In the name of god

Reza Adinepour
402131055
HW07

9.11: Hi skew Nand gate
$$\begin{cases} 1)\ g_u \ (\text{Pull up}) = g_u = (k-1)\cdot g_{on} & (\text{determind by critical Path}) \\ 2)\ g_d \ (\text{Pull down}) = S\cdot g_{on} & \to (\text{determind by noncritial Path}) \\ 3)\ g_{avg} \ (\text{average}) \end{cases}$$

Lo skew Nand gate
$$\begin{cases} 1)\ g_u = \frac{1}{S} g_{on} \\ 2)\ g_d = (k-1)\ g_{on} \\ 3)\ g_{avg} \end{cases}$$

g_{avg} for HI skew:
$$g_{avg} = \frac{2 g_u g_d}{g_u + g_d} \xrightarrow{(1),(2)} g_{avg} = \frac{2\cancel{(k-1)} g_{on}\cdot S\cdot g_{on}}{\cancel{(k-1)} g_{on} + S g_{on}}$$

mistake!

$$= \frac{2(k-1)\cdot S\cdot g_{on}^2}{(k-1) + S}$$

g_{avg} for LO skew:
$$g_{avg} = \frac{2 g_u g_d}{g_u + g_d} = \frac{\frac{2}{S} g_{on}\cdot (k-1)\cdot g_{on}}{\frac{1}{S} g_{on} + (k-1) g_{on}} = \frac{2(k-1)\cdot g_{on}^2}{(k-1)\cdot S + 1}$$

9.13: Assume Equal rise and fall delay, it means: $g_{on,PNP} \times \sqrt{P} = g_{on,npn} \times \sqrt{N}$

$$= g_{on,PNP} \times \sqrt{\left(\frac{W}{L}\right)_p} = g_{on,npn} \times \sqrt{\left(\frac{W}{L}\right)_n}$$

I know:
$$\left.\begin{array}{l} \tau_{avg} = \frac{1}{2}\left(\tau_{rise} + \tau_{fall}\right) \\ \tau_{rise} = \frac{0.69}{g_{on,p}\sqrt{P}}, \quad \tau_{fall} = \frac{0.69}{g_{on,n}\sqrt{N}} \end{array}\right) \longrightarrow \tau_{avg} = \frac{0.69}{2}\left(\frac{1}{g_{on,P}\sqrt{P}} + \frac{1}{g_{on,n}\sqrt{n}}\right)$$

Now we should find minimum of the $\tau_{avg} \xrightarrow[g_{on,n} = g_{on,P}]{\text{assume}}$

$$\frac{1}{g_{on,P}\sqrt{P}} + \frac{1}{g_{on,n}\sqrt{N}} = \frac{2}{g_{on,P}\sqrt{P}} \Rightarrow \tau_{avg} = \frac{0.69}{\cancel{2}}\left(\frac{\cancel{2}}{g_{on,P}\sqrt{P}}\right) = \boxed{\frac{0.69}{g_{on,P}\sqrt{P}}}$$

9.14: $$\rho(g,p) = \frac{g}{1+g}\left(\frac{1}{p} + \sqrt{\frac{1}{p^2}+\frac{1}{g}}\right) \xrightarrow{g \Rightarrow \frac{g}{g}} \rho(g,p) = \frac{g/g}{1+g/g}\cdot\left(\frac{1}{p} + \sqrt{\frac{1}{p^2}+\frac{1}{g/g}}\right)$$

$$\Rightarrow \rho(g,p) = \frac{1}{2}\left(\frac{1}{p} + \sqrt{\frac{1}{p^2}+1}\right)$$

Now express $\sqrt{\frac{1}{p^2}+1}$ in terms of $\rho(1,\frac{p}{g})$:

$$\sqrt{\frac{1}{p^2}+1} = \frac{1}{\sqrt{p}}\cdot\rho\left(1,\frac{1}{\sqrt{p}}\right) \Rightarrow \rho(g,p) = \frac{1}{2}\left(\frac{1}{p} + \frac{1}{\sqrt{p}}\cdot\rho\left(1,\frac{1}{\sqrt{p}}\right)\right) \Rightarrow g\cdot\rho(g,p)$$

$$= \frac{1}{2}\left(\frac{g}{p} + \frac{g}{\sqrt{p}}\cdot\rho\left(1,\frac{1}{\sqrt{p}}\right)\right) \Rightarrow \rho\left(1,\frac{1}{\sqrt{p}}\right) = \sqrt{\frac{1}{\left(\frac{1}{\sqrt{p}}\right)^2}+1}\cdot\rho\left(1,\frac{1}{\sqrt{p}\cdot\sqrt{\frac{1}{\left(\frac{1}{\sqrt{p}}\right)^2}+1}}\right)$$

$$\Rightarrow \rho\left(1,\frac{1}{\sqrt{p}}\right) = \sqrt{\frac{1}{\frac{1}{p}}+1}\cdot\rho\left(1,\frac{\sqrt{p}}{\sqrt{p+1}}\right)$$

$$\Rightarrow g\cdot\rho(g,p) = \frac{1}{2}\left(\frac{g}{p} + \frac{g}{\sqrt{p}}\cdot\sqrt{\frac{1}{\frac{1}{p}}+1}\cdot\rho\left(1,\frac{\sqrt{p}}{\sqrt{p+1}}\right)\right) \Rightarrow g.\rho(g,p) = \frac{1}{2}\left(\frac{g}{p} + g.\rho\left(1,\frac{\sqrt{p}}{\sqrt{p+1}}\right)\right)$$

$$\xrightarrow{\times 2} 2g\cdot\rho(g,p) = \frac{g}{p} + g\cdot\rho\left(1,\frac{\sqrt{p}}{\sqrt{1+p}}\right)$$

$$\Rightarrow 2g\cdot\rho(g,p) - g\cdot\rho\left(1,\frac{\sqrt{p}}{\sqrt{p+1}}\right) = \frac{g}{p} \Rightarrow g.\left(2.\rho(g,p) - \rho\left(1,\frac{\sqrt{p}}{\sqrt{p+1}}\right)\right) = \frac{g}{p}$$

$$g = \frac{1}{p\left(2.\rho(g,p) - \rho\left(1,\frac{\sqrt{p}}{\sqrt{p+1}}\right)\right)}$$