یالطیف

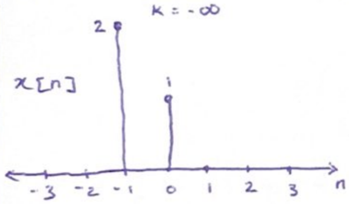
تمرین سری ۳ سیگنال سیستم

1400 / 12 / 25

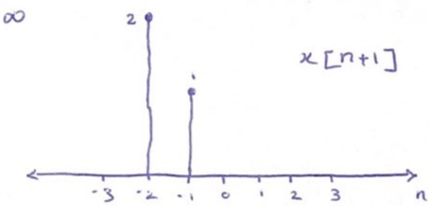
سوال ①

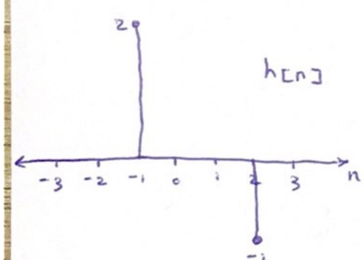
$$\begin{cases} x[n] = \delta[n] + 2\delta[n+1] \\ h[n] = 2\delta[n+1] - \delta[n-2] \end{cases} \Rightarrow y[n] = x[n+1] * h[n-1]$$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k] = \sum_{k=-\infty}^{+\infty} x[n-k]\,h[k]$$

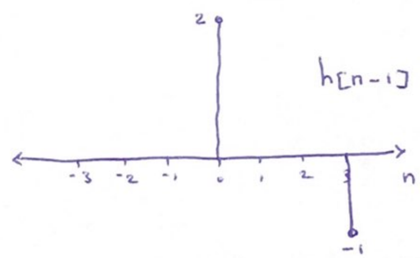


$\Rightarrow$

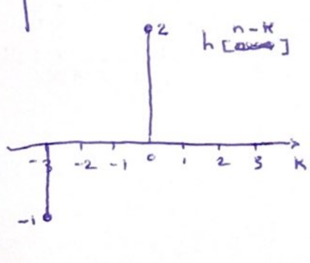




$\Rightarrow$



یکی از سیگنال های $x[n+1]$ یا $h[n-1]$ را نسبت به محور عمودی قرینه کرده و نمونه به نمونه سیگنال ها را در هم ضرب کرده و حاصل را با هم جمع می کنیم.



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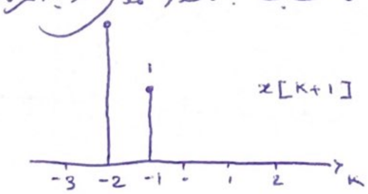

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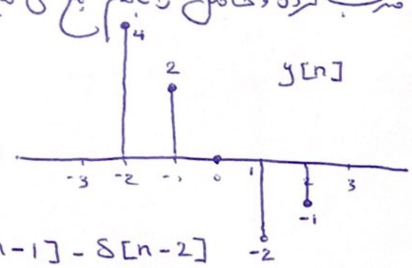


$\Rightarrow y[n] = x[n+1] * h[n-1] = 2\delta[n+1] + 4\delta[n+2] - 2\delta[n-1] - \delta[n-2]$

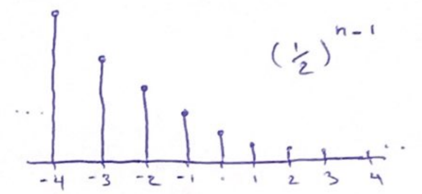
② الف

$$x[n] = \left(\tfrac{1}{2}\right)^{n-1} \cdot \left[u[n] - u[n-10]\right]$$

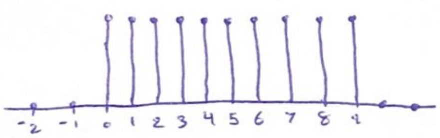
$$h[n] = 3^{n} \cdot \left(u[-n+2]\right)$$

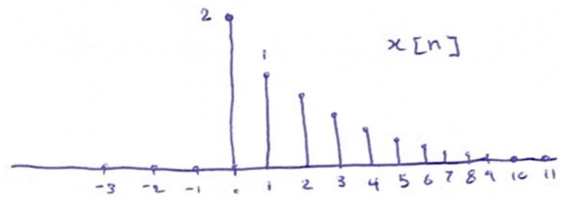
$$y[n] = x[n] * h[n]$$

$$x[n] = \left(\tfrac{1}{2}\right)^{n-1} \rightarrow \begin{cases} \vdots \\ n=-2 \longrightarrow \left(\tfrac{1}{2}\right)^{n-1} = 8 \\ n=-1 \longrightarrow \left(\tfrac{1}{2}\right)^{n-1} = 4 \\ n=0 \longrightarrow \left(\tfrac{1}{2}\right)^{n-1} = 2 \\ n=1 \longrightarrow \left(\tfrac{1}{2}\right)^{n-1} = 1 \\ \vdots \end{cases}$$

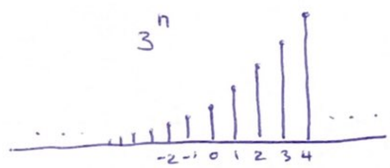
$\Rightarrow$ 

$u[n] - u[n-10]$

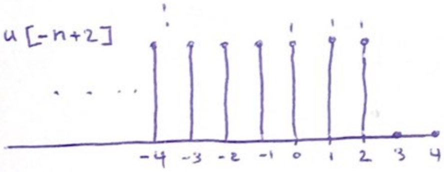


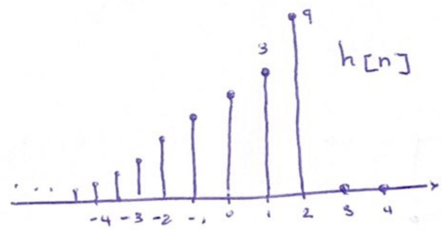
$\Rightarrow$ 

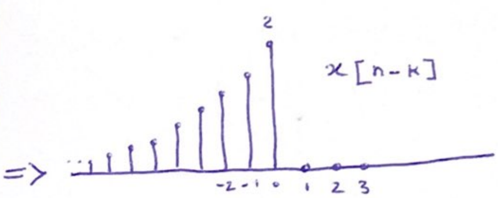
$$3^{n} : \begin{cases} \vdots \\ n=-2 \rightarrow 3^{-2} = 0.111 \\ n=-1 \rightarrow 3^{-1} = 0.333 \\ n=0 \rightarrow 3^{0} = 1 \\ n=1 \rightarrow 3^{1} = 3 \\ \vdots \end{cases}$$

$\Rightarrow$ 

$u[-n+2]$



$\Rightarrow$ 

$\Rightarrow$ 

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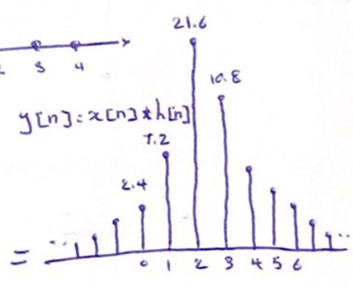
$=$ 

4 سؤال

$$\begin{cases} x(t) = 2e^{t-1}\, u(-t) \\ h(t) = e^{t+1} \cdot \left[u(t) - u(t-5) \right] \\ y(t) = x(t) * h(t) \end{cases}$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} x(t-\tau)\, h(\tau)\, d\tau$$

$$\begin{cases} x(\tau) = 2e^{\tau-1}\, u(-\tau) \\ h(t-\tau) = e^{t-\tau+1} \cdot \left(u(t-\tau) - u(t-\tau-5) \right) \end{cases} \qquad \begin{cases} x(t-\tau) = 2e^{t-\tau-1}\, u(-(t-\tau)) \\ h(\tau) = e^{\tau+1} \cdot \left(u(\tau) - u(\tau-5) \right) \end{cases}$$

$$\Rightarrow y(t) = \int_{-\infty}^{+\infty} x(t-\tau)\, h(\tau)\, d\tau = \int_{-\infty}^{+\infty} 2e^{t-\tau-1}\, u(-t+\tau) \cdot e^{\tau+1} \left(u(\tau) - u(\tau-5) \right) d\tau$$

$$= \int_{-\infty}^{+\infty} 2e^{t-\tau-1}\, u(-t+\tau)\, e^{\tau+1}\, u(\tau)\, d\tau - \int_{-\infty}^{+\infty} 2e^{t-\tau-1}\, u(-t+\tau)\, e^{\tau+1}\, u(\tau-5)\, d\tau$$

$$= \int_{0}^{t} 2e^{t-\tau-1} \cdot e^{\tau+1}\, d\tau - \int_{5}^{t} 2e^{t-\tau-1} \cdot e^{\tau+1}\, d\tau$$

$$= 2e^{t} e^{-1} e^{1} \int_{0}^{t} \underbrace{e^{-\tau} e^{\tau}}_{=1}\, d\tau - 2e^{t} e^{-1} e^{1} \int_{5}^{t} \underbrace{e^{-\tau} e^{\tau}}_{=1}\, d\tau$$

$$= 2e^{t} \int_{0}^{t} d\tau - 2e^{t} \int_{5}^{t} d\tau = 2e^{t} \left(\tau \Big|_{0}^{t} \right) - 2e^{t} \left(\tau \Big|_{5}^{t} \right)$$

$$= 2e^{t} (t) - 2e^{t} (t-5) = 10e^{t}$$