یاالطیف

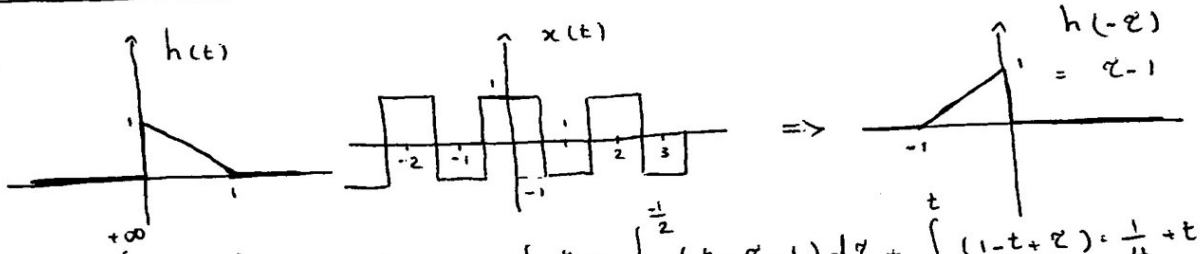
رضا آذینه پور
۹۸۱۴۳۰۳
تمرین سری ۳- سیگنال و سیستم

#1
$$\begin{cases} x[n] = \left(\frac{-1}{2}\right)^n u[n-4] \\ h[n] = 4^n u[2-n] \\ y[n] = x[n] * h[n] = ? \end{cases}$$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k] = \sum_{k=-\infty}^{+\infty} \left(\left(\frac{-1}{2}\right)^k u[k-4]\right)\cdot\left(4^{(n-k)}\, u[2-n+k]\right)$$

$$= 4^n \sum_{k=4}^{n-2} \left(\frac{-1}{2}\right)^k \cdot 4^{-k} = 4^n \sum_{k=4}^{n-2} \left(\frac{-1}{8}\right)^k = 4^n \left(\frac{\left(\frac{-1}{8}\right)^4 - \left(\frac{-1}{8}\right)^{n-1}}{\frac{9}{8}}\right)$$

#2

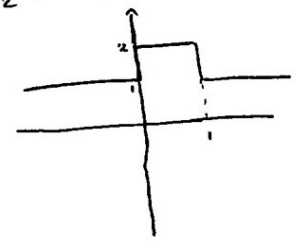


$$\Rightarrow y(t) = \int_{-\infty}^{+\infty} x(\tau)\,h(t-\tau)\,d\tau =$$

$$\begin{cases} y_1 = \int_{t-1}^{\frac{-1}{2}} (t-\tau-1)\,d\tau + \int_{-\frac{1}{2}}^{t} (1-t+\tau) = \frac{1}{4} + t - t^2 & -\frac{1}{2} < t < \frac{1}{2} \\ y_2 = \int_{t-1}^{\frac{1}{2}} (1-t+\tau)\,d\tau + \int_{\frac{1}{2}}^{t} (t-1-\tau)\,d\tau = t^2 - \frac{3t}{2} + \frac{7}{4} & \frac{1}{2} < t < \frac{3}{2} \end{cases}$$

#3 a) $h[n] = \left(\frac{-1}{2}\right)^n u[n] + (1.01)^n u[1-n]$

$\Rightarrow u[n] + u[1-n]$:



$\Rightarrow$ سیستم LTI، علی‌ست چون $h[n] \neq 0$ for $n \leq 0$

پایدار: $\sum_{n=-\infty}^{+\infty} |h[n]| < \infty$:
$$\sum_{-\infty}^{0} (1.01)^n + \sum_{0}^{1} \left(\frac{-1}{2}\right)^n + \sum_{0}^{1} (1.01)^n + \sum_{1}^{\infty} \left(\frac{-1}{2}\right)^n < \infty$$

$\Rightarrow$ سیستم LTI، پایدار است

b) $h(t) = (2e^{-t} - e^{-\frac{t-100}{100}})u(t)$

$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \Rightarrow h(t) = \begin{cases} 2e^{-t} - e^{-\frac{t-100}{100}} & t > 0 \\ 0 & t < 0 \end{cases}$ ← سیستم علی است

پایدار: $\int_{-\infty}^{+\infty} |h(t)|\,dt < \infty : \int_{0}^{\infty} 2e^{-t}\,dt - \int_{0}^{\infty} e^{\frac{-t+100}{100}}\,dt = 2 - 100e < \infty$

پس سیستم پایدار است

#4 $y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \underbrace{x(t-\tau)}_{=\lambda}\,d\tau$ $\quad \lambda = t-\tau \Rightarrow \tau = t-\lambda$

$\Rightarrow$ [illegible]

$\Rightarrow \int_{-\infty}^{\lambda+\tau} e^{-\lambda} x(\lambda)\,d(t-\lambda)$

#5 $y[n] + 2y[n-1] = x[n] + 2x[n-2]$

$y[n] = x[n] + 2x[n-2] - 2y[n-1]$

اگر $n=0 \longrightarrow y[0] = x[0] + 2x[-2] - 2y[-1] = 0 \Rightarrow y[-1] = \frac{1}{2}x[0] + x[-2]$

$\Rightarrow y[-1] = \frac{1}{2} + 1 = \frac{3}{2}$

اگر $n=1 \longrightarrow y[1] = x[1] + 2x[-1] - 2y[0] = 1 + 2 = 3$

اگر $n=2 \longrightarrow y[2] = x[2] + 2x[0] - 2y[1] = \boxed{0 + 2 - 2y[1] = y[2]} \Rightarrow y[2] = -4$

اگر $n=3 \longrightarrow y[3] = x[3] + 2x[-1] - 2y[2] \Rightarrow y[3] = 10$

اگر $n=4 \longrightarrow y[4] = -2y[+3] = -20$

اگر $n=5 \longrightarrow y[5] = -2y[4] = 40$

اگر $n=-1 \longrightarrow y[-1] = x[-1] + 2x[-3] - 2y[-2] \Rightarrow y[-2] = \frac{1}{2} + 0 + \frac{1}{2} = 0 \rightarrow y[-2] = 0$

اگر $n=-2 \longrightarrow y[-2] = x[-2] + 2x[-4] - 2y[-3] \Rightarrow y[-3] = \frac{-1}{2}$

اگر $n=-3 \longrightarrow y[-3] = x[-3] + 2x[-5] - 2y[-4] \Rightarrow y[-4] = \frac{1}{4}$

اگر $n=-4 \longrightarrow y[-4] = -2y[-5] \Rightarrow y[-5] = \frac{-1}{8}$

$\Rightarrow$