یا لطیف

دانشگاه صنعتی شاهرود
دانشکده مهندسی برق و رباتیک

مدار های الکتریکی ۱

نیم سال اول ۰۰-۹۹

پاسخ تمرین سری ششم — مدار های مرتبه اول، پاسخ پله و ضربه

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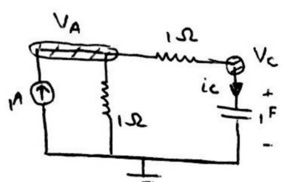


$$KCL \text{ in } V_A: \; -1 + \frac{V_A - 0}{1} + \frac{V_A - V_c}{1} = 0 \Rightarrow 2V_A - V_c = 1 \quad (I)$$

$$KCL \text{ in } V_c: \; \frac{V_c - V_A}{1} + \underbrace{C\,\frac{d(V_c - 0)}{dt}}_{= i_c} = 0 \quad (II)$$

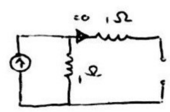
$$\xrightarrow{(I) \text{ in } II} V_c - \frac{(1-V_c)}{2} + 1 \times \frac{dV_c}{dt} = 0 \Rightarrow \frac{dV_c}{dt} + \overset{a}{\frac{1}{2}} V_c = \overset{b}{\frac{1}{2}}$$

$$\Rightarrow V_c(t) = \left(V_c(t=0) - \frac{b}{a}\right) e^{-a(t-t_0)} + \frac{b}{a} = (3-1)\, e^{-t/2} + 1 = 2e^{-t/2} + 1 = V_c(t) \;;\; t \geq 0$$

روش دوم : استفاده از فرمول : $V_c(t) = [\underbrace{V_c(t=0)}_{①} - \underbrace{V_c(t=\infty)}_{②}]\, \underbrace{e^{-\frac{t}{T}}}_{③} + V_c(t=\infty) \quad (**)$ ثابت زمانی

① $\Rightarrow V_c(t=0) = 3^v$ صورت سوال داده

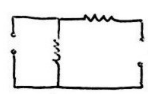
② $\Rightarrow V_c(t=\infty) \longrightarrow$ خازن در لحظه ∞ معادل با مدار باز است $\Rightarrow$



مدار معادل در $t=\infty$

$\Rightarrow V_c(t=\infty) = 1$

③ $T = R_{TH} \times C \Rightarrow$



R_{TH} $\quad R_{TH} = 2 \Rightarrow T = 2 \times 1 = 2$

①,②,③ in $(**)$ $= [3-1]\, e^{-\frac{t}{2}} + 1 = 2e^{-\frac{t}{2}} + 1 \;;\; t > 0$

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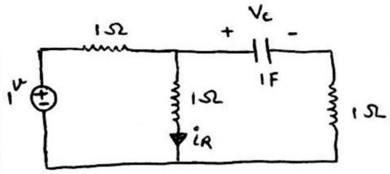


$V_c(0) = 1^V$

$i_R(0) = ?$

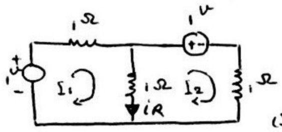
$i_R(t)\ ;\ t>0\ ?$

یادآوری :

خازن : $\begin{cases} t=0 \rightarrow \text{منبع ولتاژ به مقدار شرایط اولیه} \xrightarrow{=0} \text{اتصال کوتاه} \\ t=\infty \rightarrow \text{اتصال باز} \end{cases}$

سلف : $\begin{cases} t=0 \rightarrow \text{منبع جریان به مقدار شرایط اولیه} \xrightarrow{=0} \text{مدار باز} \\ t=\infty \rightarrow \text{اتصال کوتاه} \end{cases}$

مدار معادل در $t=0$:



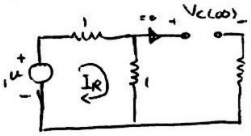
KVL in I_1 : $-1 + 1\times I_1 + 1(I_1 - I_2) = 0 \Rightarrow 2I_1 - I_2 = 1$ (I)

KVL in I_2 : $+1 + 1\times I_2 + 1(I_2 - I_1) = 0 \Rightarrow 2I_2 - I_1 = -1$ (II)

$\xrightarrow{(I),(II)} I_1 = \frac{1}{3}^A,\ I_2 = \frac{-1}{3}^A \Rightarrow i_{R(0)} = I_1 - I_2 = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ *

B) $I_R(t) = [\overset{2/3}{I_R(0)} - I_R(\infty)]\, e^{\frac{-t}{T}} + I_R(\infty)$

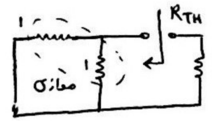
مدار معادل در $t=\infty$:



KVL : $-1 + I_{R(\infty)} + I_{R(\infty)} = 0 \Rightarrow I_{R(\infty)} = \frac{1}{2}$ **

$\Rightarrow I_R(t) = [\frac{2}{3} - \frac{1}{2}]\, e^{\frac{-t}{3/2}} + \frac{1}{2} = \frac{1}{6} e^{-\frac{2}{3}t} + \frac{1}{2}\ ;\ t>0$

$T = R_{TH} \times C \Rightarrow$



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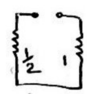

$\Rightarrow R_{TH} = \frac{3}{2} \Rightarrow T = \frac{3}{2} \times 1 = \frac{3}{2}$

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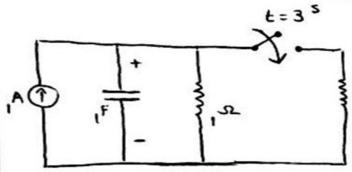
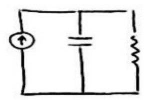


$V_c(0) = 2^v$

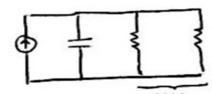
$V_c(t)\ ;\ t>0\ ?$

(a)



$0<t<3$

(b)

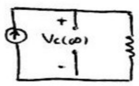


موازی

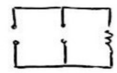
$t>3$

a) $0<t<3 \Rightarrow V_c(t) = [\cancel{V_c(0)}^{\,2} - V_c(\infty)]\, e^{-\frac{t}{T}} + V_c(\infty)$

مدار معادل $t=\infty$ ،، :



$\Rightarrow V_c(\infty) = 1^v \quad , \quad T = R_{TH} \times C$:

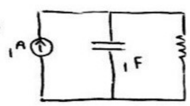


$R_{TH} = 1^{\Omega}$

$T = 1 \times 1 = 1^s$

$\Rightarrow V_c(t) = [2-1]\, e^{-t} + 1 = e^{-t} + 1 \ ;\ 0<t\leq 3$

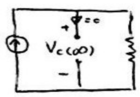
b)



$V_c(t) = [V_c(t=t_0) - V_c(t=\infty)]\, e^{\frac{-t-t_0}{T}} + V_c(t=\infty) \quad (t_0 = 3)$

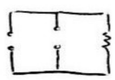
مبدأ زمانی در این مدار

مدار معادل در $t=\infty$



$\Rightarrow V_c(\infty) = 0.5^v$

$T = R_{TH} \times C$:



$R_{TH} = 0.5 \Rightarrow T = 0.5 \times 1 = 0.5 \ ,\ V_c(t=3) = ? \Rightarrow V_c(t) = e^{-t} + 1 \quad 0<t\leq 3$

$\Rightarrow V_c(t=3) = e^{-3} + 1 \Rightarrow V_c(t) = [1 + e^{-3} - 0.5]\, \exp\left(\frac{-t-3}{0.5}\right) + 0.5$

$$\Rightarrow \begin{cases} V_c(t) = \left(\frac{1}{2} + e^{-3}\right) e^{-2(t-3)} + \frac{1}{2} \ ;\ t>3 \\ V_c(t) = e^{-t} + 1 \ ;\ 0<t\leq 3 \end{cases}$$

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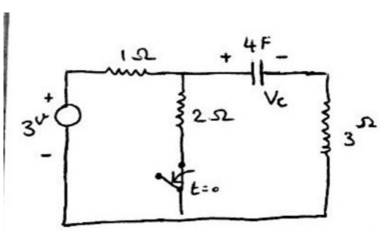
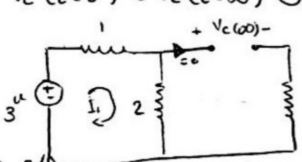


$t<0 \longrightarrow V_c(t=0^-) = V_c(t=\infty)$ چون کلید برای مدت طولانی بسته بوده است

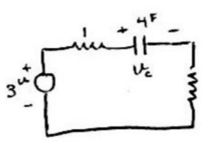
مدار معادل در لحظه $t=\infty$



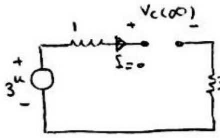
KVL: $-3 + 1 \times I_1 + 2(I_1 - \cancel{I_2}) = 0 \rightarrow I_1 = 1^A \ ,\ I_2 = 0 \Rightarrow V_c(0^-) = V_c(\infty) = 2^v$

$\Rightarrow V_c(0^-) = V_c(0^+) = V_c(\infty) = 2^v$

$t>0$:

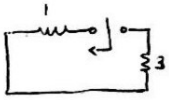


در لحظه ∞



$\Rightarrow V_c(\infty) = 3^v$

$\Rightarrow T = R_{TH} \times C$:



$\Rightarrow R_{TH} = 3+1 = 4 \Rightarrow T = 4 \times 4 = 16^s$

$\Rightarrow V_c(t) = [V_c(0^-) - V_c(\infty)]\, e^{-\frac{t}{T}} + V_c(\infty) = [2-3]\, e^{-\frac{t}{16}} + 3 = -e^{-\frac{t}{16}} + 3 \ ;\ t>0$

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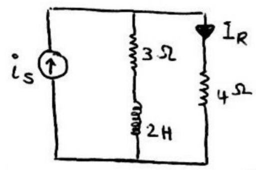
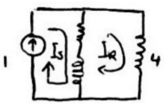


پاسخ پله → شرایط اولیه = صفر → $I_L(0) = 0$

پاسخ پله → ورودی پله واحد → $i_s = u(t)$

$$u(t) = \begin{cases} 1 & ; t>0 \\ 0 & ; t<0 \end{cases}$$

$\Rightarrow \quad t>0 :$



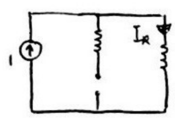
KVL in I_R : $4 \times I_R + 2\dfrac{d(I_R - I_s)}{dt} + 3(I_R - I_s)$

$$\xrightarrow{I_s = 1} \frac{dI_R}{dt} + \underbrace{\frac{7}{2}}_{a} I_R = \underbrace{\frac{3}{2}}_{b}$$

یادآوری : $i_L = L\dfrac{di_L}{dt}$

$$\Rightarrow I_R(0) = \Big(\underbrace{I_R(0^+)}_{①} - \frac{b}{a}\Big)e^{-at} + \frac{b}{a} = \Big(I_R(0) - \frac{3}{7}\Big)e^{\frac{7}{2}t} + \frac{3}{7}$$

مدار در لحظه $t = 0^+$:



سلف معادل می‌شود با یک منبع جریان با مقدار شرایط اولیه $(0 = i_L(0))$

$$\Rightarrow \underline{I_R(0^+) = 1} \quad \Rightarrow I_R(0) = \frac{4}{7}e^{-\frac{7}{2}t} + \frac{3}{7}$$

$$\Rightarrow S(t) = \Big(\frac{4}{7}e^{-\frac{7}{2}t} + \frac{3}{7}\Big) \times u(t)$$

$$\Rightarrow h(t) = \frac{dS(t)}{d(t)} = (-2e^{-\frac{7}{2}t})u(t) + \delta(t)\Big(\frac{4}{7}\cancel{e^{-\frac{7}{2}t}}^{\,1} + \frac{3}{7}\Big) \xrightarrow[f(0)\cdot\delta(t)]{f(t)\cdot\delta(t) =} \boxed{h(t) = (-2e^{-\frac{7}{2}t})u(t) + \delta(t)}$$

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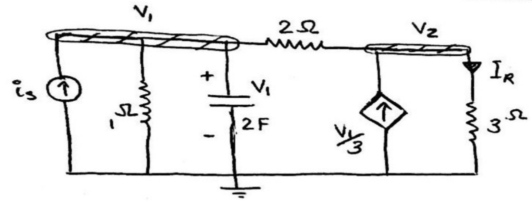
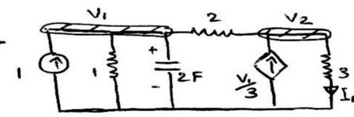


پاسخ پله : شرایط اولیه صفر $\longrightarrow V_1(0)=0$

ورودی پله واحد $\longrightarrow i_s = u(t)$

می دانیم : $t>0 \rightarrow u(t)=1$

$\Rightarrow$ 

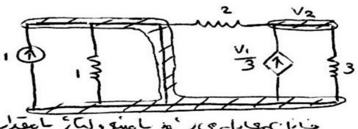
$$KCL \text{ in } V_1 : -1 + \frac{V_1 - 0}{1} + 2\frac{dV_1}{dt} + \frac{V_1 - V_2}{2} = 0 \quad (\text{II})$$

$$KCL \text{ in } V_2 : \frac{V_2 - V_1}{2} + \frac{V_2 - 0}{3} - \frac{V_1}{3} = 0 \Longrightarrow \boxed{V_1 = V_2} \quad (\text{I})$$

$$\xrightarrow{(\text{I}) \text{ in } (\text{II})} \frac{dV_2}{dt} + \frac{1}{2}V_2 = \frac{1}{2} \xrightarrow{V_2 = 3 \times I_R} 3\frac{dI_R}{dt} + \frac{3}{2}I_R = \frac{I}{2}$$

$$\Rightarrow \frac{dI_R}{dt} + \overbrace{\frac{1}{2}}^{a} I_R = \overbrace{\frac{1}{6}}^{b} \Rightarrow I_R(t) = \left(I_R(0) - \frac{b}{a}\right)e^{-at} + \frac{b}{a} = \underbrace{\left(I_R(0) - \frac{1}{3}\right)}_{①}e^{-\frac{t}{2}} + \frac{1}{3}$$

برای به دست آوردن ① ، مدار معادل را در لحظه $t=0$ می کشیم :



خازن معادل می شود با منبع ولتاژ با مقدار اولیه $(V_C(0)=0)$

$$KCL \text{ in } V_2 : \frac{V_2 - 0}{3} + \frac{V_2 - 0}{2} - \cancel{\frac{V_1}{3}} = 0$$

$$\Rightarrow V_2 = 0 \Rightarrow \boxed{i_R(0^+) = 0}$$

$$\Rightarrow I_R(t) = \left(0 - \frac{1}{3}\right)e^{-\frac{t}{2}} + \frac{1}{3} = \frac{1}{3}\left(1 - e^{-\frac{t}{2}}\right) \quad ; \ t>0$$

$$\Rightarrow S(t) = \frac{1}{3}\left(1 - e^{-\frac{t}{2}}\right) \cdot u(t) \longrightarrow h(t) = \frac{dS(t)}{dt} = \frac{1}{3}\left(\frac{1}{2}e^{-\frac{t}{2}}\right)u(t) + \cancel{\delta(t)\left(1 - e^{-\frac{t}{2}}\right)}$$

$$\xrightarrow[f(0)\cdot\delta(t)]{\delta(t)\cdot f(t) =} h(t) = \frac{1}{6}e^{-\frac{t}{2}} \cdot u(t)$$

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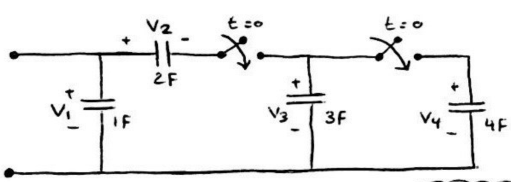
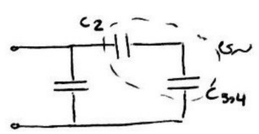


$$\left\{\begin{array}{l} \text{خازن های سری} : C_T = \frac{C_1 C_2}{C_1 + C_2} \ \left(\text{عکس رابطه مقاومت های سری}\right) \\ V_{C_T}(0) = V_{C_1}(0) + V_{C_2}(0) + \cdots \end{array}\right.$$

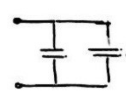
و چون در مدار تابع ضربه نداریم ، $V_C(0^-) = V_C(0^+)$

$$\left\{\begin{array}{l} \text{خازن های موازی} : C_T = C_1 + C_2 + \cdots \ \left(\text{عکس رابطه مقاومت های موازی}\right) \\ V_{C_T}(0) = \frac{C_1 V_{C_1}(0) + C_2 V_{C_2}(0)}{C_1 + C_2} \end{array}\right.$$

$$\Rightarrow V_{3,4} = \frac{C_3 V_{C_3}(0^-) + C_4 V_{C_4}(0^-)}{C_3 + C_4} = \frac{3\times3 + 4\times4}{3+4} = \frac{25}{7}^{v}$$

$\Rightarrow$ 

$$\Rightarrow V_{3,4,2} = V_{3,4} + V_{C_2} = \frac{25}{7} + 2 = \frac{39}{7}^{v} \Rightarrow$$

 $C = \frac{14}{9}$

$$\Rightarrow \left\{\begin{array}{l} C_{3,4} = C_3 + C_4 = 3 + 4 = 7\,F \\ C_{34,2} = \frac{C_{3,4} \times C_2}{C_{3,4} + C_2} = \frac{7 \times 2}{7 + 2} = \frac{14}{9}\,F \end{array}\right.$$

$$\Rightarrow V_{eq} = \frac{C_1 V_{C_1}(0^-) + C_{34,2}V_{34,2}(0^-)}{C_1 + C_{34,2}}$$

$$= \frac{1\times1 + \frac{14}{9}\times\frac{39}{7}}{1 + \frac{14}{9}} = \boxed{\frac{87}{23}\ V}$$

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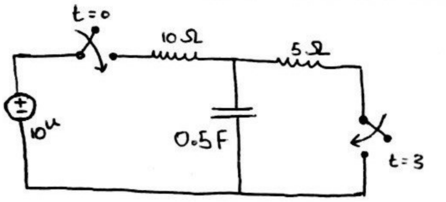
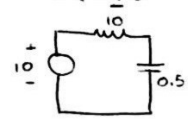
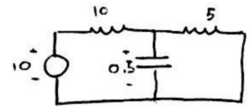
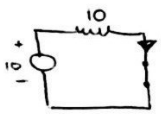






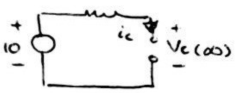
I) $0<t\leq 3$: $i_{c(t)} = \left[i_c(t=0) - i_c(t=\infty) \right] e^{-\frac{t}{T}} + i_c(t=\infty)$

مدار معادل در لحظه $t=0$: $\longrightarrow$ خازن برابر با منبع ولتاژ با شرایط اولیه ($V_{c(0)}=0$) :



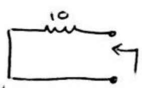
$\xrightarrow{KVL}$ $i_c(0) = \frac{10}{10} = 1^A$

مدار معادل در لحظه $t=\infty$: خازن معادل اتصال باز :



$\Rightarrow$ $\boxed{i_c(\infty) = 0}$

$\Rightarrow T = R_{TH} \times C$:

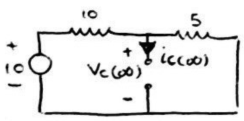


$R_{TH} = 10\Omega \Rightarrow T = 10 \times 0.5 = 5\,s$

$\Rightarrow i_c(t) = [1-0]\, e^{-\frac{t}{5}} + 0 = e^{-\frac{t}{5}} \quad ; \quad 0 < t \leq 3$

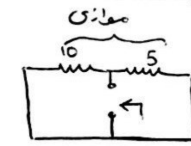
II) $t>3$: $i_c(t) = \left[i_c(t=3) - i_c(t=\infty) \right] e^{\frac{-t-3}{T}} + i_c(t=\infty)$

مدار معادل در لحظه $t=\infty$:



$\longrightarrow$ $\boxed{i_c(\infty) = 0}$, $i_c(t=3) = ?$ $\Rightarrow$ از رابطه به دست آمده از قسمت قبل استفاده می کنیم

$\Rightarrow i_c(t) = e^{-\frac{t}{3}} \xrightarrow{t=3} i_c(t=3) = e^{-\frac{3}{5}}$, $T = R_{TH} \times C$:



$R_{TH} = \frac{10}{3}$

$\Rightarrow T = \frac{10}{3} \times \frac{1}{2} = \frac{10}{6} = \frac{5}{3}\,s$

$\Rightarrow i_c(t) = \left[e^{-\frac{5}{3}} - 0 \right] e^{\frac{-t-3}{5/3}} = e^{-\frac{5}{3}} \times e^{-\frac{3}{5}(t+3)} \quad ; \quad t>3$

$$\Rightarrow i_c(t) = \begin{cases} e^{-\frac{t}{5}} & ; \ 0<t<3 \\ e^{-4} \times e^{-\frac{3}{5} t} & ; \ t>3 \end{cases}$$

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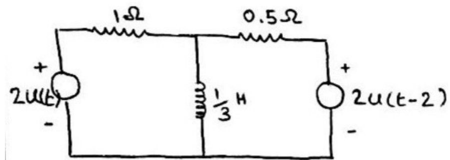
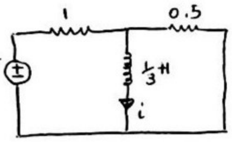


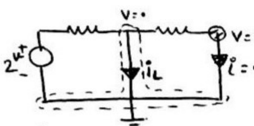
$$u(t)=\begin{cases}1 & ; t>0\\ 0 & ; t<0\end{cases}\quad,\quad u(t-2)=\begin{cases}1 & ; t>2\\ 0 & ; t<2\end{cases}$$

I) $0<t<2$: $\underbrace{2u(t)}_{=1} = 2^V$



$$i_L(t)=\left[i_L(t=0)-i_L(t=\infty)\right]e^{\frac{-t}{T}}+i_L(t=\infty)$$

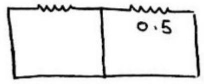
مدار معادل در لحظه $t=\infty$: سلف معادل با اتصال کوتاه :



$\Rightarrow \boxed{i_L(\infty)=2^A}$, $i_L(0)=0$

چون برای $t<0$ ، $u(t)=0$

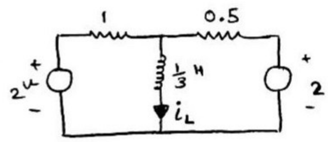
$\Rightarrow T=\dfrac{L}{R_{TH}}$:



$R_{TH}=\dfrac{1\times 0.5}{1+0.5}=\boxed{\dfrac{1}{3}}$ $\Rightarrow T=\dfrac{\frac{1}{3}}{\frac{1}{3}}=\boxed{1^S}$

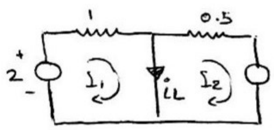
$$\Rightarrow i_L(t)=[0-2]e^{\frac{-t}{1}}+2=-2e^{-t}+2=\boxed{2(1-e^{-t})\ ;\ 0<t<2}$$

$t>2$:



$$i_L=\left[i_L(t=2)-i_L(t=\infty)\right]e^{\frac{-t-2}{T}}+i_L(\infty)$$

$i_L(t=2)=2(1-e^{-2})$, مدار معادل در لحظه $t=\infty$:

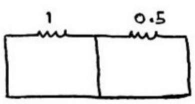


با KVL در ۱، $I_1=2^A$

با KVL در ۲، $2I_2=-4$

$\Rightarrow i_{L(\infty)}=I_1-I_2=\boxed{5A}$

$\Rightarrow T=\dfrac{L}{R_{TH}}$:



$R_{TH}=\dfrac{1}{3}$ $\Rightarrow T=\dfrac{\frac{1}{3}}{\frac{1}{3}}=1$

$$\Rightarrow i_L(t)=\left[2(1-e^{-2})-6\right]e^{-t-2}+6\quad ;\quad t>2$$

$$\Rightarrow i_L(t)=\begin{cases}2(1-e^{-t}) & ; 0<t\le 2\\ \left[2(1-e^{-2})-6\right]e^{-t-2}+6 & ; t>2\end{cases}$$