ALGEBRA

RULES

Isolating the variable

The goal is to isolate the variable:

2a + 4 = 8

a = ?

Opposite operations

|  |  |  |
| --- | --- | --- |
| **Equation** | **Operation** | **Opposite operation** |
| X + 3 = 5 | Addition | Subtraction |
| X – 3 = 2 | Subtraction | Addition |
| 3x = 6 | Multiplication | Division |
| x/2 = 3 | Division | Multiplication |

Balancing equations

Whatever you do to one side of the equation, you must do to the other side. E.g. if you add 3 to the left-hand side of the equation, then you must add 3 to the right-hand side.

ADDITION

X + 20 = -15

// minus 20 from the left hand side in order to isolate x

// minus 20 from the right hand side

X = -15 – 20

X = -35

Y+(-12) = 15

Y = 15 – (-12) // remember, subtracting a negative is like adding a positive

Y = 27

Decimals

-5.5 + c = 10

C = 10 + 5.5

C = 15.5

Challenges

1. X + 38 = -20 (answer -58)
2. 62 = y + (-10) (answer 72)
3. -14.7 + s = -18.9 (answer -4.2)
4. a + (-1/2) = -3/4 (answer -1/4)
5. 29 + c – 8 (answer -22)

SUBTRACTION

X – 20 = 40

X = 60 // x = 40 + 20

S – (-10) = -25 // AKA s + 10 = -25

S = -25 – 10

S = -35

-5.8 = z – 2.5

Z = 2.5 – 5.8

Z = -3.3

Challenges

1. X – 130 = -220 (answer -90)
2. 67 = y – 97 (answer 164)
3. -0.8 = s – 1.4 (answer 0.6)
4. Y – (-22) = -32 (answer -54)
5. D – 30 – 5 = -15 (answer 20)

MULTIPLICATION

A coefficient is a number being multiplied by a variable:

3x = -9 // 3 is a coefficient

X = -9/3

X = -3

-25p = -100

P = -100 \* - 25 = 4

2/3h = 9

// times the reciprocal *- this is flipping the fraction round so 2/3 becomes 3/2*

h *= 9 \* 3/2*

h = 9/1 \* 3/2 // put whole number over 1 to turn in to fraction

h = 27/2 or 13.5

Challenges

1. -12x = 84 (answer -7)
2. 1/4s = -2 (answer -8)
3. -3/4 = 1/3x (answer -9/(+)4)
4. -0.8x = -3.2 (answer 4)
5. 0.25x = 85 (answer 340) // tip– times decimals to make them whole numbers (e.g. x10)

DIVISION

x/5 = -95

x = -95 \* 5

x = -475

y/-6 = -17/+4

y = -17/+4 \* -6

y = -17/+4 \* -6/+1

y = 102/4

y = 51/2 or 25.5

Challenges

1. x/4 = -14 (answer -56)
2. y/-1.2 = 6 (answer -7.2)
3. x/0.3 = 4.2 (answer 1.26)
4. -3/+5 = y/-3 (answer 9/5)
5. -1/+4 = x/-8 (answer 4/2 or 2/1 or 2)

2 STEP EQUATIONS

2y + 3 = 43

In order to get the *y* by itself, we need to remove the 2 and the 3 (2 steps)

Do these in order:

1. Always remove the constant first

2y = 43 – 3

1. Remove the coefficient last

2y = 40

Y = 40 / 2

Y = 20

-r – 3 = 36

-r = 36 + 3

-r = 39

r = 39 / -1

r = -39

solving a 2 step equation with a fraction as the coefficient

1 = 2/3p + 6

2/3p + 6 = 1

2/3 p = -5

P = -5/1 \* 3/2

P = -15/2

P = 7.5

Solving a 2 step equation when you must combine like terms

3x + 5 – 8x = -24

-3x – 8x + 5 = -24

-5x = -29

X = -29/-5

Challenges

1. x/4 + 2 = -10 (answer: -48)
2. 0.6y – 2.5 = 8.9 (answer: 19)
3. 2/3x – 6 = 12 (answer: 27)
4. -18 = 3y + 5 – 6y (answer: 23/3)
5. 20 = 9 – 3x – 6 (answer: -17/3)

MULTISTEP EQUATIONS WITH DISTRIBUTIVE PROPERTY

3(x – 2) = 21

3x – 6 = 21

3x = 27

X = 9

Combining like terms

2b – 3(3b + 2) + 4 = -18

2b – 9b – 6 + 4 = -18

-7b = -18 + 6 – 4

-7b = -16

b = -16/7

double distributive property

20 = 2(2c – 4) – (3c + 1)

4c – 8 – 3c – 1 = 20 // remember *– (3c + 1) is same as –* ***1****(3c + 1)*

C = 20 + 8 +1

C = 29

Challenges

1. 3s – 5(s – 2) = -22 (answer: 16)
2. 8r – (2r + 4) + 1 = 36 (answer: 13/2)
3. -12 = 3w – 5(2w + 1) – 8 (answer: - 1/7)
4. 4(p + 3) + 3(2p – 1) = 21 (answer: 6/5)
5. 4(n + 3) – 2(4n – 1) = -42 (answer: 14)

EQUATIONS WITH DECIMALS

Times by 10, 100, 1000, ... so you don’t work with decimals!

All terms will need to be multiplied

0.7x – 3.9 + 2.45x = 21 x 100

100(0.7x – 3.9 + 2.45x) = (21)100

170x – 390 + 245x = 2100

170x + 245x – 390 = 2100

415x = 2490

X = 2490/415

X = 6

Using distributive property

0.5w – 2.1(w – 5) + 3.1w = 4.5 x10

5w – 21(w – 5) + 31w = 45

5w – 21w + 105 + 31w = 45

15w + 105 = 45

15w = -60

W = -4

Challenges

1. 2.532(w + 3) – 1.45 = 8.678 (answer: 1)
2. 0.3p + 2.54(p – 5) + 2.45 = -38.65 (answer: -10)
3. 4.5(p + 2) – 3.4(2p – 1) = 17 (answer: -2)

EQUATIONS WITH FRACTIONS

1/3y – 2/3 = -14

Multiply each term by denominator

3/1(1/3y) – 3/1(2/3) = (-14)3

Y – 2 = -42

Y = -40

Different denominators

3/4r + 2/3 = 5

Multiply common terms by the least common multiple (LCM)

4, 8, **12**, 16 - 3rd multiple

3, 6, 9, **12**, 15 – 4th multiple

12/1(3/4r) + 12/1(2/3) = (5)12

9r + 8 = 60 // numerator \* multiple

9r = 52

R = 52/9

Using distributive property

-13/6 = 2/3(x – 5) + 1/2 LCM = 6

6/1(-13/6) = 6/1(2/3(x – 5)) + 6/1(1/2)

-13 = 4(x – 5) + 3

-13 = 4x – 20 + 3

4x – 17 = -13

4x = 4

X = 1

Challenges

1. 2/5x + 4/5 = 10 (answer: 23)
2. 8 – 2/7(2a – 2) = -4 (answer: -22)
3. ¾ = 1/6y + 5/4 (answer: -3)
4. 3m + 2/3(m – 4) = -1/6 (answer: 15/22)
5. ¾(x + 4) – 1/3(2x + 1) = 19/6 (answer: 6)

LITERAL EQUATIONS

Literal equations are equations that contain mostly variables:

A = LW (area = length \* width)

To find the value for *w*:

W = A/L

You can check the solution by picking numbers for the variables

Using fractions

*Solve for f*

Y = f+g/3

3y = f + g

F = 3y – g

*Solve for t*

W = 3s+u/t

T(w) = (3s+u/t)t/1

Tw = 3s + u

T = 3s+u/w

Challenges

1. *Solve for r*

I = PRT (answer: r = I/PT)

1. *Solve for w*

P = 2L + 2W (answer: W = P-2L/2)

1. *Solve for b*

A = 1/2bh (answer: b = 2a/h)

1. *Solve for b*

6a/b = c (answer: b = 6a/c)

1. *Solve for t*

2st+u/w = r (answer: t = rw-u/2s)

VARIABLES ON BOTH SIDES

3a + 2 = 5a + 16

1. Get all variables on one side
2. Get all the constants on the other side

3a – 5a = 16 – 2

-2a = 14

a = 7

using distributive property

4x – 2(x - 2) = 5(x – 1)

4x – 2x + 4 = 5x – 5

2x + 4 = 5x – 5

2x – 5x + 4 = -5

-3x + 4 = -5

-3x = -9

X = 3

Challenges

1. 3p – 4 = 5p – 60 (answer: p = -28)
2. 3 – a = 4 + 3a (answer: a = - ¼)
3. 2.25b – 4.5 = 11.25b (answer: b = -0.5)
4. 3(w – 4) = 5 + 2(2w – 6) (answer: w = -5)
5. 2 – 2(g + 3) = 3(3g + 2) – 5 (answer: g = -5/11)

MULTISTEP EQUATIONS

2/3(x – 4) – 5 = 1/6(x + 2) – 2x

1. Fractions first – multiply each term on both sides by the LCM

6(2/3(x – 4) – 5) = (1/6(x + 2) – 2x)6

4(x – 4) – 30 = 1(x + 2) – 12x

1. Next, use distribute property

4x – 16 – 30 = x + 2 – 12x

1. Combine like terms

4x – 46 = -11x + 2

1. Add or subtract to get all variable terms on one side

4x + 11x = 2 + 46

15x = 48

1. If all variables were already on one side before step *4*, then use addition/subtraction to remove any constants from the variable side of the equation
2. Use multiplication/division to remove any coefficients from the variable side

X = 48/15

X = 16/5

1. Check answer

Challenges

1. ¾(x – 8) - 4x = 2(x – 3) – ½x (answer: x = 0)
2. 2x – 4(x – 5) = 3/2(3x – 6) + 5 (answer: x = 48/13)
3. 2/5(x – 5) + 1/3x = 1/3(x – 9) + x – 8 (answer: x = 135/9 or 15)

ABSOLUTE VALUE EQUATIONS

Absolute value is the value of how many units a number is from 0. The absolute value of any number must be positive!

|5| = 5

|-5| = 5

|-120| = 120

|120| = 120

|x| = 3 // x = 3 or x = -3

{-3, 3} // answers are written in {} and the first number must be the lowest

|x - 10| = 20 // the absolute value is always positive, so the answer could be 20 or -20

1. X – 10 = 20

X = 30

1. X – 10 = -20
2. X = -10

Answer: {-10, 30}

With absolute values it is very important to check your answer because the solution is not always right and sometimes there may only be one correct answer.

Isolate absolute value expression

|2y + 4| - 8 = 12

|2y + 4| = 12 + 8

|2y + 4| = 20

Now, assume 20 and -20 in order to work out the equation

2y + 4 = 20 (answer: y = 8)

2y + 4 = -20 (answer: y = -12)

{-12, 8}

Absolute value equations with no solutions

|x| = -4

An absolute value can never be negative, so the above results in an empty set {}

2|x – 6| + 10 = 8

2|x – 6| = -2

|x – 6| = -1

Answer: {}

-2|x – 6| + 10 = 8

-2|x – 6| = -2 // isolate the full absolute value before assuming empty set

|x – 6| = 1 // -2 / -2 is positive 1, so there lies a potential solution here

X – 6 = 1 (answer: 7)

X – 6 = -1 (answer: 5)

{5, 7}

Absolute value with one solution

|x + 3| + 2 = 3x + 1

|x + 3| = 3x – 1

X + 3 = 3x – 1 (answer: x = 2)

X + 3 = -(3x – 1) (answer: x = -1/2)

Check answer if x is 2

2 + 3 + 2 = 3(2) + 1

7 = 7 // 2 is a valid solution

Check answer is x = -1/2

-1/2 + 3 + 2 = 3(-1/2) + 1

4.5 = -0.5 // -1/2 is not a valid solution

Answer: {2}

Challenges

1. |8r – 5| - 12 = 17 (answer: {-3, 4.25})
2. 18 + |1/4x – 10| = 24 (answer: {16, 64})
3. 2|-3x – 8| + 2 = 54 (answer: {-34/3, 6})
4. |2a + 4| - 5 = 3a + 16 (answer: {-5})
5. |5a – 4| + 9 = 5 (answer: {})