

# Tutorial 5

## COMP 5361: Discrete Structures and Formal Languages

Mohammad Reza Davari

Concordia University

1 Introduction to Sets

2 Set Operations

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## 1 Introduction to Sets

## 2 Set Operations

## Definition

A **set** is an unordered collection of objects, called **elements** or **members** of the set. A set is said to **contain** its elements. We write  $a \in A$  to denote that  $a$  is an element of the set  $A$ . The notation  $a \notin A$  denotes that  $a$  is not an element of the set  $A$ .

It is common for sets to be denoted using uppercase letters. Lowercase letters are usually used to denote elements of sets.

## Definition

Two sets are **equal** if and only if they have the same elements. Therefore, if  $A$  and  $B$  are sets, then  $A$  and  $B$  are equal if and only if  $\forall x(x \in A \leftrightarrow x \in B)$ . We write  $A = B$  if  $A$  and  $B$  are equal sets.

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- **Singleton Set:** It is a set containing only one element.

## Definition

- **Subset:** The set  $A$  is a subset of  $B$  if and only if every element of  $A$  is also an element of  $B$ . We use the notation  $A \subseteq B$  to indicate that  $A$  is a subset of the set  $B$ .



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- **Proper Subset:** The set  $A$  is a proper subset of  $B$  ( $A \subset B$ ) if and only if  $A$  is a subset of  $B$  and there exists an element  $x$  of  $B$  that is not an element of  $A$ . i.e.

$$\forall x(x \in A \rightarrow x \in B) \wedge \exists x(x \in B \wedge x \notin A)$$

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## Strategies

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## Strategies

- **A Subset of B:** Show that if  $x$  belongs to  $A$  then  $x$  also belongs to  $B$ .
- **A Not Subset of B:** Find a single  $x \in A$  such that  $x \notin B$

## Definition

**Equality:** Two set  $A$  and  $B$  are equal if and only if  $A$  is a subset of  $B$  and  $B$  is a subset of  $A$ .

## Definition

- **Finite Set:** Let  $S$  be a set. If there are exactly  $n$  distinct elements in  $S$  where  $n$  is a nonnegative integer, we say that  $S$  is a finite set and that  $n$  is the cardinality of  $S$ . The cardinality of  $S$  is denoted by  $|S|$ .

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- **Infinite Set:** A set that is not finite, is said to be infinite.

## Definition

- **Power Set:** The power set of a set  $S$  is the set of all subsets of the set  $S$ . The power set of  $S$  is denoted by  $\mathcal{P}(S)$ . If a set has  $n$  elements, then its power set has  $2^n$  elements.

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**Example 1** What is the power set of the set  $\{0, 1, 2\}$ ?

**Example 2** What is the power set of the empty set? What is the power set of the set  $\{\emptyset\}$ ?

## Definition

**Cartesian product:** The Cartesian product of two sets  $A$  and  $B$ , denoted by  $A \times B$ , is the set of all ordered pairs  $(a, b)$ , where  $a \in A$  and  $b \in B$ , i.e.

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**Example 1** What is the Cartesian product  $A \times B$ ?

**Example 2** What is the Cartesian product  $B \times A$ ?

**Example 3** What is the Cartesian product  $A \times B \times C$ ?

## Definition

**Relation:** A subset  $R$  of the Cartesian product  $A \times B$  is called a **relation** from the set  $A$  to the set  $B$ . The elements of  $R$  are ordered pairs, where the first element belongs to  $A$  and the second to  $B$ . A relation from a set  $A$  to itself is called a relation on  $A$ .

## Definition

**Truth Set:** The truth set of the statement  $P$  is the set of elements  $x$  in a domain  $D$  for which  $P(x)$  is true. The truth set of  $P(x)$  is denoted by  $\{x \in D | P(x)\}$ .

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## Example

What are the truth sets of the predicates  $P(x)$ ,  $Q(x)$ , and  $R(x)$ , where the domain is the set of integers and  $P(x)$  is " $|x| = 1$ ,"  $Q(x)$  is " $x^2 = 2$ ," and  $R(x)$  is " $|x| = x$ ."



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## Definition

- **Union:** Let  $A$  and  $B$  be sets. The union of the sets  $A$  and  $B$ , denoted by  $A \cup B$ , is the set that contains those elements that are either in  $A$  or in  $B$ , or in both.

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- **Intersection** Let  $A$  and  $B$  be sets. The intersection of the sets  $A$  and  $B$ , denoted by  $A \cap B$ , is the set containing those elements in both  $A$  and  $B$ .

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- **Disjoint:** Two sets are called disjoint if their intersection is the empty set.

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**Example 2:** What is the intersection of the sets  $\{1, 3, 5\}$  and  $\{1, 2, 3\}$ ?

**Example 3:** Are two sets  $A = \{1, 3, 5, 7, 9\}$  and  $B = \{2, 4, 6, 8, 10\}$  disjoint?

## Definition

- **Difference:** Let  $A$  and  $B$  be sets. The difference of  $A$  and  $B$ , denoted by  $A - B$  or  $A \setminus B$ , is the set containing those elements that are in  $A$  but not in  $B$ .



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- **Complement:** Let  $U$  be the universal set. The complement of the set  $A$ , denoted by  $\bar{A}$ , is the complement of  $A$  with respect to  $U$ . Therefore, the complement of the set  $A$  is  $U - A$ .

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# Set Operations

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**Example 1:** What is the difference of the sets  $\{1, 3, 5\}$  and  $\{1, 2, 3\}$ ?

**Example 2:** Let  $A$  be the set of positive integers greater than 10 (with universal set the set of all positive integers), what is  $\bar{A}$ ?