$x_1(t) \to$ // $\to a_k$ -1

$$x_2(t) = 2x_1(\overset{\to T_1}{1-t}) - \frac{1}{3}x_1(\overset{\to T_1/2}{2t-1})$$

T_2? a_k?

$T_2 = \text{lcm}\{T_1, T_1/2\} = T_1$

$\Rightarrow T_2 = T_1$

$x_1(t) \longrightarrow a_{k_1}$

$x(t+1) \to a_{k_1} e^{+jk\omega_0}$

* $x(-t+1) \to a_{-k_1} e^{-jk\omega_0}$

$x_1(t-1) \to a_{k_1} e^{-jk\omega_0}$

* $x_1(2t-1) \to a_{k_1} e^{-jk2\omega_0}$ $T' = T/2 \Rightarrow \omega' = 2\omega_0$

خاصیت خطی بودن $\Rightarrow a_{k_2} = 2a_{-k_1} e^{-jk\omega_0} - \frac{1}{3} a_{k_1} e^{-jk2\omega_0}$

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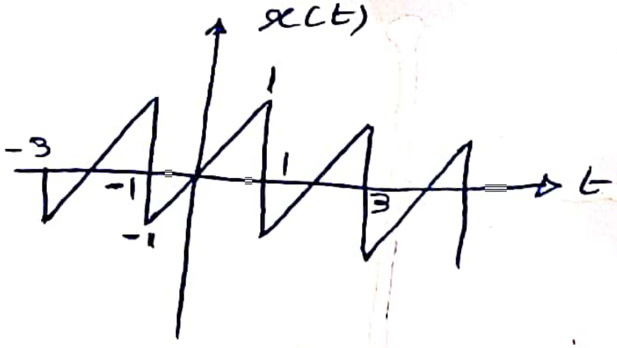


$$a_k = \begin{cases} 0 & k = 0 \\ (j)^k \dfrac{\sin k\pi/4}{k\pi} & \text{o.w.} \end{cases}$$

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt$$

انتگرال گیری :

t	$e^{-jk\omega_0 t}$
1	$\frac{-1}{jk\omega_0} e^{-jk\omega_0 t}$
0	$\frac{-1}{(k\omega_0)^2} e^{-jk\omega_0 t}$

$$a_k = \frac{1}{2}\int_{-1}^{1} t\, e^{-jk\omega_0 t}\, dt$$

$$a_k = \frac{1}{2}\left[\frac{-t}{jk\omega_0} e^{-jk\omega_0 t} + \frac{1}{(k\omega_0)^2} e^{-jk\omega_0 t} \Big|_{-1}^{1} \right]$$

$$a_k = \frac{1}{2}\left[e^{-jk\omega_0}\left(\frac{1}{(k\omega_0)^2} - \frac{1}{jk\omega_0}\right) - e^{jk\omega_0}\left(\frac{1}{(k\omega_0)^2} + \frac{1}{jk\omega_0}\right) \right]$$

$$a_k = -\frac{1}{2}\left[\frac{1}{(k\omega_0)^2}\left[e^{jk\omega_0} - e^{-jk\omega_0}\right] + \left[e^{jk\omega_0} + e^{-jk\omega_0}\right]\frac{1}{jk\omega_0} \right]$$

$$a_k = \frac{-j}{(k\omega_0)^2}\underbrace{\left[\frac{e^{jk\omega_0} - e^{-jk\omega_0}}{2j} \right]}_{\sin k\omega_0} + \underbrace{\left[\frac{e^{jk\omega_0} + e^{-jk\omega_0}}{2} \right]}_{\cos k\omega_0}\frac{-j}{k\omega_0}$$

$$\Rightarrow \quad a_k = \frac{-j}{(k\omega_0)^2}\sin k\omega_0 + \left(\frac{-j}{k\omega_0}\right)\cos k\omega_0$$

سؤال ۳-

$x(t) \to a_k \quad , T = 6$

$z(t) = 2x(\tfrac{1}{2}t - 1) + 3x(2t)$

$\quad\quad \hookrightarrow 2T \quad\quad\quad \hookrightarrow T/2$

$T_2 = lcm\{\overset{12}{2T}, \overset{3}{T/2}\} = 12 = 2T$

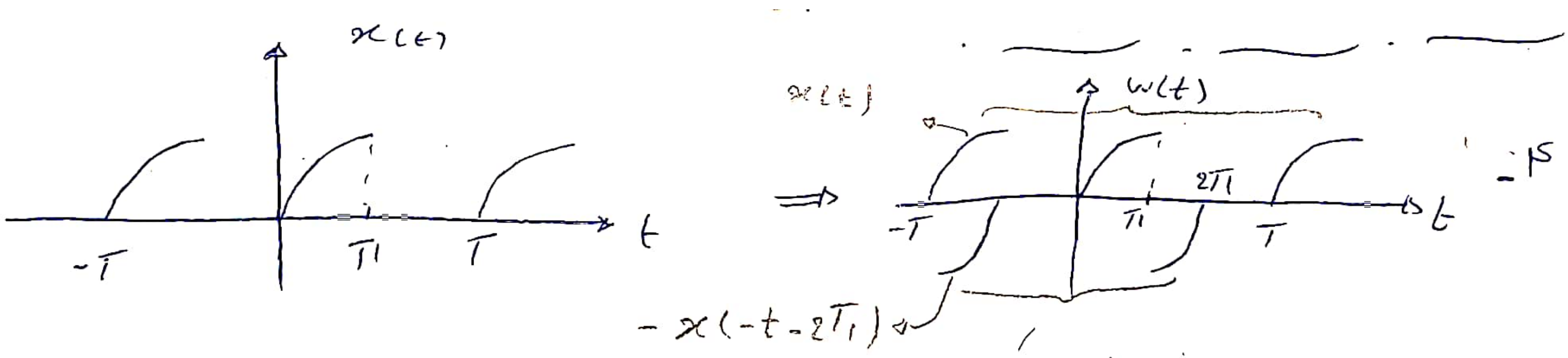
$x(t) \to a_k$

$x(t-1) \to a_k e^{-jk\omega_0}$

$x(\tfrac{1}{2}t - 1) \to a_k e^{-jk\frac{1}{2}\omega_0}$

$x(2t) \to a_k$

$\Rightarrow z(t) = 2a_k e^{-jk\frac{1}{2}\omega_0} + 3a_k$



$$w(t) = x(t) - x(-t-2T_1) \longrightarrow b_k$$

$$\begin{cases} x(t-2T_1) \longrightarrow a_k e^{-jk\omega_0 2T_1} \\ x(-t-2T_1) \longrightarrow a_{-k} e^{jk\omega_0 2T_1} \end{cases}$$

$$w(t) \longrightarrow b_k = a_k - a_{-k} e^{jk\omega_0 2T_1}$$

$a_k = a_{k+2} \qquad a_k = a_{-k} \qquad \int_{-0.5}^{0.5} x(t)\,dt = 1 \qquad \int_{0.5}^{1.5} x(t)\,dt = 2 \Rightarrow T = 3$

$$* \begin{cases} a_k = a_{-k} \Rightarrow x(t) = x(-t) \\ a_k = a_{k+2} \Rightarrow x(t) = x(t)\, e^{-j\left(\frac{4\pi}{3}\right)t} \Rightarrow t = 0, \pm 1.5, \pm 3, \pm 4.5, \ldots \end{cases}$$

$$* \int_{-0.5}^{0.5} x(t)\,dt = 1 \Rightarrow -0.5 < t < 0.5 \Rightarrow x(t) = \delta(t)$$

$$* \int_{0.5}^{1.5} x(t)\,dt = 2 \Rightarrow 0.5 < t < 1.5 \Rightarrow x(t) = 2\delta\left(t - \tfrac{3}{2}\right)$$

$$\Rightarrow x(t) = \sum_{-\infty}^{+\infty} \delta(t - 3k) + 2\sum_{-\infty}^{+\infty} \delta\left(t - 3k - \tfrac{3}{2}\right)$$

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$x(t) \longrightarrow a_k$

$T = 5$

$b_k = j(k+1)\omega_0 e^{jk\omega_0} \times a_{k+1}$

1. $\frac{dx(t)}{dt} \rightarrow jk\omega_0 a_k$

2. $e^{-j\omega_0 t} \times \frac{dx(t)}{dt} \rightarrow j(k+1)\omega_0 a_{k+1}$

3. $e^{-j\omega_0(t+1)} \times \frac{dx(t+1)}{dt} \longrightarrow j(k+1)\omega_0 a_{k+1} e^{jk\omega_0}$

$y(t) = e^{-j\omega_0(t+1)} \times \frac{dx(t+1)}{dt}$