

Worksheet-1

Monday, May 1, 2023 8:52 PM

1 Determine $x \div y$ and $x \bmod y$ for each pair of values below.

- $x = 252, y = 7$
- $x = 1398, y = 13$
- $x = -21, y = 33$
- $x = -457, y = 22$

d) $-457 \div 22 = -(20.7) = -21$
 $-457 \bmod 22 \rightarrow 457/22 = 20.7 \rightarrow -451 + 20(22) =$
 $-17 + 22 = 5$

a) $252 \div 7 = 36$ b) $1398 \div 13 = 107$ c) $-21 \div 33 = -1$ only added 1 times so -1
 $252 \bmod 7 = 0$ $1398 \bmod 13 = 7$ $-21 \bmod 33 \rightarrow -21 + 33 = 12$

2 Determine the value for each of the following. These can be done without a calculator.

- 9×3 in \mathbb{Z}_{20}
- $15^{26} \bmod 7$
- $(352 \cdot 407) \bmod 50$
- $(1302^3 + 4505^2) \bmod 10$

a) 9×3 in $\mathbb{Z}_{20} \rightarrow 9 \cdot 3 \bmod 20 = 27 \bmod 20 = 7$ b) $15^{26} \bmod 7 = (15 \bmod 7)^{26} = 1^{26} = 1$
c) $(352 \cdot 407) \bmod 50 = [352 \bmod 50 \cdot 407 \bmod 50] \bmod 50 = (2 \cdot 7) \bmod 50 = 14 \bmod 50 = 14$
d) $(1302^3 + 4505^2) \bmod 10 = [(1302 \bmod 10)^3 + (4505 \bmod 10)^2] \bmod 10 = (2^3 + 5^2) \bmod 10 = 25 + 8 \bmod 10 = 3$

3 Determine if the following values are prime.

- 157
- 481
- 1907
- 2021

a) $157 \rightarrow \sqrt{157} \approx 12$, 1-12 are not factors, so prime

b) $481 \rightarrow \sqrt{481} \approx 21$, $481 \div 13 = 37$, so Not Prime

c) $1907 \rightarrow \sqrt{1907} \approx 43$, prime

d) $2021 \rightarrow$ not prime

4 For each pair of x and y values below,

- Determine the greatest common divisor (GCD) of x and y .
- Write the $\gcd(x, y)$ as a linear combination of x and y .
- Determine the multiplicative inverse of $x \bmod y$, if it exists.

- $x = 45, y = 55$
- $x = 51, y = 72$
- $x = 39, y = 44$
- $x = 324, y = 431$

a) $55 = 45 \cdot 1 + 10$
 $45 = 10 \cdot 4 + 5$
 $10 = 5 \cdot 2 + 0$
 $5 = 45 - 4(10)$
 $= 45 - 4(55 - 45)$
 $= 45 + 4 \cdot 45 - 4 \cdot 55$
 $= 45 \cdot 5 - 4 \cdot 55$

i) $\gcd(45, 55) = 5$
ii) $5 \cdot 45 + -4 \cdot 55$
iii) D.N.E b/c $\gcd \neq 1$

b) $72 = 51 \cdot 1 + 21$ $3 = 21 - 2(9)$ i. $\gcd(51, 72) = 3$
 $51 = 21 \cdot 2 + 9$ $= 21 - 2(31 - 2 \cdot 21)$ ii. $-7 \cdot 51 + 5 \cdot 72$
 $21 = 4 \cdot 2 + 3$ $= 21 + 4 \cdot 21 - 2 \cdot 51$ iii. $\gcd \neq 1$, D.N.E.
 $9 = 3 \cdot 3 + 0$ $= 5 \cdot 21 - 2 \cdot 51 = -2 \cdot 51 + 5(72 - 51)$
 $= -2 \cdot 51 - 5 \cdot 51 + 5 \cdot 72 = -7 \cdot 51 + 5 \cdot 72$

c) $44 = 39 \cdot 1 + 5$ $1 = 5 - 1(4)$ i. $\gcd(39, 44) = 1$
 $39 = 5 \cdot 7 + 4$ $= 5 - 1(39 - 5 \cdot 7) = 5 + 7 \cdot 5 - 39 = 8 \cdot 5 - 1 \cdot 39$ ii. $-9 \cdot 39 + 8 \cdot 44$
 $5 = 4 \cdot 1 + 1$ $= -1 \cdot 39 + 8(44 - 1 \cdot 39) = -1 \cdot 39 + 8 \cdot 44 - 8 \cdot 39 = -9 \cdot 39 + 8 \cdot 44$ iii. $-9 \bmod 44 = 35$
 $4 = 1 \cdot 4 + 0$

d) $431 = 324 \cdot 1 + 107$ $1 = 3 - 1(2)$ i. $\gcd(324, 431) = 1$
 $324 = 107 \cdot 3 + 3$ $= 3 - 1(107 - 3 \cdot 107) = 3 + 3 \cdot 107 - 107 = 36 \cdot 3 - 107$ ii. $145 \cdot 324 - 109 \cdot 431$
 $107 = 3 \cdot 35 + 2$ $= -107 + 36(324 - 3 \cdot 107) = -107 + 108 \cdot 324 - 36 \cdot 324$ iii. $145 \bmod 431 = 145$
 $3 = 2 \cdot 1 + 1$ $= 36 \cdot 324 - 109(431) = 36 \cdot 324 - 109 \cdot 431$
 $2 = 1 \cdot 2 + 0$ $= 36 \cdot 324 + 109 \cdot 324 - 109 \cdot 431 = 145 \cdot 324 - 109 \cdot 431$