Growth Model with Automation and Endogenous Human Capital

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Abstract

Will automation ultimately displace or complement skilled and unskilled workers if human capital accumulation endogenously responds to the technology change? How long does it take for skilled and unskilled workers to adapt to the new equilibrium? I study the effects of the automation technology wave on the labor share, wage level, and inequality under the framework of the task model with heterogeneous workers and endogenous human capital. Even if automation ultimately benefits all types of labor, the transition can be long, unequal, and sub-optimal.

When the technology wave lowers the cost of automation, it increases automation, lowers labor share, and raises wage inequality in the short run by direct productivity effect. However, the technology wave changes the demand for production factors and factor prices; automation could decrease if the price effect dominates. The magnitude of the price effect depends on the substitutability between production factors and the comparative advantage of labor over the machine on newer tasks.

The presence of human capital amplifies the technology wave but accelerates the transition. Different skill group responds to the technology wage differently. I estimate the model by fitting the transition path to the data and discussing the optimal policy to optimize the transition. Empirical evidence is provided using Artificial Intelligence Occupational Impact (AIOI), estimated by Felten et al. (2019) and Occupational Information Network (O*NET). The development of AI increases the demand for social and technical skills, and workers respond to the technology change by increasing their content, process, and social skills.

1 Introduction

As has been argued in the literature, automation and artificial intelligence (AI) are essential in explaining the increase of college premium decrease in labor share. Automation allows the machine to replace unskilled labor and decrease the labor share. At the same time, automation complements college graduates and increases the college premium.

Human capital accumulation is a vital response channel of workers to automation that has not been discussed much in the literature. The workers can respond to automation by increasing the supply of skilled workers (extensive margin). The high school and college graduation rates have been rising over time. The education channel is very slow and can take generations. The workers can also increase their skill level by doing on-the-job training (intensive margin). One important stylized fact of job polarization is the shift of unskilled workers to the service sector. Facing the accelerating automation, unskilled workers will spend more time improving their social skills and reallocate to the sectors that are less exposed to automation. The skilled workers who already have college degrees have an incentive to increase their technical and management skills to distinguish themselves while facing a more competitive environment.

I study the effect of automation on the labor share, wage level, and inequality under the framework of the task model with heterogeneous workers and endogenous human capital. My research highlights the transition path to the new balanced growth path. Even if automation ultimately benefits all types of labor and decreases inequality, the transition path can be long, unequal, and sub-optimal.

The formal model of the economy has three sectors. The task producers use workers (skilled or unskilled) or capital (if the task has been automated) to produce the intermediate good. They have monopoly power and make positive profits by charging a markup. The final good sector is perfectly competitive and combines the tasks using a CES production function to produce the consumption good. The research sector hires scientists (skilled workers) to develop new patents. There are two types of patents: innovation patent brings new tasks to the economy, which can only be produced using labor; automaton patent allows the firm to use capital instead of labor to produce the task.

The growth of the economy is endogenous and Schumpeterian. The old tasks are destroyed when new ones are introduced, keeping the task measure constant. The innovation patent introduces new tasks and increases the economy's productivity, in which the skilled workers have a comparative advantage over the unskilled workers and capital. Tasks can only be produced by skilled or unskilled workers if it has not been automated yet. The automation patent allows the firm to produce the old task using a machine instead of a worker. The R&D investment is motivated by the monopoly rents, the direction of research is endogenous (towards innovation or automation).

The households make two decisions simultaneously. Firstly, they make consumption-saving decisions to maximize lifetime utility. Secondly, they allocate their time to working or learning to maximize their lifetime wage income. Leaning is the only way to accumulate human capital. The human capital investment is complementary to R&D investment if the capital rental rate does not respond too much to the growth rate. Physical and human capital are two alternative ways for the household to save for the future. Higher technology growth rates increase both interest rates and wage growth rates. If the wage growth rate rises more than the long-run interest rate (IES is greater than 1), saving through human capital accumulation will be preferred. As a result, more human capital investment is made. High human capital investment increases the supply of efficient labor and long-run capital rental rate, which leads to a lower automation level. The full automation of all tasks will never occur if households invest enough in human capital.

When the technology wave lowers the cost of automation, it increases automation, lowers labor share, and raises wage inequality in the short run by direct productivity effect. R&D investment is directed more the automation since the automation department is more efficient. However, the technology change requires more capital to sustain the growth, and fewer scientists in R&D, as a result, increase the capital rental rate and lowers the wage. This price effect increases the innovation, decreases the automation patent value, and decreases the automation level. These two effects work in opposite directions, and the new equilibrium automation level depends on the magnificence of the price effect. The price effect depends on the substitutability between production factors and the comparative advantage of labor over the machine on newer tasks. The prices are sensitive to the change of demand when the substitutability between production factors is high, and the wage is less sticky when labor has a higher comparative advantage over the machine.

The presence of human capital amplifies the effect of the technology wave but also accelerates the transition. Different skill groups respond to the technology wave differently. At the early stage of the transition, the human capital investment will drop since automation decreases the wage growth, and the unskilled workers will decrease it more since they are more affected by automation. Later on, automation slows down, and the technology wave benefits the workers. The human capital investment will increase, and the unskilled workers will increase it more since their acquisition is comparatively less costly. The technology wave is amplified mainly by the fluctuation of human capital and labor supply; unskilled workers respond to the wave more than skilled workers. The human capital investment increases the effective labor supply, brings down the automation patent value more quickly, and accelerates the transition.

Empirical evidence is provided using Artificial Intelligence Occupational Impact (AIOI) estimated by Felten et al. (2019)[16] and Occupational Information Network (O*NET). The development of AI increases the demand for social and technical skills, and workers respond to the technology change by increasing their content, process, and social skills.

2 Related Literature

2.1 Innovation and Technology Adoption

This paper is closely related to Acemoglu and Restrepo(2018)[3], in which they mainly examine the concerns that new technologies will render labor redundant. I endogenize the human capital following Grossman and Helpman (2020). Human capital accumulation results from time spent on learning, which increases the productivity of workers and scientists. This paper is also related to Acemoglu and Autor(2011)[2] and Acemoglu and Restrepo(2020)[4], in which they mainly discussed the impact of automation on wage inequality. Stokey (2014) [32] and Stokey (2020) [31] are also related to my work. The human capital and the technology are twin engines of growth when they are also racing each other.

Some papers focus on the process of innovation, how the new ideas are generated and how the firms make the decision to invest in R&D. Kortum (1997)[22] develops a search-theoretic model to study the technological change process. Ideas arrive to the researchers as a Poisson process, its efficiency is drawn from a probability distribution which depends on the research effort stock. The technological frontier is generated by all the past research, and will determine the productivity. Comin et al.(2006)[15] incorporate R&D and technology adoption in RBC model to understand the medium-term cycles. This framework can generate procyclical R&D investment and adoption intensities, which explains the persistent response of economic activity to the high frequency fluctuations. Acemoglu et al.(2013)[1] develop a model of endogenous reallocation and innovation with heterogeneous firms, they estimate parameters of the model using US Census micro data on firm-level output, R&D and patenting. Unskilled workers are used in the production, while skilled workers perform R&D functions and operations. Firms differ in terms of their innovative capacities, they choose their investment level on R&D. Aghion et al.(2014)[6] survey Schumpeterian growth theory, in which new innovations replace older technologies, resulting in endogenous growth, individuals can choose to allocate the labor supply between production and research. The theory provides us a framework to study macroeconomic growth while incorporating many microeconomic issues regarding incentives, policies and organizations that interact with growth. Atkeson et al.(2019)[9] nest several canonical models into one unifying model with endogenous technology adoption. Production labor hours are used to produce the intermediate good; research labor hours are used to produce the research good. Final good is a CES aggregate of the intermediate goods, intermediate good producers need to invest research good in order to enter the market, to develop new product, or to upgrade the technology. The new entries can enter the market producing new products or replacing the incumbents.

Some papers focus on the technology adoption. When the new technologies are invented, how would the firms make the decision to put the new technology into production. Hall and Khan (2003)[18] discuss the technology diffusion when facing uncertainty of benefits and costs, the demand for new technology is mainly determined by the skill level of workers and the technical capacity, customer commitment and network effects. Michelacci et al.(2007)[26] incorporate labor market search frictions and technology adoption in Solow growth model, the technology could be neutral or investment-specific, the newly created job will directly adopt the technology at the frontier, while the old job can only upgrade the technology to the frontier with certain probability. They find that advancements in the neutral technology lead to an increase in job destruction, job reallocation, and unemployment.

König et al.(2016)[21] develop a dynamic growth model where the firm can innovate to increase the productivity or imitate other firm's technology. Anzoategui et al.(2019)[8] develop a model with an endogenous TFP mechanism that allows for costly development and adoption of technologies. Innovators use skilled labor to create new intermediate goods, adopters use skilled workers to convert unadopted technologies into ones that can be used in production; the slow recovery could be explained by the sharp decline in adoption intensity during the Great Recession.

The innovation and technology adoption can generate endogenous growth in the economy. Luttmer(2012)[24] identifies the assumptions needed to guarantee a balanced growth path with stationary firm distribution. Firms differ in their productivity level z, optimizing their production size. New firms can enter with certain hazard rate by paying a cost, they will start their business with an exogenous growing productivity level Z_t, which drives up the equilibrium wage level (by labor market clearing condition). Incumbents are facing permanent productivity shocks (BM) and will leave the market if the productivity is lower than certain threshold, since they will not be able to afford the fixed cost to maintain their business. If entrants can enter with technology level slightly better than the technologies used by the least productive incumbents, the economy can sustain a balanced growth path with stationary firm distribution (Zipf's law). Perla and Tonetti(2014)[27] develop an analytically tractable model where firms at the bottom of the productivity distribution can imitate more productive firms. Firms are heterogeneous in terms of their productivity, there is no aggregate or individual exogenous growth, no idiosyncratic productivity shocks, and there is no enter and exit. The only dynamic is that each period, firms make the search decision with the opportunity cost of current period production, they can redraw their productivity level from the current distribution if they decide to search. The growth is endogenous, at each period, firms with productivity under certain threshold would decide to search; and they redraw productivity from the the part of the distribution above this threshold, since all firms below the threshold would be searching without producing. BGP would be achieved with geometric growth rate, starting with an initial Pareto distribution with certain conditions satisfied. Benhabib et al.(2017)[12] study how the interaction between adoption and innovation determines the shape of the productivity distribution, the expansion of the technology frontier, and the aggregate economic growth rate. Firms can adopt the technology to increase their productivity by paying a cost, they can also choose to innovate and charge the licensing fee. Akcigit et al.(2018)[7] bring together the knowledge diffusion models and innovation based growth models. Skilled research workers produce innovations, unskilled labor are used in production. Productivity of skilled workers evolve endogenously over time as a result of endogenous interactions or exogenous external learning. Workers choose the meeting rate with others, when they meet, they group into research team, and the innovation quality depend on the leader's productivity and the number of members. They use the European Patent Office data to estimate the model.

2.2 Human capital

The accumulation of human capital takes two forms. Firstly, the knowledge will be passed across generations through education, labor can get their initial skill level by choosing the schooling investment. Secondly, the human can be increased during the life time through learning-by-doing (the experience will be accumulated by doing the job) or learning-or-doing (workers need to spend time to learn new skills with the opportunity cost of production.

Kalemli-Ozcan et al.(2000)[20] develop a continuous time, overlapping generations model in which individuals make optimal schooling investment choices facing certain probability of death, workers only invest in education at the beginning of their lives, and work until the death. Galor and Moav (2004)[17] develop a growth model with overlapping generation, individuals acquire their human capital in the first period, which is a function of the education expenditure; individuals start to work in the second period, and allocate their wage income. The economy growth is driven by physical capital accumulation at the beginning, then human capital at the later stage. Imai et al.(2004)[19] build a dynamic life cycle model where the agents choose optimal consumption and labor supply, and human capital evolves depending on the amount of labor supply. They use NLSY79 data to estimate the intertemporal elasticity of substitution in labor supply, and get the elasticity much larger than the estimation in micro literature. Bohacek and Kapicka (2008)[13] set up a dynamic private information model where the workers can increase their human capital by investing schooling time. Individuals are heterogeneous in terms of their ability, which is private information for them; in each period, they allocate their time between work, leisure and human capital accumulation.

Burdett et al.(2011)[14] incorporates learning-by-doing and life cycle in the BM model, workers are heterogeneous in terms of productivity and accumulate experience while working. The wage variance could be decomposed into workers' heterogeneity, firms' heterogeneity, difference in experience and sorting. kapicka and Neira (2013) studies a life-cycle economy with risky human capital accumulation. Agents live for J periods, their earnings are determined by ability, human capital and labor supply, the ability is constant and is known from the beginning, human capital can be accumulated by learning with idiosyncratic shock; agents dislike working and learning. Stantcheva (2015)[29] builds a dynamic model where the workers can allocate their time between working and training, and training can increase the worker's human capital stock. The training and working could be substitute (learning or doing) or complement (learning and doing). Stantcheva (2017)[30] set up a life cycle model with risky human capital. Agents live for T periods, each period, agents can build their stock of human capital by spending money; wage rate is determined by the human capital stock and stochastic ability.

2.3 Interaction between technology and human capital

The technology and human capital are complementary component in the production, so the investment decision will depend on the level of the other component. Redding(1996)[28] discusses the strategic game between workers and entrepreneurs on decisions of human capital and R&D investment under the context of an endogenous growth model. Beaudry et al.(2006)[11] use the city level data to study the interaction between technology adoption and labor market conditions. Compared with the old technology, the new technology uses a different form of capital, and is skilled biased; the new technology is more productive than the old technology only when used with a high fraction of skill workers. The empirical results show that cities with a high fraction of college educated workers adopted PCs more intensively; and cities adopting PCs intensively witnessed greater increases in returns to skill. Adão et al.(2020)[5] develop a theory to study technological transitions driven by both worker reallocation within a generation and changes in the distribution of skills across generations. New born workers choose their skill type to maximize the expected future earning less the cost; then with fixed skill type, workers choose to work in high-tech or low-tech sector.

The interaction between technology and skill is important for the economic growth. Lloyd-Ellis and Roberts(2002)[23] develop an endogenous growth model with skill acquisition and innovation, skills are required to implement and invent new technology. The economic growth only takes the form of growth of variety in the CES production function of final good. A sustainable growth could be maintained only when technology and skill grow at the same rate. If skill grows too slow, higher technology won't be able to put into production and no labor would be available for R&D. If technology grows too slow, the workers won't have the incentive to spend time in school, since they cannot benefit from specialization when no higher technology is available. Stokey(2014)[32] develops a growth model in which heterogeneous firms invest in R&D and heterogeneous workers invest in human capital. By assuming that the cost of R&D and skill investment is scaled by the firm's or the worker's own surplus, and homothetic CES production function, the balanced growth path is easy to solve. All the firms and workers will choose the same rate of growth, and the relative skill-technology position and match will stay constant. The only parameters that capture the growth are the cost of investments for R&D and human capital, which jointly determine the growth rate of the economy. Luttmer(2015)[25] summarizes four models of knowledge diffusion and long-run growth with different ways to model long-run evolution of skill distribution, and discussed different possible distribution that could be used the describe the growth process. The distribution evolves through three channels: (1) producers could learn from each others (social learning) but with some search delay and learning delay; (2) the productivity state evolves according to Brownian motion with time trend; (3) producers under certain threshold would exit the market and redraw from the current distribution. What happens in the long run depends on initial conditions. Stokey(2020)[31] develops a model in which economic growth comes from technology and skill acquisition; while growth can take 2 forms: higher TFP and more variety. Due to the complementarity between technology and skill, the endogenous growth would happen only if both parts grow at the same time. TFP growth mainly depends on worker's incentive to acquire new skill, determined by the parameters capturing the skill growth rate. With high skill growth rate, firms have the incentive to increase their technology since the marginal gain would be high; with low skill growth rate, firms would switch to variety growth. Variety growth mainly depends on the firm's incentive to enter and stay in the market, determined by the difference between parameters capturing technology and skill growth; high variety growth could suppress TFP growth since it dilutes profit of each individual firm and reduce the technology adoption and skill acquisition incentive.

3 Model with Automation

In this part of the paper, I develop a model with automation and endogenous human capital.

3.1 Environment

The economy is populated by two types of representative household: skilled household with measure ϵ_H and unskilled household with measure ϵ_L. They supply different types of labor, but share the same preference and discount rate. There are three sectors in this economy: final good sector, task producer and research sector.

Household maximizes his lifetime utility. Low skilled workers only receive capital and wage income.

$$\max \quad \int_0^\infty e^{-\rho t} \frac{C_L(t)^{1-\theta}}{1-\theta}$$

subject to the budget constraint:

$$\dot{K}_L(t) = r_t K_L(t) + W_L(t) l_L(t) - C_L(t),$$

High skilled workers receive not only capital and wage income, but also dividends from research sector.

$$\max \quad \int_0^\infty e^{-\rho t} \frac{C_H(t)^{1-\theta}}{1-\theta}$$

subject to the budget constraint:

$$\dot{K}_H(t) = r_t K_H(t) + W_H(t) l_H(t) + \Pi(t) - C_H(t),$$

The final good sector is perfectly competitive and produce the consumption good by combining a unit measure of tasks $y(i)$, with an elasticity of substitution $\sigma \in$

$(0, \infty)$. The creation of new tasks will destroy and replace the old tasks, so the total measure of the task in this economy remains constant 1.

$$Y = \tilde{A}(\int_{N-1}^{N} y(i)^{\frac{\sigma-1}{\sigma}} di)^{\frac{\sigma}{\sigma-1}}$$

The task sector is also competitive, task producers purchase task-specific intermediates $q(i)$ at price ψ from R&D sector, which embodies the technology, then combine them with capital or labor. If the automation patent is not developed for the task ($i > I$), they use only workers (skilled $l_H(i)$ or unskilled $l_L(i)$) to produce the intermediate good $y(i)$. If the task has been automated ($i \leq I$), the task producer is able to choose between capital and labor. When the intermediate good $y(i)$ is produced with machine, the productivity is normalized to 1. The productivity of the skilled worker $\gamma_H(h_H, i)$ and the unskilled worker $\gamma_L(h_L, i)$ depends on the task index i and the worker's human capital level h_H and h_L.

$$y(i) = \begin{cases} q(i)^{\eta}(k(i) + \gamma_L(i, h_L)l(i) + \gamma_H(i, h_H)h(i))^{1-\eta}, & N-1 \leq i \leq I \\ q(i)^{\eta}(\gamma_L(i, h_L)l(i) + \gamma_H(i, h_H)h(i))^{1-\eta}, & I < i \leq N \end{cases}$$

The research sector hires scientists (skilled workers) to develop new innovation or automation patents. After developing new patent, the R&D sector produces task-specific intermediates and sell them to intermediate producer at price ψ. Innovation patent introduces new task into the economy and increases the productivity, automation patent allows the firm to produce the task using machine and improves the factor allocation. The number of new patents introduced to the economy is a function of number of scientists hired $\kappa^N(\epsilon^N)$ and $\kappa^I(\epsilon^I)$, with $\kappa' > 0$ and $\kappa'' \leq 0$.

3.2 Equilibrium in Static Model

Assume that skilled workers (H) have comparative advantage in high index tasks over capital and unskilled workers (L), all workers have absolute advantage over machine. The productivity of machine is constant over all tasks and is normalized to 1.

$$\gamma_H(i, h_H) = e^{B(i)}e^{b_H h_H}$$
$$\gamma_L(i, h_L) = e^{B(N-1)+B_L(i-(N-1))}e^{b_L h_L}$$

By solving the final good producer problem, we can derive the demand function for task i

$$y(i) = \tilde{A}^{\sigma-1} Y p(i)^{-\sigma}$$

The task producers sell the tasks to final good producers at the minimum unit cost since the sector is competitive

$$p(i) = \begin{cases} \phi \min\{R, \dfrac{W_H}{\gamma_L(i, h_H)}, \dfrac{W_L}{\gamma_L(i, h_L)}\}, & N-1 \leq i \leq I \\ \phi \min\{\dfrac{W_H}{\gamma_L(i, h_H)}, \dfrac{W_L}{\gamma_L(i, h_L)}\}, & I < i \leq N \end{cases}$$
where $\phi = (\dfrac{\psi}{\eta})^{\eta}(\dfrac{1}{1-\eta})^{1-\eta}$

Proposition 1. (Allocation) *Given automation level I, there exist cutoff point $S(h_H, h_L, I, L_H, L_L)$, such that all the tasks between $N-1$ and I are produced by machine, all the tasks between I and S are produced by unskilled workers, and all the tasks between S and N are produced by skilled workers. To have incentive for automation, I must satisfy:*

$$R < \frac{W_L}{\gamma_L(h_L, I)}$$

S must satisfy the non-arbitrage condition:

$$\frac{W_H}{\gamma_H(h_L, S)} = \frac{W_L}{\gamma_L(h_L, S)}$$

The demand of production factor in each task then can be solved as

$$k(i) = \begin{cases} A^{\hat{\sigma}-1}(1-\eta) Y R^{-\hat{\sigma}}, & N-1 < i \leq I \\ 0, & I < i \leq S \\ 0, & S < i \leq N \end{cases}$$

$$l_L(i) = \begin{cases} 0, & N-1 < i \leq I \\ A^{\hat{\sigma}-1} \dfrac{(1-\eta)Y}{\gamma_L(h_L, i)} (\dfrac{W_L}{\gamma_L(h_L, i)})^{-\hat{\sigma}}, & I < i \leq S \\ 0, & S < i \leq N \end{cases}$$

$$l_H(i) = \begin{cases} 0, & N-1 < i \leq I \\ 0, & I < i \leq S \\ A^{\hat{\sigma}-1} \dfrac{(1-\eta)Y}{\gamma_H(h_H, i)} (\dfrac{W_H}{\gamma_H(h_H, i)})^{-\hat{\sigma}}, & S < i \leq N \end{cases}$$

where $\hat{\sigma} = \sigma(1-\eta) + \eta$ and $A = (\frac{\tilde{A}}{\phi})^{\frac{\sigma-1}{\hat{\sigma}-1}}$

Proposition 2. (Equilibrium in the static model) *After solving the firm's optimization problem, the equilibrium price can be solved by applying the market clearing condition. Where H_K, Γ_H and Γ_L represent the average productivity of capital, skilled and unskilled worker:*

$$R = (1-\eta) A H_K (\frac{Y}{AHK})^{\frac{1}{\hat{\sigma}}}$$
$$W_H = (1-\eta) A \Gamma_H (\frac{Y}{A \Gamma_H L_H})^{\frac{1}{\hat{\sigma}}}, \quad W_L = (1-\eta) A \Gamma_L (\frac{Y}{A \Gamma_L L_L})^{\frac{1}{\hat{\sigma}}}$$
$$H(N, I) = \tilde{I}^{\frac{1}{\hat{\sigma}-1}}$$
$$\Gamma_H(h_H, N, I, S) = \gamma_H(h_H, N) (\frac{1 - e^{-B(1-\tilde{S})(\hat{\sigma}-1)}}{B(\hat{\sigma}-1)})^{\frac{1}{\hat{\sigma}-1}}$$
$$\Gamma_L(h_L, N, I, S) = \frac{\gamma_H(h_H, N)}{\gamma_{HL}} (\frac{e^{-B_L(1-\tilde{S})(\hat{\sigma}-1)} - e^{-B_L(1-\tilde{I})(\hat{\sigma}-1)}}{B(\hat{\sigma}-1)})^{\frac{1}{\hat{\sigma}-1}}$$

Where $\tilde{I} = I - (N-1)$ represents the automation level, and $\tilde{S} = S - (N-1)$ represents the share of tasks produced by unskilled worker. $\gamma_{HL} = e^{B - B_L + b_{hH} h_{hH} - b_{hL} h_{hL}}$ represents the the absolute advantage of skilled worker over unskilled worker on frontier task N.

Proposition 3. (Comparative statistics) *The change of allocation $\tilde{S}$ and wage premium $\omega = \frac{W_H}{W_L}$ is given by:*

$$d\tilde{S} = \frac{1}{\epsilon(\tilde{S})} (b_L dh_L - b_H dh_H + d\ln L_L - d\ln L_H) + \frac{\tilde{\Gamma}_L^{1-\sigma}}{\epsilon(\tilde{S})} \tilde{I}$$
$$d\omega = (1 - \frac{B}{\epsilon(\tilde{S})})(b_H dh_H - b_L dh_L) + \frac{B}{\epsilon(\tilde{S})} (d\ln L_H - d\ln L_L) + B \frac{\tilde{\Gamma}_L^{1-\sigma}}{\epsilon(\tilde{S})} \tilde{I}$$

An increase of automation increases the wage premium, since machine replaces the job of unskilled worker and pushes them to the tasks they have disadvantage over.

3.3 Dynamics and Balanced Growth Path

Consumption

Given the interest rate $\{r(t)\}$ and income from wage $\{W_j(t)L_j^i(t)\}$ and profit $\Pi(t)$, households maximize their lifetime utility. The consumption of household needs to satisfy:

$$\frac{\dot{C}_H(t)}{C_H(t)} = \frac{\dot{C}_L(t)}{C_L(t)} = \frac{r(t)-\rho}{\theta}$$

Capital stock follows the low of motion:

$$\begin{aligned}
\dot{K}_H(t) &= r_t K_H(t) + W_H(t) l_H(t) + \Pi(t) - C_H(t) \\
\dot{K}_L(t) &= r_t K_L(t) + W_L(t) l_L(t) - C_L(t) \\
\dot{K}(t) &= \dot{K}_H(t) + \dot{K}_L(t)
\end{aligned}$$

Human capital

Given the interest rate $\{r(t)\}$, the growth rates $\{g_N(t), g_I(t), g_{hL}(t), g_{hH}(t)\}$ and wage function $\{W_H(h_H^i, t), W_L(h_L^i, t)\}$, workers choose between doing and learning to maximize his labor income. The benefit of training is the life time increase of his labor income, and the opportunity cost of the training is the labor income he loss by spending time on training. Optimal training decision for skilled and unskilled worker can be characterized as:

$$\begin{aligned}
\frac{W_j'(h_j^i(t))}{W_j(h_j^i(t))} &> \mu_{hj}(r(t) - g_{W_j|h_j^i}(t)), \quad l_j^i(t) = 0, \quad j = \{H, L\} \\
\frac{W_j'(h_j^i(t))}{W_j(h_j^i(t))} &= \mu_{hj}(r(t) - g_{W_j|h_j^i}(t)), \quad l_j^i(t) \in (0,1), \quad j = \{H, L\} \\
\frac{W_j'(h_j^i(t))}{W_j(h_j^i(t))} &< \mu_{hj}(r(t) - g_{W_j|h_j^i}(t)), \quad l_j^i(t) = 1, \quad j = \{H, L\}
\end{aligned}$$

The physical capital and the human capital are two ways for the workers to save for the future. The training incentive is decreasing in interest rate $r(t)$, high interest rate increases the opportunity cost of training. The training incentive is increasing in the wage growth rate while keeping the human capital of individual i unchanged $g_{W_j|h_j^i}(t)$. Higher wage growth makes the household more willing to put effort on the training and take advantage of the growth.

By plugging in the production function, marginal productivity gain of training could be solved as:

$$\frac{W_j'(h_j^i(t))}{W_j(h_j^i(t))} = b_j$$

Wage growth rate depends on the technology growth rate and the aggregate human capital growth rate:

$$\begin{aligned}
g_{W_H|h_H^i}(t) &= Bg_N(t) - a(\tilde{I},t)(g_I(t) - g_N(t)) + s_{HL}(t)\frac{B}{\epsilon(\tilde{S})}(b_L g_{hL}(t) - b_H g_{hH}(t)) \\
g_{W_L|h_L^i}(t) &= Bg_N(t) - a(\tilde{I},t)(g_I(t) - g_N(t)) + (1 - s_{HL}(t))\frac{B}{\epsilon(\tilde{S})}(b_H g_{hH}(t) - b_L g_{hL}(t)) \\
a(\tilde{I},t) &= (1 - \frac{s_K(t)}{\sigma - 1})\frac{d\ln\Gamma_L}{dI}(t) + \frac{s_K(t)}{\sigma - 1}\frac{d\ln H}{dI}(t)
\end{aligned}$$

R&D

When the new task is introduced, the R&D sector produces task-specific intermediates and sell them to intermediate producers. Given the factor prices $\{r(t), W_H(t), W_L(t)\}$ and the growth rates $\{g_N(t), g_{hH}(t), g_{hL}(t)\}$. The profit of holding patent for task i at time t can be solved as:

$$\text{Automation: } \pi_I(t) = \eta(\frac{R(t)}{A})^{1-\hat{\sigma}}Y(t)$$

$$\text{Innovation: } \pi_N(i,t) = \begin{cases} \eta(\frac{W_H(t)}{A\gamma_H(h_H(t),i)})^{1-\hat{\sigma}}Y(t), & i > S(t) \\ \eta(\frac{W_L(t)}{A\gamma_L(h_L(t),i)})^{1-\hat{\sigma}}Y(t), & i \leq S(t) \end{cases}$$

The present discounted value of the profit generated by an innovation patent at time t is:

$$\begin{aligned}
V_N(N(t),t) &= \int_t^\infty e^{-\int_t^\tau r(s)ds}\pi_N(N(t),\tau)d\tau \\
&= Y(t)\int_t^\infty e^{-\int_t^\tau (r(s)-g(s))ds}\frac{\pi_N(N(t),\tau)}{Y(\tau)}d\tau
\end{aligned}$$

The present discounted value of the profit generated by an automation patent at time t is:

$$\begin{aligned} V_I(t) &= \int_t^\infty e^{-\int_t^\tau r(s)ds} \pi_I(\tau) d\tau \\ &= Y(t) \int_t^\infty e^{-\int_t^\tau (r(s)-g(s))ds} \frac{\pi_I(\tau)}{Y(\tau)} d\tau \end{aligned}$$

When the automated firm enters the market, it needs to compensate the incumbent firm which is forced to exit the market. The present discounted value of the profit for the exiting firm is

$$\begin{aligned} V_N(I(t), t) &= \int_t^\infty e^{-\int_t^\tau r(s)ds} \pi(I(t), \tau) d\tau \\ &= Y(t) \int_t^\infty e^{-\int_t^\tau (r(s)-g(s))ds} \frac{\pi(I(t), \tau)}{Y(\tau)} d\tau \end{aligned}$$

The expected profit that can be generated by each patent type can be written as:

$$\begin{aligned} P_N(t) &= V_N(N(t), t) - V_I(t) \\ P_I(t) &= V_I(t) - V_N(I(t), t) \end{aligned}$$

Assume that the production function of research sector is linear

$$\kappa_N(\epsilon_N) = \frac{\epsilon_N}{\mu_N} \quad \text{and} \quad \kappa_I(\epsilon_I) = \frac{\epsilon_I}{\mu_I}$$

Then the optimal number of scientist hired by the research sector must satisfy:

$$\frac{p_N(t)}{\mu_N Y(t)} = \frac{W_H(t)}{Y(t)} \quad \text{and} \quad \frac{p_I(t)}{\mu_I Y(t)} = \frac{W_H(t)}{Y(t)}$$

Consistency

The human capital and technology growth rate chosen by the households and the research sector need to be consistent with the given aggregate growth rate:

$$\begin{aligned} g_{hH}(t) &= \frac{1}{\mu_{hH}}(1 - l_H^i(t)) \\ g_{hL}(t) &= \frac{1}{\mu_{hL}}(1 - l_L^i(t)) \\ g_N(t) &= \frac{\epsilon_N(t)}{\mu_N} \\ g_I(t) &= \frac{\epsilon_I(t)}{\mu_I} \end{aligned}$$

The labor supply is solved as:

$$\begin{aligned} L_H(t) &= \epsilon_H l_H^i(t) - \epsilon_N - \epsilon_I \\ L_L(t) &= \epsilon_L l_L^i(t) \end{aligned}$$

The equilibrium prices $r(t)$, $W_H(t)$ and $W_L(t)$ clears the capital and labor market.

BGP

A balanced growth path (BGP) is defined as a transition path on which the growth rates $\{g_N(t), g_I(t), g_{hH}(t), g_{hL}(t)\}$ are constant over time, automation level and labor allocation $\{\tilde{I}(t), \tilde{S}(t)\}$, factor shares $\{s_K(t), s_L(t),\ s_H(t)\}$, capital productivity $\{H\}$ and the rental rate of capital $\{r(t)\}$ are constant ,the normalized factor productivity, $\tilde{\Gamma}_H = \Gamma_H e^{-\int_0^t g(\tau)d\tau}$ are constant, the normalized variables $\tilde{\Gamma}_L = \Gamma_L e^{-\int_0^t g(\tau)d\tau}$, $c_H(t) = C_H(t)e^{-\int_0^t g(\tau)d\tau}$, $c_L(t) = C_L(t)e^{-\int_0^t g(\tau)d\tau}$, $k(t) = K(t)e^{-\int_0^t g(\tau)d\tau}$, $\omega_H(t) = W_H(t)e^{-\int_0^t g(\tau)d\tau}$ and $\omega_L(t) = W_L(t)e^{-\int_0^t g(\tau)d\tau}$ are also constant. The growth rate of the economy is defined by:

$$\begin{aligned} g(t) &= Bg_N(t) + bg_h(t) \\ bg_h(t) &= s_{HL}(t)b_H g_{hH}(t) + (1 - s_{HL}(t))b_L g_{hL}(t)) \end{aligned}$$

The balanced growth path (BGP) is then characterized by constant growth rates $\{g_N, g_I, g_{hH}, g_{hL}\}$, factor supply and scientist $\{k, L_L, L_H, \epsilon_N, \epsilon_I\}$, factor price $\{r, \omega_H, \omega_L\}$, automation level and labor allocation $\{\tilde{I}, \tilde{S}\}$ and factor share $\{s_K, s_H, s_L\}$ such that the following equations need to be satisfied:

By solving the household problem, we can get the Euler equation with normalized variables:

$$\frac{\dot{c_H}}{c_H} = \frac{\dot{c_L}}{c_L} = \frac{r-\rho}{\theta} - g = 0$$

On the balanced growth path, the interest rate needs to satisfy:

$$r = \rho + \theta g \tag{3.1}$$

The optimal training is characterized by:

$$\frac{b_H}{\mu_{hH}} = r - (Bg_N - a(\tilde{I})(g_I - g_N) + s_{HL}\frac{B}{\epsilon(\tilde{S})}(b_L g_{hL} - b_H g_{hH})) \tag{3.2}$$

$$\frac{b_L}{\mu_{hL}} = r - (Bg_N - a(\tilde{I})(g_I - g_N) + (1 - s_{HL})\frac{B}{\epsilon(\tilde{S})}(b_H g_{hH} - b_L g_{hL})) \tag{3.3}$$

The firm's problem give us equilibrium factor price and factor share:

$$R = (1-\eta)\Phi\tilde{H}(\frac{y}{A\tilde{H}k})^{\frac{1}{\hat{\sigma}}} \tag{3.4}$$

$$\omega_H = (1-\eta)A\tilde{\Gamma}_H(\frac{y}{A\tilde{\Gamma}_H L_H})^{\frac{1}{\hat{\sigma}}} \tag{3.5}$$

$$\omega_L = (1-\eta)A\tilde{\Gamma}_L(\frac{y}{A\tilde{\Gamma}_L L_L})^{\frac{1}{\hat{\sigma}}} \tag{3.6}$$

$$s_K = (1-\eta)(\frac{R}{A\tilde{H}})^{1-\hat{\sigma}} \tag{3.7}$$

$$s_H = (1-\eta)(\frac{\omega_H}{A\tilde{\Gamma}_H})^{1-\hat{\sigma}} \tag{3.8}$$

$$s_L = (1-\eta)(\frac{\omega_L}{A\tilde{\Gamma}_L})^{1-\hat{\sigma}} \tag{3.9}$$

Cutoff point $\tilde{S}$ satisfies:

$$\frac{\omega_H}{\gamma_H(h_H, \tilde{S})} = \frac{\omega_L}{\gamma_L(h_L, \tilde{S})} \tag{3.10}$$

The price of innovation and automation patent with constant interest and growth rate can be normalized

$$p_I = P_I\frac{y}{Y}, \quad p_N = P_N\frac{y}{Y}$$

The optimal amount of scientist hired by research sector needs to satisfy:

$$\frac{p_N}{\mu_N} = \frac{\omega_H}{y} \tag{3.11}$$

$$\frac{p_I}{\mu_I} = \frac{\omega_H}{y} \tag{3.12}$$

$$\tag{3.13}$$

where the number of scientists is:

$$g_N = \frac{\epsilon_N}{\mu_N} \tag{3.14}$$
$$g_I = \frac{\epsilon_I}{\mu_I} \tag{3.15}$$

The automation level will converge to the level where the innovation and automation grows at the same rate:

$$g_N = g_I \tag{3.16}$$

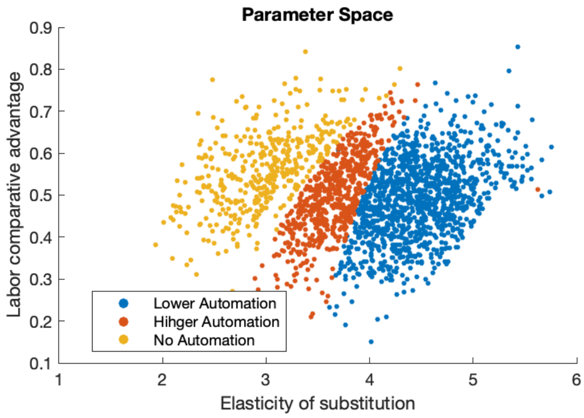
The labor supply of each skill type can be solved as:

$$L_H = (1 - \mu_{hH} g_{hH})\epsilon_H - \epsilon_N - \epsilon_I \tag{3.17}$$
$$L_L = (1 - \mu_{hL} g_{hL})\epsilon_L \tag{3.18}$$

Technology wage

How does the technology wave change the balanced growth path? Consider a decrease of automation cost by 10%. The change of automation cost has a direct productivity effect on the automation level. Since the automation patents are now easier to develop, the measure of tasks that are automated will increase. However, the technology wave has an indirect price effect. The more efficient R&D sector increases the demand for capital and the interest rate, since we need to accumulate more capital to sustain the growth of the economy. When R&D sector is more efficient, it requires less scientists to work in the sector, so it also decreases the demand for labor and the wage. The response of the price to the change of demand depends on the elasticity of substitution between production factors σ. When the elasticity of substitution is high, the price responds more to the change of the demand. The sensitivity of the wage to the change of labor demand also depends on the comparative advantage of labor B_H. When the workers have higher comparative advantage over machine on newer tasks, the wage responds less to the change of labor demand.

The value of σ and B_H can be divided into three parameter spaces. With low elasticity of substitution and high labor comparative advantage, the workers will be hard to replace, so there will be no automation in the economy. When the elasticity of substitution is higher and labor comparative advantage is low, the price effect will dominate. When the technology wave decreases the cost of automation, the economy will converge to a new BGP with lower automation level.



4 Transition Path

The transition path between two balanced growth path before and after political or technological shock are a sequence of state variables: capital stock $\{k(t)\}$, automation level $\{\tilde{I}(t)\}$, human capital gap $\{h_{HL}(t)\}$ and growth rates of the economy $\{g_N(t), g_{hH}(t), g_{hL}(t)\}$, and a set of policy functions $\{f_{Hk}(t), f_{Lk}(t) f_{Hc}(t), f_{Lc}(t)\}$ and $\{f_{Hl}(t), f_{Ll}(t)\}$ for households, $\{f_K(t), f_H(t), f_L(t)\}$ for firms, $\{f_{\epsilon_N}(t), f_{\epsilon_I}(t)\}$ for R&D sector, a set of value functions $\{V_H(t), V_L(t)\}$ for household, a set of price functions $\{r(t), \omega_H(t), \omega_L(t)\}$ and allocation $\tilde{S}(t)\}$, such that:

Given the state variable sequence and price function, household policy functions solve household's problem:

$$
\begin{aligned}
&(\rho + (\theta - 1)g)V_H(k_H, h_H, t) - \frac{dV_H(k_H, h_H, t)}{dt} \\
&= \max_{c_H, l_H} \quad u(c_H) + \frac{dV_H(k_H, h_H, t)}{dk_H}((r - g)k_H + \omega_H L_H + \pi - c_H) \\
&+ \frac{dV_H(k_H, h_H, t)}{dh_H} \frac{\epsilon_H(1 - l_H)}{\mu_{hH}} \\
&(\rho + (\theta - 1)g)V_L(k_L, h_L, t) - \frac{dV_L(k_L, h_L, t)}{dt} \\
&= \max_{c_L, l_L} \quad u(c_L) + \frac{dV_L(k_L, h_L, t)}{dk_L}((r - g)k_L + \omega_L L_L - c_L) \\
&+ \frac{dV_L(k_L, h_L, t)}{dh_L} \frac{\epsilon_L(1 - l_L)}{\mu_{hL}} \\
L_H &= (1 - l_H)\epsilon_H - \epsilon_N - \epsilon_I \\
L_L &= (1 - l_L)\epsilon_L
\end{aligned}
$$

Given the state variable sequence and price function, firm policy functions solve firm's problem:

$$
\begin{aligned}
r &= (1-\eta)A\tilde{H}(\frac{y}{A\tilde{H}k})^{\frac{1}{\sigma}} - \delta \\
\omega_H &= (1-\eta)A\tilde{\Gamma}_H(\frac{y}{A\tilde{\Gamma}_H L_H})^{\frac{1}{\sigma}} \\
\omega_L &= (1-\eta)A\tilde{\Gamma}_L(\frac{y}{A\tilde{\Gamma}_L L_L})^{\frac{1}{\sigma}} \\
\frac{\omega_H}{\gamma_H(h_L, S)} &= \frac{\omega_L}{\gamma_L(h_L, S)}
\end{aligned}
$$

Given the state variable sequence and price function, R&D sector policy functions solve R&D sector's problem:

$$
\begin{aligned}
\frac{p_N}{\mu_N} &= \frac{\omega_H}{y} \\
\frac{p_I}{\mu_I} &= \frac{\omega_H}{y}
\end{aligned}
$$

Market clear conditions are satisfied:

$$
\begin{aligned}
k(t) &= k_H(t) + k_L(t) \\
L_H(t) &= \epsilon_H l_H(t) - \epsilon_N - \epsilon_I \\
L_L(t) &= \epsilon_L l_L(t)
\end{aligned}
$$

Consistency conditions are satisfied :

$$
\begin{aligned}
g_{hH}(t) &= \frac{1}{\mu_{hH}}(1 - l_H^i(t)) \\
g_{hL}(t) &= \frac{1}{\mu_{hL}}(1 - l_L^i(t)) \\
g_N(t) &= \frac{\epsilon_N(t)}{\mu_N} \\
g_I(t) &= \frac{\epsilon_I(t)}{\mu_I} \\
\frac{d\tilde{I}(t)}{dt} &= g_I(t) - g_N(t) \\
\frac{dh_{HL}(t)}{dt} &= b_{hH}g_{hH}(t) - b_{hL}g_{hL}(t)
\end{aligned}
$$

4.1 Algorithm

To solve the transition path numerically between two balanced growth path. I firstly solve the BGP before and after political or technological shock, and use them as the starting and end point.

(1) Start with the initial guess of capital stock $\{k_0(t)\}$, labor supply $\{L_{H0}(t), L_{L0}(t)\}$, automation level $\{\tilde{I}_0(t)\}$, human capital gap $\{h_{HL0}(t)\}$ and growth rates $\{g_{N0}(t), g_{hH0}(t), g_{hL0}(t)\}$.

(2) By solving firm's problem, I can get prices $\{r(t), W_H(t), W_L(t)\}$, capital and labor share $\{s_K(t), s_H(t), s_L(t)\}$, and task allocation $\{\tilde{S}(t)\}$:

$$r = (1-\eta)A\tilde{H}(\frac{y}{A\tilde{H}k})^{\frac{1}{\hat{\sigma}}} - \delta \tag{4.1}$$

$$\omega_H = (1-\eta)A\tilde{\Gamma}_H(\frac{y}{A\tilde{\Gamma}_H L_H})^{\frac{1}{\hat{\sigma}}} \tag{4.2}$$

$$\omega_L = (1-\eta)A\tilde{\Gamma}_L(\frac{y}{A\tilde{\Gamma}_L L_L})^{\frac{1}{\hat{\sigma}}} \tag{4.3}$$

$$\frac{\omega_H}{\gamma_H(h_L, S)} = \frac{\omega_L}{\gamma_L(h_L, S)} \tag{4.4}$$

$$s_K = (1-\eta)(\frac{R}{A\tilde{H}})^{1-\hat{\sigma}} \tag{4.5}$$

$$s_H = (1-\eta)(\frac{\omega_H}{A\tilde{\Gamma}_H})^{1-\hat{\sigma}} \tag{4.6}$$

$$s_L = (1-\eta)(\frac{\omega_L}{A\tilde{\Gamma}_L})^{1-\hat{\sigma}} \tag{4.7}$$

$$s_{HL} = \frac{s_H}{s_H + s_L} \tag{4.8}$$

(3) Given factor prices $\{r(t), W_H(t), W_L(t)\}$ and growth rates $\{g_{N0}(t), g_{hH0}(t), g_{hL0}(t)\}$, I can solve consumption $\{c_H(t), c_L(t)\}$ and capital $\{k_H(t), k_L(t)\}$ by solving household's problem, then solve the new capital stock $\{k_1(t)\}$:

$$(\rho + (\theta - 1)g)V_H(k_H, t) - \frac{dV_H}{dt}(k_H, t) = \max\ u(c_H) + \frac{dV_H}{k_H}(k_H, t)\dot{k}_H \tag{4.9}$$

$$\dot{k}_H = (r-g)k_H + W_H L_H + \pi - c_H \tag{4.10}$$

$$(\rho + (\theta - 1)g)V_L(k_L, t) - \frac{dV_L}{dt}(k_L, t) = \max\ u(c_L) + \frac{dV_L}{k_L}(k_L, t)\dot{k}_L \tag{4.11}$$

$$\dot{k}_L = (r-g)k_L + W_K L_K - c_L \tag{4.12}$$

$$\dot{k} = \dot{k}_H + \dot{k}_L \tag{4.13}$$

(4) Given factor prices $\{r(t), W_H(t), W_L(t)\}$ and factor share $\{s_K(t), s_H(t), s_L(t)\}$, I can solve $\{g_I(t)\}$ and new labor supply $\{L_{H1}(t), L_{L1}(t)\}$ and new growth rates $\{g_{N1}(t), g_{hH1}(t), g_{hL1}(t)\}$ using research sector's and household's problem:

$$\frac{b_H}{\mu_{hH}} = r - Bg_N + a(\tilde{I})(g_I - g_N) - s_{HL}\frac{B}{\epsilon(\tilde{S})}(b_L g_{hL} - b_H g_{hH}) \tag{4.14}$$

$$\frac{b_L}{\mu_{hL}} = r - Bg_N + a(\tilde{I})(g_I - g_N) - (1 - s_{HL})\frac{B}{\epsilon(\tilde{S})}(b_H g_{hH} - b_L g_{hL}) \tag{4.15}$$

$$\frac{p_N}{\mu_N} = \frac{\omega_H}{y} \tag{4.16}$$

$$\frac{p_I}{\mu_I} = \frac{\omega_H}{y} \tag{4.17}$$

$$L_H = \epsilon_H(1 - \mu_{hH} g_{hH}) - \mu_N g_N - \mu_I g_I \tag{4.18}$$

$$L_L = \epsilon_L(1 - \mu_{hL} g_{hL}) \tag{4.19}$$

(5) Given growth rate $\{g_{N1}(t), g_I(t), g_{hH1}(t), g_{hL1}(t)\}$, I can solve the new automation level $\{\tilde{I}_1(t)\}$ and human capital gap $\{h_{HL1}(t)\}$ using the law of motion:

$$\frac{d\tilde{I}(t)}{dt} = g_I(t) - g_N(t)$$
$$\frac{dh_{HL}(t)}{dt} = b_H g_{hH}(t) - b_L g_{hL}(t)$$

(6) Update capital stock $\{k_0(t)\}$, labor supply $\{L_{H0}(t), L_{L0}(t)\}$, automation level $\{\tilde{I}_0(t)\}$, human capital gap $\{h_{HL0}(t)\}$ and growth rates $\{g_{N0}(t), g_{hH0}(t), g_{hL0}(t)\}$, and go back to 2 until converges.

$$k_0(t) = k_0(t) + \rho(k_1(t) - k_0(t))$$
$$L_{H0}(t) = L_{H0}(t) + \rho(L_{H1}(t) - L_{H0}(t))$$
$$L_{L0}(t) = L_{L0}(t) + \rho(L_{L1}(t) - L_{L0}(t))$$
$$\tilde{I}_0(t) = \tilde{I}_0(t) + \rho(\tilde{I}_1(t) - \tilde{I}_0(t))$$
$$g_{N0}(t) = g_{N0}(t) + \rho(g_{N1}(t) - g_{N0}(t))$$
$$g_{hH0}(t) = g_{hH0}(t) + \rho(g_{hH1}(t) - g_{hH0}(t))$$
$$g_{hL0}(t) = g_{hL0}(t) + \rho(g_{hL1}(t) - g_{hL0}(t))$$

4.2 Technology wave

Now consider a technology wave where the automation cost is lowered by 10%, with a set of parameters that the price effect dominates.

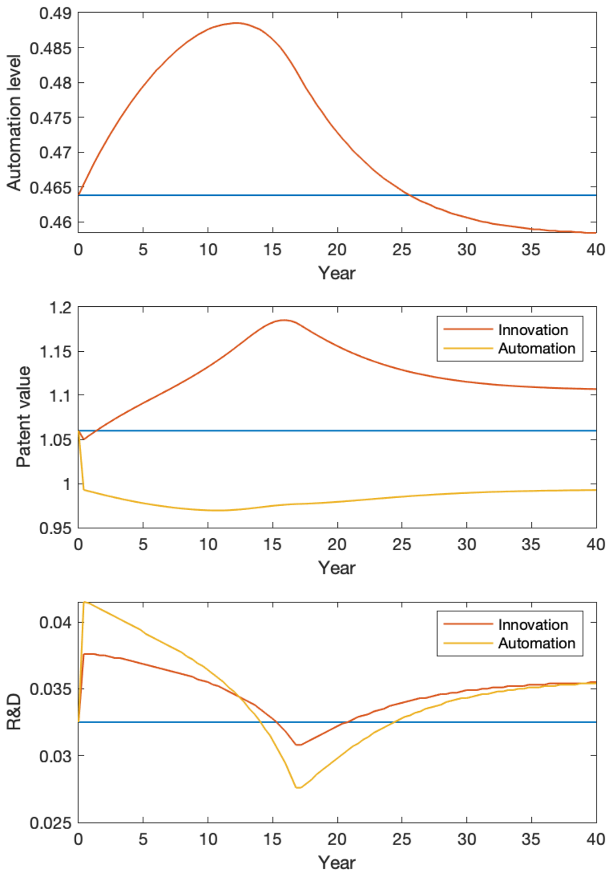


Figure 1: Automation level

The simulation result of automation level is shown in figure 1. At the beginning, the productivity effect dominates, and the automation level increases. Not only the automation is developed at a higher rate, the innovation is growing at a higher rate, since the more efficient R&D sector makes the scientists supply more abundant. But the increase of automation is not sustainable, the increase of interest rate lowers down the value of automation patent and makes the innovation patent more profitable. The high growth rate in both automation and innovation are not sustainable, the increase of human capital investment decreases the labor supply, the higher wage stops the R&D sector from hiring more scientists. The V share of the growth rate is consistent with the shape of labor supply.

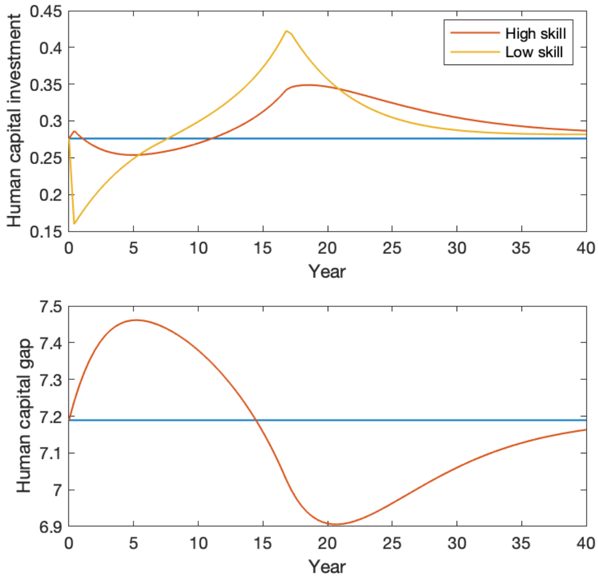


Figure 2: Human capital

The simulation result of human capital is shown in figure 2. At the early stage of the technology wave, the human capital investment increases. The automation process lowers down the wage growth, and discourage the workers from spending time on their training. The unskilled workers will decrease their investment more since they are more impacted by automation. Later on, the automation process start to slow down due to the change of factor price, the workers start to benefit from the technology change. The human capital investment start to catch up with the technology, especially unskilled workers, due to the comparatively cheaper human capital investment cost.

The simulation result of inequality is shown in figure 3. The transition of wage premium is consistent with the shape of human capital gap, but more volatile. The decrease of wage premium at the early stage is also related with the impact of automation, the increase of wage premium later is related with the shortage of unskilled labor supply. The increase of income inequality is the main driver of the increase of welfare inequality. But the welfare inequality will eventually converge to a lower level since the automation level is lowered and the utility function is concave.

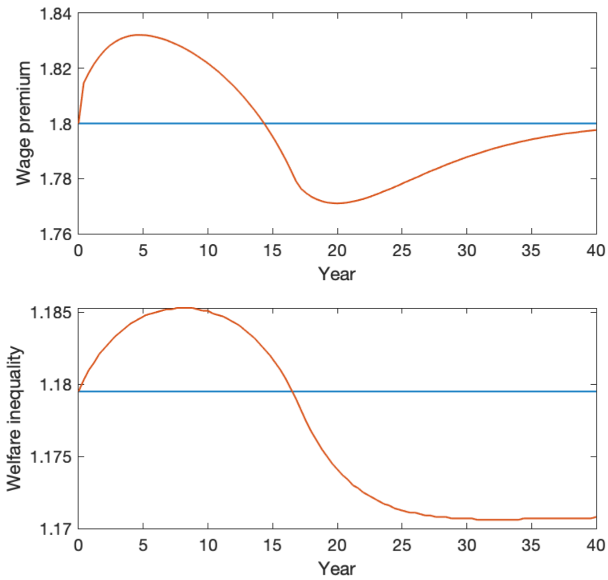


Figure 3: Inequality

4.3 Role of human capital

The comparison between models with and without human capital is shown in figure 4. Compared with the model where there is no human capital, the presence of the human capital amplifies the technology wave but also accelerates the transition. The human capital gap and different labor supply increases the income inequality, so does the welfare inequality. But the human capital accumulation also increases the effective labor supply and brings up the capital rental rate faster, which lowers down the automation level by lowering down the value of automation patent.

5 Empirical Evidence

5.1 Industry Level

In this subsection, I use EU KLEMS, an industry level panel dataset covering OECD countries since 1970, and I use September 2017 release. The dataset contains statistical data on the economic growth (value added, compensation to capital and labor, etc.) and labor market composition (compensation and employment share for high/median/low educated workers), and analytical data on the growth

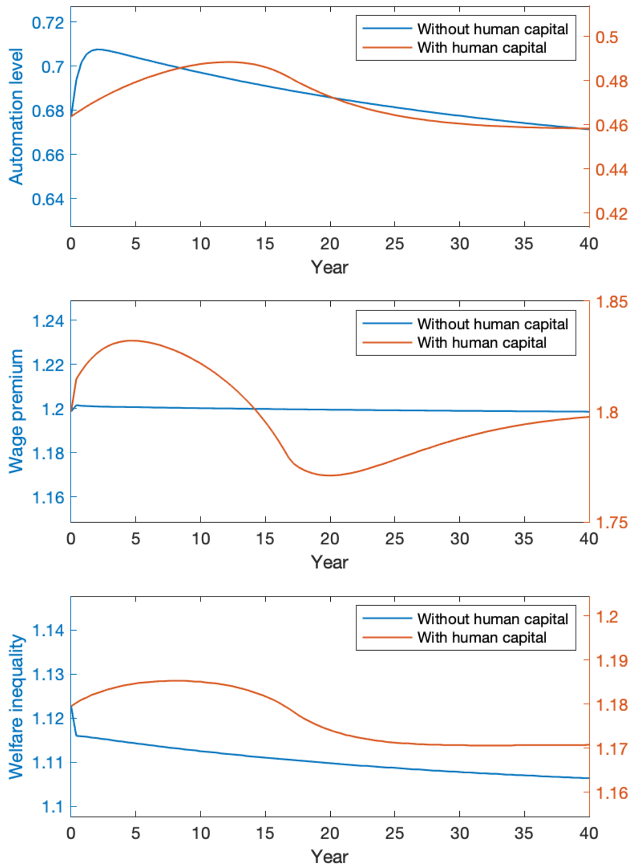


Figure 4: With and without human capital

factors (TFP, capital services, hours worked, labor composition, etc.). The main challenge in this section is that there is no direct measure of automation in the data, the TFP growth is a mixture of new innovation that favors labor and automation that replaces labor.

Based on the prediction of model, in the short run, automation increases the output but decreases the labor share; while in the long run, workers would respond to the change of technology by investing more in their human capital, so we should observe an increase of human capital or an improvement of the labor composition (higher ratio of high skilled workers). If the TFP growth comes only from the innovation, then the TFP growth should have no impact or even a positive effect on the labor share; if automation also contributes to the TFP growth, then we can observe a negative correlation between TPF growth and labor share. In the long run, TFP growth should have a impact on the human capital investment, so we should observe a positive correlation between TFP growth and the labor composition change, but with some lags.

Autor and Salomons (2018)[10] documented well in their paper the impact of automation on employment, hours worked, wage bill, and labor share. I use mainly their estimation method while the outcome of interest is the labor composition.

$$\Delta \ln Y_{ic,t} = \beta_0 + \beta_1 \Delta \ln TFP_{i,c\neq c(i),t-k} + \sum_{k=0}^{K} \beta_2^k \Delta \ln TFP_{i,c\neq c(i),k} + \beta_3 \Delta \ln TFP_{i,c\neq c(i),t-2} + \beta_4 \Delta \ln Y_{ic,t-1} + \alpha_c + \gamma_t + \epsilon_{ict}$$

where $\Delta \ln Y_{ic,t}$ reflects the log change in share of high educated labor in industry i and country c, from year t to year $t{+}1$. The impulse variable is, $\Delta \ln TFP_{i,c\neq c(i),t-k}$, the log change in other-country-industry TFP between years $t-k$ and $t-k+1$. These effects are estimated while controlling for lagged values of both TFP growth $\Delta \ln TFP_{i,c\neq c(i),t-k-1}$ and of outcome variable growth $\Delta \ln Y_{ic,t-1}$. I also control the fixed effect of country α_c and year γ_t.

The estimation result is reported in table 1. In the short run, the TFP growth does not impact the labor composition. But the labor composition will respond to it with a delay, we start to observe a positive effect of TFP on labor composition with 4 years lag.

5.2 Occupation level

In this subsection, I use AI Occupational Impact (AIOI) measured by Felten et al.(2019)[16] and Occupational Information Network (O*NET). Felten et al.(2019)[16] use Electronic Frontier Foundation (EFF) AI Progress Measurement to track the progress made on AI, and use survey date to measure impact of AI across all abilities. The AI Occupational Impact then can be measure by using the occupation ability descriptions in O*NET.

In O*NET dataset, abilities are descried as enduring attributes of the individual that influence performance, while skills are descried as developed capacities that facilitate learning or the more rapid acquisition of knowledge. While abilities take years to require, skills are easier to adjust in the short run. For each occupation, O*NET measures the importance and level of different skills, and update the date almost twice a year. To measure the response of worker's skill to the automation,

VARIABLES	(1) High educated	(2) High educated	(3) High educated	(4) High educated
TFP Growth (lag 1)	0.000100			
	(0.000752)			
TFP Growth (lag 2)	0.000213	0.000866		
	(0.000681)	(0.000720)		
TFP Growth (lag 3)		-0.00138**	-0.000724	
		(0.000696)	(0.000738)	
TFP Growth (lag 4)			0.000816	0.00166**
			(0.000729)	(0.000778)
TFP Growth (lag 5)				-0.000706
				(0.000754)
Constant	0.0493***	0.0619***	0.0767***	0.0724***
	(0.0185)	(0.0185)	(0.0188)	(0.0192)
Observations	2,993	2,619	2,245	1,871
R-squared	0.116	0.129	0.146	0.170

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 1: TFP effect on High educated labor share

I merge the date released by O*NET each year and construct a panel data.

$$y_{ijt} = t + AIOI_i + t \times AIOI_i + t \times i + t \times j + \alpha_i + \gamma_j + \epsilon_{ijt}$$

y_{ijt} represents the importance or level of a skill j for occupation i at time t. $AIOI_i$ measures the impact of AIOI on the importance or level of the skill, t capture the time trend and $t * AIOI_i$ represents how $AIOI$ affects the time trend. The effects is estimated while controlling the trend for each occupation i and skill j, and fixed effect for each occupation i and skill j.

The estimation result is reported in table 2. Development of AI increases firm's demand for technical and social skills, and the workers respond to the technological change by increasing their contend, process and social skills.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
VARIABLES	Content	Process	Social	Complex Problem	Technical	Systems	Management
edate2	-0.000415***	-0.000441***	-0.000411***	0.000592***	-0.000153	-0.000278**	-0.000164
	(0.000151)	(0.000149)	(0.000143)	(0.000200)	(0.000145)	(0.000138)	(0.000169)
c.edate2#c.AIOI	0.000464**	0.000543**	0.000485**	-0.000901***	-1.52e-06	0.000279	0.000211
	(0.000228)	(0.000225)	(0.000216)	(0.000303)	(0.000219)	(0.000209)	(0.000256)
Constant	6.289***	6.091***	5.864***	6.160***	4.612***	7.388***	6.261***
	(0.393)	(0.388)	(0.372)	(0.521)	(0.379)	(0.360)	(0.441)
Observations	174,720	116,480	174,720	29,120	320,320	87,360	116,480
R-squared	0.760	0.724	0.631	0.842	0.584	0.819	0.754

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2: Skill response to AI development

6 Conclusion

When the technology wave lowers the cost of automation, it increases automation, lowers labor share, and raises wage inequality in the short run by direct productivity effect. However, the technology wave changes the demand for production factors and factor prices; automation could decrease if the price effect dominates. The magnitude of the price effect depends on the substitutability between production factors and the comparative advantage of labor over the machine on newer tasks. When the the substitutability between production factors is high and comparative advantage of labor is low, the prices are less sticky and the price effect will dominate the productivity effect, the lower automation cost decreases the automation level in the new balanced growth path.

The presence of human capital amplifies the technology wave but accelerates the transition. Different skill group responds to the technology wage differently. The response of unskilled workers to the technology wave is more volatile, which increases the inequality during the transition.

Empirical evidence is provided using Artificial Intelligence Occupational Impact (AIOI), estimated by Felten et al. (2019) and Occupational Information Network (O*NET). The development of AI increases the demand for social and technical skills, and workers respond to the technology change by increasing their content, process, and social skills.

The biggest limit of the model is the constant elasticity of substitution between different production factors. The skilled workers are complements to capital, while unskilled workers should be substituted by machine more easily.

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