DISCRETE MATH

SETS

|  |  |  |
| --- | --- | --- |
| **Symbol** | **Meaning** | **Example** |
| { } | [Set](https://www.mathsisfun.com/sets/sets-introduction.html): a collection of elements | {1,2,3,4} |
| A ∪ B | [Union](https://www.mathsisfun.com/sets/venn-diagrams.html): in A or B (or both) | C ∪ D = {1,2,3,4,5} |
| A ∩ B | [Intersection](https://www.mathsisfun.com/sets/venn-diagrams.html): in both A and B | C ∩ D = {3,4} |
| A ⊆ B | Subset: A has some (or all) elements of B | {3,4,5} ⊆ D |
| A ⊂ B | Proper Subset: A has some elements of B | {3,5} ⊂ D |
| A ⊄ B | Not a Subset: A is not a subset of B | {1,6} ⊄ C |
| A ⊇ B | Superset: A has same elements as B, or more | {1,2,3} ⊇ {1,2,3} |
| A ⊃ B | Proper Superset: A has B's elements and more | {1,2,3,4} ⊃ {1,2,3} |
| A ⊅ B | Not a Superset: A is not a superset of B | {1,2,6} ⊅ {1,9} |
| Ac | [Complement](https://www.mathsisfun.com/sets/venn-diagrams.html): elements not in A | Dc = {1,2,6,7} When  = {1,2,3,4,5,6,7} |
| A − B | [Difference](https://www.mathsisfun.com/sets/venn-diagrams.html): in A but not in B | {1,2,3,4} − {3,4} = {1,2} |
| *a*∈ A | [Element](https://www.mathsisfun.com/sets/sets-introduction.html) of: *a* is in A | 3 ∈ {1,2,3,4} |
| *b*∉ A | Not element of: *b* is not in A | 6 ∉ {1,2,3,4} |
| ∅ | [Empty set](https://www.mathsisfun.com/sets/sets-introduction.html) = {} | {1,2} ∩ {3,4} = Ø |
| U | [Universal Set](https://www.mathsisfun.com/sets/sets-introduction.html): set of all possible values (in the area of interest) |  |
|  |  |  |
| **P**(A) | [Power Set](https://www.mathsisfun.com/sets/power-set.html): all subsets of A | P({1,2}) = { {}, {1}, {2}, {1,2} } |
| A = B | Equality: both sets have the same members | {3,4,5} = {5,3,4} |
| A×B | Cartesian Product (set of ordered pairs from A and B) | {1,2} × {3,4} = {(1,3), (1,4), (2,3), (2,4)} |
| |A| | Cardinality: the number of elements of set A | |{3,4}| = 2 |
|  |  |  |
| | | [Such that](https://www.mathsisfun.com/sets/set-builder-notation.html) | { *n* | *n* > 0 } = {1,2,3,...} |
| : | [Such that](https://www.mathsisfun.com/sets/set-builder-notation.html) | { *n* : *n* > 0 } = {1,2,3,...} |
| ∀ | For All | ∀x>1, x2>x |
| ∃ | There Exists | ∃ x | x2>x |
| ∴ | Therefore | *a=b ∴ b=a* |
|  |  |  |
| N | [Natural Numbers](https://www.mathsisfun.com/whole-numbers.html) | {1,2,3,...} or {0,1,2,3,...} |
| Z | [Integers](https://www.mathsisfun.com/whole-numbers.html) | {..., -3, -2, -1, 0, 1, 2, 3, ...} |
| Q | [Rational Numbers](https://www.mathsisfun.com/rational-numbers.html) |  |
| R | [Real Numbers](https://www.mathsisfun.com/numbers/real-numbers.html) |  |

LOGIC

|  |  |  |  |
| --- | --- | --- | --- |
| Symbol | Symbol Name | Meaning / definition | Example |
| **⋅** | and | and | *x***⋅***y* |
| ^ | caret / circumflex | and | *x* ^ *y* |
| & | ampersand | and | *x* & *y* |
| + | plus | or | *x* + *y* |
| ∨ | reversed caret | or | *x* ∨ *y* |
| | | vertical line | or | *x* | *y* |
| *x*' | single quote | not - negation | *x*' |
| *x* | bar | not - negation | *x* |
| ¬ | not | not - negation | ¬ *x* |
| ! | exclamation mark | not - negation | ! *x* |
| ⊕ | circled plus / oplus | exclusive or - xor | *x* ⊕ *y* |
| ~ | tilde | negation | ~ *x* |
| ⇒ | implies |  |  |
| ⇔ | equivalent | if and only if (iff) |  |
| ↔ | equivalent | if and only if (iff) |  |
| ∀ | for all |  |  |
| ∃ | there exists |  |  |
| ∄ | there does not exists |  |  |
| ∴ | therefore |  |  |
| ∵ | because / since |  |  |

COUNTING

|  |  |  |
| --- | --- | --- |
| **Symbol** | **Name** | **Example** |
| () | List | {a, b, c} = {c, b, a}  (a, b, c) ≠ (c, b, a) |
| n! | Factorial | 4! = 4 \* 3 \* 2 \* 1 = 24 |
| P(n, k) | Permutation  k – permutation of an n element set | P(4, 3) = 24  (2, 4, 6, 8)  |n| = 4  4 \* 3 \* 2  P(4, 2) = 12  4 \* 3 = 12 |
| [ ] | Multiset – hybrid between list and set | [1, 1, 1, 5, 10, 10] |
| [ ] (with top part missing) | Floor (round down) | [9.31] = 9 |
| [ ] (bottom part missing) | Ceiling (round up) | [9.31] = 10 |

DISCRETE MATH

**What is it?**

* The study of math that falls outside the realm of calculus
* Calculus is the study of continuous concepts, whereas this topic is discrete. The study of separable, countable, or quantified objects
* For example; a tv remote – the on/off button is discrete – either on or off. The volume button is variable and continuous

**Core concepts**

* Logic
  + If, this, then. That, and, or, not
* Set theory
  + Finite/infinite sets
* Number theory
  + Using integers to enumerate sets of data
* Graph theory
  + Model pairwise relationships between objects
* Combinatorics
  + The study of finite countable discrete structures. i.e. the study of sequences, e.g. fibinocci

**Programming**

* Computers operate in a discrete manner. Machine language is a series of bits and bytes. Logical expressions evaluate to TRUE (1), FALSE (0)

SET THEORY

* A set is a collection of objects
* A set of positive odd numbers **1, 3, 5, 7, 9, …**
* Properties:
  + The order of elements in a set is not important
  + A set with no elements is called an empty set
  + Two sets that have exactly the same elements are equal
  + Set cardinality is the number of elements in the set

**Special set names**

* N => The set of all natural numbers (1, 2, 3, 4, …)
* W => The se of whole numbers (0, 1, 2, 3, …)
* Z => The set of all integers (…, -2, -1, 0, 1, 2, …)
* Q => The set of all rational numbers
* R => The set of all real numbers (irrational, rational, integers, etc)

Venn diagram of special set names

R

Q

Z

W

N

**Set notation**

*See tables at top of document*

Subset definition

If every element in set A is also an element of set B, then A is a subset of B, written as A ⊆ B.

If A ⊆ B & there is an element of B that is not in A, then A is a proper subset of B. e.g. B = {2, 4, 6, 8, 10}, A = {2, 4, 6}, A ⊆ B

Red ∈ {red, orange, yellow, green, blue, indigo, violet}

The element red, is an element of the set containing the colours of the rainbow

{Red} ∉ {red, orange, yellow, green, blue, indigo, violet}

The set containing the word red, is not an element of the set containing the colours of the rainbow (nested set)

Set of natural numbers

N = { x ∈ Z | x > 0 }

i.e. x is an element of integers, such that (|) x is greater than 0

**Set operations**

* Set intersection
  + A ∩ B – A intersects B
    - Formal notation: { x∈A and x∈B }
* Set union
  + A ∪ B
    - Formal notation: { x∈A or x∈B }
* Set difference
  + A – B
    - Formal notation: { x∈A and x∉B }
* Set complement
  + Ac
    - Formal notation: { x∉A }
    - Let U represent the universal set, then the complement of set A is the set of all elements in U that not an element of A
    - E.g. U = { Z }, E = { x ∈ Z | x = 2i }

U = Z

Ec

Aka odd integers

All even integers

POWER SETS

* The power set is the set of all subsets for a particular set including the empty set
* The cardinality of a power set is 2n (where n is the cardinality of the original set)

If |A| = 3, then |**P**(A)| = 8

23 = 8

Given the set:

A = { 1, 2, 3 }

**P**(A) = { {}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3} }

SEQUENCES AND SUMS

Finite: { am, am+1, am+2, …, an }

Infinite: { am, am+1, am+2, …}

Given the sequence: { 21, 22, 23, … } – what is an explicit formula for this series?

dk = 2k for k ≥ 1

When working with sequences we want to know the direction:

* Increasing – ak < ak + 1 (1, 2, 3, 4, …)
* Decreasing – ak > ak + 1

**Special sequences**

Geometric sequence

Each term is derived by taking the previous term and multiplying it by a common ratio. It can be finite or infinite.

Fibonacci sequence

1, 1, 2, 3, 5, 8, 13, 21, …

A recursive sequence where the element is the sum of the previous 2 elements

DIRECT PROOF

Theorem – a mathematical statement that is true and can be (has been) verified as true

Proof – a proof of a theorem is a written verification that shows that the theorem is definitely true

Definition – an unambiguous explanation of the meaning of a mathematical word or phrase

Definitions:

* An integer is even if n = 2a | a ∈ Z
* An integer is odd if n = 2a + 1 | a ∈ Z
* a|b if b = ac | c ∈ Z // a divides b; e.g. 8|32 (8 \* 4 = 32)

{a ∈ Z | a|6} = {-6, -3, -2, -1, 1, 2, 3, 6}

Direct proof setup

1. suppose P
2. logic and definitions
3. therefore Q

example

1. proposition: if x is odd, then x^2 is odd
2. proof: suppose x is odd, then x = 2a + 1 for some ∈ Z by definition of an odd number
3. therefore x^2 is odd

Direct proofs tend to take the structure P ⇒ Q (if P then Q)

|  |  |  |
| --- | --- | --- |
| **P** | **Q** | **P ⇒ Q** |
| T | T | T |
| F | T | T |
| T | F | F |
| F | F | T |

We say, if P is true, then Q must be true, we don’t say if it is false then Q can’t be true.

Example of full direct proof

Proposition

If x is odd, then x^2 is odd

Proof.

Suppose x is odd, then x^2 = 2a + 1 for some *a* ∈ *Z*, by definition of an odd number. Thus x^2 = (2a + 1)^2 = 4a^2 + 4a + 1 = 2(2a^2 + 2a) + 1, so x^2 = 2b + 1, where b = 2a^2 + 2a ∈ Z, therefore x^2 is odd, by definition of an odd number

Main set definitions

* A×B = {(x, y) : x ∈ A, y ∈ B}
* A U B = {x: (x ∈ A) ∨ (x ∈ B)}
* A ∩ B = {x: (x ∈ A) & (x ∈ B)}
* A – B = {x: (x ∈ A) & (x ∉ B)}
* Ac = U – A

Examples:

* {n ∈ Z : n is odd} = {…, -3, -1, 1, 3, …}
* {x ∈ N : 6 | x} = {6, 12, 18, 24, …}
* {(a, b) ∈ Z \* Z : b = a + 5} = {…, (-2, 3), (-1, 4), (0, 5), … }
* {X ∈ P(Z) : |X| = 1} = {…, {-1}, {0}, {1}, {2}, …}

INDUCTION

Induction is a direct proof technique which answers the question of how to prove an infinite set is true for a specific proposition.

Suppose you can prove the first statement S1, also say you can prove Sk being true and Sk+1, then all must be true

Example:

Prove that 1 + 2 + 3 + … + n =