Tutorial 3

COMP 5361: Discrete Structures and Formal Languages

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Outline

Contents of the section

Nested Quantifiers

Definition

The case where one quantifier is within the scope of another quantifier.

Assuming the domain of discourse is $\mathbb{R}$ translate the followings to plain English:

1. $\forall x \forall y (x + y = y + x)$

Examples

Assuming the domain of discourse is $\mathbb{R}$ translate the followings to plain English:

1. $\forall x \forall y (x + y = y + x)$
2. $\forall x \exists y (x + y = 0)$

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1. $\forall x \forall y (x + y = y + x)$
2. $\forall x \exists y (x + y = 0)$
3. $\forall x \forall y \forall z (x + (y + z) = (x + y) + z)$

Examples

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1. $\forall x \forall y (x + y = y + x)$
2. $\forall x \exists y (x + y = 0)$
3. $\forall x \forall y \forall z (x + (y + z) = (x + y) + z)$
4. $\forall x \forall y ((x > 0) \wedge (y < 0) \rightarrow (xy < 0))$

Caution

Order matters...

- Be careful with the order of existential and universal quantifiers!

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- Be careful with the order of existential and universal quantifiers!
- Quantifiers of the same kind can interchange places.

Example

- $\forall x \forall y \exists z (x + y = z)$

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  - True

Example

- $\forall x \forall y \exists z (x + y = z)$
  - True
- $\exists z \forall x \forall y (x + y = z)$

Example

- $\forall x \forall y \exists z (x + y = z)$
  - True
- $\exists z \forall x \forall y (x + y = z)$
  - False

Translation to English

- Translate the statement, $\forall x(C(x) \vee \exists y(C(y) \wedge F(x,y)))$ where:
  - $C(x)$: x has a computer
  - $F(x,y)$: x and y are friends

Translation to English

- Translate the statement, $\forall x(C(x) \vee \exists y(C(y) \wedge F(x,y)))$ where:
  - $C(x)$: x has a computer
  - $F(x,y)$: x and y are friends
- Translate the statement, $\exists x \forall y \forall z((F(x,y) \wedge F(x,z) \wedge (y \neq z)) \rightarrow \neg F(y,z))$ where:
  - $F(x,y)$: x and y are friends
  - Domain: All student in the class

Translate from English

Translate the followings into logical expressions:

- If a person is female and is a parent, then this person is someone's mother.

Translate from English

Translate the followings into logical expressions:

- If a person is female and is a parent, then this person is someone's mother.
- Everyone has exactly one best friend.

Negating Nested Quantifiers

Recursive Negation

Statements involving nested quantifiers can be negated by successively applying the rules for negating statements involving a single quantifier.

Example

Negate the following statement:

$$\exists w \forall a \exists f (P(w, f) \wedge Q(f, a))$$

for some f, P, and Q.

Contents of the section

Definition

- **Argument:** An argument is a sequence of statements that end with a conclusion.

[1] Also called premise

Definition

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Definition

- **Argument:** An argument is a sequence of statements that end with a conclusion.
- **Valid Argument:** A valid argument is an argument that the conclusion, or final statement of the argument, follows from the **truth** of the preceding statements[1].
- **Fallacy:** An incorrect way of reasoning which lead to invalid arguments.

[1] Also called premise

Example

Check weather the following argument is valid or not.

$$\begin{array}{r} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Rules of Inference for Propositional Logic

The most important rule of all time

The tautology $(p \wedge (p \rightarrow q)) \rightarrow q$ is the basis of the rule of inference called modus ponens, or the law of detachment.

Rules of Inference for Propositional Logic

- **Modus tollens:** $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$
- **Hypothetical syllogism:** $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
- **Disjunctive syllogism:** $((p \vee q) \wedge \neg p) \rightarrow q$
- **Addition:** $p \rightarrow (p \vee q)$
- **Simplification:** $(p \wedge q) \rightarrow p$
- **Conjunction:** $((p) \wedge (q)) \rightarrow (p \wedge q)$
- **Resolution:** $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$

Examples

State which rule of inference is the basis of the following arguments:

1. It is below freezing now. Therefore, it is either below freezing or raining now.

Examples

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Examples

State which rule of inference is the basis of the following arguments:

1. It is below freezing now. Therefore, it is either below freezing or raining now.
   - **Rule:** Addition
   - $p \rightarrow (p \vee q)$
2. It is below freezing and raining now. Therefore, it is below freezing now.

Examples

State which rule of inference is the basis of the following arguments:

1. It is below freezing now. Therefore, it is either below freezing or raining now.
   - **Rule:** Addition
   - $p \rightarrow (p \vee q)$
2. It is below freezing and raining now. Therefore, it is below freezing now.
   - **Rule:** Simplification
   - $(p \wedge q) \rightarrow p$

Examples

State which rule of inference is the basis of the following arguments:

1. It is below freezing now. Therefore, it is either below freezing or raining now.
   - **Rule:** Addition
   - $p \rightarrow (p \vee q)$
2. It is below freezing and raining now. Therefore, it is below freezing now.
   - **Rule:** Simplification
   - $(p \wedge q) \rightarrow p$
3. If it rains today, then we will not have a barbecue today. If we do not have a barbecue today,then we will have a barbecue tomorrow. Therefore, if it rains today, then we will have a barbecue tomorrow.

Examples

State which rule of inference is the basis of the following arguments:

1. It is below freezing now. Therefore, it is either below freezing or raining now.
   - **Rule:** Addition
   - $p \to (p \vee q)$
2. It is below freezing and raining now. Therefore, it is below freezing now.
   - **Rule:** Simplification
   - $(p \wedge q) \to p$
3. If it rains today, then we will not have a barbecue today. If we do not have a barbecue today,then we will have a barbecue tomorrow. Therefore, if it rains today, then we will have a barbecue tomorrow.
   - **Rule:** Hypothetical syllogism
   - $((p \to q) \wedge (q \to r)) \to (p \to r)$

Examples

Show that the premises:

1. It is not sunny this afternoon and it is colder than yesterday.
2. We will go swimming only if it is sunny.
3. If we do not go swimming, then we will take a canoe trip.
4. If we take a canoe trip, then we will be home by sunset.

Leads to the conclusion: We will be home by sunset.

Examples

Show that the premises:

1. If you send me an e-mail message, then I will finish writing the program.
2. If you do not send me an e-mail message, then I will go to sleep early.
3. If I go to sleep early, then I will wake up feeling refreshed.

Leads to the conclusion: If I do not finish writing the program, then I will wake up feeling refreshed.

Fallacies

Definition

- **Affirming the conclusion:** Happens when we treat the proposition $((p \rightarrow q) \wedge q) \rightarrow p$ as a tautology.
- **Denying the hypothesis:** Happens when we treat the proposition $((p \rightarrow q) \wedge \neg p) \rightarrow \neg q$ as a tautology.

Examples

In the examples below indicate whether the argument is valid or not. If not what type of fallacy is being observed?

1. If you do every problem in this book, then you will learn discrete mathematics. You learned discrete mathematics. Therefore, you did every problem in this book.

Examples

In the examples below indicate whether the argument is valid or not. If not what type of fallacy is being observed?

1. If you do every problem in this book, then you will learn discrete mathematics. You learned discrete mathematics. Therefore, you did every problem in this book.
   - The argument is invalid and it is an example of fallacy of affirming the conclusion.

Examples

In the examples below indicate whether the argument is valid or not. If not what type of fallacy is being observed?

1. If you do every problem in this book, then you will learn discrete mathematics. You learned discrete mathematics. Therefore, you did every problem in this book.
   - The argument is invalid and it is an example of fallacy of affirming the conclusion.
2. If you do every problem in this book, then you will learn discrete mathematics. You did not do every problem in the book. Hence, you did not learn discrete mathematics.

Examples

In the examples below indicate whether the argument is valid or not. If not what type of fallacy is being observed?

1. If you do every problem in this book, then you will learn discrete mathematics. You learned discrete mathematics. Therefore, you did every problem in this book.
    - The argument is invalid and it is an example of fallacy of affirming the conclusion.
2. If you do every problem in this book, then you will learn discrete mathematics. You did not do every problem in the book. Hence, you did not learn discrete mathematics.
    - The argument is invalid and it is an example of fallacy of denying the hypothesis.

Rules of Inference for Quantified Statements

- **Universal instantiation**: $\forall x P(x) \rightarrow P(c)$
- **Universal generalization**: $P(c)$ for an arbitrary $c \rightarrow \forall x P(x)$
- **Existential instantiation**: $\exists x P(x) \rightarrow P(c)$ for some element c
- **Existential generalization**: $P(c)$ for some element $c \rightarrow \exists x P(x)$

Examples

Show that the following arguments are valid:

1.
    - Everyone in this discrete mathematics class has taken a course in computer science.
    - Marla is a student in this class.
    - Therefore, Marla has taken a course in computer science.

Examples

Show that the following arguments are valid:

1. - Everyone in this discrete mathematics class has taken a course in computer science.
   - Marla is a student in this class.
   - Therefore, Marla has taken a course in computer science.
2. - A student in this class has not read the book.
   - Everyone in this class passed the first exam.
   - Therefore, someone who passed the first exam has not read the book.

Combining Rules of Inference for Propositions and Quantified Statements

- **Universal modus ponens**: $[(\forall x(P(x) \to Q(x))) \wedge (P(a))] \to Q(a)$
- **Universal modus tollens**: $[(\forall x(P(x) \to Q(x))) \wedge (\neg Q(a))] \to \neg P(a)$

Examples

Show the following argument is valid:

- For all positive integers n, if n is greater than 4, then n^2 is less than 2^n.
- We can conclude, $100^2 < 2^{100}$