

## Page 335 Table 5.19

```
In [1]: import math
import numpy as np
```

```
In [2]: def f1(t,x1, x2):
        return -4.0 * x1 + 3.0 * x2 + 6.0

def f2(t,x1, x2):
    return -2.4 * x1 + 1.6 * x2 + 3.6
```

```

In [3]: def RungeKuttaSDE_Q1(a, b, n, alpha1, alpha2):
    k1a = [0.0] * n
    k2a = [0.0] * n
    k3a = [0.0] * n
    k4a = [0.0] * n

    w1 = [0.0] * (n + 1)
    w2 = [0.0] * (n + 1)

    k1b = [0.0] * n
    k2b = [0.0] * n
    k3b = [0.0] * n
    k4b = [0.0] * n

    h = (b - a)/n
    t = a

    tList = [0.0] * n

    w1[0] = alpha1
    w2[0] = alpha2

    for i in range(0, n):
        k1a[i] = h * f1(t, w1[i], w2[i])
        k1b[i] = h * f2(t, w1[i], w2[i])

        k2a[i] = h * f1((t + h/2.0), (w1[i] + k1a[i]/2.0), (w2[i] + k1b[i]/2.0))
        k2b[i] = h * f2((t + h/2.0), (w1[i] + k1a[i]/2.0), (w2[i] + k1b[i]/2.0))

        k3a[i] = h * f1((t + h/2.0), (w1[i] + k2a[i]/2.0), (w2[i] + k2b[i]/2.0))
        k3b[i] = h * f2((t + h/2.0), (w1[i] + k2a[i]/2.0), (w2[i] + k2b[i]/2.0))

        k4a[i] = h * f1((t + h), (w1[i] + k3a[i]), (w2[i] + k3b[i]))
        k4b[i] = h * f2((t + h), (w1[i] + k3a[i]), (w2[i] + k3b[i]))

        w1[i + 1] = w1[i] + (k1a[i] + (2.0 * k2a[i]) + (2.0 * k3a[i]) + k4a[i])/6.0
        w2[i + 1] = w2[i] + (k1b[i] + (2.0 * k2b[i]) + (2.0 * k3b[i]) + k4b[i])/6.0

        t = a + (i * h)

    tList[i] = round(t, 2)

```

```
return (tList[:n], w1[:n], w2[:n])
```

```
In [4]: tList, w1, w2 = RungeKuttaSDE_Q1(0.0, 0.6, 6, 0, 0)
```

```
In [5]: tList
```

```
Out[5]: [0.0, 0.1, 0.2, 0.3, 0.4, 0.5]
```

```
In [6]: w1
```

```
Out[6]: [0,  
         0.5382551999999999,  
         0.9684987375290879,  
         1.310719039205257,  
         1.5812652389631423,  
         1.793507490120283]
```

```
In [7]: w2
```

```
Out[7]: [0,  
         0.31962624,  
         0.5687821730349056,  
         0.7607331318681753,  
         0.9063206179489272,  
         1.0144024167698833]
```

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```
In [8]: def f1(t,x1, x2):  
        return x2  
  
        def f2(t,x1, x2):  
            return math.exp(2.0 * t) * math.sin(t) - 2.0 * x1 + 2.0 * x2
```

```
In [9]: def f1_Actual(t):  
        return 0.2 * math.exp(2.0 * t) * (math.sin(t) - 2 * math.cos(t))  
  
        def f2_Actual(t):  
            return 0.2 * math.exp(2.0 * t) * (4.0 * math.sin(t) - 3.0 * math.cos(t))
```

```

In [10]: def RungeKuttaSDE_Q2(a, b, n, alpha1, alpha2):
    k1a = [0.0] * n
    k2a = [0.0] * n
    k3a = [0.0] * n
    k4a = [0.0] * n

    w1 = [0.0] * (n + 1)
    w2 = [0.0] * (n + 1)

    k1b = [0.0] * n
    k2b = [0.0] * n
    k3b = [0.0] * n
    k4b = [0.0] * n

    func1Results = [0.0] * n
    func2Results = [0.0] * n

    func1Diff = [0.0] * n
    func2Diff = [0.0] * n

    h = (b - a)/n
    t = a

    tList = [0.0] * n

    w1[0] = alpha1
    w2[0] = alpha2

    for i in range(0, n):
        k1a[i] = h * f1(t, w1[i], w2[i])
        k1b[i] = h * f2(t, w1[i], w2[i])

        k2a[i] = h * f1((t + h/2.0), (w1[i] + k1a[i]/2.0), (w2[i] + k1b[i]/2.0))
        k2b[i] = h * f2((t + h/2.0), (w1[i] + k1a[i]/2.0), (w2[i] + k1b[i]/2.0))

        k3a[i] = h * f1((t + h/2.0), (w1[i] + k2a[i]/2.0), (w2[i] + k2b[i]/2.0))
        k3b[i] = h * f2((t + h/2.0), (w1[i] + k2a[i]/2.0), (w2[i] + k2b[i]/2.0))

        k4a[i] = h * f1((t + h), (w1[i] + k3a[i]), (w2[i] + k3b[i]))
        k4b[i] = h * f2((t + h), (w1[i] + k3a[i]), (w2[i] + k3b[i]))

```

```

w1[i + 1] = w1[i] + (k1a[i] + (2.0 * k2a[i]) + (2.0 * k3a[i]) + k4a[i])/6.0
w2[i + 1] = w2[i] + (k1b[i] + (2.0 * k2b[i]) + (2.0 * k3b[i]) + k4b[i])/6.0

t = a + (i * h)

tList[i] = round(t,1)

func1Results[i] = f1_Actual(t)
func2Results[i] = f2_Actual(t)

func1Diff[i] = abs(func1Results[i] - w1[i])
func2Diff[i] = abs(func2Results[i] - w2[i])

return (tList[:n], func1Results[:n], w1[:n], func2Results[:n], w2[:n], func1Diff[:n], func2Diff[:n])

```

In [11]: tList, func1Results, w1, func2Results, w2, func1Diff, func2Diff = RungeKuttaSDE\_Q2(0.0, 1.1, 11, -0.4, -0.6)

In [12]: tList

Out[12]: [0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]

In [13]: func1Results

Out[13]: [-0.4,  
-0.46173297065077745,  
-0.5255590475937428,  
-0.5886000461233488,  
-0.6466102840938321,  
-0.6935639464462552,  
-0.7211484905565834,  
-0.7181488962215788,  
-0.6697067730635202,  
-0.556438136834108,  
-0.3533943569029151]

In [14]: w1

Out[14]: [-0.4,  
-0.4617333423313103,  
-0.5262971471689236,  
-0.5921537346739594,  
-0.6562028315998559,  
-0.7139500885203098,  
-0.759094054129381,  
-0.7830196995978596,  
-0.7741834068375281,  
-0.717373554157506,  
-0.5928306418610831]

In [15]: func2Results

Out[15]: [-0.6000000000000001,  
-0.6316310507516716,  
-0.6401486642186465,  
-0.6136636069585657,  
-0.5365821965839523,  
-0.388739054825885,  
-0.14438322022753577,  
0.22899242763115088,  
0.7719838294360526,  
1.5347686210660054,  
2.578746620829612]

In [16]: w2

Out[16]: [-0.6,  
-0.6316312421166997,  
-0.6562820270160954,  
-0.6557289070515501,  
-0.6178202537682936,  
-0.5266938865583847,  
-0.36191440618428083,  
-0.09746493531136907,  
0.29941667454143495,  
0.8695624608788612,  
1.6633578196164158]

```
In [17]: func1Diff
```

```
Out[17]: [0.0,  
          3.71680532851304e-07,  
          0.000738099575180784,  
          0.0035536885506105564,  
          0.009592547506023785,  
          0.020386142074054647,  
          0.03794556357279766,  
          0.0648708033762807,  
          0.10447663377400784,  
          0.160935417323398,  
          0.23943628495816804]
```

```
In [18]: func2Diff
```

```
Out[18]: [1.1102230246251565e-16,  
          1.9136502815708667e-07,  
          0.016133362797448925,  
          0.04206530009298437,  
          0.0812380571843413,  
          0.13795483173249967,  
          0.21753118595674506,  
          0.3264573629425199,  
          0.47256715489461765,  
          0.6652061601871442,  
          0.9153888012131961]
```