Let's consider the nonlinear system

$z_k = f_k(z_{k-1}\ , u_{k-1}\ ,w_{k-1})+w_{k-1}$ $\qquad w_k \sim (0, Q_k)$

$y_k = Cz_k + v_k$ $\qquad v_k \sim (0, R_k)$

$A_k = \frac{\partial f_k}{\partial z_k}|_{z=\hat{z}_{k-1}}$

$z_k \in R^{q=12}$

$y_k \in R^{m=4}$

Objective: Stability Analysis for the EKF algorithm based on K.Reif paper in 1999

First, after running the program it is necessary to store the $P_k\ \hat{z}_k\ , z_k,\ k = 1,2, \dots, 1000$ variables, then according to the theorem 3.1 we need to satisfied the equations from 28 to 34

Theorem 3.1: Consider a nonlinear stochastic system given by (1), (2) and an extended Kalman filter as stated in Definition 3.1. Let the following assumptions hold.

1) There are positive real numbers $\bar{a}, \bar{c}, \underline{p}, \bar{p} > 0$ such that the following bounds on various matrices are fulfilled for every $n \geq 0$:

$$\|A_n\| \leq \bar{a} \tag{28}$$
$$\|C_n\| \leq \bar{c} \tag{29}$$
$$\underline{p}I \leq P_n \leq \bar{p}I \tag{30}$$
$$\underline{q}I \leq Q_n \tag{31}$$
$$\underline{r}I \leq R_n. \tag{32}$$

For equation (28) :

$\bar{a} = Max(||A_k||)$ $\quad k = 1,2, \ldots., 1000$

For equation (29) :

$\bar{c} = Max(||C_k||)$ $\quad k = 1,2, \ldots., 1000$ $\quad Note: C_k = C$

For equation (30) :

Since the P_k matrix may not be diagonal, so it is needed to be diagonal also, this matrix is symmetric and linear algebra is proved that by using eigenvalues and eigenvectors it can be converted to diagonal matrix

Let X be the matrix with eigenvectors as its columns:

$P_k{}^d = X^{-1} P_k X \quad k = 1,2, \ldots.,1000$

ppp: The estimation error covariance matrix diagonalization (PPP = $P_1{}^d, P_2{}^d, \ldots, P_{1000}{}^d$)

$\underline{p} = \min\left(min\left(diag(P_k{}^d)\right)\right) \quad k = 1,2, \ldots,1000$

$\bar{p} = \max\left(max\left(diag(P_k{}^d)\right)\right) \quad k = 1,2, \ldots,1000$

Question1: In some cases the elements of the principle diameter of diagonal matrix might be negative and it is violated the assumption $\underline{p} > 0$, thus what is your idea about that?

For equation (31) :

$\bar{q} = min(||Q_k||)$ $\quad k = 1,2, \ldots., 1000$

For equation (32) :

$\bar{r} = min(||R_k||)$ $\quad k = 1,2, \ldots., 1000$

Assumption 2: There is no problem

Assumption :

$$\kappa_\varphi = \max_{1\le i\le q} \sup_{z\in\mathcal{K}} ||\text{Hess } f_i(z)|| \qquad (100)$$

$$\kappa_\chi = \max_{1\le i\le m} \sup_{z\in\mathcal{K}} ||\text{Hess } h_i(z)||. \qquad (101)$$

Question2: Do you think the meaning of the compact subset K is exactly the subset of $Z = z_1, z_2, \ldots, z_{1000}$?

For obtaining the ϵ_φ, ϵ_χ we need to use below definition:

$$\epsilon_\varphi = \epsilon_\chi = \min\left(||z_k - \hat{z}_k||\right) \qquad k = 1,2, \ldots,1000$$

Question3: Do you think ϵ_φ, ϵ_χ are equal? Or they are dependent to k_φ, k_χ parameters?

Last but not least, I would be extremely grateful if you send me some related m.files or Matlab codes as a sample.

Thank you so much