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Functions Start

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import math

#Composite Simpson's Rule
def simpson(f, a, b, n):                      #INPUT endpoints a, b; even positive integer n.
                                              #OUTPUT approximation X I to I.
    h = (float(b) - float(a)) / float(n)      #Step 1 Set h = (b - a)/n
    XI0 = f(float(a)) + f(float(b))           #Step 2 Set XI0 = f(a) + f(b);
    XI1 = float(0)                            #XI1 = 0; (Summation of (x_2i-1))
    XI2 = float(0)                            #XI2 = 0  (Summation of (x_2i))
    for i in range(1, n - 1):                 #Step 3 For i = 1 ... n - 1 do steps 4 and 5
        X = a + i*h                           #Step 4 Set X = a + ih
        if i % 2 == 0:                        #Step 5 If i is even then set XI2 = XI2 + f(X)
            XI2 = XI2 + f(X)
        else:                                 #Else set XI1 = XI1 + f(X)
            XI1 = XI1 + f(X)
    XI = h*(XI0 + 2 * XI2 + 4 * XI1)/3        #Step 6 Set XI = h(XI0 + 2 * XI2 + 4 * XI1)/3
    return XI                                 #Step 7 OUTPUT(XI);
```

In [2]:
```
#Test to make sure function works correctly
simpson(np.sin,0,np.pi/2,1000000)
```

Out[2]: 0.9999979056048846

In [3]:
```
#Trapezoidal Rule
def trap(f, a, b, n):                                #INPUT endpoints a, b; even positive integer n.
    h = (float(b) - float(a)) / float(n)             #Set the distance of each increment
    s = 0                                            #Add a variable for summation
    for i in range(0, n-1):                          #For each increment in the range (n-1 adds to n)
        s = s + (h/2)*(f(a+i*h) + f(a+(i+1)*h))      #Add the value for each increment
    return s                                         #Return the summation
```

In [4]:
```
#Test to make sure function works correctly
trap(np.sin,0,np.pi/2,1000000)
```

Out[4]: 0.999998429203461

Questions Start

Question 01

f(x) = e^(-x^2)

f'(x) = -2x * e^(-x^2)

f''(x) = (4x^(2) - 2) * e^(-x^2)

f^(3)(x) = (-8x^(3) + 12x) * e^(-x^2)

f^(4)(x) = (16x^(4) - 48x^(2) + 12) * e^(-x^2)

Tried 0 <= x <= 1 because 1 is easy to work with

f''(0) = (2/sqrt(pi))(4(0)^(2) - 2) * e^(-(0)^2) = -2.2567

f''(2) = (2/sqrt(pi))(4(2)^(2) - 2) * e^(-(2)^2) = 0.2893

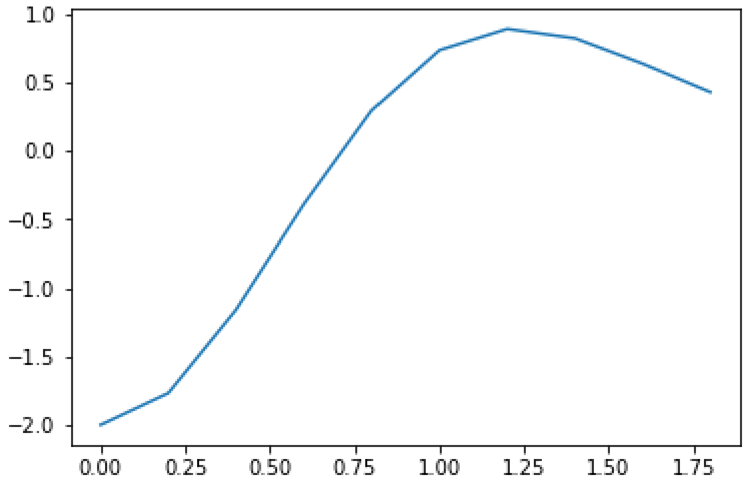
In [5]:
```
def prob1(x):
    return (4*x**(2) - 2) * math.exp(-x**(2))

prob1v = np.vectorize(prob1)

x = np.arange(0, 2, 0.2)


plt.plot(x, prob1v(x))

plt.show()
```



|-2.2567| > |0.2893|

m = 2.2567

((2 - a)/12) <= h^2 * m

((2 - 0)/12) h^2 * (2.2567) <= 10^(-6)

(1/6) h^2 * (2.2567) <= 10^(-6)

h^2 *(2.2567)*<= *(10^(-6)* 6)

h^2 <= (10^(-6) * 6) / (2.2567)

h^2 <= ((10^(-6) * 6) / (2.2567))^(1/2)

h <= 0.001630

n = pi/h

n >= 1927.3574

n >= 1928 (Next whole number)

In [6]:
```
math.erf(2)
```

Out[6]: 0.9953222650189527

In [7]:
```
def nerf(x):
    return math.exp(-x**2)
```

In [8]:
```
(2/math.sqrt(math.pi)) * (trap(nerf,0, 2, 1928))
```

Out[8]: 0.9953007742647099

Question 02

$f(x) = e^{-x^2}$

$f'(x) = -2x \cdot e^{-x^2}$

$f''(x) = (4x^{2} - 2) \cdot e^{-x^2}$

$f^{(3)}(x) = (-8x^{3} + 12x) \cdot e^{-x^2}$

$f^{(4)}(x) = (16x^{4} - 48x^{2} + 12) \cdot e^{-x^2}$

Tried $0 \le x \le 1$ because 1 is easy to work with

$f^{(4)}(0) = \frac{2}{\sqrt{\pi}}\left((16(0)^{4} - 48(0)^{2} + 12) \cdot e^{-(0)^2}\right) = 13.5405$

$f^{(4)}(1) = \frac{2}{\sqrt{\pi}}\left((16(2)^{4} - 48(2)^{2} + 12) \cdot e^{-(2)^2}\right) = 1.5706$

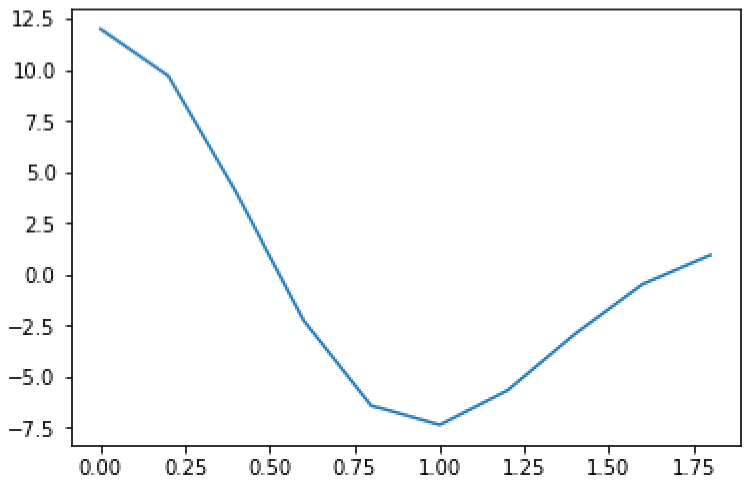
In [9]:
```python
def prob2(x):
    return (16*x**(4) - 48*x**(2) + 12) * math.exp(-x**2)

prob2v = np.vectorize(prob2)

x = np.arange(0, 2, 0.2)


plt.plot(x, prob2v(x))

plt.show()
```



|13.5405| > |1.5706|

m = 13.5405

((b - a)/90) <= h^4 * m

((2 - 0)/90) h^4 * (13.5405) <= 10^(-6)

(2/90) h^4 <= 10^(-6) / (13.5405)

(1/45) h^4 <= 10^(-6) / (13.5405)

h^4 <= 10^(-6) (45) / (13.5405)

h <= (10^(-6) (45) / (13.5405))^(1/4)

h <= 0.04269

n = pi/h

n >= 73.5908

n >= 74 (Next whole even number)

```
In [10]: (2/math.sqrt(math.pi)) * (simpson(nerf,0, 2, 74))

Out[10]: 0.9944930847717389
```

Simpson rule uses less iterations, but is not as accurate as the composite trapezoidal rule. The composite trapezoidal rule is more accurate, but goes through many more iterations (and thus uses more computer resources).

Question 03

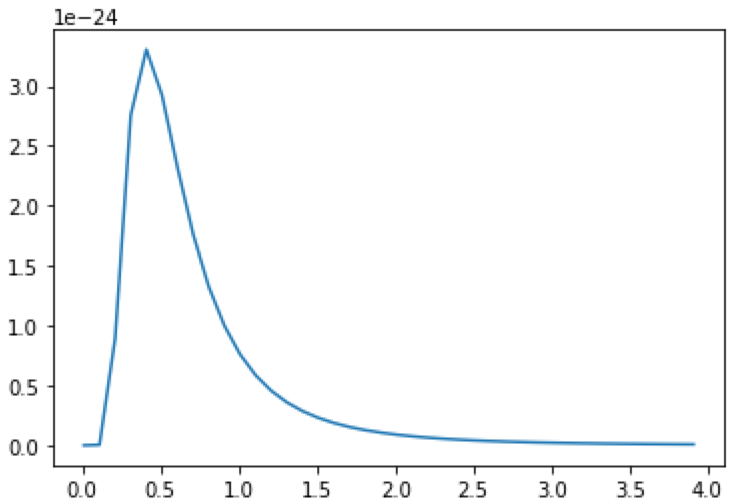
In [11]:
```
#Part 1
def spec(wavelength):
    return (8 * np.pi * (6.62607004 * 10**(-34) * (299792458))/(wavelength**(5) * (math.exp((2) / wavelength)
- 1)))

spec2 = np.vectorize(spec)

x = np.arange(0.01, 4, 0.1)

plt.plot(x, spec2(x))

plt.show()
```



In [12]:
```
#Part 2
simpson(spec,0.01,0.4,1000000)
```

Out[12]: 4.973891292665168e-25