Electric and Magnetic field equations

$$\begin{split} \overline{E}(\bar{r}) &= \int_{c'} \frac{\rho_l(\bar{r}')dl'}{4\pi\epsilon R^2} \hat{a}_R & \overline{E}(\bar{r}) &= \int_{s'} \frac{\rho_s(\bar{r}')ds'}{4\pi\epsilon R^2} \hat{a}_R & \overline{E}(\bar{r}) &= \int_{v'} \frac{\rho_v(\bar{r}')dv'}{4\pi\epsilon R^2} \hat{a}_R \\ V(\bar{r}) &= \int_{c'} \frac{\rho_l(\bar{r}')dl'}{4\pi\epsilon R} & V(\bar{r}) &= \int_{s'} \frac{\rho_s(\bar{r}')ds'}{4\pi\epsilon R} & V(\bar{r}) &= \int_{v'} \frac{\rho_v(\bar{r}')dv'}{4\pi\epsilon R} \end{split}$$

$$\oint \ \overline{E} \cdot \overline{ds} = \frac{1}{c} \int \rho_{v} \, dv \qquad \nabla \cdot \overline{E} = \frac{\rho}{\epsilon_{0}} \qquad \nabla \times \overline{E} = 0 \qquad \nabla^{2}V = -\frac{\rho_{v}}{\epsilon_{0}} \qquad \nabla \cdot \overline{D} = \rho_{v} \qquad \overline{D} = \varepsilon \overline{D}$$

$$\begin{split} &\oint_{s} \ \overline{E} \,.\, \overline{ds} = \frac{1}{\varepsilon_{0}} \int_{v} \rho_{v} \,dv \qquad \nabla.\, \overline{E} = \frac{\rho}{\varepsilon_{0}} \qquad \nabla \times \overline{E} = 0 \qquad \nabla^{2}V = -\frac{\rho_{v}}{\varepsilon_{0}} \qquad \nabla.\, \overline{D} = \rho_{v} \qquad \overline{D} = \varepsilon \overline{E} \\ &\oint_{s} \ \overline{D} \,.\, \overline{ds} = Q \qquad V = \int_{L} \ \overline{E} \,.\, \overline{dl} \qquad C = \frac{Q}{V} \qquad \qquad \rho_{ps} = \overline{P} \,.\, \widehat{a}_{n} \qquad \rho_{pv} = -\nabla.\, \overline{P} \qquad \overline{D} = \varepsilon_{0} \overline{E} + \overline{P} \qquad \overline{P} = \varepsilon_{0} \chi_{e} \overline{E} \end{split}$$

$$\overline{B}(\overline{r}) = \int_{c'} \frac{\mu_0 I \overline{dI'} \times \overline{R}}{4\pi R^3} \qquad \overline{B}(\overline{r}) = \int_{s'} \frac{\mu_0 \vec{J}_s(\overline{r'}) \times \overline{R} ds'}{4\pi R^3} \qquad \overline{B}(\overline{r}) = \int_{v'} \frac{\mu_0 \vec{J}_v(\overline{r'}) \times \overline{R} dv'}{4\pi R^3}$$

$$\overline{A}(\overline{r}) = \int_{c'} \frac{\mu_0 I \overline{dI'}}{4\pi R} \qquad \overline{A}(\overline{r}) = \int_{s'} \frac{\mu_0 \vec{J}_s(\overline{r'}) ds'}{4\pi R} \qquad A(\overline{r}) = \int_{v'} \frac{\mu_0 \vec{J}_v(\overline{r'}) dv'}{4\pi R}$$

$$\oint_{S} \overline{B} \cdot \overline{dS} = 0 \qquad \nabla \cdot \overline{B} = 0 \qquad \nabla \times \overline{B} = \mu_{0} \vec{J}_{v} \qquad \nabla^{2} \overline{A} = -\mu_{0} \vec{J}_{v} \qquad \nabla \times \overline{H} = \vec{J}_{v} \qquad \overline{B} = \mu_{0} \overline{H}$$

$$\oint_{I} \overline{H} \cdot \overline{dL} = I \qquad L = \frac{\Psi}{I} \qquad \vec{J}_{ms} = \overline{M} \times \hat{a}_{n} \qquad \vec{J}_{mv} = \nabla \times \overline{M} \qquad \overline{H} = \frac{1}{\mu_{0}} \overline{B} - \overline{M} \qquad \overline{M} = \frac{\chi_{m}}{\mu_{0}} \overline{B}$$

Boundary conditions

$$\hat{\mathbf{a}}_{n21} \times (\overline{\mathbf{E}}_1 - \overline{\mathbf{E}}_2) = 0 \qquad \hat{\mathbf{a}}_{n21}.(\overline{\mathbf{D}}_1 - \overline{\mathbf{D}}_2) = \rho_s$$

$$\hat{\mathbf{a}}_{n21} \times (\overline{\mathbf{H}}_1 - \overline{\mathbf{H}}_2) = \vec{J}_s \qquad \hat{\mathbf{a}}_{n21}.(\overline{\mathbf{B}}_1 - \overline{\mathbf{B}}_2) = 0$$

Some useful integrals

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2 (x^2 + a^2)^{1/2}} \qquad \int \frac{x dx}{(x^2 + a^2)^{3/2}} = \frac{-1}{(x^2 + a^2)^{1/2}}$$

Cylindrical to Cartesian coordinate transforms

Inner product (.)	â _x	â _y	â _z
â _r	cosφ	sinφ	0
âφ	-sinφ	cosφ	0
âz	0	0	1

Spherical to Cartesian coordinate transforms

Inner product	â _x	â _y	â _z
â _r	sinθ cosφ	$\sin\theta\sin\phi$	cosθ
âθ	cosθ cosφ	cosθ sinφ	- sinθ
âφ	- sinφ	cosφ	0

$$\begin{split} & \nabla.\overline{\mathbf{A}} = \frac{1}{h_1 h_2 h_3} \bigg[\frac{\partial}{\partial \mathbf{u}_1} (h_2 h_3 \mathbf{A}_1) + \frac{\partial}{\partial \mathbf{u}_2} (h_1 h_3 \mathbf{A}_2) + \frac{\partial}{\partial \mathbf{u}_3} (h_1 h_2 \mathbf{A}_3) \bigg] \\ & \nabla \times \overline{\mathbf{A}} = \frac{1}{h_1 h_2 h_3} \bigg[\frac{\partial}{\partial \mathbf{u}_1} & \frac{\partial}{\partial \mathbf{u}_2} & \frac{\partial}{\partial \mathbf{u}_3} \\ & \frac{\partial}{\partial \mathbf{u}_1} & \frac{\partial}{\partial \mathbf{u}_2} & \frac{\partial}{\partial \mathbf{u}_3} \\ & h_1 \mathbf{A}_1 & h_2 \mathbf{A}_2 & h_3 \mathbf{A}_3 \bigg] \\ & \nabla \mathbf{f} = \frac{1}{h_1 h_2 h_3} \bigg[\frac{\partial}{\partial \mathbf{u}_1} \left(\frac{h_2 h_3}{h_1} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_2} \right) + \frac{\partial}{\partial \mathbf{u}_2} \left(\frac{h_1 h_3}{h_2} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_2} \left(\frac{h_1 h_3}{h_2} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_2} \left(\frac{h_1 h_2}{h_2} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_2} \left(\frac{h_1 h_2}{h_2} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_2} \left(\frac{h_1 h_2}{h_2} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_2} \left(\frac{h_1 h_2}{h_2} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_2} \left(\frac{h_1 h_2}{h_2} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_2} \left(\frac{h_1 h_2}{h_2} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_2}{h_2} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_2}{h_2} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_2}{h_2} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_2}{h_2} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_2}{h_2} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_2}{h_2} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_2}{h_3} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_2}{h_3} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_2}{h_3} \frac{\partial \mathbf{f}}{\partial \mathbf{u}_3} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_2}{h_3} \frac{\partial \mathbf{f}}{\partial \mathbf{f}} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_2}{h_3} \frac{\partial \mathbf{f}}{\partial \mathbf{f}} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_2}{h_3} \frac{\partial \mathbf{f}}{\partial \mathbf{f}} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_3}{h_3} \frac{\partial \mathbf{f}}{\partial \mathbf{f}} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_3}{h_3} \frac{\partial \mathbf{f}}{\partial \mathbf{f}} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_3}{h_3} \frac{\partial \mathbf{f}}{\partial \mathbf{f}} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_3}{h_3} \frac{\partial \mathbf{f}}{\partial \mathbf{f}} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_3}{h_3} \frac{\partial \mathbf{f}}{\partial \mathbf{f}} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_3}{h_3} \frac{\partial \mathbf{f}}{\partial \mathbf{f}} \right) + \frac{\partial}{\partial \mathbf{u}_3} \left(\frac{h_1 h_3}{h_3} \frac{\partial \mathbf{f}}{\partial \mathbf{f}} \right) + \frac{\partial}{\partial \mathbf{f}} \left(\frac{h_1 h_3}{h_3} \frac{\partial \mathbf{f}}{\partial \mathbf{f}} \right)$$

Cartesian coordinate

$$\begin{split} & \overline{dl}_x = dx \hat{a}_x, \qquad \overline{dl}_y = dy \hat{a}_y, \qquad \overline{dl}_z = dz \hat{a}_z \\ & \overline{dl} = dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z \\ & \overline{ds}_x = dy dz \hat{a}_x, \quad \overline{ds}_y = dx dz \hat{a}_y, \quad \overline{ds}_z = dx dy \hat{a}_z \\ & dv = dx dy dz \\ & \overline{r} = x \hat{a}_x + y \hat{a}_y + z \hat{a}_z \end{split}$$

Cylindrical coordinate

$$\begin{split} & \vec{dl}_r = dr \hat{a}_r, \quad \vec{dl}_\phi = r d\phi \hat{a}_\phi, \quad \vec{dl}_z = dz \hat{a}_z \\ & \vec{dl} = dr \hat{a}_r + r d\phi \hat{a}_\phi + dz \hat{a}_z \\ & \vec{ds}_r = r d\phi dz \hat{a}_r, \quad \vec{ds}_\phi = dr dz \hat{a}_\phi, \quad \vec{ds}_z = r dr d\phi \hat{a}_z \\ & dv = r dr d\phi dz \\ & \vec{r} = r \hat{a}_r + z \hat{a}_z \end{split}$$

Spherical coordinate

$$\begin{split} & \overline{dl}_r = dr \hat{a}_r, \qquad \overline{dl}_\theta = r d\theta \hat{a}_\theta, \qquad \overline{dl}_\phi = r sin\theta d\phi \hat{a}_\phi \\ & \overline{dl} = dr \hat{a}_r + r d\theta \hat{a}_\theta + r sin\theta d\phi \hat{a}_\phi \\ & \overline{ds}_r = r^2 sin\theta d\theta d\phi \hat{a}_r, \quad \overline{ds}_\theta = r sin\theta dr d\phi \hat{a}_\theta, \quad \overline{ds}_\phi = r dr d\theta \hat{a}_\phi \\ & dv = r^2 sin\theta dr d\theta d\phi \\ & \overline{r} = r \hat{a}_r \end{split}$$

$$sin^{2}x = \frac{1 - \cos(2x)}{2}, \quad cos^{2}x = \frac{1 + \cos(2x)}{2}$$

$$sin^{3}x = \frac{1}{4}(-\sin(3x) + 3\sin x)$$

$$cos^{3}x = \frac{1}{4}(\cos(3x) + 3\cos x)$$