The TANK Model with CBDC, Bank and Dollarization

1 Model without Dollarization

This model is an extension of the TANK model in a small open economy, with CBDC and a monopolistic competitive bank sector. Deposit, cash and CBDC provide liquidity service and decrease the transaction cost. Banks take deposits from unconstrained households and invest in intermediate firms; constrained households do not have access to bank services and can only hold cash or CBDC.

1.1 Households

The consumption bundle is defined as follows:

$$c_t = [\gamma^{\frac{1}{\eta}} c_{Ht}^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} c_{Ft}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}},$$

where c_{Ht} and c_{Ft} denote consumption of domestic and foreign final good respectively. γ is the home bias. The representative household decides how to allocate her consumption expenditure between domestic and foreign goods.

By solving the static optimization problem, the optimal consumption of domestic and foreign final goods can be solved as

$$c_{Ht} = \gamma(\frac{P_{Ht}}{p_t})^{-\eta},$$
$$c_{Ft} = (1-\gamma)(\frac{P_{Ft}}{p_t})^{-\eta},$$

where p_t is the domestic CPI, i.e., the price of one unit of consumption:

$$p_t = [\gamma P_{Ht}^{1-\eta} + (1-\gamma) P_{Ft}^{1-\eta}]^{\frac{1}{1-\eta}}.$$

In a small open economy, the price of the foreign good coincides with the foreign CPI p_t^*, adjusted by the nominal exchange rate e_t:

$$P_{Ft} = e_t p_t^*$$

The real exchange rate is defined as the ratio of the foreign and domestic price levels, where the foreign price level is converted into domestic currency units via the nominal exchange rate:

$$s_t = e_t \frac{p_t^*}{p_t}$$

Define $p_{Ht} = P_{Ht}/p_t$ and $p_{Ft} = P_{Ft}/p_t$ as the price of domestic and foreign goods in terms of the domestic CPI, then the real exchange rate can be written as:

$$s_t = p_{Ft},$$

and the terms of trade, i.e., the ratio between the price imports and the price exports can be written as

$$tot_t = \frac{P_{Ft}}{P_{Ht}} = \frac{s_t}{p_{Ht}}$$

Unconstrained HH $(1 - \lambda)$

Unconstrained households with measure $1 - \lambda$ have access to the bank service. Each period, they choose their consumption c_{1t} and labor supply h_{1t} and receive labor income ω_t. Unconstrained households pay consumption tax at the rate τ_c. They also receive a lump-sum transfer from the government t_{1t} and profit of intermediate firms and banks Γ_{1t}.

Unconstrained households face liquidity constraints. I follow Schmitt-Grohé and Uribe (2007) and assume that the transaction cost of consumption s_{1t} is determined by the Transaction Cost Function:

$$s_{1t} = z_t A_1 \frac{c_{1t}}{l_{1t}} + B_1 \frac{l_{1t}}{c_{1t}} - 2\sqrt{A_1 B_1},$$

where the ratio between consumption and liquidity c_{1t}/l_{1t} represents the money velocity, the transaction cost is increasing in the velocity as long as the following condition is satisfied:

$$\frac{c_{1t}}{l_{1t}} > \sqrt{\frac{B_1}{z_t A_1}}.$$

Unconstrained households get liquidity services from commercial banks. There is a continuum measure of monopolistic competitive banks index by $j \in [0, 1]$. Bank deposits are substitutes with the elasticity of substitution $\epsilon_b > 1$. The total liquidity for unconstrained households can be written as:

$$l_{1t} = \int_0^1 (d_{1t}(j)^{\frac{\epsilon_b - 1}{\epsilon_b}} dj)^{\frac{\epsilon_b}{\epsilon_b - 1}} \tag{1}$$

The return to deposit $r_t^d(j)$ is determined by the bank sector optimization problem. Unconstrained households also hold one-period government bond b_{1Ht} and foreign bond b_{1Ft}, with r_t and r_t^* being the nominal interest rate of bonds.

Domestic households pay a quadratic adjustment cost when they change their financial position (foreign bond and foreign currency) with the rest of the world: this assumption ensures the existence of a determinate steady state and a stationary solution. (Schmitt-Grohé and Uribe, 2003)

The unconstrained households' problem can be defined as (all variables are in real terms):

$$\max_{c_{1t}, h_{1t}, s_{1t}, l_{1t}, d_{1t}(j), b_{1Ht}, b_{1Ft}} E_0 \sum_0^\infty \beta^t \left(\frac{c_{1t}^{1-\sigma}}{1-\sigma} - \chi \frac{h_{1t}^{1+\phi}}{1+\phi}\right),$$

subject to the budget and liquidity constraints:

$$(1+s_{1t}+\tau_c)c_{1t}+\int_0^1(d_{1t}(j)-\frac{r^d_{t-1}(j)}{\pi_t}d_{1t-1}(j))dj+(b_{1Ht}-\frac{r_{t-1}}{\pi_t}b_{1Ht-1})+s_t(b_{1Ft}-r^*_{t-1}b_{1Ft-1})$$
$$\leq w_th_{1t}+t_{1t}+\Gamma_{1t}-\frac{\kappa_B}{2}s_t((1-\lambda)b_{1Ft}-\bar{b}_F)^2 \quad (\lambda_{1t})$$
$$l_{1t}=\int_0^1(d_{1t}(j)^{\frac{\epsilon_b-1}{\epsilon_b}}dj)^{\frac{\epsilon_b}{\epsilon_b-1}} \quad (\tau_{1t}\lambda_{1t})$$
$$s_{1t}=z_tA_1\frac{c_{1t}}{l_{1t}}+B_1\frac{l_{1t}}{c_{1t}}-2\sqrt{A_1B_1} \quad (\mu_{1t}\lambda_{1t})$$

Taking first order conditions with respect to $c_{1t}, h_{1t}, s_{1t}, l_{1t}, d_{1t}(j), b_{1Ht}, b_{1Ft}$:

$$c_{1t}: \quad c_{1t}^{-\sigma}-\lambda_{1t}(1+s_{1t}+\tau_c)+\tau_{1t}\lambda_{1t}(z_tA_1\frac{1}{l_{1t}}-B_1\frac{l_{1t}}{c_{1t}^2})=0, \tag{2}$$
$$h_{1t}: \quad -\chi h_{1t}^{\phi}+\lambda_{1t}\omega_t=0, \tag{3}$$
$$s_{1t}: \quad -\lambda_{1t}c_{1t}-\tau_{1t}\lambda_{1t}=0, \tag{4}$$
$$l_{1t}: \quad \tau_{1t}\lambda_{1t}(-z_tA_1\frac{c_{1t}}{l_{1t}^2}+B_1\frac{1}{c_{1t}})-\mu_{1t}\lambda_{1t}=0, \tag{5}$$
$$d_{1t}(j): \quad -\lambda_{1t}+\beta E_t\lambda_{1t+1}\frac{r^d_t(j)}{\pi_{t+1}}+\mu_{1t}\lambda_{1t}(\frac{l_{1t}}{d_{1t}(j)})^{\frac{1}{\epsilon_b}}=0, \tag{6}$$
$$b_{1Ht}: \quad -\lambda_{1t}+\beta E_t\lambda_{1t+1}\frac{r_t}{\pi_{t+1}}=0, \tag{7}$$
$$b_{1Ft}: \quad -\lambda_{1t}+\beta E_t\lambda_{1t+1}\frac{s_{t+1}}{s_t}t^*_t-\lambda_{1t}\kappa_B(1-\lambda)((1-\lambda)b_{1Ft}-\bar{b}_F)=0, \tag{8}$$

Define the transaction wedge as:

$$\tau^c_{1t}=z_tA_1\frac{c_{1t}}{l_{1t}}-B_1\frac{l_{1t}}{c_{1t}}. \tag{9}$$

Plug (9) into first order condition (2) and (3), the intertemporal and intratemporal euler equation can be written as:

$$c_{1t}^{-\sigma}=\lambda_{1t}(1+s_{1t}+\tau_c+\tau^c_{1t})$$
$$\lambda_{1t}(1-\frac{c_{1t}}{l_{1t}}\tau^c_{1t}(\frac{l_{1t}}{d_{1t}(j)})^{\frac{1}{\epsilon_b}})=\beta\mathbb{E}_t\lambda_{1t+1}\frac{r^d_t(j)}{\pi_{t+1}}$$
$$\chi h_{1t}^{\phi}=\lambda_{1t}\omega_t$$

Combine equation (5), (6) and (7), then plug in (9), we can get:

$$\beta E_t\lambda_{1t+1}\frac{r^d_{Ht}(j)}{\pi_{t+1}}=\beta E_t\lambda_{1t+1}\frac{r_t}{\pi_{t+1}}(1-\frac{c_{1t}}{l_{1t}}\tau^c_{1t}(\frac{d_{1Ht}}{d_{1Ht}(j)})^{\frac{1}{\epsilon_b}}).$$

Then plug the equation above to equation (1), the demand function for $d_{1t}(j)$ can be derived as:

$$d_{1t}(j)=\left(\frac{(r_t-r^d_t(j))^{-\epsilon_b}}{\left(\int_0^1(r_t-r^d_t(j))^{1-\epsilon_b}dj\right)^{-\frac{\epsilon_b}{1-\epsilon_b}}}\right)d_{1t} \tag{10}$$

Constrained HH (λ)

The constrained households with measure λ don't have access to bank services, so they don't have access to bank deposits or government bonds. Each period, they choose their consumption c_{2t} and labor supply h_{2t} and receive labor income ω_t. Unconstrained households pay consumption tax at rate τ_c, but only when the payment is made by CBDC. They also receive a lump-sum transfer from the government t_{2t}.

Constrained households face the same liquidity constraints as unconstrained households. They can only get liquidity service by holding cash (m_{2t}) or CBDC ($CBDC_{2t}$). Cash users are facing an additional cost δ_m since cash is at risk of being stolen and subject to an adjustment cost. CBDC is a safer and more convenient liquid asset, so it's free of risk and adjustment costs. Cash and CBDC are substitutes with the elasticity of substitution $\epsilon_m > 1$. The total liquidity for constrained households can be written as:

$$l_{2t} = ((m_{2t})^{\frac{\epsilon_m - 1}{\epsilon_m}} + (CBDC_{2t})^{\frac{\epsilon_m - 1}{\epsilon_m}})^{\frac{\epsilon_m}{\epsilon_m - 1}}$$

The constrained households' problem can be defined as (all variables are in real terms):

$$\max_{c_{2t}, h_{2t}, l_{2t}, s_{2t}, m_{2t}, CBDC_{2t}} E_0 \sum_0^\infty \beta^t (\frac{c_{2t}^{1-\sigma}}{1-\sigma} - \chi \frac{h_{2t}^{1+\phi}}{1+\phi})$$

subject to the budget and liquidity constraints:

$$\begin{aligned}
\text{s.t.} \quad & (1 + s_{2t} + \tau_c \frac{CBDC_{2t}}{l_{2t}}) c_{2t} + (m_{2t} - \frac{1-\delta_m}{\pi_t} m_{2t-1}) + (CBDC_{2t} - \frac{r_{t-1}^{CBDC}}{\pi_t} CBDC_{2t-1}) \\
& \leq w_t h_{2t} + t_{2t} \\
& l_{2t} = ((m_{2t})^{\frac{\epsilon_m - 1}{\epsilon_m}} + (CBDC_{2t})^{\frac{\epsilon_m - 1}{\epsilon_m}})^{\frac{\epsilon_m}{\epsilon_m - 1}} \quad (\tau_{2t} \lambda_{2t}) \\
& s_{2t} = z_t A_2 \frac{c_{2t}}{l_{2t}} + B_2 \frac{l_{2t}}{c_{2t}} - 2\sqrt{A_2 B_2} \quad (\mu_{2t} \lambda_{2t})
\end{aligned}$$

Taking first order conditions with respect to $c_{2t}, h_{2t}, s_{2t}, l_{2t}, m_{2t}, CBDC_{2t}$:

$$c_{2t}: \quad c_{2t}^{-\sigma} - \lambda_{2t}(1 + s_{2t} + \tau_c \frac{CBDC_{2t}}{l_{2t}}) + \tau_{2t} \lambda_{2t} (z_t A_2 \frac{1}{l_{2t}} - B_2 \frac{l_{2t}}{c_{2t}^2}) = 0, \tag{11}$$

$$h_{2t}: \quad -\chi h_{2t}^{\phi} + \lambda_{2t} \omega_t = 0, \tag{12}$$

$$s_{2t}: \quad -\lambda_{2t} c_{2t} - \tau_{2t} \lambda_{2t} = 0, \tag{13}$$

$$l_{2t}: \quad \lambda_{2t} \tau_c \frac{CBDC_{2t}}{l_{2t}^2} c_{2t} + \tau_{2t} \lambda_{2t} (-z_t A_2 \frac{c_{2t}}{l_{2t}^2} + B_2 \frac{1}{c_{2t}}) - \mu_{2t} \lambda_{2t} = 0, \tag{14}$$

$$m_{2t}: \quad -\lambda_{2t} + \beta E_t \lambda_{2t+1} \frac{1-\delta_m}{\pi_{t+1}} + \mu_{2t} \lambda_{2t} (\frac{l_{2t}}{m_{2t}})^{\frac{1}{\epsilon_m}} = 0, \tag{15}$$

$$CBDC_{2t}: \quad -\lambda_{2t}(1 + \tau_c \frac{c_{2t}}{l_{2t}}) + \beta E_t \lambda_{2t+1} \frac{r_t^{CBDC}}{\pi_{t+1}} + \mu_{2t} \lambda_{2t} (\frac{l_{2t}}{CBDC_{2t}})^{\frac{1}{\epsilon_m}} = 0. \tag{16}$$

Similar to the unconstrained households, we can define the transaction wedge as:

$$\tau_{2t}^c = z_t A_2 \frac{c_{2t}}{l_{2t}} - B_2 \frac{l_{2t}}{c_{2t}}. \tag{17}$$

Plug (17) into first order condition (11) and (12), combine with equation (15), the intertemporal and intratemporal euler equation can be written as:

$$
\begin{aligned}
c_{2t}^{-\sigma} &= \lambda_{2t}(1 + s_{2t} + \tau_{2t}^c + \tau_c \frac{CBDC_{2t}}{l_{2t}}) \\
\lambda_{2t}(1 - \frac{c_{2t}}{l_{2t}}(\tau_{2t}^c + \tau_c \frac{CBDC_{2t}}{l_{2t}})(\frac{l_{2t}}{m_{2t}})^{\frac{1}{\epsilon_m}}) &= \beta E_t \lambda_{2t+1} \frac{1-\delta_m}{\pi_{t+1}}, \\
\chi h_{1t}^{\phi} &= \lambda_{1t}\omega_t
\end{aligned}
$$

The no-arbitrage condition for cash and CBDC can be derived from equations (15) and (16).

1.2 Bank

There is a continuum measure of monopolistic competitive banks $j \in [0,1]$. They take deposits from unconstrained households $d_t(j)$ and invest in intermediate firms $i_t(j)$ and accumulate capital stock $k_t(j)$. The deposit demand functions are solved from the household problem above.

$$
d_t(j) = \left(\frac{(r_t - r_t^d(j))^{-\epsilon_b}}{\left(\int_0^1 (r_t - r_t^d(j))^{1-\epsilon_b} dj\right)^{-\frac{\epsilon_b}{1-\epsilon_b}}} \right) d_t
$$

Banks are owned by unconstrained households, so bankers maximize its expected pre-dividend profits:

$$
\max_{r_t^d(j), i_t(j), k_t(j)} E_0 \sum_0^{\infty} \beta^t \frac{\lambda_{1t}}{\lambda_{10}} (r_t^k k_{t-1}(j) - i_t(j) + (d_t(j) - \frac{r_{t-1}^d(j)}{\pi_t} d_{t-1}(j)),
$$

subject to the law of motion of capital and balance sheet constraint:

$$
\begin{aligned}
&k_t(j) = (1-\delta)k_{t-1}(j) + (1 - \frac{\kappa_I}{2}(\frac{i_t(j)}{i_{t-1}(j)} - 1)) i_t(j), \\
&k_t(j) = d_t(j).
\end{aligned}
$$

The equilibrium is symmetric; then the first-order conditions give the supply for new investment

$$
\lambda_{1t} = q_t \lambda_{1t}(1 - \frac{\kappa_I}{2}(\frac{i_t}{i_{t-1}} - 1)^2 - \kappa_I(\frac{i_t}{i_{t-1}} - 1)\frac{i_t}{i_{t-1}}) + \beta \mathbb{E}_t q_{t+1} \lambda_{1t+1} \kappa_I (\frac{i_{t+1}}{i_t} - 1)^2 (\frac{i_{t+1}}{i_t})^2,
$$

and optimal deposit rate is chosen by the bank:

$$
r_t^{d*} = \frac{\epsilon_b}{\epsilon_b - 1} \frac{\beta E_t \lambda_{1t+1}(r_{t+1}^k + (1-\delta)q_{t+1}) - (q_t - 1)\lambda_{1t}}{\beta E_t(\lambda_{1t+1}/\pi_{t+1})} - \frac{1}{\epsilon_b - 1} r_t.
$$

When $\epsilon_b \to \infty$, then banks are perfectly competitive, then the deposit rate chosen by the bank becomes the net return to capital:

$$
r_t^{d*} = \frac{\beta E_t \lambda_{1t+1}(r_{t+1}^k + (1-\delta)q_{t+1}) - (q_t - 1)\lambda_{1t}}{\beta E_t(\lambda_{1t+1}/\pi_{t+1})}.
$$

1.3 Firm

The firm problem is standard. The representative final-good firm uses the following CES aggregator to produce the domestic final good, and the intermediate firms produce differentiated domestic input.

Final

The representative final-good firm uses the following CES aggregator to produce the domestic final good y_{Ht}:

$$y_{Ht} = (\int_0^1 y_{Ht}(i)^{\frac{\epsilon}{\epsilon-1}})^{\frac{\epsilon-1}{\epsilon}}$$

where $y_{Ht}(i)$ is an intermediate input produced by the intermediate firm i, whose price is $P_{Ht}(i)$. The final good firm maximizes the profit:

$$\max_{y_{Ht}, y_{Ht}(i)} \quad P_{Ht}(\int_0^1 y_{Ht}(i)^{\frac{\epsilon}{\epsilon-1}})^{\frac{\epsilon-1}{\epsilon}} - \int_0^1 P_{Ht}(i) y_{Ht}(i)$$

Then the first order conditions give the demand for intermediate goods:

$$y_{Ht}(i) = y_{Ht}\Big(\frac{P_{Ht}(i)}{P_{Ht}}\Big)^{-\epsilon}$$

Intermediate

There is a continuum of firms indexed by i producing a differentiated domestic input using the following Cobb-Douglas function:

$$y_{Ht}(i) = a_t(k_{t-1}(i))^{\alpha}(h_t(i))^{1-\alpha}$$

where a_t is the total factor productivity, which follows an autoregressive process:

$$\log(a_t) = (1-\rho_a)\log(\bar{a}) + \rho_a \log(a_{t-1}) + \nu_t^a, \quad \nu_t^a \sim N(0, \sigma_a^2)$$

Firms operate in monopolistic competition, so they set the price of their own good subject to the demand of final good firms. In addition, firms pay quadratic adjustment cost $AC_t(i)$ in nominal terms as in Rotemberg (1982), whenever they adjust prices with respect to the benchmark inflation rate $\bar{\pi}$:

$$AC_t(i) = \frac{\kappa_P}{2}\Big(\frac{P_{Ht}(i)}{P_{Ht-1}(i)} - \bar{\pi}\Big)^2 P_{Ht} y_{Ht}$$

The intermediate firm is owned by unconstrained households and maximizes the profit:

$$\max_{P_{Ht}(i), y_{Ht}(i), k_{t-1}(i), h_t(i)} E_0 \sum_0^{\infty} \beta^t \frac{\lambda_{1t}}{\lambda_{10}}(\frac{P_{Ht}(i)}{p_t} y_{Ht}(i) - w_t h_t(i) - r_t^k k_{t-1}(i) - \frac{AC_t(i)}{p_t}),$$

subject to the production and demand function:

$$y_{Ht}(i) = a_t(k_{t-1}(i))^{\alpha}(h_t(i))^{1-\alpha},$$
$$y_{Ht}(i) = y_{Ht}\Big(\frac{P_{Ht}(i)}{P_{Ht}}\Big)^{-\epsilon}.$$

The equilibrium is symmetric, the equilibrium interest rate and wage can be solved as:

$$r_t^k = \alpha mc_t \frac{y_{Ht}}{k_{t-1}},$$
$$\omega_t = (1-\alpha)mc_t \frac{y_{Ht}}{h_t}.$$

The marginal cost mc_t is defined by rearranging the price condition:

$$\frac{\epsilon}{\kappa_P}(\frac{mc_t}{p_{Ht}} - \frac{\epsilon - 1}{\epsilon}) = (\pi_{Ht} - \bar{\pi})\pi_t - \beta E_t \frac{\lambda_{1t+1}}{\lambda_{1t}}(\pi_{Ht+1} - \bar{\pi})\pi_{Ht+1}\frac{y_{Ht+1}}{y_{Ht}}$$

1.4 Policy

The government finances public expenditure g_t and transfers τ_{1t} and τ_{2t} by raising consumption tax and public debt:

$$g_t + (1-\lambda)t_{1t} + \lambda t_{2t} = (1-\lambda)(b_{1t} - \frac{r_{t-1}}{\pi_t}b_{1t-1}) + \lambda(CBDC_{2t} - \frac{1}{\pi_t}CBDC_{2t-1})$$
$$+ (1-\lambda)\tau_c c_{1t} + \lambda\tau_c \frac{CBDC_{2t}}{l_{2t}}c_{2t}$$

The central bank sets interest rate following the Taylor rule:

$$\frac{r_t}{\bar{r}} = (\frac{r_{t+1}}{\bar{r}})^{\rho_r}((\frac{\pi_t}{\bar{\pi}})^{\phi_\pi}(\frac{p_{Ht}y_{Ht}}{\bar{p_H y_H}})^{\phi_y}(\frac{\Delta e_t}{\bar{\Delta e}})^{\phi_e})^{1-\rho_r}\exp(\nu_t^m)$$

1.5 Foreign Sector

Given that the domestic economy is sufficiently small relatively to the foreign economy, the foreign sector is treated as endogenously given. The foreign output y_t^* and interest rate r_t^* follow the autoregressive processes:

$$\log(y_t^*) = \rho_y \log(y_{t-1}^*) + \nu_t^y$$
$$r_t^* = (1-\rho_r)\frac{1}{\beta} + \rho_r r_{t-1}^* + \nu_t^r$$

The foreign CPI is assumed to be constant over time:

$$\pi^* = \frac{p_t^*}{p_{t-1}^*} = 1$$

Given the home bias γ^* of the foreign countr, the demand for domestic good of foreign sector is

$$\gamma^*\Big(\frac{p_{Ht}}{s_t}\Big)^{-\eta}y_t^*$$

2 Model with Dollarization

In this section, I extend the model so that deposits and cash can be indexed by domestic or foreign currency. CBDC can only be indexed by domestic currency.

2.1 Households

Unconstrained HH $(1-\lambda)$

The unconstrained households now can choose the deposit to be indexed by domestic $d_{1Ht}(j)$ or foreign currency $d_{1Ft}(j)$; the deposit indexed by domestic and foreign currency are perfect substitutes. The total liquidity for unconstrained households can be written as:

$$l_{1t} = \int_0^1 (d_{1Ht}(j)^{\frac{\epsilon_b - 1}{\epsilon_b}} dj)^{\frac{\epsilon_b}{\epsilon_b - 1}} + s_t \int_0^1 (d_{1Ft}(j)^{\frac{\epsilon_b - 1}{\epsilon_b}} dj)^{\frac{\epsilon_b}{\epsilon_b - 1}}$$

The return to deposit indexed by domestic and foreign currency $r^d_{Ht}(j)$ and $r^d_{Ft}(j)$ are determined by the bank sector problem.

Domestic households pay a quadratic adjustment cost when they change their financial position (foreign bond and foreign currency) with the rest of the world: this assumption ensures the existence of a determinate steady state and a stationary solution. (Schmitt-Grohé and Uribe, 2003)

The unconstrained households' problem can be defined as (all variables are in real terms):

$$\max_{c_{1t}, h_{1t}, s_{1t}, l_{1t}, d_{1Ht}(j), d_{1Ft}(j), b_{1Ht}, b_{1Ft}} E_0 \sum_0^\infty \beta^t \left(\frac{c_{1t}^{1-\sigma}}{1-\sigma} - \chi \frac{h_{1t}^{1+\phi}}{1+\phi}\right),$$

subject to the budget and liquidity constraints:

$$\begin{aligned}
&(1 + s_{1t} + \tau_c)c_{1t} + \int_0^1 (d_{1Ht}(j) - \frac{r^d_{Ht-1}(j)}{\pi_t} d_{1Ht-1}(j))dj + s_t \int_0^1 (d_{1Ft}(j) - r^d_{Ft-1}(j) d_{1Ft-1}(j))dj \\
&+ (b_{1Ht} - \frac{r_{t-1}}{\pi_t} b_{1Ht-1}) + s_t(b_{1Ft} - r^*_{t-1} b_{1Ft-1}) \\
&\leq w_t h_{1t} + t_{1t} + \Gamma_{1t} - \frac{\kappa_D}{2} s_t((1-\lambda)\int_0^1 d_{1Ft}(j)dj - \bar{d}_F)^2 - \frac{\kappa_B}{2} s_t((1-\lambda) b_{1Ft} - \bar{b}_F)^2 \quad (\lambda_{1t}) \\
&l_{1t} = \int_0^1 (d_{1Ht}(j)^{\frac{\epsilon_b - 1}{\epsilon_b}} dj)^{\frac{\epsilon_b}{\epsilon_b - 1}} + s_t \int_0^1 (d_{1Ft}(j)^{\frac{\epsilon_b - 1}{\epsilon_b}} dj)^{\frac{\epsilon_b}{\epsilon_b - 1}} \quad (\tau_{1t}\lambda_{1t}) \\
&s_{1t} = z_t A_1 \frac{c_{1t}}{l_{1t}} + B_1 \frac{l_{1t}}{c_{1t}} - 2\sqrt{A_1 B_1} \quad (\mu_{1t}\lambda_{1t})
\end{aligned}$$

Taking first order conditions with respect to $c_{1t}, h_{1t}, s_{1t}, l_{1t}, d_{1Ht}(j), d_{1Ft}(j), b_{1Ht}, b_{1Ft}$:

$$
\begin{aligned}
c_{1t}: &\quad c_{1t}^{-\sigma} + \lambda_{1t}(1 + s_{1t} + \tau_c) - \tau_{1t}\lambda_{1t}(z_t A_1 \frac{1}{l_{1t}} - B_1 \frac{l_{1t}}{c_{1t}^2}) = 0, \\
h_{1t}: &\quad -\chi h_{1t}^{\phi} + \lambda_{1t}\omega_t = 0, \\
s_{1t}: &\quad -\lambda_{1t}c_{1t} - \tau_{1t}\lambda_{1t} = 0, \\
l_{1t}: &\quad \tau_{1t}\lambda_{1t}(-z_t A_1 \frac{c_{1t}}{l_{1t}^2} + B_1 \frac{1}{c_{1t}}) - \mu_{1t}\lambda_{1t} = 0, \\
d_{1Ht}(j): &\quad -\lambda_{1t} + \beta E_t \lambda_{1t+1} \frac{r_{Ht}^d(j)}{\pi_{t+1}} + \mu_{1t}\lambda_{1t}(\frac{d_{1Ht}}{d_{1Ht}(j)})^{\frac{1}{\epsilon_b}} = 0, \\
d_{1Ft}(j): &\quad -\lambda_{1t} + \beta E_t \lambda_{1t+1} \frac{s_{t+1}}{s_t} r_{Ft}^d(j) + \mu_{1t}\lambda_{1t}(\frac{d_{1Ft}}{d_{1Ft}(j)})^{\frac{1}{\epsilon_b}} \\
&\quad -\lambda_{1t}\kappa_D(1-\lambda)((1-\lambda)\int_0^1 d_{1Ft}(j)dj - \bar{d}_F) = 0, \\
b_{1Ht}: &\quad -\lambda_{1t} + \beta E_t \lambda_{1t+1} \frac{r_t}{\pi_{t+1}} = 0, \\
b_{1Ft}: &\quad -\lambda_{1t} + \beta E_t \lambda_{1t+1} \frac{s_{t+1}}{s_t} t_t^* - \lambda_{1t}\kappa_B(1-\lambda)((1-\lambda)b_{1Ft} - \bar{b}_F) = 0,
\end{aligned}
$$

with

$$
\begin{aligned}
d_{1Ht} &= \int_0^1 (d_{1Ht}(j)^{\frac{\epsilon_b - 1}{\epsilon_b}} dj)^{\frac{\epsilon_b}{\epsilon_b - 1}}, \\
d_{1Ft} &= \int_0^1 (d_{1Ft}(j)^{\frac{\epsilon_b - 1}{\epsilon_b}} dj)^{\frac{\epsilon_b}{\epsilon_b - 1}}.
\end{aligned}
$$

Similar to the case without dollarization, the demand function for $d_{1Ht}(j)$ and $d_{1Ft}(j)$ can be derived as:

$$
\begin{aligned}
d_{1Ht}(j) &= \left(\frac{(r_t - r_{Ht}^d(j))^{-\epsilon_b}}{\left(\int_0^1 (r_t - r_{Ht}^d(j))^{1-\epsilon_b} dj\right)^{-\frac{\epsilon_b}{1-\epsilon_b}}} \right) d_{1Ht} \\
d_{1Ft}(j) &= \left(\frac{(\frac{r_t^*}{1+\tau_{Bt}} - \frac{r_{Ft}^d(j)}{1+\tau_{Dt}})^{-\epsilon_b}}{\left(\int_0^1 (\frac{r_t^*}{1+\tau_{Bt}} - \frac{r_{Ft}^d(j)}{1+\tau_{Dt}})^{1-\epsilon_b} dj\right)^{-\frac{\epsilon_b}{1-\epsilon_b}}} \right) d_{1Ht}
\end{aligned}
$$

where τ_{Bt} and τ_{Dt} are wedges due to the quadratic adjustment costs:

$$
\begin{aligned}
\tau_{Bt} &= \kappa_B(1-\lambda)((1-\lambda)b_{1Ft} - \bar{b}_F) \\
\tau_{Dt} &= \kappa_D(1-\lambda)((1-\lambda)d_{1Ft} - \bar{d}_F)
\end{aligned}
$$

Constrained HH (λ)

The constrained households now can choose to hold domestic m_{2Ht} or foreign cash m_{2Ft}. CBDC is only indexed in domestic currency. Domestic cash users are facing an additional cost δ_m, since cash is under the risk of being stolen and subject to an adjustment cost.

Domestic and foreign currency are perfect substitutes. Cash and CBDC are substitutes with elasticity of substitution $\epsilon_m > 1$. The total liquidity for constrained households can be written as:

$$l_{2t} = ((m_{2Ht})^{\frac{\epsilon_m - 1}{\epsilon_m}} + (s_t m_{2Ft})^{\frac{\epsilon_m - 1}{\epsilon_m}} + (CBDC_{2t})^{\frac{\epsilon_m - 1}{\epsilon_m}})^{\frac{\epsilon_m}{\epsilon_m - 1}}$$

The constrained households' problem can be defined as (all variables are in real terms):

$$\max_{c_{2t}, h_{2t}, l_{2t}, s_{2t}, m_{2Ht}, m_{2Ft}} E_0 \sum_0^\infty \beta^t \left(\frac{c_{2t}^{1-\sigma}}{1-\sigma} - \chi \frac{h_{2t}^{1+\phi}}{1+\phi}\right)$$

subject to the budget and liquidity constraints:

$$\begin{aligned}
\text{s.t.} \quad & (1 + s_{2t} + \tau_c \frac{CBDC_{2t}}{l_{2t}}) c_{2t} + (m_{2Ht} - \frac{1-\delta_m}{\pi_t} m_{2Ht-1}) + s_t(m_{2Ft} - m_{2Ft-1}) \\
& + (CBDC_{2t} - \frac{r_{t-1}^{CBDC}}{\pi_t} CBDC_{2t-1}) \leq w_t h_{2t} + t_{2t} - \frac{\kappa_M}{2} s_t (\lambda m_{2Ft} - \bar{m}_F)^2 \quad (\lambda_{2t}) \\
& l_{2t} = ((m_{2Ht})^{\frac{\epsilon_m - 1}{\epsilon_m}} + (s_t m_{2Ft})^{\frac{\epsilon_m - 1}{\epsilon_m}} + (CBDC_{2t})^{\frac{\epsilon_m - 1}{\epsilon_m}})^{\frac{\epsilon_m}{\epsilon_m - 1}} \quad (\tau_{2t}\lambda_{2t}) \\
& s_{2t} = z_t A_2 \frac{c_{2t}}{l_{2t}} + B_2 \frac{l_{2t}}{c_{2t}} - 2\sqrt{A_2 B_2} \quad (\mu_{2t}\lambda_{2t})
\end{aligned}$$

Taking first order conditions with respect to $c_{2t}, h_{2t}, s_{2t}, l_{2t}, m_{2Ht}, m_{2Ft}, CBDC_{2t}$:

$$\begin{aligned}
c_{2t}: \quad & c_{2t}^{-\sigma} - \lambda_{2t}(1 + s_{2t} + \tau_c \frac{CBDC_{2t}}{l_{2t}}) + \tau_{2t}\lambda_{2t}(z_t A_2 \frac{1}{l_{2t}} - B_2 \frac{l_{2t}}{c_{2t}^2}) = 0, \\
h_{2t}: \quad & -\chi h_{2t}^{\phi} + \lambda_{2t}\omega_t = 0, \\
s_{2t}: \quad & -\lambda_{2t} c_{2t} - \tau_{2t}\lambda_{2t} = 0, \\
l_{2t}: \quad & \lambda_{2t}\tau_c \frac{CBDC_{2t}}{l_{2t}^2} c_{2t} + \tau_{2t}\lambda_{2t}(-z_t A_2 \frac{c_{2t}}{l_{2t}^2} + B_2 \frac{1}{c_{2t}}) - \mu_{2t}\lambda_{2t} = 0, \\
m_{2Ht}: \quad & -\lambda_{2t} + \beta E_t \lambda_{2t+1} \frac{1-\delta_m}{\pi_{t+1}} + \mu_{2t}\lambda_{2t}(\frac{l_{2t}}{m_{2Ht}})^{\frac{1}{\epsilon_m}} = 0, \\
m_{2Ft}: \quad & -\lambda_{2t} + \beta E_t \lambda_{2t+1} \frac{s_{t+1}}{s_t} + \mu_{2t}\lambda_{2t}(\frac{l_{2t}}{s_t m_{2Ft}})^{\frac{1}{\epsilon_m}} - \lambda_{2t}\kappa_M \lambda(\lambda m_{2Ft} - \bar{m}_F) = 0, \\
CBDC_{2t}: \quad & -\lambda_{2t}(1 + \tau_c \frac{c_{2t}}{l_{2t}}) + \beta E_t \lambda_{2t+1} \frac{r_t^{CBDC}}{\pi_{t+1}} + \mu_{2t}\lambda_{2t}(\frac{l_{2t}}{CBDC_{2t}})^{\frac{1}{\epsilon_m}} = 0.
\end{aligned}$$

2.2 Bank

There is a continuum measure of monopolistic competitive banks $j \in [0,1]$. They take deposits from unconstrained households indexed by domestic or foreign currency $d_{Ht}(j)$ and $d_{Ft}(j)$, and invest in intermediate firms $i_t(j)$ and accumulate capital stock $k_t(j)$. The deposit demand functions are solved from the household problem above.

$$d_{Ht}(j) = \left(\frac{(r_t - r_t^d(j))^{-\epsilon_b}}{\left(\int_0^1 (r_t - r_t^d(j))^{1-\epsilon_b} dj \right)^{-\frac{\epsilon_b}{1-\epsilon_b}}} \right) d_{Ht}$$

$$d_{Ft}(j) = \left(\frac{(\frac{r_t^*}{1+\tau_{Bt}} - \frac{r_t^d(j)}{1+\tau_{Dt}})^{-\epsilon_b}}{\left(\int_0^1 (\frac{r_t^*}{1+\tau_{Bt}} - \frac{r_t^d(j)}{1+\tau_{Dt}})^{1-\epsilon_b} dj \right)^{-\frac{\epsilon_b}{1-\epsilon_b}}} \right) d_{Ht}$$

Banks are owned by unconstrained households, so bankers solve the following optimization problem:

$$\max_{r_t^d(j), i_t(j), k_t(j)} E_0 \sum_0^\infty \beta^t \frac{\lambda_{1t}}{\lambda_{10}} (r_t^k k_{t-1}(j) - i_t(j)$$
$$+ (d_{Ht}(j) - \frac{r_{Ht-1}^d(j)}{\pi_t} d_{Ht-1}(j)) + s_t(d_{Ft}(j) - r_{Ft-1}^d(j) d_{Ft-1}(j)),$$

subject to the law of motion of capital and balance sheet constraint:

$$k_t(j) = (1-\delta) k_{t-1}(j) + (1 - \frac{\kappa_I}{2}(\frac{i_t(j)}{i_{t-1}(j)} - 1)) i_t(j)$$
$$k_t(j) = d_{Ht}(j) + s_t d_{Ft}(j).$$

The equilibrium is symmetric and the optimal deposit rate chosen by bank is

$$r_{Ht}^{d\ *} = \frac{\epsilon_b}{\epsilon_b - 1} \frac{\beta E_t \lambda_{1t+1}(r_{t+1}^k + (1-\delta) q_{t+1}) - (q_t - 1)\lambda_{1t}}{\beta E_t(\lambda_{1t+1}/\pi_{t+1})} - \frac{1}{\epsilon_b - 1} r_t$$
$$r_{Ft}^{d\ *} = \frac{\epsilon_b}{\epsilon_b - 1} \frac{\beta E_t \lambda_{1t+1}(r_{t+1}^k + (1-\delta) q_{t+1}) - (q_t - 1)\lambda_{1t}}{\beta E_t(\lambda_{1t+1} s_{t+1}/s_t)} - \frac{1}{\epsilon_b - 1} \frac{1+\tau_D}{1+\tau_B} r_t^*$$

3 Calibration

	Description	Value	Reference
Household			
β	Discount factor	0.990	Gali and Monacelli (2015)
σ	Intertemporal elasticity of substitution	2	
χ	Leisure weight	1	Gali and Monacelli (2015)
ϕ	Frisch elasticity	3	Gali and Monacelli (2015)
A_1	Transaction cost (unconstraint)	0	
B_1	Transaction cost (unconstraint)	0	
Firm			
α	Capital share	0.33	Gali and Monacelli (2015)
κ_P	Price adjustment cost	2	
ϵ	Intermediate-good elasticity of substitution	6	Gali and Monacelli (2015)
δ	Capital depreciation rate	0.025	Gertler and Karadi (2011)
Bank			
κ_I	Investment adjustment cost	1.728	Gertler and Karadi (2011)
ϵ_b	Bank elasticity of substitution	∞	
Open Economy			
η	Domestic and foreign good ES	2	Gali and Monacelli (2015)
$\bar{y}^*$	Foreign output	1	
$\bar{r}^*$	Foreign bond interest rate	$1/\beta$	
ρ_{y^*}	y^* autocorrelation	0.6031	George et al. (2020)
ρ_{r^*}	r^* autocorrelation	0.5374	George et al. (2020)
σ_{y^*}	Standard deviation of y^* innovation	0.0788	George et al. (2020)
σ_{r^*}	Standard deviation of r^* innovation	0.0799	George et al. (2020)

Table 1: Parameters directly calibrated from literature

	Description	Value	Implied steady state	Value
Household				
λ	Share of constrained households	20%	Share of unbanked households	20%
δ_m	Cost of carrying cash	0.2	Time spent on getting cash	
ϵ_m	Cash-CBDC elasticity of substitution	2	CBDC adoption rate	
A_2	Transaction cost (constraint)	0.9	Money velocity	
B_2	Transaction cost (constraint)	0.7	Money velocity	1.25
Government				
$\bar{r}$	Taylor rule: interest rate target	$\bar{\pi}/\beta$	Nominal interest rate	
$\bar{\pi}$	Taylor rule: inflation target	1.03	Inflation rate	3%
$\overline{\Delta e}$	Taylor rule: real exchange rate target	1.03	Exchange rate	1
τ_C	Consumption tax	0.12		
$\bar{B}$	Government bond	2	National debt/GDP	100%
$\bar{G}$	Government purchase	0.3	Government purchase/GDP	15%
Open economy				
γ	Domestic home bias	0.58	Import/GDP ratio	42.13%
γ^*	Foreign home bias	0.27	Export/GDP ratio	27.49%
$\bar{B}_F$	Steady state foreign bond level	0.05	Financial account balance/GDP	0.05
κ_B	Foreign bond adjustment cost	5	Financial account volatility	
$\bar{D}_f$	Steady state foreign deposit level	0.5		
κ_D	Foreign deposit adjustment cost	3		
$\bar{M}_f$	Steady state foreign currency level	0.05	Dollar/GDP	
κ_M	Foreign currency adjustment cost	3		

Table 2: Parameters with implied steady state implications

	Description	Value
Shocks		
ρ_a	a autocorrelation	0.8552
ρ_z	z autocorrelation	0.7217
ρ_g	g autocorrelation	0.8
σ_a	Standard deviation of a shock innovation	0.0711
σ_z	Standard deviation of z innovation	0.0694
σ_g	Standard deviation of g innovation	0.05
σ_m	Standard deviation of m innovation	0.25
Taylor Rule		
ρ_r	Interest rate elasticity	0.5
ϕ_π	Inflation elasticity	10
ϕ_y	Output elasticity	10
ϕ_e	Exchange rate elasticity	2

Table 3: Estimation

4 Welfare analysis

Welfare change	Social	Unconstrained	Constrained
Dollarization	0.0086	0.0065	0.0142

Table 4: Welfare change by adding dollarization

Table 4 shows the welfare change if the economy is dollarized. Constrained households can use the dollar as an alternative payment method; unconstrained households can choose to index their deposit using the dollar. The steady state of the economy with and without dollarization is presented in the first and third columns of Table 6. Dollarization provides additional liquidity services to constrained households. It increases the welfare of constrained households by decreasing the transaction cost.

Welfare change	Social	Unconstrained	Constrained
TANK	0.0126	0.0117	0.0151
TANK with dollarization	0.0073	0.0068	0.0087

Table 5: Welfare change by introducing CBDC

Table 5 shows the welfare change if the CBDC is introduced. CBDC increases the welfare of constrained households by decreasing the transaction cost and cost of getting cash. The steady state of the economy with and without CBDC is presented in Table 6. The introduction of CBDC decreases the use of both cash and dollar, even though constrained households can avoid paying taxes by using cash or dollar.

The use of CBDC decreases the dead weight loss associated with the use of cash. The real exchange rate decreases, and the terms of trade improve. The welfare of both constrained and unconstrained households improve.

Table 6 compares the steady state of four scenarios. The adoption of CBDC increases the domestic GDP when the economy is dollarized. Also, by promoting CBDC, the government can collect more tax revenue by monitoring better the transaction. By using CBDC, the dead weight loss associated with cash is reduced significantly.

5 Variance Decomposition

Table 7 presents the variance decomposition of the benchmark TANK model. The consumption fluctuation of unconstrained households is mainly contributed to TFP shocks (over 90%), while consumption fluctuation of constrained households is affected more by liquidity (18%) and monetary shocks (8%). The terms of trade are affected by the foreign output shocks; over 30% of the volatility of the inflation rate is attributed to monetary policy shocks. The volatility of terms of trade is determined mainly by TFP and foreign shocks.

Figure 1 shows the variance decomposition of consumption for the constrained households. Both introduction of CBDC and the dollar decrease the effect of TFP on the volatility of the consumption of constrained households. Dollarization increases the exposure of

	TANK		Dollarization	
	no CBCD	CBDC	no CBCD	CBDC
Consumption	1.2671	1.281	1.2776	1.2872
Labor	0.883	0.8788	0.8819	0.8807
Capital	15.0352	15.0265	15.0496	15.0604
Investment	0.3759	0.3757	0.3762	0.3765
GDP	1.9195	1.9180	1.9211	1.9223
Consumption (unconstrained)	1.3392	1.3527	1.3467	1.3545
Labor (unconstrained)	0.8453	0.8409	0.8428	0.8402
Deposit	18.794	18.7831	10.6317	10.67
Deposit indexed in dollar	0	0	6.25	6.25
Consumption (constrained)	0.9787	0.9941	1.0014	1.0179
Labor (constrained)	1.0339	1.0307	1.0383	1.0427
Cash	1.0161	0.2728	0.3408	0.0715
Dollar	0	0	0.1581	0.0565
CBDC	0	0.7334	0	0.284
Transaction cost	0.0062	0.0006	0.0021	0
Cost of cash	0.1016	0.0273	0.0341	0.0071
Return to capital	0.0351	0.0351	0.0351	0.0351
Wage	1.2134	1.2185	1.2161	1.2187
Domestic price	0.8528	0.8552	0.8541	0.8553
Real exchange rate	1.313	1.3052	1.3088	1.3049
Tax revenue	0.1286	0.1458	0.1293	0.1361

Table 6: Steady State Comparison

	TFP	Liquidity	Fiscal purchase	Foreign output	Foreign interest rate	Monetary
Output	96.9318	0.006	0.2548	2.6956	0.012	0.0998
Consumption	90.6252	2.483	0.6722	1.6683	3.9913	0.56
Consumption (unconstrained)	91.9247	0.0778	0.4511	1.4554	6.0061	0.0849
Consumption (constrained)	67.8614	17.6443	0.555	1.1615	4.4085	8.3692
Interest rate	84.8279	0.0572	0.0152	2.0398	12.9978	0.062
Deposit rate	84.8279	0.0572	0.0152	2.0398	12.9978	0.062
Wage	88.5733	0.2514	0.0052	4.1626	0.699	6.3085
Domestic inflation	70.4137	0.0701	0.0588	1.6873	0.0716	27.6985
CPI inflation	65.7572	0.0027	0.0852	1.0509	0.3899	32.714
TOT	47.692	1.6742	0.2693	44.8187	5.5368	0.0089
Deposit	98.1528	0.0282	0.3555	0.6492	0.7673	0.0469
Cash	80.5723	0.8232	0.3528	1.2652	6.6471	10.3393

Table 7: Variance decomposition of TANK model

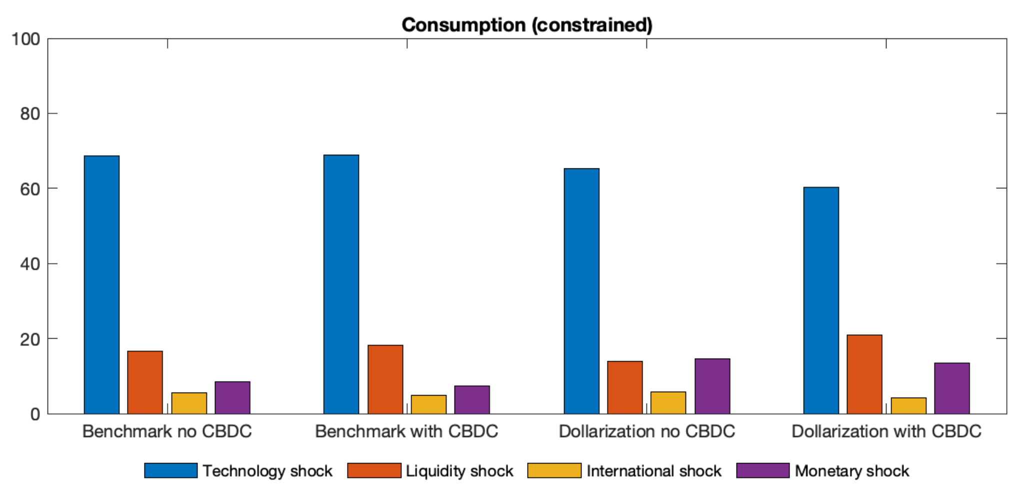

Figure 1: Variance decomposition of consumption (constrained)

constrained households to international shocks, while CBDC decreases it. The adoption of CBDC decreases the volatility attributable to policy shocks.

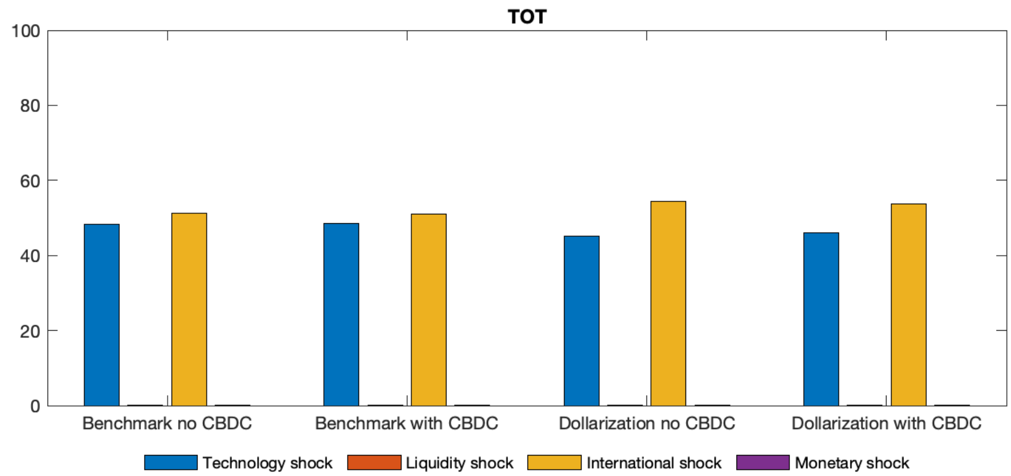

Figure 2: Variance decomposition of terms of trade

Figure 2 shows the variance decomposition of terms of trade. Dollarization amplifies the effect of international shocks on the terms of trade. Adoption of CBDC decreases the responses of terms of trade to international shocks.

6 Impulse Response

Figure 3 plots the responses of key variables following a negative TFP shock. The introduction of CBDC decreases the responses of consumption to TFP shocks. CBDC promotes the financial inclusion of constrained households and decreases the volatility of consumption. The constrained households switch more to CBDC following a negative TFP shock since they are more sensitive to the cost of cash and rely more on CBDC to smooth out their consumption. The terms of trade respond less to TFP shocks when CBDC is introduced as a result of the portfolio adjustment.

Figure 4 plots the responses of key variables following a positive monetary shock. A positive monetary policy shock favors unconstrained households since they receive a higher payoff to their deposit. The introduction of CBDC decreases the inequality effect of monetary policy.

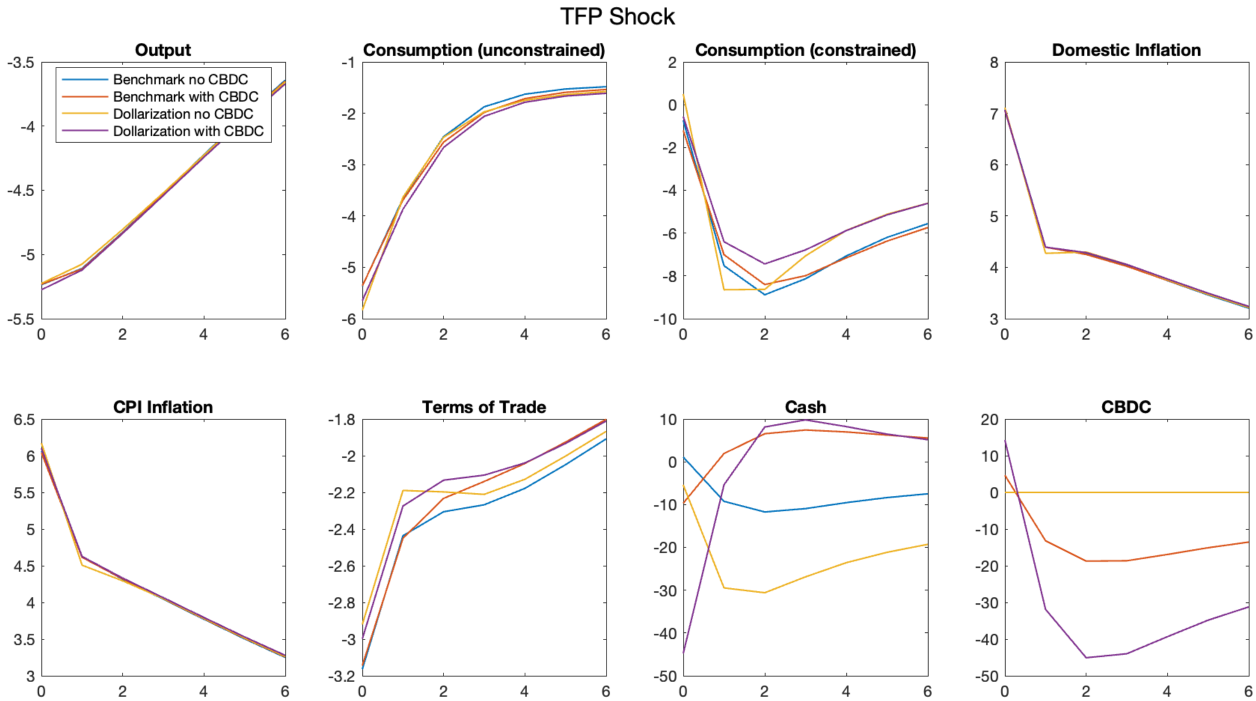

Figure 3: Impulse Responses of TFP Shock

Notes: Response of key variables to a negative 1 standard deviation TFP shock. The Vertical axis indicates the percentage deviation from the steady state. The horizontal axis indicates years after the shock.

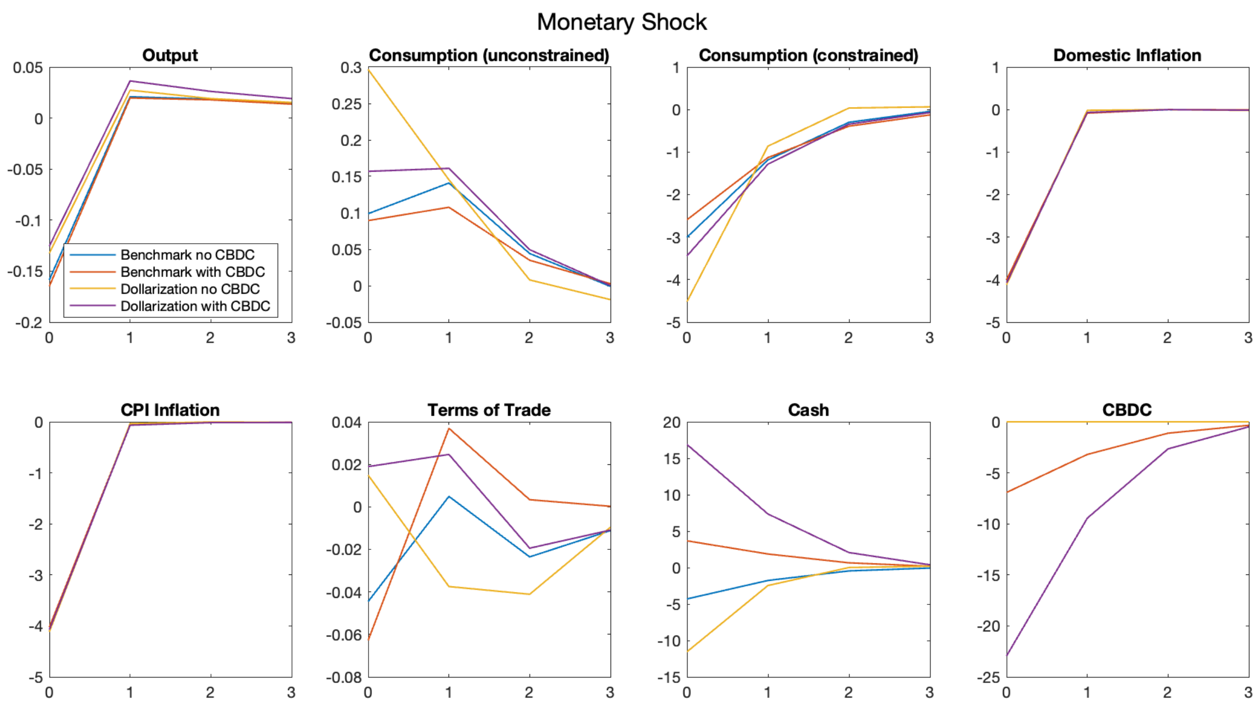

Figure 4: Impulse Responses of Monetary Policy Shock

Notes: Response of key variables to a positive 1 standard deviation monetary policy shock. The Vertical axis indicates the percentage deviation from the steady state. The horizontal axis indicates years after the shock.

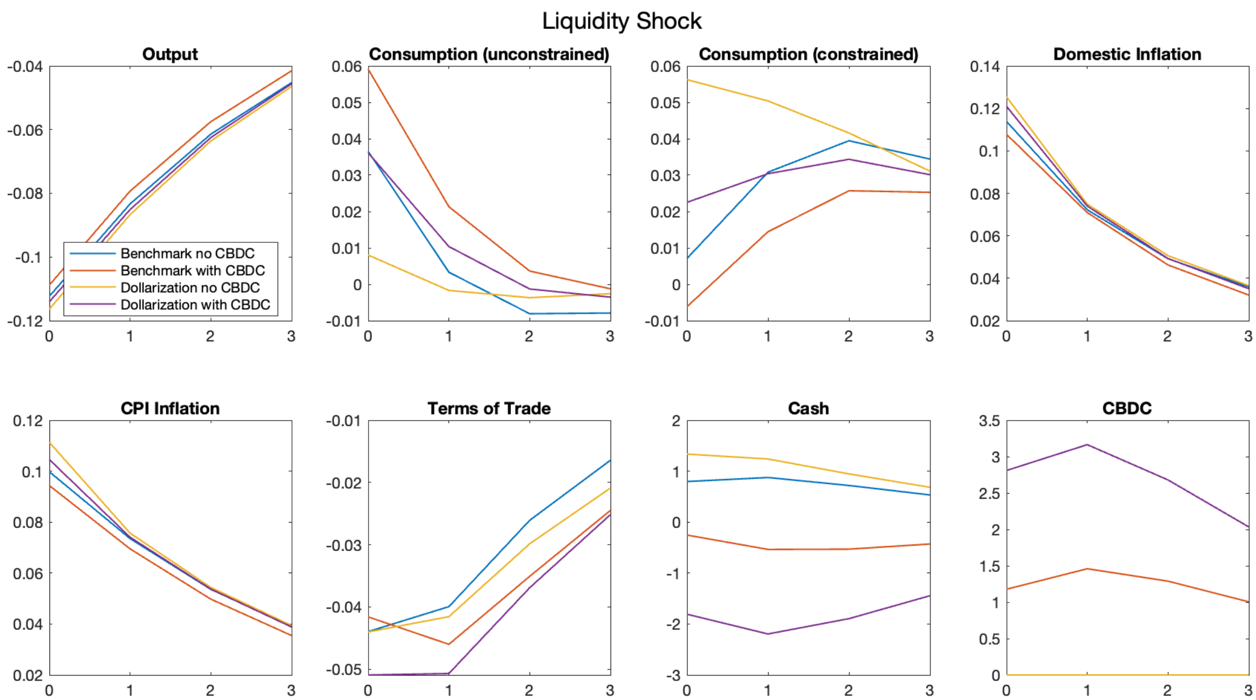

Figure 5: Impulse Responses of Liquidity Shock

Notes: Response of key variables to a positive 1 standard deviation liquidity shock. The Vertical axis indicates the percentage deviation from the steady state. The horizontal axis indicates years after the shock.