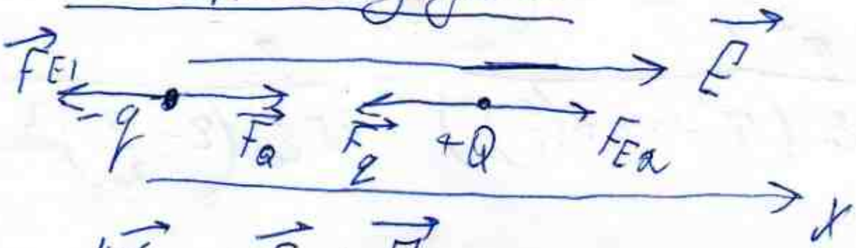
Зачётные задачи

1) Дано:
$-q, +Q$
m, M
E

$d - ?$



$$m\vec{a}_1 = \vec{F}_Q + \vec{F}_{E1}$$

$$x:\ ma_{1x} = \frac{Qq}{4\pi\varepsilon_0 d^2} - qE$$

$$M\vec{a}_2 = \vec{F}_q + \vec{F}_{E2}$$

$$x:\ Ma_{2x} = QE - \frac{qQ}{4\pi\varepsilon_0 d^2}$$

$$a_{1x} = a_{2x}$$

$$\frac{Qq}{4\pi\varepsilon_0 d^2 m} - \frac{qE}{m} = \frac{QE}{M} - \frac{qQ}{4\pi\varepsilon_0 d^2 M}$$

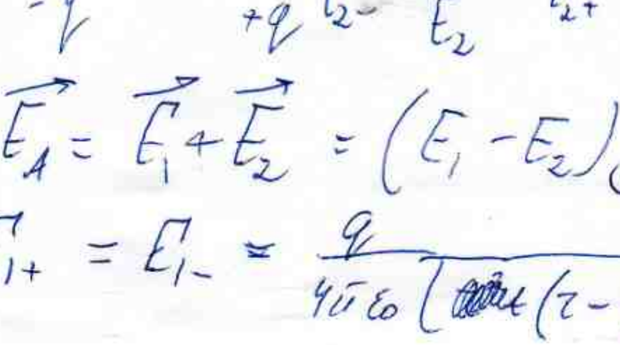
$$\frac{qQ}{4\pi\varepsilon_0 d^2}\left(\frac{1}{m} + \frac{1}{M}\right) = E\left(\frac{Q}{M} + \frac{q}{m}\right)$$

$$\frac{qQ}{4\pi\varepsilon_0 d^2}\left(\frac{M+m}{mM}\right) = E\left(\frac{Qm + qM}{Mm}\right)$$

$$d^2 = \frac{Qq(M+m)}{4\pi\varepsilon_0 E(Qm+qM)};\quad d = \sqrt{\frac{Qq(M+m)}{4\pi\varepsilon_0 E(Qm+qM)}}$$

2) Дано: $q, r, a, r \gg a$

$E_a - ?$



$$\vec{E}_A = \vec{E}_1 + \vec{E}_2 = (E_1 - E_2)\vec{j}$$

$$E_{1+} = E_{1-} = \frac{q}{4\pi\varepsilon_0\left[\left(r - \frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right]}$$

$$\left(r-\frac{a}{2}\right)^2 = r^2\left(1-\frac{a}{2r}\right)^2 \cong r^2\left(1-\frac{a}{r}\right) = r^2 - ar = r(r-a)$$

$$E_{1+} = E_{1-} \cong \frac{q}{4\pi\varepsilon_0\left(r^2 - ar + \left(\frac{a}{2}\right)^2\right)} = \frac{q}{4\pi\varepsilon_0\left(r-\frac{a}{2}\right)^2} =$$

$$\cong \frac{q}{4\pi\varepsilon_0 r(r-a)}.$$

$$E_1 = 2E_{1+}\cos\alpha$$

$$\cos\alpha = \frac{a}{2\left(\left(r-\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right)^{1/2}} \cong \frac{a}{2\left(r^2 - ar + \left(\frac{a}{2}\right)^2\right)^{1/2}} =$$

$$= \frac{a}{2\left(r-\frac{a}{2}\right)} = \frac{a}{(2r-a)}$$

$$E_1 = \frac{qa}{2\pi\varepsilon_0 r(r-a)(2r-a)}$$

$$E_{2+} = E_{2-} = \frac{q}{4\pi\varepsilon_0\left(\left(r+\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right)}$$

$$\left(r+\frac{a}{2}\right)^2 = r^2\left(1+\frac{a}{2r}\right)^2 \cong r^2\left(1+\frac{a}{r}\right) = r^2 + ar = r(r+a)$$

$$E_{2+} = E_{2-} \cong \frac{q}{4\pi\varepsilon_0\left(r^2 + ar + \left(\frac{a}{2}\right)^2\right)} = \frac{q}{4\pi\varepsilon_0\left(r+\frac{a}{2}\right)^2} = \frac{q}{4\pi\varepsilon_0 r(r+a)}$$

$$E_2 = 2E_{2+}\cos\beta$$

$$\cos\beta = \frac{a}{2\left(\left(r+\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right)^{1/2}} = \frac{a}{2\left(r^2 + ar + \left(\frac{a}{2}\right)^2\right)^{1/2}} = \frac{a}{2\left(r+\frac{a}{2}\right)} =$$

$$= \frac{a}{(2r+a)}.$$

$$E_2 = \frac{qa}{2\pi\varepsilon_0 r(r+a)(2r+a)}$$

$$E = E_1 - E_2 = \frac{qa}{2\pi\varepsilon_0 r}\left(\frac{1}{(r-a)(2r-a)} - \frac{1}{(r+a)(2r+a)}\right) =$$

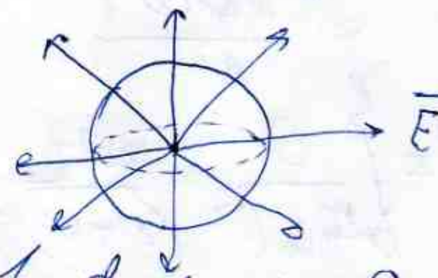
$$= \frac{qa}{2\pi\varepsilon_0 r}\left(\frac{(r+a)(2r+a)-(r-a)(2r-a)}{(r-a)(2r-a)(r+a)(2r+a)}\right) = \frac{qa}{2\pi\varepsilon_0 r}\left(\frac{\cancel{2r^2}+2ar+ar+\cancel{a^2}-\cancel{2r^2}+2ar+ar-\cancel{a^2}}{(r^2-a^2)(4r^2-a^2)}\right) = \frac{qa\cdot 6ar}{2\pi\varepsilon_0 r(4r^4-4r^2a^2-a^2r^2+a^2)} =$$

$$= \frac{3qa^2}{\varepsilon_0\pi(4r^4-5r^2a^2+a^2)} \approx \boxed{\frac{3qa^2}{4\varepsilon_0\pi r^4}}$$

3) Дано:

ρ

$\vec{E}(\vec{r})$ - ?



$\vec{E} \uparrow\uparrow \vec{r}$.

$\operatorname{div}\vec{E} = \frac{\rho}{\varepsilon_0}$.

$$\frac{1}{r^2}\frac{d}{dr}(Er^2) = \frac{\rho}{\varepsilon_0}$$

$$\frac{d}{dr}(Er^2) = \frac{\rho r^2}{\varepsilon_0}$$

$$Er^2 = \frac{\rho r^3}{3\varepsilon_0} + C; \quad E = \frac{\rho r}{3\varepsilon_0} + \frac{C}{r^2}$$

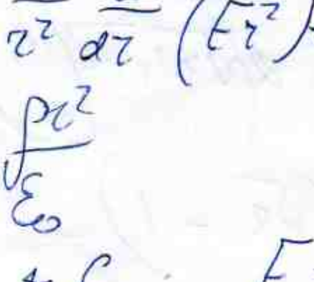
$$E(0) = 0; \quad C = 0$$

$$E = \frac{\rho r}{3\varepsilon_0}; \quad \boxed{\vec{E} = \frac{\rho\vec{r}}{3\varepsilon_0}}$$

4) Дано:

R, Q

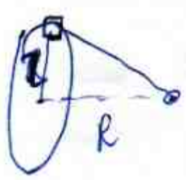
$q_ш$ - ?



$\varphi_ш = 0$

$\varphi_ш = \frac{q_ш}{4\pi\varepsilon_0 R} + \varphi_{диска} = 0.$

Потенциал кольца



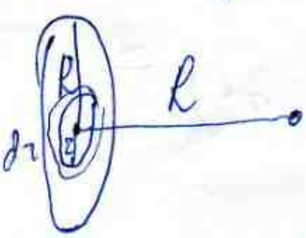
$$d\varphi = \frac{dq}{4\pi\varepsilon_0(r^2+R^2)^{1/2}}$$

$dq = \lambda dl; \quad \lambda = \frac{q}{2\pi r}; \quad dl = r\,d\beta$

$dq = \frac{q}{2\pi r} r\,d\varphi = \frac{q\,d\beta}{2\pi}$

$d\varphi = \frac{q\,d\beta}{8\pi^2\varepsilon_0 (r^2+R^2)^{1/2}}$; $\varphi = \int_0^{2\pi} \frac{q\,d\beta}{8\pi^2\varepsilon_0 (r^2+R^2)^{1/2}} = \frac{q}{4\pi\varepsilon_0 (r^2+R^2)^{1/2}}$

Потенциал диска



$d\varphi = \frac{dq}{4\pi\varepsilon_0 (r^2+R^2)^{1/2}}$

$dq = \sigma\,dS$; $dS = 2\pi r\,dr$; $\sigma = \frac{Q}{\pi R^2}$

$dq = \frac{Q}{\pi R^2}\cdot 2\pi r\,dr = \frac{2Q r\,dr}{R^2}$

$d\varphi = \frac{Q r\,dr}{2\pi\varepsilon_0 R^2 (r^2+R^2)^{1/2}}$; $\varphi = \int_0^R \frac{Q}{2\pi\varepsilon_0 R^2}\frac{r\,dr}{(r^2+R^2)^{1/2}} =$

$= \frac{Q}{4\pi\varepsilon_0 R^2}\int_0^R \frac{d(r^2+R^2)}{(r^2+R^2)^{1/2}} = \frac{Q}{4\pi\varepsilon_0 R^2}\, 2\left(\sqrt{r^2+R^2}\Big|_0^R\right) =$

$= \frac{Q}{2\pi\varepsilon_0 R^2}(R\sqrt{2} - R) = \frac{Q}{2\pi\varepsilon_0 R}(\sqrt{2}-1)$

$\frac{q_ш}{4\pi\varepsilon_0 R} + \frac{Q}{2\pi\varepsilon_0 R}(\sqrt{2}-1) = 0$

$\boxed{q_ш = -2Q(\sqrt{2}-1)}$

5) Дано:

d_1, q_1

d_2, q_2

Δq - ?

$\begin{cases} q_1 + q_2 = q_1' + q_2' \\ \varphi_1' = \varphi_2' \end{cases}$

$\begin{cases} q_1 + q_2 = q_1' + q_2' \\ \frac{2q_1'}{4\pi\varepsilon_0 d_1} = \frac{2q_2'}{4\pi\varepsilon_0 d_2} \end{cases}$ $\qquad \frac{q_1'}{d_1} = \frac{q_2'}{d_2}$

$$q_2' = q_1' \frac{d_2}{d_1}$$

$$q_1 + q_2 = q_1'\left(1 + \frac{d_2}{d_1}\right); \quad q_1 + q_2 = q_1'\left(\frac{d_1 + d_2}{d_1}\right)$$

$$q_1' = \frac{(q_1 + q_2)}{(d_1 + d_2)} d_1$$

$$\Delta q = |q_1' - q_1| = \left|\frac{(q_1 + q_2)d_1}{d_1 + d_2} - q_1\right| = \frac{|\cancel{q_1 d_1} + q_2 d_1 - \cancel{q_1 d_1} - q_1 d_2|}{d_1 + d_2} =$$

$$= \boxed{\frac{|q_2 d_1 - q_1 d_2|}{d_1 + d_2}}$$

6.

Дано:

r, R, ε

$C - ?$

$$C = \frac{q}{\varphi_{пров}}$$

$$\varphi(r) = \int_x^{\infty} E(x)dx =$$

$$= \int_x^R \frac{q}{4\pi\varepsilon\varepsilon_0 x^2} dx + \int_R^{\infty} \frac{q\,dx}{4\pi\varepsilon_0 x^2} =$$

$$= \frac{q}{4\pi\varepsilon\varepsilon_0}\left(\frac{1}{x} - \frac{1}{R}\right) + \frac{q}{4\pi\varepsilon_0 R} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\varepsilon x} - \frac{1}{\varepsilon R} + \frac{1}{R}\right)$$

$$\varphi_{пров} = \varphi(r) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\varepsilon r} - \frac{1}{\varepsilon R} + \frac{1}{R}\right) = \frac{q}{4\pi\varepsilon_0 \varepsilon R r}(R - r + \varepsilon r) =$$

$$= \frac{q}{4\pi\varepsilon_0\varepsilon R r}(R + r(\varepsilon - 1)) = \frac{q}{4\pi\varepsilon\varepsilon_0 r}\left(1 + \frac{r}{R}(\varepsilon - 1)\right)$$

$$C = \frac{q}{\varphi_{пров}} = \frac{4\pi\varepsilon\varepsilon_0 r}{\left(1 + \frac{r}{R}(\varepsilon - 1)\right)}.$$

7. Дано:

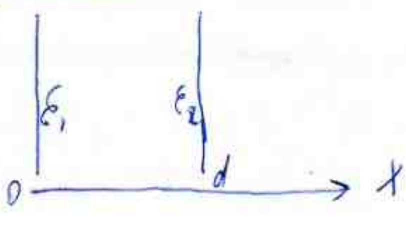
$\varepsilon_1, \varepsilon_2$

$\varepsilon_2 < \varepsilon_1$

d, S

$C - ?$

Решение:

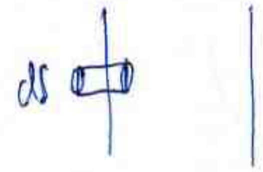


$$\varepsilon(x) = m - kx$$

$$\varepsilon_1 = m$$

$$\varepsilon_2 = \varepsilon_1 - kd; \quad k = \frac{\varepsilon_1 - \varepsilon_2}{d}.$$

$$\varepsilon(x) = \varepsilon_1 - \frac{(\varepsilon_1 - \varepsilon_2)}{d}x = \frac{\varepsilon_1 d - (\varepsilon_1 - \varepsilon_2)x}{d}$$



$$\oint \vec{D}\, d\vec{S} = q_{св}$$

$$(D_{2n} - D_{1n})\, dS = \sigma_{св}\, dS$$

$$D_{1n} = 0$$

$$D = \sigma_{св}$$

$$D = \frac{Q}{S}; \quad E = \frac{D}{\varepsilon\varepsilon_0}$$

$$E = \frac{Q}{S\varepsilon\varepsilon_0} = \frac{Qd}{S\varepsilon_0(\varepsilon_1 d - (\varepsilon_1 - \varepsilon_2)x)}$$

$$\Delta\varphi = \int_0^d \frac{Qd\, dx}{S\varepsilon_0(\varepsilon_1 d - (\varepsilon_1 - \varepsilon_2)x)} = -\frac{Qd}{S\varepsilon_0(\varepsilon_1 - \varepsilon_2)}\left(\ln(\varepsilon_1 d - (\varepsilon_1 - \varepsilon_2)x)\Big|_0^d\right) =$$

$$= -\frac{Qd}{\varepsilon_0 S(\varepsilon_1 - \varepsilon_2)} \ln\frac{\varepsilon_2 d}{\varepsilon_1 d} = \frac{Qd}{\varepsilon_0 S(\varepsilon_1 - \varepsilon_2)} \ln\frac{\varepsilon_1}{\varepsilon_2}$$

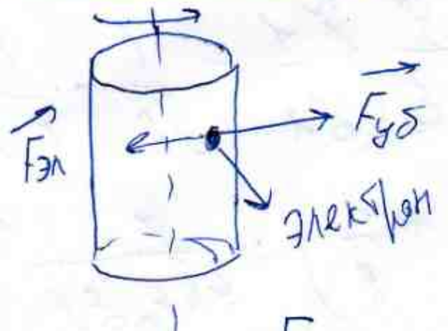
$$\boxed{C = \frac{Q}{\Delta\varphi} = \frac{S\varepsilon_0(\varepsilon_1 - \varepsilon_2)}{d\ln(\varepsilon_1/\varepsilon_2)}}$$

(8) Дано:

n, D

$E - ?$

$U - ?$



$$F_{уб} = F_{эл}$$

$$F_{уб} = m\omega^2 r$$

$$\omega = \frac{2\pi}{T} = 2\pi n.$$

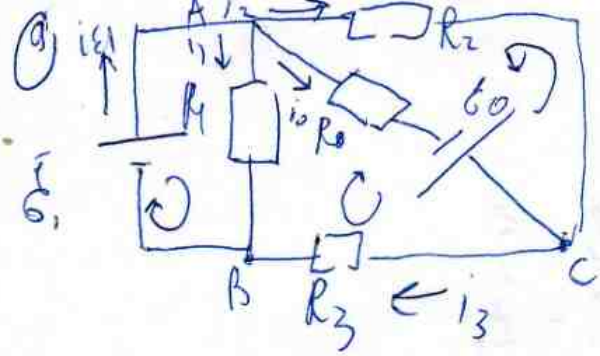
$$F_{эл} = mr \cdot 4\pi^2 n^2.$$

$$\boxed{E = \frac{F_{эл}}{e} = \left(\frac{m}{e}\right) 4\pi^2 r n^2}$$

$$U = \int_0^{D/2} \frac{m}{e} \cdot 4\pi^2 n^2 r\, dr = \frac{m}{e} 4\pi^2 n^2 \frac{1}{2}\frac{D^2}{4} = \frac{mD^2}{8e} 4\pi^2 n^2 =$$

$$= \frac{mD^2}{8e}\omega^2.$$

9



I_{R_0} - ?

A: $i_{\mathcal{E}_1} = i_0 + i_1 + i_2$

B: $i_3 = i_0 + i_2$

$AB\mathcal{E}_1A$: $i_1 R_1 = \mathcal{E}_1$

$A\mathcal{E}_0CBA$: $i_0 R_0 + i_3 R_3 - i_1 R_1 = \mathcal{E}_0$

$CA\mathcal{E}_0C$: $i_0 R_0 - i_2 R_2 = \mathcal{E}_0$.

$$\begin{cases} i_0 + i_1 + i_2 - i_{\mathcal{E}_1} = 0 \\ i_0 + i_2 - i_3 = 0 \\ i_1 R_1 = \mathcal{E}_1 \\ i_0 R_0 - i_1 R_1 + i_3 R_3 = \mathcal{E}_0 \\ i_0 R_0 - i_2 R_2 = \mathcal{E}_0 \end{cases} ; \quad \begin{vmatrix} 1 & 1 & 1 & 0 & -1 \\ 1 & 0 & 1 & -1 & 0 \\ 0 & R_1 & 0 & 0 & 0 \\ R_0 & -R_1 & 0 & R_3 & 0 \\ R_0 & 0 & -R_2 & 0 & 0 \end{vmatrix} =$$

$$= - \begin{vmatrix} 1 & 0 & 1 & -1 \\ 0 & R_1 & 0 & 0 \\ R_0 & -R_1 & 0 & R_3 \\ R_0 & 0 & -R_2 & 0 \end{vmatrix} = -R_1 \begin{vmatrix} 1 & 1 & -1 \\ R_0 & 0 & R_3 \\ R_0 & -R_2 & 0 \end{vmatrix} = -R_1 (R_0 R_3 + R_0 R_2 + R_2 R_3) =$$

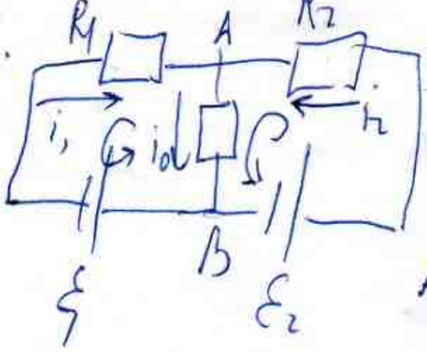
$$= R_1 R_2 \left(R_0 \left(1 + \frac{R_3}{R_2} \right) + R_3 \right)$$

$$\begin{vmatrix} 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 \\ \mathcal{E}_1 & R_1 & 0 & 0 & 0 \\ \mathcal{E}_0 & -R_1 & 0 & R_3 & 0 \\ \mathcal{E}_0 & 0 & -R_2 & 0 & 0 \end{vmatrix} = - \begin{vmatrix} 0 & 0 & 1 & -1 \\ \mathcal{E}_1 & R_1 & 0 & 0 \\ \mathcal{E}_0 & -R_1 & 0 & R_3 \\ \mathcal{E}_0 & 0 & -R_2 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & -1 \\ \mathcal{E}_1 & R_1 & 0 & 0 \\ \mathcal{E}_0 & -R_1 & R_3 & R_3 \\ \mathcal{E}_0 & 0 & -R_2 & 0 \end{vmatrix} =$$

$$= \begin{vmatrix} \mathcal{E}_1 & R_1 & 0 \\ \mathcal{E}_0 & -R_1 & R_3 \\ \mathcal{E}_0 & 0 & -R_2 \end{vmatrix} = -(\mathcal{E}_1 R_1 R_2 + \mathcal{E}_0 R_1 R_3 + \mathcal{E}_0 R_1 R_2) = -R_1 R_2 \left(\mathcal{E}_0 \left(1 + \frac{R_3}{R_2} \right) + \mathcal{E}_1 \right)$$

$$\boxed{i_0 = \frac{\mathcal{E}_0 \left(1 + \frac{R_3}{R_2} \right) + \mathcal{E}_1}{R_0 \left(1 + \frac{R_3}{R_2} \right) + R_3}}$$

(10).



$P = J^2 R$

$A: \; i_0 - i_1 - i_2 = 0$

$A\xi_1 BA: \; -i_0 R - i_1 R_1 = \xi_1$

$A\mathcal{E}_2 BA: \; i_0 R + i_2 R_2 = \mathcal{E}_2$

$$\begin{vmatrix} 1 & -1 & -1 \\ -R & -R_1 & 0 \\ R & 0 & R_2 \end{vmatrix} = -R_1R_2 - R_1R - R_2R.$$

$$\begin{vmatrix} 0 & -1 & -1 \\ \xi_1 & -R_1 & 0 \\ \mathcal{E}_2 & 0 & R_2 \end{vmatrix} = -\mathcal{E}_2 R_1 + \xi_1 R_2$$

$$i_0 = \frac{\mathcal{E}_2 R_1 - \mathcal{E}_1 R_2}{R_1R_2 + R(R_1+R_2)}$$

$$P = \frac{(\mathcal{E}_2 R_1 - \mathcal{E}_1 R_2)^2 R}{\left(R_1R_2 + R(R_1+R_2)\right)^2}$$

$$\frac{dP}{dR} = \frac{(\mathcal{E}_2 R_1 - \mathcal{E}_1 R_2)^2}{\left(R_1R_2 + R(R_1+R_2)\right)^4}\left(\left(R_1R_2 + R(R_1+R_2)\right)^2 - 2R(R_1+R_2)\left(R_1R_2 + R(R_1+R_2)\right)\right) = 0$$

$$R_1R_2 + R(R_1+R_2) - 2R(R_1+R_2) = 0$$

$$R(R_1+R_2) = R_1R_2$$

$$\boxed{R = \frac{R_1R_2}{R_1+R_2}}$$

$i_0 = 0 \Rightarrow \mathcal{E}_2 R_1 - \xi_1 R_2 = 0; \; \mathcal{E}_2 R_1 = \xi_1 R_2 \Rightarrow$

$$\boxed{\frac{\xi_1}{\mathcal{E}_2} = \frac{R_1}{R_2}}$$

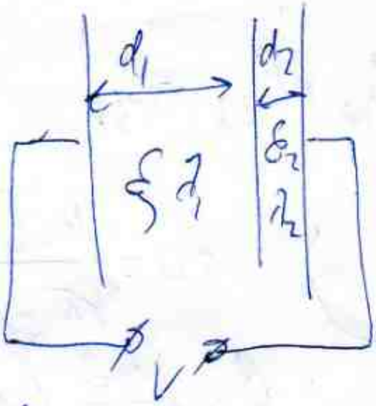
(11) Дано:

$d_1\ d_2$

$\varepsilon_1 \lambda_1,\ \varepsilon_2 \lambda_2$

V

$\sigma - ?$



$$\sigma = D_2 - D_1$$

$$j_1 = j_2$$

$$\begin{cases} \lambda_1 E_1 = \lambda_2 E_2. \\ E_1 d_1 + E_2 d_2 = V \end{cases}$$

$$E_2 = \frac{\lambda_1}{\lambda_2} E_1$$

$$E_1\left(d_1 + \frac{\lambda_1}{\lambda_2} d_2\right) = V \ ; \ E_1\left(\frac{d_1\lambda_2 + d_2\lambda_1}{\lambda_2}\right) = V.$$

$$E_1 = \frac{V\lambda_2}{\lambda_2 d_1 + \lambda_1 d_2}$$

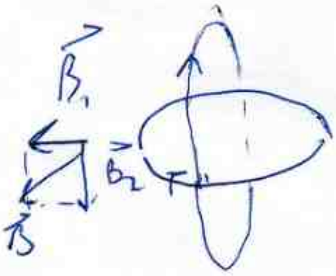
$$E_2 = \frac{V\lambda_1}{\lambda_1 d_2 + \lambda_2 d_1}$$

$$\sigma = \varepsilon_0(\varepsilon_2 E_2 - \varepsilon_1 E_1) = \frac{\varepsilon_0 V}{(\lambda_1 d_1 + \lambda_2 d_1)}(\varepsilon_2 \lambda_1 - \varepsilon_1 \lambda_2)$$

(12) Дано:

$R,\ I$

$B_0 - ?$



$$B_1 = B_2 ; \quad B_0 = B_1\sqrt{2}.$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{[d\vec{l}\ \vec{r}]}{r^3}$$

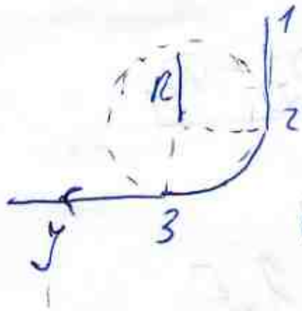
$$dB = \frac{\mu_0 I}{4\pi} \frac{R\,d\varphi\,R}{R^3} = \frac{\mu_0 I}{4\pi R} d\varphi$$

$$B = \frac{\mu_0 I}{4\pi R} \int_0^{2\pi} d\varphi = \frac{\mu_0 I}{2R}.$$

$$B_0 = \frac{\mu_0 I}{2R}\sqrt{2} = \boxed{\frac{\mu_0 I}{R\sqrt{2}}}.$$

13. Дано:
J, R

B_0 - ?



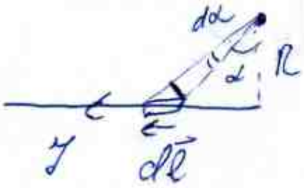
$B_0 = B_{12} + B_{23} + B_{34}$.

2-3:

$dB = \frac{\mu_0 J}{4\pi}\frac{d\varphi}{R}$;

$B = \frac{\mu_0 J}{4\pi R}\int_0^{\pi/2} d\varphi = \frac{\mu_0 J}{8R}$.

$B_{12} = B_{34}$



$d\vec{B} = \frac{\mu_0 J}{4\pi}\frac{[d\vec{l}\,\vec{r}]}{r^3}$

$r = \frac{R}{\cos\alpha}$

$dl = \frac{r\,d\alpha}{\cos\alpha} = \frac{R}{\cos^2\alpha}\,d\alpha$

$dB = \frac{\mu_0 J}{4\pi}\frac{R}{\cos^2\alpha}\,d\alpha\cdot\sin(\pi/2+\alpha) = \frac{\mu_0 J R \cancel{\cos\alpha}\,d\alpha}{4\pi \cancel{\cos^2\alpha}\cdot R^3}\cdot\cos^3\alpha\,\frac{R}{\cancel{\cos\alpha}} =$

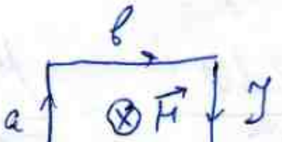
$= \frac{\mu_0 J}{4\pi R}\cos\alpha\,d\alpha$

$B = \frac{\mu_0 J}{4\pi R}\int_0^{\pi/2}\cos\alpha\,d\alpha = \frac{\mu_0 J}{4\pi R}$.

$B = \frac{\mu_0 J}{2\pi R} + \frac{\mu_0 J}{8R} = \frac{\mu_0 J}{8\pi R}(\pi + 4)$

14. Дано:
a, b, J

H_0 - ?



$\vec{B} = \mu\mu_0\vec{H}$, $\mu = 1$.

$\vec{H} = \frac{\vec{B}}{\mu_0}$.

$H = \frac{1}{\mu_0}(2B_b + 2B_a) = \frac{2}{\mu_0}(B_b + B_a)$.

b:

$d\vec{B} = \frac{\mu_0 J}{4\pi}\frac{[d\vec{l}\,\vec{r}]}{r^3}$.

$r = \frac{a}{2\cos\alpha}$; $dl = \frac{r\,d\alpha}{\cos\alpha} = \frac{a\,d\alpha}{2\cos^2\alpha}$.

$$dB = \frac{\mu_0 J}{4\pi} \cdot \frac{a\,d\alpha}{2\cos^2\alpha} \cdot \frac{a}{2\cos\alpha} \cdot \sin(\pi/2+\alpha) \cdot \frac{8\cos^3\alpha}{a^3} =$$

$$= \frac{\mu_0 J}{2\pi a} \cos\alpha\, d\alpha$$

$$B = \frac{\mu_0 J}{2\pi a} \int\limits_{-\arcsin\frac{b}{\sqrt{a^2+b^2}}}^{\arcsin\frac{b}{\sqrt{a^2+b^2}}} \cos\alpha\, d\alpha = \frac{\mu_0 J}{\pi a} \frac{b}{\sqrt{a^2+b^2}}$$

a: $dB = \frac{\mu_0 J}{2\pi b} \cos\alpha\, d\alpha$

$$B = \frac{\mu_0 J}{2\pi b} \int\limits_{-\arcsin\frac{a}{\sqrt{a^2+b^2}}}^{\arcsin\frac{a}{\sqrt{a^2+b^2}}} \cos\alpha\, d\alpha = \frac{\mu_0 J a}{\pi b\sqrt{a^2+b^2}}$$

Вых $H_0 = \frac{2}{\mu_0}\left(\frac{\mu_0 J a}{\pi b\sqrt{a^2+b^2}} + \frac{\mu_0 J b}{\pi a\sqrt{a^2+b^2}}\right) =$

$$= \frac{2J}{\pi ab}\left(\frac{a^2+b^2}{\sqrt{a^2+b^2}}\right) = \frac{2J}{\pi ab}\sqrt{a^2+b^2}.$$

15. Дано:
R, ω, Q

p_m - ?

$p_m = \int J(S)\, dS$

$S = \frac{\pi r^2}{2}$; $dS = \pi r\, dr$

$q(r) = \frac{Q}{\pi R^2} \cdot \frac{\pi r^2}{2} = \frac{Q r^2}{R^2 \cdot 2}$

$T = \frac{2\pi}{\omega}$

$J(r) = \frac{Q r^2 \omega}{4\pi R^2}$.

$dp_m = \frac{Q r^3 \omega\, dr}{4\pi R^2 \cdot R}$

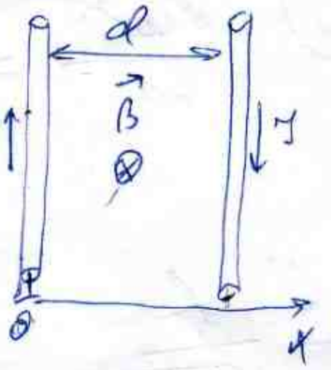
$p_{m1} = \frac{Q\omega}{4\pi R^2} \int\limits_0 r^3 dr = \frac{Q\omega R^2}{16}$; $p = 2p_{m1} = \frac{1}{8} Q\omega R^2$.

(16). Дано:

r, d

$I_1 = I_2$

$L - ?$



$$L = \frac{\Phi}{I}$$

$$\Phi = \int_S B\,dS$$

$$B = 2B_1.$$

$\underline{B_1(r)}$: $2\pi r B_1(r) = \mu_0 I$

$$B_1(r) = \frac{\mu_0 I}{2\pi r}; \quad B_1(x) = \frac{\mu_0 I}{2\pi x}.$$

$$B(x) = \frac{\mu_0 I}{\pi x}; \quad dS = 1\cdot dx = dx.$$

$$\Phi = \int_r^{d+r} \frac{\mu_0 I\,dx}{\pi x} = \frac{\mu_0 I}{\pi} \ln\frac{d+r}{r}$$

$$\boxed{L = \frac{\mu_0}{\pi} \ln\frac{d+r}{r}}$$

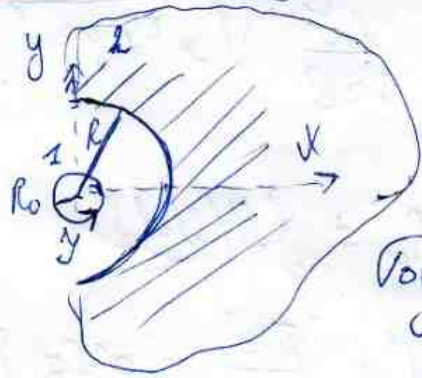
(17) Дано:

R_0

I

$R = 10R_0$

$\Phi - ?$



Пусть в контуре 2 радиусом R течёт такой же ток I. Тогда по теореме взаимности

$$L_{12} = L_{21} \; |\cdot I$$

$$\underline{\Phi_1 = \Phi_2}.$$

$$\Phi = B_2 \cdot \pi R_0^2$$

B_2:

$$dB = \frac{\mu_0 I}{4\pi}\,\frac{R\,d\varphi\,R}{R^3} = \frac{\mu_0 I\,d\varphi}{4\pi R}.$$

$$B = \frac{\mu_0 I}{4\pi R}\int_0^{\pi} d\varphi = \frac{\mu_0 I}{4R}$$

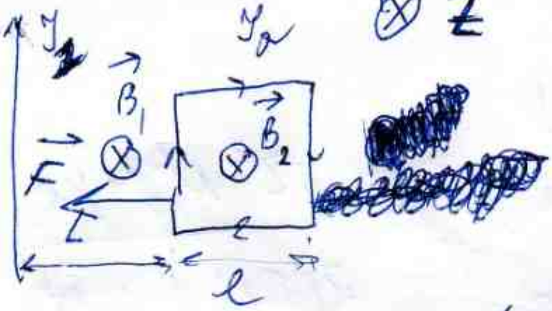
$$\boxed{\Phi = \frac{\mu_0 I \pi R_0^2}{4R} = \frac{\mu_0 I \pi R_0}{40}}$$

(18) Дано:

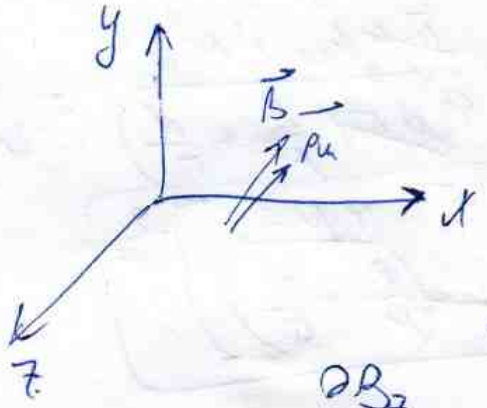
L, J_1,
l, J_2

F - ?

Решение



$$\vec{F} = p_m \frac{\partial \vec{B}}{\partial n}$$



$$B(r) = \frac{\mu_0 J_1}{2\pi r}$$

$$B_z = -\frac{\mu_0 J_1}{2\pi z}$$

$$\frac{\partial B_z}{\partial n} = \frac{\partial B_z}{\partial z}$$

$$\frac{\partial B_z}{\partial z} = \frac{\mu_0 J_1}{2\pi z^2}$$

$$p_m = J_2 S = J_2 l^2$$

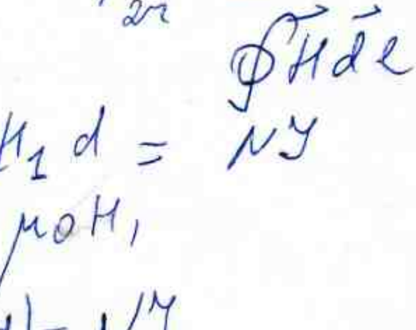
$$F_L = p_m \cdot \frac{\partial B_z}{\partial z}\Big|_L = \frac{\mu_0 J_1 J_2}{2\pi}\left(\frac{l}{L}\right)^2$$

(19) Дано:

N, μ
R, r,
$r \ll R$
d, $d \ll r$
J

F - ?

Решение



$$F = +\frac{\partial W}{\partial d}\Big|_{J = const}$$

$$W = \frac{\Phi J}{2}$$

$$\oint \vec{H}\, d\vec{l} = J$$

$$\begin{cases} H \cdot 2\pi R + 2H_1 d = NJ \\ \mu\mu_0 H = \mu_0 H_1 \end{cases}$$

$$2H(\pi R + \mu d) = NJ$$

$$H = \frac{NJ}{2(\pi R + \mu d)}$$

$$B = \mu \mu_0 H$$

$$B = \frac{\mu \mu_0 N J}{2(\pi R + \mu d)}$$

$$\Phi = B \cdot S = B \cdot \pi r^2 N = \frac{\pi r^2 N^2 \mu \mu_0 J}{2(\pi R + \mu d)}$$

$$W = \frac{\Phi J}{2} = \frac{\pi r^2 N^2 J^2 \mu \mu_0}{4(\pi R + \mu d)} = \frac{\cancel{\pi \mu} \mu_0 r^2 N^2 J^2}{4 \cancel{\mu \pi}\left(\frac{R}{\mu} + \frac{d}{\pi}\right)} = \frac{\mu_0 (r N J)^2}{4\left(\frac{R}{\mu} + \frac{d}{\pi}\right)}$$

$$\boxed{F = + \frac{\partial W}{\partial d}\Big|_{J = \text{Const}} = - \frac{\mu_0}{4\pi} \left[\frac{r N J}{\left(\frac{R}{\mu} + \frac{d}{\pi}\right)} \right]^2}$$