# Types of Matrices with Expressions

Here is a summary of important matrices, their defining expressions, and examples:

| Matrix Type | Expression / Condition | Example |
|---|---|---|
| **Row Matrix** | Only one row: $A = [a_1\ a_2\ \dots\ a_n]$ | $A = [2\ \ 5\ \ 7]$ |
| **Column Matrix** | Only one column: $A = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$ | $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$ |
| **Square Matrix** | Rows = Columns: $A \in M_{n\times n}$ | $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ |
| **Zero (Null) Matrix** | All entries zero: $A = 0$ | $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ |
| **Diagonal Matrix** | $a_{ij} = 0$ for $i \neq j$ | $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ |
| **Scalar Matrix** | Diagonal with equal elements: $a_{ii} = k$ | $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ |
| **Identity Matrix** | $a_{ii} = 1,\ a_{ij} = 0$ for $i \neq j$ | $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ |
| **Upper Triangular** | $a_{ij} = 0$ for $i > j$ | $\begin{bmatrix} 2 & 3 & 1 \\ 0 & 5 & 7 \\ 0 & 0 & 6 \end{bmatrix}$ |
| **Lower Triangular** | $a_{ij} = 0$ for $i < j$ | $\begin{bmatrix} 4 & 0 & 0 \\ 8 & 5 & 0 \\ 2 & 9 & 7 \end{bmatrix}$ |
| **Symmetric Matrix** | $A^T = A$ | $\begin{bmatrix} 1 & 3 & 2 \\ 3 & 4 & -1 \\ 2 & -1 & 5 \end{bmatrix}$ |
| **Skew-Symmetric** | $A^T = -A$ | $\begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix}$ |
| **Orthogonal Matrix** | $AA^T = I$ | $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ |
| **Idempotent Matrix** | $A^2 = A$ | $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ |
| **Involutory Matrix** | $A^2 = I$ | $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ |
| **Singular Matrix** | $\det(A) = 0$ | $\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$ |
| **Non-Singular Matrix** | $\det(A) \neq 0$ | $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ |
| **Hermitian Matrix** | $A^* = A$ (complex conjugate transpose) | $\begin{bmatrix} 2 & 3+i \\ 3-i & 4 \end{bmatrix}$ |
| **Skew-Hermitian** | $A^* = -A$ | $\begin{bmatrix} 0 & 2+i \\ -2+i & 0 \end{bmatrix}$ |