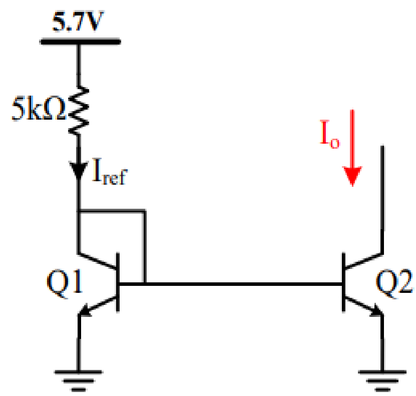
1- For the following circuits, the transistors are the same and $V_{BE} = 0.7V$.

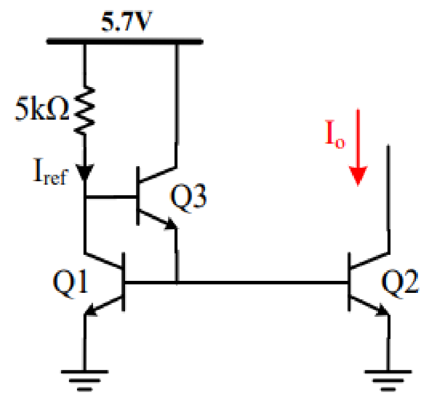
a) Calculate I_{ref} in fig. 1.a.

b) Determine I_o / I_{ref} in terms of β and compute its value for $\beta = 50, \beta = 200, \beta = \infty$. Discus about the results (Fig. 1.a).

c) In order to alleviate the undesirable effect of β in BJT current mirrors, the circuit whic is depicted in fig. 1.b can be used. For this circuit, calculate I_o / I_{ref} and compare th results with those in (b). Assume $\beta = 50$.

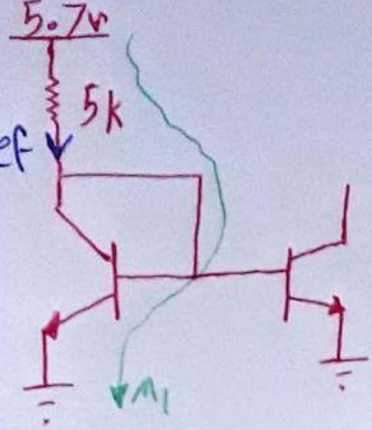


(1.a)



(1.b)

$v_{BE}=0.7v$ و $i_{ref}=?$ (a

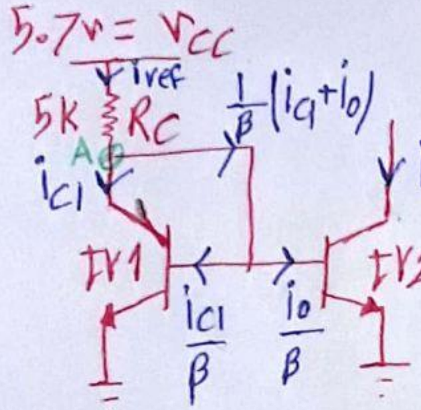


$$KVL@M_1: -5.7+5i_{ref}+0.7=0$$

$$\boxed{i_{ref}=1mA}$$

$$? = \frac{i_o}{i_{ref}}$$ (b

$\beta=50, 200, \infty$



$i_{s1}=i_{s2}$ ← ترانزیستورها مشابه

$v_{T1}=v_{T2}$

$$i_C=i_s\exp\left(\frac{v_{BE}}{nv_T}\right)$$

$$v_{BE1}=v_{BE2}\rightarrow i_{C1}=i_o \ (\mathrm{I})$$

$$KCL@A: i_{ref}=i_{C1}+\frac{1}{\beta}(i_{c1}+i_o)\xrightarrow{(\mathrm{I})}$$

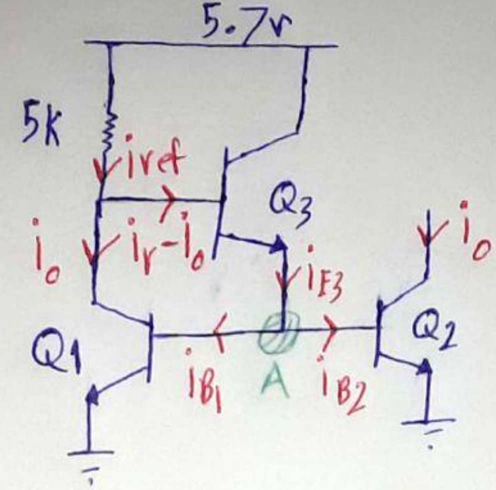
$$i_{ref}=\frac{2i_o}{\beta}+i_o=\left(\frac{\beta+2}{\beta}\right)i_o=i_{ref}\rightarrow$$

$$\frac{i_o}{i_{ref}}=\frac{\beta}{\beta+2}$$

$$\beta=50\rightarrow\frac{i_o}{i_{ref}}=0.96$$

$$\beta=200\rightarrow\frac{i_o}{i_{ref}}=0.99$$

$$\beta=\infty\rightarrow\frac{i_o}{i_{ref}}=1$$



$\frac{i_o}{i_{ref}} = ?\ ,\ \beta = 50$ |C

$v_{BE1} = v_{BE2} \rightarrow i_o = i_{C1}$

$$i_{E3} = (\beta+1)\, i_{B3} = 51(i_r - i_o)$$

$$i_{B1} = \frac{i_{C1}}{\beta} = \frac{i_o}{\beta} = \frac{i_o}{50}$$

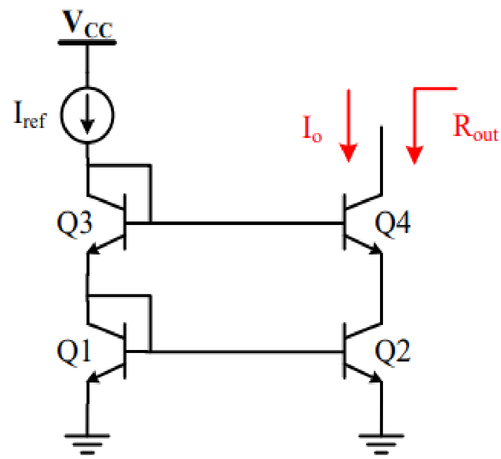
$$i_{B2} = \frac{i_o}{\beta} = \frac{i_o}{50}$$

$$KCl@A:\ i_{B1} + i_{B2} - i_{E3} = 0 \rightarrow \frac{i_o}{50} + \frac{i_o}{50} - 51(i_r - i_o) = 0$$

$$51 i_r = \frac{i_o}{25} + 51 i_o \rightarrow 51 i_r = \frac{1276\, i_o}{25} \rightarrow \frac{i_o}{i_r} = 0.9992$$

0.08% خطا

2- The following circuit is known as the “cascade” current-mirror. Determine the output current and the output resistance (Assume $\lambda \neq 0$ and neglect β effect). What are the advantages of this configuration over the simple current-mirror scheme?

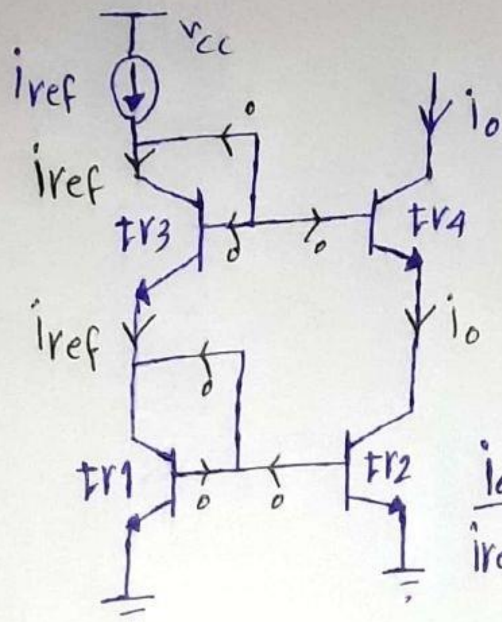


$i_o = ?$ (2

$R_{out} = ?$

$v_A \neq \infty$

$\beta \simeq \infty$



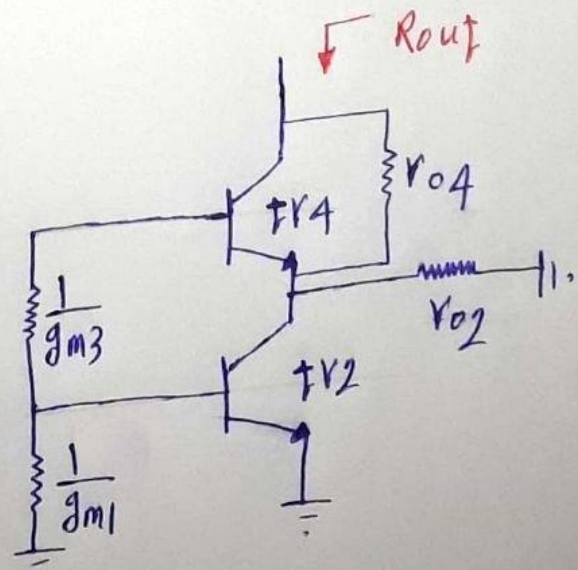
$$i_C = i_s \exp\left(\frac{v_{BE}}{n v_T}\right)\left(1 + \frac{v_{CE}}{v_A}\right)$$

$$\frac{i_o}{i_{ref}} = \frac{i_{s2}}{i_{s1}} \times \frac{1 + \frac{v_{CE2}}{v_A}}{1 + \frac{v_{CE1}}{v_A}}$$

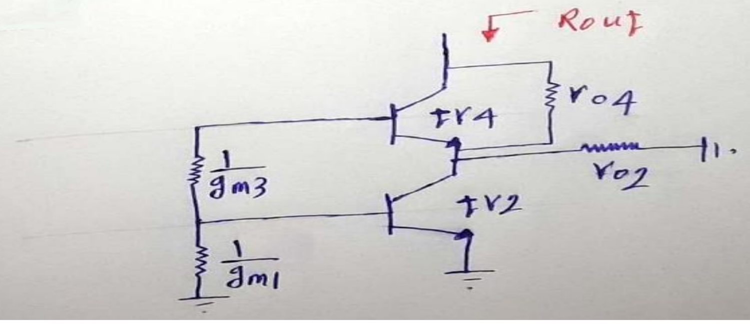
$$\text{if } \{ v_{BE3} = v_{BE4} \} \therefore v_{CE1} = v_{CE2}$$

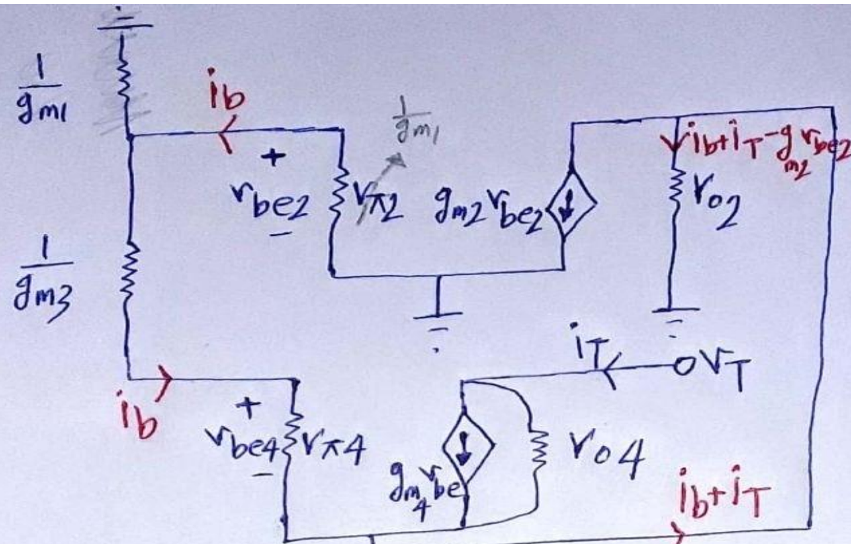
$$\boxed{\frac{i_o}{i_{ref}} = \frac{i_{s2}}{i_{s1}}}$$

5.5)



5.5)





$r_{\pi 2} \gg \frac{1}{g_{m1}}$

$\beta \simeq \infty$

$$\frac{1}{g_{m1}} i_b + \frac{1}{g_{m3}} i_b + r_{\pi 4} i_b + r_{o2}\left(i_T + i_b - g_{m2} v_{be2}\right) = 0$$

$$v_{be2} = -\frac{1}{g_{m1}} i_b \rightarrow -\left(\frac{1}{g_{m1}} + \frac{1}{g_{m3}}\right) i_b = r_{\pi 4} i_b + r_{o2}\left(i_T + i_b + \frac{g_{m2}}{g_{m1}} i_b\right)$$

$$\xrightarrow{\simeq} r_{\pi 4} i_b + r_{o2}\left(i_T + \frac{g_{m1} + g_{m2}}{g_{m1}} i_b\right) = 0 \rightarrow i_b = \frac{-r_{o2} i_T}{r_{\pi 4} + \frac{g_{m1} + g_{m2}}{g_{m1}} r_{o2}}$$

$$\text{KVL}: v_T = r_{o4}\left(i_T - g_{m4} r_{\pi 4} i_b\right) + r_{o2}\left(i_T + \frac{g_{m1} + g_{m2}}{g_{m1}} i_b\right)$$

$$v_T = r_{o4} i_T + r_{o2} i_T + \left(\frac{(g_{m1} + g_{m2}) r_{o2}}{g_{m1}} - g_{m4} r_{\pi 4} r_{o4}\right) \frac{-r_{o2} i_T}{r_{\pi 4} + \frac{g_{m1} + g_{m2}}{g_{m1}} r_{o2}}$$

$$R_o = r_{o2} + r_{o4} + \frac{\beta_4 r_{o4} r_{o2}}{r_{\pi 4} + \frac{g_{m1} + g_{m2}}{g_{m1}} r_{o2}}$$

$$i_{c2} \simeq i_{c4} \rightarrow r_{o2} \simeq r_{o4} = r_o$$

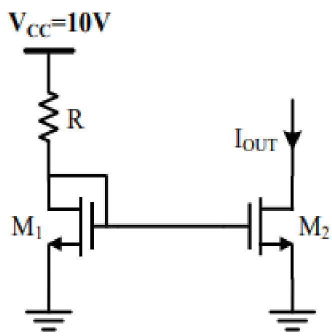
$$R_{out} = 2r_o + \frac{\beta_4 r_o^2}{r_{\pi 4} + \frac{g_{m1} + g_{m2}}{g_{m1}} r_o}$$

$$R_{out} \simeq 2r_o + \frac{\beta_4 r_o^2}{\frac{g_{m1} + g_m}{g_{m1}} r_o} \simeq 2r_o + \frac{g_{m1} \beta_4 r_o}{g_{m1} + g_{m2}}$$

$$\boxed{R_{out} \simeq \beta_4 r_o \left(\frac{g_{m1}}{g_{m1} + g_{m2}} \right)}$$

$$\text{if } \{ g_{m1} = g_{m2} \} : R_{out} = \frac{\beta r_o}{2}$$

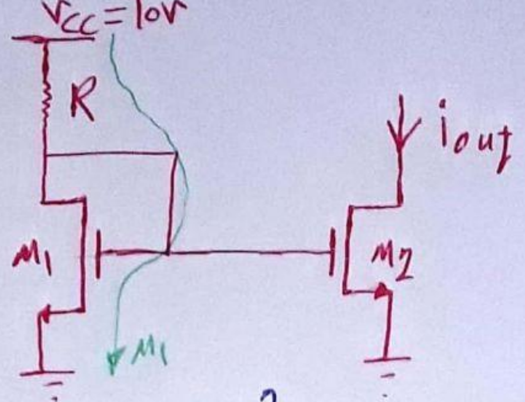
3- Specify the value of R so that $I_{OUT} = 5$ mA.



$$\begin{cases} \beta_1 = 2 \ \frac{mA}{V^2} \\ \beta_2 = 10 \ \frac{mA}{V^2} \\ V_{TH1} = V_{TH2} = 2V \end{cases}$$

$R = ?$

$i_{out} = 5\,mA$



$\beta_1 = 2$

$\beta_2 = 10$

$V_{TH1} = V_{TH2} = 2v$

$$i_D = \frac{\beta}{2}\left(V_{GS} - V_{TH}\right)^2$$

$$V_{GS1} = V_{GS2} \qquad V_{TH1} = V_{TH2}$$

$$\frac{i_{D2}}{i_{D1}} = \frac{\beta_2}{\beta_1} = \frac{10}{2} = 5 = \frac{i_{D2}}{i_{D1}} \quad (I)$$

$$i_{D2} = i_{out} = 5mA \xrightarrow{(I)} i_{D1} = 1mA$$

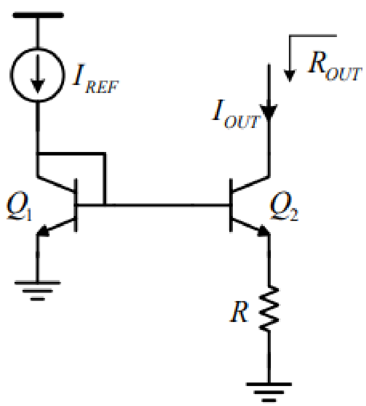
$$V_{GS1} = V_{TH} + \sqrt{\frac{i_{D1}}{k_1}} = 2 + \sqrt{\frac{1}{1}} = 3v$$

$$KVL @ M_1: -10 + R \times i_{D1} + V_{GS1} = 0 \rightarrow R = 10 - 3 = 7\,k\Omega$$

4- In the following circuit,

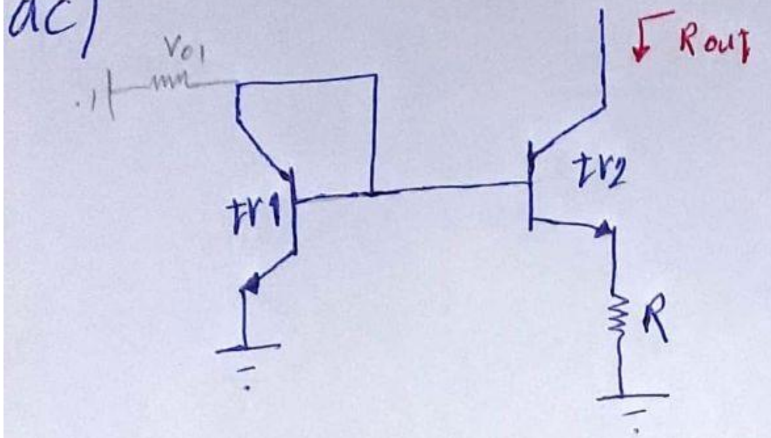
a) Determine the output resistance. Assume that the current source is ideal.

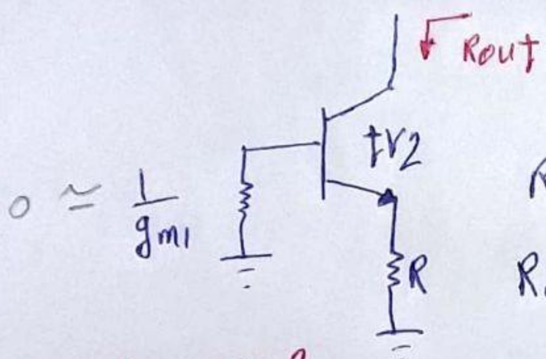
b) Specify R such a way that $I_{REF} = 2 \times I_{OUT}$. The transistors are the same and $\beta \succ\succ 1$.



a) $R_{out} = ?$

ac)

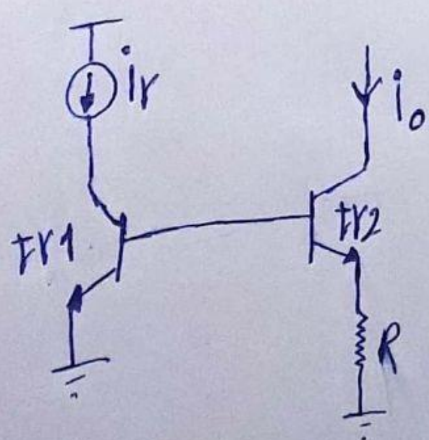




$$R_{out} \simeq r_{o2}\left(1 + g_{m2}\left(R \,\|\, r_{\pi 2}\right)\right)$$

$$R_{out} \simeq \beta r_{o2}$$

b) $i_r = 2 i_o \rightarrow R = ?$



$$-v_{BE1} + v_{BE2} + R i_o = 0$$

$$-v_T \ln\left(\frac{i_r}{i_s}\right) + v_T \ln\left(\frac{i_o}{i_s}\right) = -R i_o$$

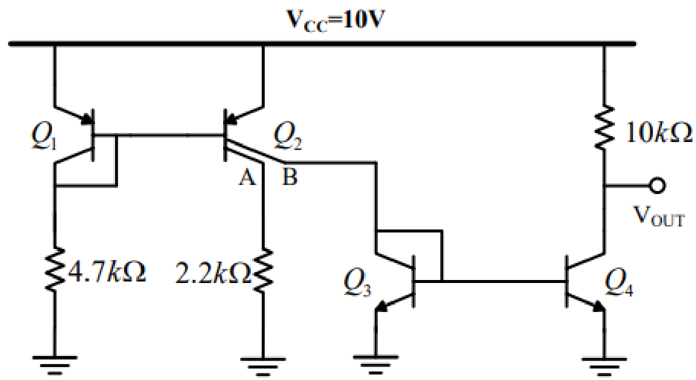
$$v_T \ln\left(\frac{i_o}{i_s} \times \frac{i_s}{i_r}\right) = -R i_o$$

$$v_T \ln\left(\frac{i_r}{i_o}\right) = R i_o \rightarrow R = \frac{v_T \ln(2)}{i_o}$$

$$R = \frac{v_T \ln(4)}{i_r}$$

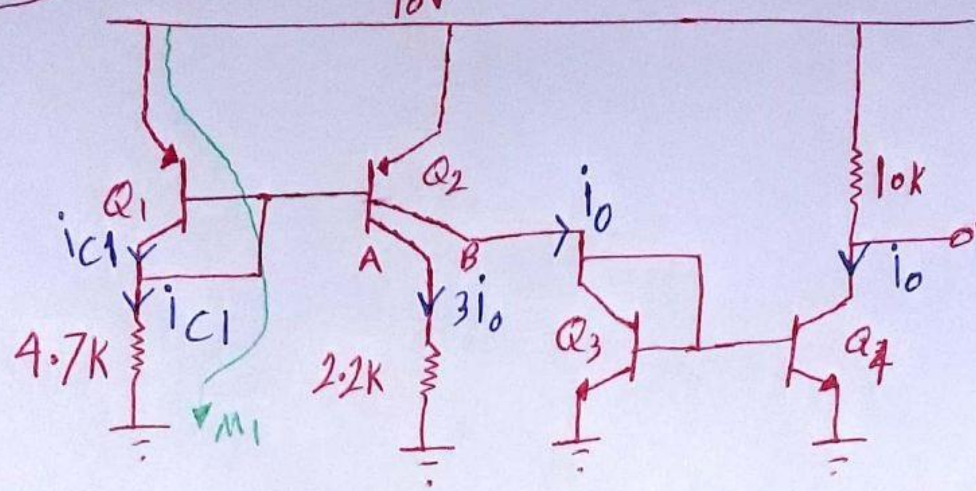
5- In the following circuit, all of the transistors are the same. The effective area of the collector "A" of Q_2 is 3 times larger than the effective area of the collector "B" of Q_2. Calculate the output voltage.

(Hint: Q_2 is a transistors with two collector terminals, which, their currents are proportional to their effective areas. The total effective collector area of Q_2 is the same as that of Q_1).



$v_0 = ?$

15



$A_{S,A} = 3A_{S,B}$

$A_{S_1} = A_{S_2} = A_{S3} = A_{S4}$

$v_{BE3} = v_{BE4} \rightarrow i_{C_3} = i_{C4}$

$i_{C2,A} = 3\, i_{C2,B}$

$i_{C2,A} = 3i_0$

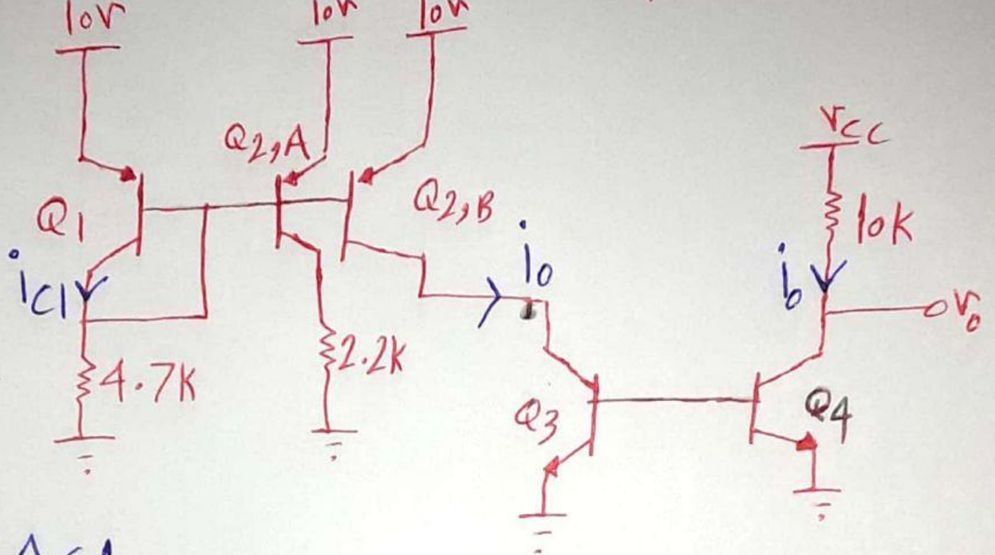
$v_{EB1} = v_{EB2} \rightarrow i_{C1} = i_{C2} = 4i_0$

$KVL @ M_1: -10 + 0.7 + 4.7 i_{C1} = 0$

$4.7 \times 4 i_0 = 9.3 \rightarrow i_0 = 0.5\,mA$

$v_0 = 10 - 10 i_0 = 5v = v_0$

روش دوم حل سوال 5)



$A_{S2} = A_{S1}$

$A_{S2} = A_{S,A} + A_{S,B} = 3A_{S,B} + A_{S,B} = 4A_{S,B}$

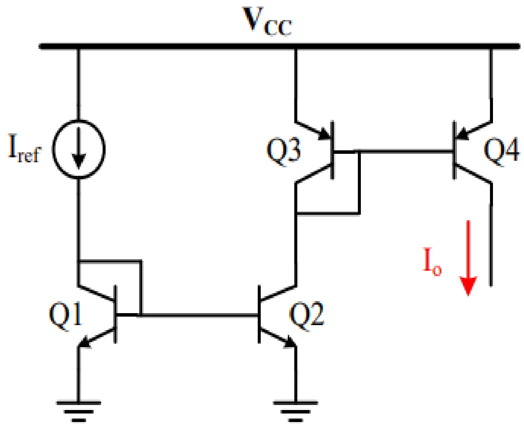
$A_{S1} = 4A_{S,B}$

$v_{EB1} = v_{EB2,B} \longrightarrow i_{C1} = 4\, i_{C2,B} = 4 i_o$

$-10 + 0.7 + 4.7 i_{C1} = 0 \longrightarrow 4.7 \times 4 i_o = 9.3 \longrightarrow$

$i_o \simeq 0.5mA \longrightarrow v_o = 5V$

6- In the following circuit, specify a relation for I_o in terms of I_{ref} (neglect β and λ effects). The collector areas of Q_2 and Q_4 is 2 times larger than Q_1 and the collector area of Q_3 is 3 times larger than Q_1.

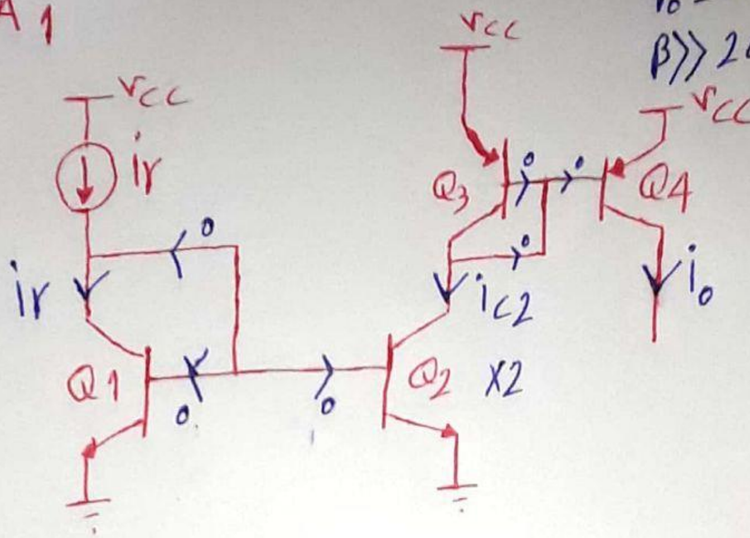


$A_{2,4} = 2A_1$

$A_3 = 3A_1$

$V_o = \infty$

$\beta >> 20$



$V_{BE1} = V_{BE2} \longrightarrow i_{C2} = 2\, i_{C1} = 2\, i_r$

$i_{C3} = i_{C2} = 2\, i_r$

$V_{EB3} = V_{EB4} \qquad \dfrac{A_3}{A_4} = \dfrac{3A_1}{2A_1} = \dfrac{3}{2} \rightarrow i_{C4} = \dfrac{2}{3}\, i_{C3}$

$i_{C4} = \dfrac{2}{3} \times 2\, i_r = \dfrac{4}{3} i_r = i_o \rightarrow i_o = \dfrac{4}{3}\, i_r$