یالطیف

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تمرین سری ۴ - محاسبات عددی
گروه ۵

#5

$$\begin{cases} E(t) = L\dfrac{dI}{dt} + RI \\ L = 0.98 \text{ H} \\ R = 0.142\ \Omega \end{cases}$$

t	1.00	1.01	1.02	1.03	1.04
$I(t)$	3.10	3.12	3.14	3.18	3.24

$$E(t)\Big|_{t=1} \Rightarrow 0.98\,\frac{dI}{dt}(1) + 0.142\,I(1) \Rightarrow I'(1) \cong \frac{I(1.01) - I(1)}{0.01} = \frac{3.12 - 3.10}{0.01} = 2$$

$$\Rightarrow E(1) \cong 0.98 \times 2 + 0.142 \times 3.10 = \underline{2.4002}$$

$$E(t)\Big|_{t=1.02} \Rightarrow 0.98\,I'(1.02) + 0.142\,I(1.02) \Rightarrow I'(1.02) \cong \frac{I(1.03) - I(1.01)}{2 \times 0.01} = \frac{3.18 - 3.12}{0.02} = 3$$

$$\Rightarrow E(1.02) \cong 0.98 \times 3 + 0.142 \times 3.14 = \underline{3.38588}$$

$$E(t)\Big|_{t=1.04} \Rightarrow 0.98\,I'(1.04) + 0.142\,I(1.04) \Rightarrow I'(1.04) = \frac{I(1.04) - I(1.03)}{0.01} = \frac{3.24 - 3.18}{0.01} = 6$$

$$\Rightarrow E(1.04) = 0.98 \times 6 + 0.142 \times 3.24 = \underline{6.34008}$$

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$$\begin{cases} f(x) = (\sin x)^{\frac{9}{2}} \rightarrow [0, \frac{\pi}{2}] \\ |f^{(4)}(x)| \leq 60 \\ \int_0^{\frac{\pi}{2}} f(x)\,dx \end{cases}$$

$$f'(x) = \frac{9}{2}\cos x(\sin x)^{\frac{7}{2}}$$

$$f''(x) = -\frac{9}{2}(\sin x)^{\frac{9}{2}} + \frac{63}{4}\cos^2 x(\sin x)^{\frac{5}{2}}$$

$$f^{(3)}(x) = -\frac{81}{4}\cos x(\sin x)^{\frac{7}{2}} - \frac{63}{2}\cos x(\sin x)^{\frac{7}{2}} + \frac{315}{8}\cos^3 x(\sin x)^{\frac{3}{2}} \Rightarrow$$

$$f^{(3)}(x) = \frac{-207}{4}\cos x(\sin x)^{\frac{7}{2}} + \frac{315}{8}\cos^3 x(\sin x)^{\frac{3}{2}}$$

$$f^{(4)}(x) = \frac{207}{4}(\sin x)^{\frac{9}{2}} \times \frac{-1197}{4}\cos^2 x(\sin x)^{\frac{5}{2}} + \frac{945}{16}\cos^4 x \times (\sin x)^{\frac{1}{2}}$$

$$\xrightarrow{\text{با سرچ در اینترنت}} |f^{(4)}|_{max} \rightarrow x = \frac{\pi}{2} \quad \text{در نقطه}$$

$$\Rightarrow \forall x \in [0, \frac{\pi}{2}] \Rightarrow |f^{(4)}(x)| \leq \frac{207}{4} < 60$$

$$\Rightarrow |E| \leq \frac{(b-a)}{180}h^4 M \xrightarrow{|f^{(4)}(x)| < M} \frac{b-a}{180}h^4 M \leq 10^{-4}$$

$$\Rightarrow \frac{\pi}{2 \times 180}h^4 \times 60 \leq 10^{-4} \Rightarrow h \leq 0.1\sqrt[4]{\frac{30}{\pi}} \Rightarrow \frac{b-a}{n} \leq 0.1 \times \sqrt[4]{\frac{30}{\pi}}$$

$$\Rightarrow \frac{2n}{\pi} \geq 10\sqrt[4]{\frac{\pi}{30}} \Rightarrow n \geq 5\pi\sqrt[4]{\frac{\pi}{30}} = 5.93 \longrightarrow$$

n حداقل باید 7 باشد

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$$\begin{cases} erf = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_0^x e^{-t^2} dt \\ \text{الف)}\ erf(0.5) \xrightarrow{\text{سیمپسون}} h = \frac{1}{8} \\ \text{ب)}\ erf(0.5) \xrightarrow[{[0,\,0.5]}]{\text{دو نقطه‌ای}} \text{4 رقم اعشار درست} \end{cases}$$

الف) $erf(0.5) = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_0^{0.5} e^{-t^2} dt \xrightarrow{h=\frac{1}{8}} \int_0^{0.5} e^{-t^2} dt = \frac{1}{24}\left[f(0) + 4f(\tfrac{1}{8}) + 2f(\tfrac{2}{8}) + 4f(\tfrac{3}{8}) + 4f(\tfrac{4}{8})\right] = 0.4612864$

$\Rightarrow erf(0.5) \cong \dfrac{2}{\sqrt{\pi}}(0.4612864) = \underline{0.5205059}$

ب) $\dfrac{a-b}{12} h^2 f''(\eta) = |E| < 0.5 \times 10^{-4} \Rightarrow |f''(t)| = \left|\dfrac{2}{\sqrt{\pi}}(4t^2-2)e^{-t}\right| = g(t)$

$\longrightarrow |g(t)|_{max} \longrightarrow t = 0$ در نقطه $\longrightarrow \dfrac{4}{\sqrt{\pi}}$

$\Rightarrow \dfrac{0.5}{12} h^2 \times \dfrac{4}{\sqrt{\pi}} < \dfrac{1}{2} \times 10^{-4} \Rightarrow \dfrac{h^2}{6\sqrt{\pi}} < \dfrac{1}{2} \times 10^{-4} \Rightarrow h = \dfrac{1}{2n} < 0.023 \Rightarrow n > 21.65$

حداقل n باید برابر با $\underline{22}$ باشد

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$$\begin{cases} f(x) = x^4 + ax^3 + bx^2 + cx + d \\ I = \displaystyle\int_{-1}^{+1} f(x)\,dx \end{cases}$$

$I = \displaystyle\int_{-1}^{+1} f(x)dx \cong f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right) = \frac{2}{9} + \frac{2b}{3} + 2d \qquad (I)$

از طرفی: $\displaystyle\int_{-1}^{1} f(x)dx = \int_{-1}^{1}(x^4 + ax^3 + bx^2 + cx + d)\,dx = \frac{2}{5} + \frac{2b}{3} + 2d \qquad (II)$

$\xrightarrow{(I),\,(II)} E = I - \left(\dfrac{2}{9} + \dfrac{2b}{3} + 2d\right) = \dfrac{8}{45}$

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$I = \displaystyle\int_0^{\infty} \frac{x}{e^x+1} dx = ? \qquad \varepsilon < 0.001$

$I = I_1 + E \longrightarrow I_1 = \displaystyle\int_0^a \frac{x}{e^x+1} dx \quad , \quad E = \int_a^{\infty} \frac{x}{e^x+1} dx$

$\Rightarrow E \le \displaystyle\int_a^{\infty} \frac{x}{e^x} dx = \int_a^{\infty} x e^{-x} dx = e^{-a} \times (1+a) \xrightarrow{E<0.001} e^{-a}(1+a) < 0.001$

$\xrightarrow{a=10} I \simeq I_1 = \displaystyle\int_0^{10} \frac{x}{e^x+1} dx \xrightarrow[h=0.5]{f(x)=\frac{x}{e^x+1}} I_1 \simeq \frac{1}{4}\left[f(0) + 2\sum_{i=1}^{19} f(0.5i) + f(10)\right]$

$= \underline{0.81154}$