

# Smoke rings in atmospheric science

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## I. INTRODUCTION

In *dynamic meteorology* one applies the fundamental principles of physics to understand and predict the *motion* of the atmosphere. Focusing on the motions of the atmosphere, this subject does *not* address the interactions of electromagnetic radiation or the chemistry with the atmosphere or the microscopic processes involved in the formation of precipitation [1].

Consider a velocity profile  $\mathbf{u}$  of the atmosphere. Its *circulation* is defined as

$$C = \oint \mathbf{u} \cdot d\ell, \quad (1.1)$$

and the *vorticity* is the circulation for unit area.

The time derivative of the circulation

$$\frac{dC}{dt} = \frac{d}{dt} \oint \mathbf{u} \cdot d\ell = \oint \left[ \frac{d\mathbf{u}}{dt} \cdot d\ell + \mathbf{u} \cdot d\mathbf{u} \right] = \oint \frac{d\mathbf{u}}{dt} \cdot d\ell. \quad (1.2)$$

To study the dynamics of smoke rings we may use the momentum equation in an inertial frame and neglect friction (assuming a negligible viscosity)

$$\frac{d\mathbf{u}}{dt} = -g\hat{\mathbf{z}} - \frac{\nabla P}{\rho}, \quad (1.3)$$

where  $g$  is the gravity constant,  $\rho$  is the density of the air, and  $P$  is the atmospheric pressure. We then find

$$\frac{dC}{dt} = - \oint \frac{\nabla P}{\rho} \cdot d\ell = - \oint \frac{dP}{\rho}. \quad (1.4)$$

In an incompressible fluid  $\rho$  is constant and we find that  $dC/dt = 0$ , i.e.  $C$  is a constant. This is also known as Kelvin circulation theorem.

In Fig. 1 we show a vertical section cutting a smoke ring along its diameter and the velocity profile of the air. From the close up of Fig. 2 we can say that  $C = 2\pi r\bar{u}$  with  $r$  the internal radius of the smoke torus and  $\bar{u}$  the air velocity at the torus surface. Since  $C$  is constant as the smoke ring evolves in time its final fate is to disappear with  $R \rightarrow \infty$ , the ring radius, and  $r \rightarrow 0$ , the torus internal radius. So that we conclude that the speed of the air at the torus surface must

$$\bar{u} \longrightarrow \infty. \quad (1.5)$$

## AUTHOR DECLARATIONS

### Conflicts of interest

None declared.

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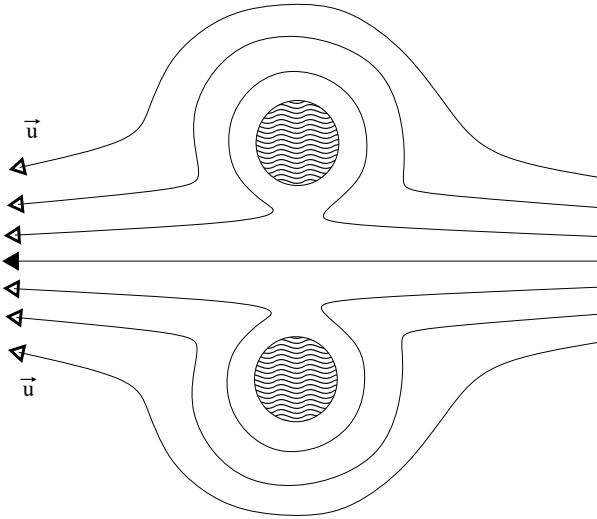


FIG. 1. Vertical section cutting a smoke ring along its diameter and the velocity profile of the air.

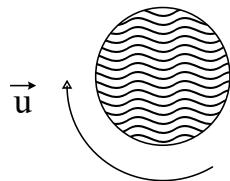
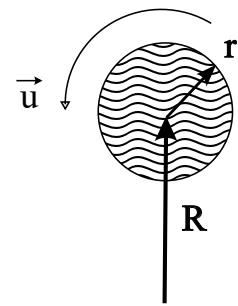


FIG. 2. Close up of the vertical section cutting a smoke ring along its diameter.  $R$  is the outer radius of the torus and  $r$  its inner radius. The velocity  $\bar{u}$  at the torus surface has magnitude  $\bar{u}$ .

#### Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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- [1] R. Fantoni, Two-phase coexistence for hydrogen-helium mixtures, Phys. Rev. E **92**, 012133 (2015).