MORE ALGEBRA

**Simultaneous equations**

2x + y = 10 // x & y are both unknown

Equations that have more than one unknown could have an infinite number of solutions, e.g. in the example above:

* X = 1, y = 8
* X = 2, y = 6
* X = 3, y = 4

To be able to solve an equation like this, another equation has to be used alongside it

3x + y = 11 and 2x + y = 8

1. Identify which unknown has the same coefficient, in this example, y has coefficient of 1 in both equations
2. Add or subtract the 2 equations from each other to eliminate y. If the signs in front of the coefficient are different, add the 2 equations; if they are the same, subtract them.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 3x | + | Y | = | 11 |
| 2x | + | Y | = | 8 |
| X |  |  | = | 3 |

Now we have the value for *x*, we can substitute this in one of the equations to find the value for *y*.

3x + y = 11

3(3) + y = 11

9 + y = 11

Y = 11 - 9

Y = 2

Now, check the answer of x and y against the other equation to confirm.

No common coefficient

3a + 2b = 17 and 4a – b = 30

1. One or both equations must be multiplied to create a common coefficient.

Remember, a common coefficient is regardless of sign. -1 and 1 are common coefficients

1. In this example, we can multiply the second equation by 2, in order to make the b variable, 2b, which would match the coefficient of b in the first equation

3a + 2b = 17 and 8a – 2b = 60 // multiple to get a common coefficient

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 3a | + | 2b | = | 17 |
| 8a | - | 2b | = | 60 |
| 11a |  |  | = | 77 |
| a |  |  | = | 7 |

Now, use *a* as 7 to work out the value for *b*

3(7) + 2b = 17

21 + 2b = 17

2b = -4

b = -2

// now check the answer against the other equation, substituting *a* and *b* with their found values.

**Sequences**

A sequence is a set of numbers that follow a pattern or rule. If the rule is to add or subtract a number each time, it is called an *arithmetic sequence.* If the rule is to multiply or divide, it is called a *geometric sequence.*

3, 7, 11, 15, … // the rule is +4

1, 2, 4, 8 // the rule is x2

Each term in a sequence has a position, starting at 1. *Position to term rules* use algebra to work out what number is in a sequence if the position in the sequence is known, also called the *nth term*.

Sequence: 5 6 7 8

Position: 1 2 3 4

Operation: +4 +4 +4 +4

= n+4

Example: 3, 5, 7, 9, … = 2n + 1 (position 2 is 5, 2(2) + 1 = 5)

Quadratic sequences

n^2 + 3n – 5

when *n* is:

* 1: 1^2 + 3(1) – 5 = -1
* 2: 2^2 + 3(2) – 5 = 5
* 3: 3^2 + 3(3) – 5 = 13

Finding the nth term of a quadratic sequence

+2 +2 +2

+3 +5 +7 +9

**2 5 10 17 26**

The second difference, the top line, shows that the sequence is quadratic. The coefficient of n^2 is always half of the second difference, so in this case it is 1. Now work out the *nth term*.

n^2: 1 4 9 16

operation: +1 +1 +1 +1

sequence: 2 5 10 17

answer: n^2 + 1

challenge:

1. What is the *nth term* of the sequence; 5, 11, 21, 35 (answer: 2n^2 + 3)

How to work out a number belongs to a sequence

Put the *nth* term equal to the number you want to figure out to be part of a sequence and solve the equation.

Example: does 14 belong to 4n + 2?

4n + 2 = 14

4n = 12

n = 3

yes, it does, position 3 equals 14 in the sequence.

The sequence would look like this: 6, 10, 14, 18

**Laws of logarithms & exponents**

Logarithms come in the form *log*ax. We say this as ‘log to the base *a* of *x*’.

*Log525 means: ‘what power of 5, gives 25?’ –* the answer is 2, because

Both *a* & *x* must be positive.

*Logax = y*

*X =*

Laws of logarithms

|  |  |  |
| --- | --- | --- |
|  | **Law** | **Example** |
| 1 | Logaa = 1 | Log77 = 1 |
| 2 | Loga1 = 0 | Log201 = 0 |
| 3 | Logap + logaq = logapq | Log32 + log35 = log310 |
| 4 | Logap - logaq = loga | Log654 + log69 = log6 = log66 = 1 |
| 5 | Loga = nlogaP | Log25^3 = 3log25 |

Thus: log101000 = log1010^3 = 3log1010 = 3 x 1 = 3

Example

Write as a log statement.

log39 = 2

challenges

1. Write *log5125 = 3* as a power statement (answer )
2. Simplify 5log82 + log84 – log816

(answer = 5log82 + 2log82 – 4log82

= 3log82 = log82^3 = log88 = 1)