Growth Model with Automation and Endogenous Human Capital

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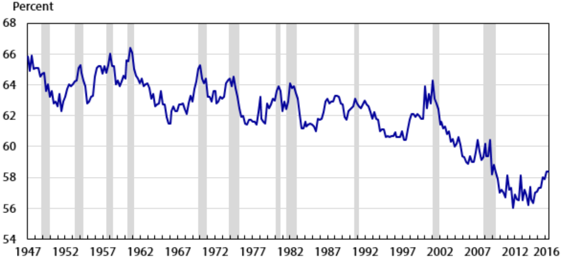
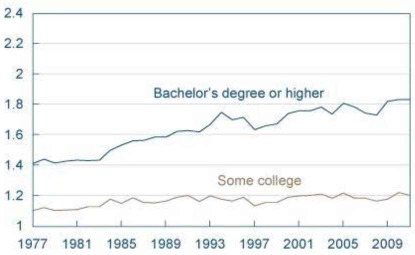
Motivation

Figure 1. Labor's share of output in the nonfarm business sector, first quarter 1947 through third quarter 2016



Source: Bureau of Labor Statistics

Figure 1. Wage Premiums for College



Source: Federal Reserve Bank of Cleveland

- Declining labor share
- Increasing college premium

Related literature

Automation

- Generalized CES: Prettner and Strulikc (2020)
- Task model: Acemoglu and Autor (2011), Acemoglu and Restrepo (2018, 2019, 2020, 2021)

Education and technology

- Twin engines: Stokey (2018), Adao et al. (2020)
- Race: Goldin and Katz (2009), Grossman et al. (2020)

Empirical evidence

- AI Occupational Impact (AIOI) measured by Felten et al.(2019)
- Occupational Information Network (O*NET): bi-annually

$$\underset{\substack{\downarrow \\ \text{Skill level } j \text{ for occupation } i}}{y_{ijt}} = \underbrace{t + t \times AIOI_i}_{\text{Time Trend}} + \underbrace{t \times i + t \times j + \alpha_i + \gamma_j}_{\text{Fixed effect}} + \epsilon_{ijt}$$

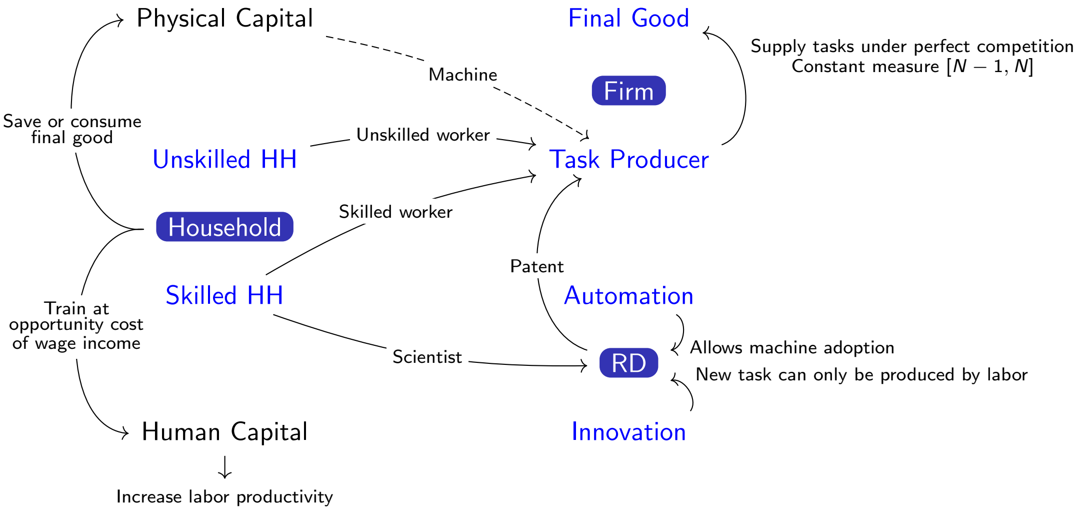
Skill	(1) Content	(2) Process	(3) Social	(4) Complex Problem
Time	-0.000415***	-0.000441***	-0.000411***	0.000592***
	(0.000151)	(0.000149)	(0.000143)	(0.000200)
Time × AIOI	0.000464**	0.000543**	0.000485**	-0.000901***
	(0.000228)	(0.000225)	(0.000216)	(0.000303)
Fixed effect	Yes	Yes	Yes	Yes
Observations	174,720	116,480	174,720	29,120
R-squared	0.760	0.724	0.631	0.842

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Research question

- **How does automation affect skilled and unskilled workers?**
  Direct effect on unskilled workers and ripple effect on skilled workers
- **How do workers respond to automation?**
  Uneven human capital investment
- **Is human capital important for understanding automation?**
  Wage premium and inequality

Model


Physical Capital
Final Good
Machine
Firm
Supply tasks under perfect competition
Constant measure [$N-1$, N]
Save or consume final good
Unskilled HH
Unskilled worker
Task Producer
Skilled worker
Household
Patent
Skilled HH
Automation
Train at opportunity cost of wage income
Scientist
RD
Allows machine adoption
New task can only be produced by labor
Innovation
Human Capital
Increase labor productivity

Human capital investment

Law of motion:

$$\underbrace{\dot{h}_j}_{\text{Change in human capital}} = \frac{\overbrace{(1-l_j)^{\alpha_j}}^{\text{Training time}}}{\underbrace{\mu_{hj}}_{\text{Training cost}}}, \quad \underbrace{j = \{H, L\}}_{\text{Skilled or Unskilled}}$$

Human capital investment

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- $\alpha_j \to 1$, learning or doing
- $\alpha_j \to 0$, learning by doing
- $\alpha_H > \alpha_L$, different learning ability

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Euler:
Trade off between physical and human capital

$$\underbrace{\frac{\delta \log \omega_j(h_j) l_j}{\delta h_j} \frac{\delta \dot{h}_j}{\delta(1-l_j)}}_{\text{Direct wage gain}} + \underbrace{\frac{\delta \log \omega_j(h_j)}{\delta h_j} \dot{h}_j + \frac{\delta \log \omega_j(h_j)}{\delta t}}_{\text{Return to human capital}} = \underbrace{r}_{\text{Return to phisical capital}}$$

$$\frac{\delta \log \omega_j(h_j)}{\delta t} = g_{\omega|h}(\underbrace{g_N}_{\substack{\text{Growth} \\ (+)}}, \underbrace{g_I - g_N}_{\substack{\text{Automation} \\ \text{(-)}}}, \underbrace{g_{h-j}}_{\substack{\text{Human capital} \\ (+)}})$$

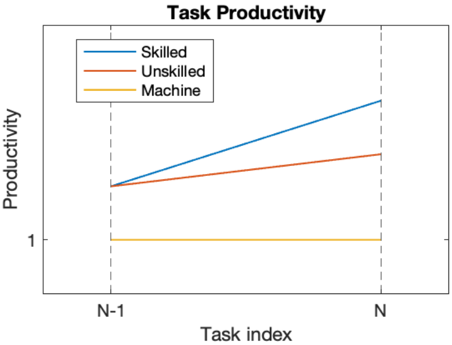
Production factor allocation

Factor productivity:

$$\text{Machine: } \eta(i) = 1$$

$$\text{Skilled worker: } \gamma_H(i, h_H) = e^{B_H(i-1)+B_H(i-(N-1))} e^{b_H h_H}$$

$$\text{Unskilled worker: } \gamma_L(i, h_L) = \underbrace{e^{B_H(i-1)+B_L(i-(N-1))}}_{\downarrow \text{ Task}} \underbrace{e^{b_L h_L}}_{\downarrow \text{ Human capital}}$$



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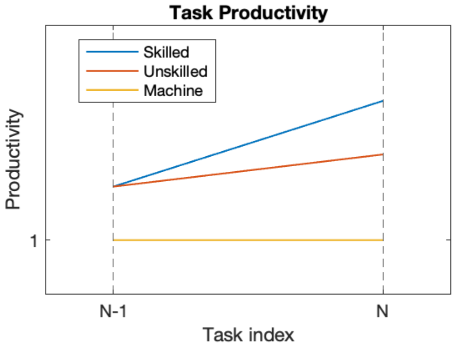
$\downarrow$ Task $\qquad \downarrow$ Human capital



Equilibrium allocation:

Capital | Unskilled | Skilled

$N-1 \rightarrow$ Schumpeterian | $I \rightarrow$ Automation | S Non arbitrage | $N \rightarrow$ Innovation

Directed research

Scientist productivity:

Innovation: $\dot{N} = \frac{1}{\mu_N} {\epsilon_N}^{\lambda}$

Automation: $\dot{I} = \frac{1}{\mu_I} {\epsilon_I}^{\lambda}, \lambda < 1$

$\dot{I}$ ↓ Output

ϵ_I ↓ Scientist

Cost = Return

$\omega_H = \frac{\lambda}{\mu_N} \epsilon_N^{\lambda - 1} P_N$

$\omega_H = \frac{\lambda}{\mu_I} \epsilon_I^{\lambda - 1} P_I$

ω_H ↓ High skill wage

P_I ↓ Patent value

Patent value:

Innovation: $P_N = \underbrace{V_N(N)}_{\text{Newest task profit}} - \underbrace{V_I(N-1)}_{\text{Oldest task profit}}$

Automation: $P_I = \underbrace{V_I(I)}_{\text{Automated task profit}} - \underbrace{V_N(I)}_{\text{Unautomated task profit}}$

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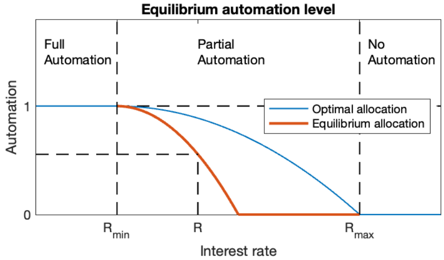


Patent value:

Innovation: $P_N = V_N(N) - V_I(N-1)$

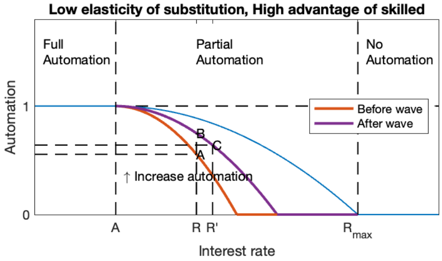
$V_N(N)$ ↓ Newest task profit; $V_I(N-1)$ ↓ Oldest task profit

Automation: $P_I = V_I(I) - V_N(I)$

$V_I(I)$ ↓ Automated task profit; $V_N(I)$ ↓ Unautomated task profit

- At optimal allocation, $P_I = 0$
- Equilibrium < Optimal → $P_I > 0$
- Equilibrium allocation is determined by non-arbitrage condition

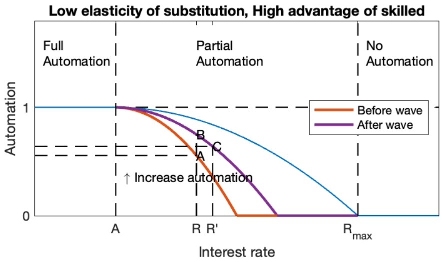
Technology wave

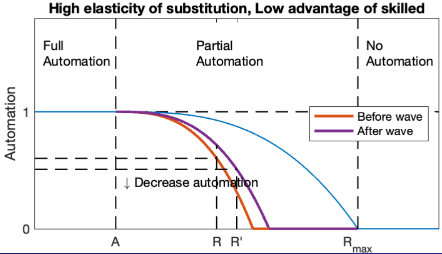


Lower automation cost: $\mu_I \downarrow$

- Productivity effect: $A \to B$
  Automation $\uparrow$, Wage premium $\uparrow$
- Price effect: $B \to C$
  Capital demand $\uparrow$, $R \uparrow$
  Automation $\downarrow$, Wage premium $\downarrow$

Technology wave





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  Capital demand $\uparrow$, $R \uparrow$
  Automation $\downarrow$, Wage premium $\downarrow$

Strong price effect with strong ripple effect

- Unskilled workers can relocate more easily
- Higher elasticity of substitution between production factors
- Lower comparative advantage of skilled worker (productivity, human capital)

Calibration: external

Parameters calibrated externally

Parameter	Description	Value
θ	Intertemporal elasticity of substitution	0.8
δ	Depreciation rate	0.12
ϵ_H	High skill workers share	0.3
ϵ_L	Low skill workers share	0.7
μ_N	Innovation cost	1
μ_I	Automation cost	1
μ_h	Training cost	1

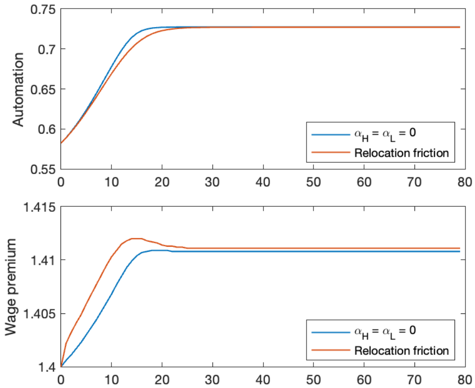
Calibration: internal

Parameters calibrated internally

Parameter	Description	Value	Targeted moment	Value
ρ	Discount rate	0.0196	Long run interest rate	0.04
η	Patent share	0.0684	RD and GDP ratio (1980)	0.0245
σ	Elasticity of substitution	3.112	RD growth rate	0.02
A	Capital productivity	0.1359	Short run interest rate	0.04
b_h	Human capital productivity	0.1885	Human capital growth rate	0.0055
B_H	High skill CA	0.6727	Wage premium (1980)	1.4
B_N	Low skill CA	0.4968	Labor share (1980)	0.64
λ	RD Decreasing return	0.7786	RD and GDP ratio (2000)	0.0265
z	Technology shock size	0.3472	Labor share (2000)	0.6

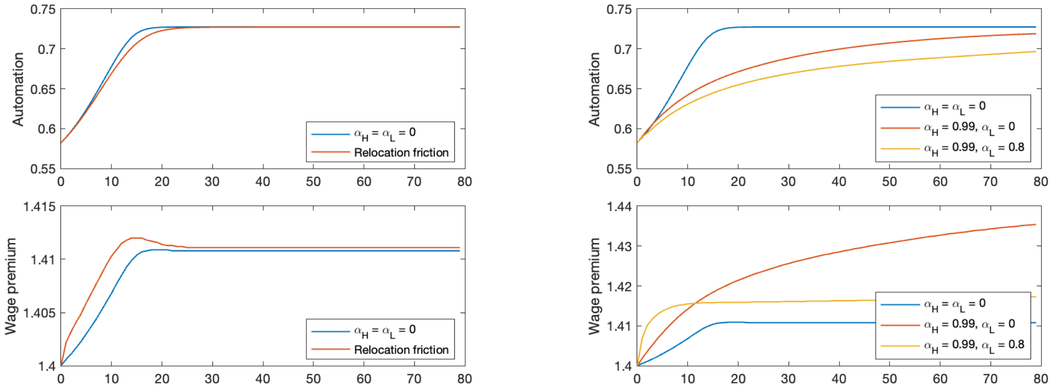
Data source: FRED

Transition: Automation and wage premium

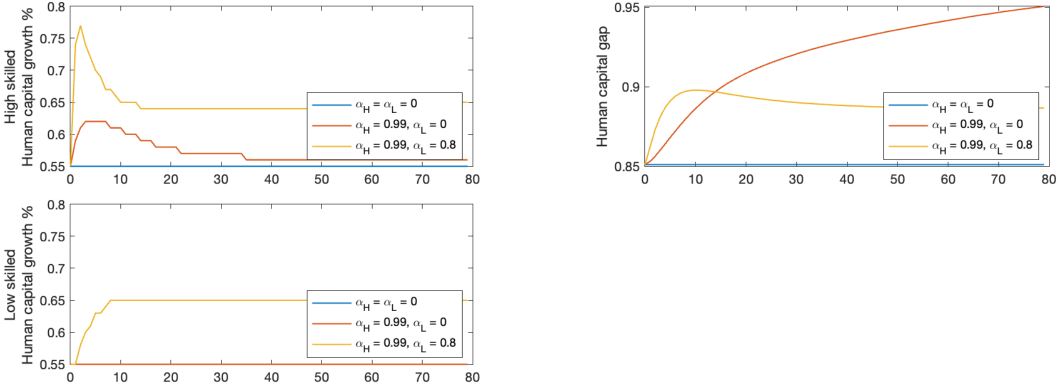


- Small wage premium change without human capital response.
- Limited improvement with relocation friction.

Transition: Automation and wage premium

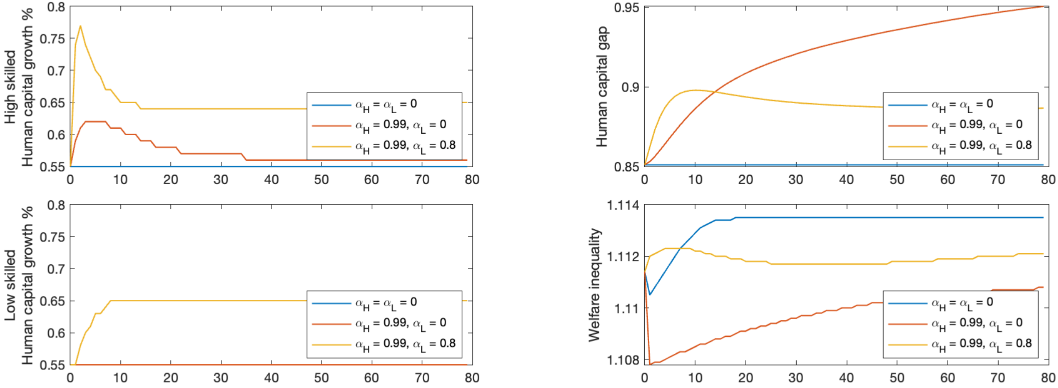


- Small wage premium change without human capital response.
- Limited improvement with relocation friction.
- Human capital investment lowers the automation level but raises the wage premium
- Change in wage premium depends on $\alpha_H - \alpha_L$.



- Workers increase their human capital investment after the technology shock.
- Skilled worker respond more.

Transition: Human capital



- Workers increase their human capital investment after the technology shock.
- Skilled worker respond more.
- Human capital decreases the welfare inequality.

Conclusion

Main takeaways

- The uneven impacts of automation depend on the magnitude of ripple effect.
- Skilled workers respond more to a technology wave.
- Human capital investment explains the rise in the wage premium.
- Human capital investment decreases the welfare inequality.

Skilled HH problem

Skilled workers/Scientist: ϵ_H

$$(\rho + (\theta - 1)g)V_{Ht}(k_H, h_H) - \frac{dV_{Ht}(k_H, h_H)}{dt}$$
$$= \max_{c_H, l_H} \; u(c_H) + \frac{dV_{Ht}(k_H, h_H)}{dk_H}\dot{k}_H + \frac{dV_{Ht}(k_H, h_H)}{dh_H}\dot{h}_H$$

Physical capital: $\dot{k}_H = (r - g)k_H + \omega_H L_H + \pi - c_H$

Human capital: $\dot{h}_H = \dfrac{(1 - l_H)^{\alpha_H}}{\mu_{hH}}, \quad 0 \leq \alpha_H \leq 1$

Labor supply: $\underbrace{L_H}_{\text{Workers}} = \epsilon_H(1 - l_H) - \underbrace{(\epsilon_N + \epsilon_I)}_{\text{Scientists}}$

Cost of human capital investment:

$$\mu_{hH} = f(h_{hH} - h_{hL})$$

back

Unskilled HH problem

Unskilled workers: ϵ_L

$$
\begin{aligned}
(\rho + (\theta - 1)g) V_{Lt}(k_L, h_L) - \frac{dV_{Lt}(k_L, h_L)}{dt} \\
= \max_{c_L, l_L} \ u(c_L) + \frac{dV_{Lt}(k_L, h_L)}{dk_L} \dot{k}_L + \frac{dV_{Lt}(k_L, h_L)}{dh_L} \dot{h}_L \\
\text{Physical capital: } \dot{k}_L = (r - g) k_L + \omega_L L_L - c_L \\
\text{Human capital: } \dot{h}_L = \frac{(1 - l_L)^{\alpha_L}}{\mu_{hL}}, \quad 0 \leq \alpha_L \leq \alpha_H \\
\text{Labor supply: } L_L = \epsilon_L (1 - l_L)
\end{aligned}
$$

Cost of human capital investment:

$$\mu_{hL} = \mu_h$$

back

Firm problem

Final good

$$\underbrace{Y}_{\text{Output}} = \underbrace{A}_{\text{TFP}} \left(\int_{N-1}^{N} \underbrace{y(i)^{\frac{\sigma-1}{\sigma}}}_{\text{task}} di \right)^{\frac{\sigma}{\sigma-1}}$$

Task: With automation constraint I

$$y(i) = \begin{cases} q(i)^{\eta} \Big(k(i) + \gamma_L(i, h_L) l(i) + \gamma_H(i, h_H) h(i) \Big)^{1-\eta}, & N-1 \leq i \leq I \\ \underbrace{q(i)^{\eta}}_{\text{Patent}} \underbrace{\Big(\gamma_L(i, h_L) l(i) + \gamma_H(i, h_H) h(i) \Big)^{1-\eta}}_{\text{Production factor}}, & I < i \leq N \end{cases}$$

back

Firm problem

Demand function

$$y(i) = A^{\sigma-1} Y p(i)^{\sigma}$$

Price = Cost of production

$$p(i) = \begin{cases} \phi \min\{R^{1-\eta}, \left(\frac{W_H}{\gamma_L(i,h_H)}\right)^{1-\eta}, \left(\frac{W_L}{\gamma_L(i,h_L)}\right)^{1-\eta}\}, & N-1 \leq i \leq I \\ \phi \min\{\left(\frac{W_H}{\gamma_L(i,h_H)}\right)^{1-\eta}, \left(\frac{W_L}{\gamma_L(i,h_L)}\right)^{1-\eta}\}, & I < i \leq N \end{cases}$$

where $\phi = (\frac{\psi}{\eta})^{\eta}(\frac{1}{1-\eta})^{1-\eta}$

back

R&D problem

Patent Value

$$\text{Innovation: } P_N(t) = V_N(N(t), t) - V_I(N(t) - 1, t)$$
$$\text{Automation: } P_I(t) = V_I(I(t), t) - V_N(I(t), t)$$

Present discounted value of future profit

$$\text{Task N using labor: } V_N(N, t) = \int_t^\infty e^{-\int_t^\tau r(s)ds} \pi(N, \tau) d\tau$$
$$\text{Task I using machine: } V_I(I, t) = \int_t^\infty e^{-\int_t^\tau r(s)ds} \pi(I, \tau) d\tau$$

Scientist productivity

$$\dot{N} = \frac{1}{\mu_N} \epsilon_N^\lambda \qquad \dot{I} = \frac{1}{\mu_I} \epsilon_I^\lambda$$

back

Ripple effect

$$\underbrace{d\tilde{S}}_{\text{Ripple effect}} = \underbrace{\frac{1}{(\sigma-1)\Lambda(\tilde{S})}(b_L dh_L - b_H dh_H)}_{\text{Human capital}} + \underbrace{\frac{1}{\Lambda(\tilde{S})}\delta_L(\tilde{I})d\tilde{I}}_{\text{Automation}}$$

$$\Lambda(\tilde{S}) = \underbrace{(\frac{d\ln\Gamma_L}{d\tilde{S}} - \frac{d\ln\Gamma_H}{d\tilde{S}})}_{\text{Non arbitrage}} + \underbrace{\frac{\sigma}{\sigma-1}(B_H - B_L)}_{\text{ES and CA}}$$

$$\underbrace{d\log\omega}_{\text{Wage premium}} = (B_H - B_L)d\tilde{S}$$

back