

یا الطیف

رضا اکبریان

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#1

$$\begin{cases} x[n] = 3^n u[n-1] + \left(\frac{1}{3}\right)^n u[n] \\ h[n] = \left(\frac{1}{2}\right)^n u[n+3] \end{cases} \quad y[n] = x[n] * h[n]$$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k] \cdot h[n-k] = \sum_{k=-\infty}^{+\infty} \left(3^k u[k-1] + \left(\frac{1}{3}\right)^k u[k] \right) \cdot \left(\frac{1}{2}\right)^{n-k} u[n-k+3]$$

$$= \sum_{k=-\infty}^{+\infty} 3^k u[k-1] \cdot \left(\frac{1}{2}\right)^{n-k} u[n-k+3] + \sum_{k=-\infty}^{+\infty} \left(\frac{1}{3}\right)^k u[k] \cdot \left(\frac{1}{2}\right)^{n-k} u[n-k+3]$$

$$= \sum_{k=-\infty}^{n+3} 3^k \cdot \left(\frac{1}{2}\right)^{n-k} + \sum_{k=0}^{n+3} \left(\frac{1}{3}\right)^k \cdot \left(\frac{1}{2}\right)^{n-k} = \left(\frac{1}{2}\right)^n \sum_{k=-\infty}^{n+3} 6^k + \left(\frac{1}{2}\right)^n \sum_{k=0}^{n+3} \left(\frac{2}{3}\right)^k$$

$$= \left(\frac{1}{2}\right)^n \left(\sum_{k=-\infty}^{n+3} 6^k + \sum_{k=0}^{n+3} \left(\frac{2}{3}\right)^k \right) = \left(\frac{1}{2}\right)^n \left[\left(\frac{6^{n+4} - 6}{1-6} \right) + \left(\frac{\left(\frac{2}{3}\right)^{n+4} - \left(\frac{2}{3}\right)^0}{1 - \frac{2}{3}} \right) \right]$$

$$= \left(\frac{1}{2}\right)^n \cdot \left(\frac{6^{n+4}}{5} + \frac{1 - \left(\frac{2}{3}\right)^{n+4}}{\frac{1}{3}} \right) = \left(\frac{1}{2}\right)^n \cdot \left(\frac{1}{5} 6^{n+4} + 3 \left(1 - \left(\frac{2}{3}\right)^{n+4}\right) \right)$$

ب) علی: $h[n] = 0$; $\forall n < 0 \rightarrow$ علی

$$h[n] = \left(\frac{1}{2}\right)^n u[n+3] \quad u[n+3] \Rightarrow \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad h[n] \neq 0 \quad n < 0$$

$$\sum_{n=-\infty}^{+\infty} |h[n]| < \infty ? \Rightarrow \sum_{n=-3}^{\infty} \left(\frac{1}{2}\right)^n = \frac{\left(\frac{1}{2}\right)^{-3} - \left(\frac{1}{2}\right)^{\infty}}{1 - \frac{1}{2}} = \frac{\left(\frac{1}{2}\right)^{-3}}{\frac{1}{2}} = 16 < \infty$$

سیستم علی

پایداری

$$h[n] = K \delta[n] \Rightarrow h[n] = \left(\frac{1}{2}\right)^n u[n+3] \rightarrow$$

پایه ضربی هم $K \delta[n]$ نیست
پس سیستم حافظه دار است