یالطیف

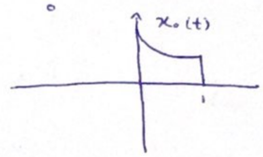
رضا ادیب پور

٩٨١٤٣٠٣

تمرین سری ۶ سیگنال سیستم (تبدیل فوریه پیوسته)

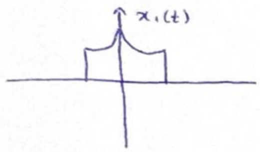
#1

$$x_0(t) = \begin{cases} e^{-t} & ; \ 0 \le t \le 1 \\ 0 & ; \ \text{other} \end{cases} \Rightarrow X_0(jw) = \int_{-\infty}^{+\infty} x(t) e^{-jwt} dt = \int_0^1 e^{-t} e^{-jwt} dt = \int_0^1 e^{-(1+jw)t} dt$$



$$= \frac{-1}{1+jw} e^{-(1+jw)t} \Big|_0^1 = \frac{1}{1+jw}\left(1 - e^{-(1+jw)}\right)$$

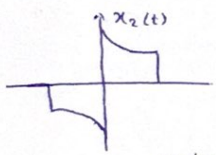
a)



$$x_1(t) = x_0(t) + x_0(-t) \xleftrightarrow{F} X_0(jw) + X_0(-jw)$$

$$X_1(jw) = \frac{1}{1+jw}\left(1 - e^{-(1+jw)}\right) + \frac{1}{1-jw}\left(1 - e^{-(1-jw)}\right)$$

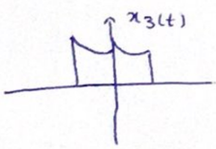
b)



$$x_2(t) = x_0(t) - x_0(-t) \xleftrightarrow{F} X_0(jw) - X_0(-jw)$$

$$X_2(jw) = \frac{1}{1+jw}\left(1 - e^{-(1+jw)}\right) - \frac{1}{1-jw}\left(1 - e^{-(1-jw)}\right)$$

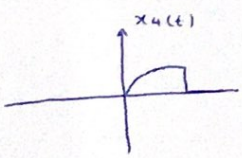
c)



$$x_3(t) = x_0(t) + x_0(t+1) \xleftrightarrow{F} X_0(jw) + X_0(jw)e^{jw}$$

$$X_3(jw) = \frac{1}{1+jw}\left(1 - e^{-(1+jw)}\right) + \frac{1}{1+jw}\left(1 - e^{-(1+jw)}\right) \cdot e^{jw} = \frac{1}{1+jw}\left(1 - e^{-(1+jw)}\right) \cdot \left(1 + e^{jw}\right)$$

d)



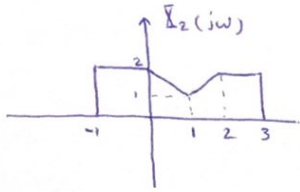
$$x_4(t) = t\,x_0(t) \xleftrightarrow{F} X_4(jw) = j\frac{d}{dw} X_0(jw)$$

$$X_4(jw) = j\frac{d}{dw}\left[\frac{1}{1+jw}\left(1 - e^{-(1+jw)}\right)\right] = \frac{1 - 2e^{-1}e^{-jw} - jwe^{-1}e^{-jw}}{(1+jw)^2}$$

#3

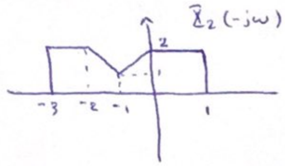
a) $$X_1(jw) = \frac{2\sin(3(w-2\pi))}{(w-2\pi)}$$

b)

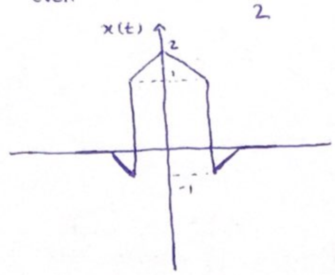


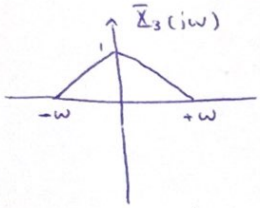
if $x(t)$ is Real $\Rightarrow \underset{\text{even}}{x(t)} \xleftrightarrow{F} Re\{\bar{X}_2(j\omega)\}$

$x_{even}(t) = \dfrac{\bar{X}_2(j\omega) + \bar{X}_2(-j\omega)}{2}$



$\Rightarrow$



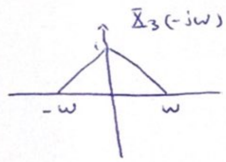


b)

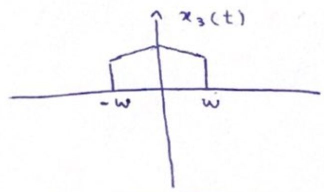
if $x_3(t)$ is Real $\Rightarrow \underset{\text{even}}{x_3(t)} \xleftrightarrow{F} Re\{\bar{X}_3(j\omega)\}$

$x_{even}(t) = \dfrac{\bar{X}_3(j\omega) + \bar{X}_3(-j\omega)}{2}$

$\Rightarrow$



$\Rightarrow$



#4 a) $\begin{cases} x(t) = te^{-2t}u(t) \\ h(t) = e^{-4t}u(t) \end{cases}$

$\bar{X}(j\omega) = \int_{-\infty}^{+\infty} te^{-2t}u(t)e^{j\omega t}\,dt = \int_0^{\infty} te^{-2t}e^{j\omega t}\,dt$

$= \int_0^{\infty} te^{-(2-j\omega)t}\,dt$

$\bar{X}(j\omega) = \left[\cancel{\dfrac{-t}{2-j\omega}e^{-(2-j\omega)t}} - \dfrac{1}{4-\omega^2}e^{-(2-j\omega)t}\right]_0^{\infty}$

مشتق	انتگرال
t	$e^{-(2-j\omega)t}$
1	$\oplus\ \dfrac{-1}{2-j\omega}e^{-(2-j\omega)}$
0	$\ominus\ \dfrac{1}{4-\omega^2}e^{-(2-j\omega)}$

$= \left(\dfrac{1}{4-\omega^2} - 0\right) = \boxed{\dfrac{1}{4-\omega^2} = \bar{X}(j\omega)}$

$H(j\omega) = \dfrac{1}{4+j\omega} \Rightarrow y(t) = x(t) * h(t) = \bar{X}(j\omega)H(j\omega)$

$\Rightarrow Y(j\omega) = \dfrac{1}{4+j\omega} \times \dfrac{1}{4-\omega^2} = \dfrac{-1}{4\omega^2 - 16 + j(\omega^3 - 4\omega)}$

b) $\begin{cases} x(t) = e^{-t}u(t) \longrightarrow \bar{X}(j\omega) = \dfrac{1}{1+j\omega} \\ h(t) = e^{t}u(-t) \longrightarrow h(t) \to h(-t) : \bar{X}(j\omega) \to \bar{X}(-j\omega) \Rightarrow \boxed{H(j\omega) = \dfrac{1}{1-j\omega}} \end{cases}$

$\Rightarrow y(t) = x(t) * h(t) \Rightarrow Y(j\omega) = \bar{X}(j\omega)H(j\omega) = \dfrac{1}{1+j\omega} \times \dfrac{1}{1-j\omega} = \dfrac{1}{1+\omega^2}$

5

$$\begin{cases} x(t) = \cos t = \overset{=a_1}{\tfrac{1}{2}} e^{jt} + \overset{=a_{-1}}{\tfrac{1}{2}} e^{-jt} \Rightarrow \bar{X}(j\omega) = 2\pi\left(a_1 \delta(\omega-\omega_0) + a_{-1}\delta(\omega+\omega_0)\right) \\ \Rightarrow \bar{X}(j\omega) = \pi\delta(\omega-1) + \pi\delta(\omega+1) \\ a)\ h_1(t) = u(t) \\ b)\ h_2(t) = -2\delta(t) + 5e^{-2t}u(t) \quad , \quad c)\ h_3(t) = 2te^{-t}u(t) \end{cases}$$

$$H_1(j\omega) = \int_0^\infty e^{-j\omega t}\,dt = \frac{-1}{j\omega} e^{-j\omega t}\Big|_0^\infty = \frac{1}{j\omega}$$

$$H_2(j\omega) = \int_{-\infty}^{+\infty} -2\delta(t) e^{-j\omega t}\,dt + 5\int_0^\infty e^{-2t} e^{-j\omega t}\,dt = -2 + 5\int_0^\infty e^{-(2+j\omega)t}\,dt = -2 + \frac{-5}{2+j\omega} e^{-(2+j\omega)t}\Big|_0^\infty$$

$$= \boxed{\frac{5}{2+j\omega} - 2}$$

$$H_3(j\omega) = \int_0^\infty 2te^{-t}e^{-j\omega t}\,dt = 2\int_0^\infty t e^{-(1+j\omega)t}\,dt = 2\left[\cancel{\frac{-t}{1+j\omega} e^{-(1+j\omega)t}} - \frac{1}{1-\omega^2} e^{-(1+j\omega)t}\right]_0^\infty$$

t	$e^{-(1+j\omega)t}$
1 (+)	$\frac{-1}{1+j\omega} e^{-(1+j\omega)t}$
0 (−)	$\frac{1}{1-\omega^2} e^{-(1+j\omega)t}$

$$= \boxed{\frac{1}{1-\omega^2} = H_3(j\omega)}$$

$$x(t) \longrightarrow \boxed{h(t)} \xrightarrow{} y(t) \qquad \bar{X}(\omega) \longrightarrow \boxed{H(j\omega)} \longrightarrow Y(j\omega)$$

$$y(t) = x(t) * h(t) \qquad Y(j\omega) = \bar{X}(j\omega) H(j\omega)$$

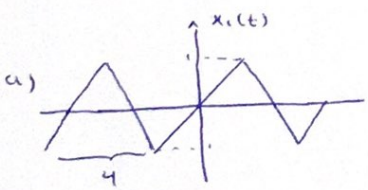
$$Y_1(j\omega) = \bar{X}(j\omega) H_1(j\omega) = \left(\pi\delta(\omega-1) + \pi\delta(\omega+1)\right)\cdot \frac{1}{j\omega} = \frac{\pi}{j\omega}\delta(\omega-1) + \frac{\pi}{j\omega}\delta(\omega+1)$$

$$Y_2(j\omega) = \bar{X}(j\omega) H_2(j\omega) = \left(\pi\delta(\omega-1) + \pi\delta(\omega+1)\right)\cdot\left(\frac{5}{2+j\omega} - 2\right)$$

$$Y_3(j\omega) = \bar{X}(j\omega) H_3(j\omega) = \left(\pi\delta(\omega-1) + \pi\delta(\omega+1)\right)\cdot\left(\frac{1}{1-\omega^2}\right)$$

6

a)



الف) x_1 is Real and odd $\Rightarrow \bar{X}_1(j\omega)$ موهومی خالص و فرد

$\Rightarrow \bar{X}_1(\omega=0) = 0$

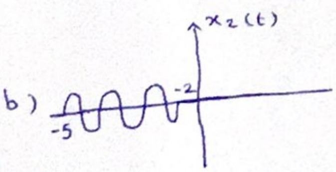
ب) بخش موهومی تبدیل فوریه صفر نیست

پ) $\bar{X}_1(j\omega) \rightarrow$ موهومی خالص است پس رابطه داده شده برقرار نیست

ت) $\int_{-\infty}^{+\infty} \bar{X}_1(j\omega)\,d\omega = 0 \rightarrow$ چون سیگنال فرد است

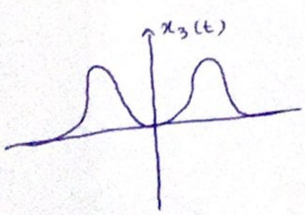
ث) برقرار نیست

b)



اگر $x_2(t)$ را به سمت راست شیفت دهیم، سیگنال فردی شود. پس قسمت الف برقرار است
قسمت ب برقرار نیست چون سیگنال موهومی خالص و فرد است، قسمت پ برقرار نیست
قسمت ت صحیح است و قسمت ث برقرار نیست

c)



الف) $x_3(t)$ is Real and even $\Rightarrow \bar{X}_3(j\omega)$: Real and even

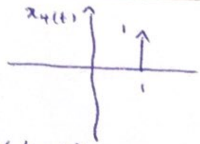
$\Rightarrow$ قسمت الف الزاما برقرار نیست

ب) بخش موهومی $\bar{X}_3$ صفر است

پ) $\bar{X}_3(j\omega)$ همواره حقیقی است

ت) برقرار نیست چون $\bar{X}_3(j\omega)$ زوج است

ث) برقرار است

d) 

$x(t) = \delta(t-1)$

$\longrightarrow \bar{X}_4(j\omega) = \frac{1}{2\pi} e^{-j\omega} \longrightarrow \frac{1}{2\pi}\left(\cos(\omega) - j\sin(\omega)\right)$

الف) در صفر ۰ $\bar{X}_4(j\omega) =$

ب) بخش موهومی $\bar{X}_4(j\omega) \neq 0$

پ) درست است

ت) نادرست است

ث) برقرار نیست

#7 $|H(j\omega)| = \begin{cases} \frac{1}{2000\pi}|\omega| & |\omega| < 2000\pi \\ 4000\pi & 2000\pi \leq |\omega| \leq 3000\pi \\ 0 & \text{other} \end{cases}$, $\measuredangle H(j\omega) = \begin{cases} \frac{\pi}{2} & 0 < \omega < 2000\pi \\ \frac{\omega}{6000} & 2000\pi \leq \omega \leq 3000\pi \\ 0 & \omega > 3000\pi \end{cases}$

$\measuredangle H(j\omega) = -\measuredangle H(-j\omega)$

$x(t) = \frac{1}{2} + 2\sin\left(1000\pi t + \frac{\pi}{4}\right) - 3\cos\left(2500\pi t - \frac{\pi}{4}\right) + 4\sin(4000\pi t)$