Interaction between Automation and Human Capital: Labor Share and Inequality

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Rong Fan (IU)

Motivation: Automation

Automation

Productivity effect

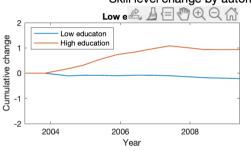
Motivation: Automation

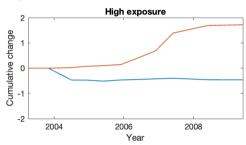
Automation

- Productivity effect
- Displacement effect
 - Labor share and employment
 - Wage premium and inequality

Motivation: Human Capital

Skill level change by automation exposure and education





- O*NET occupational skill level: by automation exposure and education level
- Different skill level change at low and high automation exposure
- Different responses of skilled and unskilled workers

Motivation: Automation and Human Capital

Automation Automation

- Productivity effect
- Displacement effect
 - Labor share and employment
 - Wage premium and inequality

Human capital

- Labor share and employment: race between human capital and automation
- Wage premium and inequality: heterogeneous human capital responses

Research Question

Research questions

- Interaction between automation and human capital?
- Heterogeneous responses of skilled and unskilled workers?
- Labor share, wage premium, and inequality change in the era of automation?

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- Heterogeneous responses of skilled and unskilled workers?
- Labor share, wage premium, and inequality change in the era of automation?

Methodology

- Task model with endogenous automation and human capital
- Calibrate to match the data 1980-2005
- Two scenarios: with and without endogenous human capital
- Empirical evidence



• Automation:

• Acemoglu and Restrepo (2018); Aghion et al. (2021); Acemoglu and Autor (2011)

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- Endogenous human capital

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Automation and human capital:

• Sachs and Kotlikoff (2012); Athreya and Eberly (2015)

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- After-school human capital accumulation
- Heterogeneous workers

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Interaction between technology and human capital:

• Stokey (2014); Stokey (2020); Beaudry et al. (2006)



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Interaction between technology and human capital:

- Stokey (2014); Stokey (2020); Beaudry et al. (2006)
- Two types of R&D: automation and innovation



Task Model

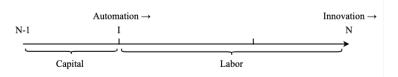
- Household: skilled and unskilled
 - Consumption/Saving
 - Working/Training

Task Model

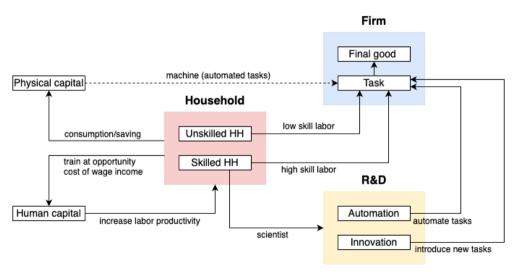
- Household: skilled and unskilled
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- Production: task and final good producer
 - Tasks \in [N-1,N], constant measure 1
 - Automation $\in [N-1, I]$
 - $\bullet \ \, \mathsf{Tasks} \to \mathsf{Final} \; \mathsf{good} \, \,$

Task Model

- Household: skilled and unskilled
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- Production: task and final good producer
 - Tasks $\in [N-1, N]$, constant measure 1
 - Automation $\in [N-1, I]$
 - ullet Tasks o Final good
- R&D: automation and innovation
 - Automation: $I \rightarrow$
 - Innovation: N →



Model Flowchart



Tasks are produced by combining:

Patent intermediates

Production factors

Tasks are produced by combining:

Patent intermediates $\begin{cases} \text{capital} \\ \text{low-skill labor} \\ \text{high-skill labor} \end{cases}$

Tasks are produced by combining:

| | | Input | Price |
|----------------------|------------------|-------|--------|
| Patent intermediates | | q(i) | ψ |
| Production factors | (capital | k(i) | R |
| | low-skill labor | I(i) | W_L |
| | high-skill labor | h(i) | W_H |

Tasks are produced by combining:

The task producers solve the following problem:

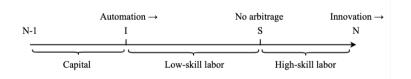
$$\max_{\boldsymbol{y}(i) = \begin{pmatrix} q(i)^{\eta} \left(\gamma_{K} k(i) + \gamma_{L}(i, h_{L}) I(i) - W_{H} h(i) \right) \\ \psi(i) = \begin{cases} q(i)^{\eta} \left(\gamma_{K} k(i) + \gamma_{L}(i, h_{L}) I(i) + \gamma_{H}(i, h_{H}) h(i) \right)^{1-\eta} \\ q(i)^{\eta} \left(\gamma_{L}(i, h_{L}) I(i) + \gamma_{H}(i, h_{H}) h(i) \right)^{1-\eta} \end{cases}, \text{ not automated}$$
Task index

Task Price and Factor Allocation

$$p(i) = \begin{cases} \Psi \min\{ {\color{red}R^{1-\eta}}, \left(\frac{W_L}{\gamma_L(i,h_L)}\right)^{1-\eta}, \left(\frac{W_H}{\gamma_L(i,h_H)}\right)^{1-\eta} \} & \text{, automated} \\ \downarrow & \downarrow \\ \text{effective cost} \\ \psi \min\{ \left(\frac{W_L}{\gamma_L(i,h_L)}\right)^{1-\eta}, \left(\frac{W_H}{\gamma_L(i,h_H)}\right)^{1-\eta} \} & \text{, not automated} \end{cases}$$

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- Technology frontier N and automation I are determined by R&D sector
- S is determined by no arbitrage condition



Final Good Producer

The final good producers solve the following problem:

$$\max \quad Y - \int_{N-1}^{N} p(i)y(i)di$$

$$Y = \tilde{A} \Big(\int_{N-1}^{N} y(i)^{\frac{\sigma-1}{\sigma}} di \Big)^{\frac{\sigma}{\sigma-1}}$$
final goods tasks

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final goods tasks

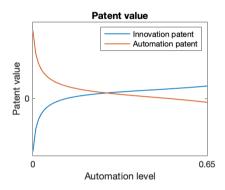
Task demand function:

$$y(i) = \tilde{A}^{\sigma-1} Y p(i)^{-\sigma}.$$



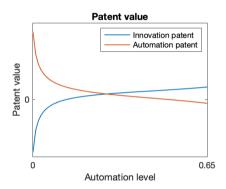
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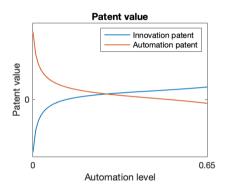
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- \bullet Automation $\uparrow,$ R $\uparrow,$ W \downarrow
- Innovation patent ↑, Automation patent ↓

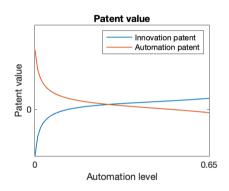


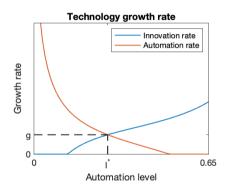


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•
$$\kappa(\epsilon) = \epsilon^{\lambda}/\mu$$
 \downarrow

Rate Scientist



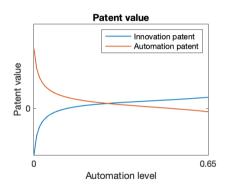


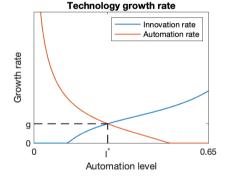
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Rate Scientist





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$$\bullet \ \kappa(\epsilon) = \epsilon^{\lambda}/\mu$$

$$\downarrow \qquad \downarrow$$

Rate Scientist

• No arbitrage: $\kappa'(\epsilon)P = W_H$

Households: Skilled and Unskilled

Households maximize their lifetime utility by making:

- Consumption and saving decision
- Working and training decision

$$\rho V_j(K_j,h_j) = \max_{C_j,l_j} \quad \frac{C_j^{1-\theta}}{1-\theta} + V_{jK}(K_j,h_j)\dot{K}_j + V_{jh}(K_j,h_j)\dot{h}_j, \quad j = \{H,L\}$$

$$\downarrow$$
skilled or unskilled

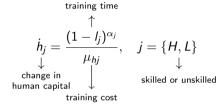
subject to the laws of motion:

Physical capital:
$$\dot{K}_j = rK_j + W_j I_j + \Pi_j - C_j$$
 Π_j : skilled workers receive patent profits Human capital: $\dot{h}_j = \frac{(1-I_j)^{\alpha_j}}{\mu_{hi}}$



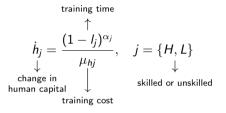
Human Capital Investment

Law of motion:



Human Capital Investment

Law of motion:



Grossman et al. (2021) Stantcheva (2015)

- ullet $\alpha_j
 ightarrow 1$, learning or doing
- ullet $\alpha_j o 0$, learning by doing
- $\alpha_H > \alpha_L$, different learning ability

Human Capital Investment

Law of motion:

training time
$$\dot{h}_j = \frac{(1-l_j)^{\alpha_j}}{\mu_{hj}}, \quad j = \{H, L\}$$
 change in human capital training cost

Grossman et al. (2021) Stantcheva (2015)

- $\alpha_j \to 1$, learning or doing
- $\alpha_j \rightarrow 0$, learning by doing
- $\alpha_H > \alpha_L$, different learning ability

Euler: Trade off between physical and human capital

$$\underbrace{\frac{\delta \log W_j(h_j)l_j}{\delta h_j} \frac{\delta \dot{h}_j}{\delta (1 - l_j)}}_{\text{Direct wage gain}} + \underbrace{\frac{\delta \log W_j(h_j)}{\delta h_j} \dot{h}_j + \frac{\delta \log W_j(h_j)}{\delta t}}_{\text{Return to human capital}} = r$$

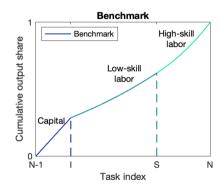
Return to physical capital

Balanced Growth Path

- Normalized variables $x = Xe^{-\int_0^t g(\tau)d\tau}$ are constant on BGP
 - Growth rates $\{g_N, g_I, g_{hH}, g_{hL}\}$
 - Automation level and the labor allocation $\{\tilde{I}, \tilde{S}\}$
 - Factor shares $\{s_K, s_L, s_H\}$, the rental rate of capital $\{r\}$, labor supply $\{L_H, L_L\}$
 - Normalized capital, output and consumption $\{k, y, c_H, c_L\}$, labor wages $\{\omega_H, \omega_L\}$
- Optimization problem
- Market clear

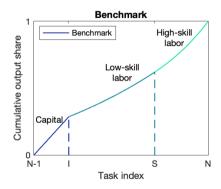


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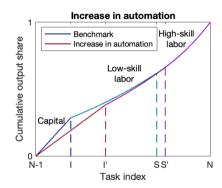






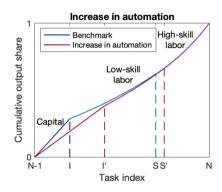
Automation ↑

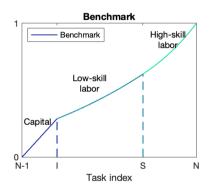




- Automation ↑
- Labor share ↓: displacement effect
- Wage premium ↑: relocation effect



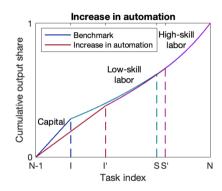




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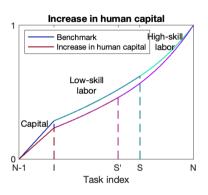
Human capital of skilled workers ↑





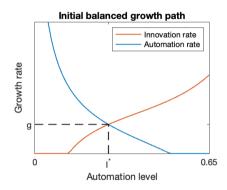


- ullet Labor share \downarrow : displacement effect
- Wage premium ↑: relocation effect

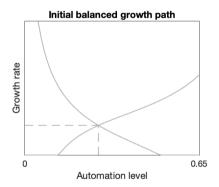


- Human capital of skilled workers ↑
- Labor share ↑: labor supply
- Wage premium ↑: productivity



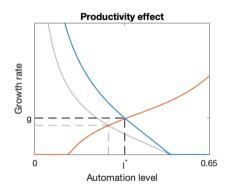






Automation cost ↓



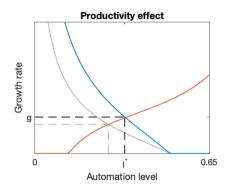


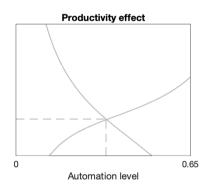
Automation cost ↓

- Automation curve ↑
- Automation ↑







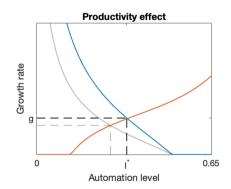


Automation cost \downarrow

- Automation curve ↑
- Automation ↑

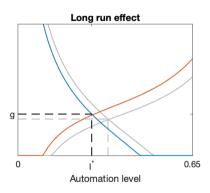
Long run effect





Automation cost ↓

- Automation curve ↑
- Automation ↑



Long run effect

- \bullet Long run k \downarrow
- Innovation patent ↑
- Automation patent ↓



Return to human capital =
$$\underbrace{\frac{\delta \log W_j(h_j)}{\delta h_j}}_{\text{direct retrun}} + \underbrace{\frac{\delta \log W_j(h_j)}{\delta t}}_{\text{technological environment}}$$

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$$\frac{dW_{H}(h_{H}, t)}{dt} = g_{WN} + g_{WI} + \frac{s_{L}}{s_{H} + s_{L}}g_{WS}$$
Unskilled:
$$\frac{dW_{L}(h_{L}, t)}{dt} = g_{WN} + g_{WI} - \frac{s_{H}}{s_{H} + s_{L}}g_{WS}$$

$$\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \uparrow \qquad \qquad \downarrow \qquad \qquad \uparrow \qquad \qquad \downarrow \qquad \qquad \uparrow \qquad \qquad \uparrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \uparrow \qquad \qquad \uparrow \qquad \qquad \uparrow \qquad \qquad \downarrow \qquad \qquad \uparrow \qquad \qquad \downarrow \qquad \qquad \uparrow \qquad \qquad \uparrow \qquad \qquad \downarrow \qquad \qquad \uparrow \qquad \qquad \uparrow \qquad \qquad \downarrow \qquad \qquad \uparrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \uparrow \qquad \qquad \downarrow \qquad \qquad$$

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Technological revolution



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Technological revolution

ullet Productivity effect: better allocation and higher innovation rate $g_{WN}\uparrow$



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productivity rellocation

displacement

Technological revolution

- Productivity effect: better allocation and higher innovation rate $g_{WN} \uparrow$
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Technological revolution

- Productivity effect: better allocation and higher innovation rate $g_{WN} \uparrow$
- ullet Displacement effect: automation replaces labor $g_{WI}\downarrow$
- Relocation effect: unskilled workers relocate to higher index tasks $g_{WS} \uparrow$

Equation

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Internal Calibration

| Parameter | Description | Value | Targeted moment | Value |
|------------------|-----------------------------|----------|-----------------------------|-------|
| ρ | Discount rate | 0.0121 | Long run interest rate | 4.0% |
| η | Patent share | 0.1125 | RD and GDP ratio | 2.8% |
| \boldsymbol{A} | Capital productivity | 0.1190 | Short run interest rate | 4.0% |
| B_H | Skill comparative advantage | 2.2651 | Wage premium (1980) | 1.4 |
| μ_{N} | Cost of innovation | 2.3619 | RD growth rate | 2.8% |
| μ_I | Cost of automation | 9.0135 | Labor share (1980) | 0.625 |
| μ_h | Cost of h_L accumulation | 193.5608 | Human capital growth rate | 0.3% |
| α_H | h_H accumulation function | 0.9026 | Change of training time (H) | 0.141 |
| $lpha_{L}$ | h_L accumulation function | 0.2797 | Change of training time (L) | 0.160 |
| λ | RD Decreasing return | 0.7675 | Wage premium (2005) | 1.6 |
| Z | Technological revolution | 0.7853 | Labor share (2005) | 0.605 |

Balanced Growth Path

| Moment | 1980 | 2005 | |
|---------------------------|---------|---------------|---------------|
| | | Fixed | Endogenous |
| | | human capital | human capital |
| Automation level | 0.6818 | 0.7796 | 0.7623 |
| Labor share | 0.6174 | 0.5906 | 0.5939 |
| Skilled labor share | 0.2814 | 0.2758 | 0.2840 |
| Unskilled labor share | 0.3360 | 0.3148 | 0.3099 |
| Wage premium | 1.4012 | 1.4454 | 1.5897 |
| Technology growth rate | 4.2036% | 5.5177% | 5.2122% |
| Human capital growth rate | 0.2612% | 0.2612% | 0.2743% |
| Welfare inequality | 1.0087 | 1.0902 | 1.1859 |

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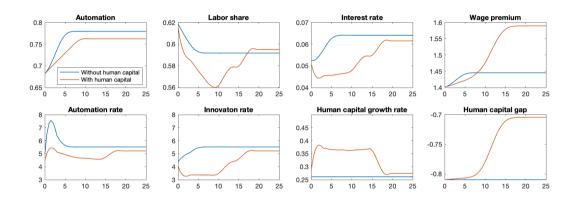
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- Endogenous human capital increases labor share
- Uneven responses increase the inequality



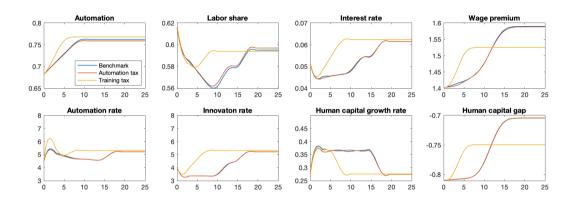
Transition



- Endogenous human capital amplifies the effect of automation
- Labor share drops more in the short run: human capital adjustment
- It takes longer for innovation rate to catch up



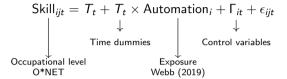
Policy implications



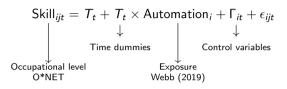
- Automation tax: increase labor share
- Training tax on skilled workers: decrease inequality

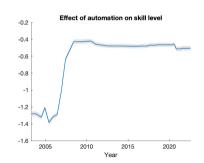


Empirical Evidence: O*NET



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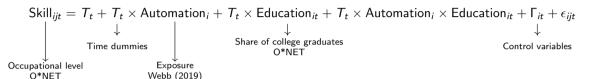


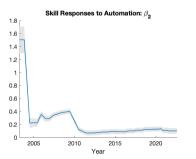


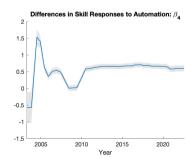
Automation

- Automation ↑ Occupational skill ↑
- Productivity effect: improve allocation
- Complement human capital: adaptation

Different Responses



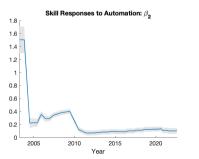




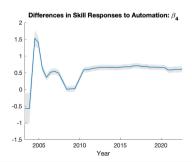
Different Responses

O*NET





Webb (2019)



- After control for education, automation decreases skill growth
- Skilled workers increase skill level more



Main takeaways

• A reduction in automation costs increases productivity

Main takeaways

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- Human capital investment complements innovation but substitutes automation

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Main takeaways

- A reduction in automation costs increases productivity
- Human capital investment complements innovation but substitutes automation
- Endogenous human capital amplifies the effect of the technological revolution
- An automation tax decreases automation and increases labor share. A training tax on skilled workers is more efficient in reducing inequality and accelerating the recovery.

Comparative Statistics 1

The change in labor share is given by the following equation:

$$d \ln(s_L + s_H) = \frac{\hat{\sigma} - 1}{\hat{\sigma}} \frac{s_K}{1 - \eta} \underbrace{\left(\frac{s_L}{s_L + s_H} \frac{d \ln \tilde{\Gamma}_L}{d\tilde{I}} - \frac{d \ln H}{d\tilde{I}}\right) d\tilde{I}}_{\text{Automation}}$$

$$- \frac{\hat{\sigma} - 1}{\hat{\sigma}} \frac{s_K}{1 - \eta} \underbrace{\left(d \ln K - BdN - bdh - d \ln L\right)}_{\text{Capital deepening}}$$

where the average change of human capital and labor supply is defined as:

$$d \ln h = \frac{s_H}{s_H + s_L} d \ln h_H + \frac{s_L}{s_H + s_L} d \ln h_L$$
$$d \ln L = \frac{s_H}{s_H + s_L} d \ln L_L + \frac{s_L}{s_H + s_L} d \ln L_L$$

back



Comparative Statistics 2

The change of allocation \tilde{S} can be written as:

$$\underbrace{d\tilde{S}}_{\text{Allocation}} = \frac{1}{\epsilon(\tilde{S})} \left(\underbrace{\frac{1}{\hat{\sigma}} (d \ln L_L - d \ln L_H - b d h_{HL})}_{\text{Labor supply}} - \underbrace{\frac{\hat{\sigma} - 1}{\hat{\sigma}} \frac{d \ln \tilde{\Gamma}_L}{d\tilde{I}} d\tilde{I}}_{\text{Automaton}} \right)$$

where $\epsilon(\tilde{S})$ is the inverse of allocation elasticity taking the form:

$$\epsilon(\tilde{S}) = \underbrace{\frac{\hat{\sigma} - 1}{\hat{\sigma}} (\frac{d \ln \tilde{\Gamma}_L}{d \tilde{S}} - \frac{d \ln \tilde{\Gamma}_H}{d \tilde{S}})}_{\text{Wage elasticity}} + \underbrace{(B - B_L)}_{\text{Comparative advantage}}$$

back



Comparative Statistics 3

The change of wages can be written as:

$$d \ln W_L = \underbrace{d \ln Y}_{\text{Productivity}} + \underbrace{\frac{\hat{\sigma} - 1}{\hat{\sigma}} \frac{d \ln \tilde{\Gamma}_L}{d\tilde{I}} d\tilde{I}}_{Displacement} + \underbrace{\frac{\hat{\sigma} - 1}{\hat{\sigma}} \frac{d \ln \tilde{\Gamma}_L}{d\tilde{S}} d\tilde{S}}_{\text{Relocate}}$$

$$d \ln W_H = \underbrace{d \ln Y}_{\text{Productivity}} + \underbrace{\frac{\hat{\sigma} - 1}{\hat{\sigma}} \frac{d \ln \tilde{\Gamma}_H}{d\tilde{S}} d\tilde{S}}_{\text{Relocate}}$$

$$d \ln \omega = (B - B_L) d\tilde{S} + b dh_{HL}$$

back



R&D Problem 1

Patent Value

Innovation:
$$P_N(t) = V_N(N(t), t) - V_I(N(t) - 1, t)$$

Automation: $P_I(t) = V_I(I(t), t) - V_N(I(t), t)$

Present discounted value of future profit

Task N using labor:
$$V_N(N,t) = \int_t^\infty e^{-\int_t^\tau r(s)ds} \pi(N,\tau) d\tau$$
Task I using machine: $V_I(I,t) = \int_t^\infty e^{-\int_t^\tau r(s)ds} \pi(I,\tau) d\tau$

back



R&D Problem 2

Scientist productivity

$$\dot{N} = rac{1}{\mu_N} \epsilon_N^{\lambda} \qquad \quad \dot{I} = rac{1}{\mu_I} \epsilon_I^{\lambda}$$

No arbitrage condition

$$\frac{\lambda \epsilon_I(t)^{\lambda-1}}{\mu_I} P_I(t) = \frac{\lambda \epsilon_N(t)^{\lambda-1}}{\mu_N} P_N(t) = W_H(t).$$

Technology growth rate

$$g_N = \frac{1}{\mu_N} \left(\frac{\mu_N W_H}{\lambda P_N}\right)^{\frac{\lambda}{\lambda - 1}} \qquad g_I = \frac{1}{\mu_I} \left(\frac{\mu_I W_H}{\lambda P_I}\right)^{\frac{\lambda}{\lambda - 1}}$$





Wage Growth

$$g_{WN} = \underbrace{\frac{Bg_N}{Productivity}}_{Productivity} + \underbrace{\frac{a(\tilde{I})(g_K - g_L - Bg_N - bg_h)}{Capital deepening}}_{Capital deepening}$$

$$g_{WI} = \underbrace{\frac{a(\tilde{I})\frac{d\ln H}{d\tilde{I}}(g_I - g_N)}{Productivity}}_{Productivity} + \underbrace{\frac{s_L}{s_H + s_L}(1 - a(\tilde{I}))\frac{d\ln \tilde{\Gamma}_L}{d\tilde{I}}(g_I - g_N)}_{Displacement}$$

$$g_{WS} = \underbrace{\frac{B - B_L}{\epsilon(\tilde{S})}}_{Labor supply} (\underbrace{\frac{1}{\hat{\sigma}}(g_{LL} - g_{LH} + b(g_{hL} - g_{hH}))}_{Labor supply} - \underbrace{\frac{\hat{\sigma} - 1}{\hat{\sigma}}\frac{d\ln \tilde{\Gamma}_L}{d\tilde{I}}(g_I - g_N)}_{Relocation})$$

back



Rong Fan (IU)

External Calibration

| Parameter | Description | Value | Reference |
|--------------|--|-------|--------------------------------|
| θ | Intertemporal elasticity of substitution | 0.9 | Beaudry and Van Wincoop (1996) |
| σ | Factor elasticity of substitution | 2 | |
| δ | Depreciation rate | 0.1 | BEA Depreciation Estimates |
| B_L | Comparative advantage of unskilled workers | 1 | |
| Ь | Return to human capital | 1 | |
| ϵ_H | high-skill workers share | 0.3 | FRED |
| ϵ_L | Low skill workers share | 0.7 | FRED |
| λ | R&D production function | 0.5 | Prettner and Strulik (2020) |

Table: External Calibration



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Social Planner's Problem

Innovation patent

$$\begin{split} \text{SPP: } & \frac{\lambda \epsilon_N^{\lambda-1}}{\mu_N} \frac{(1-\eta) P_N^{SPP}}{Y} = \frac{W_H}{Y} \\ & \text{CE: } & \frac{\lambda \epsilon_N^{\lambda-1}}{\mu_N} \frac{P_N}{Y} = \frac{W_H}{Y} \end{split}$$

Training for skilled workers

SPP:
$$\frac{L_H}{\epsilon_H} \frac{b\delta \dot{h}_H}{\delta (1 - l_H)} + g_{WH} = \frac{R}{1 - \eta} - \delta$$
CE: $l_H \frac{b\delta \dot{h}_H}{\delta (1 - l_H)} + g_{WH} = r(1 + \tau_{hH})$

back



Empirical Evidence: ATES

| | (1) | (2) | | |
|---------------------------------|-----------------------|-----------------------|--|--|
| VARIABLES | Training hours | | | |
| High school \times AOE | -4.544*** | -7.138*** | | |
| Associate × AOE | (0.0672) 7.670*** | (0.0671) 4.465*** | | |
| Bachelor or higher \times AOE | (0.0863) 5.722*** | (0.0862) 3.765*** | | |
| AOE | (0.0794) -2.314*** | (0.0793) -2.857*** | | |
| High school | (0.0604) 2.959*** | (0.0606) 2.438*** | | |
| Associate | (0.0325) 4.718*** | (0.0324) 4.356*** | | |
| | (0.0386) 10.36*** | (0.0386) 10.16*** | | |
| Bachelor or higher | (0.0342) | (0.0342) | | |
| Constant | 18.28*** (0.0299) | 36.38*** (0.0344) | | |
| Observations | 495,266,531 | 495,266,531 | | |
| R-squared | 0.018 | 0.022 | | |
| Time dummies | Yes | Yes | | |
| Control variable | No | Yes | | |

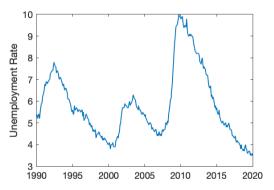
Skill Mismatch and Jobless Recovery

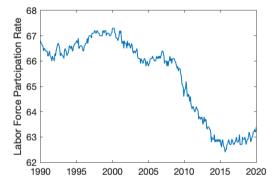
Rong Fan

Indiana University

November 10, 2022

Motivation





- Unemployment Rate (UR): excessive response and slow recovery
- Unemployment Rate (UR): 3.5% in February 2020
- \bullet Labor Force Participation Rate (LFPR): $66\% \rightarrow 63.3\%$

Research Question

Research questions

- Excessive response of unemployment rate and labor force participation rate?
- Decline trend of labor force participation rate?
- Skill mismatch and labor market structural change?

Research Question

Research questions

- Excessive response of unemployment rate and labor force participation rate?
- Decline trend of labor force participation rate?
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Methodology

- Search-and-matching model with heterogeneous skill and technology
- Sequential auction: on- and off- the job search
- Benchmark: calibrate to match the data 1987-1993
- Structural change in 2007-2013: three scenarios
- Empirical evidence



Skill Mismatch

Match output f(z, a, b)

- a: skill
- b: technology

$$f(z, a, b) = z\underbrace{(\kappa a + (1 - \kappa)b)}_{\text{productivity}} - \underbrace{\{a < b\}\alpha_u \frac{(b - a)^2}{b}}_{\text{Underqualification penalty}}$$

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Overqualification disutility d(z, a, b)

$$d(a,b) = 1\{a > b\}\alpha_o \frac{(b-a)^2}{a}$$



Labor Market Dynamics: Sequential Auction

$$\bigvee_{\substack{\downarrow \\ \text{unemployment}}} \underbrace{\frac{O(a,b)}{\text{disutility}}} \xrightarrow{\substack{\text{On the job search} \\ \text{Wage bargaining}}} \underbrace{V_e(z,a,b',b)} \xrightarrow{\substack{\text{Climb job ladder} \\ \downarrow \\ \text{outside option}}} \dots$$

$$\xrightarrow{\mathsf{Climb\ job\ ladder}} V_e(z,a,b'',b) \xrightarrow{\mathsf{endogenous\ separation}} V_u + D(a,b)$$

Extension

Home productivity shock

$$\epsilon_n \sim Pareto(\epsilon_{min}, \lambda_n)$$

Endogenous skill and technology

$$\bar{S}(z,a,b) = \max_{a^*,b^*} \quad S(z,a^*,b^*) - \phi_a \frac{(a^*-a)^2}{a} - \phi_b \frac{(b^*-b)^2}{b}.$$



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Calibration

| | Benchmark (1987-1993) | Scenario 1 | Scenario 2 (2007-2013) | Scenario 3 |
|------------------------------|--------------------------|------------------|---------------------------|----------------------|
| Structural change | (, | Technology level | Skill dispersion | Higher training cost |
| Matching moment | | Labor share | Skill variance | Training/GDP |
| Unemployment rate | 0.0541 | 0.0554 | 0.0668 | 0.0512 |
| Labor for participation rate | 0.6637 | 0.6313 | 0.6175 | 0.6488 |
| Mismatch | 0.2643 | 0.3080 | 0.3144 | 0.3013 |
| Unemployment volatility | 0.0756 | 0.0942 | 0.0835 | 0.0940 |

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- \bullet Mismatch $\uparrow,$ U volatility $\uparrow,$ LFPR \downarrow
- \bullet Training cost \uparrow , U \downarrow , LFPR \uparrow



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Conclusion

• Skill mismatch increases unemployment rate and volatility



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Conclusion

- Skill mismatch increases unemployment rate and volatility
- Endogenous skill and technology amplifies volatility

Conclusion

- Skill mismatch increases unemployment rate and volatility
- Endogenous skill and technology amplifies volatility
- Mismatch can be a result of
 - Higher technology level
 - Higher skill heterogeneity
 - Higher training cost

Central Bank Digital Currency in Small Open Economies

Rong Fan, Todd Walker, Wayne Robinson and Allan Wright

Indiana University

November 11, 2022

Motivation



- 17.9% of Bahamians are unbanked. (Central Bank of the Bahamas)
- Electronic payment: only Nassau and Freeport
- Over 90% of Bahamians have used internet in last three months. (World Bank)

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Sand Dollar

- Transaction efficiency
- Financial inclusion
- Regulated payment
- Dollarization

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Research Question

Research questions

- CBDC and transaction frictions?
- CBDC and social welfare?
- CBDC and fiscal and monetary policy efficiency?

Research Question

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- CBDC and transaction frictions?
- CBDC and social welfare?
- CBDC and fiscal and monetary policy efficiency?

Methodology

- TANK model with constrained and unconstrained households
- Households are facing liquidity constraints
- CBDC as a more efficient and safer asset than cash
- Dollarization: deposit and cash can be indexed by domestic or foreign currency

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Unconstrained Household

$$\max_{c_{1t},h_{1t},s_{1t},h_{1t},d_{1t}(j),b_{1Ht},b_{1Ft}} E_0 \sum_{0}^{\infty} \beta^t (\frac{c_{1t}^{1-\sigma}}{1-\sigma} - \chi \frac{h_{1t}^{1+\phi}}{1+\phi}),$$

subject to the budget and liquidity constraints:

$$\begin{aligned} \mathsf{BC:} \ &(1+s_{1t}+\tau_c)c_{1t} + \int_0^1 (d_{1t}(j) - \frac{r_{t-1}^d(j)}{\pi_t} d_{1t-1}(j))dj + (b_{1Ht} - \frac{r_{t-1}}{\pi_t} b_{1Ht-1}) \\ &+ s_t(b_{1Ft} - r_{t-1}^* b_{1Ft-1}) \leq w_t h_{1t} + t_{1t} + \Gamma_{1t} - \frac{\kappa_B}{2} s_t ((1-\lambda)b_{1Ft} - \bar{b}_F)^2 \\ \mathsf{Liquidity:} \ & l_{1t} = \int_0^1 (d_{1t}(j)^{\frac{\epsilon_b - 1}{\epsilon_b}} dj)^{\frac{\epsilon_b}{\epsilon_b - 1}} \end{aligned}$$

$$\mathsf{Transaction \ cost:} \ & s_{1t} = z_t A \frac{c_{1t}}{l_{1t}} + B \frac{l_{1t}}{c_{1t}} - 2\sqrt{AB} \end{aligned}$$

Constrained Households

$$\max_{c_{2t},h_{2t},l_{2t},s_{2t},m_{2t},CBDC_{2t}} E_0 \sum_{0}^{\infty} \beta^t (\frac{c_{2t}^{1-\sigma}}{1-\sigma} - \chi \frac{h_{2t}^{1+\phi}}{1+\phi})$$

subject to the budget and liquidity constraints:

BC:
$$(1 + s_{2t} + \underbrace{\tau_c \frac{CBDC_{2t}}{l_{2t}}}_{\text{consumption tax}})c_{2t} + (m_{2t} - \underbrace{\frac{1 - \delta_m}{\pi_t}}_{\text{cost of cash}} m_{2t-1}) + (CBDC_{2t} - \frac{1}{\pi_t}CBDC_{2t-1})$$

$$\leq w_t h_{2t} + t_{2t}$$

Liquidity:
$$I_{2t} = ((m_{2t})^{\frac{\epsilon_m-1}{\epsilon_m}} + (CBDC_{2t})^{\frac{\epsilon_m-1}{\epsilon_m}})^{\frac{\epsilon_m}{\epsilon_m-1}}$$

Transaction cost:
$$s_{2t} = z_t A \frac{c_{2t}}{l_{2t}} + B \frac{l_{2t}}{c_{2t}} - 2\sqrt{AB}$$



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Dollarization

Unconstrained households

$$I_{1t} = \int_0^1 (d_{1Ht}(j)^{rac{\epsilon_b-1}{\epsilon_b}} dj)^{rac{\epsilon_b}{\epsilon_b-1}} + s_t \int_0^1 (d_{1Ft}(j)^{rac{\epsilon_b-1}{\epsilon_b}} dj)^{rac{\epsilon_b}{\epsilon_b-1}}$$

Dollarization

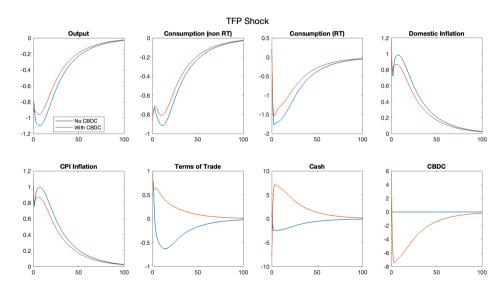
Unconstrained households

$$I_{1t} = \int_0^1 (d_{1Ht}(j)^{rac{\epsilon_b-1}{\epsilon_b}} dj)^{rac{\epsilon_b}{\epsilon_b-1}} + s_t \int_0^1 (d_{1Ft}(j)^{rac{\epsilon_b-1}{\epsilon_b}} dj)^{rac{\epsilon_b}{\epsilon_b-1}}$$

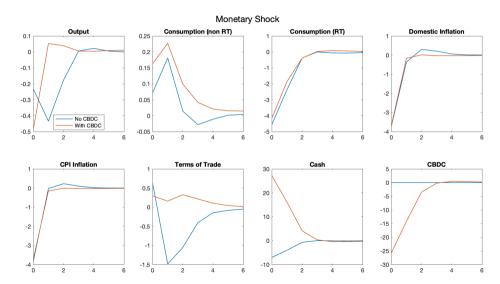
Constrained households

$$\mathit{I}_{2t} = (\left(m_{2Ht}\right)^{\frac{\epsilon_{m}-1}{\epsilon_{m}}} + \left(s_{t}m_{2Ft}\right)^{\frac{\epsilon_{m}-1}{\epsilon_{m}}} + \left(\mathit{CBDC}_{2t}\right)^{\frac{\epsilon_{m}-1}{\epsilon_{m}}})^{\frac{\epsilon_{m}}{\epsilon_{m}-1}}$$

Impulse Response: TFP



Impulse Response: Monetary



Preliminary Results

Main takeaways:

- CBDC increases the welfare of constrained households by 2.73%
 - Transaction friction
 - Financial inclusion

Preliminary Results

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- CBDC increases the welfare of constrained households by 2.73%
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- CBDC increases the tax revenue

Preliminary Results

Main takeaways:

- CBDC increases the welfare of constrained households by 2.73%
 - Transaction friction
 - Financial inclusion
- CBDC increases the tax revenue
- CDBC increases the monetary policy efficiency when the SOE is dollarized