

generalizing approximate contraction for arbitrary tensor networks

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Group Meeting

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Tensor network contraction

Examples

- Quantum many-body
- Classical stat-mech
- SAT / cover counting
- Bayesian Inference

$$Z = \sum_{\{x\}} \prod_i T_i[\bar{x}_i]^{d^{|E|} |V|}$$

For anything
not pseudo-
1D need to
do
approximatel
y

- All ‘useful’ and have their own approx methods

8 cm. from the primary. Reverse the wires in the secondary circuit, reverse the wires in the primary circuit, how you please, the mercury always moves towards the point of the capillary.

8. Shouting or singing (excepting the above-mentioned note) produces no visible effect under the conditions mentioned in Experiments 5, 6, and 7.

9. If the secondary coil be now moved close up, so as to cover as completely as possible the primary, talking to the telephone with the ordinary voice, i.e. with moderate strength and at any pitch, produces a definite movement of the mercury column for each word, some sounds of course giving more movement than others, but the movement is always towards the end of the capillary. Singing the note mentioned in Experiments 5, 6, and 7 loudly, produces a movement too large to be measured with the electrometer.

Reversing the poles of the magnet in the telephone does not alter the results of Experiments 5, 6, 7, and 9.

On mentioning the above results to Dr. Burdon Sanderson, he suggested that the apparently anomalous behaviour of the electrometer might be accounted for, by supposing that the mercury moved quicker when a current passed towards the point of the capillary than when it flowed in the opposite direction; so that if a succession of rapidly alternating currents be passed through the instrument, the mercury will always move towards the point of the capillary, the movement away from the point being masked by the sluggishness of the instrument in that direction. That this explanation is the correct one is proved by the following experiment:—The current from two Grove's cells is sent through a metal reed vibrating 100 times a second, the contact being made and broken at each vibration, the primary wire of a Du Bois Reymond's induction-coil is also included in the circuit; on connecting the electrometer with the secondary coil placed at an appropriate distance the mercury always moves to the point of the tube whatever be the direction of the current.

F. J. M. PAGE
Physiological Laboratory, University College,
London, February 2

NOTE.—On February 4 Prof. Graham Bell kindly placed at my disposal a telephone much more powerful than any of those I had previously used. On speaking to this instrument, the electrometer being in the circuit, movements of the mercury column as considerable as those in Experiment 9 were observed.—F. J. M. P.

CHEMISTRY AND ALGEBRA

I may not be wholly without interest to some of the readers of NATURE to be made acquainted with an analogy that has recently forcibly impressed me between branches of human knowledge apparently so dissimilar as modern chemistry and modern algebra. I have found it of great utility in explaining to non-mathematicians the nature of the investigations which algebraists are at present busily at work upon to make out the so-called *Grundformen* or irreducible forms appurtenant to binary quantics taken singly or in systems, and I have also found that it may be used as an instrument of investigation in purely algebraical inquiries. So much is this the case that I hardly ever take up Dr. Frankland's exceedingly valuable "Notes for Chemical Students," which are drawn up exclusively on the basis of Kekulé's exquisite conception of *valence*, without deriving suggestions for new researches in the theory of algebraical forms. I will confine myself to a statement of the grounds of the analogy, referring those who may feel an interest in the subject and are desirous for further information about it to a memoir which I have written upon it for the new *American Journal of Pure and Applied Mathematics*, the first number of which will appear early in February.

The analogy is between atoms and binary quantics exclusively.

I compare every binary quantic with a chemical atom. The number of factors (or rays, as they may be regarded by an obvious geometrical interpretation) in a binary quantic is the analogue of the number of bonds, or the *valence*, as it is termed, of a chemical atom.

Thus a linear form may be regarded as a monad atom, a quadratic form as a duad, a cubic form as a triad, and so on.

An invariant of a system of binary quantics of various degrees is the analogue of a chemical substance composed of atoms of corresponding *valences*. The order of such invariant in each set of coefficients is the same as the number of atoms of the corresponding *valence* in the chemical compound.

A co-variant is the analogue of an (organic or inorganic) compound radical. The orders in the several sets of coefficients corresponding, as for invariants, to the respective valences of the atoms, the free valence of the compound radical then becomes identical with the degree of the co-variant in the variables.

The weight of an invariant is identical with the number of the bonds in the chemicograph of the analogous chemical substance, and the weight of the leading term (or basic differential) of a co-variant is the same as the number of bonds in the chemicograph of the analogous compound radical. Every invariant and covariant thus becomes expressible by a graph precisely identical with a Kekuléan diagram or chemicograph. But not every chemicograph is an algebraical one. I show that by an application of the algebraical law of reciprocity every algebraical graph of a given invariant will represent the constitution in terms of the roots of a quantic of a type reciprocal to that of the given invariant of an invariant belonging to that reciprocal type. I give a rule for the geometrical multiplication of graphs, i.e. for constructing a graph to the product of in- or co-variants whose separate graphs are given. I have also ventured upon a hypothesis which, whilst in nowise interfering with existing chemicographical constructions, accounts for the seeming anomaly of the isolated existence as "monad molecules" of mercury, zinc, and arsenic—and gives a rational explanation of the "mutual saturation of bonds."

I have thus been led to see more clearly than ever I did before the existence of a common ground to the new mechanism, the new chemistry, and the new algebra. Underlying all these is the theory of pure colligation, which applies undistinguishably to the three great theories, all initiated within the last third of a century or thereabouts by Eisenstein, Kekulé, and Peacock.

Baltimore, January 1 J. J. SYLVESTER

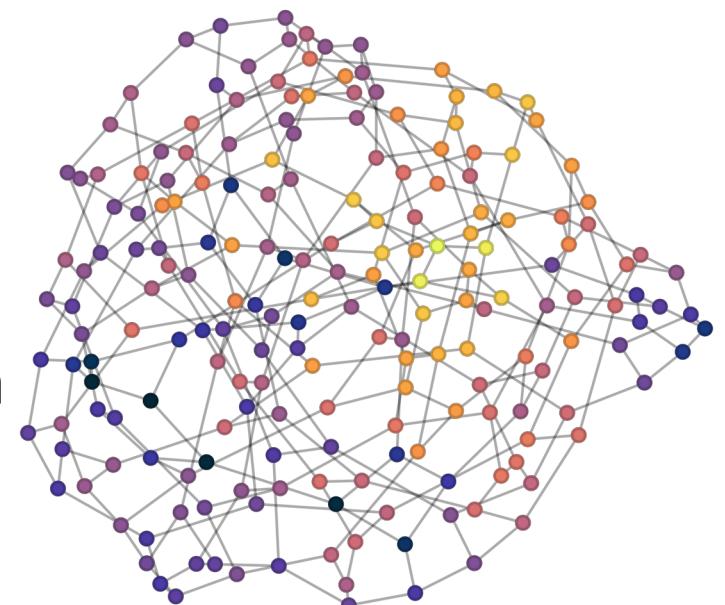
PALMEN ON THE MORPHOLOGY OF THE TRACHEAL SYSTEM

DR. PALMEN, of Helsingfors, has recently published an interesting memoir on the tracheal system of insects. He observes that although the gills of certain aquatic larve are attached to the skin very near to the points at which the spiracles open in the mature insects, and though spiracles and gills do not co-exist in the same segment, yet the point of attachment of the gills never exactly coincides with the position of the future spiracle. Moreover, he shows that even during the larval condition, although the spiracles are not open, the structure of the stigmatic duct is present, and indeed that it opens temporarily at each moult, to permit the inner tracheal membrane to be cast, after which it closes again. In fact, then, he urges, the gills and spiracles do not correspond exactly, either in number or in position, and there can therefore be between them no genetic connection. He concludes that the insects with open tracheæ are not derived from ancestors provided with gills,

Why arbitrary geometry?

- For many problems geometry is instance specific:
 - Circuits, inference and counting problems...
- Easier transference of methods to new geometry
 - Treat geometry just as another input rather than
 - Might be that ansatz geometry changes even in physical geometry remains the same

$$Z = \sum_{\{x\}} \prod_i T_i[\bar{x}_i]$$



Exact vs approximate complexity

- We know exact is one of the hardest computational problems (generally much harder than NP problems)
- Assuming TN contraction $\sim \#P$ =complete, interesting result:
 - "... every $\#P$ -complete problem either has an FPRAS, or is essentially impossible to approximate ..."

Fully Polynomial Randomized Approximation Scheme – algorithm runs in time scaling polynomially with **both size and error**

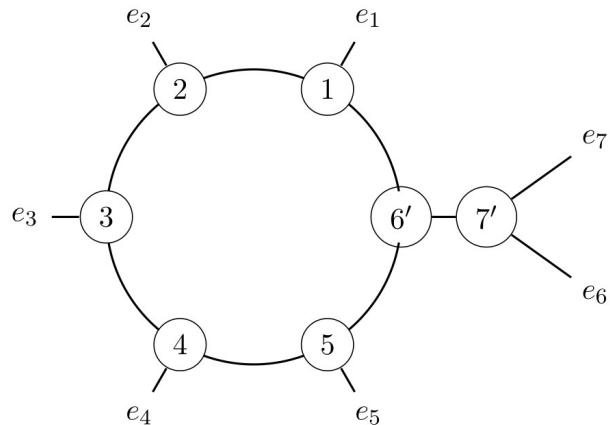
Random generation of combinatorial structures from a uniform distribution

Mark R. Jerrum ^a, Leslie G. Valiant ^{b, *}, Vijay V. Vazirani ^{c, **}

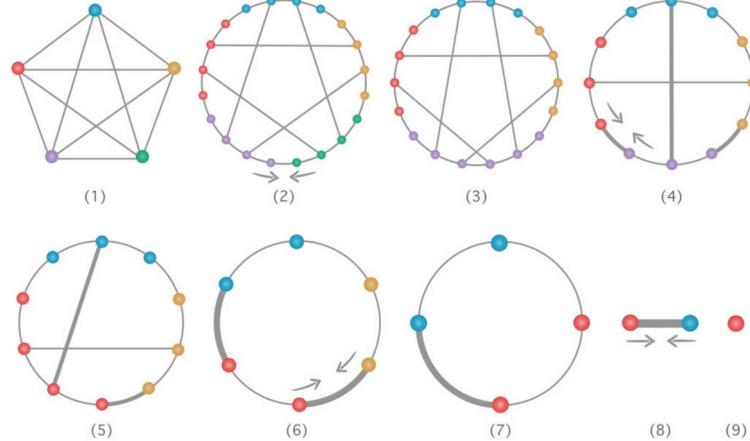
Existing Approaches

Automatic Contraction of Unstructured Tensor Networks

Adam S. Jermyn¹,



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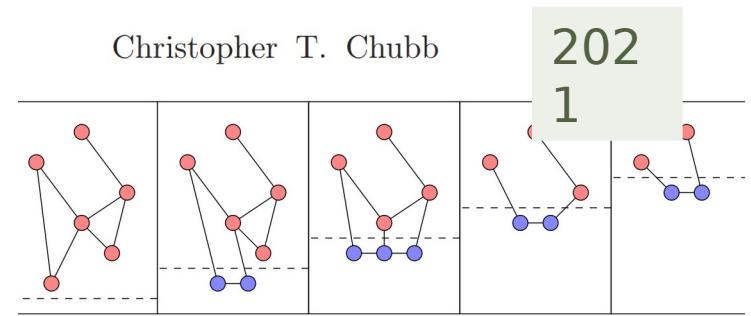


Contracting Arbitrary Tensor Networks: General Approximate Algorithm and Applications in Graphical Models and Quantum Circuit Simulations

Feng Pan,^{1,2,*} Pengfei Zhou,^{1,2,*} Sujie Li,^{1,2,*} and Pan Zhang^{1,3,4,†}

General tensor network decoding of 2D Pauli codes

Christopher T. Chubb



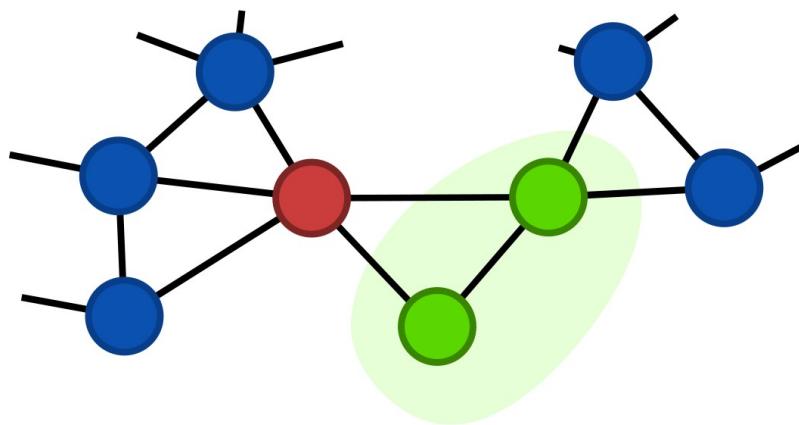
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Planar contractor for decoding quantum error correcting codes (only low accuracy required)

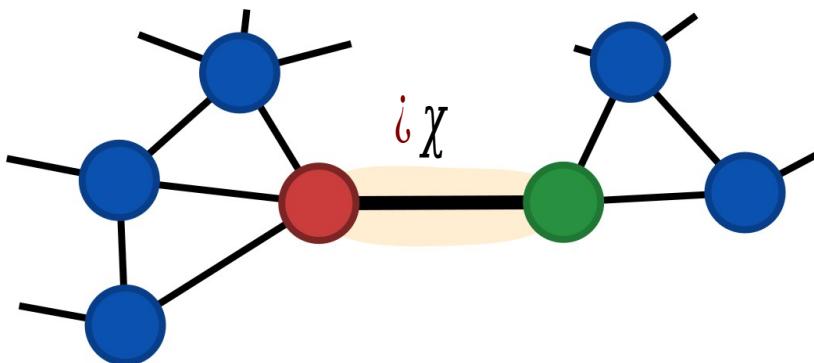
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General strategy / algorithm

- Don't keep boundary (intermediate tensors) in specific ansatz, just let it be defined by contraction tree, 'simply' compress whenever we can as we go along:



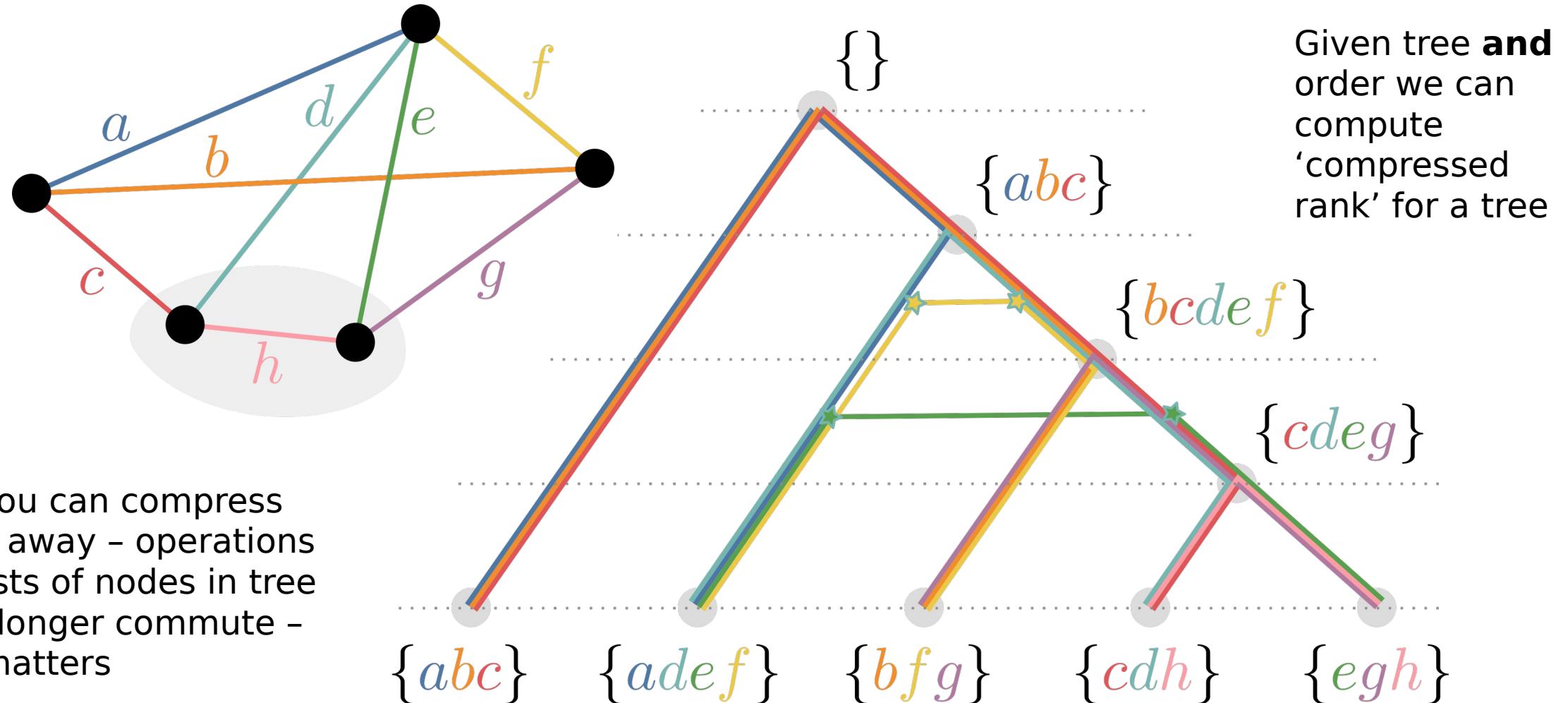
Contract according to
series of merges (the
tree)



Compress whenever
shared bonds grow beyond

1. Take **TN geometry**
2. Find **contraction tree**
3. Find **ordering** for tree
4. Choose **gauging** method
5. Choose **compression** method

Cost: Tree Ordering & ‘Compressed rank’

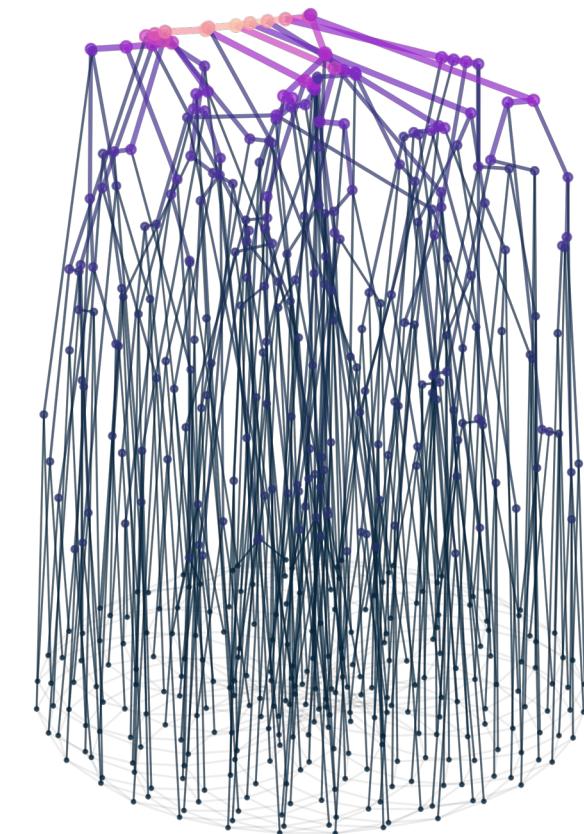


Compressable Contraction Trees

- Want to generate ‘compressable’ trees
- Is harder task than exact trees...
- But we can take Bayesian optimization approach:
 1. Supply a few heuristic, tunable algorithms
 2. Based on compressed cost, let an optimizer learn to sample good candidates

TN Contraction Tree - Compressed Greedy

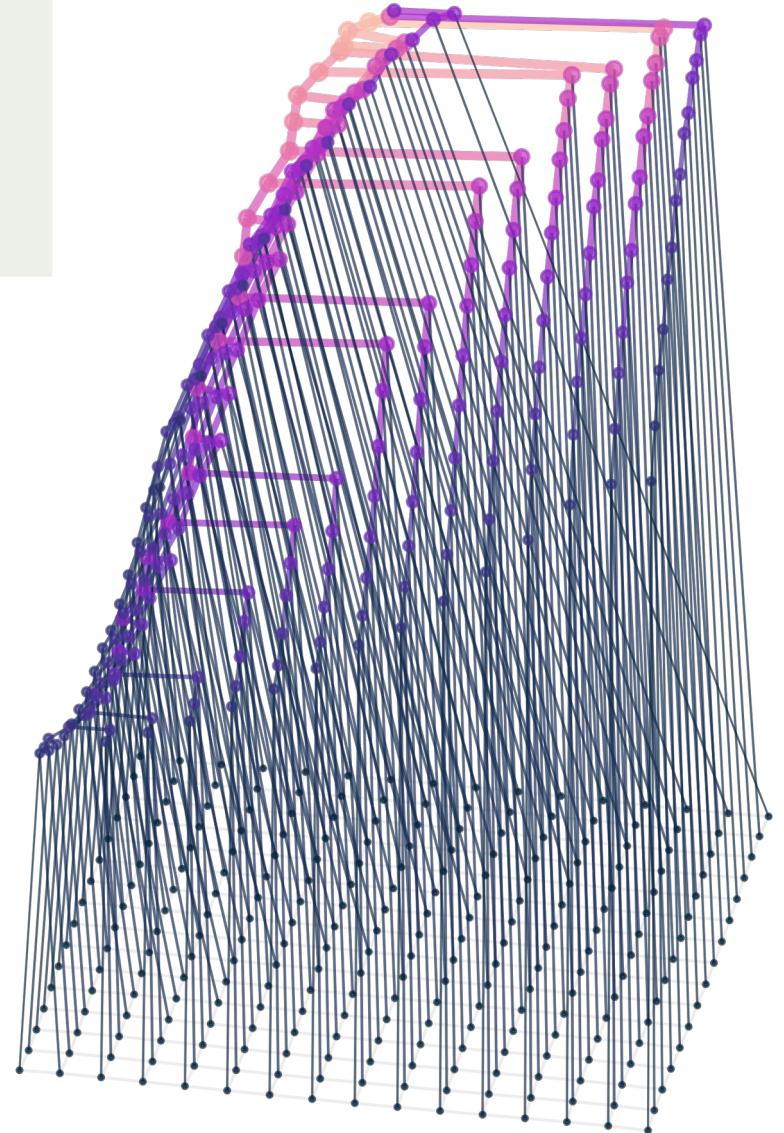
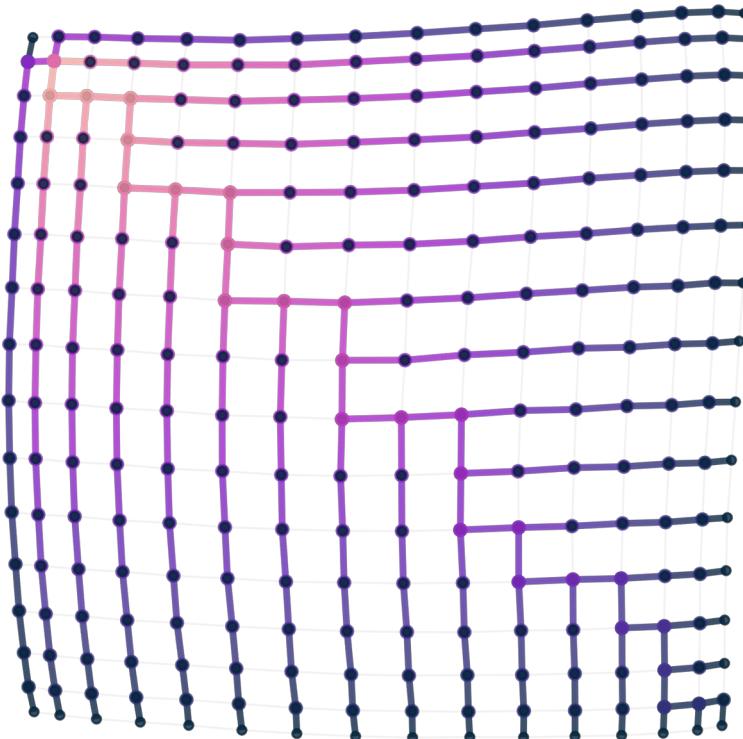
- Generic approach, assess all possible contractions, and greedily select based on
 1. Compressed output size
 2. Centrality
 3. Subgraph size
 4. ...
- Can't see beyond a single move



TN Contraction Tree - Span 'Boundary Style'

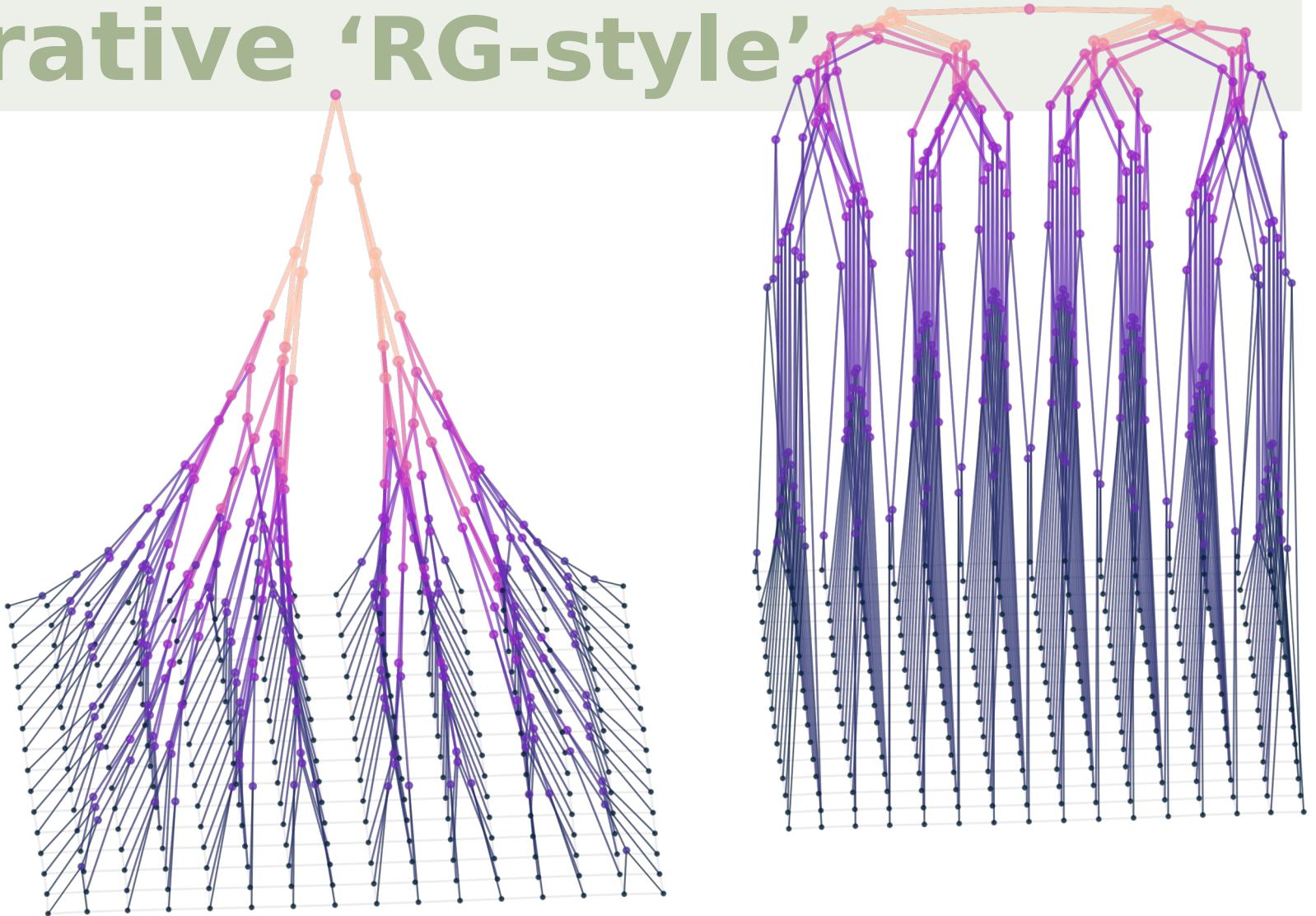
Generate tree from
span of original graph:

- Feed a few heuristics like centrality into span generator, then check quality and repeat
- Restricted set of contraction trees, works well for regular lattices

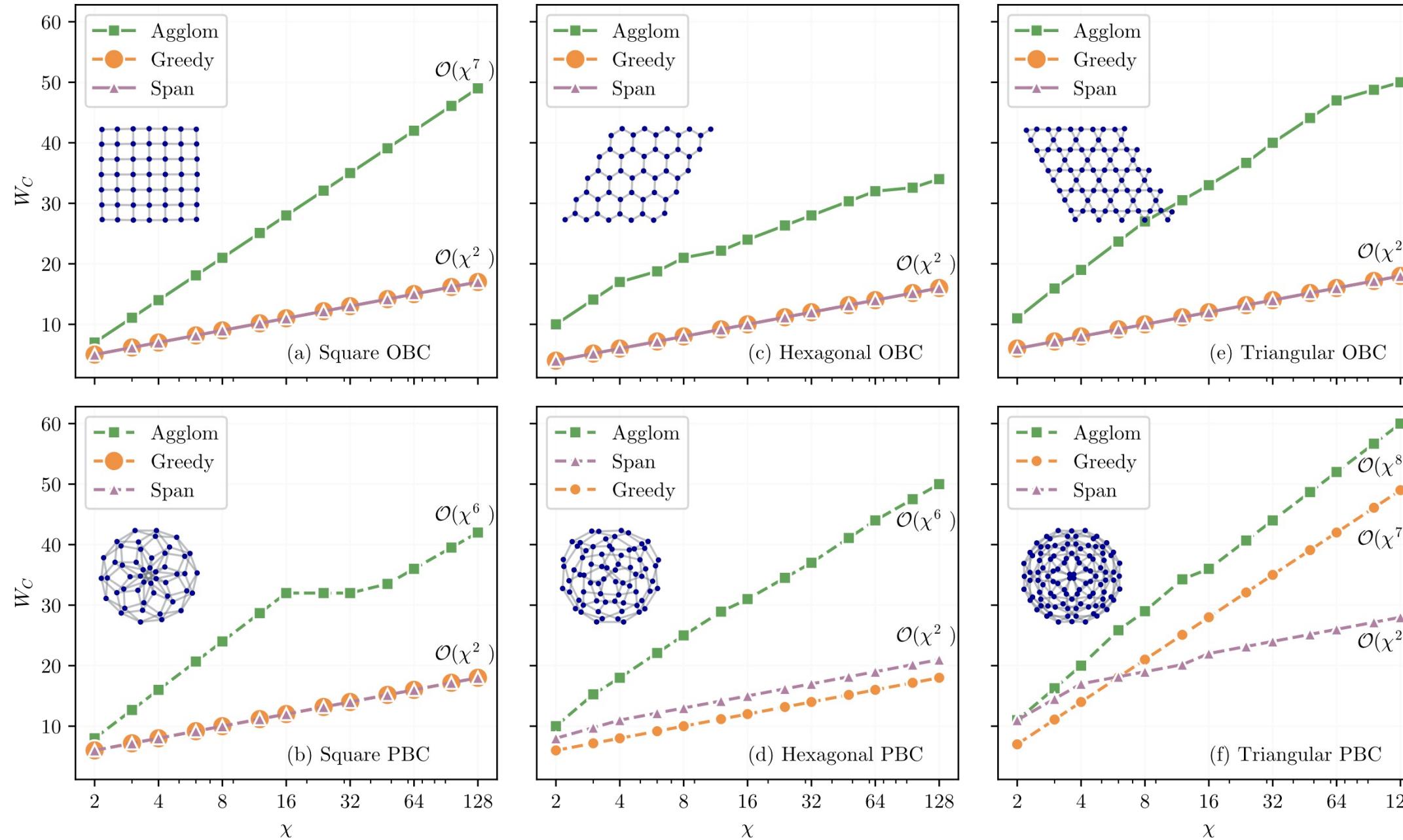


TN Contraction Tree - Agglomerative ‘RG-style’

- Generate tree by grouping together small communities of tensors, compressing between these essentially coarse grains the lattice
- Seems good for cases where no obvious boundary or regularity



Results - compressed trees vs 2D geometry

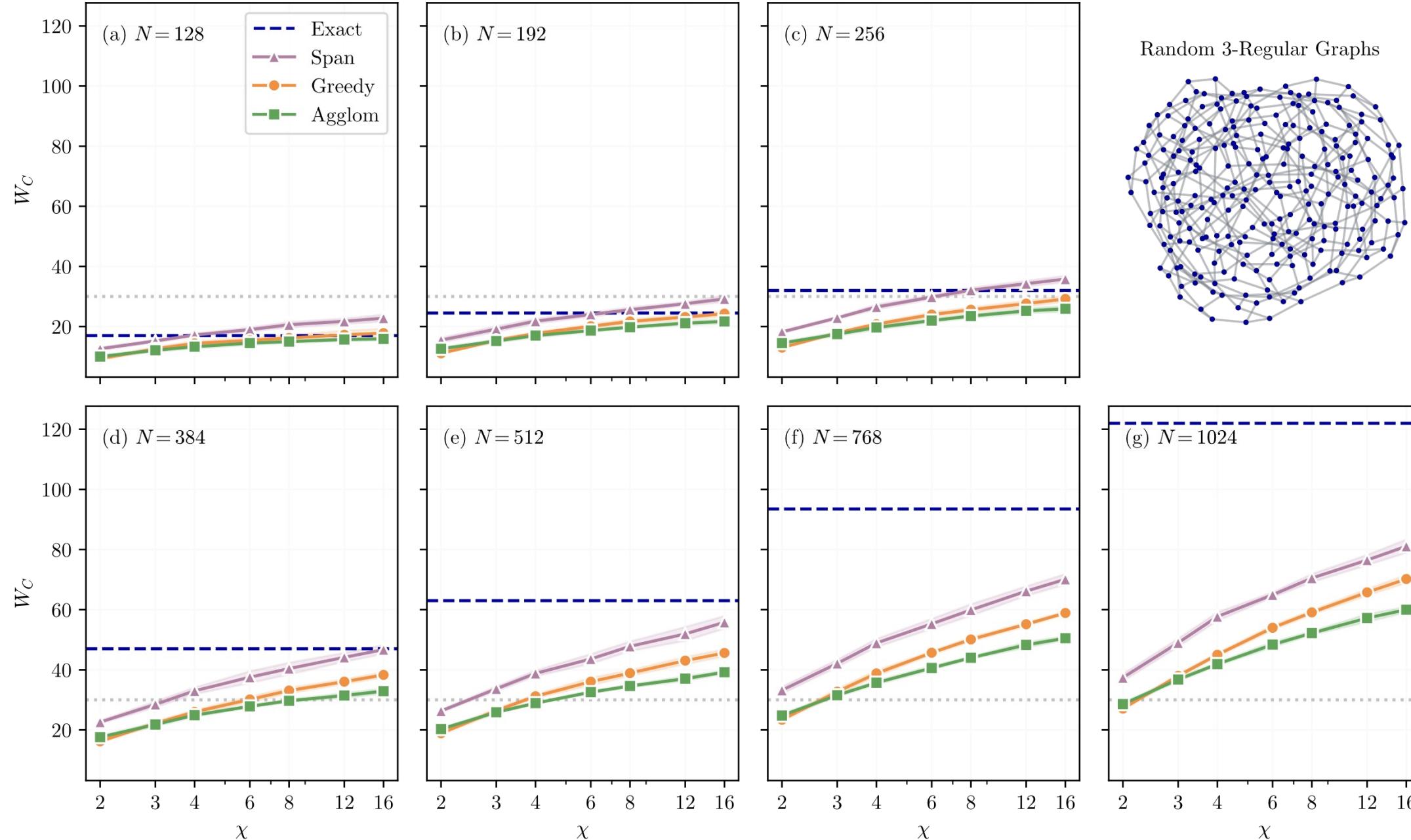


Sensible scaling with found in all cases

Spanning tree and greedy approach both good but neither 'all-round' winner

Agglomerative (partitioning) approach clearly bad in terms of scaling with

Results - compressed trees vs random regular

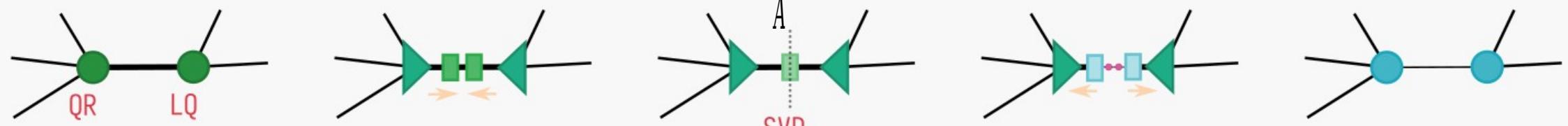


Agglomerative method comes into its own – suitable for geometries with no natural boundary?

Whilst can compressed contract larger TNs than exactly can't scale indefinitely with size – connectivity is issue.

Compression Strategies

basic reduced compress

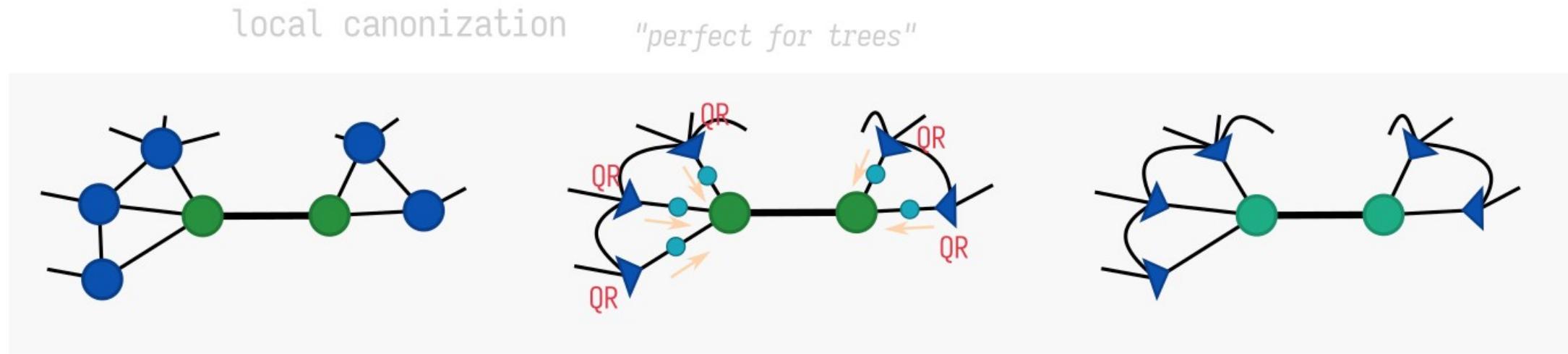


- However, if we assume global TN is scalar and has loops, important to consider we only have half the information:

$$Z = \text{Tr}(AB) \approx \text{Tr}(\tilde{A}B)$$

i.e. approximating Z has no guarantees on accuracy given codependence on environment

Simplest approach - local 'canonization'



- Can understand two ways:
 - Assuming Simple Update style gauge, like absorbing the product envs
 - Treat local tree as boundary-ansatz - orthogonalize it

Optimal approach - 'full-bond' gauging

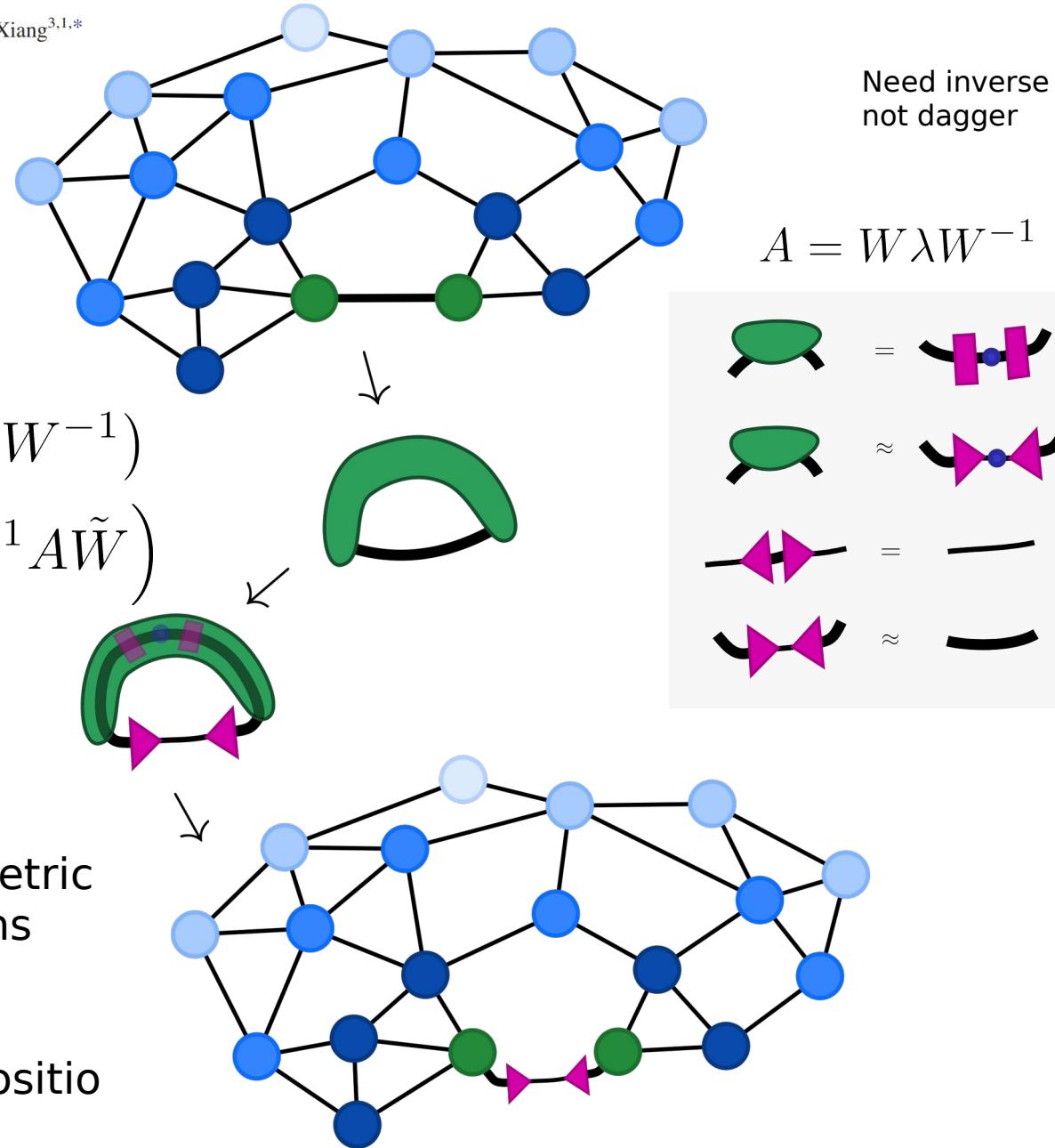
What if we can (approximately) contract the full environment of a bond?

Want to insert a pair of isometries that *would* act as gauges if full rank, that preserve the trace value

Trace value is not guaranteed to be positive

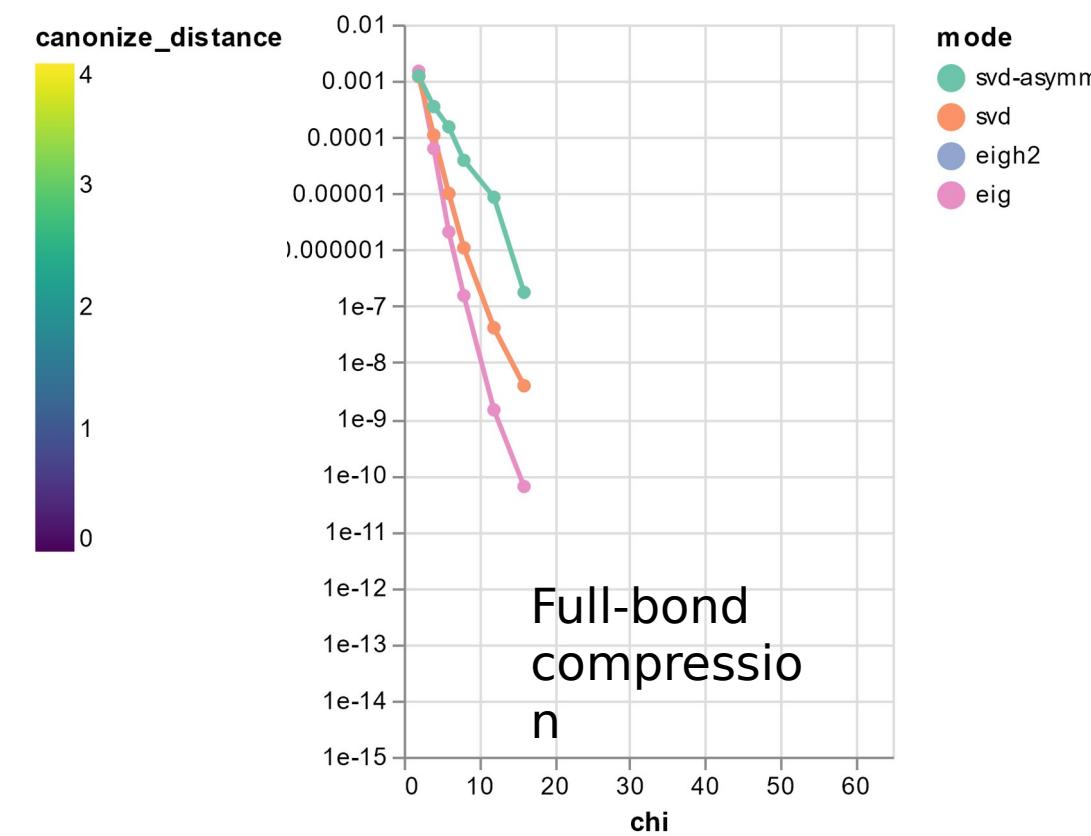
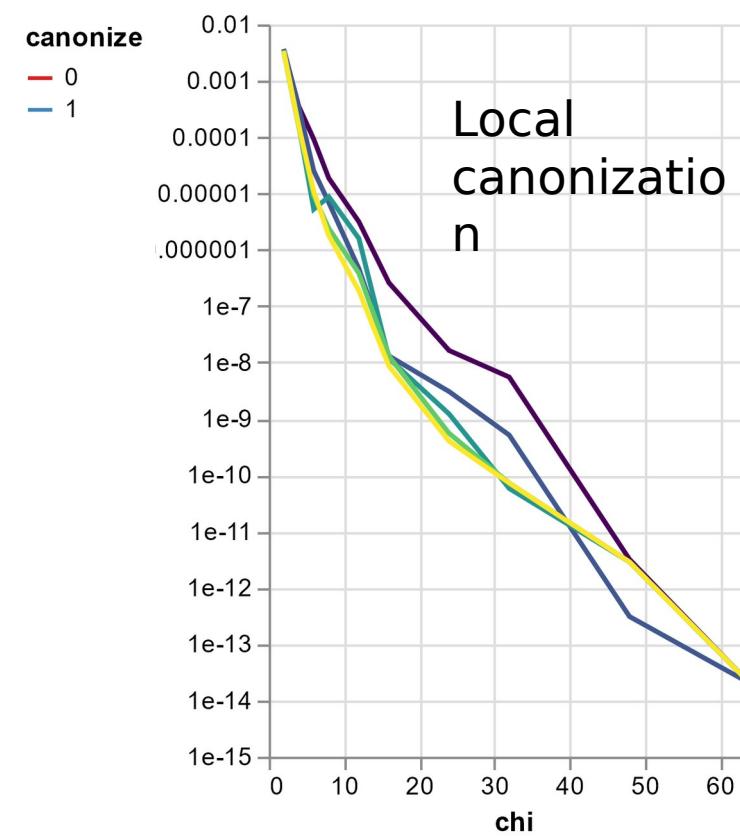
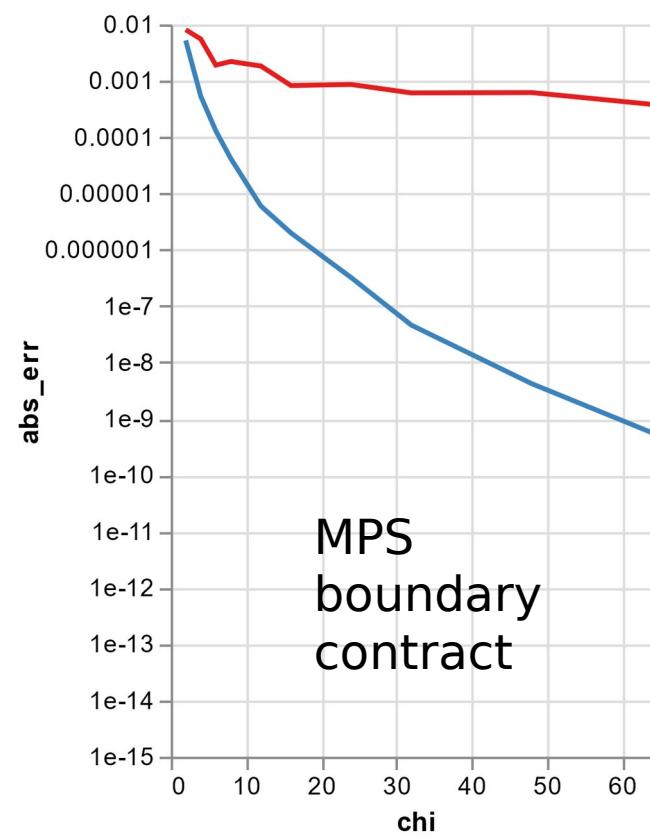
$$\begin{aligned} z &= \text{Tr}(A) \\ &= \text{Tr}(AWW^{-1}) \\ &\approx \text{Tr}(\tilde{W}^{-1}A\tilde{W}) \end{aligned}$$

Various symmetric decompositions possible, non-Hermitian eigendecomposition basic choice

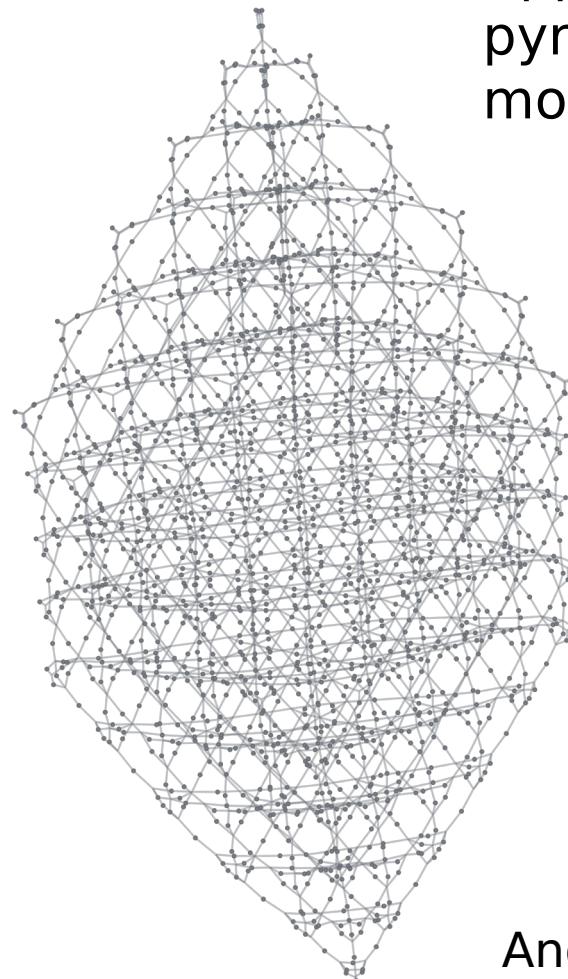


Compression type vs accuracy

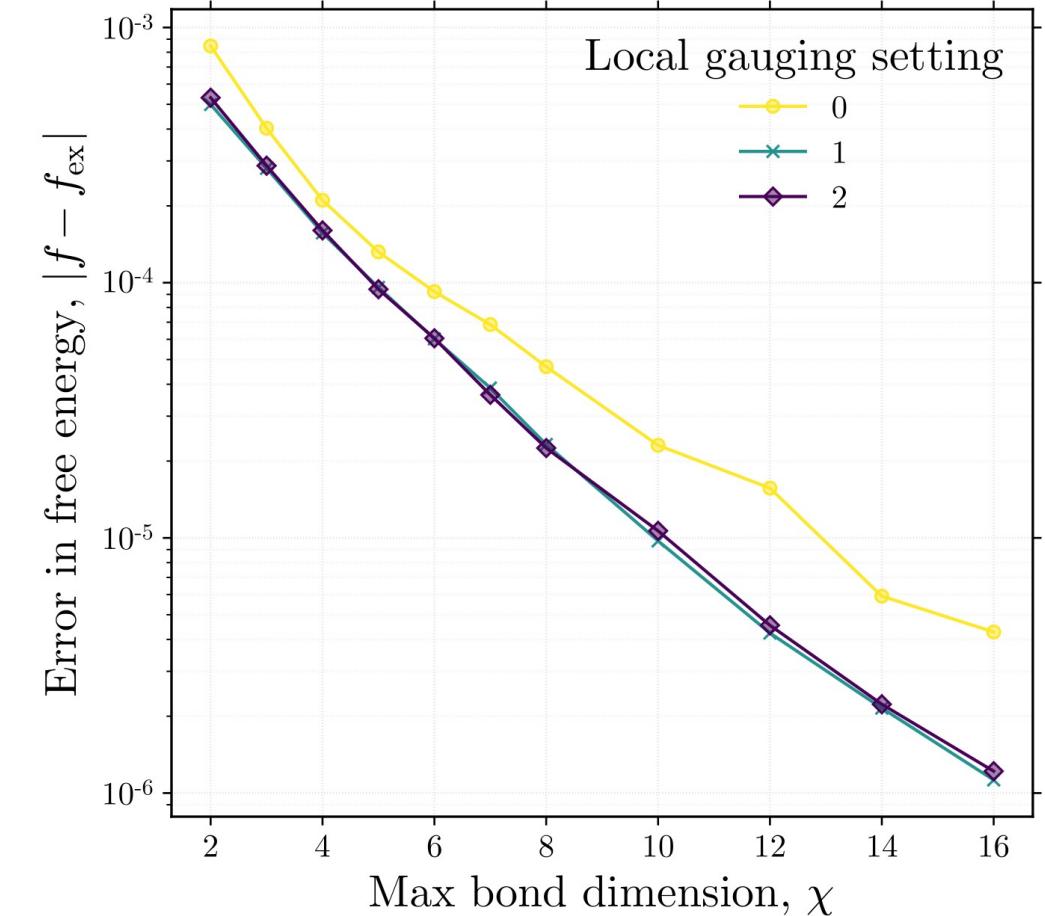
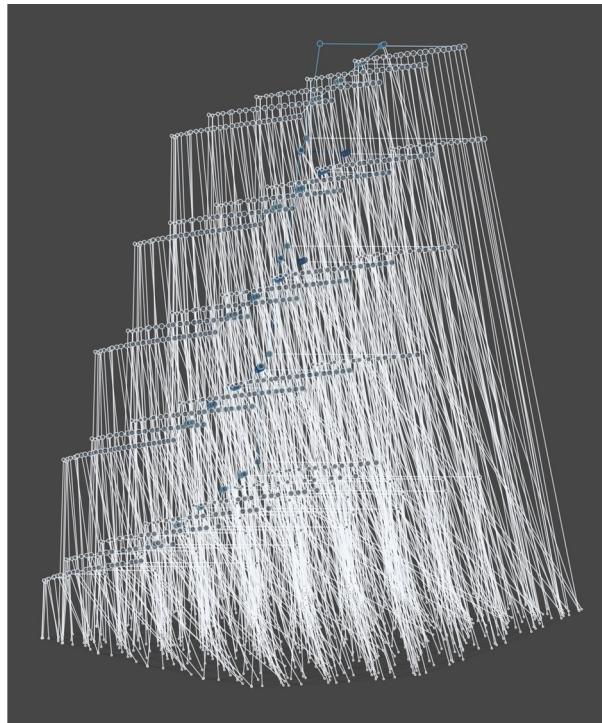
For simple cases (critical classical 2D Ising model here), compression works as expected:



Example high accuracy for complex geometry



Approximately critical 3D
pyrochlore (frustrated) Ising
model



And seemingly well for various local tensors: classical spins, dimer covers, SAT, XORSAT

Remaining to-dos / outstanding questions

- Different trees produce different scalings with , however if we plot *accuracy vs* are methods roughly similar?
 - Unlikely – but would be nice / good to know scale of variance
- How practical for 3D quantum systems with
 - Can we embed real geometry in sparser graph with many tensors for each site?
- How best to integrate into quantum algorithms with optimization
 - ‘Single-shot’ contraction on its own is insufficient / inefficient

