CSCI 520, Assignment 3

REZAUL AKRAM BARBHUIYA

USC ID: 4811-2972-49

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Please use as "ConstrainedParticleSystem.exe”

Implemented following as basic requirement :

Implemented a Constrained Particle System with a deformable chain-like structures. Each of the mass points are connected and a constraint is added to maintain the length to constant. The first particle is assumed to be fixed to the circular torus to give an impression of hanging from the object. The last mass is again constrained to stick to the torus only. We use gsl library to use the SVD(Singlar Matrix Decomposition) method to solve the linear system of equations.

Implemented BaumGarte stabilization with different values of alpha and beta.

Checked underwater effect by adding a damping force dependent of velocity.

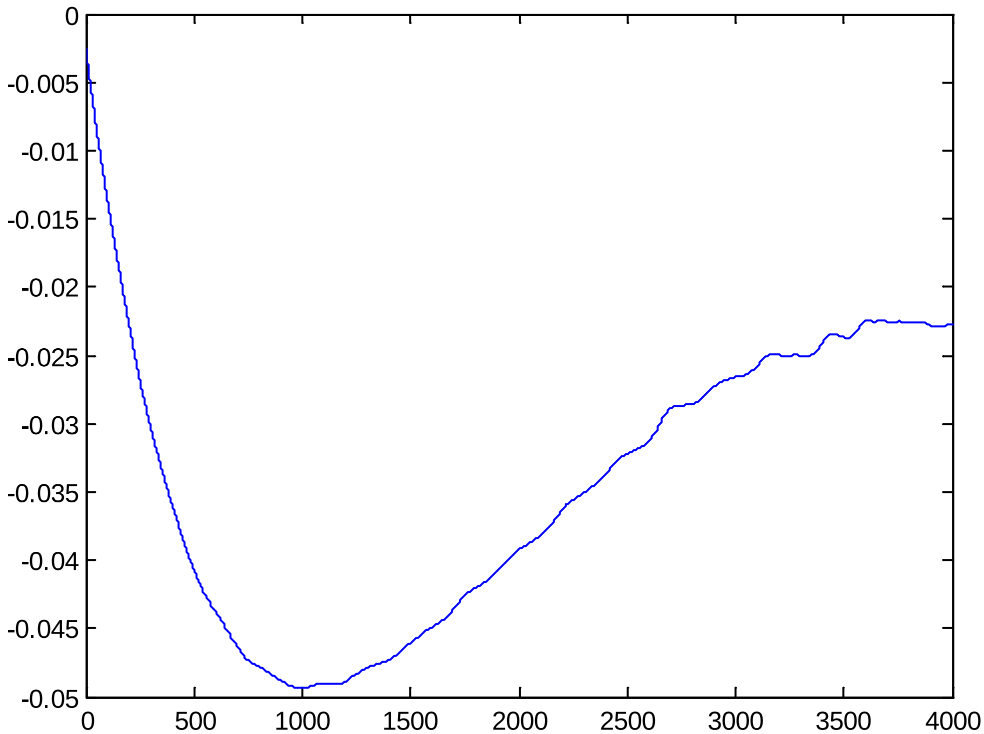
Implemented following for extra credit :

1. Dynamic no of beads(increase by pressing i, decrease by pressing o)
2. Rotate camera by pressing v
3. Add force to left and right by pressing a/d
4. Implemented RK4. Toggle between Euler and RK4
5. Single Frame (by pressing j)
6. Press c to remove the CRing constraint so that last bead is no longer attached to the torus
7. Print Frame rate and time elapsed like details on screen.
8. Add Textures to the system to have better looks.
9. Perlin Force
10. Plotted graphs of sum of squared error on constraints for different values of alpha and beta and underdamped and overdamped scenarios.

Other Features :

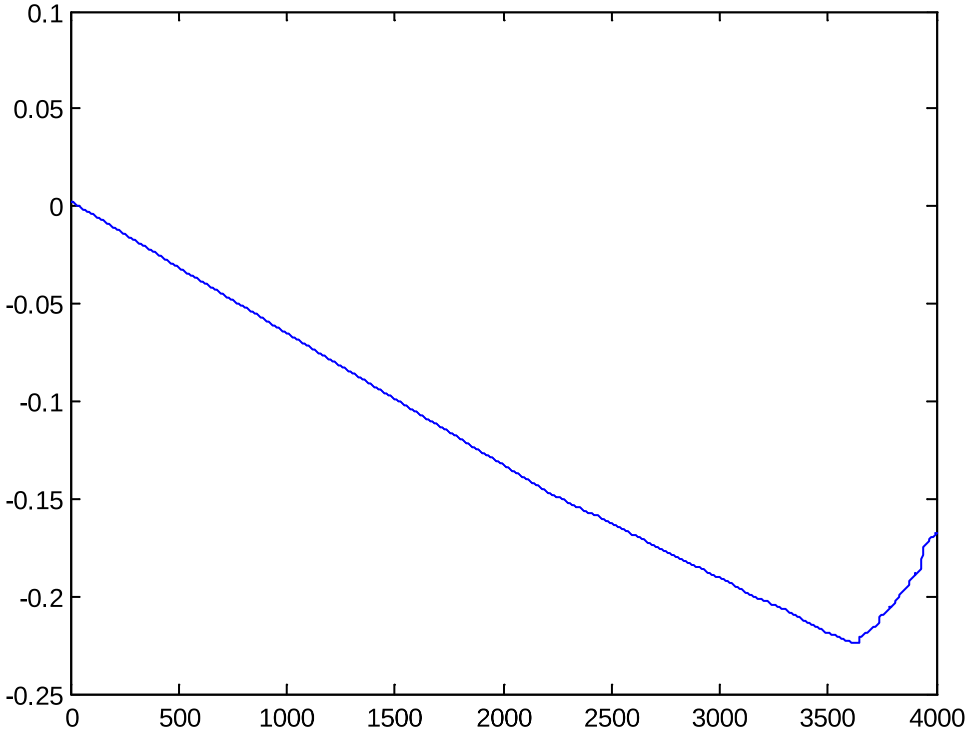
Three types of force field :

a. Normal Constant Force
b. Perlin Force
c. Underwater Force



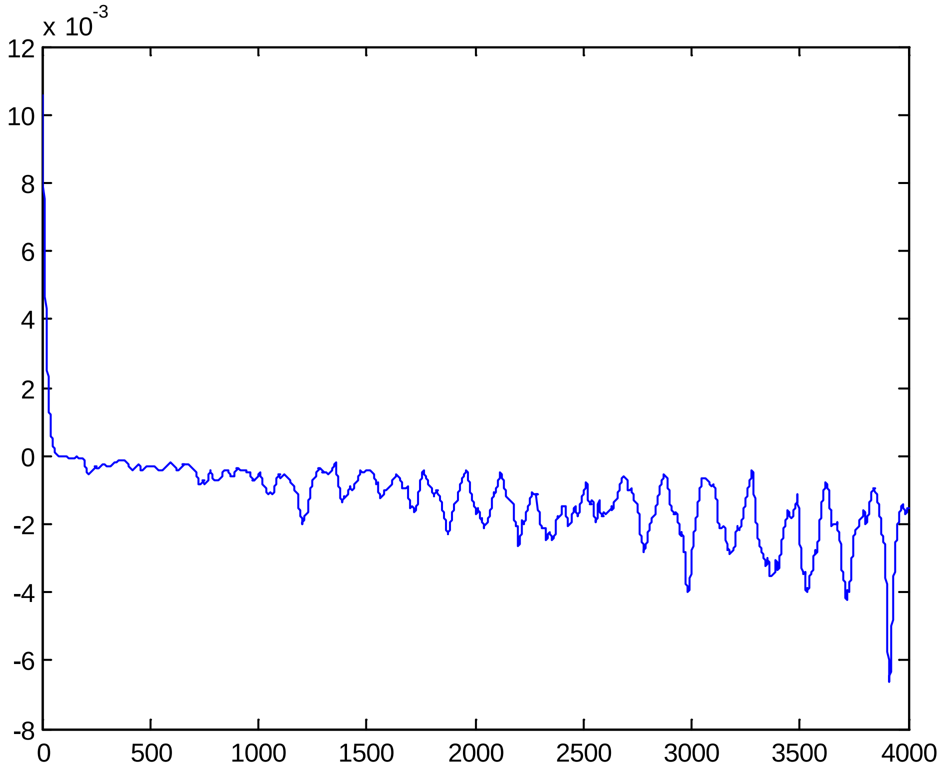
Alpha : 0.2

Beta : 0.01



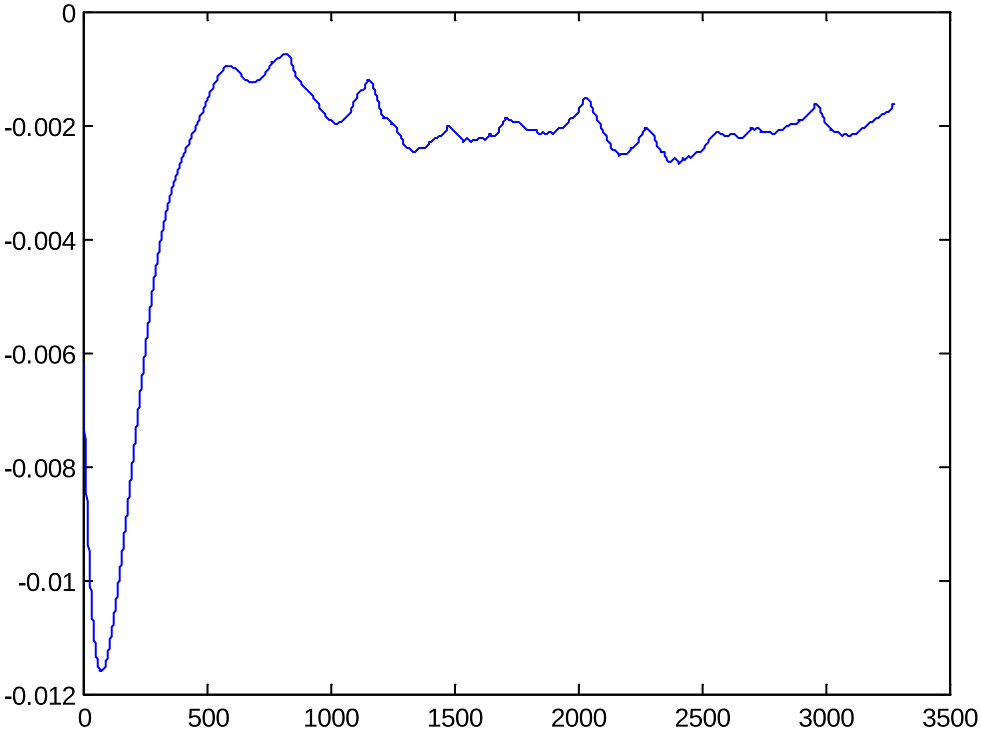
Alpha : 0

Beta : 0



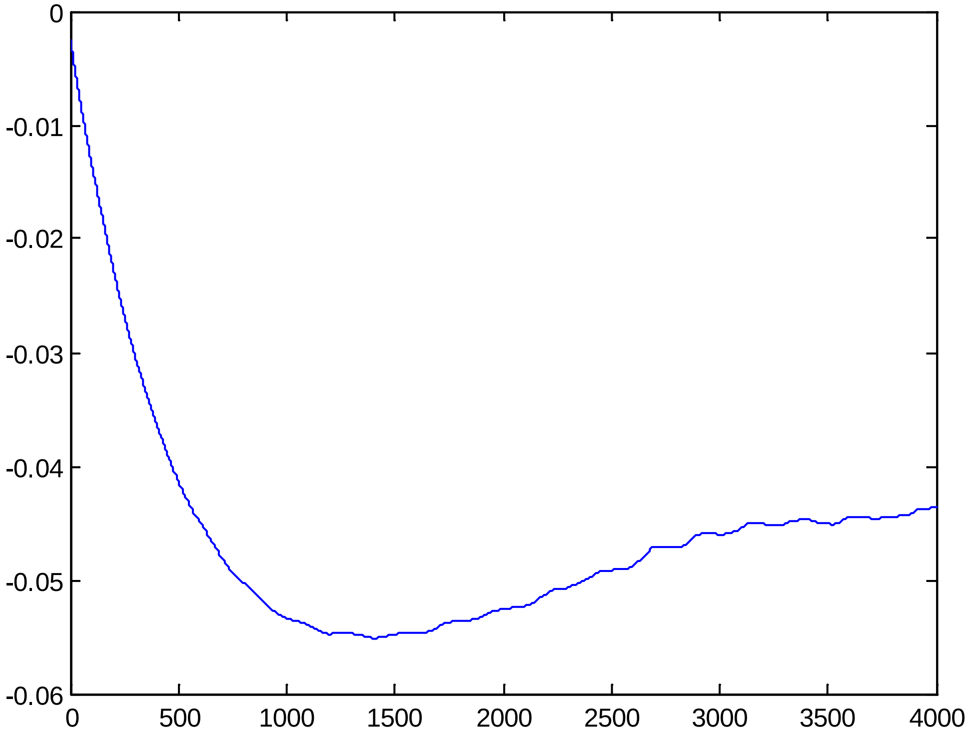
Alpha – 20

Beta -100



Alpha – 2

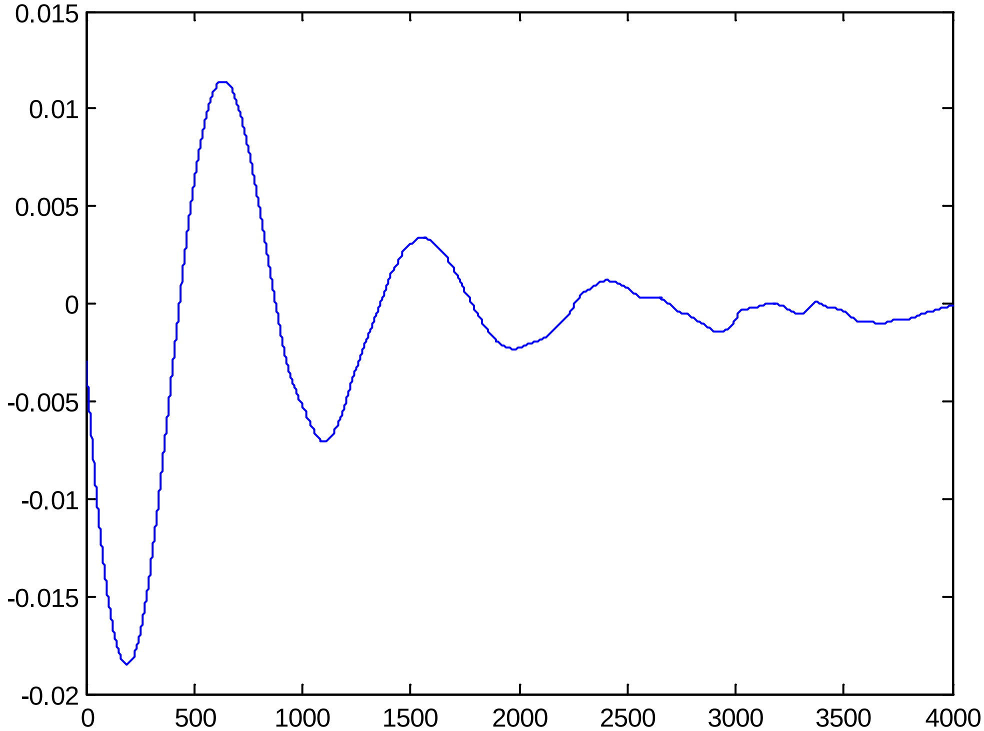
Beta – 1



Alpha - 0.2

Beta -0.005(overrdamped)

The above graph shows that the effect of changing alpha and beta on the stabilization of particle system.

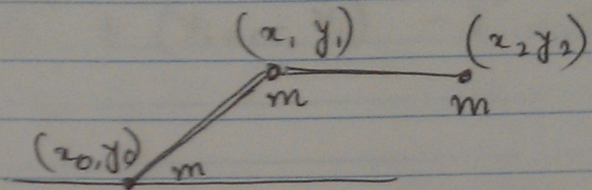


Alpha – 0.2

Beta – 0.5

CONSTRAINTS

Two types → Maximal Coordinates
Reduced Coordinates



$$M(q)\,\ddot{q} = F(q, \dot{q}, t)$$

$$M = \begin{bmatrix} m & & & \\ & m & & \\ & & \ddots & \\ & & & m \end{bmatrix}$$

Diagon

$$F\,\cancel{q} = \begin{bmatrix} F_x^0 \\ F_y^0 \\ F_x^1 \\ F_y^1 \\ F_x^2 \\ F_y^2 \end{bmatrix} + F_c$$

Constraints $\rightarrow$

$$x_0 = y_0 = 0$$

for $i = 1$ to $(n-1)$

Dist $(i+1, i) = L$

$$\Rightarrow (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 - l^2 = 0.$$

$$C(q) = \phi$$

$$\Rightarrow C(q) = \begin{bmatrix} x_0 \\ y_0 \\ (x_1 - x_0)^2 + (y_1 - y_0)^2 - l^2 \\ (x_2 - x_1)^2 + (y_2 - y_1)^2 - l^2 \end{bmatrix}$$

$$M\ddot{q} = F_{ext} + F_c.$$

such that $C(q) = 0.$

$$\nabla C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 2(x_0 - x) & 2(y_0 - y_1) & 2(x_1 - x_0) & 2(y_1 - y_0) & 0 & 0 \\ 0 & 0 & 2(x_1 - x_2) & 2(y_1 - y_2) & 2(x_2 - x_1) & 2(y_2 - y_1) \end{bmatrix}$$

$F_c = (\nabla C)^T \lambda \qquad \lambda \in R^4$

$$M\ddot{q} = F_{ext} - (\nabla C)^T \lambda \quad \text{— (1)}$$

$$C(q) = 0.$$

$$\dot{C} = \frac{\partial C}{\partial q} \cdot \dot{q} = \nabla C \cdot \dot{q}$$

$$\Rightarrow \ddot{C} = 0 = \nabla C \ddot{q} + \frac{d}{dt}\left(\frac{\partial C}{\partial q}\right)\dot{q} \quad \text{— (ii)}$$

Hence from (1) and (ii)

$$\begin{bmatrix} M & (\nabla C)^T \\ \nabla C & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} F_{ext} \\ -\frac{\partial \dot{C}}{\partial q}\dot{q} \end{bmatrix}.$$

$\alpha^2 < 4\beta$ UNDERDAMPED

$\alpha^2 > 4\beta$ OVERDAMPED

$\alpha^2 = 4\beta$ CRITICALLY DAMPED