Page 1

Wednesday, April 26, 2023 8:23 PM

1 With the pseudo-code below, determine the final value of sum when n=23.

Algorithm 1

```
procedure
    sum = 0
    for (i=2, i<n, i=i+3) do
        if i mod 2 == 0
            sum = sum + i
```

2 → 0+2=2
5
8 → 2+8=10
11
14 → 10+14=24
17
20 → 24+20=44
23

44

2 Use the pseudo-code below to answer the following question.

(a) Without working through the pseudo-code, how many times with val be updated in Line 7?

(b) What is the final value of val.

Algorithm 2

```
1: procedure
2:     dFour = [1, 2, 3, 4]
3:     dSix = [1, 2, 3, 4, 5, 6]
4:     val = 0
5:     for (x in dFour) do
6:         for (y in dSix) do
7:             val = val + x·y
```

$0 + 1\cdot1 + 1\cdot2 + 1\cdot3 + 1\cdot4 + 1\cdot5 + 1\cdot6$
$+ 2\cdot1 + 2\cdot2 + 2\cdot3 + 2\cdot4 + 2\cdot5 + 2\cdot6$
$+ 3\cdot1 + 3\cdot2 + 3\cdot3 + 3\cdot4 + 3\cdot5 + 3\cdot6$
$+ 4\cdot1 + 4\cdot2 + 4\cdot3 + 4\cdot4 + 4\cdot5 + 4\cdot6 = 210$

a) 4 loops · 6 loops = 24

b) 210

Page 2

Wednesday, April 26, 2023 8:38 PM

3 Using the pseudo-code below, what is the ending value for `count` when `n = 100`? When `n = 10,000`? When `n =` 10^{42}?

Algorithm 3

```
procedure
    count=0
    while (n>1) do
        count++
        n = n/10
```

$n_1 = 100 = 10^2 \rightarrow count_1 = \underline{2}$

$n_2 = 10{,}000 = 10^4 \rightarrow count_2 = \underline{4}$

$n_3 = 10^{42} \rightarrow count_3 = \underline{42}$

4 This question assumes familiarity with modular arithmetic discussed in Unit 2. In the pseudo-code below, Line 7 performs string addition (concatenation), for example: `"a" + "b" = "ab"`. Even if the addends look like numbers, it will still do string addition: `"2" + "3" = "23"`. Answer the questions below.

a. What is the output of the algorithm when $x = 99$ and $b = 2$?

b. What is the output of the algorithm when $x = 1642$ and $b = 8$?

Algorithm 4

```
1: procedure EXPAND(x, b)
2:     Input: integers x and b
3:     Output: outVal, x written in base b
4:
5:     outVal = an empty string
6:     while (x>0) do
7:         outVal = string(x mod b) + outVal
8:         x = x div b
9:     return(outVal)
```

a) x = 99 b = 2

- (99 div 2 = 49, 99 mod 2 = 1) → (49 div 2 = 24, 49 mod 2 = 1) → (24 div 2 = 12 24 mod 2 = 0) → (12 div 2 = 6, 12 mod 2 = 0) → (6 div 2 = 3 6 mod 2 = 0) → (3 div 2 = 1 3 mod 2 = 1) → (1 div 2 = 0 1 mod 2 = 1)

Blank String

- OutVal = (1 + "") → (1 + 1) → (0 + 11) → (0 + 011) → (0 + 0011) → (1 + 00011) → (1 + 100011)

= <u>1100011</u>

b) x = 1642 b = 8

- (1642 div 8 = 205 1642 mod 8 = 2) → (205 div 8 = 25 205 mod 8 = 5) → (25 div 8 = 3 25 mod 8 = 1) → (3 div 8 = 0 3 mod 8 = 3)

- OutVal = <u>3152</u>

Page 3

Wednesday, April 26, 2023 9:09 PM

5 Order each function according to their growth from slowest to fastest (This is the same as execution speed from fastest to slowest for an algorithm).

(a) $n \cdot \log n$

(b) $\log_{10} n$

(c) 10^n

(d) n^{10}

(e) $10n$

(f) 100

(g) n^2

f) 100 b) $\log_{10} n$ e) $10n$ a) $n\log n$ g) n^2 d) n^{10} c) 10^n

6 The provided expressions are the processing time of an algorithm for problems of size n. For each expression, find the *lowest possible* Big-$\mathcal{O}$ complexity stated in simplest form.

(a) $100n^2 + 0.0002n^3 + 10,000$

(b) $20n + 2n\log_{10} n + 30$

(c) $\log_{10} 5 + \log_2 7$

(d) $5n + \sqrt{n}$

(e) $0.01(2^n) + n^5 + \ln n$

(f) $100 \cdot 4^n + 200 \cdot 3^n$

a) $O(n^3)$ b) $O(n\log n)$ c) $O(1)$ d) $O(n)$ e) $O(2^n)$ f) $O(4^n)$

7 $f(x) = 6x \log_4 x$ is which of the following? **Mark all that apply**.

○ $\mathcal{O}(6)$ ○ $\mathcal{O}(\log x)$ ○ $\mathcal{O}(6\log x)$ ○ $\mathcal{O}(x\log x)$ ○ $\mathcal{O}(x^2)$ ○ $\mathcal{O}(2^x)$

Since Big-O is upperbound, $f(x) \in O(x\log x) \subset O(x^2) \subset O(2^x)$, so
$O(x\log x)$, $O(x^2)$, $O(2^x)$

8 Let $h(x) = 6x^4 + 2x^3 + x + 100$. Which of the following is true? **Mark all that apply**.

○ $h(x) \in \mathcal{O}(x^4)$ ○ $h(x) \in \Theta(x^4)$ ○ $h(x) \in \Omega(x^4)$

$O(x^4)$ upper bound $\Omega(x^4)$ lower bound $\Theta(x^4)$ tight bound

Page 4

Thursday, April 27, 2023 11:12 PM

9 Let $h(x) = 6\log_2 x$. Which of the following is true? **Mark all that apply**.

○ $h(x) \in \mathcal{O}(4x^2)$ ○ $h(x) \in \Theta(4x^2)$ ○ $h(x) \in \Omega(4x^2)$

✓

6logx is faster than 4x

10 Let $f(x) = x^2 \log_3 x$. Which of the following is true? **Mark all that apply**.

○ $f(x) \in \mathcal{O}(2x\log_{10} x)$ ○ $f(x) \in \Theta(2x\log_{10} x)$ ○ $f(x) \in \Omega(2x\log_{10} x)$

✓ $x^2 > 2x$

11 Let $g(x) = 2x\log_{10} x$. Which of the following is true? **Mark all that apply**.

○ $g(x) \in \mathcal{O}(x^2\log_3 x)$ ○ $g(x) \in \Theta(x^2\log_3 x)$ ○ $g(x) \in \Omega(x^2\log_3 x)$

✓ $2x < x^2$

Note: $\Theta \rightarrow$ upper bigger slower $\Omega \rightarrow$ Lower smaller faster

12 Using the provided pseudo-code, find the worst case performance in Big-$\mathcal{O}$ notation.

Algorithm 5 Some more messy pseudocode

```
procedure SomeProcedure2
    j=0
    for i=0; i<n; i++ do
        j=i+j
```

A. $\mathcal{O}(\log n)$

B. $\mathcal{O}(n\log n)$

C. $\mathcal{O}(n^2)$

D. $\mathcal{O}(n)$ D

Page 5

Thursday, April 27, 2023 11:33 PM

13 Using the provided pseudo-code, find the worst case performance in Big-$\mathcal{O}$ notation.

Algorithm 6 Some messy pseudo-code

```
procedure SOMEPROCEDURE
    for i=1 and i<=n do
        j=1
        while j<n do
            j=j+2
```

A. $\mathcal{O}(\log n)$

B. $\mathcal{O}(n \log n)$

C. $\mathcal{O}(n^2)$

D. $\mathcal{O}(n)$

C

14 Using the provided pseudo-code, find the worst case performance in Big-$\mathcal{O}$ notation. Assume we know that **someMethod(n)** is $\mathcal{O}(\log n)$.

Algorithm 7 Some pseudocode with a method in it

```
procedure SOMEPROCEDURE3
    j=0
    for i=0; i<n; i++ do
        j=someMethod(n)
```

A. $\mathcal{O}(\log n)$

B. $\mathcal{O}(n \log n)$

C. $\mathcal{O}(n^2)$

D. $\mathcal{O}(n)$

B

15 Using the provided pseudo-code, find the worst case performance in Big-$\mathcal{O}$ notation.

Algorithm 8 Some more messy pseudocode

```
procedure SOMEPROCEDURE4
    while n> 1 do
        n=n/2
```

A. $\mathcal{O}(\log n)$

B. $\mathcal{O}(n \log n)$

C. $\mathcal{O}(n^2)$

D. $\mathcal{O}(n)$

A

16 Using the provided pseudo-code, find the worst case performance in Big-$\mathcal{O}$ notation.

Algorithm 9

```
procedure
    a = 1
    c = 0
    while a < n do
        for i = 0; i < n; i++ do
            c = c + 1
        a = a * 3
```

- While loop runs $\log_3 n$ times
- for loop runs n times

$\rightarrow$ $n \log n$

A. $\mathcal{O}(\log n)$

B. $\mathcal{O}(n \log n)$

C. $\mathcal{O}(n^2)$

D. $\mathcal{O}(n)$

B