Day 1 — Week 4 Chapter 13: Fluids and Density

13.1 Fluids and Density

What is a fluid?

Fluids are materials that **flow**: they deform continuously under shear stress.

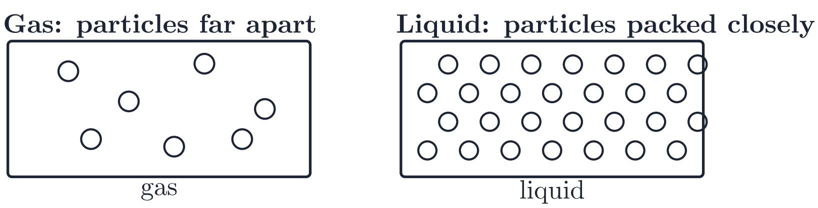
- **Gases and liquids** are fluids.
- Many introductory problems treat liquids as **incompressible**: their density is approximately constant.

Compressibility (idea)

For an **incompressible** fluid, density does not change appreciably with pressure:

$$\Delta\rho \approx 0 \quad \text{even if} \quad \Delta P \neq 0.$$

(Real fluids compress slightly, but often the effect is negligible for liquids.)



Density

Definition

$$\rho \equiv \frac{m}{V}, \qquad \text{so} \qquad m = \rho V.$$

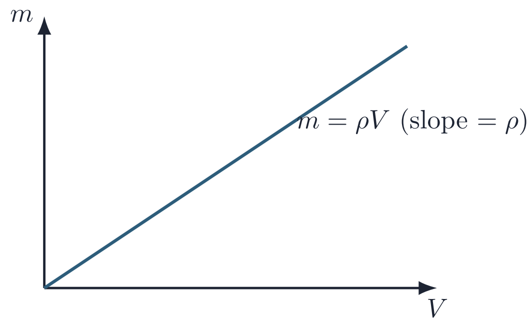
If ρ is constant (incompressible), then $m \propto V$.

Unit conversion example (water)

Water has density approximately

$$\rho_{\text{water}} \approx 1\,\text{g/cm}^3 = 1000\,\text{kg/m}^3.$$

Therefore, 1 m^3 of water has mass $\approx 1000\,\text{kg}$.



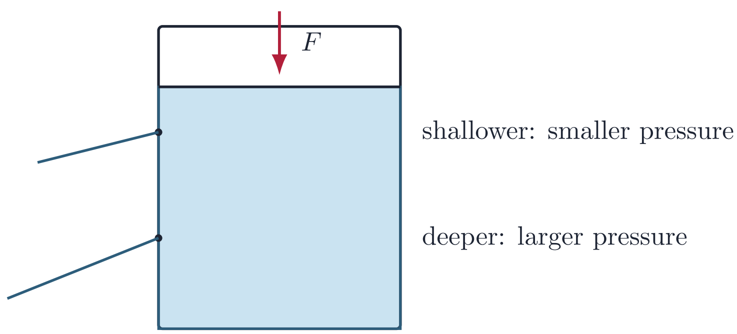
Pressure

Definition

$$P \equiv \frac{F}{A}.$$

SI units:

$$1\,\mathrm{Pa} = 1\,\mathrm{N/m^2} = 1\,\mathrm{kg\,m^{-1}\,s^{-2}}.$$



Pressure increases with depth

Key idea: pressure in a static fluid

In a fluid at rest, pressure depends mainly on **depth** (and density), not on horizontal position.

Hydrostatic Pressure (Pressure vs. Depth)

Result

At depth d below the surface of a fluid open to the atmosphere,

$$P = P_0 + \rho g d,$$

where P_0 is the pressure at the surface (often P_{atm}).

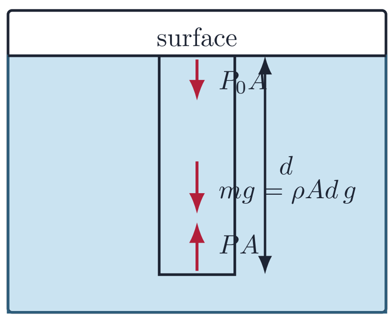
Quick derivation (force balance on a fluid column)

Consider a vertical column of fluid of cross-sectional area A and height d.

$$\sum F_y = 0 \quad \Rightarrow \quad P_{\text{bottom}}A - P_0 A - mg = 0.$$

With $m = \rho V = \rho(Ad)$, we get

$$P_{\text{bottom}} = P_0 + \rho g d.$$

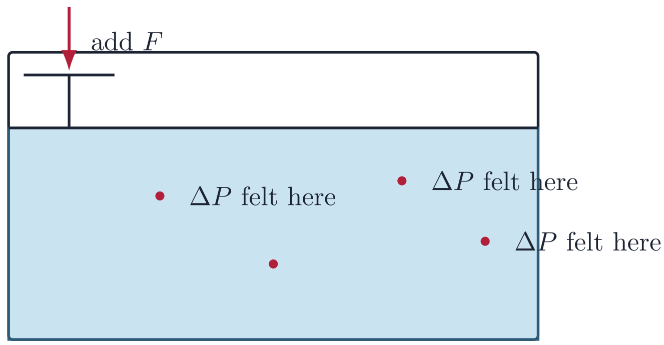


Static equilibrium: $PA = P_0 A + \rho A d g$

Pascal's Principle

Statement

If an **external pressure increase** ΔP is applied to a confined fluid, that increase is transmitted **undiminished** to **all points** in the fluid and to the container walls.



All points in the fluid experience the same pressure increase ΔP

Example: Pressure at a Point in a Connected Fluid

Setup

A static fluid open to the atmosphere: pressure at depth d is

$$P = P_{\text{atm}} + \rho g d.$$

Only the **vertical depth** below the free surface matters (not the shape of the container).

Numerical example (illustrative)

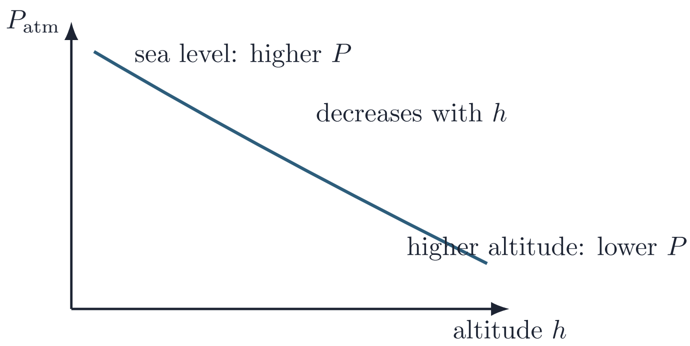
If $d = 0.60\,\text{m}$ below the water surface and $\rho = 1000\,\text{kg/m}^3$, $g = 9.8\,\text{m/s}^2$:

$$P \approx P_{\text{atm}} + (1000)(9.8)(0.60) \approx 1.0 \cdot 10^5\,\text{Pa} + 5.9 \cdot 10^3\,\text{Pa} \approx 1.06 \cdot 10^5\,\text{Pa}.$$

Atmospheric Pressure

Basic facts

- Atmospheric pressure generally **decreases with altitude**.
- Air tends to flow from **high pressure** to **low pressure**.
- Weather connection (qualitative): low pressure is often associated with stormy conditions; high pressure often with clearer weather.

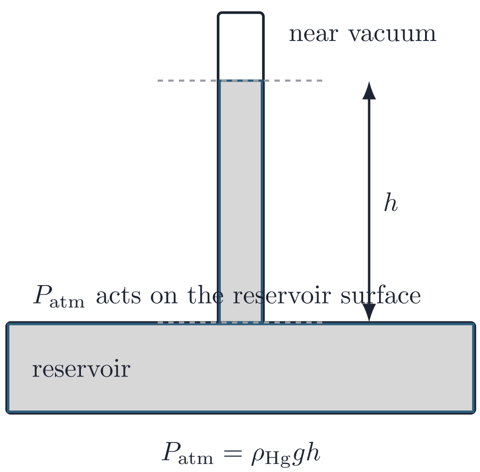


Barometer (Mercury Column)

Concept

A mercury barometer balances atmospheric pressure with the hydrostatic pressure of a mercury column:

$$P_{\text{atm}} = \rho_{\text{Hg}} g h.$$



Typical value

With $\rho_{\text{Hg}} \approx 13\,600\,\text{kg/m}^3$ and $P_{\text{atm}} \approx 1.01 \cdot 10^5\,\text{Pa}$,

$$h \approx \frac{P_{\text{atm}}}{\rho_{\text{Hg}} g} \approx 0.76\,\text{m} \approx 76\,\text{cm}.$$

Manometers (U-tubes)

Rule for static connected fluids

At the **same horizontal level** in the **same connected fluid**, pressures are equal.