

Spectral index:

$$\alpha = \frac{\log\left(\frac{S_{4.8}}{S_{1.4}}\right)}{\log\left(\frac{4.8}{1.4}\right)}$$

$$\sigma_\alpha = \sqrt{\left(\frac{\partial \alpha}{\partial S_{4.8}}\right)^2 \cdot (\sigma_{S_{4.8}})^2 + \left(\frac{\partial \alpha}{\partial S_{1.4}}\right)^2 \cdot (\sigma_{S_{1.4}})^2}$$

$$\frac{\partial \alpha}{\partial S_{4.8}} = \frac{\partial}{\partial S_{4.8}} \left(\frac{\log\left(\frac{S_{4.8}}{S_{1.4}}\right)}{\log\left(\frac{4.8}{1.4}\right)} \right) = \frac{1}{S_{4.8} \cdot \ln\left(\frac{4.8}{1.4}\right)}$$

$$\frac{\partial \alpha}{\partial S_{1.4}} = \frac{\partial}{\partial S_{1.4}} \left(\frac{\log\left(\frac{S_{4.8}}{S_{1.4}}\right)}{\log\left(\frac{4.8}{1.4}\right)} \right) = -\frac{1}{S_{1.4} \cdot \ln\left(\frac{4.8}{1.4}\right)}$$

$$\Rightarrow \sigma_\alpha = \sqrt{\left(\frac{\sigma_{S_{4.8}}}{S_{4.8}} \cdot \frac{1}{\ln\left(\frac{4.8}{1.4}\right)}\right)^2 + \left(-\frac{\sigma_{S_{1.4}}}{S_{1.4}} \cdot \frac{1}{\ln\left(\frac{4.8}{1.4}\right)}\right)^2}$$

$$= \frac{1}{\ln\left(\frac{4.8}{1.4}\right)} \cdot \sqrt{\left(\frac{\sigma_{S_{4.8}}}{S_{4.8}}\right)^2 + \left(\frac{\sigma_{S_{1.4}}}{S_{1.4}}\right)^2}$$

