یا لطیف

رضا ادیب پور

۹۸۱۴۳۰۳

تمرین سری ۷ سیگنال سیستم

#1 a) $\sum_{k=0}^{\infty} a^k \delta(t-kT) = \delta(t) + a\delta(t-T) + a^2\delta(t-2T) + \cdots$

$\xleftrightarrow{L} X(s) = 1 + ae^{-sT} + a^2(e^{-sT})^2 + \cdots = 1 + \sum_{k=1}^{\infty} a^k e^{-kST}$ ROC: کل صفحه

b) $te^{-at}u(t) \quad a>0 \quad \Rightarrow \quad x_1(t) = e^{-at}u(t) \xleftrightarrow{L} X_1(s) = \frac{1}{s+a} \quad Re\{s\} > -a$

$\xrightarrow[\text{مشتق}]{\text{خاصیت}} \quad t\,x_1(t) \xleftrightarrow{L} -\frac{d}{ds}X_1(s) \quad \Rightarrow \quad X(s) = -\left(\frac{-1}{(s+a)^2}\right) = \frac{1}{(s+a)^2} \quad Re\{s\} > -a$

c) $x(t) = \cos(\omega_0 t + \varphi)u(t) \quad : \quad \Rightarrow \quad x_1(t) = \cos\omega_0 t\, u(t) \xleftrightarrow{L} X_1(s) = \frac{s}{s^2+\omega_0^2} \quad Re\{s\} > 0$

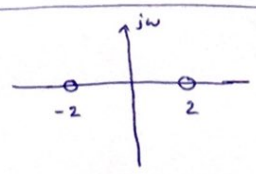
$\Rightarrow x(t) = x_1(t+\varphi) \xleftrightarrow{L} X(s) = e^{s\varphi}X_1(s) = e^{s\varphi}\frac{s}{s^2+\omega_0^2} \quad Re\{s\} > 0$

d) $x(t) = e^{-at}\cdot\sin\omega_0 t\cdot u(t) \quad a>0 \quad : \quad x_1(t) = \sin\omega_0 t\, u(t)$

$\xleftrightarrow{L} X_1(s) = \frac{\omega_0}{s^2+\omega_0^2} \quad Re\{s\} > 0 \quad \longrightarrow \quad x(t) = x_1(t)e^{-at} \xleftrightarrow{L} X(s) = X_1(s+a)$

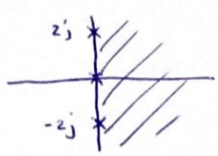
$\Rightarrow X(s) = \frac{\omega_0}{(s+a)^2+\omega_0^2} \quad Re\{s\} > -a$

#2 a)



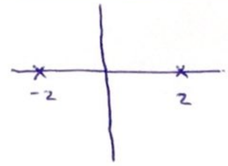
$\longrightarrow X(s) = \frac{(s-2)(s+2)(s-k)}{(s-k)} \xleftrightarrow{L^{-1}} x(t) =$

b)

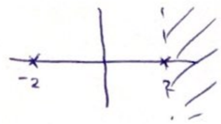


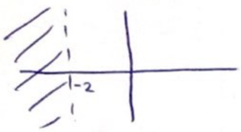
$\rightarrow X(s) = \frac{1}{s(s-2j)(s+2j)} \xleftrightarrow{L^{-1}} x(t) = \frac{A}{s} + \frac{B}{(s-2j)} + \frac{C}{(s+2j)}$

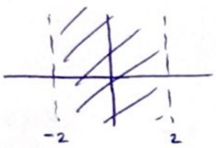
$\Rightarrow \begin{cases} A = \frac{-1}{4} \\ B = \frac{-1}{8} \\ C = \frac{-1}{8} \end{cases} \Rightarrow X(s) = \frac{-\frac{1}{4}}{s} + \frac{-\frac{1}{8}}{(s-2j)} + \frac{-\frac{1}{8}}{(s+2j)} \xleftrightarrow{L^{-1}} x(t) = -\frac{1}{4}u(t) - \frac{1}{8}(e^{2jt})u(t) - \frac{1}{8}e^{-2jt}u(t)$

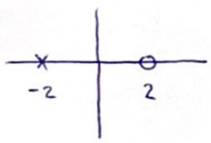
c)  $\longrightarrow \underline{X}(s) = \frac{1}{(s-2)(s+2)} = \frac{A}{(s-2)} + \frac{B}{(s+2)} \longrightarrow \begin{cases} A = \frac{1}{4} \\ B = \frac{-1}{4} \end{cases}$

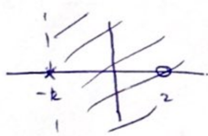
$\Rightarrow \underline{X}(s) = \frac{\frac{1}{4}}{(s-2)} - \frac{\frac{1}{4}}{(s+2)} \xleftrightarrow{L^{-1}}$

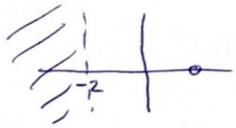
$\Rightarrow$  $: \; x(t) = \frac{1}{4} e^{2t} u(t) - \frac{1}{4} e^{-2t} u(t) \qquad Re\{s\} > 2$

 $: \; x(t) = -\frac{1}{4} e^{2t} u(-t) + \frac{1}{4} e^{-2t} u(-t) \qquad Re\{s\} < -2$

 $: \; x(t) = -\frac{1}{4} e^{-2t} u(t) - \frac{1}{4} e^{2t} u(-t) \qquad -2 < Re\{s\} < 2$

d)  $: \; \underline{X}(s) = \frac{(s-2)}{(s+2)} = \frac{A}{(s+2)} \longrightarrow A = -4$

 $: \; x(t) = -4 e^{-2t} u(t)$

 $: \; x(t) = 4 e^{-2t} u(-t)$

#3 $\quad \underline{X}(s) = \frac{s^2 - s + 1}{(s+1)^2} \; : \; Re\{s\} > -1 \quad \Rightarrow \underline{X}(s) = \frac{A}{(s+1)} + \frac{B}{(s+1)^2} \longrightarrow \begin{cases} A = \frac{d}{ds}\left((s+1)^2 \underline{X}(s)\right) \\ \longrightarrow A = -3 \\ B = +2 \end{cases}$

$\Rightarrow \underline{X}(s) = \frac{-3}{(s+1)} + \frac{2}{(s+1)^2} = \underline{X}_1(s) + \underline{X}_2(s)$

$\begin{cases} \underline{X}_1(s) \xleftrightarrow{L^{-1}} x_1(t) = -3 e^{-t} u(t) \\ \underline{X}_2(s) = \frac{2}{(s+1)^2} \longrightarrow \text{if } \underline{X}_3(s) = \frac{2}{(s+1)} \xleftrightarrow{L^{-1}} x_3(t) = 2 e^{-t} u(t) \end{cases}$

$\Rightarrow +t\, x_3(t) \xleftrightarrow{L} -\frac{d \underline{X}_3(s)}{ds} \quad \Rightarrow \quad x_2(t) = t\, x_3(t) = 2t e^{-t} u(t)$

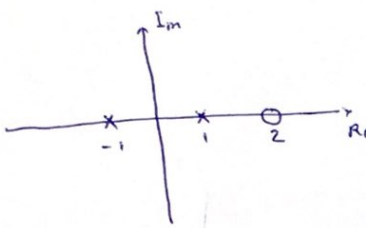
$\Rightarrow x(t) = x_1(t) + x_2(t) = -3 e^{-t} u(t) + 2t e^{-t} u(t)$

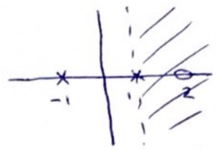
b) $X(s) = \dfrac{s+1}{(s+1)^2+4} \qquad Re\{s\} > -1$

$\therefore \; X_1(s) = \dfrac{s}{s^2+4} \xleftrightarrow{L^{-1}} x_1(t) = \cos 2t \, u(t)$

$\Rightarrow \; X(s) = X_1(s+1) \xleftrightarrow{L^{-1}} x(t) = e^{-t} x_1(t) = e^{-t} \cos 2t \, u(t)$

#4

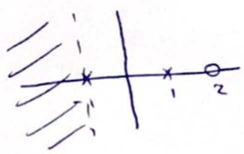




→ سیستم ناپایدار و ضد علی است

البته اگر می‌دانستیم سیستم گویا است، می‌توان گفت سیستمی با این ROC، حتما علی است

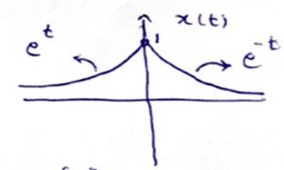
→ سیستم پایدار و ضد علی است



→ سیستم ناپایدار و ضد علی است

#5

$$\begin{cases} H(s) = \dfrac{s+1}{s^2+2s+2} \\ x(t) = e^{-|t|} \xleftrightarrow{L} \cancel{X(s)} \; x(t) = e^{t}u(-t) + e^{-t}u(t) \end{cases}$$



$\xleftrightarrow{L} X(s) = \dfrac{-1}{(s-1)} + \dfrac{1}{s+1} \quad Re\{s\} > 1 \Rightarrow X(s) = \dfrac{-2}{(s-1)(s+1)} \quad Re\{s\} > 1$

$\Rightarrow y(t) = x(t) * h(t) \xleftrightarrow{L} Y(s) = X(s)H(s) = \dfrac{-2}{(s-1)(s+1)} \times \dfrac{s+1}{s^2+2s+2} = \dfrac{-2}{(s-1)(s^2+2s+2)}$

$= \dfrac{A}{(s-1)} + \dfrac{Bs+C}{(s^2+2s+2)} \rightarrow \begin{cases} A+B=0 \\ 2A-B+C=0 \\ 2A-C=2 \end{cases} \rightarrow \begin{cases} A = \frac{2}{5} \\ B = \frac{-2}{5} \\ C = \frac{-6}{5} \end{cases}$

$\Rightarrow Y(s) = \dfrac{\frac{2}{5}}{(s-1)} + \dfrac{-\frac{2}{5}s - \frac{6}{5}}{(s^2+2s+2)} = \dfrac{\frac{2}{5}}{(s-1)} - \dfrac{\frac{2}{5}s}{s^2+2s+2} - \dfrac{\frac{6}{5}}{s^2+2s+2} = \dfrac{\frac{2}{5}}{s-1} - \dfrac{\frac{2}{5}s}{(s+1)^2+1}$

$- \dfrac{\frac{6}{5}}{(s+1)^2+1} \xleftrightarrow{L^{-1}} y(t) = \frac{2}{5}e^{t}u(t) - \frac{2}{5}e^{-t}\cos t \, u(t) - \frac{6}{5}e^{-t}\sin t \, u(t)$

#6 LTI سیستم علی ، :

$$\begin{cases} \text{الف)} \quad x(t) = e^{2t} \longrightarrow \boxed{LTI} \longrightarrow y(t) = \frac{1}{6} e^{2t} \\ \text{ب)} \quad \frac{dh(t)}{dt} + 2h(t) = e^{-4t} u(t) + b u(t) \\ H(s) \,,\; b = ? \end{cases}$$

$$e^{st} \xrightarrow{x(t)} \boxed{LTI} \longrightarrow H(s) e^{st} \Rightarrow \boxed{H(s) = \frac{1}{6}} \Rightarrow H(s) = \int_{-\infty}^{+\infty} h(\tau) e^{-s\tau} d\tau$$

$$\Rightarrow \frac{1}{6} = \int_{-\infty}^{+\infty} h(\tau) e^{-2\tau} d\tau$$

$$\text{ب)} \quad \frac{dh(t)}{dt} + 2h(t) = e^{-4t} u(t) + b u(t) \xleftrightarrow{L} S H(s) + 2 H(s) = \frac{1}{S+4} + \frac{b}{S}$$

$$\xrightarrow{H(s) = \frac{1}{6}} \frac{1}{6}(S+2) = \frac{1}{S+4} + \frac{b}{S} \Rightarrow \frac{S + bS + 4b}{S(S+4)} = \frac{S\cancel{2}+2}{6}$$

$$\Rightarrow b = \frac{S(S+4)(S+2) - 6S}{6S + 24} = \boxed{S(S+2) - \frac{6S}{(S+4)}}$$