Tutorial 3

COMP 5361: Discrete Structures and Formal Languages

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Outline

Contents of the section

Nested Quantifiers

Definition

The case where one quantifier is within the scope of another quantifier.

Examples

Assuming the domain of discourse is $\mathbb{R}$ translate the followings to plain English:

1. $\forall x \forall y (x + y = y + x)$

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3. $\forall x \forall y \forall z (x + (y + z) = (x + y) + z)$

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Assuming the domain of discourse is $\mathbb{R}$ translate the followings to plain English:

1. $\forall x \forall y (x + y = y + x)$
2. $\forall x \exists y (x + y = 0)$
3. $\forall x \forall y \forall z (x + (y + z) = (x + y) + z)$
4. $\forall x \forall y ((x > 0) \land (y < 0) \rightarrow (xy < 0))$

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Caution

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- Be careful with the order of existential and universal quantifiers!
- Quantifiers of the same kind can interchange places.

Example

- $\forall x \forall y \exists z (x + y = z)$

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- $\forall x \forall y \exists z (x + y = z)$
  - True

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- $\forall x \forall y \exists z (x + y = z)$
  - True
- $\exists z \forall x \forall y (x + y = z)$

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- $\forall x \forall y \exists z (x + y = z)$
  - True
- $\exists z \forall x \forall y (x + y = z)$
  - False

Translation to English

- Translate the statement, $\forall x(C(x) \vee \exists y(C(y) \wedge F(x, y)))$ where:
  - $C(x)$: x has a computer
  - $F(x, y)$: x and y are friends

- Translate the statement, $\forall x(C(x) \vee \exists y(C(y) \wedge F(x, y)))$ where:
  - $C(x)$: x has a computer
  - $F(x, y)$: x and y are friends
- Translate the statement, $\exists x \forall y \forall z((F(x, y) \wedge F(x, z) \wedge (y = z)) \rightarrow \neg F(y, z))$ where:
  - $F(x, y)$: x and y are friends
  - Domain: All student in the class

Translate from English

Translate the followings into logical expressions:

- If a person is female and is a parent, then this person is someone's mother.

Translate the followings into logical expressions:

- If a person is female and is a parent, then this person is someone's mother.
- Everyone has exactly one best friend.

Recursive Negation

Statements involving nested quantifiers can be negated by successively applying the rules for negating statements involving a single quantifier.

Example

Negate the following statement:

$$\exists w \forall a \exists f (P(w, f) \land Q(f, a))$$

for some f, P, and Q.

Definition

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- **Argument:** An argument is a sequence of statements that end with a conclusion.
- **Valid Argument:** A valid argument is an argument that the conclusion, or final statement of the argument, follows from the **truth** of the preceding statements[1].
- **Fallacy:** An incorrect way of reasoning which lead to invalid arguments.

[1] Also called premise

Check weather the following argument is valid or not.

$$\begin{array}{r} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

The most important rule of all time

The tautology $(p \wedge (p \to q)) \to q$ is the basis of the rule of inference called modus ponens, or the law of detachment.

- **Modus tollens:** $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$
- **Hypothetical syllogism:** $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
- **Disjunctive syllogism:** $((p \vee q) \wedge \neg p) \rightarrow q$
- **Addition:** $p \rightarrow (p \vee q)$
- **Simplification:** $(p \wedge q) \rightarrow p$
- **Conjunction:** $((p) \rightarrow (q)) \rightarrow (p \wedge q)$
- **Resolution:** $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$