Tutorial 8

COMP 5361: Discrete Structures and Formal Languages

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Outline

Contents of the section

Alphabets

Definition

An alphabet is a finite, nonempty set of symbols. Conventionally, we use the symbol Σ for an alphabet.

Alphabets

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Example

Example 1: $\Sigma = \{0, 1\}$

Example 2: $\Sigma = \{a, b, c\}$

Example 3: $\Sigma = \{1, a, 2, b, \#\}$

Word

Definition

- **Word**: A word (or sometimes string) is a finite sequence of symbols chosen from some alphabet.
- **Empty String**: The empty string is the string with zero occurrences of symbols. This string, denoted ϵ, is a string that may be chosen from any alphabet whatsoever.
- **Length of a Word**: Length of a Word is equal to the number of symbols in the word.
- **Powers of an Alphabet**: The set of strings of length k, each of whose symbols is in Σ, is the k^{th} power of the alphabet, i.e. Σ^k.

Word

Example

Let $\Sigma = \{0, 1\}$

Example 1: $\Sigma^1 = \{0, 1\}$

Example 2: $\Sigma^2 = \{00, 01, 10, 11\}$

Example 3: $|01| = 2$

Example 4: $|\epsilon| = 0$

Word

Definition

Let Σ be set of the alphabet, then:

- **Set of all words over the alphabet**:

$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \dots$$

- **Set of nonempty strings over the alphabet**:

$$\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \dots$$

Word

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Let Σ be set of the alphabet, then:

- **Concatenation**: Let x and y be words, then xy denotes the concatenation of x and y.

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Example

Let $x = 01101$ and $y = 110$, then:

1. $xy = 01101110$
2. $yx = 11001101$

Language

Definition

Let Σ be set of the alphabet, then:

- **Language**: A set of words all of which are chosen from some $\Sigma*$ is called a language.

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Example

Example 1: The language of all words consisting of n 0's followed by n 1's, for some $n \geq 0$: $\{\epsilon, 01, 0011, 000111, \dots\}$

Example 2: The empty language: $\emptyset$

Example 3: The language consisting of only the empty string: $\{\epsilon\}$

Contents of the section

Deterministic Finite Automata (DFA)

Definition

- **DFA**: A deterministic finite automaton is a five-tuple consists of:

$$M = (Q, \Sigma, \delta, q_0, F)$$

1. A finite set of states, often denoted Q.
2. A finite set of input symbols, often denoted Σ.
3. A start state q_0, one of the states in Q.
4. A set of final or accepting states F. The set F is a subset of Q.
5. A transition function δ that takes as arguments a state and an input symbol and returns a state.

Definition

- **Transition diagram for a DFA**: A transition diagram for a DFA is a graph defined as follows:
  1. For each state in Q there is a node.
  2. For each state q in Q and each input symbol a in Σ, let $\delta(q, a) = p_q$. Then the transition diagram has an arc from node q to node p_q, labeled a.
  3. There is an arrow into the start state q_0 , labeled Start.
  4. Nodes corresponding to accepting states (those in F) are marked by double circle

Definition

- **Transition table for a DFA**: A transition table is a conventional, tabular representation of a function like δ that takes two arguments and returns a value. The rows of the table correspond to the states, and the columns correspond to the inputs. The entry for the row corresponding to state q and the column corresponding to input a is the state $\delta(q, a)$.

Deterministic Finite Automata (DFA)

Definition

- **Extended Transition Function**: The extended transition function is a function that takes a state q and a word w and returns a state p, the state that the automaton reaches when starting in state q and processing the sequence of inputs w. If the transition function is denoted by δ, the extended transition function is denoted by $\hat{\delta}$.

Definition

- **The Language of the DFA**: The language of the DFA is the set of all strings that the DFA accepts. This language is denoted by $L(A)$:

$$L(A) = \{w | \hat{\delta}(q_0, w) \text{ is in } F\}$$

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Example 1

For the following DFA determine:

- The alphabet set.

Examples

Example 1

For the following DFA determine:

- The alphabet set.
- Its transition table.

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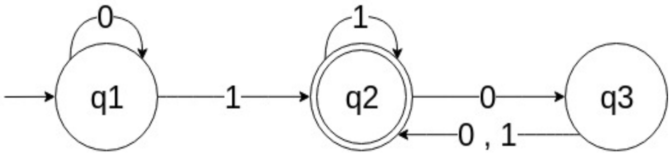

Figure: Example 1 DFA

Examples

Example 2

Build a DFA that identifies the non-negative multiples of 3.

- What is the set of alphabet?

Example 2

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- What is the transition diagram?

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Build a DFA that identifies the non-negative multiples of 3.

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- What is the transition diagram?
- What is the transition table?

Example 3

Build a DFA that identifies the non-negative multiples of 5.

- What is the set of alphabet?

Examples

Example 3

Build a DFA that identifies the non-negative multiples of 5.

- What is the set of alphabet?
- What is the transition diagram?

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Build a DFA that identifies the non-negative multiples of 5.

- What is the set of alphabet?
- What is the transition diagram?
- What is the transition table?

Example 4

Build a DFA that identifies the non-negative powers of 2.

- What is the set of alphabet?

Examples

Example 4

Build a DFA that identifies the non-negative powers of 2.

- What is the set of alphabet?
- What is the transition diagram?

Example 4

Build a DFA that identifies the non-negative powers of 2.

- What is the set of alphabet?
- What is the transition diagram?
- What is the transition table?