BGCE Honours Project
Winter Term 2022/23

Surrogate Models in Structural Dynamics

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Abstract

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Keywords: this, that.

Introduction

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1 Random Dynamic System

The steady-state response of a Linear Time-Invariant (LTI) structural dynamic system can be found through solving the discretized equation of motion in the frequency domain instead of time domain. The equation of motion in time and frequency domain are given below.

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t) \quad \xrightarrow{\mathscr{F}} \quad \left(\mathbf{K} - \omega^2\mathbf{M} + j\omega\mathbf{C}\right)\mathbf{U}(\omega) = \mathbf{F}(\omega) \tag{1}$$

Equation 1 depends on structural and load parameters, which are subject to uncertainties. To account for this, the problem can be stated in a probabilistic setting, and the system of equations is reformulated as a function of a set of random variables $\mathbf{x}$:

$$\left[\mathbf{K}(\mathbf{x}) - \omega^2\mathbf{M}(\mathbf{x}) + j\omega\mathbf{C}(\mathbf{x})\right]\mathbf{U}(\omega, \mathbf{x}) = \mathbf{F}(\omega, \mathbf{x}) \tag{2}$$

One can obtain distribution of $\mathbf{U}(\omega, \mathbf{x})$ using Monte Carlo Simulation (MCS) i.e. to solve the equations multiple times, each time using a different sample of $\mathbf{x}$. For large and complex system, the computational cost of MCS is too high. Thus, one usually create a meta / surrogate model that approximate the response and is cheaper to compute.

2 Response Approximation

In the deterministic finite element method, the solution of the boundary value problem is approximated in functional space spanned by the shape functions, which reduces the problem to determining the coefficients of these functions. In a similar way, one can approximate a function of random variables in a functional space spanned by basis functions in terms of the input random variables.

2.1 Polynomial Chaos Expansion

Polynomial Chaos Expansion (PCE) approximates the response as a linear combination of a set of basis functions:

$$\mathbf{U}(\omega, \mathbf{x}) = \sum_{p=1}^{P} \mathbf{U}_p(\omega) \cdot \Psi_p(\mathbf{x}) \tag{3}$$

The basis functions are products of polynomial functions. The polynomials are chosen such that each pair is statistically independent. In addition, the polynomials can be normalised such that their covariance is the identity matrix.

$$\Psi_p(\mathbf{x}) = \prod_{\alpha=1}^{d} \psi_{p_\alpha}(x_\alpha) \quad ; \quad \int_{\Omega_x} \psi_i(x)\, \psi_j(x) \cdot f_X(x)\, dx = \delta_{ij} \tag{4}$$

2.2 Rational Polynomial Chaos Expansion

Another approach is to approximate the response as a ratio between two linear combinations of a set of basis functions:

$$U_i(\omega, \mathbf{x}) = \frac{\sum_{p=1}^{P} U_{ip}(\omega) \cdot \Psi_p(\mathbf{x})}{\sum_{q=1}^{Q} U_{iq}(\omega) \cdot \Psi_q(\mathbf{x})} \tag{5}$$

The basis functions used in Rational Polynomial Chaos Expansion (RPCE) is the same functions used in PCE.

Conclusion

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