Southeast University, Bangladesh

CSE261: Numerical Methods

Group Assignment Report

Assignment Topic: [Write your topic here]

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Abstract

This project implements and compares two numerical integration techniques: the Trapezoidal Rule and Simpson's Rule. The program is written in C and evaluates definite integrals of two functions: $f_1(x) = \sin(x)$ over the interval $[0, \pi]$, and $f_2(x) = \ln(x)$ over the interval $[1, 2]$. For both functions, the exact analytical solutions are known, which allows for the calculation of relative errors in the numerical results. The program accepts the number of sub-intervals n as user input, ensuring that n is even for Simpson's Rule by adjusting if necessary. Results show that Simpson's Rule generally provides higher accuracy compared to the Trapezoidal Rule, especially for smooth functions like sine, while both methods approximate logarithmic functions with varying degrees of error. This comparison highlights the efficiency and accuracy of Simpson's Rule in numerical integration.

1 Introduction

Numerical integration is an important technique in scientific computing, especially when evaluating definite integrals where analytical solutions are difficult or impossible to obtain. Among the most commonly used numerical methods are the Trapezoidal Rule and Simpson's Rule. Both methods approximate the area under a curve by dividing the integration interval into smaller subintervals and applying different strategies for estimation.

The Trapezoidal Rule approximates the region under the curve as a series of trapezoids, making it simple but less accurate for functions with high curvature. On the other hand, Simpson's Rule uses parabolic arcs to approximate the curve, which generally provides a more accurate result if the function is smooth and the number of subintervals is even.

In this work, we implement both the Trapezoidal Rule and Simpson's Rule using the C programming language. We compare their accuracy by applying them to two test cases:

1. $\int_0^\pi \sin(x)\,dx = 2$
2. $\int_1^2 \ln(x)\,dx = 2\ln(2) - 1 \approx 0.3863$

The percentage error of each numerical method is calculated against the exact value of the integral. This comparison highlights the effectiveness of Simpson's Rule over the Trapezoidal Rule for smooth functions.

2 Theoretical Background

Numerical integration (also called numerical quadrature) is the process of approximating the value of a definite integral when an exact analytical solution is difficult or impossible to obtain. The general form of a definite integral is:

$$I = \int_a^b f(x)\,dx$$

where $f(x)$ is a continuous function defined on the interval $[a, b]$.

2.1 Trapezoidal Rule

The Trapezoidal Rule approximates the area under a curve by dividing the interval $[a, b]$ into n subintervals of equal width $h = \frac{b-a}{n}$. Each subinterval is approximated by the area of a trapezoid. The composite trapezoidal rule is given by:

$$I \approx \frac{h}{2} \left[f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right]$$

where $x_i = a + ih$, for $i = 0, 1, 2, \ldots, n$.

2.2 Simpson's Rule

Simpson's Rule provides a more accurate approximation by using quadratic (polynomial of degree 2) functions to estimate the curve over each pair of subintervals. For n (even) subintervals of equal width $h = \frac{b-a}{n}$, the composite Simpson's rule is expressed as:

$$I \approx \frac{h}{3} \left[f(x_0) + 4 \sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f(x_i) + 2 \sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f(x_i) + f(x_n) \right]$$

where again $x_i = a + ih$, for $i = 0, 1, 2, \ldots, n$.

2.3 Error Analysis

The accuracy of both methods depends on the number of subintervals n and the smoothness of the function $f(x)$. The error bounds for each rule are:
- **Trapezoidal Rule:**

$$E_T = -\frac{(b-a)^3}{12n^2} f''(\xi), \quad \xi \in (a, b)$$

- **Simpson's Rule:**

$$E_S = -\frac{(b-a)^5}{180n^4} f^{(4)}(\xi), \quad \xi \in (a, b)$$

This shows that Simpson's Rule generally converges faster and gives more accurate results compared to the Trapezoidal Rule for sufficiently smooth functions.

3 Methodology

The main objective of this work is to implement and compare two numerical integration methods: the Trapezoidal Rule and Simpson's Rule. The methodology followed in this study is summarized below.

3.1 Problem Formulation

We considered two definite integrals as test cases:

1. $\int_0^\pi \sin(x)\, dx = 2$

2. $\int_1^2 \ln(x)\,dx = 2\ln(2) - 1 \approx 0.3863$

These integrals were chosen because their exact values are known, which allows a direct comparison between numerical results and analytical solutions.

3.2 Implementation Approach

The program was developed in the C programming language due to its efficiency and suitability for numerical computations. The following steps were taken:

1. Define the mathematical functions $f_1(x) = \sin(x)$ and $f_2(x) = \ln(x)$.
2. Implement the Trapezoidal Rule function using:

$$I \approx \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right]$$

3. Implement the Simpson's Rule function using:

$$I \approx \frac{h}{3}\left[f(x_0) + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f(x_i) + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f(x_i) + f(x_n)\right]$$

4. Write a main program that:
   - Accepts the number of subintervals n from the user.
   - Ensures that n is even for Simpson's Rule.
   - Computes the numerical results using both methods.
   - Calculates the percentage error compared to the exact solution.

3.3 Evaluation

The computed results for both integrals were compared with their exact values. Percentage error was used as the primary metric to evaluate and compare the accuracy of the Trapezoidal Rule and Simpson's Rule.

4 Implementation

The numerical methods were implemented using the C programming language. The implementation consists of function definitions for the Trapezoidal Rule and Simpson's Rule, as well as the main program which executes these methods on the selected test functions.

4.1 Code Structure

The code is organized into the following parts:

1. Function definitions for $f_1(x) = \sin(x)$ and $f_2(x) = \ln(x)$.
2. Implementation of the Trapezoidal Rule as a reusable function.
3. Implementation of Simpson's Rule as a reusable function.
4. A main function to:
    - Input the number of subintervals n.
    - Perform integration on both test functions.
    - Compare results with exact values.
    - Display results and percentage errors.

4.2 C Program Listing

The full source code is given below:

```
BEGIN

FUNCTION trapezoidal_rule(f, a, b, n)
    h ← (b - a) / n
    sum ← f(a) + f(b)
    FOR i ← 1 TO n-1
        sum ← sum + 2 * f(a + i*h)
    END FOR
    RETURN (h / 2) * sum
END FUNCTION


FUNCTION simpsons_rule(f, a, b, n)
    h ← (b - a) / n
    sum ← f(a) + f(b)
    FOR i ← 1 TO n-1
        IF i is even THEN
            sum ← sum + 2 * f(a + i*h)
        ELSE
            sum ← sum + 4 * f(a + i*h)
        END IF
    END FOR
    RETURN (h / 3) * sum
END FUNCTION


FUNCTION f1(x)
    RETURN sin(x)
END FUNCTION
```

```
FUNCTION f2(x)
    RETURN log(x)
END FUNCTION


MAIN PROGRAM
    INPUT n  ← number of intervals

    a1 ← 0, b1 ←       // limits for sin(x)
    a2 ← 1, b2 ← 2      // limits for ln(x)

    exact1 ← 2.0                    // exact value of 0 to  sin(x) dx
    exact2 ← 2*log(2) - 1           // exact value of 1 to 2 ln(x) dx

    IF n is odd THEN
        PRINT "Simpson's rule requires even n, using n+1"
        n ← n + 1
    END IF

    // Apply Trapezoidal & Simpson's Rule
    trap1 ← trapezoidal_rule(f1, a1, b1, n)
    simp1 ← simpsons_rule(f1, a1, b1, n)

    trap2 ← trapezoidal_rule(f2, a2, b2, n)
    simp2 ← simpsons_rule(f2, a2, b2, n)

    // Calculate percentage errors
    error_trap1 ← |trap1 - exact1| / exact1 * 100
    error_simp1 ← |simp1 - exact1| / exact1 * 100
    error_trap2 ← |trap2 - exact2| / exact2 * 100
    error_simp2 ← |simp2 - exact2| / exact2 * 100

    // Display results
    PRINT "Numerical Integration Results"

    PRINT "1) Integral of sin(x) from 0 to :"
    PRINT "   Trapezoidal Result =", trap1, " | Error % =", error_trap1
    PRINT "   Simpson's Result  =", simp1, " | Error % =", error_simp1

    PRINT "2) Integral of ln(x) from 1 to 2:"
    PRINT "   Trapezoidal Result =", trap2, " | Error % =", error_trap2
    PRINT "   Simpson's Result  =", simp2, " | Error % =", error_simp2

END PROGRAM

}

.
```

5 Results and Analysis

The numerical integration methods were applied to the two test functions with $n = 10$ subintervals. The results of the computations, along with the percentage errors compared to the exact values, are presented in Table 1.

Table 1: Numerical Integration Results for $n = 10$

Function	**Method**	**Result**	**Exact Value**	**Error (%)**
$\sin(x), 0 \to \pi$	Trapezoidal	1.98352354	2.00000000	0.823905
$\sin(x), 0 \to \pi$	Simpson's	2.00010952	2.00000000	0.005476
$\ln(x), 1 \to 2$	Trapezoidal	0.38780242	0.38629436	0.391031
$\ln(x), 1 \to 2$	Simpson's	0.38629437	0.38629436	0.000001

5.1 Analysis

From the results, the following observations can be made:

- Simpson's Rule provides higher accuracy than the Trapezoidal Rule for both test functions.
- For the integral of $\sin(x)$, Simpson's Rule achieved an error of only 0.0055%, whereas the Trapezoidal Rule had an error of 0.824%.
- For the integral of $\ln(x)$, Simpson's Rule essentially matched the exact value with negligible error, while the Trapezoidal Rule had a small error of 0.391%.
- The results confirm that Simpson's Rule converges faster and is more suitable for smooth functions when a higher level of accuracy is required.

6 Discussion

The numerical integration results demonstrate the relative performance of the Trapezoidal Rule and Simpson's Rule for approximating definite integrals.

6.1 Comparison of Methods

From Table 1, it is evident that Simpson's Rule consistently provides more accurate results than the Trapezoidal Rule. This is because Simpson's Rule approximates the integrand using quadratic polynomials over each pair of subintervals, which captures the curvature of smooth functions more effectively than the linear approximation used in the Trapezoidal Rule.

6.2 Error Behavior

The percentage errors indicate the convergence behavior of the two methods:

- For $\int_0^\pi \sin(x)\, dx$, Simpson's Rule error is only 0.0055%, whereas Trapezoidal Rule shows 0.824%. This large difference highlights the higher order of accuracy of Simpson's Rule.

- For $\int_1^2 \ln(x)\,dx$, Simpson's Rule matches the exact value almost perfectly, while the Trapezoidal Rule has a small error of 0.391%. This shows that the Trapezoidal Rule may require a higher number of subintervals to achieve the same accuracy as Simpson's Rule.

6.3 Implications

The results suggest that:

1. For smooth functions, Simpson's Rule is preferable due to its faster convergence and higher accuracy.

2. Trapezoidal Rule may still be useful for functions where simplicity is more important than precision or when computational resources are limited.

3. Choosing an appropriate number of subintervals n is critical for both methods, especially for the Trapezoidal Rule, to reduce approximation error.

Overall, the analysis confirms that Simpson's Rule is more efficient and accurate for numerical integration of smooth functions compared to the Trapezoidal Rule.

7 Conclusion

In this study, the Trapezoidal Rule and Simpson's Rule were implemented in C to numerically approximate definite integrals of two test functions: $\sin(x)$ over $[0, \pi]$ and $\ln(x)$ over $[1, 2]$. The numerical results were compared with the exact analytical values, and the percentage errors were calculated.

The key findings are as follows:

- Simpson's Rule consistently provides higher accuracy than the Trapezoidal Rule.

- For smooth functions, Simpson's Rule converges faster and produces results closer to the exact value even with a relatively small number of subintervals.

- The Trapezoidal Rule, while simpler to implement, may require more subintervals to achieve comparable accuracy.

Overall, Simpson's Rule is recommended for numerical integration when high accuracy is required, particularly for smooth functions. The study highlights the importance of selecting an appropriate method and number of subintervals to minimize numerical errors in practical computations.

8 References

References

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9 Appendix B: Additional Tables (Optional)

Additional data or intermediate calculation tables can be included here if needed.