1) If $\boldsymbol{A}$ is nonsingular, explain why $\det(\boldsymbol{A}^{-1}) = 1/\det(\boldsymbol{A})$.

2) If $\boldsymbol{A}$ is $n \times n$, explain why $\det(\alpha\boldsymbol{A}) = \alpha^n \det(\boldsymbol{A})$ for all scalars α.

3) Find all the matrix solutions of the matrix equation $\boldsymbol{X^2} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ where a is any number different from 0.

4) Compute the determinant of the following matrix theoretically. In addition, obtain it by the MATLAB or Python.

$$\begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 2 & 7 \\ 2 & 4 & -3 & 2 \\ 3 & 0 & 15 & 3 \end{vmatrix}$$

5) Given the matrix $\boldsymbol{A}$, find $det(\boldsymbol{A}^{-1}\boldsymbol{A}^T\boldsymbol{A})$.

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 7 & 0 \\ 4 & 0 & 5 \end{pmatrix}$$

6) Pick any numbers that add to $x + y + z = 0$. Find the angle between your vector $v = (x, y, z)$ and the vector $w = (z, x, y)$. Explain why $v.w/\|v\|\|w\|$ is always $-\frac{1}{2}$.