Politecnico di Torino

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Operations Research: theory and applications to networking

Lesson 1 – Warm-Up

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ICT4SS-CE

Mathematical Optimization

= Select the “best” element from a set of available alternatives

Simplest case: minimizing a real function

- choosing input values from an allowed input set
- computing the value of the function

Area of Applied Mathematics

Structure of an Optimization Problem

Minimize or Maximize

Objective Function

Subject to

Constraints

With Variables

Variables

Parameters vs Variables

Parameters

Constant terms
The input of the program

Variables

What we want to optimize
The output of the program

Taxonomy of Mathematical Optimization

Convex Programming

- Objective function and constraints set are convex
- The variables are continuous
- Includes linear programming

Linear Programming

- Objective function and constraints are linear

Integer Linear Programming

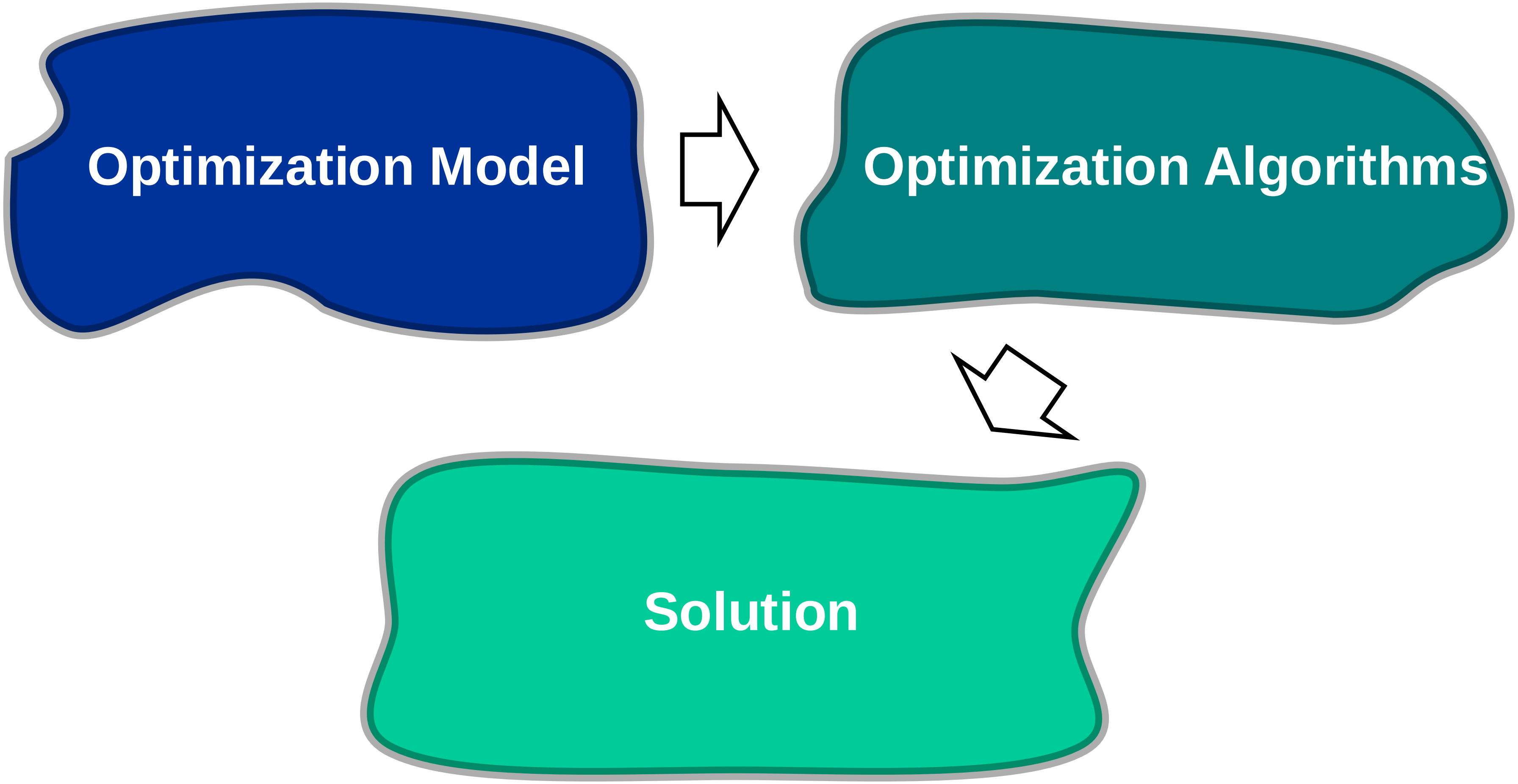
- Some variables take INTEGER values

  example c = {0,1,2,3,4}

General Non-Linear Programming

- Non-linear objective functions and constraints

Building Blocks of Optimization



Optimization & Networks

Telecommunication Networks are one of the most important application fields for optimization

Internet: moves information from one point to another in an efficient way

Choosing how to move traffic in the network (routing)

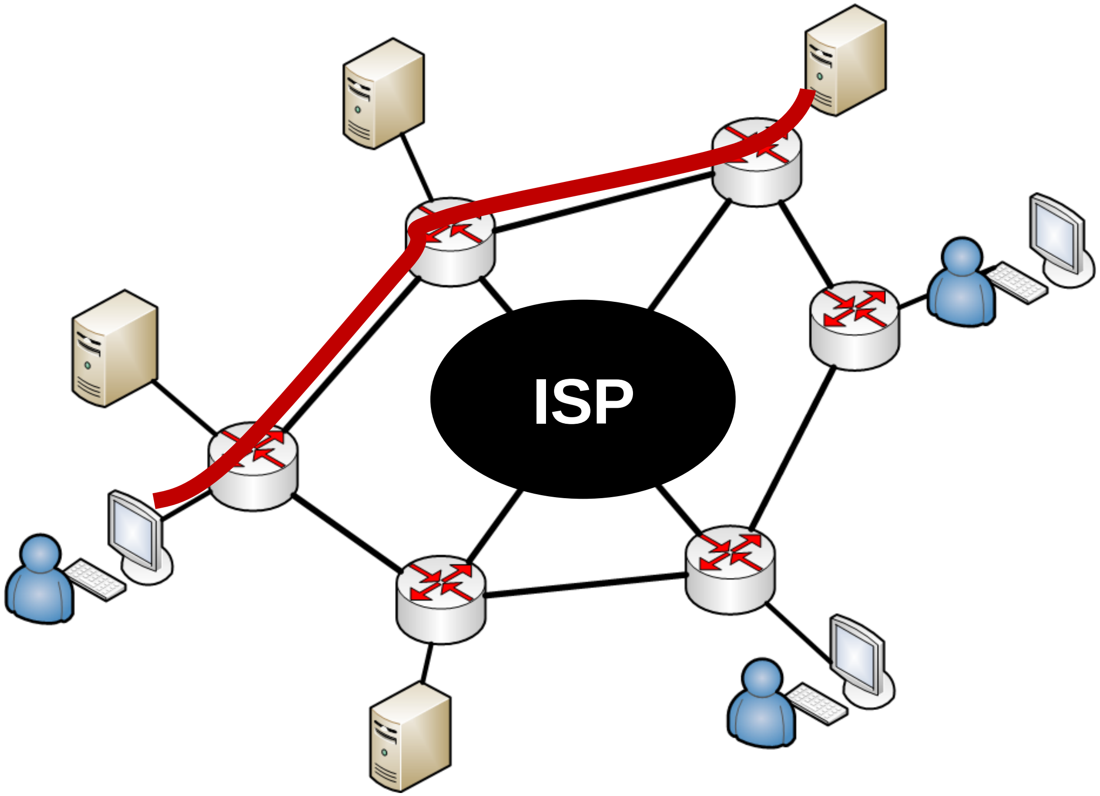
Dimensioning the network resources (design)

Guaranteeing user performance (Quality of Service)

Example I: routing

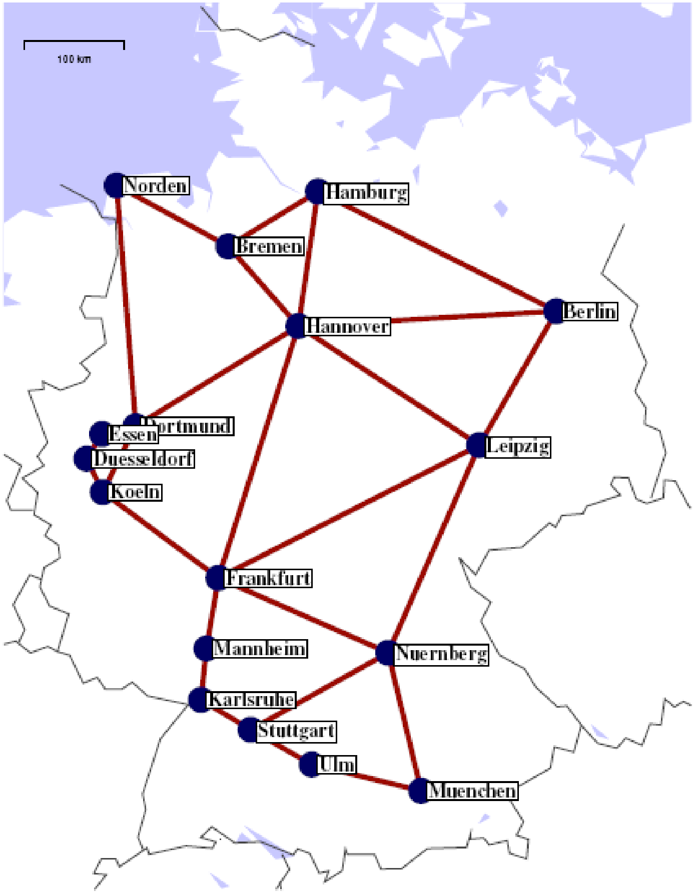
Route traffic between servers and users over an Internet Service Provider Network.

Choose the best route that minimizes the network delay.



Example II: design

Design the Internet Service Provider Core Network



IP-over-WDM architecture

Minimize the Capital Expenditures (CAPEX)

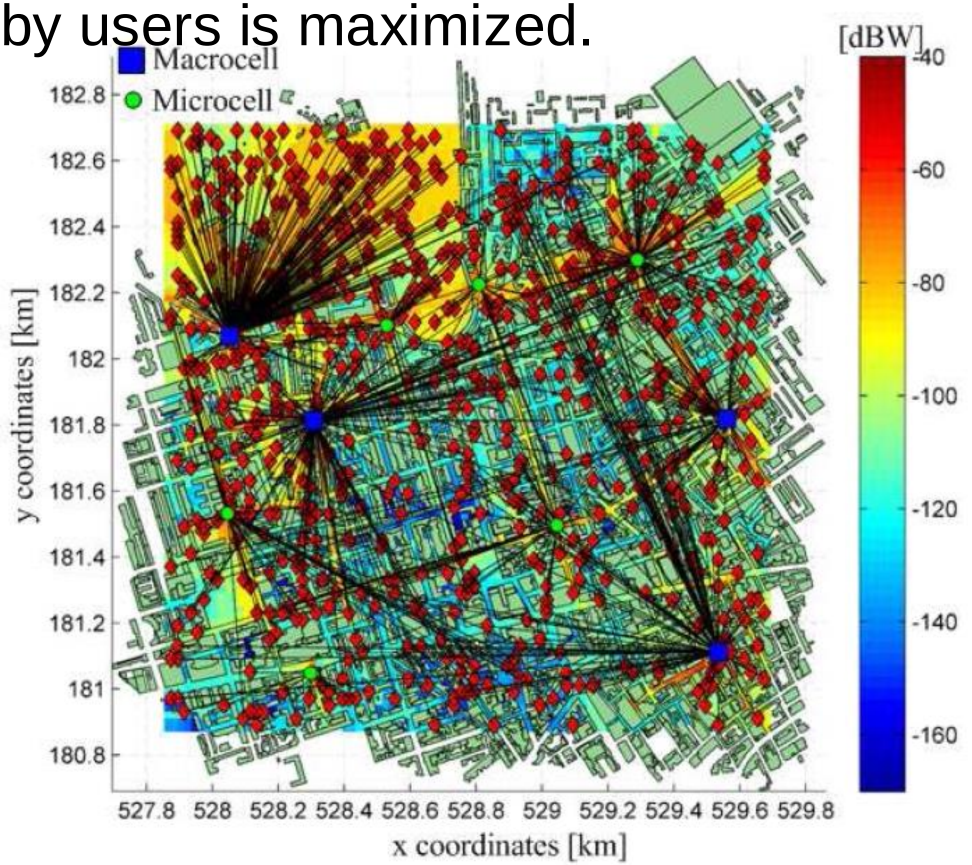
How many fibers on physical links?

How many routers?

Example III (Quality of Service)

Associate mobile phones to Base Stations in a cellular network

Best association so that the Quality of Service experienced by users is maximized.



The decision process

Problem Definition

“I want to minimize the total delay in my network”

Mathematical model

Minimize

$$D_T = \sum_{(i,k)\in L} D_{ik}(f_{ik})$$

Computer Program

```
delay=minimize_delay(flows);
```

Optimization Solver

Solution

```
Minimized Delay = 100 ms
```

Optimization Solvers

IBM ILOG CPLEX Optimizer

Solution of IBM

Gurobi Optimization

Very Powerful

neos SOLVERS

Available on-line

MathWorks

Optimization Toolbox

FICO® Xpress Optimization Suite

The product of choice for leading companies like: AA AVIS Europe Honeywell Nestlé NFL

Used in this Lab

You can download the student version

Interaction with solvers

System Calls

- Use a standard language, like “C”, “Java”, or scripting language like “python”
- Functions to send/receive information from the solver
- Less easy to use
  - Many functions to be invoked
  - Interaction with the solver need to be managed

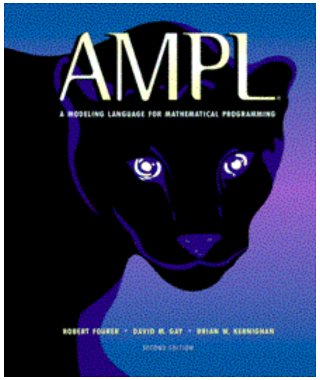
Modeling Language

- Use a proprietary language
- Compact structures to define the problem
- Easy to formalize complex problems
- Syntax is different from standard languages

Modeling Languages

AMPL

- A Modeling Language for Mathematical Programming
- Developed at Bell Laboratories



GAMS

- General Algebraic Modeling System

GAMS

MOSEL

- Used in this Lab together with XPress^{MP}

Mosel

What you will learn with the lab activities

Formulate an optimization problem with a mathematical formulation

Translate the mathematical formulation to a problem written in MOSEL language

Learn how to configure the XPress^{MP} solver to run the problem

Solve the MOSEL problem with XPress^{MP}

Analyze the results

Compare the optimal results with custom heuristics

Questions?