۱- حقیقی . $x(t+T)=x(t) \quad T=8$

$a_1=a_{-1}=2$

$a_3=a_{-3}^*=-4j \qquad x(t)=a_0+\sum_{k=1}^{\infty}a_k\sin(\omega_k t+\phi_k)=?$

$a_1=2\to 2e^{j\omega t} \qquad a_3=4je^{3j\omega t}$

$a_{-1}=2\to 2e^{-j\omega t} \qquad a_{-3}^*=a_3\Rightarrow a_{-3}=a_3^*=-4je^{-3j\omega t}$

$x(t)=-4je^{-3j\omega t}+2e^{-j\omega t}+2e^{j\omega t}+4je^{3j\omega t}$

$4\left(\frac{e^{j\omega t}+e^{-j\omega t}}{2}\right)-8\left(\frac{e^{3j\omega t}-e^{-3j\omega t}}{2j}\right)=4\cos\omega t-8\sin 3\omega t \qquad \omega_0=\frac{2\pi}{T}=\frac{2\pi}{8}=\frac{\pi}{4}$

$4\cos(\frac{\pi}{4})(t)-8\sin(\frac{3\pi}{4})(t)$

$\to -8\cos(\frac{3\pi}{4}+\frac{\pi}{2})t$ ✓

۲- $x(t+T)=x(t)\Rightarrow T=8$

$a_1=a_{-1}^*=j \qquad x(t)=a_0+\sum_{k=1}^{\infty}a_k\sin(\omega_k t+\phi_k)$

$a_5=a_{-5}=2 \qquad \omega_0=\frac{2\pi}{T}=\frac{2\pi}{8}=\frac{\pi}{4}$ ✓

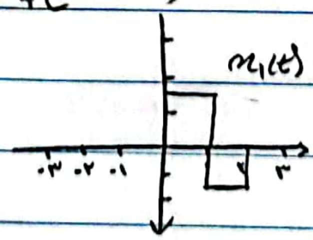
$a_1=j\to je^{j\omega t}$

$a_{-1}^*=a_{+1}\to a_{-1}=a_1^*=-j\to -je^{-j\omega t}$

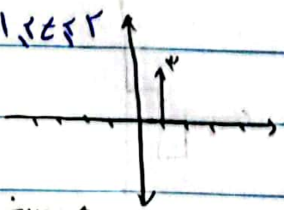
$a_5\to 5e^{5j\omega t} \qquad \Rightarrow x(t)=5e^{-5j\omega t}-je^{-j\omega t}+je^{j\omega t}+5e^{5j\omega t}$

$a_{-5}\to 5e^{-5j\omega t} \qquad 5e^{-j\frac{5\pi}{4}t}-je^{-j\frac{\pi}{4}t}+je^{+j\frac{\pi}{4}t}+5e^{j\frac{5\pi}{4}t}$

$j\left(e^{j\frac{\pi}{4}t}-e^{-j\frac{\pi}{4}t}\right)+5\left(e^{j\frac{5\pi}{4}t}+e^{-j\frac{5\pi}{4}t}\right)$



$x(t)\begin{cases}\frac{3}{2} & 0<t<1\\ -\frac{3}{2} & 1<t<2\end{cases}$ $\qquad$ δ a_k -۳



$\frac{d}{dt}x(t) \qquad \frac{3}{2}-(-\frac{3}{2})=3$

$x'(t)=3\delta(t-1)$

$\frac{1}{T}\int_T x'(t)e^{-jk\omega_0 t}dt=$

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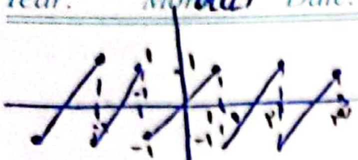
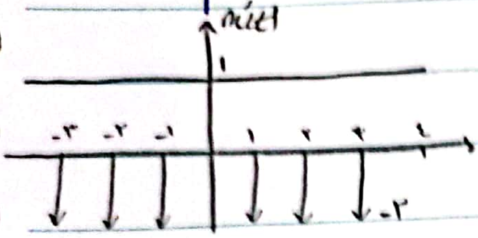


$\frac{d}{dt} x(t) = x'(t)$

a_k - ع

$\omega_0 = \frac{2\pi}{T} = \pi$



$x'(t) = -2\delta(t-1) + 1$

$$a_k = \begin{cases} 0 & k=0 \\ jk\omega_0 a_k = (j)^k \dfrac{\sin\frac{k\pi}{4}}{k\pi} & o.w. \end{cases}$$

$$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t}\, dt$$

$$\begin{array}{c|l} t & e^{-j\omega_0 kt} \\ \hline 1 & \frac{-1}{j\omega_0 k} e^{-j\omega_0 kt} \\ 0 & \frac{-1}{(k\omega_0)^2} e^{-jk\omega_0 t} \end{array}$$

$T = 2$

$$\frac{1}{2}\left[\frac{-t}{jk\omega_0}e^{-jk\omega_0 t} + \frac{e^{-jk\omega_0 t}}{(k\omega_0)^2}\Big|_{-1}^{1}\right] = \frac{1}{2}\left(e^{-jk\omega_0}\left(\frac{1}{(k\omega_0)^2} - \frac{1}{jk\omega_0}\right) - e^{jk\omega_0}\left(\frac{1}{(k\omega_0)^2} + \frac{1}{jk\omega_0}\right)\right)$$

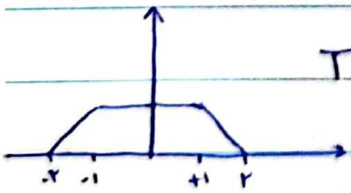
$$a_k = -\frac{1}{2}\left[\frac{1}{(k\omega_0)^2}\left(e^{+jk\omega_0} - e^{-jk\omega_0}\right) + \left(e^{jk\omega_0} + e^{-jk\omega_0}\right)\frac{1}{jk\omega_0}\right]$$

$$a_k = \frac{-j}{k\omega_0} \sin k\omega_0 + \left(\frac{-j}{k\omega_0}\right) \cos k\omega_0$$

$e^{-jk\pi} = (-1)^k$

$$a_k = \begin{cases} 0 & k=0 \\ \dfrac{-2(-1)^k}{jk\pi} \end{cases}$$

ب)



$T = 4 \to \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \pi/2$

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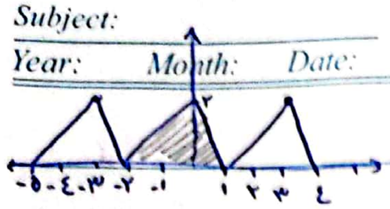
ب)

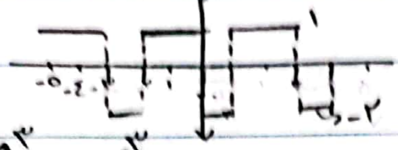


$T = ۳ \qquad S = \frac{1}{۲} \times ۲ \times ۳ = ۳$

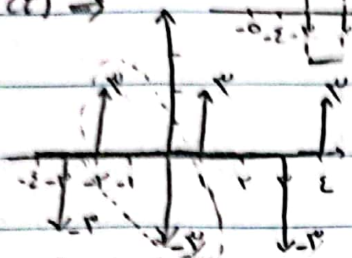
$\omega_0 = \frac{۲\pi}{T} = \frac{۲\pi}{۳}$

$x''(t) = g(t)$

$\frac{d}{dt} x(t) = x'(t) \rightarrow$



$x''(t) = g(t)$



$x''(t) = ۳\delta(t-1) - ۳\delta(t) \qquad -۲ \rightarrow ۱ \qquad ۳(\delta(t-1) - \delta(t))$

$$b_k = \frac{1}{T}\int_T x''(t) e^{-jk\omega_0 t} dt = \frac{1}{۳}\int_{-۲}^{۱} x''(t) e^{-jk\omega_0 t} dt = \frac{1}{۳}\int_{-۲}^{۱} ۳\delta(t-1) - ۳\delta(t) e^{-jk\omega_0 t}$$

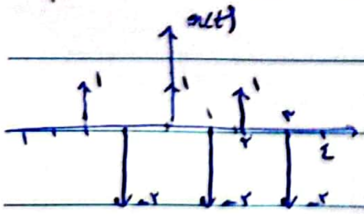
$$\frac{۳}{۳}\int_{-۲}^{۱} \delta(t-1) \cancel{e^{-jk\omega_0 t}}^{\,e^{-jk\omega_0}} dt - \frac{۳}{۳}\int_{-۲}^{۱} \delta(t) \cancel{e^{-jk\omega_0 t}}^{\,1} dt$$

$$\int_{-۲}^{۱} \delta(t-1) e^{-jk\omega_0} dt + \int_{-۲}^{۱} \delta(t) dt = \int_{-۲}^{۱} \delta(t-1) e^{-jk\omega_0} dt + 1$$

$$e^{-jk\frac{۲\pi}{۳}} \int_{-۲}^{۱} \cancel{\delta(t-1)dt}^{\,1} + 1 = e^{-jk\frac{۲\pi}{۳}} + 1$$

$$b_k = (jk\omega_0)^۲ a_k \rightarrow a_k = \frac{b_k}{(jk\omega_0)^۲} = \frac{e^{-jk\frac{۲\pi}{۳}} + 1}{-k^۲\left(\frac{۲\pi}{۳}\right)^۲} = \frac{e^{-jk\frac{۲\pi}{۳}} + 1}{-k^۲ \frac{۴\pi^۲}{۹}} \checkmark$$

ب)



$T = ۲ \rightarrow \omega_0 = \frac{۲\pi}{T} = \pi \checkmark$

$x(t) = \delta(t) - ۲\delta(t-1)$

$$b_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t} dt \Rightarrow \frac{1}{۲}\int_{0}^{۲} (\delta(t) - ۲\delta(t-1)) e^{-jk\omega_0 t} dt =$$

$$\frac{1}{۲}\int_{0}^{۲} \delta(t) \cancel{e^{-jk\pi t}}^{\,1} dt - \frac{1}{۲}\int ۲\delta(t-1) e^{-jk\pi t} dt = \frac{1}{۲}\cancel{\int_{0}^{۲}\delta(t)}^{\,1} - \frac{۲}{۲}\int_{0}^{۲} \delta(t-1) e^{-jk\pi} dt$$

$$\frac{1}{۲} - e^{-jk\pi} \int_{0}^{۲} \cancel{\delta(t-1)dt}^{\,1} = \frac{1}{۲} - e^{-jk\pi} \rightarrow \boxed{\frac{1}{۲} - (-1)^k}$$

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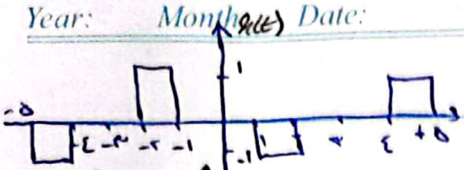
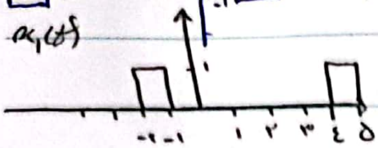


$T=4$ $\quad \omega_0 = \frac{2\pi}{T} = \pi/2$ ت)



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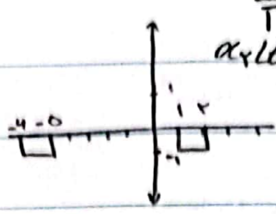
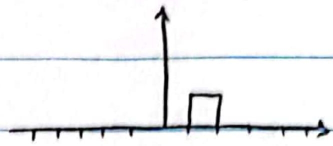
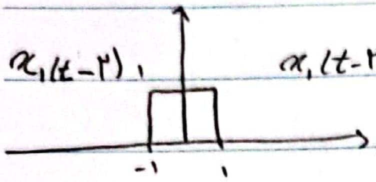


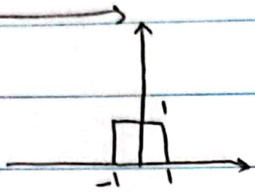
$= x(t) = x_1(t) + x_2(t)$



$x_1(t-2)$



$x_1(t-2) =$ زوج $\quad -x_2(t)$



$-x_2(t+2) =$ زوج

$-x_2(t+2)$

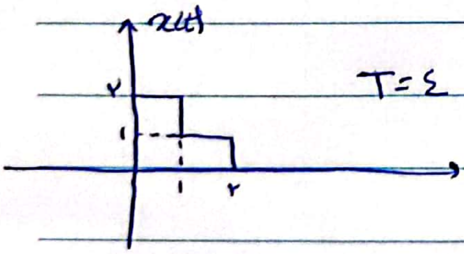
$x_1(t-2) - x_2(t+2) \xleftrightarrow{F.S} a_k e^{-jk\omega_0 t_0} - b_k e^{-jk\omega_0 t_0}$

$d_1 = \frac{2T_1}{T} = \frac{2}{4} = 1/2 \quad d_1 = d_2 = 1/2$

$a_k e^{-jk\frac{\pi}{2}(2)} - b_k e^{-jk\frac{\pi}{2}(-2)} = \frac{1}{2}\,\text{sinc}(k/2)\, e^{-jk\frac{2\pi}{2}} - \frac{1}{2}\,\text{sinc}(k/2)\, e^{+jk\frac{2\pi}{2}}$

$\frac{1}{2}\,\text{sinc}(k/2)\left(e^{-jk\frac{2\pi}{2}} - e^{jk\frac{2\pi}{2}}\right)$

ث)



$T = 4$

$x(t) = $ ؟

$T = 4$

$$a_k = \begin{cases} 0 & k = 0 \\ (j)^k \dfrac{\sin \frac{k\pi}{4}}{k\pi} & o.w \end{cases}$$

$$\frac{\sin \pi (\frac{k}{4})}{k\pi} \cdot \frac{1}{4} = \frac{1}{4}\operatorname{sinc}(\frac{k\pi}{4}) \qquad a_k = d\operatorname{sinc}(kd)$$

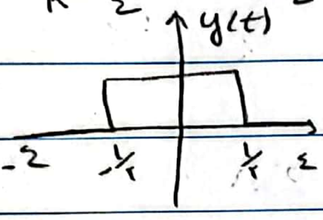
$$\downarrow$$

$$\frac{2T_1}{T} = \frac{1}{4}$$

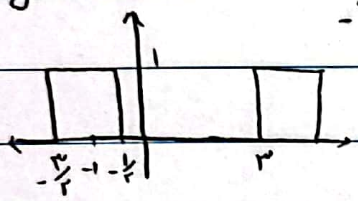
$(j)^k$ $\qquad b_k = \frac{1}{4}\operatorname{sinc}(\frac{k}{4})$

$c_k = (e^{j\frac{\pi}{2}})^k b_k \qquad \omega_0 = \frac{\pi}{2}$

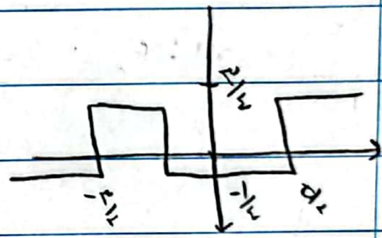
$\xrightarrow{F.S} z(t) = y(t+1)$



$z(t) =$



$x(t) = z(t) - \frac{1}{4}$

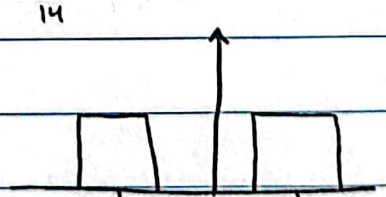


b) $b_k = (-1)^k \dfrac{\sin \frac{k\pi}{8}}{2k\pi}$ $\qquad \omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$

$(-1)^k = e^{-jk\pi}$ $\qquad$ شیفت $\qquad x(t - t_0) \xleftrightarrow{F.S} a_k e^{-jk\omega_0 t_0}$

$$\frac{\sin \pi(\frac{k}{8})}{\pi k \times 2 \times} \cdot \frac{1}{\frac{1}{16}} = \frac{\sin \frac{k\pi}{8}}{\frac{k\pi}{8}} = \frac{1}{16}\operatorname{sinc}(\frac{k}{8}) \qquad d = \frac{1}{8} \qquad \frac{1}{2} \text{ دامنه}$$

$\frac{1}{2}(\frac{1}{8}\operatorname{sinc}(\frac{k}{8}))$



ج) $c_k = \begin{cases} j^k & |k| \le 3 \\ 0 & o.w \end{cases}$

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d) $d_k = \begin{cases} 1 & k=2n \\ 2 & k=2n-1 \end{cases} \to 1+\begin{cases} 0 & k=\text{زوج} \\ 1 & k=\text{فرد} \end{cases} = \sum_{k=-\infty}^{+\infty} 4\delta(t-4N)$

$z(t)=4e^{+j\frac{\pi}{2}t}\,2\delta(t-2n) \to x(t)=y(t)+z(t)=\sum_{k=-\infty}^{+\infty}\delta(t-4n)+\sum_{k=-\infty}^{+\infty}e^{j\frac{\pi}{2}t}\,2\delta(t-2n)$

۶- فرض:

a_0؟

سری فوریه $\frac{d}{dt}x(t)$؟

$x(t)=\begin{cases} t & 0\le t\le 1 \\ 2-t & 1\le t\le 2 \end{cases} \qquad T=2$

$a_0=\frac{1}{T}\int_T x(t)\,dt=\frac{1}{2}\int_0^1 t\,dt+\frac{1}{2}\int_1^2(2-t)\,dt=\frac{1}{2}$

$\frac{d}{dt}x(t)=\begin{cases} 1 & 0<t<1 \\ -1 & 1<t<2 \end{cases} \qquad \omega_0=\pi \qquad b_k=\frac{1}{T}\int_T x(t)e^{-j\omega_0 kt}\,dt=$

$\frac{1}{2}\int_0^2 x(t)e^{-jkt\pi}\,dt=\frac{1}{2jk\pi}\left(e^{-jk\pi}-1-e^{-jk\pi}+e^{-jk\pi}\right)=\frac{2e^{-jk\frac{\pi}{2}}}{k\pi}$

$\sin\left(\frac{k\pi}{2}\right)$

۷-

$x(t+T)=x(t) \qquad a_k=\begin{cases} j\left(\frac{1}{2}\right)^{|k|} & o.w \\ 2 & k=0 \end{cases}$

$x(t)$ حقیقی؟ خیر

$x(t)$ زوج؟ زوج است

$\frac{d}{dt}x(t)$ زوج؟ فرد است.

$a_k=a_{-k}^*$ حقیقی $\Rightarrow a_1=j\left(\frac{1}{2}\right)$

$a_{-1}^*=-j\left(\frac{1}{2}\right)$

$|a_k|=|a_{-k}| \qquad j\left(\frac{1}{2}\right)^{|k|}=j\left(\frac{1}{2}\right)^{|-k|}$ ✓

$+j^2=-1$

$b_k=jk\omega_0 a_k \to jk\omega_0\, j\left(\frac{1}{2}\right)^{|k|}=-k\omega\left(\frac{1}{2}\right)^{|k|}$

$|a_k|=|a_{-k}| \to \begin{array}{l} k>0 \to -k\omega\left(\frac{1}{2}\right)^{|k|} \\ k<0 \to +k\omega\left(\frac{1}{2}\right)^{|k|} \end{array}$ فرد α

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$a_{۱۵} = j$ $a_0 \; a_{-1} \; a_{-2} \; a_{-3} = ?$ ۸ $x[n]$ حقیقی فرد $N=۷$

$a_{۱۷} = ۳j$ $a_k = a_{-k}^* \rightarrow a_{-k} = a_k^*$

$a_{۱۹} = ۲j$

$a_{-۱۵} = a_{۱۵}^* = -j$ $a_{-۱۴} = -۲j$ $a_{-۱۷} = -۳j$

سیگنال فرد است

$a_0 = \frac{1}{T}\int_T x(t)dt = 0$ مقدار دی سی صفر است.

$a_{-1} = a_{-۱۵} = -j$ $a_{-2} = a_{-۱۴} = -۲j$ $a_{-3} = a_{-۱۷} = -۳j$

$a_{11} = ۵$ ۹- حقیقی و زوج است $N=۱۰$

$\frac{1}{۱۰}\sum_{n=0}^{۹} |x[n]|^2 = ۵۰$ $x[n] = A\cos(Bn+C)$

$a_{11} = a_{-11}^*$, $a_{-11} = a_{11}^* = ۵$