$a_1 = a_{-1} = 2 \quad , \quad a_2 = a_{-2} = 4j \quad . \quad T = 8 \longrightarrow \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$ ۱-

$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin(\omega_k t + \phi_k)$

$x(t) = a_1 e^{j\omega t} + a_{-1} e^{-j\omega t} + a_2 e^{2j\omega t} + a_{-2} e^{-2j\omega t} = 2e^{j\frac{\pi}{4}t} + 2e^{-j\frac{\pi}{4}t} + 4je^{j\frac{2\pi}{4}t} - 4je^{-j\frac{2\pi}{4}t}$

$\Rightarrow 4\sin(\frac{\pi}{4}t + \frac{\pi}{2}) - 8\sin(\frac{2\pi}{4}t)$

$a_1 = a_{-1} = j \quad , \quad a_{-5} = a_5 = 2 \qquad T = 8 \longrightarrow \frac{\pi}{4}$ ۲-

$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin(\omega_k t + \phi_k)$

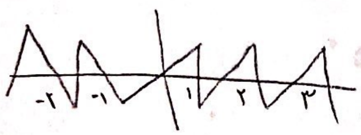
$x(t) = a_1 e^{j\omega t} + a_{-1} e^{-j\omega t} + a_5 e^{5j\omega t} + a_{-5} e^{-5j\omega t} = je^{j\frac{\pi}{4}t} - je^{-j\frac{\pi}{4}t} - je^{-j\frac{\pi}{4}t} + 2e^{5j\frac{\pi}{4}t} + 2e^{-5j\frac{\pi}{4}t}$

$= 4\sin(\frac{5\pi}{4}t + \frac{\pi}{2}) - 2\sin(\frac{\pi}{4}t)$

$x(t) = \begin{cases} \frac{3}{2} & 0 \le t < 1 \\ -\frac{3}{2} & 1 \le t < 2 \end{cases} \qquad T = \frac{2\pi}{\omega_0} = 2$ ۳-

$a_k = \frac{1}{T}\int_0^T x(t) e^{-jk\omega_0 t} dt = \frac{1}{2}\left[\int_0^1 \frac{3}{2} e^{-jk\pi t} dt - \int_1^2 \frac{3}{2} e^{-jk\pi t}\right] = \frac{3}{4}\left[\frac{1}{-jk\pi} e^{-jk\pi t}\Big|_0^1\right.$

$\left. - \frac{1}{jk\pi} e^{-jk\pi t}\Big|_1^2\right] = \frac{3}{k\pi} e^{-jk\frac{\pi}{2}} \left(\frac{e^{jk\frac{\pi}{2}} - e^{-jk\frac{\pi}{2}}}{2j}\right) = \frac{3}{k\pi} e^{-jk\frac{\pi}{2}} \sin(\frac{k\pi}{2})$



تابع فرد

(قرینه فرد نسبت به مبدا)

$a_0 = 0$

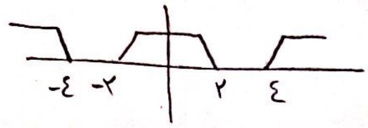
$T = 2 \rightarrow \omega_0 = \frac{2\pi}{T} = \pi$ ۴-

$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 k} dt = \frac{1}{2}\int_{-1}^{1} t e^{-jk\pi t} dt \xrightarrow{\int u dv = uv - \int v du} \begin{cases} u = t \\ dv = e^{-jk\pi t} dt \end{cases}$

$\rightarrow a_k = \frac{1}{2}\left[-t\frac{e^{-jk\pi t}}{-jk\pi}\Big|_{-1}^{1} - \frac{e^{-jk\pi t}}{(+jk\pi)^2}\Big|_{-1}^{1}\right] = \frac{1}{2}\left[\frac{1}{-jk\pi}(+e^{-jk\pi} + e^{jk\pi})\right) - \frac{1}{(jk\pi)^2}(e^{-jk\pi} - e^{jk\pi})$

$= \frac{-1}{jk\pi}\cos(k\pi) + \frac{1}{(jk\pi)^2}\sin(k\pi) = \frac{-1}{jk\pi}(-1)^k$

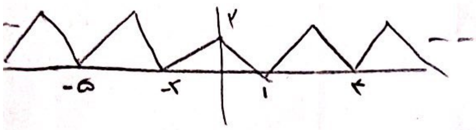
$$x(t) = \begin{cases} t+۲ & -۲<t<-۱ \\ ۱ & -۱<t<۱ \\ ۲-t & ۱<t<۲ \end{cases}$$



$$T=۴ \longrightarrow \omega_0 = \frac{\pi}{۲}$$

$$a_0 = \frac{۱}{۴}\int_{-۲}^{۲} x(t)\,dt = \frac{۱}{۴}\left[\int_{-۲}^{-۱} t+۲\,dt + \int_{-۱}^{۱} dt + \int_{۱}^{۲} ۲-t\,dt\right]$$

$$a_k = \frac{۱}{T}\int_T x(t)\, e^{-jk\omega_0 t}\,dt \longrightarrow \frac{۱}{۴}\left[\int_{-۲}^{-۱} (t+۲)\, e^{-jk\frac{\pi}{۲}t} + \int_{-۱}^{۱} e^{-jk\frac{\pi}{۲}t} + \int_{۱}^{۲} (۲-t)\, e^{-jk\frac{\pi}{۲}t}\right]$$

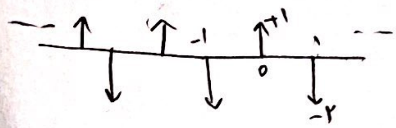


$$T=۳ \longrightarrow \omega_0 = \frac{۲\pi}{۳}$$

$$x(t) = \begin{cases} t+۲ & -۲<t<۰ \\ ۲-۲t & ۰<t<۱ \end{cases}$$

$$a_0 = \frac{۱}{T}\int_T x(t)\,dt = \left[\frac{۱}{۳}\int_{-۲}^{۰} (t+۲)\,dt + \int_{۰}^{۱} (۲-۲t)\,dt\right]$$

$$a_k = \frac{۱}{T}\int_T x(t)\, e^{-jk\omega_0 t}\,dt = \frac{۱}{۳}\left[\int_{-۲}^{۰} (t+۲)\, e^{-jk\frac{۲\pi}{۳}t}\,dt + \int_{۰}^{۱} (۲-۲t)\, e^{-jk\frac{۲\pi}{۳}t}\,dt\right]$$

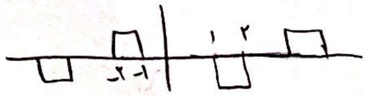


$$T=۲ \qquad \omega_0 = \frac{۲\pi}{T} = \pi$$

$$x(t) = \delta(t) - ۲\delta(t-۱)$$

$$a_0 = \frac{۱}{۲}\int_{-۱}^{۱} [\delta(t) - ۲\delta(t-۱)]\,dt$$

$$a_k = \frac{۱}{۲}\int_{-۱}^{۱} [\delta(t) - ۲\delta(t-۱)]\, e^{-jk\pi t}\,dt = \frac{۱}{۲}\left[\int_{-۱}^{۱} \delta(t)\, e^{-jk\pi t}\,dt - \int_{-۱}^{۱} ۲\delta(t-۱)\, e^{-jk\pi t}\,dt\right] =$$

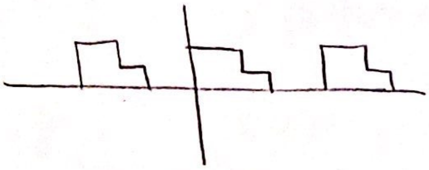


$$T=۴ \qquad \omega_0 = \frac{\pi}{۲}$$

پس $a_0 = ۰$

$$x(t) = \begin{cases} ۱ & -۲<t<-۱ \\ -۱ & ۱<t<۲ \end{cases}$$

$$a_k = \frac{۱}{۴}\left[\int_{-۲}^{-۱} e^{-jk\frac{\pi}{۲}t}\,dt + \int_{۱}^{۲} -e^{-jk\frac{\pi}{۲}t}\,dt\right]$$



$T=4 \quad \omega_0=\frac{\pi}{2} \quad x(t)=\begin{cases} 2 & 0<t<1 \\ 1 & 1<t<2 \end{cases}$

$$a_0=\frac{1}{4}\left[\int_0^1 2\,dt+\int_1^2 dt\right]$$

$$a_k=\frac{1}{4}\left[\int_0^1 2e^{-jk\frac{\pi}{2}t}dt+\int_1^2 e^{-jk\frac{\pi}{2}k}dt\right]\dots$$

(5

الف) $a_k=\begin{cases} (j)^k\frac{\sin k\frac{\pi}{4}}{k\pi} & ow \\ 0 & k=0 \end{cases}$

$$x(t)=a_1e^{j(\frac{2\pi}{T})t}+a_{-1}e^{-j(\frac{2\pi}{T})t}+a_5e^{j5(\frac{2\pi}{T})t}+a_{-5}e^{-j\omega(\frac{2\pi}{T})t}$$

$$x(t)=je^{j(\frac{2\pi}{8})t}+je^{-j(\frac{2\pi}{8})t}+2e^{j5(\frac{2\pi}{8})t}+2e^{-j5(\frac{2\pi}{8})t}=-2\sin(\frac{\pi}{4}t)+4\cos(\frac{5\pi}{4}t)$$

ب) $b_k=\frac{\sin(\frac{k\pi}{8})}{2k\pi} \rightarrow \frac{1}{2}$ [illegible]

$$y(t)=\begin{cases} \frac{1}{2} & |t|<\frac{1}{4} \\ 0 & \frac{1}{4}<|t|<2 \end{cases}$$

$$a_k=b_ke^{j\pi k} \rightarrow x(t)=y(t+4)$$

ج) $c_k=\begin{cases} jk & , |k|<3 \\ 0 & ow \end{cases} \qquad a_1=a_1^*=j \qquad a_2=a_{-2}^*=2j$

$$x(t)=a_1e^{j(\frac{2\pi}{T})t}+a_{-1}e^{-j(\frac{2\pi}{T})t}+a_2e^{j2(\frac{2\pi}{T})t}+a_{-2}e^{j2(\frac{2\pi}{T})t}$$

$$=je^{j(\frac{2\pi}{4})t}-je^{-j(\frac{2\pi}{4})t}+2je^{j2(\frac{2\pi}{4})t}-2je^{-j2(\frac{2\pi}{4})t}=-2\sin(\frac{\pi}{2}t)-4\sin(\pi t)$$

د) $d_k=\begin{cases} 1 & k=\text{even} \\ 2 & k=\text{odd} \end{cases}$

$$x(t)=\sum_{k=-\infty}^{\infty}e^{j(\frac{\pi}{2})t}\delta(t-2k)$$

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۶-

$$x(t)=\begin{cases} t & 0\le t\le 1 \\ 2-t & 1\le t\le 2\end{cases} \qquad T=2 \quad w_0=2\pi$$

$$a_0=\frac{1}{2}\int_0^1 t\,dt+\frac{1}{2}\int_1^2 2-t\,dt$$

$$\frac{dx(t)}{dt}=\begin{cases} 1 & 0\le t\le 1 \\ -1 & 1\le t\le 2\end{cases} \qquad \text{میانگین} = a_0=0$$

$$a_k=\frac{1}{2}\left[\int_0^1 e^{-jk\pi t}-\int_1^2 e^{-jk\pi t}\right]$$

۷-

$$a_k=\begin{cases} j(\frac{1}{2})^{|k|} & else \\ 2 & k=0\end{cases}$$

$a_{-1}^*=-j\frac{1}{2}$, $a_1=j\frac{1}{2}$ و $a_k\neq a_k^*$ ← اگر تابع حقیقی باشد داریم

$$b_k=\begin{cases} j(\frac{1}{2})^{|k|}(jk\frac{2\pi}{T})=-k(\frac{1}{2})^{|k|}\frac{2\pi}{T} & k\neq 0 \\ 0 & k=0\end{cases}$$

$x(t)=x(-t)$ ← اگر تابع زوج باشد

$$\frac{dx(t)}{dt}\longrightarrow b_k=a_k\,jk\frac{2\pi}{T} \qquad b_k\neq -b_k$$

۸-

$a_1=a_8=a_{15}=j$ $\quad a_2=a_{16}=2j$ $\quad a_3=a_{17}=3j$

$x(t)=R$ و فرد $=x(-t)=-x(t)$

$a_k=I_m$ و فرد $=a_{-k}=-a_k$ $\longrightarrow$ $a_{-3}=-a_3=-3j$

$a_{-2}=-a_2=-2j$

$a_{-1}=-a_1=-j$

۹-

$x[n]\longrightarrow R$ زوج $\quad T=10 \quad a_{11}=5 \quad \frac{1}{10}\sum_{n=0}^{9}|x[n]|^2=50$

$x[n]=A\cos(Bn+C)$

$a_k=R$ زوج $\rightarrow a_{-k}=a_k\longrightarrow a_1=a_{11}=a_{-1}=5 \qquad w_0=\frac{\pi}{5}$

$$\frac{1}{N}\sum_{k=N}|x[n]|^2=\sum_{k=N}|a_k|^2=50\longrightarrow\sum_{k=-4}^{5}|a_k|^2=|a_{-4}|^2+\dots+|a_5|^2=50$$

$$x[n]=\sum_{k=N}a_k e^{jwkn}=a_1e^{jwn}+a_{-1}e^{-jwn}=10\cos(\frac{\pi}{5}n)\longrightarrow\begin{cases}A=10\\B=\frac{\pi}{5}\\C=0\end{cases}$$