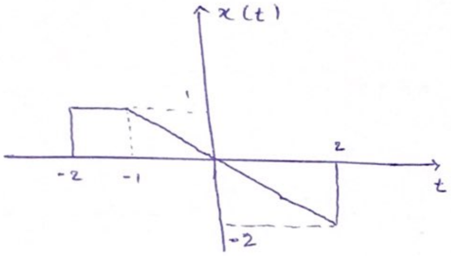
به نام خدا

تمرین سری ۱ سیگنال سیستم

① بخش زوج و فرد سیگنال های زیر را مشخص کنید ؟

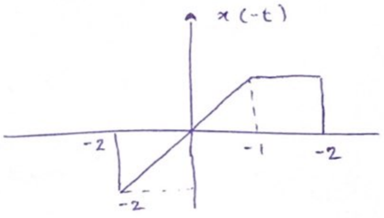
الف)



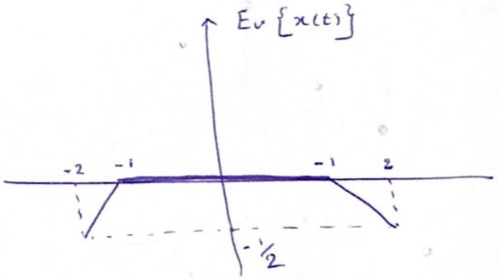
$$x(t) = \begin{cases} -t & ; \ -1 < t \le 2 \\ 1 & ; \ -2 < t \le -1 \end{cases}$$

روش اول : نوشتن معادلات سیگنال : $E_v\{x(t)\} = \dfrac{\overbrace{x(t)}^{\text{داریم}} + \overbrace{x(-t)}^{\text{نداریم}}}{2}$ ← بدست میاریم

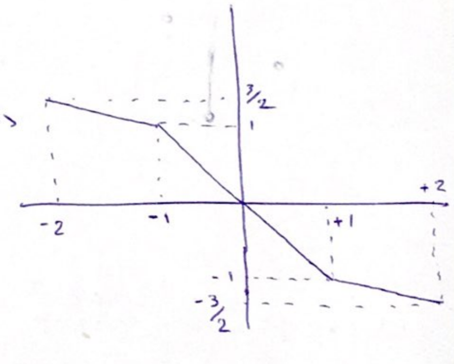
$$\Rightarrow x(-t) = \begin{cases} t & ; \ -2 < t \le 1 \\ 1 & ; \ 1 < t \le 2 \end{cases}$$



$$\Rightarrow \frac{x(t)+x(-t)}{2} = \begin{cases} 0 & ; \ 0 < t \le 1 \\ \frac{1-t}{2} & ; \ 1 < t \le 2 \\ 0 & ; \ -1 < t \le 0 \\ \frac{1+t}{2} & ; \ -2 < t \le -1 \end{cases} \Rightarrow$$

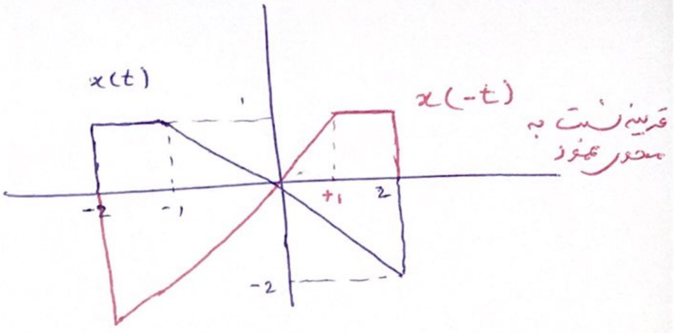


$$\mathrm{Odd}\{x(t)\} = \frac{x(t)-x(-t)}{2} = \begin{cases} -t & ; \ 0 < t < 1 \\ \frac{-t-1}{2} & ; \ 1 < t < 2 \\ -t & ; \ -1 < t < 0 \\ \frac{1-t}{2} & ; \ -2 < t < -1 \end{cases} \Rightarrow$$

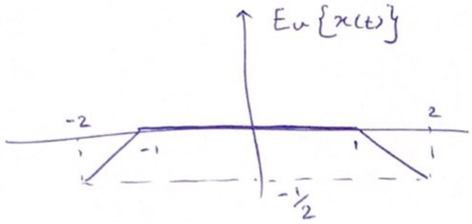


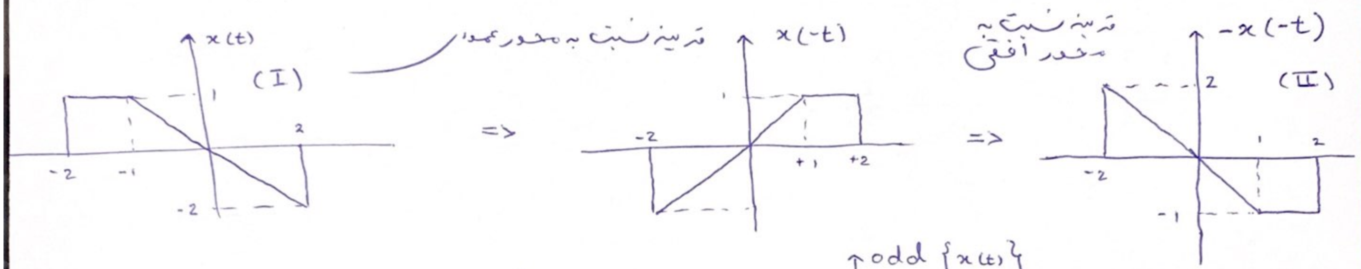
روش دوم

بدون نوشتن معادلات به صورت مستقیم : از روی گراف سیگنال بخش های زوج و فرد نوشته شود =>

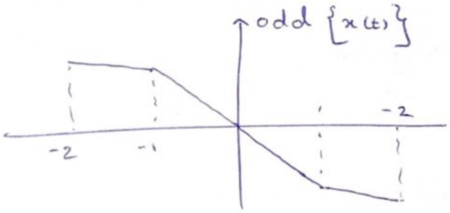


=>

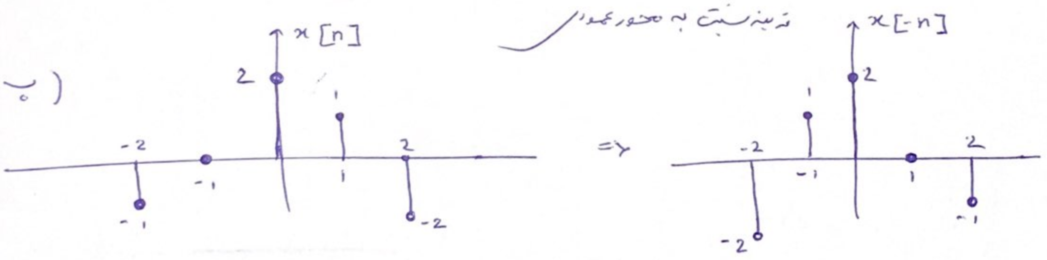


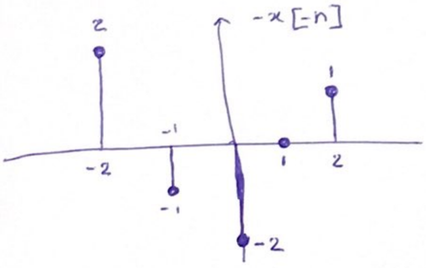


$$=> \text{odd}\{x(t)\} = \frac{x(t) - x(-t)}{2}$$

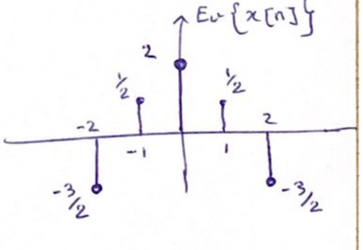


ب)

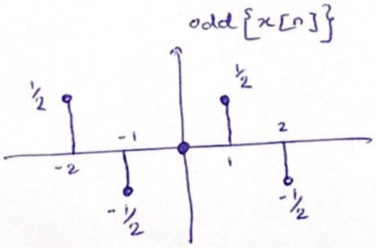




$$=> Ev\{x[n]\} = \frac{x[n] + x[-n]}{2}$$



$$\text{odd}\{x[n]\} = \frac{x[n] - x[-n]}{2}$$



② بخش حقیقی را به صورت $Ae^{-\alpha t}\cos(\omega t+\varphi)$ و بخش موهومی: $Ae^{-\alpha t}\sin(\omega t+\varphi)$

$$a)\ x_1(t) = \underbrace{j}\, e^{(-2+j100)t} = \underbrace{e^{j\pi/2}}\cdot e^{(-2+j100)t} = e^{j\pi/2}\, e^{j100t}\cdot e^{-2t}$$

$$= e^{-2t}\cdot e^{\overbrace{j(100t+\pi/2)}^{=\theta}} \xrightarrow{\text{با اویلر: } e^{j\theta} = \overbrace{\cos\theta}^{Re} + \overbrace{j\sin\theta}^{Im}} e^{-2t}\cdot\left[\cos(100t+\pi/2) + j\sin(100t+\pi/2)\right]$$

$$Re\{x(t)\} = \overset{A}{\uparrow}2\, e^{\overset{\alpha}{\uparrow}-2t} \cos(\overset{\omega}{\uparrow}100t + \overset{\varphi}{\uparrow}\pi/2)$$

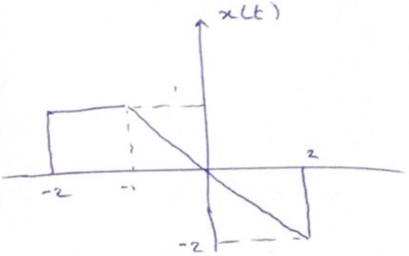
$$Im\{x(t)\} = 2e^{-2t}\sin(100t+\pi/2)$$

$$b)\ \underbrace{\sqrt{2}\, e^{j\pi/2}}\cos(3t+\pi/3) : \sqrt{2}\cdot\left[\overset{=0}{\cancel{\cos(\pi/2)}} + j\overset{=1}{\cancel{\sin(\pi/2)}}\right] = j\sqrt{2}$$

$\Rightarrow x(t) = j\sqrt{2}\cos(3t+\pi/3)$ موهومی خالص

$$\begin{cases} Re\{x(t)\} = 0\cdot e^{-\overset{0}{\cancel{\alpha}}t}\cos(3t+\pi/3) = 0 \\ Im\{x(t)\} = \sqrt{2}\, e^{-0t}\sin(3t \cancel{+} - \pi/6) \end{cases}$$

$Z(t) = ? \longleftarrow x(t) = Z(-\tfrac{1}{2}t+1)$ ③

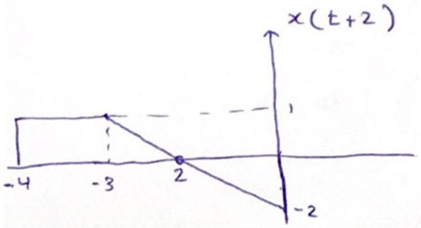


$$-\tfrac{1}{2}t + 1 = \tau \Rightarrow t = -2(\tau-1) = -2\tau+2$$

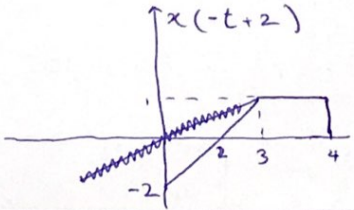
$$\Rightarrow Z(\tau) = x(-2\tau+2) \xrightarrow{\text{به جای } \tau\text{، } t \text{ می گذاریم}}$$

$$Z(t) = x(-2t+2)$$

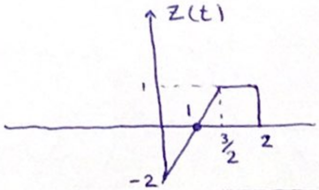
x(t) را دو واحد به چپ شیفت می دهیم سپس سیگنال بدست آمده را نسبت به محور عمودی قرینه کرده و آن را با ضریب ½ فشرده می کنیم



$\Rightarrow$



$\Rightarrow$



④ تعیین دوره تناوب سیگنال های زیر ؟

a) $x[n] = 3e^{j3\pi(n+\frac{1}{2})/5} = 3e^{j\frac{3\pi}{5}(n+\frac{1}{2})}$

روش اول

$3e^{j\frac{3\pi}{5}(n+\frac{1}{2})} = 3e^{j(\frac{3\pi n}{5}+\frac{3\pi}{10})} = \cancel{3e^{j\frac{3\pi}{5}n} \cdot e^{j\frac{3\pi}{10}}}$

$= 3e^{j\frac{3\pi}{10}} \cdot e^{j\frac{3\pi}{5}(n+N_0)} = \cancel{e^{j\frac{3\pi}{10}}} \cdot \cancel{e^{j\frac{3\pi}{5}n}} \cdot e^{j\frac{3\pi}{5}N_0}$

$\Rightarrow e^{j\frac{3\pi}{5}N_0} = \underbrace{1}_{e^{j2k\pi}} \Rightarrow e^{j\frac{3\pi}{5}N_0} = e^{j2k\pi} \Rightarrow \frac{3\pi}{5}N_0 = 2K\pi$

$\Rightarrow N_0 = \frac{10}{3}K \xrightarrow{K=3} \underline{N_0 = 10}$

روش دوم

برای پیدا کردن دوره تناوب یک سیگنال گسسته اگر بتوان W_0 را به صورت گویا زیر نوشت به نحوی که m و N ساده شوند (به عبارتی دیگر W_0 باید مضرب گویایی از π باشد) آنگاه $\underline{N}$ پریود اصلی است.

$$\boxed{W_0 = 2\pi\left(\frac{m}{N}\right)}$$

در این مثال : $W_0 = \frac{3\pi}{5} \longrightarrow 2\pi\left(\frac{3}{\boxed{10}}\right) \longrightarrow \boxed{N_0 = 10}$

b) $w(t) = \underbrace{2\cos(10t+1)}_{T_1} - \underbrace{\sin(4t-1)}_{T_2}$

روش اول

$\cos(10t+1) = \cos(10(t+T_1)+1) \Rightarrow 10T_1 = 2k_1\pi \Rightarrow T_1 = \frac{k_1\pi}{5} \rightarrow T_{10} = \frac{\pi}{5}$

$\sin(4t-1) = \sin(4(t+T_2)-1) \Rightarrow 4T_2 = 2k_2\pi \Rightarrow T_2 = \frac{k_2\pi}{2} \Rightarrow T_{20} = \frac{\pi}{2}$

$\Rightarrow T = \text{kmm}\left(\frac{\pi}{5}, \frac{\pi}{2}\right) = \pi$

راه ساده‌تر

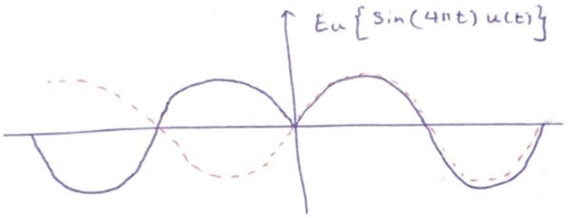
$$\begin{cases} x(t) = A\cos(w_0 t+\varphi) \rightarrow 2\cos(10t+1) \rightarrow \begin{cases} w_0 = 10 \\ \varphi = 1 \end{cases} \\ \sin(4t-1) \longrightarrow \begin{cases} w_0 = 4 \\ \varphi = -1 \end{cases} \end{cases}$$

e) $Q(t) = Ev\left\{\sin(4\pi t)\, u(t)\right\} = \dfrac{\sin(4\pi t)\,u(t) + \sin(-4\pi t)\,u(-t)}{2}$

$$\xrightarrow{\sin(-\alpha) = -\sin\alpha} \quad = \frac{1}{2}\Big(\underbrace{\sin(4\pi t)\,u(t)}_{(I)} - \underbrace{\sin(4\pi t)\,u(-t)}_{(II)}\Big)$$

یادآوری : $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$, $u(-t) = \begin{cases} 1 & t < 0 \\ 0 & t > 0 \end{cases}$

$(I) : \begin{cases} \text{for } t > 0 \Rightarrow \sin(4\pi t) \\ \text{for } t < 0 \Rightarrow 0 \end{cases}$, $(II) \begin{cases} \text{for } t < 0 \Rightarrow -\sin(4\pi t) \\ \text{for } t > 0 \Rightarrow 0 \end{cases}$



پریودیک نیست

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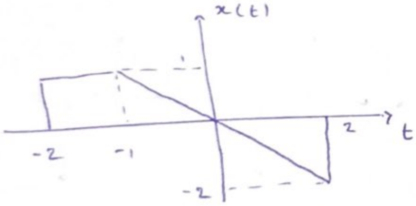
⑤ انرژی و توان متوسط سیگنال های سؤال ① با $T = 5$:

a)



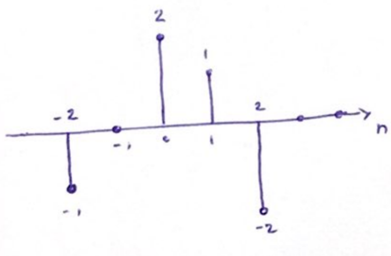
$$x(t) = \begin{cases} -t & ; -1 < t < 2 \\ 1 & ; -2 < t < -1 \end{cases}$$

$$E_\infty = \int_{-\infty}^{+\infty} |x(t)|^2\, dt = \int_{-2}^{-1} dt + \int_{-1}^{+2} t^2\, dt = t\Big|_{-2}^{-1} + \frac{1}{3}t^3\Big|_{-1}^{+2} = \underline{4}\,\Big|$$

$P = \dfrac{E_{\text{در یک پریود}}}{T} = \dfrac{4}{5}$???

دقت شود که این توان سیگنالی x(t) است که با پریود 5 پریودیک باشد. اگر صرفا توان سیگنال x(t) بالا را می خواستیم، توان ∞ می شد.

b)



$$E_\infty = \sum_{k=-\infty}^{+\infty} |x[n]|^2 = \sum_{k=-2}^{+2} |x[n]|^2 = (1)^2 + 0^2 + 2^2 + 1^2$$

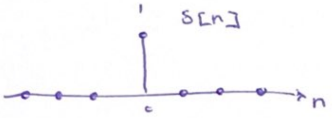
$$+ 2^2 = 10$$

$$P = \frac{E_{\text{در یک پریود}}}{T} = \frac{10}{5} = \underline{2}\,\Big|$$

$$\Rightarrow \omega_0 = 2\pi f = \frac{2\pi}{T} \Rightarrow \left| \begin{array}{l} T_1 = \frac{2\pi}{\omega_0} = \frac{2\pi}{10} = \frac{\pi}{5} \\ T_2 = \frac{2\pi}{\omega_0} = \frac{2\pi}{4} = \frac{\pi}{2} \end{array} \right.$$

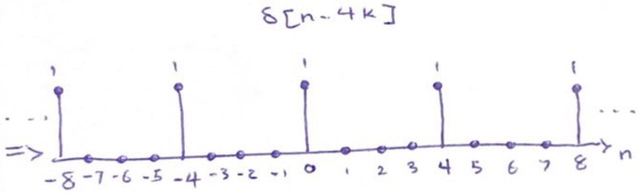
$$\Rightarrow T = \text{kmm}\left(\pi/5 , \pi/2\right) = \underline{\pi}\,\rfloor$$

c) $Z[n] = \sum_{k=-\infty}^{+\infty} \left\{ \delta[n-4k] - \delta[n-1-4k] \right\}$; $\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$

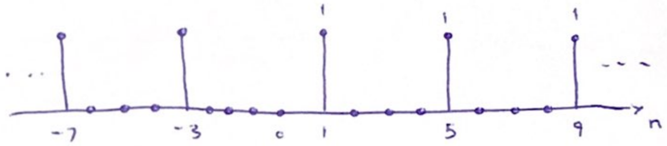


$$\Rightarrow \delta[n-4k] : \begin{cases} \text{if } k=0 \rightarrow \delta[n] \\ \text{if } k=1 \rightarrow \delta[n-4] \\ \text{if } k=2 \rightarrow \delta[n-8] \\ \text{if } k=-1 \rightarrow \delta[n+4] \\ \text{if } k=-2 \rightarrow \delta[n+8] \end{cases}$$

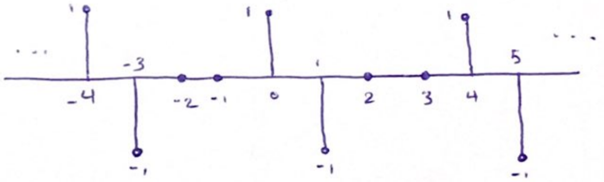
$\Rightarrow$



$\delta[n-1-4k] = \delta[(n-4k)-1] \Rightarrow$



$$\Rightarrow \sum_{k=-\infty}^{+\infty} \left(\delta[n-4k] - \delta[(n-4k)-1] \right) :$$



$\underline{N_0 = 4}\,\rfloor$ ⟵ 4 متناوب با پریود

یادآوری $\cos\alpha \cdot \cos\beta$

$= \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$

d) $S[n] = \cos\left[\frac{\pi}{2} n\right] \cdot \cos\left[\frac{\pi}{4} n\right]$

$$\Rightarrow \frac{1}{2}\left[\cos\left(\frac{\pi}{2} n + \frac{\pi}{4} n\right) + \cos\left(\frac{\pi}{2} n - \cos \frac{\pi}{4} n\right)\right]$$

$$= \frac{1}{2}\left[\underbrace{\cos\left(\overbrace{\frac{3\pi}{4}}^{\omega_0} n\right)}_{(I)} + \underbrace{\cos\left(\overbrace{\frac{\pi}{4}}^{\omega_0} n\right)}_{(II)}\right]$$

$$\Rightarrow \begin{cases} I)\ \omega_0 = 2\pi\left(\frac{3}{8}\right) \xrightarrow[\text{از } \pi \text{ است}]{\omega_0 \text{ مضرب گویایی}} N_0 = 8 \\ II)\ \omega_0 = 2\pi\left(\frac{1}{8}\right) \xrightarrow{\hspace{2cm}} N_0 = 8 \end{cases}$$

$$\Rightarrow N_0 = \text{kmm}(8 , 8) = \underline{8}\,\rfloor$$

④ توان ؟؟

$\rightarrow N=10$

a) $$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \cdot \sum_{n=-N}^{+N} |x[n]|^2 = \lim_{N\to\infty} \frac{1}{\cancel{2N+1}} (\cancel{2N+1})\, 9 = \boxed{9}$$

$\rightarrow T=\pi$

b) $$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} |x(t)|^2 dt \longrightarrow$$ راه ساده تر : $\left\{ \right.$ توان سیگنال سینوسی $= \frac{(\text{دامنه})^2}{2}$

$$\Rightarrow \quad w(t) = \underbrace{2\cos(10t+1)}_{P_1} - \underbrace{\sin(4t-1)}_{P_2} \longrightarrow P_\infty = P_1 + P_2$$

$$P_1 = \frac{2^2}{2} = 2 \quad , \quad P_2 = \frac{1^2}{2} = \frac{1}{2} \Rightarrow P_\infty = P_1 + P_2 = 2 - \frac{1}{2} = \boxed{1.5}$$

$\rightarrow N=4$

c) $$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \cdot \sum_{n=-N}^{+N} |x[n]|^2 = \frac{E_{[0,4]}}{4} = \frac{1}{4}\left(1^2+1^2+1^2\right) = \boxed{\frac{3}{4}}$$

d) $$P_\infty = P_1 + P_2 \longrightarrow S_n = \frac{1}{2}\Big(\underbrace{\cos[\tfrac{3\pi}{4}n]}_{P_1} + \underbrace{\cos[\tfrac{\pi}{4}n]}_{P_2}\Big)$$

$$\Rightarrow P_\infty = \frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}\right) = \frac{1}{2}$$

فرم سیگنال داده شده در صورت سوال

$S[n] = \cos(\frac{\pi}{2}n) \cdot \cos(\frac{\pi}{4}n)$ بود

$$\not\Downarrow$$

$P_\infty = P_1 \cdot P_2$ ✗ کاملا اشتباه => توان سیگنال ها در هم ضرب نمی شوند

یادآوری نکات مهم :

1) $\sin(\alpha) = \cos(\alpha - \frac{\pi}{2})$

2) دستور محاسبه کمم در متلب --> lcm()

3) $\cos(-\alpha) = \cos(\alpha)$

4) $\sin(-\alpha) = -\sin(\alpha)$

5) $\sin a \cdot \cos b = \frac{1}{2}\left(\sin(a+b) + \sin(a-b)\right)$

6) $\cos a \cdot \cos b = \frac{1}{2}\left(\cos(a+b) + \cos(a-b)\right)$

7) $\sin a \cdot \sin b = \frac{-1}{2}\left(\cos(a+b) - \cos(a-b)\right)$

8) $\cos^2 x = \frac{1+\cos 2x}{2}$

9) $\sin^2 x = \frac{1-\cos 2x}{2}$

14) $\sin\alpha = \cos(\alpha - \frac{\pi}{2})$

15) $\cos\alpha = \sin(\frac{\pi}{2} - \alpha)$

10) $e^{j\theta} = \cos\theta + j\sin\theta$

11) $\cos x = \frac{e^{jx} + e^{-jx}}{2}$

12) $\sin x = \frac{e^{jx} - e^{-jx}}{2j}$

12) $e^{\pm j\pi} = -1$

13) $j = e^{j\frac{\pi}{2}}$

14) $e^{\pm jk\pi} = (-1)^k$

13) $= e^{j2k\pi} = 1$