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Highlights

► CBS entropy is compared to Tsallis entropy for image recognition. ► A multi-q feature vector is proposed to improve pattern recognition. ► The results show multi-q approach boost $3\times$ the CBS entropy.



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Physica A

journal homepage: www.elsevier.com/locate/physa

Multi- q pattern analysis: A case study in image classification

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ARTICLE INFO

Article history:

Received 28 December 2011

Received in revised form 7 April 2012

Available online xxxx

Keywords:

Image pattern classification

Texture

Tsallis entropy

Non-additive entropy

ABSTRACT

This paper compares the effectiveness of the Tsallis entropy over the classic Boltzmann–Gibbs–Shannon entropy for general pattern recognition, and proposes a multi- q approach to improve pattern analysis using entropy. A series of experiments were carried out for the problem of classifying image patterns. Given a dataset of 40 pattern classes, the goal of our image case study is to assess how well the different entropies can be used to determine the class of a newly given image sample. Our experiments show that the Tsallis entropy using the proposed multi- q approach has great advantages over the Boltzmann–Gibbs–Shannon entropy for pattern classification, boosting image recognition rates by a factor of 3. We discuss the reasons behind this success, shedding light on the usefulness of the Tsallis entropy and the multi- q approach.

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1. Introduction

Pattern classification is an important problem for a number of fields, e.g., for recognizing embryo development stages [1], determining protein profile from fluorescence microscopy images [2], and identifying cellular structures [3]. This can be a challenging problem, especially for a large number of classes, with many different solutions proposed in the literature [4]. In this paper, however, our primary goal is to study the usefulness of the Tsallis entropy [5] by comparing it to the classic Boltzmann–Gibbs–Shannon (BGS) entropy when applied to the specific case of classifying image patterns. The approach readily applies to any signal beyond images, as from biometrics, sound databases, and biological experiments.

In statistical mechanics, the concept of entropy is related to the distribution of states, which can be characterized by the system energy levels. From an information-theoretic point of view, entropy is related to the lack of information of a system. It also represents how close a given probability distribution is to the uniform distribution, i.e., it is a measure of randomness, peaking at the uniform distribution itself. For pattern classification, this interpretation can be useful since, for example, a symmetric, periodic or smooth signal has less “possible states” than more uniformly random signals. A direct link between probability distributions and such concept of entropy was proposed by Boltzmann and is the foundation of what is now known as the Boltzmann–Gibbs–Shannon (BGS) entropy. A well-know generalization of this concept, the Tsallis entropy [5], extends its application to so-called non-extensive systems using a new parameter q . It recovers the classic entropy for $q \rightarrow 1$, and is better suited for long-range interactions between states (e.g., in large pixel neighborhoods) and long-term memories. The Tsallis generalization of entropy has a vast spectrum of applications, ranging from physics and

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chemistry to computer science. For instance, using the non-extensive entropy instead of the BGS entropy can produce gains in the results and efficiency of optimization algorithms [6], image segmentation [7–9] or edge detection algorithms [10].

In this paper we study the power of the Tsallis entropy in comparison to classic BGS entropy for classifying signal patterns and proposes a multi- q approach to the construction of good feature vectors for pattern analysis. An experimental study has been carried out in the specific case of images, to compare the different entropy approaches. Given a dataset of image patterns, typically comprising 40 classes, the goal of our case study is to assess how well the different entropies can be used to determine the class of a newly given image sample. The experiments show that the Tsallis entropy has great advantages over the BGS entropy – the multi- q analysis proposed by this paper boosts recognition rates by a factor of 3.

This paper is organized as follows. A review of definitions and notation of fundamental concepts is given in Section 2. Details about the problem of pattern classification, our multi- q approach based on the Tsallis entropy, and a case-study in image pattern recognition are described in Section 3. The basic experimental setup is provided in Section 4. In Section 5 experimental results towards analyzing the power of the Tsallis entropy in the case of image pattern classification are described. Section 6 summarizes and concludes the paper.

2. Formulation and notation

Let p_i be the estimated probability distribution of certain pattern properties of a signal. A well-known example is the histogram of intensities or graylevels $g_i = 1, \dots, W$ of a grayscale image I , but more sophisticated examples include histograms of gradient orientations such as HoG and other SIFT-like descriptors of image patches [11,12]. Since our goal is to study the power of different entropies, we chose a simple intensity histogram in our case-study, i.e., for simplicity p_i equals the number of pixels having intensity i divided by the total number of pixels. We assume $g_1 = 0$ stands for black, and $g_{256} = 255$ for white. The number W of different graylevels is typically 256 for 8-bit images. The BGS entropy is defined as²:

$$S_{BG} = - \sum_{i=1}^W p_i \log p_i, \quad \sum p_i = 1. \quad (1)$$

In the special case of a uniform distribution, $p_i = 1/W$, so that $S_{BG} = \log W$. Similarly, the Tsallis entropy is defined as

$$S_q = \frac{1 - \sum p_i^q}{q - 1}, \quad (2)$$

which recovers BGS entropy in the limit for $q \rightarrow 1$. The relation to BGS entropy is made clearer by rewriting this definition in the form:

$$S_q(p) = - \sum_i p_i^q \ln_q p_i, \quad (3)$$

where

$$\ln_q(x) \doteq \frac{x^{1-q} - 1}{1 - q} \quad (4)$$

is called the q -logarithm, with $\ln_q(x) \rightarrow \ln x$ for $q \rightarrow 1$. For any value of $q > 0$, S_q satisfies similar properties to the BGS entropy; for instance, $S_q \geq 0$, and S_q attains its maximum at the uniform distribution.

The BGS entropy is additive in the sense that the entropy of the whole system (the entropy of the sum) coincides with the sum of the entropies of the parts. This is not the case for the Tsallis entropy when $q \neq 1$, however. Formally,

$$S_{BG}(A + B) = S_{BG}(A) + S_{BG}(B), \quad (5)$$

while

$$S_q(A + B) = S_q(A) + S_q(B) + (1 - q)S_q(A)S_q(B), \quad (6)$$

where A and B are two probabilistically independent subsystems.

3. Multi- q analysis

In order to concretely assess the performance of using the Tsallis entropy, through a multi- q analysis, versus the classic BGS entropy for pattern recognition, this work involves a case-study in image pattern classification, but we stress that any signal and associated probability distribution could be used. The primary goal of image pattern classification is to assign a class label to a given image sample or window, the label being chosen among a predefined set of classes in a dataset. Fig. 1 shows a schematic of the classification process.

² We have dropped a constant of direct proportionality for the purpose of this paper.

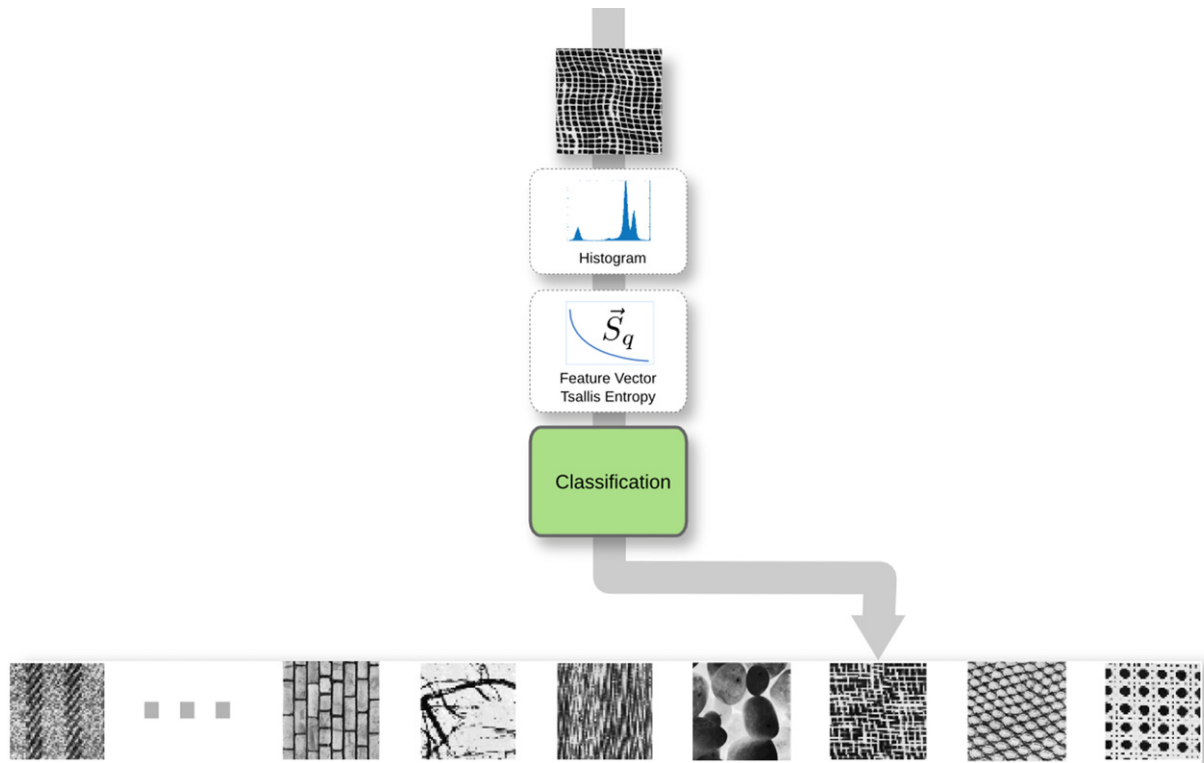


Fig. 1. Our approach to studying the effectiveness of the Tsallis entropy for pattern recognition works by estimating a probability distribution of a series of properties of an input image sample (e.g., intensity or gradient orientation histograms), computing a vector of Tsallis Entropies (the multi- q vector), and using that in a classifier to produce a class label among 40 classes stored in a dataset. The image patterns in this figure are real samples illustrating the Brodatz dataset used in this paper. Note that the approach readily applies to any signal or temporal series beyond images.

In supervised classification, the classifier is trained from a set of samples that are known to belong to the classes (*a priori* knowledge). The classifier is then validated by another set of samples. This methodology can be used for pattern recognition tasks as well as for mathematical modeling. Traditionally, in image analysis, a feature vector is extracted from the image and used to train and validate the classifier. The feature vector is expected to capture the most relevant information about the image.

In this work we investigate the Tsallis entropy as a tool to analyze signal information and propose the use of a feature vector using the so-called multi- q approach. The single q and the multi- q approaches are compared to the traditional BGS entropy. Beyond statistical mechanics, the classic BGS entropy is also traditionally used in information theory and is present as a metric in many image analysis methods, for instance Gabor texture analysis, Fourier analysis, wavelet, shape analyses among many others.

The concept of Tsallis entropy (and, in particular, BGS entropy) defined in Section 2 provides ways to abstract the information of a probability distribution of some pattern property, producing smaller and more efficient feature vectors than the entire probability distribution (e.g., histogram). For $q = 1$ we have the classic BGS entropy, strongly abstracting the distribution (e.g., the empirical 256-dimensional histogram representation) into the extreme case of a single number S_1 . This paper explores multiple q parameters towards forming better feature vectors for classification. We construct feature vectors of the form $\vec{S}_q = (S_{q_1}, \dots, S_{q_n})$, Fig. 1, whose dimension n (typically 4–20) provides a middle ground between the total abstraction of 1D BGS entropy and the full distribution. The experiments show that very few dimensions of Tsallis-entropy values in \vec{S}_q are already enough to outperform BGS entropy by a large factor.

A classical and simple approach image analysis is considering the distribution of pixel intensities in an image as a representation of texture, e.g., by analyzing its graylevel histogram to generate a feature vector. Such an approach has been used since the 1970's and, despite its simplicity, provides good results and is still the subject of active research [13]. The simple intensity-based texture analysis approach elected for our case study begins by computing the image histogram p_i of graylevels, where p_i is the number of pixels in the image for each intensity i . Assuming 8-bit grayscale images, it abstracts the image information into a feature vector of 256 dimensions. The histogram encodes a mixture of multiple intensity distributions representing luminosity patterns of image subsets, therefore being a clear candidate for image pattern representation for a number of classification applications. To classify an image or an image sample based on the histogram, statistical metrics are traditionally employed, such as mean, mode, kurtosis and BGS entropy.

Although the histogram is largely used in image analysis, it is limited, due to its simplicity. One important limitation is that the spatial information is not preserved by the histogram. Different images that have the same distribution of pixels have the same histogram; for instance, consider two images: a checkerboard pattern and an image split in the middle in black and white. While the visual information presented is quite different, they have the same histogram.

Despite its limitations, the image histogram has been used for different purposes, achieving good results, e.g., in image segmentation, image thresholding and pattern recognition. The inherent drawbacks of intensity histograms, such as the lack of spatial information, can be overcome through the use of a series of more sophisticated techniques, such as multiresolution histogram analysis [14] or by using the probability distributions of other image properties beyond intensity, such as HoGs and other SIFT-like descriptors of image patches [11,12]. However, these improvements could preclude the present study since our interest lies on clearly assessing the power of the Tsallis entropy relative to the BGS entropy for recognition, rather than of strategies for building distributions from image patterns. Therefore, we have decided to use simple intensity distributions to investigate the potentiality of Tsallis entropy applied to information theory in the context of images, and comparing its results to the those obtained with BGS entropy alone.

Note that in using a simple intensity-based strategy for building the probability distribution from data, the experimental results bear a worst-case scenario for pattern classification. In a final production system, the absolute performance results for the multi- q approach are likely to be superior than those reported here. Therefore, the simplicity and popularity of the image histogram can help focus the results of the classification on the entropy analysis itself. We show that even with a simple and limited approach to the probability distributions of images patterns, using the Tsallis entropy provides feasible results.

4. Experimental setup

The dataset used to evaluate the Tsallis entropy for recognition was created from Brodatz's art book [15]. This book is a black and white photography study for art and design and it was carried out on different patterns from wood, grass, fabric, among others. The Brodatz dataset became popular in the imaging sciences and is widely used as a benchmark for the visual attribute of texture. The dataset used in our case-study consists of 40 classes of texture, where each class is represented by a prototypical photograph of the texture containing no other patterns. Such 512×512 images are scans of glossy prints that were acquired from the author. A given image sample to be classified is much smaller than the class prototype image, 200×200 in this paper. The 512×512 prototype image can generate numerous 200×200 image samples representing the same class using a sliding window scheme. To construct our final dataset we perform this sliding window process to extract 10 representative image samples for each class. Therefore, any incoming sample to be classified is of the same size as the training windows. A few samples from this dataset are illustrated in Fig. 1.

Several approaches for the task of classification have been proposed in the literature. Since our focus is on comparing different approaches to entropy, we used the simple and well-known Naive Bayes classifier rather than a more sophisticated classifier. Although more sophisticated classifiers have been shown to produce superior results (e.g. multilayer perceptron and support vector machine), we are interested in showing the discrimination power provided by the features themselves rather than showing the classification power of classifiers.

In order to objectively evaluate the performance of the Tsallis entropy versus BGS entropy for classification, we use the stratified 10-fold cross-validation scheme [16]. In this scheme, the samples are randomly divided into 10 folds, considering that each fold contains the same proportions of the classes (i.e. for the Brodatz dataset, each fold contains 40 samples, one sample of each class). At each run of this scheme, the classifier is trained using all but one fold and then evaluated on how it classifies the samples from the separated fold. This process is repeated such that each fold is used once as validation. The performance is averaged, generating a single number for classification rate which represents the overall proportion of success over all runs. A standard deviation is also computed and displayed when significant. In the next section, moreover, a confusion matrix is eventually generated to analyze the performance of a specific classifier strategy. The confusion matrix is very well known in statistical classification and artificial intelligence. It is an $n \times n$ matrix where n is the number of classes, and whose entry (i, j) expresses how many patterns of class i were labeled as class j . It allows analysis of the error of the classification and which class was most wrongly classified.

5. Experimental results

We have conducted two sets of experiments to evaluate the performance of Tsallis entropy against the classic BGS entropy for texture recognition. The aim of the first set of experiments is to analyze the power of the Tsallis entropy with only one q value and compare its performance against that of the BGS entropy. The second set of experiments is devised to evaluate the use of the Tsallis entropy with a multi- q approach, including analysis of different sets of q 's.

5.1. Classification results: single q

To fairly compare the BGS and Tsallis entropies, we conducted an experiment using a single value of q , that is, the feature vector was reduced to a single number for the purpose of our study. Note, however, that in a practical system one would

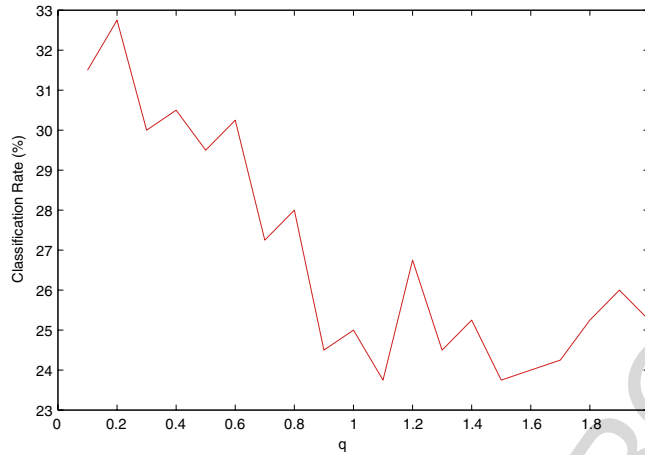


Fig. 2. Classification rate using a single value of q at a time.

Table 1

Classification results for different texture image datasets and the best value of q . The Tsallis entropy outperforms the BGS entropy for all datasets.

Dataset	best q	Classification rate	BGS entropy
Brodatz	0.2	32.8%(±6.2)	25.0%(±5.8)
VisTex	0.1	21.3%(±3.3)	16.9%(±3.3)
CURET	0.2	10.1%(±0.8)	08.6%(±0.8)
Outex	0.2	14.2%(±2.4)	13.4%(±2.0)

use a higher-dimensional feature vector (i.e., more numbers) to represent an image sample. In the Brodatz dataset, using the BGS entropy alone yielded a classification rate of 25.0%(±5.8). To compare it to the Tsallis entropy, we need to choose an appropriate value of q . The classification rate of different values of q are presented in the plot of the Fig. 2. The best result of 32.8%(±6.2) was obtained by the Tsallis entropy with $q = 0.2$. Notice that any Tsallis entropy with $q < 1$ outperforms the BGS entropy. Moreover, note that in the most cases, the classification rate decreases as the value of q increases.

We expected that for texture images in general (i.e., beyond Brodatz) the highest classification rates are also obtained for values of q close to 0.2. Table 1 presents the best values of q for different texture image datasets, including CURET [17] (Columbia-Utrecht Reflectance and Texture Database), Outex from the University of Oulu [18], and VisTex from MIT. For all but the VisTex dataset the best value of q was 0.2. For VisTex the highest classification rate of 21.3%(±3.3) was obtained by $q = 0.1$ while $q = 0.2$ achieved a correct classification rate of 20.7%(±3.2). Moreover, the Tsallis entropy using $q = 0.2$ outperforms the BGS entropy for all datasets. Note that these results concern the effectiveness of a *single number* to abstract the information of an image window. We stress that on a practical system one would likely represent a 200×200 image with a larger set of numbers (features).

5.2. Classification results: multiple q

One hypothesis for the power of the Tsallis entropy concerning pattern recognition is that each S_q could hold different information about the pattern. Therefore, different values of q used together could improve classification rates. Indeed, Fig. 3 shows that the S_q curve can help in distinguishing the patterns – the feature vector $\vec{S}_q = (S_{0.1}, S_{0.2}, \dots, S_2)$ is plotted for four different textures, giving an idea of the discriminating power of the Tsallis entropy.

Since the nature of the S_q curve is exponential, it is difficult to grasp the differences between the patterns. In order to improve the visualization of pattern behavior through the S_q curve, we calculated the mean vector $\vec{\mu}$, that is, the average S_q curve of the 400 samples of the image dataset, and plotted the difference of \vec{S}_q and $\vec{\mu}$ for 10 patterns picked at random, Fig. 4. Notice that the first values of q present the best pattern discrimination, which agrees with the results of the experiments for a single q , where a q around 0.2 performs best.

To use the Tsallis entropy curve as a pattern recognition tool, we composed a feature vector \vec{S}_q in the interval $q=0.1:0.1:2$ (i.e., q from 0.1 to 2 in increments of 0.1). Using this feature vector of 20 elements, a classification rate of 73.8%(±6.2) was achieved. We also constructed feature vectors using different intervals of values of q . The classification results are shown in Table 2. These results corroborate the hypothesis that a multi- q approach can improve the power of the Tsallis entropy applied to pattern recognition. The multi- q strategy using only 20 elements results in a gain of 41% compared to best value of q and a gain of 48.8% compared to the BGS entropy.

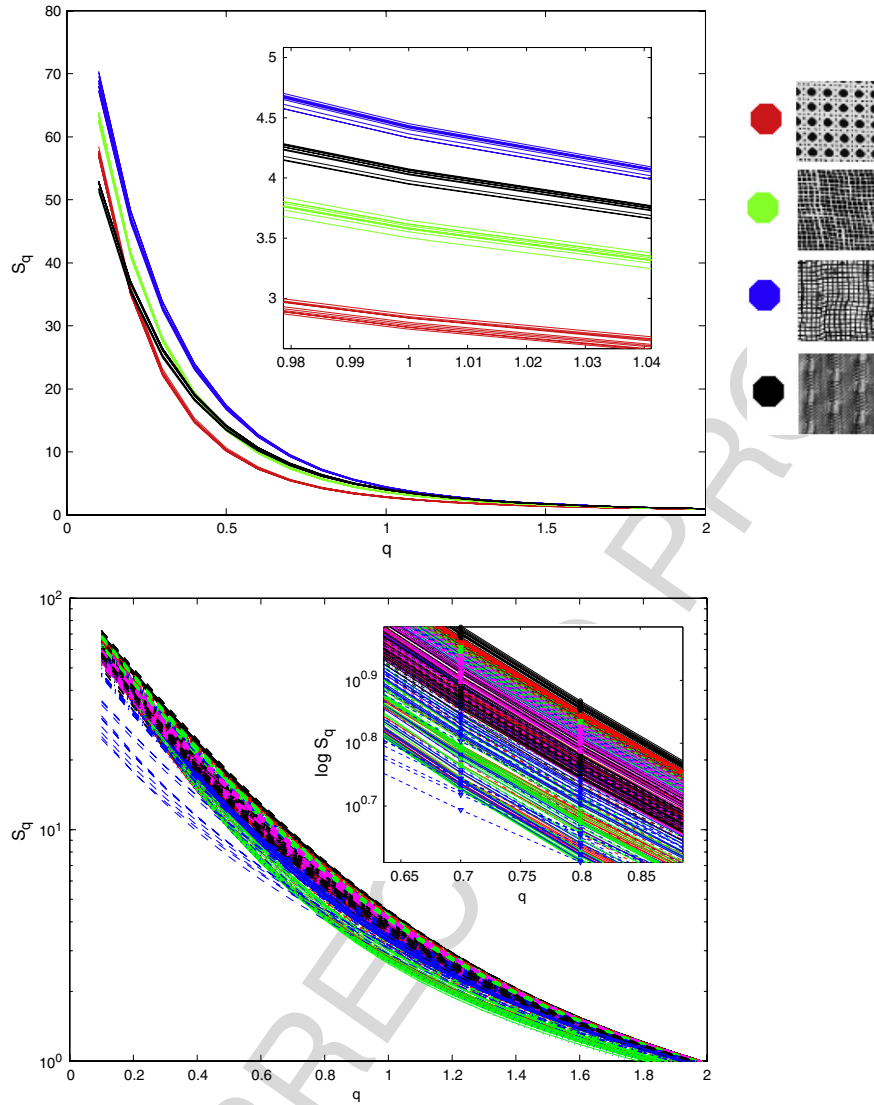


Fig. 3. The multi- q feature vectors ($S_{0.1}, \dots, S_2$) for four different textures (top) and for all textures in the dataset (bottom) using their intensity distributions, giving an idea of the discriminating power of the Tsallis entropy.

Table 2

Classification results for different sets of q .

Range of q	#Features	Classification rate %
0.2	1	32.8(± 6.2)
0.5:0.5:2	5	52.8(± 6.3)
0.2:0.2:2	10	67.0(± 6.4)
0.1:0.1:2	20	73.8(± 6.2)
0.05:0.05:2	40	75.8(± 6.5)
0.01:0.01:2	200	77.3(± 6.5)
0.005:0.005:2	400	77.5(± 6.0)
0.001:0.001:2	2000	78.5(± 6.2)

To visualize the behavior of the texture classes, we use a Karhunen–Loève transform (or principal components analysis, PCA). This allows projecting the feature vectors onto a lower-dimensional space which is easier to visualize, and where the variance is higher as possible. The PCA was applied to the feature vectors of the 400 samples (40 classes) of the Brodatz dataset and a scatter plot was obtained, Fig. 5. As we can see, the classes become organized in distinguished cluster, illustrating the power of classification of the multi- q approach.

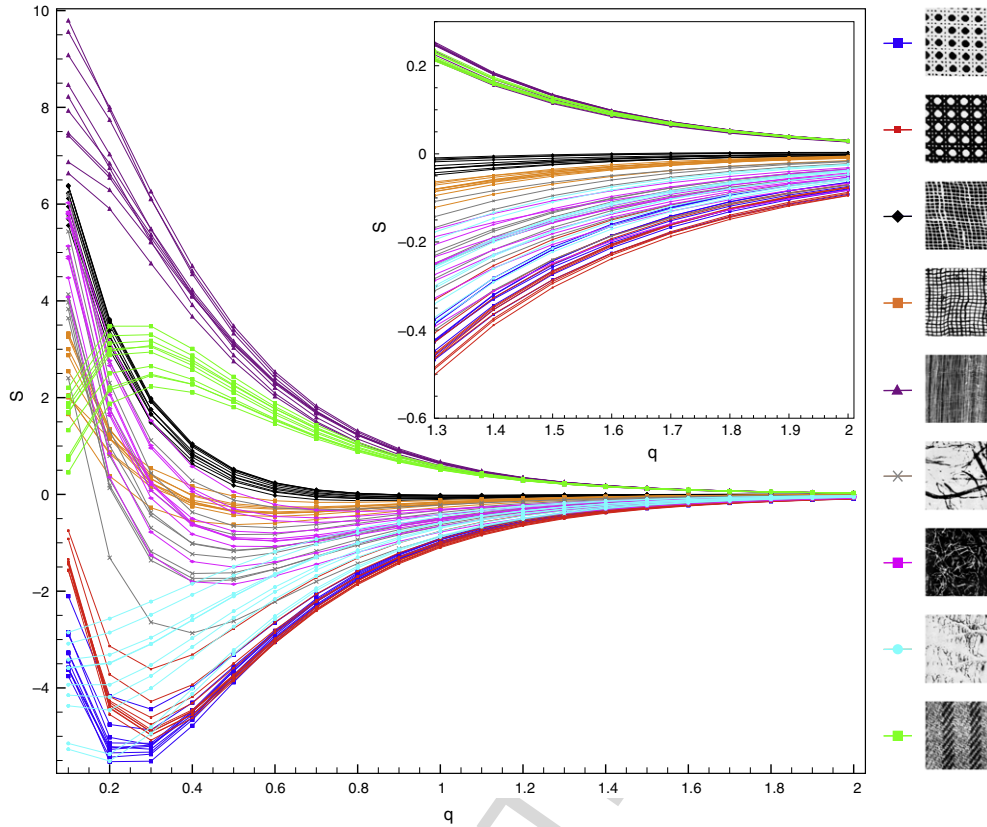


Fig. 4. Tsallis entropy versus q for different patterns using their intensity distributions. Each color and symbol combination represents a class of texture pattern.

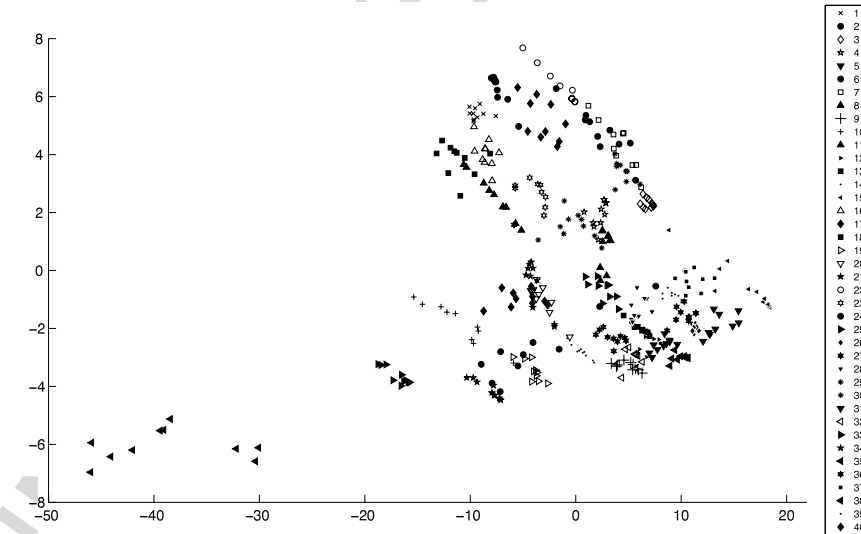


Fig. 5. To give a visual idea of the discriminating power of the Tsallis entropy, a principal components analysis (PCA) was performed on Brodatz's 40 classes considering a 20-dimensional feature vector ($S_{0.1}, \dots, S_2$), and visualized as a 2-dimensional scatter plot of the first two principal components. In other words, this shows the features projected into the plane of highest variance.

5.3. Feature selection: enhancing the discriminating power of the S_q curve

We have noticed that composing a feature vector with different S_q 's can boost the discriminative power of the Tsallis entropy. Nevertheless, the feature vector was composed by q in the range 0.1, 0.2, \dots , 2 and, as remarked in Fig. 2, there

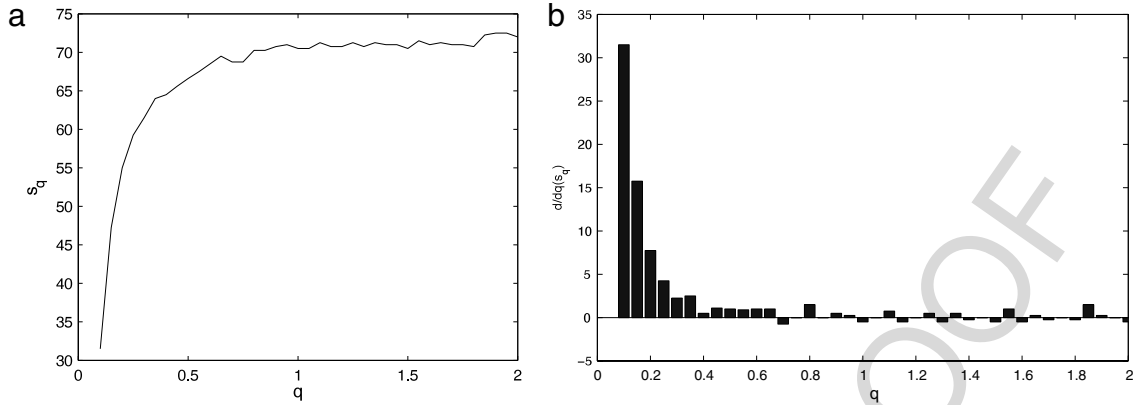


Fig. 6. (a) Classification rate for feature vectors using the first S_q elements: for a value of q , the curve gives the classification when using $\vec{S}_q = (S_{0.1}, \dots, S_q)$ as a feature vector. The number of elements in the feature vector increases for each element of x axis. (b) the derivative of the previous curve.

Table 3

Classification results for feature selection from different step sizes of q .

Range of q	#Features	Classification rate %
0.5:0.5:2	4	52.8
0.2:0.2:2	4	73.8
0.1:0.1:2	6	80
0.05:0.05:2	7	80.3
0.01:0.01:2	21	81.5
0.005:0.005:2	26	81.8
0.001:0.001:2	27	82

are q values in the interval that do not achieve the maximum classification. Therefore, a question arises: is the information of the entire interval $0 < q \leq 2$ aiding to distinguish the image patterns?

Feature selection is a technique used in multivariate statistics and in pattern recognition that selects a subset of relevant features with the aim of improving the classification rate and also its robustness. There are several algorithms for feature selection, the reader can find a feature selection survey in Ref. [19]. We have used a very simple strategy in the present work to clarify the influence of the different q in the classification process. The main idea is use only the S_q values that presents significant contribution to the classification rate. Fig. 6(a) plots the classification rate using the first S_q 's, taking different quantities of these; e.g., for the first datapoint of the curve a single S_q was used, $(S_{0.1})$, in the second datapoint two S_q 's, $(S_{0.1}, S_{0.2})$, and at position n on the x axis, nS_q 's. The curve shows there are values of S_q that improve the classification rate but there are also values of q that do not increase the classification rate or even decrease it. To make the contribution of each S_q easier to see, we take the derivative of the curve of the Fig. 6(a), shown in the Fig. 6(b). We performed feature selection by picking the q 's whose values in the derivative curve are greater than t .

The feature selection reduces the number of features and increases the classification power of the multiple q entropy approach. Table 3 shows results of the feature selection over multi- q approach for different range of q at the interval 0 to 2. As can be observed with just 4 elements, the result is equivalent to a feature vector with size 20 without the feature selection (see Table 2), and with 27 elements, the feature selection approach overcome the performance of a 2000 size feature vector (Table 2). The results demonstrate that the feature selection strategy presents a much better performance for the multi- q approach. The main reason of the performance increase is that the algorithm can select the S_q significant elements. For comparison, the 27 features obtained through the selection strategy were classified using two other approaches: Support Vector Machines and Neural Networks, achieving classification rates of 86.3% and 83%, respectively.

In order to get further insight into the nature of the classification errors on a class-by-class basis, the confusion matrices for different representative approaches to entropy-based image classification investigated in this paper are shown in Fig. 7, with a three-dimensional representation. Confusion matrices are a standard tool in machine learning and pattern recognition. An arbitrary entry (x, y) of a confusion matrix expresses how many patterns of class x were labeled as class y , and this is visualized as height z in the figure. This visualization of the classification error allows one to see which class were most wrongly classified, and which two classes are being most confused by each feature vector representation. The main diagonal of each matrix expresses the classes that were correctly classified. An ideal 100% correct classification should result into a diagonal matrix. As can be noticed, the mistakes decreasing from Fig. 7(a)–(d). Most of the classification mistakes are corrected from (a) to (d), which exception of some classes which are problematic for almost all the approaches.

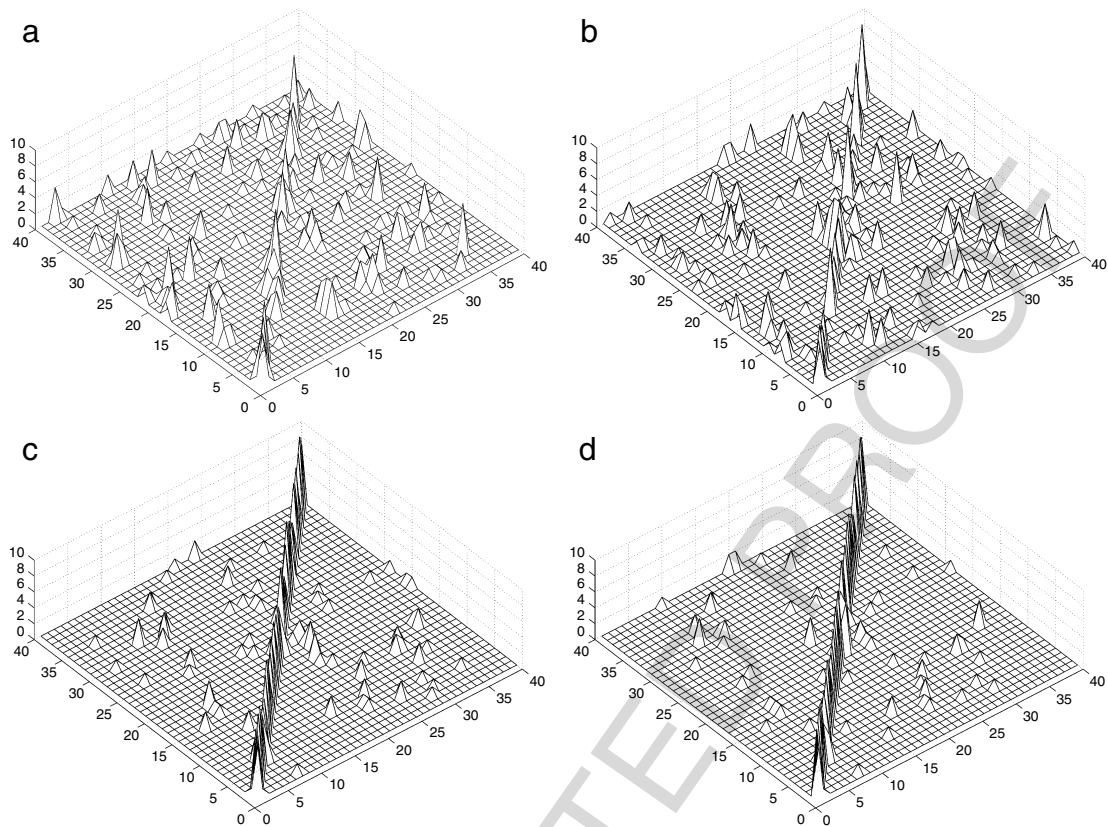


Fig. 7. Confusion matrices for (a) BGS entropy alone, (b) Tsallis entropy with $q = 0.2$, (c) multi- q approach using the feature vector $\vec{S}_q = (S_{0.1}, \dots, S_2)$, and (d) feature vector constructed with the feature selection technique with intensity distributions as described in the paper.

6. Conclusion

In this paper we proposed a multi- q pattern analysis approach based on the Tsallis entropy and conducted an image-based case study showing that the Tsallis entropy applied to pattern classification (texture image analysis) has great **advantages** over the classic BGS entropy. The multi- q Tsallis entropy enables **the extraction** of richer feature vectors using different values of q , which in fact yields vastly better performance than using BGS entropy alone, even with simple probability distributions and classifier strategies. This points to the fact that the Tsallis entropy for different q (the multi- q approach) does encode much more information from a given probability distribution than the BGS entropy. In fact, one of the results **shows** that as little as 4 values of q , together, are enough to outperform the BGS entropy by about $3\times$. Work to further **analyze** the implications of these results within a deeper information-theoretic framework is underway, shedding light into the **usefulness** of the Tsallis entropy for general problems of pattern recognition.

Acknowledgments

R.F. acknowledges support from the UERJ visiting professor grant. W.N.G. acknowledges support from FAPESP (2008/03253-9). O.M.B. acknowledges support from CNPq (Grant #308449/2010-0 and #473893/2010-0) and FAPESP (Grant #2011/01523-1).

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