I)} V_{TH} - \frac{V_{TH}}{9} = 1 \Rightarrow V_{TH} = \frac{9}{8}^v$$

$$\text{kcl in } V_A , V_T : \frac{V_A - 0}{1} + \frac{V_T - 0}{1} - I_T = 0 \Rightarrow V_A + V_T = I_T \quad (I)$$

$$\Rightarrow V_A - V_T = 10 ix = 10 V_A \quad (II) \xrightarrow{(II) \text{ in } (I)} \frac{-1}{9} V_T + V_T = I_T$$

$$\Rightarrow \boxed{\frac{V_T}{I_T} = R_{TH} = \frac{9}{8}\Omega} \Rightarrow \bar{V}_{TH} = R_{TH} \times I_N \Rightarrow \boxed{I_N = \frac{9/8}{9/8} = 1^A}$$

$$\frac{V_T}{I_T} = R_{TH} = ?$$

KVL in I_T : $-V_T + 2 I_T - V + 10 (I_T - I_1) = 0 \xrightarrow{V = -6 I_1} 12 I_T - 4 I_1 = V_T$ (I)

KVL in I_1 : $+4i + 10(I_1 - I_T) + 6 I_1 = 0 \xrightarrow{i = I_T - I_1} 10 I_1 - 10 I_T + 6 I_1 + 4 I_T - 4 I_1 = 0$

$= 12 I_1 = 6 I_T \Rightarrow I_T = 2 I_1$ (II) $\xrightarrow{(II) \, in \, (I)} 12 I_T - 4 \left(\frac{I_T}{2} \right) = V_T \Rightarrow 10 I_T = V_T$

$\Rightarrow \frac{V_T}{I_T} = R_{TH} = 10 \, \Omega$