1

$x(t) = a_{-2}e^{-2jt\omega} + a_{-1}e^{-jt\omega} + a_1 e^{jt\omega} + a_2 e^{2jt\omega}$

$x(t) = -4je^{-3j\omega t} + 2e^{-j\omega t} + 2e^{j\omega t} + 4je^{3j\omega t}$

$4\left(\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right) - 8\left(\frac{e^{3j\omega t} - e^{-3j\omega t}}{2j}\right) = 4\cos\omega t - 8\sin 3\omega t$

$4\cos\frac{\pi}{4}t + 8\cos\left(\frac{2\pi}{4} + \frac{\pi}{2}\right)$

3 $\omega = \frac{\pi}{3}$

$x(t) = 2 + \frac{e^{j\frac{2\pi}{3}} + e^{-j\frac{2\pi}{3}}}{2} + 4\left(\frac{e^{\frac{5\pi}{3}tj} - e^{\frac{5\pi}{3}jt}}{j}\right)$

$x(t) = 2 + \frac{1}{2}e^{j\frac{2\pi}{3}t} + e^{-\frac{2\pi}{3}t} - j2e^{\frac{5}{3}jt} + 2je^{-j\frac{5\pi}{3}t}$

$a_0 = 2$ $a_5 = -2j$

$a_2 = \frac{1}{2} = a_{-2}$ $a_{-5} = 2j$

۵ چون سیگنال حقیقی است

$a_{-15} = a_{15}^* = -j$ $a_{-14} = a_{14}^* = -2j$ $a_{17} = a_{17}^* = -2j$

سیگنال فرد $a_0 = 0$

$a_{-1} = a_{-15} = -j$ $a_{-2} = a_{14} = -2j$ $a_{-2} = a_{-17} = -2j$

$x(t) = \sum_{0}^{\infty} A_k \cos(\omega_k t + \phi_k)$ $\qquad \omega_0 = \frac{٢\pi}{٨} = \frac{\pi}{٤}$ ٢

$x(t) = a_{-٥} e^{-j\frac{٥\pi}{٤}t} + a_{-1} e^{-j\frac{\pi}{٤}t} + a_1 e^{j\frac{\pi}{٤}t} + a_٥ e^{j\frac{٥\pi}{٤}t}$

$x(t) = ٢ e^{-j\frac{٥\pi}{٤}t} + a_{-1} e^{-j\frac{\pi}{٤}t} + a_1 e^{j\frac{\pi}{٤}t} + a_٥ e^{j\frac{٥\pi}{٤}t}$

$= j(e^{j\frac{\pi}{٤}t} - e^{-j\frac{\pi}{٤}t}) + ٢(e^{j\frac{٥\pi}{٤}t} + e^{-j\frac{٥\pi}{٤}t})$

$= -٢\sin\frac{\pi}{٤}t + ٤\cos\frac{٥\pi}{٤}t = ٢\cos(\frac{\pi}{٤}t + \frac{\pi}{٢}) + ٤\cos\frac{٥\pi}{٤}t$

$a_0 = \frac{1}{٢}\int x(t)\,dt = \frac{1}{٢}\int_0^1 t\,dt + \frac{1}{٢}\int_1^٢ (٢-t)\,dt = \frac{1}{٢}$ ٤

$\frac{dx(t)}{dt} = \begin{cases} 1 & 0 \leq t \leq 1 \\ -1 & 1 \leq t \leq ٢ \end{cases}$ $\qquad b_k = \frac{1}{٢}\int x(t) e^{-jk\omega_0 t}\,dt, \ \omega_0 = \pi$

$b_k = \frac{1}{٢}\int_0^٢ x(t) e^{-jkt\pi}\,dt = -\frac{1}{٢jk\pi}(e^{-jk\pi} - 1 - e^{jk\pi} + e^{-jk\pi})$

$= \frac{٢e^{-jk\frac{\pi}{٢}}}{k\pi} \sin\frac{k\pi}{٢}$

$$y(t) = \begin{cases} 1 & |t| < \frac{1}{2} \\ 0 & \frac{1}{2} < |t| < 2 \end{cases} \qquad z(t) = -\frac{1}{4}$$

$$y(t) = \begin{cases} \frac{1}{2} & |t| < \frac{1}{4} \\ 0 & \frac{1}{4} < |t| < 2 \end{cases} \qquad \frac{\sin \frac{k\pi}{4}}{2k\pi} \qquad (-1)^k = e^{jk\pi} \qquad x(t) = y(t+2)$$

$$x(t) = 2je^{-j\pi t} - je^{-j\frac{\pi}{2}t} + je^{j\frac{\pi}{2}t} + j2e^{j\pi t}$$

$$= -4\left(\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right) - 2\left(\frac{e^{\frac{\pi}{2}jt} - e^{-\frac{\pi}{2}jt}}{2j}\right)$$

$$x(t) = -4\sin(\pi t) - 2\sin\left(\frac{\pi}{2}t\right)$$

$$a_k = 1 + \begin{cases} 0 & k = \text{even} \\ 1 & k = \text{odd} \end{cases}$$

$$y(t) = \sum_{-\infty}^{\infty} 4\delta(t - 4n) \Rightarrow z(t) = \sum e^{j\frac{\pi}{2}t}\, 2\delta(t - 2n)$$

$$x(t) = y(t) + z(t) = \sum_{-\infty}^{\infty} \delta(t - 4n) + \sum_{-\infty}^{\infty} e^{\frac{\pi}{2}tj}\, 2\delta(t - 2n)$$

۷   x(t) حقیقی نمی باشد

$a_k = a^*_{-k}$ $j[\frac{1}{2}]^{|k|} \neq -j[\frac{1}{2}]^{|-k|}$

سیگنال زوج $j[\frac{1}{2}]^{|k|} = j[\frac{1}{2}]^{|-k|}$

پس مشتق فرد است

$b_k = jk\omega_0 a_k = jk\omega j(\frac{1}{2})^{|k|} = -k\omega^{|k|}$

$b_k = b_{-k} \Rightarrow -k\omega(\frac{1}{2})^{|k|} \neq -(-k)\omega(\frac{1}{2})^{|k|}$