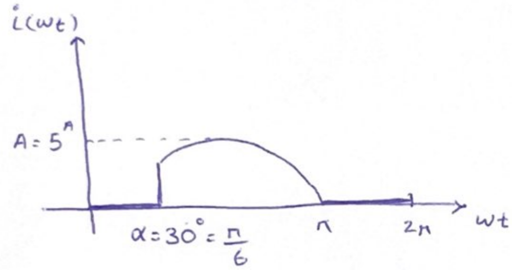
یالطیف

رضا ادینه‌پور
۹۸۱۳۴۳۰۳
امتحان اول الک صنعتی

$K = 0$

$T = 2\pi$

الف)



$$30^\circ \times \frac{\pi}{180} = \frac{\pi}{6}$$

$$i(\omega t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega t) + b_n \sin(n\omega t) \right)$$

$$a_0 = \frac{1}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} i(\omega t)\, dt = \frac{1}{2\pi} \int_{\frac{\pi}{6}}^{\pi} 5 \sin(\omega t)\, d\omega t = \frac{5}{2\pi} \left[-\cos(\omega t) \Big|_{\frac{\pi}{6}}^{\pi} \right] = \frac{(2+\sqrt{3}) \times 5}{4\pi} = \underbrace{1.48^A}_{\text{مقدار dc جریان}}$$

$$a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} i(\omega t) \cos(n\omega t)\, d\omega t = \frac{1}{\pi} \cdot \left[\int_{\frac{\pi}{6}}^{\pi} 5 \sin(\omega t) \cdot \cos(n\omega t)\, d\omega t \right] =$$

$$\Rightarrow a_n = \frac{5}{4\pi} \cdot \left[\frac{(-1)^n + \cos\left(\frac{\pi + n\pi}{6}\right)}{1+n} + \frac{(-1)^n + \cos\left(\frac{\pi - n\pi}{6}\right)}{1-n} \right]$$

$$b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} i(\omega t) \sin(n\omega t)\, d\omega t = \frac{1}{2\pi} \int_{\frac{\pi}{6}}^{\pi} 5 \sin(\omega t) \sin(n\omega t)\, d\omega t$$

$$\Rightarrow b_n = \frac{5}{4\pi} \cdot \left[\frac{\sin\left(\frac{\pi + n\pi}{6}\right)}{1+n} - \frac{\sin\left(\frac{\pi - n\pi}{6}\right)}{1-n} \right]$$

$$i(\omega t) = \frac{5(2+\sqrt{3})}{4\pi} + \sum_{n=1}^{\infty} \cos(n\omega t) \times \frac{5}{4\pi} \left(\frac{(-1)^n + \cos\left(\frac{\pi + n\pi}{6}\right)}{1+n} + \frac{(-1)^n + \cos\left(\frac{\pi - n\pi}{6}\right)}{1-n} \right)$$

$$+ \sin(n\omega t) \times \frac{5}{4\pi} \left(\frac{\sin\left(\frac{\pi + n\pi}{6}\right)}{1+n} - \frac{\sin\left(\frac{\pi - n\pi}{6}\right)}{1-n} \right)$$

$$\Rightarrow i(\omega t) = a_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \theta_n)$$

$$C_n = \sqrt{a_n^2 + b_n^2} \quad \& \quad \theta_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

$n=1 : i_1(\omega t) = 0$

$n=2 : -0.15\cos(2\omega t) + 0.04\sin(2\omega t) \Rightarrow$

$n=3 : -0.01\cos(3\omega t) + 0.007\sin(3\omega t)$

$$\text{-) } RMS = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt} = \sqrt{\frac{1}{T}\left[\int_{\frac{\pi}{6}}^{\pi} 25\sin^2(t)\,dt\right]} = \sqrt{\frac{25}{2\pi}\int_{\frac{\pi}{6}}^{\pi} \sin^2(t)\,dt} = 6.06\ A$$

$DC = a_0 = 1.48\ A$

$$\text{c.) } V(t) = 220\sin(\omega t)$$

$$P = \sum_{n=0}^{\infty} P_n = V_0 I_0 \sum_{n=1}^{\infty} V_{n,rms} I_{n,rms} \cos(\theta_n - \varphi_n) = V_0 I_0 + \sum_{n=1}^{\infty}\left(\frac{V_{n,max} I_{n,max}}{2} \cdot \cos(\theta_n - \varphi_n)\right)$$

$$= 0\,(1.48) + 0\left(\frac{220}{\sqrt{2}}\right) + 0 = 0$$

$$Q = V_0 I_0 \sum_{n=1}^{\infty}\left(\frac{V_{n,max} I_{n,max}}{2}\right)\sin(\theta_n - \varphi_n) = 0\,(1.48) + 0 = 0$$

$$S = V_{rms} I_{rms} = \left(\frac{220}{\sqrt{2}}\right) \times \left(\frac{6.06}{\sqrt{2}}\right) = 666.6 \Rightarrow S = \sqrt{\cancel{P}^2 + \cancel{Q}^2 + D^2} \Rightarrow \boxed{S = D}$$

$$PF = \frac{P}{S} = \frac{0}{666.6} = 0$$