-۱

a) $(\frac{1}{۲})^{n-1}u[n] \longrightarrow X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty}(\tfrac{1}{۲})^{n-1}u[n]e^{-j\omega n} = \sum_{n=0}^{\infty}(\tfrac{1}{۲})^{n-1}e^{-j\omega n} = ۲\sum_{n=0}^{\infty}(\tfrac{1}{۲}e^{-j\omega})^{n} = \frac{۲}{۱-\frac{1}{۲}e^{j\omega}}$$

b) $(\frac{1}{۲})^{|n-1|}$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty}(\tfrac{1}{۲})^{|n-1|}e^{-j\omega n} = \sum_{n=-\infty}^{0}(\tfrac{1}{۲})^{-(n-1)}e^{-j\omega n} + \sum_{n=1}^{\infty}(\tfrac{1}{۲})^{n-1}e^{-j\omega n}$$

c) $\delta(n-1)+۲\delta(n+۲)$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty}(\delta[n-1]+۲\delta[n+۲])e^{-j\omega n} = e^{-j\omega}+۲e^{-j\omega(-۲)}$$

d) $(\frac{1}{۲})^{-n}u(-n-1)$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty}(\tfrac{1}{۲})^{-n}u(-n-1)e^{-j\omega n} = \sum_{n=-\infty}^{-1}(\tfrac{1}{۲})^{-n}e^{-j\omega n} = \sum_{n=1}^{\infty}(\tfrac{1}{۲})^{n}e^{j\omega n} = \sum_{n=0}^{\infty}(\tfrac{1}{۲})^{n+1}e^{j\omega(n+1)}$$

e) $\sin(\frac{\pi}{۲}n)+\cos(n)$

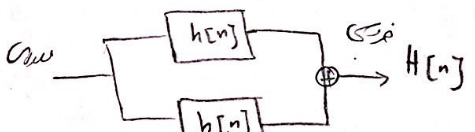
$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty}[\sin[\tfrac{\pi}{۲}n]+\cos[n]]e^{-j\omega n} = \sum_{n=-\infty}^{\infty}\left(\frac{e^{\frac{j\pi n}{۲}}-e^{-\frac{j\pi n}{۲}}}{۲j}+\frac{e^{jn}+e^{-jn}}{۲}\right)e^{-j\omega n}$$

-۲

$h[n] = (\frac{1}{۴})^{n}u[n]$

$b[n] = ?$

$H(\omega) = \dfrac{-۱۲+۵e^{j\omega}}{۱۲-۷e^{-j\omega}+e^{-۲j\omega}}$



$H[n] = h[n]+b[n]$

$$h[n] = (\tfrac{1}{۴})^{n}u[n] \xrightarrow{F} H(e^{j\omega}) = \sum_{n=-\infty}^{\infty}(\tfrac{1}{۴})^{n}u[n]e^{-j\omega n} = \sum_{n=0}^{\infty}(\tfrac{1}{۴})^{n}e^{-j\omega n} = \frac{1}{1-\frac{1}{۴}e^{-j\omega}}$$

$$b[n] = H[n]-h[n] = \frac{-۱۲+۵e^{j\omega}}{۱۲-۷e^{-j\omega}+e^{-۲j\omega}} - \frac{1}{1-\frac{1}{۴}e^{-j\omega}}$$

۳)

$$y[n]-\frac{1}{9}y[n-1]-\frac{1}{9}y[n-2]=x[n]$$

(پایدار، LTI) $x[n]$ ورودی $y[n]$ خروجی   الف)

$$Y(e^{j\omega})-\frac{1}{9}e^{-j\omega}Y(e^{j\omega})-\frac{1}{9}e^{-2j\omega}Y(e^{j\omega})=X(e^{j\omega})$$

$$H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1}{1-\frac{1}{9}e^{-j\omega}-\frac{1}{9}e^{-2j\omega}}$$

ب) $$H(e^{j\omega})=\frac{1}{1-\frac{1}{9}e^{-j\omega}-\frac{1}{9}e^{-2j\omega}}=\frac{A}{(1-\frac{1}{2}e^{-j\omega})}+\frac{B}{(1+\frac{1}{3}e^{-j\omega})}\xrightarrow{F^{-1}}$$

۴)

$$(\tfrac{4}{5})^n u[n]\longrightarrow n(\tfrac{4}{5})^n u[n]$$

الف) $$X(e^{j\omega})=\frac{1}{1-\frac{4}{5}e^{-j\omega}}\qquad H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{\frac{4}{5}e^{-j\omega}}{1-\frac{4}{5}e^{-j\omega}}$$

$$Y(e^{j\omega})=j\frac{d}{d\omega}\left(\frac{1}{1-\frac{4}{5}e^{-j\omega}}\right)=j\frac{-(-\frac{4}{5}-je^{-j\omega})}{(1-\frac{4}{5}e^{-j\omega})^2}=\frac{\frac{4}{5}e^{-j\omega}}{(1-\frac{4}{5}e^{-j\omega})^2}$$

ب) $$Y(e^{j\omega})-\frac{4}{5}e^{-j\omega}Y(e^{j\omega})=\frac{4}{5}e^{-j\omega}X(e^{j\omega})\xrightarrow{F^{-1}}y[n]-\frac{4}{5}y[n-1]=\frac{4}{5}x[n-1]$$

۵)

$$h[n]=(\tfrac{1}{2})^n u[n]$$

1) $x[n]=(\frac{3}{4})^n u[n]$ 2) $x[n]=(n+1)(\frac{1}{4})^n u[n]$

1) $$H(e^{j\omega})=\frac{1}{1-\frac{1}{2}e^{-j\omega}}\qquad X(e^{j\omega})=\frac{1}{1-\frac{3}{4}e^{-j\omega}}$$

$$Y(e^{j\omega})=H(e^{j\omega})X(e^{j\omega})=\frac{1}{1-\frac{1}{2}e^{-j\omega}}\times\frac{1}{1-\frac{3}{4}e^{-j\omega}}=\frac{A}{1-\frac{1}{2}e^{-j\omega}}+\frac{B}{1-\frac{3}{4}e^{-j\omega}}\xrightarrow{F^{-1}}$$

2) $$H(e^{j\omega})=\frac{1}{1-\frac{1}{2}e^{-j\omega}}\qquad X(e^{j\omega})=j\frac{d}{d\omega}\frac{1}{1-\frac{1}{4}e^{-j\omega}}+\frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

$$Y(e^{j\omega})=H(e^{j\omega})X(e^{j\omega})=\frac{1}{1-\frac{1}{2}e^{-j\omega}}\cdot\frac{1}{(1-\frac{1}{4}e^{-j\omega})^2}=\frac{A}{1-\frac{1}{2}e^{-j\omega}}+\frac{B}{1-\frac{1}{4}e^{-j\omega}}+\frac{C}{(1-\frac{1}{4}e^{-j\omega})^2}\xrightarrow{F^{-1}}$$

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$y[n] + \frac{1}{2} y[n-1] = x[n] \xrightarrow{F} Y(e^{j\omega}) + \frac{1}{2} e^{-j\omega} Y(e^{j\omega}) = X(e^{j\omega})$ ۶-

الف) $H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{1}{1+\frac{1}{2}e^{-j\omega}}$

ب) $x[n] = (-\frac{1}{2})^n u[n] \longrightarrow X(e^{j\omega}) = \dfrac{1}{1+\frac{1}{2}e^{-j\omega}}$

$Y(e^{j\omega}) = H(e^{j\omega}) X(e^{j\omega}) = \dfrac{1}{1+\frac{1}{2}e^{-j\omega}} \cdot \dfrac{1}{1+\frac{1}{2}e^{-j\omega}} = \dfrac{1}{(1+\frac{1}{2}e^{-j\omega})^2} \xrightarrow{F^{-1}} (n+1)(-\frac{1}{2})^n u[n] = y[n]$

$x[n] = \delta[n] - \frac{1}{2}\delta[n-1] \longrightarrow Y(e^{j\omega}) = H(e^{j\omega}) X(e^{j\omega}) = \dfrac{1}{1+\frac{1}{2}e^{-j\omega}}(1-\frac{1}{2}e^{-j\omega}) \xrightarrow{F^{-1}}$

۷-

$H_1(j\omega) = \dfrac{2-e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}} \qquad H_2(j\omega) = \dfrac{1}{1-\frac{1}{2}e^{-j\omega}+\frac{1}{4}e^{-j2\omega}}$

$— \boxed{H_1(e^{j\omega})} — \boxed{H_2(e^{j\omega})} —$

الف) $H(e^{j\omega}) = H_1(e^{j\omega}) H_2(e^{j\omega}) = \dfrac{2-e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}} \times \dfrac{1}{1-\frac{1}{2}e^{-j\omega}+\frac{1}{4}e^{-j\omega}} = \dfrac{2-e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}+\frac{1}{4}e^{-j2\omega}+\frac{1}{2}e^{-j\omega}-\frac{1}{4}e^{-j2\omega}+\frac{1}{8}e^{-j3\omega}}$

$\dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{2-e^{-j\omega}}{1-\frac{1}{8}e^{-3j\omega}} \longrightarrow Y(e^{j\omega}) - \frac{1}{8}e^{-3j\omega}Y(e^{j\omega}) = 2X(e^{j\omega}) - e^{-j\omega}X(e^{j\omega}) \xrightarrow{F^{-1}} y[n] - \frac{1}{8}y[n-3] = 2x[n] - x[n-1]$

ب) $H(e^{j\omega}) = \dfrac{2}{1+\frac{1}{8}e^{-3j\omega}} - \dfrac{e^{-j\omega}}{1+\frac{1}{8}e^{-3j\omega}} \xrightarrow{3\omega = b} H(e^{jb}) = \dfrac{2}{1+\frac{1}{8}e^{-jb}} - \dfrac{e^{-j\frac{b}{3}}}{1+\frac{1}{8}e^{-jb}} \xrightarrow{F^{-1}}$

$h[n] = 2(-\frac{1}{8})^{n'} u[n'] - (-\frac{1}{8})^{(n'-\frac{1}{3})} u[n'-\frac{1}{3}] \qquad n' = \frac{n}{3}$