

# Tutorial 8

## COMP 335: Introduction to Theoretical Computer Science

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- 1 Derivation Trees
- 2 Simplifications of Context-Free Grammars
- 3 Chomsky Normal Form

# Contents of the section

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## Question 1

Let  $G$  be:

$$S \rightarrow I|S + S|S \times S|(S) \quad (1)$$

$$I \rightarrow a|b|a|b|0|1 \quad (2)$$

- 1 Show the left most derivation of  $a \times (a + b000)$  and draw its derivation tree.
- 2 Show the right most derivation of  $a \times (a + b000)$  and draw its derivation tree.

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## Recipe

- ➊ Remove nullable variables and  $\lambda$ -productions
- ➋ Remove Unit-productions
  - Remove repeated productions.
- ➌ Remove Useless productions
  - Find every variable that produces strings with only terminals and keep them.
  - Find all the variables that are reachable from  $S$  and keep them.

# Simplifications of Context-Free Grammars

## Question 1

Simplify the following grammar:

$$S \rightarrow XYX \quad (1)$$

$$X \rightarrow 0X | \lambda \quad (2)$$

$$Y \rightarrow 1Y | \lambda \quad (3)$$

## Question 2

Simplify the following grammar:

$$S \rightarrow 0A0|1B1|BB \quad (1)$$

$$A \rightarrow C \quad (2)$$

$$B \rightarrow S|A \quad (3)$$

$$C \rightarrow S|\lambda \quad (4)$$



## Question 3

Simplify the following grammar:

$$S \rightarrow XY|W \quad (1)$$

$$X \rightarrow aXb|\lambda \quad (2)$$

$$Y \rightarrow cY|\lambda \quad (3)$$

$$W \rightarrow aWc|Z \quad (4)$$

$$Z \rightarrow bZ|\lambda \quad (5)$$

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# Chomsky Normal Form

## Recipe

- 1 Remove nullable variables unit productions.
- 2 For every terminal  $t$  add a rule  $T \rightarrow t$  and replace all occurrences of  $t$  with  $T$ .
- 3 Replace each production that leads to 3 or more variables by a production that leads to exactly 2 variables via introducing new variables.

## Question 1

Transform the following grammars into CNF:

$$S \rightarrow XYX \quad (1)$$

$$X \rightarrow 0X|\lambda \quad (2)$$

$$Y \rightarrow 1Y|1 \quad (3)$$

## Question 2

Transform the following grammars into CNF:

$$S \rightarrow 0A0|1B1|B0B \quad (1)$$

$$A \rightarrow C \quad (2)$$

$$B \rightarrow S|A \quad (3)$$

$$C \rightarrow S|\lambda \quad (4)$$

## Question 3

Transform the following grammars into CNF:

$$S \rightarrow XaY|Wb \quad (1)$$

$$X \rightarrow aXb|\lambda \quad (2)$$

$$Y \rightarrow cY|\lambda \quad (3)$$

$$W \rightarrow aWc|Z \quad (4)$$

$$Z \rightarrow bZ|\lambda \quad (5)$$