رضا ... پور

۹۸۱۴۳۴۵۳

امتحان شماره! محاسبات عددی

شماره ۳، گروه ۴

#1 $\begin{cases} x = \sqrt{2} \\ \\ x^* = \dfrac{1154}{816} \end{cases}$ $\Rightarrow$ $e = |x - x^*| = \left|\sqrt{2} - \dfrac{1154}{816}\right| = +0.0000021239 < 0.5 \times 10^{-5}$ ⑤

پس دارای ۵ رقم معتبر درست است.

$\dfrac{e}{|x|} = \dfrac{\left|\sqrt{2} - \dfrac{1154}{816}\right|}{\sqrt{2}} = 0.00000150182 < 0.5 \times 10^{-6}$ ⑥

۶ رقم با معنی درست داریم

#2 $\begin{cases} \sqrt[3]{9} \simeq ? \\ f(x) = \sqrt[3]{x} \\ x_0 = 8, \ x = 9 \end{cases}$

$f(x) = P_n(x) + R_n(x)$ $\underline{\underline{n=3}}$

$P_n(x) = f(x_0) + (x-x_0) f'(x_0) + \dfrac{(x-x_0)^2}{2!} f''(x_0) + \cdots$

$+ \dfrac{(x-x_0)^n}{n!} f^{(n)}(x_0)$

$\Rightarrow P(9) = \sqrt[3]{9} = f(8) + f'(8) + \dfrac{f''(8)}{2!} + \dfrac{f'''(8)}{3!} + \underbrace{\dfrac{f^{(4)}(\xi)}{4!}}_{R_n}$

$\begin{cases} f'(x) = \dfrac{1}{3x^{2/3}} \rightarrow f'(8) = 0.083 \\[2mm] f''(x) = \dfrac{-2}{9x^{5/3}} \rightarrow f''(8) = -0.006 \\[2mm] f'''(x) = \dfrac{10}{27x^{8/3}} \rightarrow f'''(8) = 0.001 \\[2mm] f^{(4)}(x) = \dfrac{-80}{81x^{11/3}} \rightarrow f^{(4)}(8) = -0.00048 \end{cases}$

$|R_n(9)| < 0.5 \times 10^{-3}$

$\Rightarrow \dfrac{f^{(4)}(\xi)}{4!} < 0.5 \times 10^{-3}$; $8 < \xi < 9$

$\Rightarrow \dfrac{-80}{81(8)^{11/3}} \times \dfrac{1}{4!} < \dfrac{-10}{81 \times 3} = -0.04 < 0.5 \times 10^{-3}$

$\Rightarrow P(9) = \sqrt[3]{9} \simeq 2 + 0.083 + \dfrac{-0.006}{2} + \dfrac{0.001}{6} = 2.08$

#3

x	0	0.2	0.4	0.6	0.8	1
$f(x)$	0.3	0.4	0.7	1	0.8	0

$$y = \sin(a + cx^2)$$
$$\underbrace{\quad\quad}_{P}$$

$$\Rightarrow P = a + cx^2 \quad \Rightarrow \quad E(a,c) = \sum_{i=1}^{6}(y_i - P(x_i))^2 = \sum_{i=1}^{6}(y_i - cx_i^2 - a)^2$$

$$\begin{cases} \dfrac{\partial E}{\partial a} = -2\sum_{i=1}^{6} x_i^2\left(y_i - cx_i^2 - a\right) = 0 \\[4mm] \dfrac{\partial E}{\partial c} = -2\sum_{i=1}^{6}\left(y_i - cx_i^2 - a\right) = 0 \end{cases}$$

$$\Rightarrow$$

$$\begin{cases} a\sum_{i=1}^{6} x_i^4 + c\sum_{i=1}^{6} x_i^2 = \sum_{i=1}^{6} x_i^2 y_i \\[4mm] a\sum_{i=1}^{6} x_i^2 + 6c = \sum_{i=1}^{6} y_i \end{cases}$$

i	x_i	x_i^2	x_i^4	y_i	$x_i^2 y_i$
1	0	0	0	0.3	0
2	0.2	0.04	0.0016	0.4	0.016
3	0.4	0.16	0.0256	0.7	0.112
4	0.6	0.36	0.1296	1	0.36
5	0.8	0.64	0.4096	0.8	0.512
6	1	1	1	0	0
Σ	3	2.2	1.5664	3.2	1

$$\Rightarrow$$

$$\begin{cases} 1.5664\,a + 2.2\,c = 1 \\[3mm] 2.2\,a + 6c = 3.2 \end{cases}$$

$$\begin{cases} a = \dfrac{-1900}{891} \simeq 2.132 \\[4mm] c = \dfrac{799}{405} \simeq 1.972 \end{cases}$$

$$\Rightarrow y = \sin\left(2.132 + 1.972\,x^2\right)$$