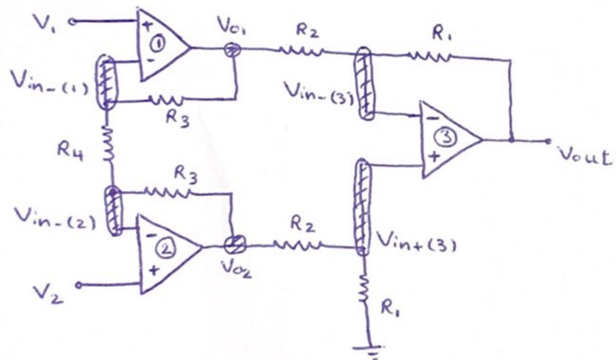
In the name of God

Assignment 9 :

1. The configuration shown below is known as the "Instrumentation Amplifier" or "IA". Determine the output voltage in term of the input voltages. How much is the input resistance seen from the input terminals (V_1 and V_2). Assume ideal op-Amps.



op Amp are Ideal $\begin{cases} V_{in+} = V_{in-} \\ R_{in} = \infty \\ I_{in} = 0 \end{cases}$

$\begin{cases} V_1 = V_{in-(1)} \\ V_2 = V_{in-(2)} \end{cases}$

KCL @ $V_{in-(1)}$: $\dfrac{V_{in-(1)} - V_{in-(2)}}{R_4} + \dfrac{V_{in-(1)} - V_{o1}}{R_3} = 0 \xrightarrow[V_{in-(2)} = V_2]{V_{in-(1)} = V_1} \dfrac{V_1 - V_2}{R_4} + \dfrac{V_1 - V_{o1}}{R_3} = 0$

$$\Rightarrow \boxed{V_{o1} = V_1\left(\frac{R_3}{R_4}+1\right) - \frac{R_3}{R_4}V_2} \quad (\mathrm{I})$$

KCL @ $V_{in-(2)}$: $\dfrac{V_2 - V_1}{R_4} + \dfrac{V_2 - V_{o2}}{R_3} = 0 \Rightarrow \boxed{V_{o2} = V_2\left(\dfrac{R_3}{R_4}+1\right) - \dfrac{R_3}{R_4}V_1} \quad (\mathrm{II})$

KCL in $V_{in-(3)}$: $\dfrac{V_{in-(3)} - V_{out}}{R_1} + \dfrac{V_{in-(3)} - V_{o1}}{R_2} = 0 \xrightarrow{(\mathrm{I})} \dfrac{V_{in-(3)} - V_{out}}{R_1} + \dfrac{V_{in-(3)}}{R_2} - \dfrac{1}{R_2}\left(V_1\left(\dfrac{R_3}{R_4}+1\right) - \dfrac{R_3}{R_4}V_2\right) \Rightarrow \dfrac{V_{out}}{R_1} = \dfrac{V_{in-(3)}}{R_2} - V_1\left(\dfrac{R_3}{R_2R_4} + \dfrac{1}{R_2}\right) + \dfrac{R_3}{R_2R_4}V_2 + \dfrac{V_{in-(3)}}{R_1}$

KCL @ $V_{in+(3)}$: $\dfrac{V_{in+(3)} - 0}{R_1} + \dfrac{V_{in+(3)} - V_{o2}}{R_2} = 0 \xrightarrow{(\mathrm{II})} \dfrac{V_{in+(3)}}{R_1} + \dfrac{V_{in+(3)}}{R_2} - \dfrac{1}{R_2}\left(V_2\left(\dfrac{R_3}{R_4}+1\right) - \dfrac{R_3}{R_4}V_1\right)$

$$\Rightarrow V_{in+(3)}\left[\frac{1}{R_1}+\frac{1}{R_2}\right] = V_2\left(\frac{R_3}{R_2R_4}+\frac{1}{R_2}\right) - \frac{R_3}{R_2R_4}V_1 \Rightarrow V_{in+(3)} = \frac{V_2\left(\frac{R_3}{R_2R_4}+\frac{1}{R_2}\right) - \frac{R_3}{R_2R_4}V_1}{\frac{1}{R_1}+\frac{1}{R_2}}$$

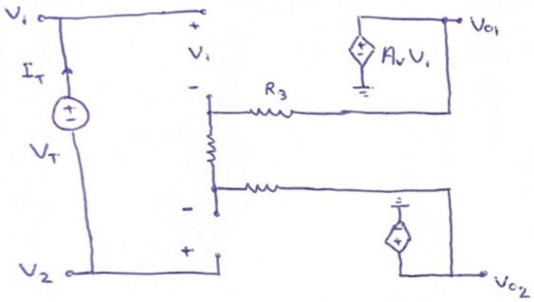
$$\Rightarrow V_{in+(3)} = V_{in-(3)} : \frac{V_2\left(\frac{R_3}{R_2R_4}+\frac{1}{R_2}\right) - \frac{R_3}{R_2R_4}V_1}{\cancel{\frac{1}{R_1}+\frac{1}{R_2}}} = \frac{\frac{V_{out}}{R_1} + V_1\left(\frac{R_3}{R_2R_4}+\frac{1}{R_2}\right) - \frac{R_3}{R_2R_4}V_2}{\cancel{\frac{1}{R_2}+\frac{1}{R_1}}}$$

$$V_2\left(\frac{R_3}{R_2R_4}+\frac{1}{R_2}\right)-\frac{R_3}{R_2R_4}V_1 = V_1\left(\frac{R_3}{R_2R_4}+\frac{1}{R_2}\right)-\frac{R_3}{R_2R_4}V_2+\frac{V_{out}}{R_1}$$

$$V_2\left[\frac{R_3}{R_2R_4}+\frac{1}{R_2}+\frac{R_3}{R_2R_4}\right]-V_1\left[\frac{R_3}{R_2R_4}+\frac{R_3}{R_2R_4}+\frac{1}{R_2}\right]=\frac{V_{out}}{R_1}$$

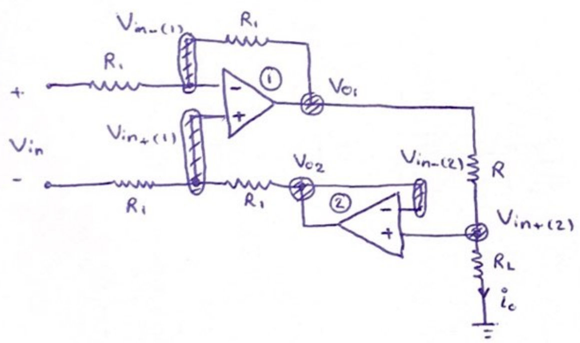
$$V_2\left[\frac{1}{R_2}+\frac{2R_3}{R_2R_4}\right]-V_1\left[\frac{1}{R_2}+\frac{2R_3}{R_2R_4}\right]=\frac{V_{out}}{R_1}\xrightarrow{\times R_1} V_{out}=V_2\left[\frac{R_1}{R_2}+\frac{2R_1R_3}{R_2R_4}\right]-V_1\left[\frac{R_1}{R_2}+\frac{2R_1R_3}{R_2R_4}\right]$$

$$V_{out}=V_1-V_2\left[\frac{R_1}{R_2}+\frac{2R_1R_3}{R_2R_4}\right] \Rightarrow \boxed{\frac{V_{out}}{V_1-V_2}=\frac{R_1}{R_2}+\frac{2R_1R_3}{R_2R_4}}$$



$$\frac{V_T}{I_T}=R_{in}=\infty$$

2. Calculate the output current (i_o) in the following Circuit. the op-Amps are supposed to be ideal one.



$$KCL @ V_{in-(1)}: \frac{V_{in-(1)}-V_{in}}{R_1}+\frac{V_{in-(1)}-V_{o1}}{R_1}=0 \xrightarrow{V_{in-(1)}=V_{in+(1)}} \frac{V_{in-(1)}-V_{in}}{R_1}+\frac{V_{in-(1)}-V_{o1}}{R_1}=0$$

$$V_{in-(1)}\left(\frac{1}{R_1}+\frac{1}{R_1}\right)=\frac{V_{in}}{R_1}+\frac{V_{o1}}{R_1} \Rightarrow \boxed{V_{in-(1)}=\tfrac{1}{2}\left(V_{in}+V_{o1}\right)} \quad (I)$$

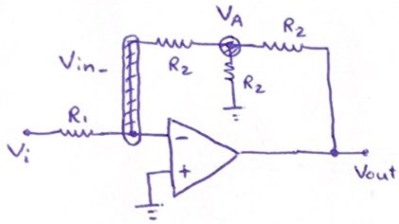
$$KCL @ V_{in+(1)}: \frac{V_{in+(1)}+V_{in}}{R_1}+\frac{V_{in+(1)}-V_{o2}}{R_1}=0 \xrightarrow[(I)]{V_{in-(1)}=V_{in+(1)}} \frac{1}{R_1}\left[\tfrac{1}{2}(V_{in}+V_{o1})\right]+\frac{V_{in}}{R_1}+\frac{1}{R_1}\left[\tfrac{1}{2}(V_{in}+V_{o1})\right]$$

$$-\frac{V_{o2}}{R_1}=0 \Rightarrow \frac{1}{R_1}\left[V_{in}+V_{o1}\right]+\frac{V_{in}}{R_1}-\frac{V_{o2}}{R_1}=0 \xrightarrow[=V_{in+(2)}]{V_{o2}=V_{in-(2)}} \boxed{V_{o2}=2V_{in}+V_{o1}} \quad (III)$$

$$KCL @ V_{in+(2)}: \frac{V_{in+(2)}-V_{o1}}{R}+i_o=0 \xrightarrow[V_{in-(2)}=V_{in+(2)}=V_{o2}]{(I)} \frac{1}{R}\left[2V_{in}+V_{o1}\right]-\frac{V_{o1}}{R}+i_o=0$$

$$\Rightarrow \boxed{i_o=\frac{-2V_{in}}{R}}$$

3. Determine the Voltage gain $\frac{V_o}{V_i}$ in the following Circuit. Assume that the op Amp is ideal.



$$\text{KCL @ } V_{in-}: \frac{\cancel{V_{in-}} - V_i}{R_1} + \frac{\cancel{V_{in-}} - V_A}{R_2} = 0$$

$$\Rightarrow \frac{V_i}{R_1} = \frac{-V_A}{R_2} \Rightarrow \boxed{V_A = \frac{-R_2}{R_1} V_i}$$

$$\text{KCL @ } V_A: \frac{V_A - V_{out}}{R_2} + \frac{V_A - 0}{R_2} + \frac{V_A - \cancel{V_{in-}}}{R_2} = 0$$

(I)

$$\Rightarrow \frac{1}{R_2}\left(\frac{-R_2}{R_1} V_i\right) + \frac{1}{R_2}\left(\frac{-R_2}{R_1} V_i\right) + \frac{1}{R_2}\left(\frac{-R_2}{R_1} V_i\right) = \frac{V_{out}}{R_2}$$

$$\frac{-3}{\cancel{R_2}} \cdot \frac{\cancel{R_2}}{R_1} V_i = \frac{V_{out}}{R_2} \Rightarrow \boxed{\frac{V_{out}}{V_i} = -3 \frac{R_2}{R_1}}$$

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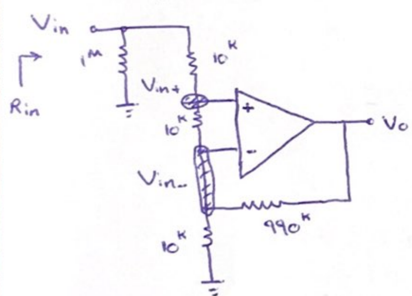
4. calculate the input resistance and the Voltage gain of the following structure. the op amp is ideal.



$$\text{KCL in } V_{in+}: \cancel{\frac{V_{in+} - V_{in-}}{10^K}} + \frac{V_{in+} - V_{in}}{10^K} = 0 \xrightarrow{V_{in+} = V_{in-}}$$

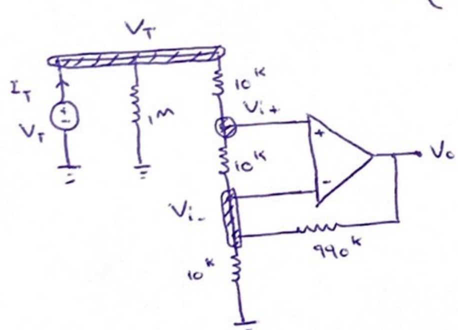
$$\Rightarrow V_{in+} = V_{in-} = V_{in}$$

$$\text{KCL @ } V_{in-}: \frac{V_{in-} - 0}{10^K} + \frac{V_{in-} - V_o}{990^K} + \cancel{\frac{V_{in-} - V_{in+}}{10^K}} = 0$$

$$\Rightarrow \frac{V_{in}}{10^K} + \frac{V_{in}}{990^K} = \frac{V_o}{990^K}$$

$$V_{in}\left(\frac{1}{10^K} + \frac{1}{990^K}\right) = \frac{V_o}{990^K} \Rightarrow A_v = \frac{V_o}{V_{in}} = \frac{\frac{1}{10} + \frac{1}{990}}{\frac{1}{990}} = \boxed{100}$$

$R_{in} = ?$



$$\text{KCL @ } V_T: -I_T + \frac{V_T}{1000^K} + \frac{V_T - V_{i+}}{10^K} = 0$$

$$\boxed{V_T\left(\frac{1}{1000^K} + \frac{1}{10^K}\right) - \frac{V_{i+}}{10^K} = I_T} \quad (I)$$

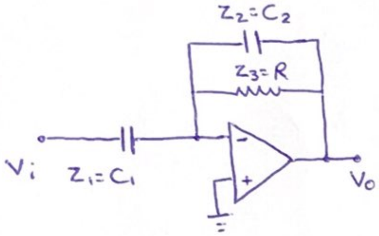
$$\text{KCL @ } V_{i+}: \frac{V_{i+} - V_T}{10^K} + \cancel{\frac{V_{i+} - V_{i-}}{10^K}} = 0$$

$$\Rightarrow \boxed{V_{i+} = V_T} \quad (II)$$

$$(II) \text{ in } (I): V_T\left(\frac{1}{1000^K} + \frac{1}{10^K} - \frac{1}{10^K}\right) = I_T$$

$$\frac{V_T}{I_T} = R_{in} = \frac{1}{\frac{1}{1000^K}} = \boxed{1000^K = 1^M}$$

5. Determine the transfer function of the following circuit and prove that it acts like a filtering circuit. Specify the type of the filter. Assume ideal op amp.



$$V_{in+} = V_{in-} = 0$$

$$\text{KCL @ } V_{in-}: \quad \frac{\cancel{V_{in-}} - V_i}{\frac{1}{C_1 S}} + \frac{\cancel{V_{i-}} - V_o}{R} + \frac{\cancel{V_{i-}} - V_o}{\frac{1}{C_2 S}} = 0$$

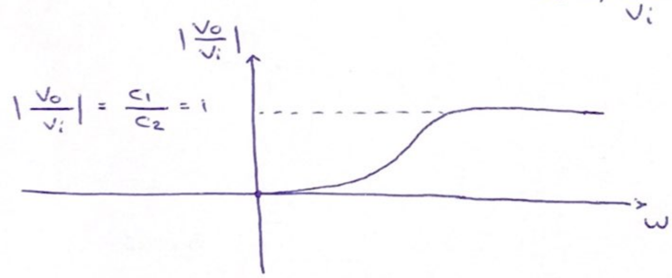
$$-C_1 S V_i = V_o\left(\frac{1}{R} + C_2 S\right)$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{-C_1 S}{\frac{1}{R} + C_2 S} \xrightarrow{\times R} \frac{-RC_1 S}{1 + RC_2 S}$$

$$\xrightarrow{S = j\omega} \frac{V_o}{V_i} = \frac{-RC_1 j\omega}{1 + RC_2 j\omega}$$

$$\begin{cases} \left|\frac{V_o}{V_i}\right| = \frac{\sqrt{(RC_1\omega)^2}}{\sqrt{1+(RC_2\omega)^2}} = \frac{RC_1\omega}{\sqrt{1+(RC_2\omega)^2}} \\ \measuredangle \frac{V_o}{V_i} = -\frac{\pi}{2} - \tan^{-1}\left(\frac{RC_2\omega}{1}\right) \end{cases}$$

* $Z_C = \frac{1}{CS}$
* $Z_R = R$
* $Z_L = LS$



High pass Filter

$$\left|\frac{V_o}{V_i}\right|^2 = \frac{(RC_1\omega)^2}{1+(RC_2\omega)^2} \xrightarrow{\text{if } C_1 = C_2} \left|\frac{V_o}{V_i}\right| = 1$$