Smoke rings in atmospheric science

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I. INTRODUCTION

In *dynamic meteorology* one applies the fundamental principles of physics to understand and predict the *motion* of the atmosphere. Focusing on the motions of the atmosphere, this subject does *not* address the interactions of electromagnetic radiation or the chemistry with the atmosphere or the microscopic processes involved in the formation of precipitation [1].

Consider a velocity profile $\boldsymbol{u}$ of the atmosphere. Its *circulation* is defined as

$$C = \oint \boldsymbol{u} \cdot d\boldsymbol{\ell}, \tag{1.1}$$

and the *vorticity* is the circulation for unit area.

The time derivative of the circulation

$$\frac{dC}{dt} = \frac{d}{dt}\oint \boldsymbol{u} \cdot d\boldsymbol{\ell} = \oint \left[\frac{d\boldsymbol{u}}{dt} \cdot d\boldsymbol{\ell} + \boldsymbol{u} \cdot d\boldsymbol{u}\right] = \oint \frac{d\boldsymbol{u}}{dt} \cdot d\boldsymbol{\ell}. \tag{1.2}$$

To study the dynamics of smoke rings we may use the momentum equation in an inertial frame and neglect friction (assuming a negligible viscosity)

$$\frac{d\boldsymbol{u}}{dt} = -g\hat{\boldsymbol{z}} - \frac{\boldsymbol{\nabla} P}{\rho}, \tag{1.3}$$

where g is the gravity constant, ρ is the density of the air, and P is the atmospheric pressure. We then find

$$\frac{dC}{dt} = -\oint \frac{\boldsymbol{\nabla} P}{\rho} \cdot d\boldsymbol{\ell} = -\oint \frac{dP}{\rho}. \tag{1.4}$$

In an incompressible fluid ρ is constant and we find that $dC/dt = 0$, i.e. C is a constant. This is also known as Kelvin circulation theorem that holds more generally for a fluid with $P = P(\rho)$ or $\rho = \rho(P)$.

In Fig. 1 we show a vertical section cutting a smoke ring along its diameter and the velocity profile of the air. From the close up of Fig. 2 we can say that $C = 2\pi r\bar{u}$ with r the internal radius of the smoke torus and $\bar{u}$ the air velocity at the torus surface. Since C is constant as the smoke ring evolves in time its final fate is to disappear with $R \to \infty$, the ring radius, and $r \to 0$, the torus internal radius. So that we conclude that the speed of the air at the torus surface must

$$\bar{u} \longrightarrow \infty. \tag{1.5}$$

AUTHOR DECLARATIONS

Conflicts of interest

None declared.

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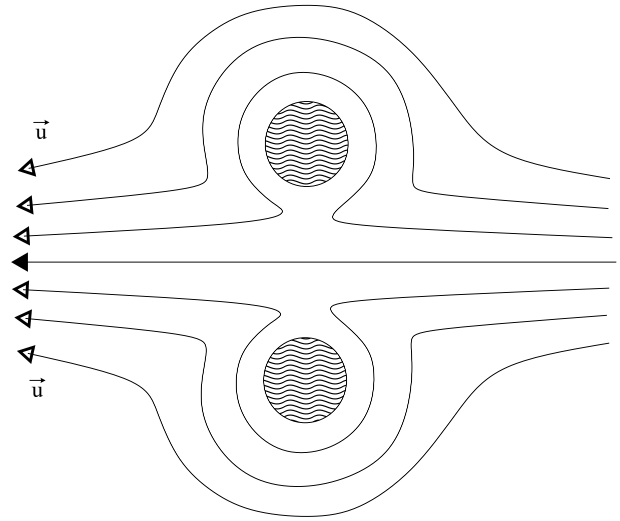

FIG. 1. Vertical section cutting a smoke ring along its diameter and the velocity profile of the air.

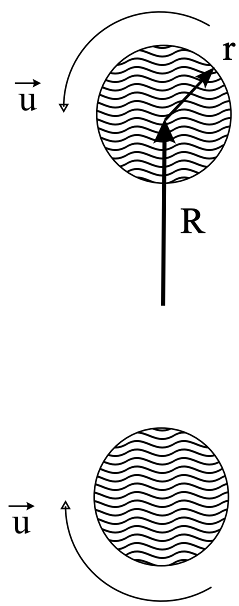

FIG. 2. Close up of the vertical section cutting a smoke ring along its diameter. R is the outer radius of the torus and r its inner radius. The velocity $\boldsymbol{u}$ at the torus surface has magnitude $\bar{u}$.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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[1] R. Fantoni, Two-phase coexistence for hydrogen-helium mixtures, Phys. Rev. E **92**, 012133 (2015).