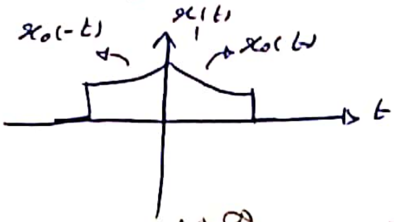
1-

$$x_0(t) = \begin{cases} e^{-t} & 0 \le t \le 1 \\ 0 & o.w. \end{cases} \qquad X(j\omega) = ?$$

الف)



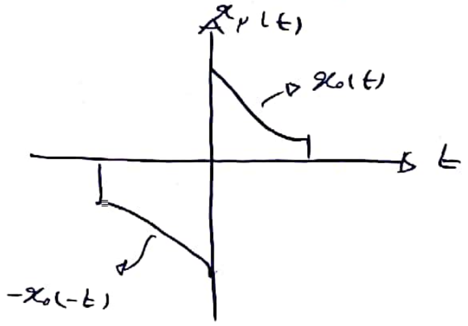
$$X_0(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt = \int_0^1 e^{-t} e^{-j\omega t}\, dt = \int_0^1 e^{-t(j\omega+1)}\, dt$$

$$\Rightarrow X_0(j\omega) = \frac{-1}{j\omega+1} e^{-t(j\omega+1)} \Big|_0^1 = \frac{-e^{-(j\omega+1)}}{j\omega+1} + \frac{1}{j\omega+1} = \frac{1-e^{-(j\omega+1)}}{j\omega+1}$$

$$x_1(t) = x_0(t) + x_0(-t) \xrightarrow{\mathcal{F}} X_1(j\omega) = X_0(j\omega) + X_0(-j\omega)$$

$$\Rightarrow X_1(j\omega) = \frac{1-e^{-(j\omega+1)}}{j\omega+1} + \frac{1-e^{-(-j\omega+1)}}{-j\omega+1} = \frac{2-2e^{-1}\cos\omega - 2\omega e^{-1}\sin\omega}{1+\omega^2}$$

ب)

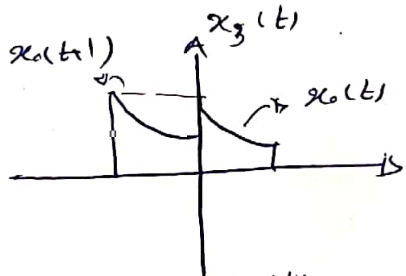


$$x_2(t) = x_0(t) - x_0(-t)$$

$$\Rightarrow X_2(j\omega) = X_0(j\omega) - X_0(-j\omega)$$

$$\Rightarrow X_2(j\omega) = j\left[\frac{-2\omega + 2e^{-1}\sin\omega + 2\omega e^{-1}\cos\omega}{1+\omega^2}\right]$$

ج)

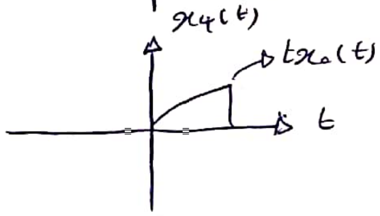


$$x_3(t) = x_0(t) + x_0(t+1)$$

$$\Rightarrow X_3(j\omega) = X_0(j\omega) + X_0(j\omega)\, e^{j\omega}$$

$$\Rightarrow X_3(j\omega) = \frac{1+e^{j\omega} - e^{-1}(1+e^{-j\omega})}{1+j\omega}$$

د)



$$x_4(t) = t\,x_0(t) \qquad \Rightarrow -jt\,x(t) \longleftrightarrow \frac{d}{d\omega} X(j\omega)$$

$$\Rightarrow X_4(j\omega) = j\frac{d}{d\omega} X_0(j\omega)$$

$$\Rightarrow X_4(j\omega) = \frac{1-2e^{-(j\omega+1)} - j\omega e^{-(j\omega+1)}}{(1+j\omega)^2}$$

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$$x(t) = \frac{t^{n-1}}{(n-1)!} e^{-at} u(t) \quad a>0 \Rightarrow X(j\omega) = \frac{1}{(a+j\omega)^n}$$ (2

$$n=1 \Rightarrow \frac{t^0}{0!} e^{-t}u(t) = \overbrace{e^{-at}u(t)}^{x_1(t)} \to \overbrace{\frac{1}{a+j\omega}}^{X_1(j\omega)}$$

$$n=2 \Rightarrow t e^{-at}u(t) = \overbrace{t x_1(t)}^{x_2(t)} \to \overbrace{j\frac{d}{d\omega}X_1(j\omega) = \frac{1}{(a+j\omega)^2}}^{X_2(j\omega)}$$

$$n=3 \Rightarrow \frac{t^2}{2}e^{-at}u(t) = \frac{t}{2}x_2(t) \to \frac{1}{2} j\frac{d}{d\omega}X_1(j\omega) = \frac{j}{2}\left(\frac{-2j(a+j\omega)}{(a+j\omega)^4}\right) = \frac{1}{(a+j\omega)^3}$$

$$\Rightarrow \frac{t^{n-1}}{(n-1)!}e^{-at}u(t) = \frac{t}{(n-1)}x_{n-1} \Rightarrow \frac{j}{(n-1)}\cdot\frac{d}{d\omega}X_{n-1}(j\omega) = j\left[\frac{-(n-1)j(a+j\omega)^{n-2}}{(n-1)\cdot(a+j\omega)^{2(n-1)}}\right]$$

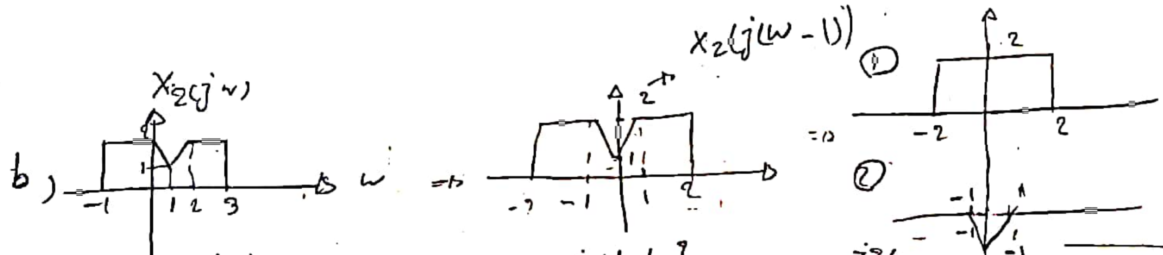
$$\frac{e^{-at}u(t)}{(n-2)!} = x_{n-1}$$

$$\Rightarrow \frac{1}{(a+j\omega)^n}$$

$$a)\ X_1(j\omega) = \frac{2\sin(3\overbrace{(\omega-2\pi)}^{\alpha})}{\underbrace{(\omega-2\pi)}_{\alpha}} = \frac{2\sin 3\alpha}{\alpha} = \begin{cases} 1\times e^{j2\pi t} & |t|<3 \\ 0 & o.w.\end{cases}$$ (3

$$\alpha = \omega - 2\pi$$

b)



$$① : \frac{2}{2\pi}\int_{-2}^{2} e^{j\omega t}d\omega = \frac{1}{\pi}\times\frac{1}{jt}e^{j\omega t}\Big|_{-2}^{2} = \frac{1}{\pi jt}(e^{j2t}-e^{-j2t}) = \frac{1}{\pi t}\times 2\sin(2t)$$

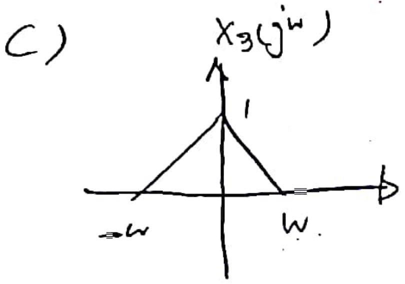
$$② : \frac{-2\sin^2 t/2}{\pi t^2} + ②$$

$$\Rightarrow ①+② \left[\frac{1}{\pi t}2\sin(2t) - \frac{2\sin^2 t/2}{\pi t^2}\right] e^{jt}$$

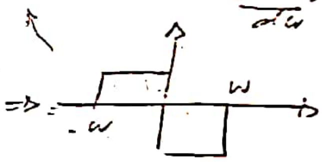
$$\frac{1}{2\pi}\int_{-\omega/2}^{\omega/2} 1\times e^{j\omega t}d\omega = \frac{1}{2\pi}\times\frac{1}{jt}e^{j\omega t}\Big|_{-\omega/2}^{\omega/2} = \frac{\sin \frac{\omega}{2}t}{\pi t}$$

$$\frac{d X_3(j\omega)}{d\omega}$$

C)



$\Rightarrow$



$$\Rightarrow f\begin{cases}1 & -\frac{\omega}{2}<\omega+\frac{\omega}{2}<\frac{\omega}{2}\\ -1 & -\frac{\omega}{2}<\omega-\frac{\omega}{2}<\frac{\omega}{2}\end{cases} \xrightarrow{F^{-1}} \frac{1}{jt}\left\{\begin{array}{l}\frac{\sin\frac{\omega}{2}t}{\pi t}e^{j\frac{\omega}{2}t}\\ -\frac{\sin\frac{\omega}{2}t}{\pi t}e^{-j\frac{\omega}{2}t}\end{array}\right.$$

$$\Rightarrow \frac{\sin\frac{\omega}{2}t}{\pi t^2}\left(\frac{e^{j\frac{\omega}{2}t}-e^{-j\frac{\omega}{2}t}}{j}\right) \Rightarrow \frac{2\sin^2\frac{\omega}{2}t}{\pi t^2}$$

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-۳

a) $x(t) = te^{-2t}u(t)$

$h(t) = e^{-4t}u(t)$

$$X(j\omega) = \frac{1}{(2+j\omega)^2} \qquad H(j\omega) = \frac{1}{4+j\omega}$$

$$x(t) * h(t) \to X(j\omega)\cdot H(j\omega) = \frac{1}{(2+j\omega)^2(4+j\omega)} = \frac{1}{-j\omega^3 - 8\omega^2 + 20j\omega + 16}$$

b) $x(t) = e^{-t}u(t) \quad h(t) = e^{t}u(-t)$

$$X(j\omega) = \frac{1}{1+j\omega} \qquad H(j\omega) = \frac{1}{1-j\omega}$$

$$X(j\omega)\cdot H(j\omega) = \frac{1}{(1-j\omega)(1+j\omega)} = \frac{1}{1+\omega^2}$$

-d

$$x(t) = \cos(t) \to X(j\omega) = \pi[\delta(\omega-1) + \delta(\omega+1)]$$

a) $h_1(t) = u(t) \to H(j\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$

$$X(j\omega)\cdot H_1(j\omega) = \pi\left[\tfrac{1}{j}\delta(\omega-1) + j\delta(\omega+1)\right] = \frac{\pi}{j}\left[\delta(\omega-1) - \delta(\omega+1)\right] \xrightarrow{\mathcal{F}^{-1}} \sin(t)$$

b) $h_2(t) = -2\delta(t) + 5e^{-2t}u(t) \Rightarrow H_2(j\omega) = -2 + \frac{5}{2+j\omega} = \frac{1-2j\omega}{2+j\omega}$

$$\frac{1-2j\omega}{2+j\omega} \times \frac{2-j\omega}{2-j\omega} = \frac{2-j\omega-4j\omega-2\omega^2}{4+\omega^2} = \frac{-2\omega^2-5j\omega+2}{4+\omega^2}$$

$$X(j\omega)\times H_2(j\omega) = \pi\left[\frac{-2-5j+2}{4+1}\delta(\omega-1) + \frac{-2+5j+2}{4+1}\delta(\omega+1)\right]$$

$$\Rightarrow \frac{\pi}{j}\left[\delta(\omega-1) - \delta(\omega+1)\right] \xrightarrow{\mathcal{F}^{-1}} \sin(t)$$

c) $h_3(t) = 2te^{-t}u(t) = \frac{2}{(1+j\omega)^2}$

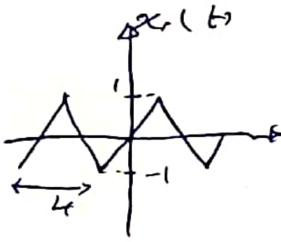
$$X(j\omega)\times H_3(j\omega) = \pi\left[\frac{2}{(1+j)^2}\delta(\omega-1) + \frac{2}{(1-j)^2}\delta(\omega+1)\right]$$

$$= \pi\left[\frac{2}{1-1+2j}\delta(\omega-1) + \frac{2}{1-1-2j}\delta(\omega+1)\right]$$

$$= \frac{\pi}{j}\left[\delta(\omega-1) - \delta(\omega+1)\right] \xrightarrow{\mathcal{F}^{-1}} \sin(t)$$

ب) همه شکل و ترکیب تغییر $h_3 = h_2 = h_1$

①



$\left\{\begin{array}{l}\text{سیگنال حقیقی و فرد = موهومی خالص} \Rightarrow Re\{X(j\omega)\} = 0 \\ \text{سیگنال حقیقی و زوج} \Rightarrow \text{حقیقی و زوج} \Rightarrow Im\{X(j\omega)\} = 0\end{array}\right.$

$x_1 \Rightarrow$ حقیقی و فرد $\Rightarrow Re\{X(j\omega)\} = 0$ ✓

$*\ Im\{X(j\omega)\} \neq 0$ ✗

$e^{j\alpha\omega}X(j\omega) =$ حقیقی و زوج $\Rightarrow x(t+\alpha)$ حقیقی و زوج باشد

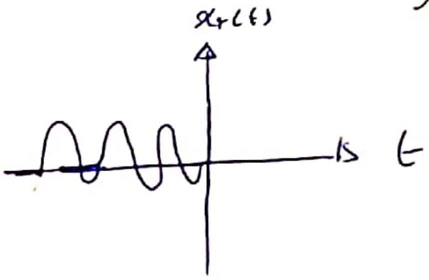
* مثلاً $\alpha = -1 \Rightarrow x_1(t-1) =$ حقیقی و زوج می‌شود ✓

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}\,d\omega$$

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\,d\omega = 0 \Rightarrow x(0) = 0$$

$*\ x_1(0) = 0$ ✓

$\int_{-\infty}^{+\infty} \omega X(j\omega) = 0 \Rightarrow$ شیب $x'(0) = 0$ ✗



الف) ✗

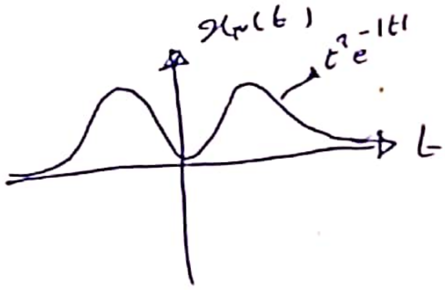
ب) ✗

پ) $x_2(t-3.5) \Rightarrow$ حقیقی و زوج ✓

$\alpha = -3.5$

ت) ✓

ث) ✓



الف) ✗

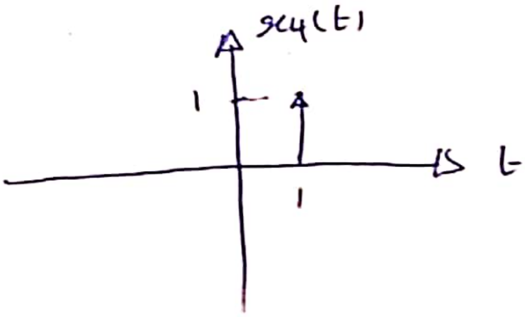
ب) ✓

پ) $x_3(t) \Rightarrow$ حقیقی و زوج ✓

$\alpha = 0$

ت) ✓

ث) ✗



الف) ✗

ب) ✗

پ) $x(t+1)$ ✓

$\alpha = 1$

ت) ✓

ث) ✓

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$$x(t) = \frac{1}{2} + 2\sin(1000\pi t + \pi/4) - 3\cos(2500\pi t - \pi/4) + 4\sin(4000\pi t)$$

$$X(j\omega) = \left(\frac{1}{2} \times 2\pi\delta(\omega)\right) + \left(\frac{2\pi}{j}\left[\delta(\omega - 1000\pi) - \delta(\omega + 1000\pi)\right] e^{j\pi/4\,\omega}\right)$$

$$+ \left(-3\pi\left[\delta(\omega - 2500\pi) + \delta(\omega + 2500\pi)\right] e^{-j\pi/4\,\omega}\right) +$$

$$\left(4\pi\left[\delta(\omega - 4000\pi) - \delta(\omega + 4000\pi)\right]\right)$$

$$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

$$|H(j\omega)| = \begin{cases} \frac{1}{2000\pi}|\omega| & |\omega| < 2000\pi \\ 4000\pi & 2000\pi \leq |\omega| \leq 3000\pi \\ 0 & \text{o.th} \end{cases}$$

$$\measuredangle H(j\omega) = \begin{cases} \pi/2 & 0 < \omega < 2000\pi \\ \omega/\cancel{4000} & 2000\pi \leq \omega \leq 3000\pi \\ 0 & \omega > 3000\pi \end{cases}$$

$$|Y(j\omega)| = \begin{cases} \left(\pi\delta(\omega) + \frac{2\pi}{j}\left[\delta(\omega - 1000\pi) - \delta(\omega + 1000\pi)\right]\right) \times \frac{|\omega|}{2000\pi} & |\omega| < 2000\pi \\ 4000\pi \times \left(-3\pi\left[\delta(\omega - 2500\pi) + \delta(\omega + 2500\pi)\right]\right] & 2000\pi \leq |\omega| \leq 3000\pi \\ 0 & \text{o.th.} \end{cases}$$

$$\measuredangle Y(j\omega) = \begin{cases} \pi/2 + \pi/4\,\omega & 0 < \omega < 2000\pi \\ \omega/6000 + (-\pi/4\,\omega) & 2000\pi \leq \omega \leq 3000\pi \\ 0 & \omega > 3000\pi \end{cases}$$

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