یالغیب

تمرین سری 5

۱۴۰۰ / ۱ / ۳۱

#1

1) $x[n]$ Periodic with $N=10$

2) $x[n] \xleftrightarrow{F.S} a_k$

3) $x[n]$: حقیقی و فرد

4) $P = 8$

5) $2|a_3| = |a_4|$

$0 < \measuredangle a_3 < \pi$

$0 < \measuredangle a_4 < \pi$

از ③ می دانیم : a_k ها موهومی خالص و فرد هستند

$$P = \frac{1}{N} \cdot \sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2 = \sum_{k=0}^{10} |a_k|^2 = |a_0|^2 + |a_1|^2 + \dots + |a_{10}|^2$$

فقط ضرایب a_3 و a_4 غیرصفر هستند .

$$\Rightarrow |a_3|^2 + |a_4|^2 \xrightarrow{(5)} |a_3|^2 + \left[2|a_3|\right]^2 = 5|a_3|^2 = 8 \Rightarrow |a_3|^2 = \frac{8}{5}$$

$$\begin{cases} |a_3| = 2\sqrt{\frac{2}{5}} \\ |a_4| = 4\sqrt{\frac{2}{5}} \end{cases} \xrightarrow{\text{طبق دیتا ③}} a_k = -a_{-k} \Rightarrow \begin{cases} a_3 = -a_{-3} = 2j\sqrt{\frac{2}{5}} \\ a_4 = -a_{-4} = 4j\sqrt{\frac{2}{5}} \end{cases}$$

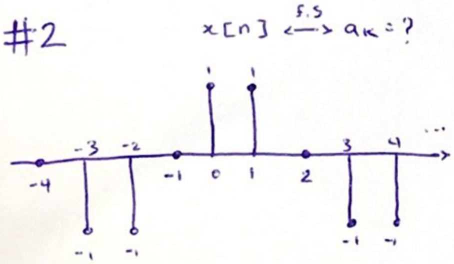
$$\Rightarrow x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\frac{2\pi}{10}n} = \sum_{k=0}^{10} a_k e^{jk\frac{\pi}{5}n} = a_3 e^{j\frac{3\pi}{5}n} + a_4 e^{j\frac{4\pi}{5}n}$$

$$= 2j\sqrt{\frac{2}{5}}\, e^{j\frac{3\pi}{5}n} + 4j\sqrt{\frac{2}{5}}\, e^{j\frac{4\pi}{5}n} = x[n]$$

پریودیک با $N=10$

#2

$x[n] \xleftrightarrow{F.S} a_k = ?$



$$a_k = \frac{1}{N} \cdot \sum_{k=\langle N\rangle} x[n] e^{-jk\left(\frac{2\pi}{N}\right)n} \xrightarrow{N=6}$$

$$= \frac{1}{6} \cdot \sum_{k=-3}^{3} x[n] e^{-jk\frac{\pi}{3}n} = \frac{1}{6}\Big[x[-3]e^{+j\pi n} + x[-2]e^{j\frac{2\pi}{3}n}$$

$$+ \cancel{x[-1]}_{0} e^{j\frac{\pi}{3}n} + x[0]e^{0} + x[1]e^{-j\frac{\pi}{3}n} + \cancel{x[2]}_{0} e^{-j\frac{2\pi}{3}n} + x[3] e^{-j\pi n}\Big]$$

$$\Rightarrow a_k = \frac{1}{6} \cdot \left[-e^{j\pi n} - e^{j\frac{2\pi}{3}n} + 1 + e^{-j\frac{\pi}{3}n} - e^{-j\pi n}\right]$$

3

$$\begin{cases} \xrightarrow{x[n]} \boxed{\text{علی}} \xrightarrow{y[n]} \\ y[n] - \frac{1}{4} y[n-1] = x[n] \\ \text{if} \quad x[n] = \cos\left(\frac{\pi}{4}n\right) + 3\sin\left(\frac{2\pi}{3}n\right) \Rightarrow y[n] \xleftrightarrow{F.S} a_k = ? \end{cases}$$

با فرض اینکه سیستم LTI است :

$$x[n] = e^{j\omega n} \longrightarrow \boxed{LTI} \xrightarrow{H(j\omega)e^{j\omega n}}$$

$\Rightarrow$ طبق معادله دیفرانسیل داده شده اگر ورودی سیستم را $e^{j\omega n}$ فرض کنیم :

$$H(e^{j\omega})e^{j\omega n} - \frac{1}{4}H(e^{j\omega})e^{-j\omega} \cdot e^{j\omega n} = e^{j\omega n}$$

$$\Rightarrow H(e^{j\omega}) \cdot \left[e^{j\omega n} - \frac{1}{4}e^{-j\omega}e^{j\omega n}\right] = e^{j\omega n}$$

$$\Rightarrow H(\cancel{j}e^{j\omega}) = \frac{e^{j\omega n}}{e^{j\omega n} - \frac{1}{4}e^{j\omega(n-1)}} \xrightarrow{\div e^{j\omega n}} H(e^{j\omega}) = \frac{1}{1 - \frac{1}{4}e^{-j\omega}}$$

$$y[n] = \sum_{k=\langle N \rangle} a_k H\left(e^{j\frac{2\pi k}{N}}\right) e^{jk\left(\frac{2\pi}{N}\right)n} \quad , \quad x[n] = \underbrace{\cos\left(\frac{\pi}{4}n\right)}_{N_1 = 8} + \underbrace{3\sin\left(\frac{2\pi}{3}n\right)}_{N_2 = 3}$$

$$\Rightarrow \text{KMM}\{N_1, N_2\} = \boxed{24}$$

$$x[n] = \underset{a_3}{\frac{1}{2}} e^{j\frac{\pi}{4}n} + \underset{a_{-3}}{\frac{1}{2}} e^{-j\frac{\pi}{4}n} + \underset{a_8}{\frac{3}{2j}} e^{j\frac{2\pi}{3}n} - \underset{a_{-8}}{\frac{3}{2j}} e^{-j\frac{2\pi}{3}n} \longrightarrow \begin{array}{|l} a_3 = a_{-3} = \frac{1}{2} \\ a_8 = \frac{3}{2j} \\ a_{-8} = -\frac{3}{2j} \end{array}$$

$$b_3 = a_3 H\left(e^{j\frac{\pi}{4}}\right) = \frac{1}{2} \times \frac{1}{1 - \frac{1}{4}e^{-j\frac{\pi}{4}}} \quad , \quad b_{-3} = a_{-3} H\left(e^{-j\frac{\pi}{4}}\right) = \frac{1}{2} \times \frac{1}{1 - \frac{1}{4}e^{j\frac{\pi}{4}}}$$

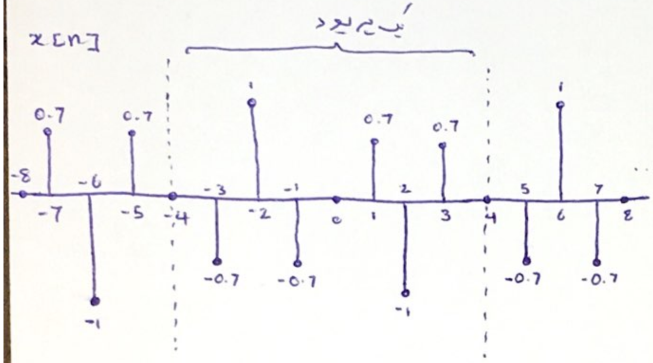
$$b_8 = a_8 H\left(e^{j\frac{2\pi}{3}}\right) = \frac{3}{2j} \times \frac{1}{1 - \frac{1}{4}e^{-j\frac{2\pi}{3}}} \quad , \quad b_{-8} = a_{-8} H\left(e^{-j\frac{2\pi}{3}}\right) = \frac{-3}{2j} \times \frac{1}{1 - \frac{1}{4}e^{j\frac{2\pi}{3}}}$$

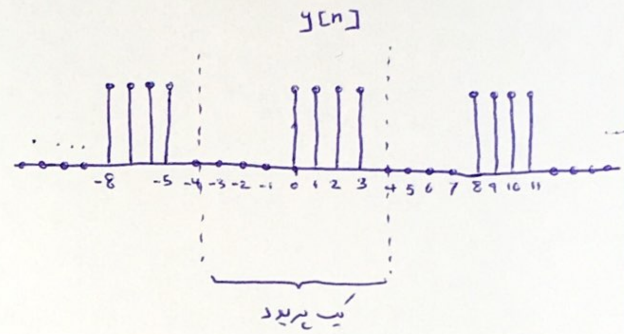
#4

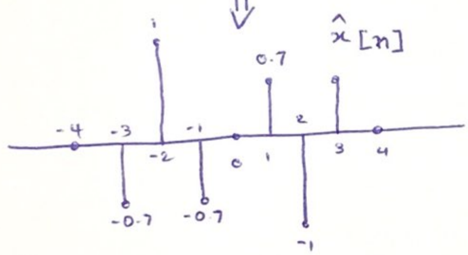
$$x[n] = \sin\left(\frac{3\pi}{4}n\right) \qquad \Rightarrow \quad x[n] \circledast y[n] = ?$$

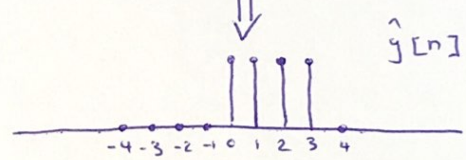
$$y[n] = \begin{cases} 1 & ; \quad 0 \le n \le 3 \\ 0 & ; \quad 4 \le n \le 7 \end{cases}$$

$\underline{N = 8}$

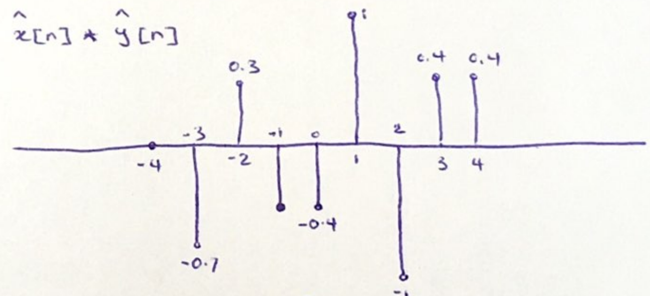




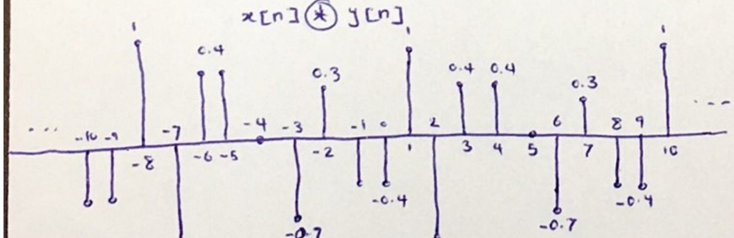




$$\hat{x}[n] * \hat{y}[n] = \sum_{k=-\infty}^{+\infty} \hat{x}[k]\,\hat{y}[n-k] \quad \Rightarrow$$



حاصل کانولوشن معمولی فقط در یک پریود ==> حاصل کانولوشن پریودیک این دو سیگنال، متناوب شده سیگنال
کانولوشن شده با پریود 8 است یعنی:



$$a_k = \frac{1}{8} \cdot \sum_{k=-4}^{4} x[n] e^{-jk\frac{2\pi}{8}n} = \frac{1}{8}\left[x[-4] e^{-jk\frac{\pi}{4}(-4)} + x[-3] e^{-jk\frac{\pi}{4}(-3)} + \cdots + x[3] e^{-jk\frac{\pi}{4}(3)} \right]$$

$$= \frac{1}{8}\left[-0.7 e^{-jk\frac{3\pi}{4}} + 0.3 e^{jk\frac{\pi}{2}} - 0.4 e^{jk\frac{\pi}{4}} - 1 + e^{-jk\frac{\pi}{4}} - e^{-jk\frac{\pi}{2}} + 0.4 e^{-jk\frac{3\pi}{4}} \right]$$