Subject: HW₀Z

Year: Month: Date: ABBASI

۱- تبدیل فوریه 8

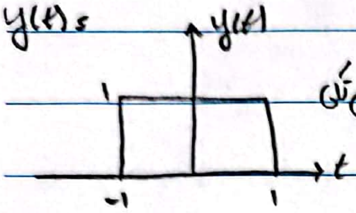
A) $\delta(t+1)+\delta(t-۳)$

$x_1(t)=\delta(t+1) \xleftrightarrow{f} e^{-j\omega t_0}$ $X_1(\omega)=e^{j\omega} \rightarrow X_1(\omega)+X_2(\omega)=e^{j\omega}+e^{-۳j\omega}$

$x_2(t)=\delta(t-۳) \xleftrightarrow{f} e^{-j\omega t_0}$ $X_2(\omega)=e^{-۳j\omega}$

B) $1+\cos(۷\pi t+\frac{\pi}{۸})$

$y(t)=$



پالس گیت $\xleftrightarrow{f} \tau\,\mathrm{sinc}(\frac{\omega\tau}{۲\pi})=۲\,\mathrm{sinc}(\frac{\omega}{\pi})$ ✓

$\underbrace{1+\cos(۷\pi t+\frac{\pi}{۸})}_{z(t)}\,y(t) \rightarrow x(t)=z(t)\cdot y(t) \xleftrightarrow{f} X(\omega)=\frac{Z(\omega)*Y(\omega)}{۲\pi}$

$z(t)=1+\cos(۷\pi t+\frac{\pi}{۸})=1+\frac{1}{۲}e^{j(۷\pi t+\frac{\pi}{۸})}+\frac{1}{۲}e^{-j(۷\pi t+\frac{\pi}{۸})}=1+\frac{1}{۲}e^{j۷\pi t}e^{j\frac{\pi}{۸}}+\frac{1}{۲}e^{-j۷\pi t}e^{-j\frac{\pi}{۸}}$

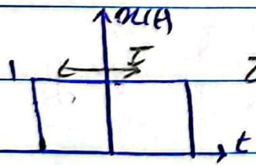
$\xleftrightarrow{f} ۲\pi\delta(\omega)+۲\pi\times\frac{1}{۲}\delta(\omega-۷\pi)e^{j\frac{\pi}{۸}}+۲\pi\times\frac{1}{۲}\delta(\omega+۷\pi)e^{-j\frac{\pi}{۸}}$

$۲\pi\delta(\omega)+\pi e^{j\frac{\pi}{۸}}\delta(\omega-۷\pi)+\pi e^{-j\frac{\pi}{۸}}\delta(\omega+۷\pi)$ ✓

C) $e^{-۲|t-1|} \longrightarrow X(\omega)=\int_{-\infty}^{+\infty}x(t)e^{-j\omega t}dt=\int_{-\infty}^{1}e^{+۲(t-1)}e^{-j\omega t}dt+\int_{1}^{+\infty}e^{-۲(t-1)}e^{-j\omega t}dt$

$\frac{e^{-j\omega}}{۲+j\omega}+\frac{e^{-j\omega}}{۲-j\omega}=\frac{۴e^{-j\omega}}{۴+\omega^{۲}}$

D)



$\tau\,\mathrm{sinc}(\frac{\omega\tau}{۲\pi})=۲\,\mathrm{sinc}(\frac{\omega}{\pi})$

$X(\omega)=\int_{-\infty}^{+\infty}x(t)e^{-j\omega t}dt=\int_{-1}^{1}1e^{-j\omega t}dt=\frac{-1}{j\omega}e^{-j\omega t}\Big|_{-1}^{1}=+\frac{1}{j\omega}(\underbrace{e^{j\omega}-e^{-j\omega}}_{۲j\sin(\omega)})$

$\frac{۲\cancel{j}\sin(\omega)}{\cancel{j}\omega}\rightarrow\frac{۲\sin\omega}{\omega}\rightarrow\frac{۲\sin(\pi\frac{\omega}{\pi})}{\pi\frac{\omega}{\pi}}=\boxed{۲\,\mathrm{sinc}(\frac{\omega}{\pi})}$ ✓

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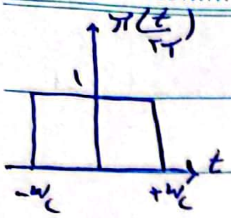


۲ـ با استفاده از دوگانی نشان دهید $\frac{\sin \omega_c t}{\pi t}$ تبدیل فوریه پالس است؟

خاصیت دوگانی:

$x(t) \xleftrightarrow{f} X(\omega)$

$X(t) \xleftrightarrow{f} 2\pi x(-\omega)$

$x(t) \xleftrightarrow{f} X(\omega) \rightarrow X(\omega) = \frac{\sin \omega_c t}{\pi (t)}$

$X(t) \xleftrightarrow{f} 2\pi x(-\omega)$

$2\pi X(t) \longleftrightarrow x(-\omega) \xrightarrow{x(\omega) = x(-\omega)} 2\pi X(t) \xleftrightarrow{f} x(\omega)$ ✓

تبدیل فوریه پالس ← sinc

تبدیل فوریه sinc ← پالس ✓

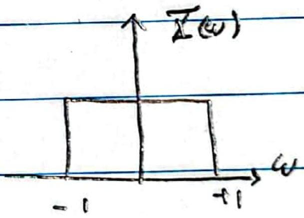
۳ـ تبدیل فوریه؟

$x(t) = t\left(\frac{\sin t}{\pi t}\right)^2 \rightarrow x(t) = t \frac{\sin^2 t}{\pi^2 t^2} \rightarrow \frac{\sin^2 t}{\pi t}$

$\frac{\sin^2\left(\pi \frac{t}{\pi}\right)}{\pi \frac{t}{\pi}} \cdot \frac{1}{\pi} = \frac{1}{\pi} \operatorname{sinc}^2\left(\frac{t}{\pi}\right) \qquad \tau = \frac{1}{\pi} \rightarrow t \operatorname{sinc}\left(\frac{\omega\tau}{2\pi}\right)$

$x(t) = \underbrace{\frac{\sin t}{\pi t}}_{z(t)} \underbrace{\sin t}_{y(t)} \rightarrow x(t) = z(t).y(t) \xleftrightarrow{f} \frac{1}{2\pi}\left(Z(\omega) * Y(\omega)\right) = X_1(\omega)$

$\frac{\sin t}{\pi t} \xleftrightarrow{f} \begin{cases} 1 & |\omega| < 1 \\ 0 & |\omega| > 1 \end{cases} = X(\omega)$

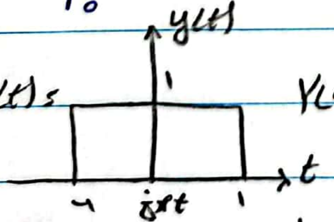


$X(\omega) = j\frac{d}{dt} X_1(\omega) \rightarrow \begin{cases} \frac{\delta}{2\pi} & -2 < \omega < 0 \\ -\frac{\delta}{2\pi} & 0 < \omega < 2 \\ 0 & o.w \end{cases}$ ✓

۴ـ تبدیل فوریه؟

$x(t) = \begin{cases} 1 + \cos \pi t & |t| \leq 1 \\ 0 & |t| > 1 \end{cases}$

$y(t) =$



$Y(\omega) = 2\operatorname{sinc}\left(\frac{\omega}{\pi}\right) \rightarrow x(t) y(t) = 1 + \cos \pi t [y(t)] =$

نیست،

$y(t) + y(t)\cos \pi t$

$y(t) + \frac{1}{2}e^{j\pi t} y(t) + \frac{1}{2}e^{-j\pi t} y(t) \rightarrow X(\omega) = Y(\omega) + \frac{1}{2}e^{j\pi t} Y(\omega) + \frac{1}{2}e^{-j\pi t} Y(\omega)$

$Y(\omega) + \frac{1}{2} Y(\omega - \pi) + \frac{1}{2} Y(\omega + \pi)$

$2\operatorname{sinc}\left(\frac{\omega}{\pi}\right) + \operatorname{sinc}\left(\frac{\omega - \pi}{\pi}\right) + \operatorname{sinc}\left(\frac{\omega + \pi}{\pi}\right)$ ✓

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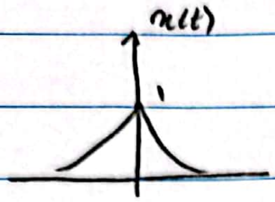
۵- تبدیل فوریه؟ ۸

$x_۲(t) = \frac{۴t}{(1+t^۲)^۲}$

$x(t) = te^{-a|t|}$



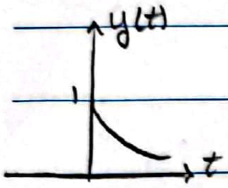
$y(t) = x_1(t) = e^{-\alpha t}u(t) \quad \alpha > 0$

$x(t) = y(t) + y(-t) \xleftrightarrow{f} X(\omega) = Y(\omega) + Y(-\omega)$

$Y(\omega) = \frac{1}{a+j\omega}$

$\frac{1}{\alpha+j\omega} + \frac{1}{\alpha-j\omega} = \frac{۲}{1+\omega^۲}$

$\alpha = 1$



$x(t) \xleftrightarrow{f} X(\omega)$

$x(t) = te^{-|t|} \xleftrightarrow{f} X(\omega) = \frac{۲}{1+\omega^۲}$

$x_۲(t) = \frac{۴t}{(1+t^۲)^۲}$

$X(t) \xleftrightarrow{f} x(-\omega)۲\pi$

$\frac{۲}{1+t^۲} \xleftrightarrow{f} -۲\pi\omega e^{-|\omega|}$

$\boxed{-۲\pi\omega e^{-|\omega|}}$ ✓

$1 \leftarrow$

$te^{-a|t|} = te^{-|t|} \xleftrightarrow{f} \frac{۲}{1+\omega^۲} \longleftrightarrow \frac{۲}{(1+t^۲)} \times \frac{۲t}{(1+t^۲)} \nearrow$

۶- عکس تبدیل فوریه؟ ۸

a) $X(\omega) = \frac{۴\sin(۳\omega-۶\pi)}{\omega-۲\pi} \rightarrow \frac{۴\sin(۳(\omega-۲\pi))}{\omega-۲\pi} \quad (\omega-۲\pi) = S$

$\frac{۴\sin ۳S}{S} = \frac{۴\sin\frac{۳\pi S}{\pi}}{\frac{\pi ۳S}{\pi}} \cdot \frac{۳}{1} \nearrow ۴\text{sinc}\left(\frac{۳S}{\pi}\right) \rightarrow ۴\text{sinc}\left(\frac{۳\omega-۶\pi}{\pi}\right)$

پالس مستطیل به ارتفاع ۱ و $t=\frac{۴}{۶}$ $T=۳$

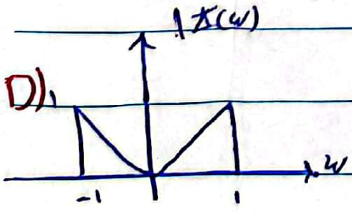
b) $۲(\delta(\omega-1) - \delta(\omega+1)) + ۳(\delta(\omega-۲\pi) + \delta(\omega+۲\pi))$

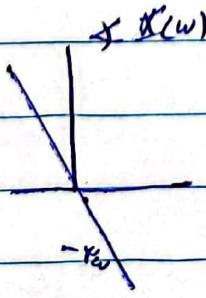
$۲\delta(\omega-1) = X(\omega) \quad \delta(t) \xleftrightarrow{f} 1 \quad 1 \xleftrightarrow{f} ۲\pi\delta(\omega) \xrightarrow{\div ۲\pi} \frac{1}{۲\pi} \xleftrightarrow{f} \delta(\omega)$

$\frac{1}{۲\pi} \times ۲e^{jt} - \frac{1}{۲\pi}۲e^{-jt} + ۳\times\frac{1}{۲\pi}e^{j۲\pi t} + ۳\times\frac{1}{۲\pi}e^{-j۲\pi t} =$

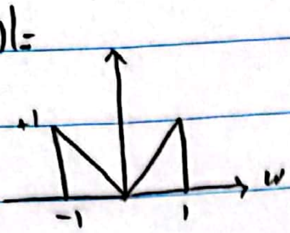
$\frac{1}{\pi}e^{jt} - \frac{1}{\pi}e^{-jt} + \frac{۳}{۲\pi}e^{j(۲\pi t-۲\pi t)} = \boxed{\frac{۲}{\pi}\sin t + \frac{۳}{\pi}\cos(۲\pi t)}$ ✓

D)





$Y(w) = |X(w)| =$



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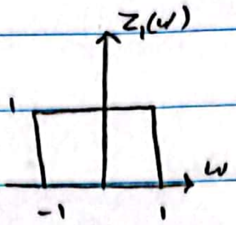

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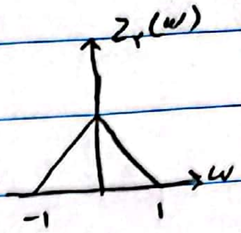


$Y(w) = Z_1(w) - Z_2(w)$

$Z_1(w) \xleftrightarrow{f} \tau\,\text{sinc}(\frac{w\tau}{۲\pi}) = ۲\,\text{sinc}(\frac{w}{\pi}) \xleftrightarrow{f} ۲\pi \times Z_1(-w)$ تقارن دارد $\xrightarrow{\div ۲\pi}$

$\frac{۲}{۲\pi}\text{sinc}(\frac{w}{\pi}) \xleftrightarrow{f} Z_1(w) \Rightarrow \boxed{\frac{1}{\pi}\text{sinc}(\frac{t}{\pi})}$ ✓

$Z_2(w) = \xleftrightarrow{f} \text{sinc}^۲(\frac{w}{۲\pi}) \longrightarrow ۲\pi Z_2(w) \longrightarrow \frac{1}{۲\pi}\text{sinc}^۲(\frac{t}{۲\pi}) \xleftrightarrow{f} Z_2(w)$

$Z_1(t) - Z_2(t) =$

$\boxed{\frac{1}{\pi}\text{sinc}(\frac{t}{\pi}) - \frac{1}{۲\pi}\text{sinc}^۲(\frac{t}{۲\pi})}$ ✓

۷- بدون یافتن $X(w)$ پاسخ دهید. 8

$\measuredangle X(w)$

$X(0)$

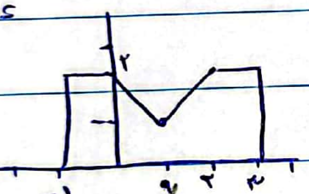
$\int_{-\infty}^{+\infty} X(w)dw$

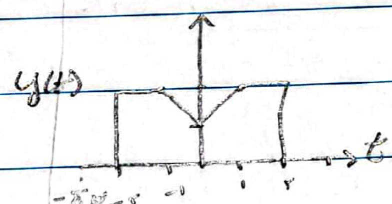
$\int_{-\infty}^{+\infty} X(w)\frac{۲\sin w}{w}e^{۲jw}dw$

عکس تبدیل $Re\{X(w)\}$

$\int_{-\infty}^{+\infty} |X(w)|^۲dw$

$x(t) =$





$x(t) = y(t-1) \xleftrightarrow{f} X(w) = Y(w)e^{-jw}$

$\measuredangle X(w) = -w$ (الف)

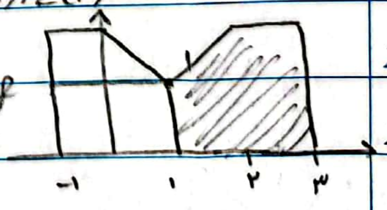
$X(0) = \int_{-\infty}^{+\infty} x(t)dt = (۴\times۲) - (\frac{1}{۲}\times۲\times1) = ۸-1 = \boxed{۷}$ ب

$x(t) = \frac{1}{۲\pi}\int_{-\infty}^{+\infty} X(w)e^{jwt}dw \xrightarrow{\times ۲\pi} ۲\pi x(t) = \int_{-\infty}^{+\infty} X(w)e^{jwt}dw$

$t=0 \longrightarrow ۲\pi x(0) = \int_{-\infty}^{+\infty} X(w)dw = ۲\pi\times۲ = \boxed{۴\pi}$ 8 ج

$\underbrace{\int_{-\infty}^{+\infty} X(w)\underbrace{\frac{۲\sin w}{w}}_{Y(w)}}_{Z(w)} e^{۲jw}dw \Rightarrow \int_{-\infty}^{+\infty} Z(w)e^{۲jw}dw = Z(t)۲\pi = ۲\pi Z(۲)$

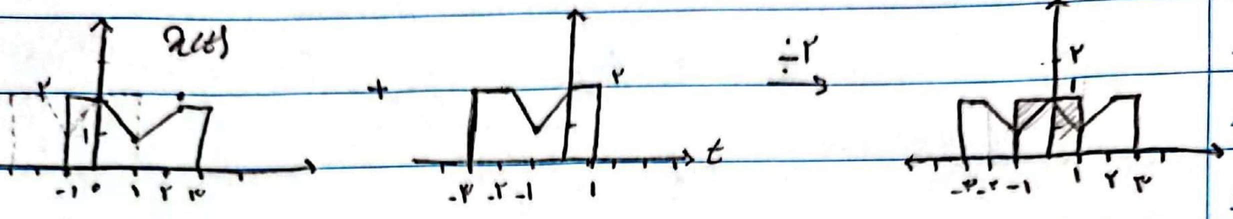
$Y(w) = \frac{۲\sin(\pi\frac{w}{\pi})}{\pi\frac{w}{\pi}} = ۲\,\text{sinc}(\frac{w}{\pi}) \xrightarrow{f}$



$Z(w) = Y(w).X(w) \xleftrightarrow{f} z(t) = x(t) * y(t) = \int_{-\infty}^{+\infty} x(\tau)y(۲-\tau)d\tau = ۴-(\frac{1}{۲}\times1\times1) =$ د

$\frac{۷}{۲}\times۲\pi = \boxed{۷\pi}$ ✓

$\mathrm{Re}\{X(\omega)\} = \mathcal{E}v\{x(t)\} = \frac{x(t)+x(t)}{۲}$



$\int_{-\infty}^{+\infty} |X(\omega)|^۲ d\omega = ۲\pi \int_{-\infty}^{+\infty} |x(t)|^۲ dt = \overline{۲۴\pi}$ ۱۲

۸-

$y(t) = x(t) * h(t) \quad g(t) = x(۳t) * h(۳t) \quad g(t) = Ay(Bt)$

$x(۳t) \xleftrightarrow{f} \frac{۱}{۳} X(\frac{\omega}{۳}) \quad G(\omega) = \frac{۱}{۳} X(\frac{\omega}{۳}) \cdot \frac{۱}{۳} H(\frac{\omega}{۳}) = \frac{۱}{۹} X(\frac{\omega}{۳}) . H(\frac{\omega}{۳})$

$h(۳t) \xleftrightarrow{f} \frac{۱}{۳} H(\frac{\omega}{۳})$

$Y(\omega) = X(\omega) . H(\omega) \rightarrow Y(\frac{\omega}{۳}) = X(\frac{\omega}{۳}) . H(\frac{\omega}{۳}) \rightarrow \frac{۱}{۹} Y(\frac{\omega}{۳}) \xleftrightarrow{f} \boxed{\frac{۱}{۳} y(۳t)}$ ۱۷

۹-

$\frac{d^۲}{dt^۲} y(t) + ۶ \frac{d}{dt} y(t) = ۲x(t)$

(۴

الف) پاسخ ضربه؟

ب) $y(t) = ?$ $x(t) = te^{-۲t}$

$(j\omega)^۲ Y(\omega) + ۶ j\omega Y(\omega) + ۸ Y(\omega) = ۲ X(\omega)$

$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{۲}{j\omega^۲ + ۶ j\omega + ۸} \quad j\omega = t$ تفکیک $\Rightarrow \frac{۲}{t^۲ + ۶t + ۸} = \frac{۲}{(t+۴)(t+۲)}$

$t = -۴ \quad t = -۲ : \frac{۲}{(t+۴)(t+۲)} = \frac{A}{(t+۴)} + \frac{B}{(t+۲)} \quad A = t+۴ \, H(\omega) \Big|_{t=-۴} = \frac{۲}{-۲} = \boxed{-۱}$

$B = (t+۲) H(\omega) \Big|_{t=-۲} = \frac{۲}{۲} = ۱ \Rightarrow -\frac{۱}{t+۴} + \frac{۱}{t+۲} = \frac{۱}{j\omega+۲} - \frac{۱}{j\omega+۴}$

$e^{-۲t} u(t) - e^{-۴t} u(t) = h(t)$

۱۰-

$x(t) \longrightarrow \boxed{LTI} \longrightarrow y(t)$

$x(t) = (e^{-t} - e^{-۳t}) u(t)$ الف) $h(t)$؟

$y(t) = (۲e^{-t} - ۲e^{-۴t}) u(t)$ ب) $H(\omega)$؟

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$H(\omega) = \frac{Y(\omega)}{X(\omega)} \Rightarrow X(\omega) = \frac{1}{1+j\omega} + \frac{1}{۳+j\omega} \quad Y(\omega) = \frac{۲}{1+j\omega} - \frac{۲}{٤+j\omega}$

$H(\omega) = \frac{۳(۳+j\omega)}{(٤+j\omega)(۲+j\omega)} \rightarrow H(\omega) = \frac{۱٫۵}{۲+j\omega} + \frac{۱٫۵}{٤+j\omega} \rightarrow h(t) = ۱٫۵(e^{-۲t} + e^{-٤t})u(t)$