یا لطیف

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1401/3/18

#1 تبدیل لاپلاس؟ ROC؟ صفر و قطب؟

a) $\sum_{k=0}^{\infty} a^k \delta(t-kT) = \delta(t) + a\delta(t-T) + a^2\delta(t-2T)\ldots$

$\xleftrightarrow{L} X(s) = 1 + ae^{-sT} + a^2(e^{-sT})^2 + \ldots = 1 + \sum_{k=1}^{\infty} a^k e^{-ksT}$

صفر : صورت $= 0$

قطب : مخرج $= 0$

ROC : کل صفحه

قطب در بی نهایت

b) $te^{-at}u(t)\,,\ a>0 \Rightarrow x_1(t) = e^{-at}u(t) \xleftrightarrow{L} X_1(s) = \dfrac{1}{s+a} \quad Re\{s\} > -a$

$\xrightarrow[\text{مشتق}]{\text{خاصیت}} t\,x_1(t) \xleftrightarrow{L} \dfrac{-d}{ds}X_1(s) \Rightarrow X(s) = -\left(\dfrac{-1}{(s+a)^2}\right) = \dfrac{1}{(s+a)^2} \quad Re\{s\} > -a$

قطب ها : $(s+a)^2 = 0 \Rightarrow s = -a$

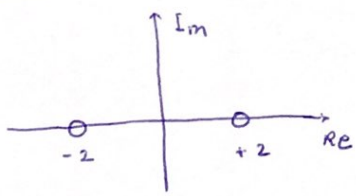
c) $\cos(\omega_0 t + \varphi)u(t) \Rightarrow x_1(t) = \cos\omega_0 t\ u(t) \xleftrightarrow{L} X_1(s) = \dfrac{s}{s^2+\omega_0^2} \quad Re\{s\} > 0$

$\Rightarrow x(t) = x_1(t+\varphi) \xleftrightarrow{L} X(s) = e^{s\varphi}X_1(s) = e^{s\varphi}\dfrac{s}{s^2+\omega_0^2} \quad Re\{s\} > 0$

d) $e^{-at}\sin(\omega_0 t)u(t)\,,\ a>0 \Rightarrow x_1(t) = \sin\omega_0 t\ u(t) \xleftrightarrow{L} X_1(s) = \dfrac{\omega_0}{s^2+\omega_0^2} \quad Re\{s\} > 0$

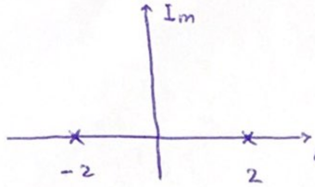
$\Rightarrow x(t) = x_1(t)e^{-at} \xleftrightarrow{L} X(s) = X_1(s+a) = \dfrac{\omega_0}{(s+a)^2+\omega_0^2}\ ; \quad Re\{s\} > -a$

#2



$\dfrac{(s-2)(s+2)}{1} = s^2 - 4 \overset{L^{-1}}{\Rightarrow} \dfrac{d^2\delta(t)}{dt^2} - 4\delta(t)$

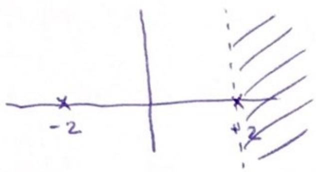
b)



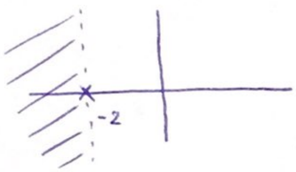
$$\frac{1}{(s-2)(s+2)} = \frac{A}{(s-2)} + \frac{B}{(s+2)} \longrightarrow \begin{cases} A = \frac{1}{4} \\ B = \frac{-1}{4} \end{cases}$$

$$\Rightarrow \bar{X}(s) = \frac{\frac{1}{4}}{(s-2)} - \frac{\frac{1}{4}}{(s+2)} \xleftrightarrow{L^{-1}}$$

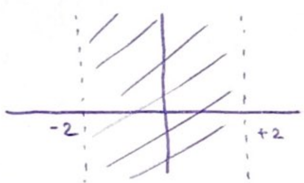
(I)



$$: \quad x(t) = \frac{1}{4}e^{2t}u(t) - \frac{1}{4}e^{-2t}u(t) \quad ; \quad Re\{s\} > 2$$

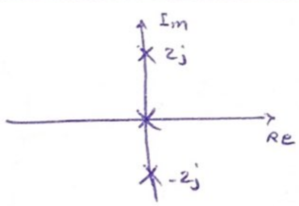


$$: \quad x(t) = -\frac{1}{4}e^{2t}u(-t) + \frac{1}{4}e^{-2t}u(-t) \quad ; \quad Re\{s\} < -2$$



$$: \quad x(t) = \frac{-1}{4}e^{-2t}u(t) - \frac{1}{4}e^{2t}u(-t) \quad ; \quad -2 < Re\{s\} < +2$$

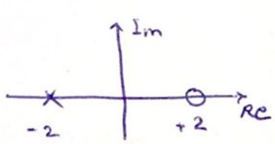
c)



$$\bar{X}(s) = \frac{1}{s(s-2j)(s+2j)} \xleftrightarrow{L^{-1}} x(t) = \frac{A}{s} + \frac{B}{s-2j} + \frac{C}{s+2j} \longrightarrow \begin{cases} A = \frac{-1}{4} \\ B = \frac{-1}{8} \\ C = \frac{-1}{8} \end{cases}$$

$$\Rightarrow \bar{X}(s) = \frac{-\frac{1}{4}}{s} + \frac{-\frac{1}{8}}{s-2j} + \frac{-\frac{1}{8}}{s+2j} \xleftrightarrow{L^{-1}} \bar{X}(s) = -\frac{1}{4}u(t) - \frac{1}{8}e^{2jt}u(t) - \frac{1}{8}e^{-2jt}u(t)$$

d)



$$\bar{X}(s) = \frac{s-2}{s+2} \xleftrightarrow{L^{-1}} \bar{X}(s) = 1 + \frac{A}{(s+2)} \longrightarrow A = -4$$

$$\bar{X}(s) = 1 + \frac{-4}{(s+2)} \xleftrightarrow{L^{-1}} x(t) = \delta(t) - 4e^{-2t}u(t)$$

3 عکس لاپلاس؟

a) $X(s) = \dfrac{s^2 - s + 1}{(s+1)^2}$, $Re\{s\} > -1$

$\Rightarrow X(s) = \dfrac{s^2 - s + 1 \overbrace{+2s - 2s}}{s^2 + 2s + 1} = 1 + \dfrac{-3s}{s^2 + 2s + 1} = 1 + \dfrac{A}{(s+1)^2} + \dfrac{B}{(s+1)} \longrightarrow \begin{cases} A = 3 \\ B = -3 \end{cases}$

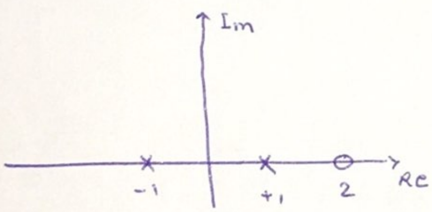
$X(s) = 1 + \dfrac{3}{(s+1)^2} + \dfrac{-3}{(s+1)} \xleftrightarrow{L^{-1}} x(t) = \delta(t) + 3te^{-t}u(t) - 3e^{-t}u(t)$

b) $X(s) = \dfrac{s+1}{(s+1)^2 + 4}$, $Re\{s\} > -1$

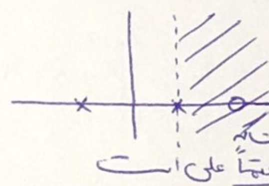
if $X_1(s) = \dfrac{s}{s^2 + 4} \xleftrightarrow{L^{-1}} x_1(t) = \cos 2t \, u(t) \Rightarrow X(s) = X_1(s+1) \xleftrightarrow{L^{-1}}$

$x(t) = e^{-t} x_1(t) = e^{-t} \cos 2t \, u(t)$

4



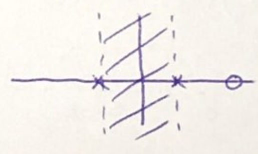
(I)



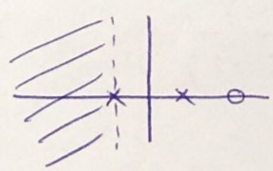
ناپایدار و غیرعلی

البته اگر بدانیم تابع سیستم گویا است، می‌توان گفت که این سیستم با این ROC حتماً علی است

(II)



پایدار و غیرعلی



غیرعلی و ناپایدار

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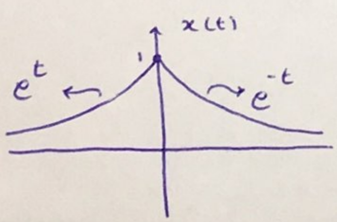
5 LTI & Causal : $H(s) = \dfrac{s+1}{s^2 + 2s + 2}$. if $x(t) = e^{-|t|} \Rightarrow y(t) = ?$



$x(t) = e^{t}u(-t) + e^{-t}u(t) \xleftrightarrow{L} X(s) = \dfrac{-1}{s-1} + \dfrac{1}{s+1}$

$Re\{s\} > 1$

$= \dfrac{-2}{(s-1)(s+1)}$

پاسخ : $y(t) = x(t) * h(t) \xleftrightarrow{L} Y(s) = X(s)\,H(s)$

$$\Rightarrow Y(s) = \frac{-2}{(s-1)(s+1)} \times \frac{s+1}{s^2+2s+2} = \frac{-2}{(s-1)(s^2+2s+2)}$$

$$= \frac{A}{(s-1)} + \frac{Bs+C}{s^2+2s+2} \longrightarrow \left| \begin{array}{l} A = \frac{2}{5} \\ B = \frac{-2}{5} \\ C = \frac{-6}{5} \end{array} \right.$$

$$\Rightarrow Y(s) = \frac{\frac{2}{5}}{(s-1)} + \frac{-\frac{2}{5}s - \frac{6}{5}}{s^2+2s+2} = \frac{\frac{2}{5}}{(s-1)} - \frac{\frac{2}{5}s}{s^2+2s+2} - \frac{\frac{6}{5}}{s^2+2s+2} = \frac{\frac{2}{5}}{s-1} - \frac{\frac{2}{5}s}{(s+1)^2+1}$$

$$- \frac{\frac{6}{5}}{(s+1)^2+1} \xleftrightarrow{L^{-1}} y(t) = \frac{2}{5}e^{t}u(t) - \frac{2}{5}e^{-t}\cos t\, u(t) - \frac{5}{6}e^{-t}\sin t\, u(t)$$

#6 LTI & causal system
$$\begin{cases} a) \text{ if } x(t) = e^{2t} \longrightarrow y(t) = \frac{1}{6}e^{2t} \\ b) \frac{dh(t)}{dt} + 2h(t) = e^{-4t}u(t) + b\,u(t) \\ H(s) = ? \qquad b = ? \end{cases}$$

پاسخ : $e^{st} \longrightarrow \boxed{LTI} \longrightarrow H(s)e^{st} \Rightarrow H(s) = \frac{1}{6}$

$$\frac{dh(t)}{dt} + 2h(t) = e^{-4t}u(t) + b\,u(t) \xleftrightarrow{L} sH(s) + 2H(s) = \frac{1}{s+4} + \frac{b}{s}$$

$$\overset{H(s) = \frac{1}{6}}{\Longrightarrow} \quad s\left(\frac{1}{6}\right) + \frac{2}{6} = \frac{1}{s+4} + \frac{b}{s} \Rightarrow \frac{s + b(s+4)}{s(s+4)} = \frac{1}{2}$$

$$H(s) = \frac{s + b(s+4)}{s(s+4)(s+2)}$$