یالطیف

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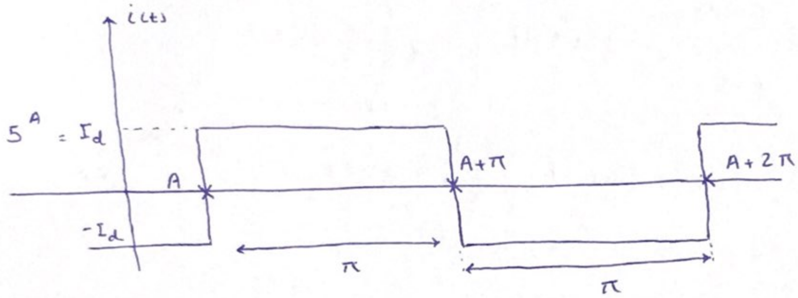
۹۸۱۴۳۰۳
تمرین سری ۱ الکترونیک صنعتی

a)



$$T = 2\pi$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n w_0 t) + b_n \sin(n w_0 t) \right)$$

$$a_0 = \frac{2}{T} \int_T f(t)\, dt$$

$$a_n = \frac{2}{T} \int_T f(t) \cos(n w_0 t)\, dt \quad , \quad b_n = \frac{2}{T} \int_T f(t) \sin(n w_0 t)\, dt$$

$$a_0 = \frac{2}{2\pi} \int_A^{A+\pi} I_d(t)\, dt + \int_{A+\pi}^{A+2\pi} -I_d(t)\, dt = \frac{1}{\pi} \int_A^{A+\pi} 5\, dt - \int_{A+\pi}^{A+2\pi} 5\, dt$$

$$= \frac{1}{\pi} \left[5t \Big|_A^{A+\pi} - 5t \Big|_{A+\pi}^{A+2\pi} \right] = \frac{1}{\pi} \left[5\pi - 5\pi \right] = 0 \Rightarrow \boxed{a_0 = 0}$$

$$a_n = \frac{1}{\pi} \cdot \left[\int_A^{A+\pi} I_d(t) \cdot \cos(n w_0 t)\, dt + \int_{A+\pi}^{A+2\pi} -I_d(t) \cdot \cos(n w_0 t)\, dt \right] \xrightarrow{w_0 = \frac{2\pi}{T_0} = 1}$$

$$a_n = \frac{1}{\pi} \left[\int_A^{A+\pi} 5 \cos(nt)\, dt - \int_{A+\pi}^{A+2\pi} 5 \cos(nt)\, dt \right] = \frac{1}{\pi} \left[\frac{5}{n} \sin(nt) \Big|_A^{A+\pi} - \right.$$

$$\left. \frac{5}{n} \sin(nt) \Big|_{A+\pi}^{A+2\pi} \right] = \frac{1}{\pi} \cdot \left[\frac{5}{n} \left(\sin(n(A+\pi)) - \sin(nA) \right) - \frac{5}{n} \left(\sin(n(A+2\pi)) - \sin(n(A+\pi) \right. \right.$$

$$\Rightarrow \boxed{a_n = \frac{1}{n\pi}\left[10\sin(n(\pi+A)) - 5\sin(n(2\pi+A)) - 5\sin(An) \right]}$$

$$b_n = \frac{1}{\pi}\left[\int_A^{A+\pi} I_d(t)\cdot \sin(n\omega_0 t)\,dt + \int_{A+\pi}^{A+2\pi} I_d(t)\cdot \sin(n\omega_0 t)\,dt \right]$$

$$= \frac{1}{\pi}\cdot\left[\int_A^{A+\pi} 5\sin(nt)\,dt - \int_{A+\pi}^{A+2\pi} 5\sin(nt)\,dt \right] = \frac{1}{\pi}\left[-\frac{5}{n}\cos(nt)\Big|_A^{A+\pi} + \frac{5}{n}\cos(nt)\Big|_{A+\pi}^{A+2\pi} \right]$$

$$\Rightarrow \boxed{b_n = \frac{1}{n\pi}\cdot\left[-10\cos(n(\pi+A)) + 5\cos(n(2\pi+A)) + 5\cos(An) \right]}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt} = \left[\frac{1}{2\pi}\int_A^{A+\pi} 25\,dt + \frac{1}{2\pi}\int_{A+\pi}^{A+2\pi} 25\,dt \right]^{1/2} = \sqrt{25} = \underline{5}$$

$$THD = \sqrt{\frac{I_{rms}^2 - I_{1,rms}^2}{I_{1,rms}^2}} = \left[\frac{25 - \left(\frac{5}{\sqrt{3}}\right)^2}{\left(\frac{5}{\sqrt{3}}\right)^2} \right]^{1/2} = \underline{1.41}$$

$$P = \frac{1}{T}\int_0^T v(t)\,i(t)\,dt = \frac{1}{2\pi}\cdot\left[\int_A^{A+\pi} 220\times 5\,dt - \int_{A+\pi}^{A+2\pi} 220\times 5\,dt \right] = 0$$

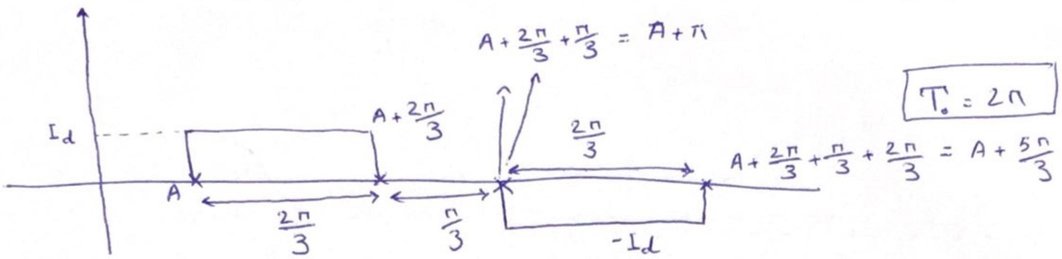
$$S = \bar{V}_{rms} I_{rms} = \frac{220}{\sqrt{2}} \times 5 = 777.8$$

$$Q = \sum_{n=1}^{\infty} \bar{V}_n I_n \sin(\theta_n - \delta_n) = 220\times 5 + 220(-5) = 0$$

$$\Rightarrow S^2 = P^2 + Q^2 + D^2 \Rightarrow D^2 = S^2 - P^2 - Q^2 = (777.8)^2 - 0 - 0 = \underline{777.8}$$

$$PF = \frac{P}{S} = 0$$

b)



$$a_0 = \frac{2}{T}\int_T f(t)\,dt = \frac{1}{\pi}\cdot\left[\int_A^{A+\frac{2\pi}{3}} 5\,dt - \int_{A+\pi}^{A+\frac{5\pi}{3}} 5\,dt\right] = 0$$

$$a_n = \frac{2}{T}\int_T f(t)\cdot\cos(n\omega_0 t)\,dt = \frac{1}{\pi}\left[\int_A^{A+\frac{2\pi}{3}} 5\cos(nt)\,dt - \int_{A+\pi}^{A+\frac{5\pi}{3}} 5\cos(nt)\,dt\right]$$

$$= \frac{1}{n\pi}\cdot\left[5\left(\sin\left(n\left(A+\frac{2\pi}{3}\right)\right) - \sin(An)\right) - 5\left(\sin\left(n\left(A+\frac{5\pi}{3}\right)\right) - \sin\left(n(A+\pi)\right)\right)\right]$$

$$b_n = \frac{2}{T}\int_T f(t)\cdot\sin(n\omega_0 t\,dt) = \frac{1}{\pi}\left[\int_A^{A+\frac{2\pi}{3}} 5\sin(nt)\,dt - \int_{A+\pi}^{A+\frac{5\pi}{3}} 5\sin(nt)\,dt\right]$$

$$= \frac{1}{n\pi}\cdot\left[5\left(-\cos\left(n\left(A+\frac{2\pi}{3}\right)\right) + \cos(An)\right) - 5\left(-\cos\left(n\left(A+\frac{5\pi}{3}\right)\right) + \cos\left(n(A+\pi)\right)\right)\right]$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt} = \left[\frac{1}{2\pi}\int_A^{A+\frac{2\pi}{3}} 25\,dt - \frac{1}{2\pi}\int_{A+\pi}^{A+\frac{5\pi}{3}} 25\,dt\right] = 0$$

$$THD = \sqrt{\frac{I_{rms}^2 - I_{1,rms}^2}{I_{1,rms}^2}} = \left[\frac{0-\left(\frac{5}{\sqrt{3}}\right)^2}{\left(\frac{5}{\sqrt{3}}\right)^2}\right]^{\frac{1}{2}}$$

$$P = \frac{1}{T}\int_0^T v(t)\,i(t)\,dt = \frac{1}{2\pi}\int_A^{A+\frac{2\pi}{3}} 220\times 5\,dt - \frac{1}{2\pi}\int_{A+\pi}^{A+\frac{5\pi}{3}} 220\times 5\,dt = 0$$

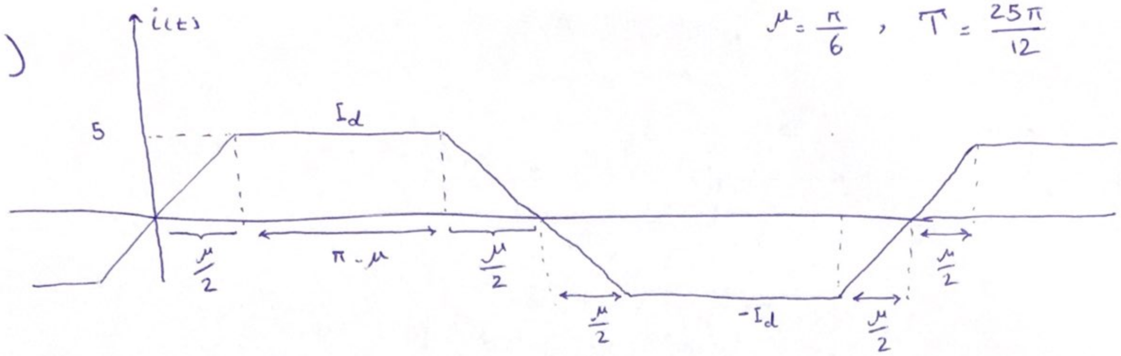
$$S = V_{rms}\,I_{rms} = \frac{220}{\sqrt{2}}\times 0 = 0$$

$$Q = \sum_{n=1}^{\infty} V_n I_n \sin(\theta_n - \delta_n) = 220(5) + 220(-5) = 0$$

$$D^2 = S^2 - P^2 - Q^2 = 0 \quad \Rightarrow \quad PF = \frac{P}{S} = 0$$

c)

$$\mu = \frac{\pi}{6} \quad , \quad T = \frac{25\pi}{12}$$



$a_n = 0$ ← تابع فرد است

$$b_n = \frac{2}{T}\int_T f(t)\cdot \sin(n\omega_0 t)\,dt = \frac{24}{25\pi}\cdot\left[\int_0^{\frac{\pi}{12}} \frac{\pi}{60} t \sin\left(\frac{24}{25}nt\right)dt + \int_{\frac{\pi}{12}}^{\frac{11\pi}{12}} 5\sin\left(\frac{24}{25}nt\right)dt\right.$$

$$\left. + \int_{\frac{11\pi}{12}}^{\frac{13\pi}{12}} -\frac{\pi}{60} t \sin\left(\frac{24}{25}nt\right)dt - \int_{\frac{13\pi}{12}}^{2\pi} 5\sin\left(\frac{24}{25}nt\right)dt + \int_{2\pi}^{\frac{25\pi}{12}} \frac{\pi}{60} t \sin\left(\frac{24}{25}nt\right)dt\right]$$

$$= \frac{24}{25\pi}\cdot\left[\frac{5\pi}{6912n^2}\cdot\left(25\sin\left(\frac{2\pi n}{25}\right) - 2\pi n\cos\left(\frac{2\pi n}{25}\right)\right) + \frac{125}{24n}\cdot\left(-\cos\left(\frac{22\pi n}{25}\right) + \cos\left(\frac{2\pi n}{25}\right)\right)\right.$$

$$- \frac{5\pi}{6912n^2}\cdot\left(22\pi n\cos\left(\frac{22\pi n}{25}\right) - 26\pi n\cos\left(\frac{26}{25}\pi n\right) + 25\sin\left(\frac{26\pi n}{25}\right) - 25\sin\left(\frac{22\pi n}{25}\right)\right)$$

$$+ \frac{125}{24n}\cdot\left(-\cos\left(\frac{48}{25}\pi n\right) + \cos\left(\frac{26}{25}\pi n\right)\right) + \frac{5\pi}{6912n^2}\cdot\left(48\pi n\cos\left(\frac{48}{25}\pi n\right) - 50\pi n\right.$$

$$\left.\left.\cos(2\pi n) + 25\sin(2\pi n) - 25\sin\left(\frac{48}{25}\pi n\right)\right)\right]$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2_{(t)}\,dt} = \left[\frac{12}{25\pi}\int_0^{\frac{\pi}{12}}\left(\frac{\pi}{60}t\right)^2 dt + \frac{12}{25\pi}\int_{\frac{\pi}{12}}^{\frac{11\pi}{12}} 25\,dt + \frac{12}{25\pi}\int_{\frac{11\pi}{12}}^{\frac{13\pi}{12}}\left(\frac{\pi}{60}t\right)^2 dt\right.$$

$$\left. + \int_{\frac{13\pi}{12}}^{2\pi} 25\,dt + \int_{2\pi}^{\frac{25\pi}{12}}\left(\frac{\pi}{60}t\right)^2 dt\right] = \frac{816480000\pi + 867\pi^5}{38880000\pi} + \frac{1801\pi^4}{38880000}$$

$$\simeq 21$$

$$THD = \sqrt{\frac{I_{rms}^2 - I_{1,rms}^2}{I_{1,rms}^2}} = \left[\frac{21 - \left(\frac{5}{\sqrt{3}}\right)^2}{\left(\frac{5}{\sqrt{3}}\right)^2}\right]^{\frac{1}{2}} = \boxed{1.23}$$

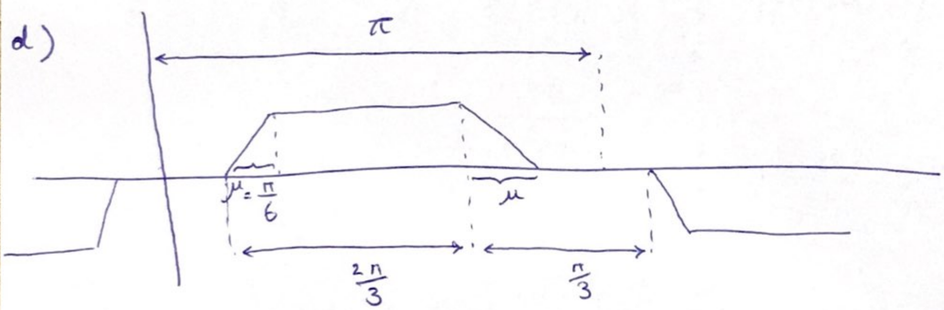
$$P = \frac{1}{T}\int_0^T v(t)\, i(t) = \frac{12}{25\pi}\int_0^{\frac{\pi}{12}} 220 \times 5 + \frac{12}{25\pi}\int_{\frac{\pi}{12}}^{\frac{11\pi}{12}} 25 \times 5 + \cdots = 0$$

$$S = V_{rms} I_{rms} = \frac{220}{\sqrt{2}} \times 21 = 3266.8$$

$$Q = \sum_{n=1}^{\infty} V_n I_n \sin(\theta_n - \delta_n) = 0 \quad , \quad D^2 = S^2 - P^2 - Q^2 = (3266.8)^2$$

$$PF = \frac{P}{S} = 0$$

d) $T = \frac{13\pi}{6} \rightarrow \omega_0 = \frac{12}{13}$



سیگنال فرد است $\Rightarrow a_n = 0$

$$b_n = \frac{2}{T}\int_T f(t)\sin(n\omega_0 t)\, dt = \frac{12}{13\pi}\cdot\left[\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{30}{\pi}\left(t - \frac{\pi}{6}\right)\cdot \sin\left(\frac{12}{13}nt\right) dt + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 5\sin\left(\frac{12}{13}nt\right) dt\right.$$

$$- \int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} +\frac{30}{\pi}\left(t - \frac{\pi}{6}\right)\cdot \sin\left(\frac{12}{13}nt\right) dt + \int_{\frac{5\pi}{6}}^{\pi} -\frac{30}{\pi}\left(t - \frac{\pi}{6}\right)\cdot \sin\left(\frac{12}{13}nt\right) dt$$

$$- \cancel{\int_{\pi}^{\frac{3\pi}{2}} 5\sin\left(\frac{12}{13}nt\right) dt} \; 0 + \int_{\frac{3\pi}{2}}^{\frac{5\pi}{3}} \frac{30}{\pi}\left(t - \frac{\pi}{6}\right)\cdot \sin\left(\frac{12}{13}nt\right) dt$$

$$= \frac{12}{13\pi}\cdot\left[\frac{1}{24\pi n^2}\cdot\left(65\left(-2\pi n\cos\left(\frac{4\pi n}{13}\right)\right) + 13\sin\left(\frac{4\pi n}{13}\right) - 13\sin\left(\frac{2\pi n}{13}\right)\right) + \frac{1}{12n}\cdot\left(+65\left(\cos\left(\frac{6\pi n}{13}\right)\right.\right.\right.$$

$$\left.\left. + \cos\left(\frac{4\pi n}{13}\right)\right)\right) - \frac{1}{24\pi n^2}\cdot\left(65\left(4\pi n\cos\left(\frac{6\pi n}{13}\right) - 6\pi n\cos\left(\frac{8\pi n}{13}\right) + 13\sin\left(\frac{8\pi n}{13}\right) - 13\sin\left(\frac{6\pi n}{13}\right)\right)\right)$$

$$-\frac{1}{24\pi n^2}\left(+65\left(8\pi n\cos\left(\frac{10\pi n}{13}\right)-10\pi n\cos\left(\frac{12}{13}\pi n\right)+13\sin\left(\frac{12}{13}\pi n\right)-13\sin\left(\frac{10}{13}\pi n\right)\right)\right)$$

$$-\frac{12}{n}\left(65\left(-\cos\left(\frac{18\pi n}{13}\right)+\cos\left(\frac{12}{13}\pi n\right)\right)\right)+\frac{1}{24\pi n^2}\left(65\left(16\pi n\cos\left(\frac{18}{13}\pi n\right)-18\pi n\right.\right.$$

$$\left.\left.\cos\left(\frac{20}{13}\pi n\right)+13\sin\left(\frac{20}{13}\pi n\right)-13\sin\left(\frac{18}{13}\pi n\right)\right)\right)\Bigg]$$

$$I_{rms}=\sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}=\left[\frac{6}{13\pi}\int_{\pi/6}^{\pi/3}\left(\frac{30}{\pi}\left(t-\frac{\pi}{6}\right)\right)^2 dt+\frac{6}{13\pi}\int_{\pi/3}^{\pi/2}25\,dt+\frac{6}{13\pi}\int_{\pi/2}^{2\pi/3}\left(\frac{30}{\pi}\left(t-\frac{\pi}{6}\right)\right)^2 dt\right.$$

$$+\int_{5\pi/6}^{\pi}\left(\frac{30}{\pi}\left(t-\frac{\pi}{6}\right)\right)^2 dt+\int_{\pi}^{3\pi/2}25\,dt+\int_{3\pi/2}^{5\pi/3}\left(\frac{30}{\pi}\left(t-\frac{\pi}{6}\right)\right)^2 dt$$

$$=\frac{25}{39}+\frac{25}{13}+\frac{475}{39}+\frac{1525}{39}+\frac{75}{13}+\frac{5425}{39}\simeq\boxed{198.71}$$

$$\Rightarrow THD=\sqrt{\frac{I_{rms}^2-I_{1,rms}^2}{I_{1,rms}}}=\left[\frac{(198.71)^2-\left(\frac{5}{\sqrt{3}}\right)^2}{\left(\frac{5}{\sqrt{3}}\right)^2}\right]=\boxed{68.82}$$

$$P=\frac{1}{T}\int_0^T v(t)\,i(t)\,dt=\frac{6}{13\pi}\int_{\pi/6}^{\pi/3}220\times5\,dt+\frac{6}{13\pi}\int_{\pi/3}^{\pi/2}220\times5\,dt+\cdots=0$$

$$S=V_{rms}I_{rms}=\frac{220}{\sqrt{2}}\times198.71=30912$$

$$Q=\sum_{n=1}^{\infty}V_nI_n\sin(\theta_n-\delta_n)=220(5)+220(-5)=0$$

$$D^2=S^2-P^2-Q^2=(30912)^2\quad\Rightarrow\quad PF=\frac{P}{S}=0$$