Homework 0

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1 Theoretical Part

1. (i) Expectation of X

$$E(X) = \sum_{i=1}^{6} x_i p(x_i)$$

For all x_i; $p(x_i) = 1/6$

$$E(X) = \sum_{i=1}^{6} x_i * 1/6$$

$$E(X) = (1 + 2 + 3 + 4 + 5 + 6) * 1/6$$

$$E(X) = (21) * 1/6 = 3.5$$

(ii) Variance of X

$$Var(X) = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}$$

We know for dice: $\mu = 3.5$

$$Var(X) = \frac{\sum_{i=1}^{6} (x_i - 3.5)^2}{6}$$

$$Var(X) = \frac{(-2.5)^2 + (-1.5)^2 + (-0.5)^2 + (0.5)^2 + (1.5)^2 + (2.5)^2}{6}$$

$$Var(X) = \frac{17}{6} \simeq 2.92$$

2. (i) Euclidean norm of v

$$EN(v) = \sqrt{\sum |v_i|^2}$$

(ii) Euclidean inner product (aka dot product) between u and v

$$u^T v = \sum_{i=1}^{n} u_i v_i$$

(iii) Matrix-vector product Au The result is $y = Au \in \mathbb{R}^n$

$$y = Au = \begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ \vdots & & \\ - & a_n^T & - \end{bmatrix} u = \begin{bmatrix} a_1^T u \\ a_2^T u \\ \vdots \\ a_n^T u \end{bmatrix}$$

3. Both algorithms do the same thing, they compute sum of all the numbers from 1 to n.

   The order of ALGO1 algorithm is $O(n)$, because it adds up all the numbers from 1 to n one by one, so this line "$result = result + i$" computes for n times.

   But ALGO2 uses the Gauss Summation formula, which is faster than the ALGO1 because its order is $O(1)$.

4. (i)

$$\frac{df}{dx} = ?, \quad \text{where } f(x, \beta) = x^2 \exp(-\beta x)$$

$$\frac{df}{dx} = \frac{dx^2}{dx} \times \exp(-\beta x) + \frac{d \exp(-\beta x)}{dx} \times x^2$$

$$\frac{df}{dx} = 2x \times \exp(-\beta x) - \beta \exp(-\beta x) \times x^2$$

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(ii)

$$\frac{df}{d\beta} = ?, \quad \text{where } f(x, \beta) = x \exp(-\beta x)$$

$$\frac{df}{d\beta} = \frac{dx}{d\beta} \times \exp(-\beta x) + \frac{d \exp(-\beta x)}{d\beta} \times x$$

$$\frac{df}{d\beta} = -x \exp(-\beta x) \times x$$

$$\frac{df}{d\beta} = -x^2 \exp(-\beta x)$$

(iii)

$$\frac{df}{dx} = ?, \quad \text{where } f(x) = \sin(\exp(x^2))$$

$$\frac{df}{dx} = \frac{d \exp(x^2)}{dx} \times \cos(\exp(x^2))$$

$$\frac{df}{dx} = 2x \exp(x^2) \times \cos(\exp(x^2))$$

5.

$$Var(X) = E[(X - E[X])^2]$$

$$Var(X) = E[X^2 - 2XE[X] + E[X]^2]$$

$$Var(X) = E[X^2] - 2E[X]E[X] + E[X]^2$$

$$Var(X) = E[X^2] - E[X]^2$$

$$Second\,Moment\,of\;X = Var(X)$$

$$1 = E[X^2] - E[X]^2$$

$$1 = E[X^2] - \mu^2$$

$$E[X^2] = 1 + \mu^2$$