Worksheet-1

Monday, May 1, 2023 8:52 PM

1 Determine $x \operatorname{div} y$ and $x \bmod y$ for each pair of values below.

a. $x = 252, \quad y = 7$

b. $x = 1398, \quad y = 13$

c. $x = -21, \quad y = 33$

d. $x = -457, \quad y = 22$

d) $-457 \text{ div } 22 = -(20+1) = \underline{-21}$

$-457 \bmod 22 \rightarrow 457/22 = 20.7 \rightarrow -457 + 20(22) = -17 + 22 = \underline{5}$ $+1$

a) $252 \text{ div } 7 = \underline{36}$
$252 \bmod 7 = \underline{0}$

b) $1398 \text{ div } 13 = \underline{107}$
$1398 \bmod 13 = \underline{7}$

c) $-21 \text{ div } 33 = \underline{-1}$ only added 1 times so -1
$-21 \bmod 33 \rightarrow -21 + 33 = \underline{12}$

2 Determine the value for each of the following. These can be done without a calculator.

a. 9×3 in $\mathbb{Z}_{20}$

b. $15^{26} \bmod 7$

c. $(352 \cdot 407) \bmod 50$

d. $(1302^3 + 4505^2) \bmod 10$

a) 9×3 in $Z_{20} \rightarrow 9 \cdot 3 \bmod 20 = 27 \bmod 20 = \underline{7}$ b) $15^{26} \bmod 7 = (15 \bmod 7)^{26} = 1^{26} = \underline{1}$

c) $(352 \cdot 407) \bmod 50 = [352 \bmod 50 \cdot 407 \bmod 50] \bmod 50 = (2 \cdot 7) \bmod 50 = 14 \bmod 50 = \underline{14}$

d) $(1302^3 + 4505^2) \bmod 10 = [(1302 \bmod 10)^3 + (4505 \bmod 10)^2] \bmod 10 = (2^3 + 5^2) \bmod 10 = 25 + 8 \bmod 10 = \underline{3}$

3 Determine if the following values are prime.

a. 157

b. 481

c. 1907

d. 2021

a) $157 \rightarrow \sqrt{157} \approx 12$, 1-12 are not factors, so <u>prime</u>

b) $481 \rightarrow \sqrt{481} \approx 21$, $481 \div 13 = 37$, so <u>Not Prime</u>

c) $1907 \rightarrow \sqrt{1907} \approx 43$, <u>prime</u>

d) $2021 \rightarrow$ <u>not prime</u>

4 For each pair of x and y values below,

i) Determine the greatest common divisor (GCD) of x and y.

ii) Write the $\gcd(x, y)$ as a linear combination of x and y.

iii) Determine the multiplicative inverse of $x \bmod y$, if it exists.

a. $x = 45, \quad y = 55$

b. $x = 51, \quad y = 72$

c. $x = 39, \quad y = 44$

d. $x = 324, \quad y = 431$

a) $45 = 3^2 \cdot 5^1 \cdot 11^0$
$55 = 3^0 \cdot 5^1 \cdot 11^1$

i. $3^{\min(0,2)} \cdot 5^{\min(1,1)} \cdot 11^{\min(0,1)} = \underline{5}$

ii. $\underline{45a + 55b}$

iii. $\gcd(x, y) \neq 1$, <u>D.N.E</u>