Temperature of the Vacuum

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In a recent trilogy we proposed a Statistical Theory of General Relativity spacetime. Here we apply our new theory to determine the (energy) "density" and (virial) "temperature" dependence of the structure of the spacetime quantum vacuum working on the simple case of a real massless scalar field in a local Lorentz frame.

Keywords: Quantum Vacuum; General Relativity; Temperature; Pair Correlation Function; Structure

I. INTRODUCTION

In a recent trilogy [1–3] we recently proposed a Statistical Theory of Gravity. This allowed us to determine a "virial temperature" of the spacetime metric tensor field. Albeit still under refinement the theory is already able to offer some measurable predictions as will become clear at the end of the paper.

Our virial temperature is conceptually different from the Davies-Unruh [4, 5] local temperature. The latter is in fact defined as $\mathscr{T}_{DU} = \hbar a/2\pi c k_B \approx 4.06 \times 10^{-21}$ Ks^{2}m^{-1} $\times$ a where a is the proper uniform acceleration of a detector in vacuum. Therefore while our virial temperature is a gravitational one ascribed to the spacetime by the stress-energy tensor, the one of Davies-Unruh is not, it cannot be derived from the Einstein field equations since the detector is not following a geodetic of the spacetime.

The fluids in nature (photon liquid, electron liquid, neutron liquid, ...) carry a temperature which through the stress-energy tensor determines the "virial temperature" of spacetime [1] which in turn excites the pure state of the quantum vacuum stimulating particle-antiparticle production and recombination. We can then talk of the temperature of the quantum vacuum of spacetime.

In Ref. [6] for example the simple case of a real massless scalar field in flat Minkowski spacetime is considered [1]

$$\phi(t,\boldsymbol{r}) = \int \frac{d\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2\omega}} \left[a_{\boldsymbol{k}} e^{-i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})} + a^\dagger_{\boldsymbol{k}} e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}\right], \tag{1.1}$$

where the temporal frequency ω and the spatial frequency $\boldsymbol{k}$ are related by $\omega = |\boldsymbol{k}|$ in natural units $\hbar = c = 1$.

The vacuum pure state $|0\rangle$ is defined by

$$a_{\boldsymbol{k}}|0\rangle = 0 \quad \forall \boldsymbol{k} \tag{1.2}$$

and $a^\dagger_{\boldsymbol{k}}|0\rangle = |\omega,\boldsymbol{k}\rangle$ with $\langle\omega,\boldsymbol{k}'|\omega,\boldsymbol{k}\rangle = \delta(\boldsymbol{k}-\boldsymbol{k}')$ so that the vacuum expectation value of the square modulus of the field, $\langle\phi^2(t,\boldsymbol{r})\rangle_0 = \langle 0|\phi^2(t,\boldsymbol{r})|0\rangle = \Lambda^2/8\pi^2$ with $|\boldsymbol{k}| = \Lambda$ a high-energy (ultraviolet) cutoff. In Eq. (1.1) the first term creates an antiparticle in the sense of Dirac and the second a particle.

The vacuum state is an eigenstate of the Hamiltonian $\mathscr{H} = \int d\boldsymbol{r}\, T_{00} = \frac{1}{2}\int d\boldsymbol{k}\omega\, (a_{\boldsymbol{k}} a^\dagger_{\boldsymbol{k}} + a^\dagger_{\boldsymbol{k}} a_{\boldsymbol{k}})$, where $T_{00}(t,\boldsymbol{r}) = \frac{1}{2}[\dot{\phi}^2 + (\boldsymbol{\nabla}\phi)^2]$. But it is *not* an eigenstate of the energy density T_{00}. This fact gives rise to a non trivial vacuum structure. Direct calculation (See appendix A of Ref. [6]) shows that the pair correlation function

$$g(x,x') = 1 - \frac{\rho^{(2)}(x,x')}{\frac{2}{3}\rho^2}, \tag{1.3}$$

where $x = (x^0, x^1, x^2, x^3) = (t,\boldsymbol{r})$, $x' = (t',\boldsymbol{r}')$ are two spacetime events and

$$\rho = \rho^{(1)}(x) = \langle T_{00}(x)\rangle_0, \tag{1.4a}$$

$$\text{covariance}(\rho) = \rho^{(2)}(x,x') = \langle\{[T_{00}(x) - \rho^{(1)}(x)][T_{00}(x') - \rho^{(1)}(x)]\}\rangle_0. \tag{1.4b}$$

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[1] Note that the integral measure here is chosen to be not Lorentz invariant in order to simplify the later structure calculation of the vacuum.

where $\{AB\} = \frac{1}{2}(AB+BA)$ for any two operators A and B, and a simple calculation shows that $\rho = \Lambda^4/16\pi^2$ is a constant over spacetime and can be considered as the energy "density" of spacetime vacuum. After a lenghty calculation we [2] find the following result

$$\rho^{(2)}(x,x') = \frac{1}{2}\int \frac{d\boldsymbol{k}\,d\boldsymbol{k}'}{(2\pi)^6}\frac{(\omega\omega'-\boldsymbol{k}\cdot\boldsymbol{k}')^2}{2\omega 2\omega'}\cos[(\omega-\omega')\Delta t-(\boldsymbol{k}+\boldsymbol{k}')\cdot\Delta\boldsymbol{r}], \tag{1.5}$$

where $\Delta t = t-t'$ and $\Delta\boldsymbol{r} = \boldsymbol{r}-\boldsymbol{r}'$.

As can be seen from Fig. 1 the pair correlation function of (1.3) reveals an inhomogeneous and unisotropic spacetime vacuum. It can also be easily shown that $\rho^{(2)}(x,x) = \frac{2}{3}\rho^2$ so that $g(0) = 0$ which can be pictured as a spacetime vacuum *hole* at events contact and on the other hand $g \to 1$ at large events separation which can be interpreted as a decorrelation among spacetime events of the vacuum which becomes *uniform* and *isotropic* on a large spacetime scale. From the figure we see how both the time like pair correlation function at $|\boldsymbol{r}-\boldsymbol{r}'| = 0$ and the space like one for $t-t' = 0$ grow monotonously towards the uniform and isotropic spacetime at large events separation.

In this picture there is no space left for a "temperature" of the vacuum. Instead we expect the structure of the spacetime vacuum to feel and depend on temperature too.

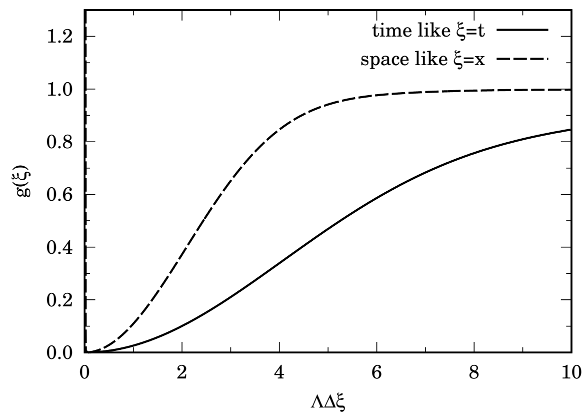

FIG. 1. The LLF pair correlation function $g(\xi)$ of Eq. (1.3): When the separation of the events x and x' is time-like for $\boldsymbol{r} = \boldsymbol{r}'$, $\Delta\xi = |t-t'|$ and when it is space-like for $t = t'$, $\Delta\xi = |\boldsymbol{r}-\boldsymbol{r}'|$.

In a recent work [1] we introduced and defined a "virial" temperature of General Relativity (GR) spacetime. Aim of the present work is to determine how that temperature can affect the structure of the quantum vacuum of spacetime.

Note that the result of Eq. (1.3) and Fig. 1 looses any value in GR. In fact the covariance of Eq. (1.4b) is inherently non local and its calculation in a Local Lorentz Frame (LLF) will not be useful in GR.

Now in GR the stress-energy tensor for the massless scalar field ϕ becomes, for a generic spacetime metric $g_{\mu\nu}$,

$$T_{\mu\nu} = \phi_{,\mu}\phi^\dagger_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi^{,\alpha}\phi^\dagger_{,\alpha} \tag{1.6}$$

where a comma stands for a partial derivative and we allow the field to be complex for the sake of more generality.

According to Einstein field equations the stress-energy tensor of the scalar field will induce a curvature of the spacetime

$$\langle G_{\mu\nu}\rangle_0 = 8\pi\langle T_{\mu\nu}\rangle_0, \tag{1.7}$$

where $G_{\mu\nu}$ is Einstein tensor and we are using Planck natural units $\hbar = c = G = k_B = 1$. Once again $\langle\ldots\rangle_0$ stands for a quantum vacuum expectation value $\langle 0|\ldots|0\rangle$. In our previous work [1] we defined the most natural thermal average for spacetime that we will here indicate with the notation $\langle\ldots\rangle_\beta$ where $\beta = 1/\mathscr{T}$ is the inverse *temperature* [3] . We will then more correctly need to average Eq. (1.7) like so

$$\langle\langle G_{\mu\nu}\rangle_0\rangle_\beta = 8\pi\langle\langle T_{\mu\nu}\rangle_0\rangle_\beta. \tag{1.8}$$

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[2] We found a sign error in their Eq. (A3).

[3] In Planck units the spacetime temperature $\mathscr{T}$ varies on Planck energy scale $\sqrt{\hbar c^5/G} = 1.9561\times 10^9$ J.

Note that while the thermal average $\langle\ldots\rangle_\beta$ acts only on the spacetime metric $g_{\mu\nu}$ the vacuum expectation value $\langle\ldots\rangle_0$ acts only on the field on the right hand side of Eq. (1.7). On the left it has no effect and we can then rewrite

$$\langle G_{\mu\nu}\rangle_\beta = 8\pi\langle\langle T_{\mu\nu}\rangle_0\rangle_\beta. \tag{1.9}$$

For example for the energy density we will find

$$\langle\langle T_{00}\rangle_0\rangle_\beta = \langle|\dot{\phi}|^2\rangle_0 - \frac{1}{2}\langle g_{00}\, g_{\mu\nu}\rangle_\beta\langle\phi^{\mu}_{,}\phi^{\dagger\nu}_{,}\rangle_0, \tag{1.10}$$

where as usual there is a hidden summation over repeated lower and upper indexes.

In Ref. [1] we were also able to render explicit the temperature. One simply has to trace out the stress-energy tensor like so

$$\mathscr{T} = -\frac{\bar{\upsilon}}{4}\langle T^{\mu}_{\mu}\rangle_\beta, \tag{1.11}$$

where $\bar{\upsilon}$ is a constant carrying dimensions of length squared divided by energy and $T^{\mu}_{\mu} = -g_{\mu\nu}\phi^{\mu}_{,}\phi^{\dagger\nu}_{,}$ is the stress-energy tensor trace. Taking a vacuum expectation value of this expression we find

$$\mathscr{T} = \frac{\bar{\upsilon}}{4}\langle g_{\mu\nu}\rangle_\beta\langle\phi^{\mu}_{,}\phi^{\dagger\nu}_{,}\rangle_0. \tag{1.12}$$

We will then redefine the first two $n-$points energy density correlation functions, now in GR

$$\rho = \rho^{(1)}(x) = \langle\langle T_{00}(x)\rangle_0\rangle_\beta, \tag{1.13a}$$

$$\mathrm{covariance}(\rho) = \rho^{(2)}(x,x') = \langle\langle\{[T_{00}(x)-\rho^{(1)}(x)][T_{00}(x')-\rho^{(1)}(x)]\}\rangle_0\rangle_\beta, \tag{1.13b}$$

which extend Eqs. (1.4a)-(1.4b) to full GR. We will assume that the field can still be written as in Eq. (1.1). We can then again easily calculate the two vacuum expectation values in Eq. (1.10)

$$\langle|\dot{\phi}|^2\rangle_0 = \frac{\Lambda^4}{16\pi^2}, \tag{1.14}$$

$$\langle\phi^{\mu}_{,}\phi^{\dagger\nu}_{,}\rangle_0 = \int\frac{d\boldsymbol{k}}{(2\pi)^3}\frac{p^\mu p^\nu}{2p^0}, \tag{1.15}$$

where $p = (p^0, p^1, p^2, p^3) = (\omega, \boldsymbol{k})$ is the four momentum. But we will now follow a different route. We will make the following approximation in Eq. (1.10)

$$\langle g_{00}\, g_{\mu\nu}\rangle_\beta \approx \langle g_{00}\rangle_\beta\langle g_{\mu\nu}\rangle_\beta \tag{1.16}$$

which allows to use the result of Eq. (1.12) to find

$$\rho = \langle|\dot{\phi}|^2\rangle_0 - \frac{2\mathscr{T}}{\bar{\upsilon}}\langle g_{00}\rangle_\beta. \tag{1.17}$$

Assuming furthermore that

$$\frac{\langle g_{00}\rangle_\beta}{\bar{\upsilon}} \approx \kappa, \tag{1.18}$$

a constant independent from temperature, we finally reach the following result [4]

$$\rho = \frac{\Lambda^4}{16\pi^2} - 2\kappa\mathscr{T}, \tag{1.19}$$

where the two approximations (1.16) and (1.18) follow a mean tensor field spirit.

Repeating now the calculation carried on for the LLF we now find from Eq. (1.13b)

$$\rho^{(2)}(x,x') = -\int\frac{d\boldsymbol{k}\,d\boldsymbol{k}'}{(2\pi)^6}\frac{\left(\omega\omega' - \frac{1}{2}\langle g_{00}\rangle_\beta\langle g_{\mu\nu}\rangle_\beta p^\mu p'^\nu\right)^2}{2\omega 2\omega'}\cos[(\omega-\omega')\Delta t - (\boldsymbol{k}-\boldsymbol{k}')\cdot\Delta\boldsymbol{r}], \tag{1.20}$$

[4] Note that our virial temperature is a local quantity which can only [1] depend on space, so $\mathscr{T} = \mathscr{T}(\boldsymbol{r})$ in the most general case.

which correctly reduces to (1.5) when $g_{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}\{-1, 1, 1, 1\}$.

We can further think about a third approximation in order to make some progress towards an insight on the pair correlation function of the quantum vacuum of full GR spacetime. a first guess could be for example the following

$$\rho^{(2)}(x, x') \approx \int \frac{d\boldsymbol{k}\, d\boldsymbol{k}'}{(2\pi)^6} \frac{\left[\omega\omega' - 2\bar{\kappa}\mathscr{T}\boldsymbol{k}\cdot\boldsymbol{k}'/(kk')\right]^2}{2\omega 2\omega'} \cos[(\omega - \omega')\Delta t - (\boldsymbol{k} + \boldsymbol{k}')\cdot\Delta\boldsymbol{r}], \tag{1.21}$$

where $\bar{\kappa}$ is a constant of dimension of energy. The pair correlation function,

$$g(x, x') = 1 - \frac{\rho^{(2)}(x, x')}{\rho^{(2)}(x, x)}, \tag{1.22}$$

$$\rho^{(2)}(x, x) \approx \left(\frac{\Lambda^4}{16\pi^2}\right)^2 + \frac{\Lambda^4(\bar{\kappa}\mathscr{T})^2}{48\pi^4}. \tag{1.23}$$

is shown in Fig. 2. From the figure we see how at low temperature the temporal structure of the spacetime quantum vacuum starts oscillating around the uniform and isotropic large separation limit. On the other hand the spatial structure remains monotonic. We then see how GR allows for a “density” and “temperature” dependence of the spacetime quantum vacuum.

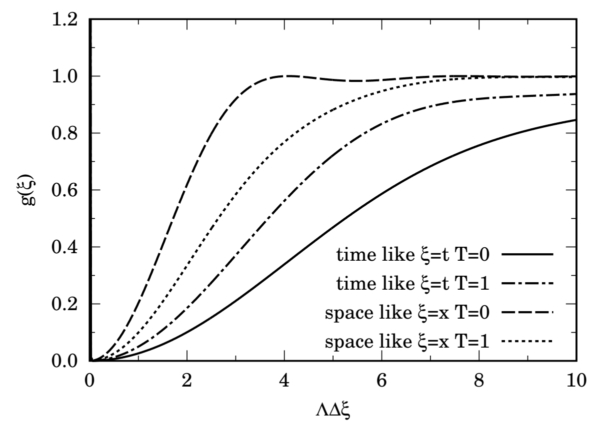

FIG. 2. The GR pair correlation function $g(\xi)$ of Eq. (1.22) at two temperatures $\mathscr{T} = 0, 1$: When the separation of the events x and x' is time-like for $\boldsymbol{r} = \boldsymbol{r}'$, $\Delta\xi = |t - t'|$ and when it is space-like for $t = t'$, $\Delta\xi = |\boldsymbol{r} - \boldsymbol{r}'|$.

II. CONCLUSIONS

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AUTHOR DECLARATIONS

Conflicts of interest

None declared.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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