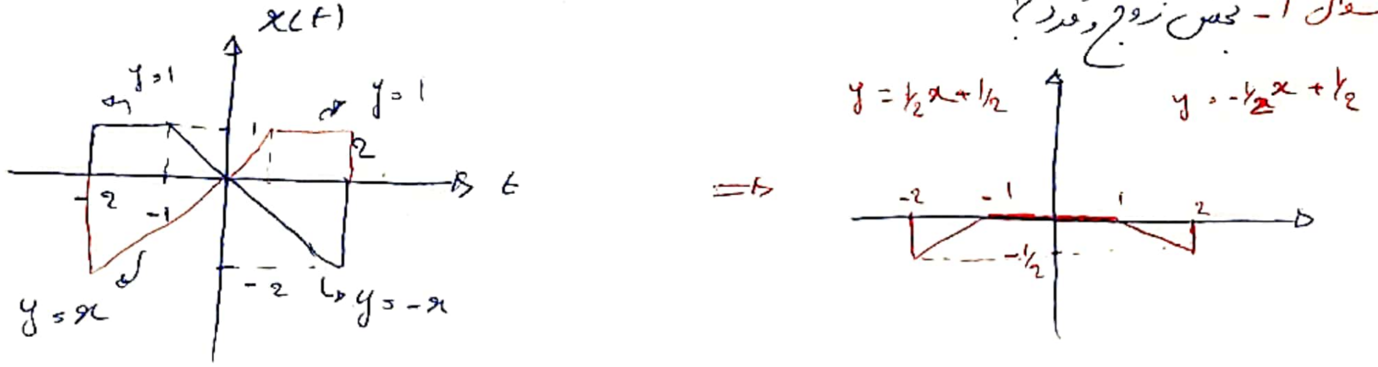
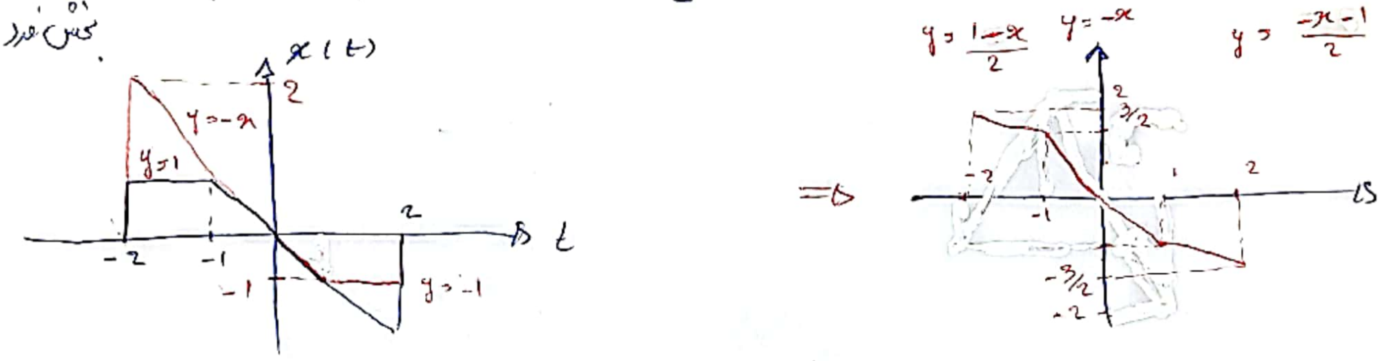
سوال ۱ - بخش زوج و فرد؟

بخش زوج :



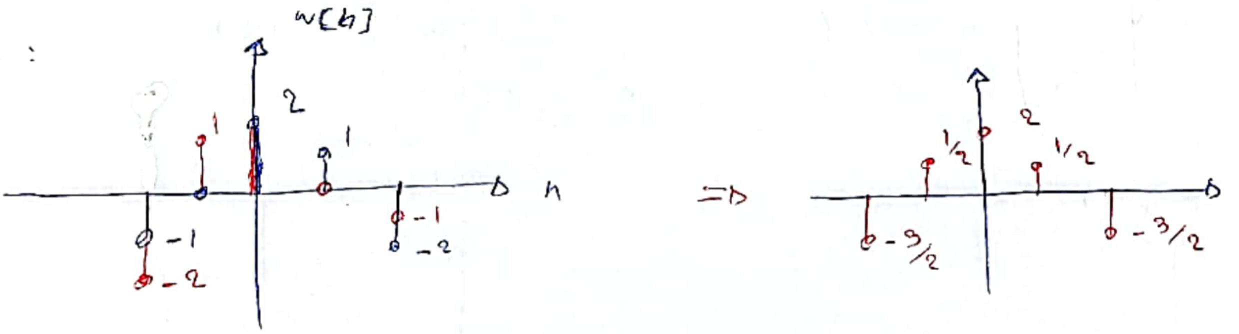
$$Ev\{x(t)\} = \frac{x(t) + x(-t)}{2}$$

بخش فرد :

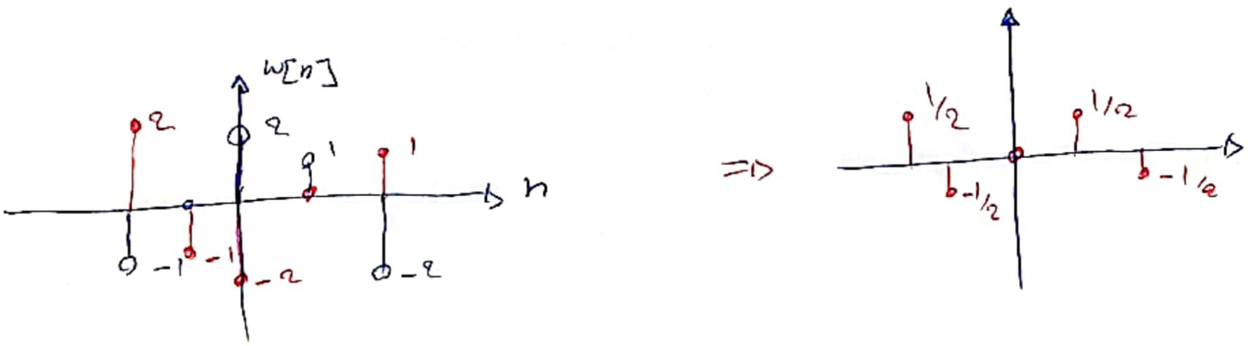


$$odd\{x(t)\} = \frac{x(t) - x(-t)}{2}$$

بخش زوج :



بخش فرد :



سؤال 2-

$$e^{\pm j\beta} = \cos\beta \pm j\sin\beta$$

$$\begin{cases} \sin\alpha = \dfrac{e^{j\alpha}-e^{-j\alpha}}{2} \\ \cos\alpha = \dfrac{e^{j\alpha}+e^{-j\alpha}}{2} \end{cases}$$

الف) $j e^{(-2t+j100t)}$

$$j e^{-2t} \cdot e^{j100t} = e^{-2t} j(\cos 100t + j\sin 100t)$$

$$\Rightarrow e^{-2t}\sin(100t+\pi) + e^{-2t}\cos(100t+\pi/2)$$

ب) $\sqrt{2}\, e^{j\pi/4}\cos(3t+\pi/3)$

$$\Rightarrow e^{0t}\cos(3t+0)$$

سؤال 3-

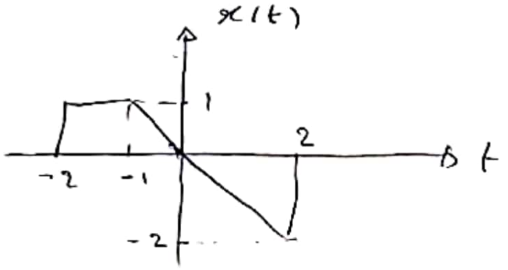
$z(t)=?$ ⟸ $x(t)=z(-\frac{1}{2}t+1)$

$-\frac{1}{2}t+1=\alpha \Rightarrow t=-2\alpha+2$

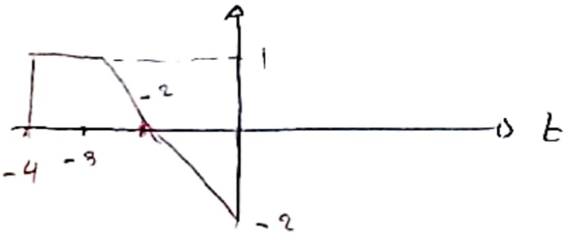
$z(\alpha)=x(-2\alpha+2)$

$\Rightarrow z(t)=x(-2t+2)$

$x(t)$

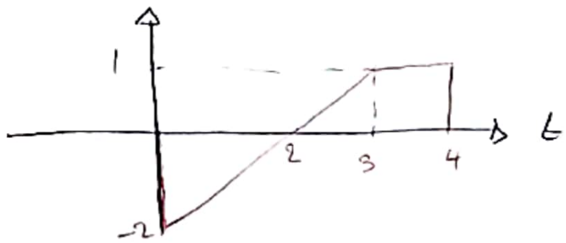


$x(t+2)$

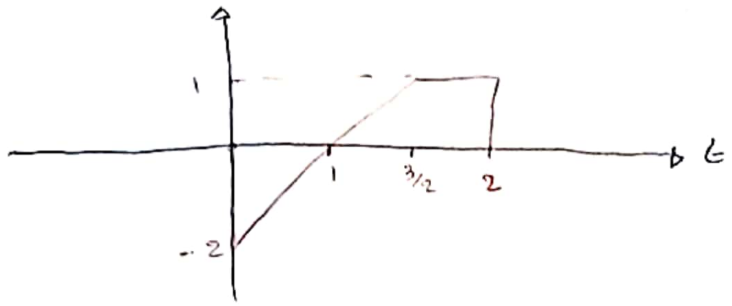


$\Rightarrow$

$x(-t+2)$



$z(t)=x(-2t+2)$



$$\sqrt{2}\, e^{j\frac{\pi}{2}} \cdot \cos\left(3t+\frac{\pi}{3}\right) = \sqrt{2}\, e^{j\frac{\pi}{2}} \cdot \left(\frac{e^{j(3t+\frac{\pi}{3})} + e^{-j(3t+\frac{\pi}{3})}}{2}\right)$$

$$= \frac{\sqrt{2}}{2}\left[e^{j\left(3t+\frac{5\pi}{6}\right)} + e^{-j\left(3t+\frac{\pi}{3}-\frac{\pi}{2}\right)}\right]$$

$$= \frac{\sqrt{2}}{2}\left[e^{j(3t+\frac{\pi}{6})} + e^{-j(3t-\frac{\pi}{6})}\right] = \frac{\sqrt{2}}{2}\left(\cos\left(3t+\frac{\pi}{6}\right)+j\sin\left(3t+\frac{\pi}{6}\right)\right)$$

$$+ \frac{\sqrt{2}}{2}\left(\cos\left(3t-\frac{\pi}{6}\right)+j\sin\left(3t-\frac{\pi}{6}\right)\right) = \sqrt{\frac{2}{2}}\left[\left(\cos\left(3t+\frac{\pi}{6}\right)+\cos\left(3t-\frac{\pi}{6}\right)\right)+\right.$$

$$\left. j\left(\sin\left(3t+\frac{\pi}{6}\right)+\sin\left(3t-\frac{\pi}{6}\right)\right)\right]$$

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سؤال 4 و 5

الف) $x[n] = 3e^{j3\pi(n+1/2)/5}$

$3e^{j(\frac{3\pi}{5}n + \frac{3\pi}{10})} = 3(e^{j\frac{3\pi}{5}n} \times e^{j\frac{3\pi}{10}}) = 3e^{j\frac{3\pi}{10}}(e^{j\frac{3\pi}{5}(n+N_0)})$

$\Rightarrow e^{j\frac{3\pi}{5}N_0} = e^{j2k\pi} \Rightarrow \frac{3\pi}{5}N_0 = 2k\pi \Rightarrow N_0 = \frac{10}{3}k \xrightarrow{k=3} N_0 = 10$

سیگنال متناوب است.

$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \Rightarrow |x[n]|^2 = 9$

$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \infty \Rightarrow P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} 9 = 9$

ب) $x(t) = 2\cos(10t+1) - \sin(4t-1)$

$\cos(10(t+T_{0_1})+1) \rightarrow 10T_{0_1} = 2k\pi \Rightarrow T_{0_1} = \frac{k\pi}{5} \quad k=1 \rightarrow T_{0_1} = \frac{\pi}{5}$

$\sin(4(t+T_0)-1) \Rightarrow 4T_{0_2} = 2k\pi \Rightarrow T_{0_2} = \frac{k\pi}{2} \quad k=1 \rightarrow T_{0_2} = \frac{\pi}{2}$

سیگنال متناوب است.

$k.m.m\{\frac{\pi}{5}, \frac{\pi}{2}\} = \pi$

$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = 2\int_{-\infty}^{+\infty} \cos^2(10t+1)dt - \int_{-\infty}^{+\infty} \sin^2(4t+1)dt = \infty$

$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} 2\cos^2(10t+1)dt - \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \sin^2(4t-1)dt$

$2\left[\frac{1+\overset{0}{\cancel{\cos(20t+2)}}}{2}\right] \qquad \left[\frac{1-\overset{0}{\cancel{\cos(8t-2)}}}{2}\right]$

$P = 1 - 1/2 = 1/2$

2.) $z[n] = \sum_{k=-\infty}^{+\infty} \{\delta[n-4k] - \delta[n-1-4k]\}$

متناوب $\Rightarrow$ $\underline{N_0 = 4}$

$$E = \sum_{n=-\infty}^{+\infty} |x[n]|^2 = \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} \infty = 1$$

ج) $\delta[n] = \begin{cases} 1 & n=0 \\ 0 & n\neq 0 \end{cases}$

غیر متناوب

$$E = \sum_{n=-\infty}^{+\infty} |x[n]|^2 = 1$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} 1 = 0$$

ه) $s[n] = \cos[\pi/2\, n] \cdot \cos[\pi/4\, n]$

$\cos a \cdot \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$

$\Rightarrow s[n] = \frac{1}{2}[\cos(3\pi/4\, n) + \cos(\pi/4\, n)]$

$\cos(3\pi/4(n+N_0)) \Rightarrow 3\pi/4\, N_0 = 2k\pi \Rightarrow N_{0_1} = 8/3\, k \to k=3 \to N_{0_1} = 8$

$\cos(\pi/4(n+N_0)) \Rightarrow \pi/4\, N_0 = 2k\pi \Rightarrow N_{0_2} = 8k \to k=1 \to N_{0_2} = 8$

سیگنال متناوب است

$\text{lcm}\{8, 8\} = 8$

$$E = \sum_{n=-\infty}^{+\infty} |s[n]|^2 = \frac{1}{2}\sum_{N=-\infty}^{+\infty} \cos^2[3\pi/4\, n] + \frac{1}{2}\sum_{N=-\infty}^{+\infty} \cos^2[\pi/4\, n] = \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \frac{1}{2}\left[\sum_{-N}^{N} \frac{1+\cos 2[3\pi/4\, n]}{2} + \sum_{-N}^{N} \frac{1+\cos 2[\pi/4\, n]}{2}\right] = \frac{1}{2}$$

9)

$$x(t) = \mathcal{E}v\{\sin(4\pi t)\,u(t)\}$$

$$x(t) = \frac{1}{2}\left[\sin(4\pi t)\,u(t) - \sin(4\pi t)\,u(-t)\right]$$

متناوب نیست

$$E = \int_{-\infty}^{+\infty} |x(t)|^2\, dt = \infty$$