Unicast Barrage Relay Networks: Outage Analysis and Optimization

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***Abstract*—*Barrage relays networks* (BRNs) are ad hoc networks built on a rapid cooperative flooding primitive as opposed to the traditional point-to-point link abstraction. *Controlled barrage regions* (CBRs) can be used to contain this flooding primitive for unicast and multicast, thereby enabling *spatial reuse*. In this paper, the behavior of individual CBRs is described as a Markov process that models the potential cooperative relay transmissions. The outage probability for a CBR is found in closed form for a given topology, and the probability takes into account fading and co-channel interference (CCI) between adjacent CBRs. Having adopted this accurate analytical framework, this paper proceeds to optimize a BRN by finding the optimal size of each CBR, the number of relays contained within each CBR, the optimal relay locations when they are constrained to lie on a straight line, and the code rate that maximizes the transport capacity.**

I. Introduction

A significant research investment has been made over the past decade on the topic of *cooperative communications* for wireless networks in general, and mobile ad hoc networks (MANETs) in particular. The improvements provided by it have been widely noted in the literature [1], [2]. See, for instance, [3]–[7] for example protocols involving single-antenna mobiles that cooperatively transmit and/or receive. *Barrage relay networks (BRNs)* are a kind of cooperative MANET designed for use at the tactical edge [8]. BRNs utilize time division multiple access (TDMA) and cooperative communications as the basis of an efficient flooding protocol wherein packets ripple out from sources in pipelined spatial waves. In a BRN, simultaneous transmissions of the same packet are not suppressed but rather exploited for the resulting diversity gains. The spatial extent of unicast and multicast transmissions can be contained in BRNs via *controlled barrage regions (CBRs)*. Briefly, a CBR is established by identifying a ring of buffer nodes around a portion of the network containing a source and its destination(s). Within the CBR, the barrage flooding primitive is used to transport data. The buffers suppress their relay function, thereby enabling multiple CBRs to be active at the same time. The increase in network capacity afforded by this *spatial reuse* was analyzed in [9] under standard information theoretic assumptions.

BRNs resemble the opportunistic large arrays (OLAs) introduced by Scaglione and Hong in [6]. Indeed, analogs to CBRs in OLAs have been proposed [10]. Both OLAs and BRNs exploit the diversity that can be obtained when multiple nodes transmit identical packets. BRNs can be distinguished from early OLA descriptions by the time synchronization and receiver signal processing employed in BRNs which enables packets to be longer and data rates to be larger than the inverse of the maximum relative delay spread between cooperating transmitters. The most important difference between more recent OLA descriptions (e.g., [11]) and BRNs is the latter's use of *autonomous* cooperative communications [7] as opposed to distributed space-time coding. This feature makes BRNs more suited for use in highly dynamic, tactical environments.

In this paper, unicast data transport in a CBR is modeled as a Markov process. Since relaying in BRNs is entirely opportunistic, a Markov chain is used to track every possible cooperative link transmission. The transition probabilities are evaluated for a given network topology via a closed-form expression for the outage probability of each cooperative transmission [12], which takes into account path loss, Rayleigh fading, noise, and co-channel interference (CCI) from adjacent CBRs. A key analytical challenge is that on the one hand, CCI influences which nodes successfully receive and therefore affects the transition probabilities of the Markov process, yet on the other hand, the transition probabilities of the Markov process determine which nodes transmit and therefore affect the interference. This coupling between the transition and outage probabilities is solved using an iterative approach, whereby the transition probabilities of the local CBR are used to seed the transition probabilities of neighboring CBRs, assumed to be similarly configured. On the best of our knowledge, Markov processes were also used to analyze a cooperative system in other papers, but the interference was neglected, e.g [13], [14], or, when it is considered, simplified assumptions, which are accurate only for high density networks, were made [15] in order to facilitate the analysis.

Having established an analysis that can obtain exact expressions for the throughput of a given network configuration, this paper proceeds to optimize the network with respect to the *transport capacity (TC)*, which is a measure of forward progress. In particular, the optimal CBR size is identified, along with the optimal configuration (number and location) of relays within each CBR, and the optimal code rate for each transmission.

II. Barrage Relay Network

A BRN is a simplified abstraction of a tactical MANET architecture that is currently being used in the field [8], [9]. BRNs employ TDMA and cooperative communications. All nodes utilize a common frame format that requires coarse

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DOI 10.1109/MILCOM.2014.96

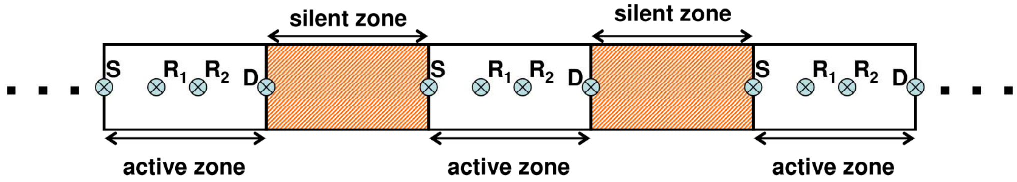


Fig. 1. Example of BRNs, composed of multiple CBRs. Each CBR is composed a four-node network.

slot-level synchronization. This can be accomplished via a distributed network timing protocol when a satellite reference is unavailable. BRNs use autonomous cooperative communications rather than distributed beamforming or space-time coding to (i) minimize the communications overhead required for cooperation and (ii) enable multiple transmitters to participate in cooperative links to multiple destinations without knowing which other nodes are transmitting. As detailed in [7], each node applies an independent, random phase-dithering pattern to each relayed message. This induces a time-varying fading characteristic at each receiver. By changing the phase dither *within* a packet, destructive interference across an entire packet can be avoided. When modern codes with long block lengths are used, the phase-dithering technique effectively translates spatial diversity from multiple transmitters into time diversity that can be captured via maximal-likelihood sequence estimation (MLSE) equalization and iterative detection.

The BRN flooding primitive works as follows. During the first slot of a frame, the source broadcasts its message. During the second slot, any node that received and successfully decoded the initial transmission will rebroadcast it. If more than one node concurrently transmits the message, then the superposition of their signals is received. The diverse signal components are effectively maximal-ratio combined at each receiver. During each subsequent slot, all messages that were successfully decoded during the previous slot are rebroadcast. The process repeats until either the destination is reached, no receiver successfully decodes a transmission, or a maximum number of transmissions is reached. Packets thus propagate outward from the source via a *decode-and-forward* approach. To prevent relay transmissions from propagating back towards the source, each node relays a given packet only once.

III. Controlled Barrage Regions

In a traditional network, a link is defined by a transmit/receive radio pair that share a suitably reliable point-to-point communications channel. A unicast route is simply a series of these links connecting a source-destination node pair. In a BRN, however, links are cooperative and comprise multiple transmitters and receivers. A cooperative path is made up of series of cooperative links between the source and destination nodes. A CBR is simply the union of one or more such cooperative paths within some subregion or *zone* of the overall network. CBRs afford a mechanism for unicast transport that is more robust than both traditional unicast routing and non-cooperative multipath variants [16].

CBRs can be established by specifying a set of *buffer* nodes around a set of cooperating *interior* nodes. Each buffer node acts as the source for transmissions into a CBR and the destination for transmissions emanating from within the CBR. External packets may enter the CBR only through a buffer node, and packets internal to a CBR may only propagate to the rest of the network through a buffer node. In this way, multiple unicast transmissions may be established in different portions of the network. This is referred to as *spatial reuse*. The buffer and interior nodes corresponding to a given source-destination pair can be specified via the broadcast of request-to-send (RTS) / clear-to-send (CTS) packets [16].

An example of linear BRN is shown in Fig. 1. Each CBR in the example is made up of four nodes. The two interior nodes of each CBR, R_1 and R_2, act as relays, while the other two nodes are buffers, acting as the source S and destination D of the transmissions within the CBR, respectively. In the case of a linear BRN, each CBR is defined to be a segment of length d, here assumed to be the same for all CBRs.

Let N be the number of interior nodes in a CBR. It is assumed that the transmission from a CBR's source to its destination must take place within one radio frame or else an *outage* occurs[1]. Since each node transmits each packet at most once, the maximum number of transmissions per frame is $F = N + 1$. Since each transmission requires one slot, the maximum number of slots per radio frame is also $N + 1$. While variable-length frames could be supported, this would complicate the synchronization of neighboring CBRs. It is thus assumed that all CBRs have N interior nodes and $F = N+1$.

A packet may be broadcast by the source of a CBR during only the first time slot of a frame. In ideal channel conditions, it is possible that the destination receives the initial source transmission in the first slot, though more generally, additional transmissions from the interior nodes may be required for successful reception. The buffer node acting as the source for one CBR also acts as the destination for the previous CBR. Because the nodes operate in a half-duplex mode, it is not possible for the buffer to simultaneously transmit into one CBR while receiving from another CBR. Thus, it is advisable to only allow one of the two CBRs serviced by a given buffer node to be active; i.e., either the CBR that the buffer node is transmitting into is active or the CBR that the buffer node is receiving from is active, but not both. More generally, this

[1]Note that in practice, BRNs often employ a fixed frame length of $F = 4$ and a fixed maximum number of relays that is greater than F [8]. In such networks, spatial pipelining enables multiple packets to be active within a given CBR at the same time. In the interest of analytical tractability, this paper considers only interference from transmissions in different CBRs. This is motivated by the observations that (i) in practice, most tactical unicast traffic is local so that CBRs will typically be less that 4 hops long and (ii) inter-CBR interference is more problematic than intra-CBR interference when $F = 4$.

suggests that CBRs can be divided into two types during a particular radio frame: 1) *active zones* and 2) *silent zones*. During a given radio frame, only the nodes in active zones may transmit. As illustrated in Fig. 1, zones will typically alternate between active zones and silent zones, though it is feasible to have even further spatial separation between active zones. A key advantage to alternating between active and silent zones is that it provides spatial separation between CCI, as the CCI in a given active zone is due to transmissions in other active zones, which are at least two zones away.

IV. Network Model

Consider a BRN composed of K CBRs. The BRN is a set of M mobile radios or *nodes* $\mathcal{X} = \{X_1, ..., X_M\}$. The variable X_i represents both the i^{th} node and its location. In the following, X_i is typically used to denote a transmitting node and X_j used to describe a receiving node. During the t^{th} time slot of a frame, $t \in \{1, ..., F\}$, node X_i transmits with probability $p_i^{(t)}$. Any node in a silent zone will be characterized by $p_i^{(t)} = 0$ for all slots of the frame. Since the source of an active CBR may only transmit in the first slot of the frame, it is characterized by $p_i^{(1)} = 1$ and $p_i^{(t)} = 0$ for $t \in \{2, ..., F\}$.

Let $\mathcal{X}_j^{(t)} \subset \mathcal{X}$ be the set of cooperating or *barraging* nodes that transmit identical packets to X_j during the t^{th} time slot and $\mathcal{G}_j^{(t)}$ the set of the indexes of the nodes in $\mathcal{X}_j^{(t)}$. Nodes within the same CBR as X_j but not in $\mathcal{X}_j^{(t)}$ do not transmit; i.e., there is no intra-CBR interference. However, nodes in other active CBRs are potential sources of interference, and each such node will generate CCI with probability $p_i^{(t)}$. The value of $p_i^{(t)}$ for each of these nodes will depend on the dynamics of the protocol, as will be described shortly.

When X_i transmits, it broadcasts a signal whose average received power in the absence of fading is P_i at a reference distance d_0. For ease of exposition, it is assumed that all the X_i that transmit do so with a common power $P_i = P$. However, this assumption could be relaxed at the expense of slightly complicating the notation and analysis. X_i's power at receiver X_j during time slot t is

$$\rho_{i,j}^{(t)} = P g_{i,j}^{(t)} f(d_{i,j}) \tag{1}$$

where $g_{i,j}^{(t)}$ is the power gain due to fading, $d_{i,j} = ||X_i - X_j||$ is the distance from X_j to X_i, and $f(\cdot)$ is a path-loss function. The $\{g_{i,j}^{(t)}\}$ are independent and exponentially distributed with unit mean, corresponding to Rayleigh fading. It is assumed that the $\{g_{i,j}^{(t)}\}$ remain fixed for the duration of a time slot, but vary independently from slot to slot. For $d \geq d_0$, the path-loss function is expressed as the attenuation power law

$$f(d) = \left(\frac{d}{d_0}\right)^{-\alpha} \tag{2}$$

where $\alpha > 2$ is the attenuation power-law exponent, and d_0 is sufficiently large that the signals are in the far field.

A commonly accepted approach to analyzing diversity combining is to assume that the interference realizations in the different branches are the same; i.e., the interference is fully-correlated among the branches (cf., [17]). While strictly speaking, this condition is not always met, the assumption makes the analysis manageable. Furthermore, as a worst-case scenario, the analysis makes it possible to obtain an upper bound on outage probability, which happens to be tight [18]. Under this assumption and by using (1) and (2), the instantaneous signal-to-interference-plus-noise ratio (SINR) at mobile X_j during slot t is

$$\gamma_j^{(t)} = \frac{|\mathcal{X}_j^{(t)}| \sum\limits_{k \in \mathcal{G}_j^{(t)}} g_{k,j}^{(t)} \Omega_{k,j}}{\Gamma^{-1} + \sum\limits_{i \notin \mathcal{G}_j^{(t)}} I_i^{(t)} g_{i,j}^{(t)} \Omega_{i,j}} \tag{3}$$

where $\Gamma = d_0^{\alpha} P/\mathcal{N}$ is the signal-to-noise ratio (SNR) of a unit-distance transmission when fading is absent, $|\mathcal{X}_j^{(t)}|$ is the number of barraging transmitters, $\Omega_{i,j} = d_{i,j}^{-\alpha}$ is the relative path gain, $I_i^{(t)}$ is a Bernoulli variable indicating the X_i is a source of interference during slot t, and $P[I_i^{(t)} = 1] = p_i^{(t)}$.

Let β denote the minimum SINR required by X_j for reliable reception and $\mathbf{\Omega}_j = \{\Omega_{1,j}, ..., \Omega_{M,j}\}$ represent the set of relative path gains from all $\{X_i\}$ to X_j. An *outage* occurs when the SINR falls below β. Conditioning on the path gains $\mathbf{\Omega}_j$ and the set of barraging nodes $\mathcal{X}_j^{(t)}$, the outage probability of mobile X_j during slot t is

$$\epsilon_j^{(t)} = P\left[\gamma_j^{(t)} \leq \beta \middle| \mathbf{\Omega}_j, \mathcal{X}_j^{(t)}\right]. \tag{4}$$

Because it is conditioned on $\mathbf{\Omega}_j$, the outage probability depends on the particular network realization, which has dynamics over timescales that are much slower than the fading. From [12], the outage probability in Rayleigh fading is

$$\epsilon_j^{(t)} = 1 - \sum_{k \in \mathcal{G}_j^{(t)}} \exp\left(-\frac{\beta}{\Omega_{k,j}\Gamma|\mathcal{X}_j^{(t)}|}\right) \prod_{s \in \mathcal{G}_j^{(t)}, s \neq k} \frac{\Omega_{k,j}}{\Omega_{k,j} - \Omega_{s,j}} \times \prod_{i \notin \mathcal{G}_j^{(t)}} \frac{|\mathcal{X}_j^{(t)}|\Omega_{k,j} + \beta\left(1 - p_i^{(t)}\right)\Omega_{i,j}}{|\mathcal{X}_j^{(t)}|\Omega_{k,j} + \beta\Omega_{i,j}}. \tag{5}$$

The outage probability is conditioned on the node locations (represented by the $\{\Omega_{i,j}\}$) and by the particular set of barraging transmitters (represented by $\mathcal{X}_j^{(t)}$). Notice that if $|\mathcal{X}_j^{(t)}| = 1$, (5) coincides with (30) in [19] and (13) in [20].

V. CBR as a Markov Process

Consider a single packet being transmitted through a single CBR composed of a source (denoted S), a destination (D) and N relays ($\{R_1, ..., R_N\}$). At the boundary between any two time slots, each node can be in one of the three following states, which we refer to as the *node state*:

- Node state 0: The node has not yet successfully decoded the packet.
- Node state 1: It has just decoded the packet received during previous slot, and it will transmit on next slot.
- Node state 2: It has decoded the packet in an earlier slot and it will no longer transmit or receive that packet.

The state of the CBR is the concatenation of the states of the individual nodes within the CBR. Hereafter, we refer to this as the *CBR state*. The CBR state can be compactly represented by a vector of the form $[S, R_1, ..., R_N, D]$ containing the states of the source, relays, and destination.

The behavior of the CBR can be described as an *absorbing* Markov process, which describes the dynamics of how the CBR states evolve over a radio frame. Define $\boldsymbol{s} = \{s_1, s_2, \ldots, s_\varrho\}$, which is the *state space* of the process composed of ϱ *Markov* states s_i. An absorbing Markov process is characterized by τ *transient states* and r *absorbing states*. A state s_i of a Markov chain is called *absorbing* if once in that state, it is impossible to leave it, while a state that is not absorbing is called a *transient* state. We note that it is often possible to group several CBR states into a single Markov state. For instance, we group all of the CBR states corresponding to successful decoding by the destination into a single absorbing Markov state.

The probability that the process moves from (Markov) state s_i to state s_j is denoted by $p_{i,j}$ and the probabilities $\{p_{i,j}\}$ are called *transition probabilities*. Fig. 2 shows the Markov chain for a CBR with $N = 2$ relays (a four-node network). The CBR states are shown as a 4-element vector, along with the transition probabilities. The message is successfully delivered whenever the destination receives the message, which is indicated by a 1 in the last position of the CBR-state vector. Such states are marked in green, and hereafter we refer to this condition as a *CBR success*. The transmission fails whenever all entries in the CBR-state vector are either 0 or 2, indicating that none of the nodes that have not yet transmitted have successfully received the message. Such states are marked in red, and hereafter we refer to this condition as a *CBR outage*. Each transient state in the Markov process corresponds to a single CBR state, and such states are numbered 1 through 6 in the diagram. The 4 CBR states that correspond to a CBR outage are collapsed into a single absorbing state of the Markov process (state 7), and similarly the 9 CBR states corresponding to a CBR success are collapsed into a single absorbing state of the Markov process (state 8).

Consider how the CBR state may change from time t to time $t+1$. All nodes at time t with node state 1 transmit, so at time $t+1$, those nodes will now be in state 2. All nodes that have node state 0 at time t will receive the transmission, so at time $t+1$ these nodes will be either in state 1 (if the transmission was successful) or 0 (if the transmission failed). Since the channels from transmitting to receiving nodes are independent, the state transition probability is the product of the individual transmission probabilities. For instance, the probability of going from state [1100] to [2210] is the product of the probability that R_2 successfully decodes the joint transmission (from S and R_1) and the probability that D does not successfully decode the transmission. The individual probability of successful decoding at each node is found using the outage probability of (5). While there is a one-to-one correspondence between CBR states and the transient Markov states, there are several CBR states associated with each of the two absorbing Markov states. Thus, for each absorbing Markov state, the probability of transitioning into it is equal to the sum of the probabilities of transitioning into the constituent

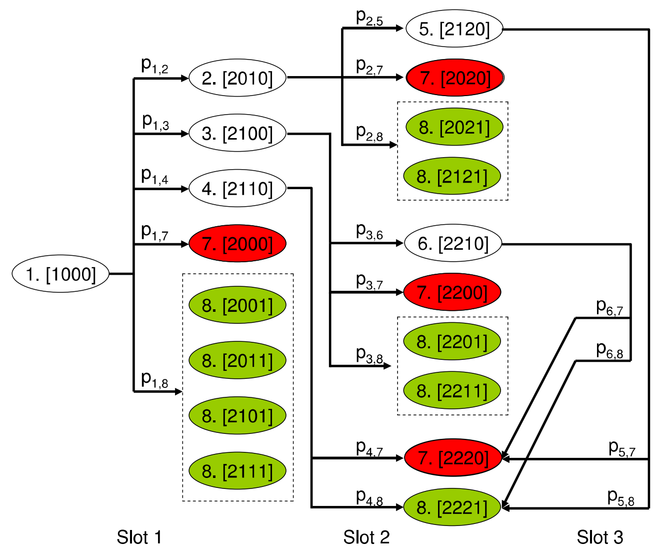


Fig. 2. Markov chain for a CBR composed of four nodes ($N = 2$). Transient states are in white, while the CBR success absorbing state is in green and the CBR outage absorbing state is in red. Each of the two absorbing states is the union of several CBR states.

CBR states.

The Markov transition probabilities are placed into a *state transition matrix* $\boldsymbol{P}$, whose $(i, j)^{th}$ entry is $\{p_{i,j}\}$. The Markov states are ordered such that the τ transient states and indexed before the r absorbing states. With this ordering, the state transition matrix assumes the following canonical form [21]

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{Q} & \boldsymbol{R} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \tag{6}$$

where $\boldsymbol{Q}$ is the $\tau \times \tau$ *transient matrix*, $\boldsymbol{R}$ is the $\tau \times r$ *absorbing matrix* and $\boldsymbol{I}$ is an $r \times r$ identity matrix.

Let $b_{i,j}$ be the probability that the process will be absorbed in the absorbing state s_j if it starts in the transient state s_i. The *absorbing probability* $b_{i,j}$ is the $(i, j)^{th}$ entry of the matrix $\boldsymbol{B}$, which can be computed by (see, for instance [21])

$$\boldsymbol{B} = \boldsymbol{N}\boldsymbol{R} \tag{7}$$

where $\boldsymbol{N} = (\boldsymbol{I} - \boldsymbol{Q})^{-1}$ is the *fundamental matrix*. The two absorbing states are indexed so that the first absorbing state corresponds to a CBR outage, while the second corresponds to a CBR success. Since the process always starts in state s_1, it follows that $b_{1,1}$ is the *CBR outage probability* (which is indicated by ϵ_{CBR}) and $b_{1,2}$ is the *CBR success probability* (which is $\hat{\epsilon}_{CBR} = 1 - \epsilon_{CBR}$).

Example # 1 Consider the four-node CBR whose nodes are equally spaced along a line, as shown in the inset in Fig. 3. In this example, the CCI from adjacent CBRs is neglected, and the path-loss exponent is $\alpha = 3.5$. Fig. 3 shows the CBR outage probability as a function of the SNR Γ for three values of the threshold β. Solid curves are obtained analytically according to the methodology of this section, while the markers correspond to a simulation using the methodology introduced in [22]. In particular, the simulation works by first computing the outage probabilities using (5), using these probabilities to determine the state-transition probabilities, and

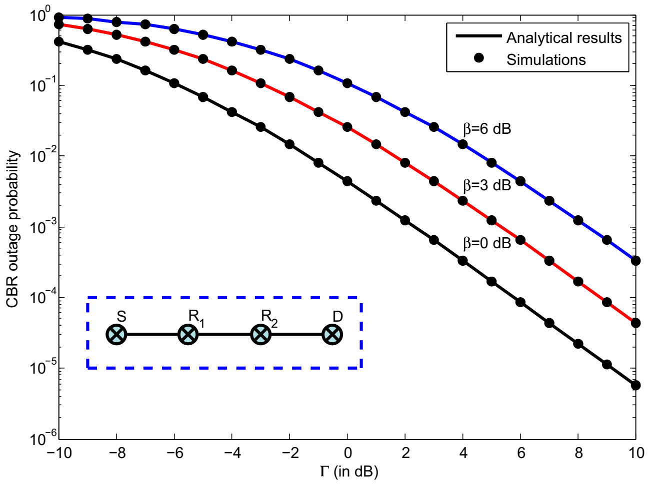


Fig. 3. CBR outage probability as a function of Γ when CCI is neglected. The solid lines are obtained analytically while the markers are obtained from Monte Carlo simulation. The network topology is a four node line network shown in the inset, and $\alpha = 3.5$.

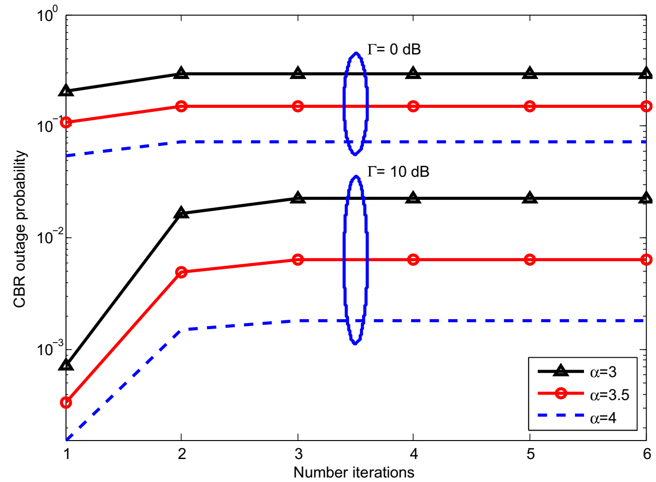


Fig. 4. CBR outage probability for the k^{th} CBR as function of the number of iterations used. Set of curves at the top: $\Gamma = 0$ dB. Set of curves at the bottom: $\Gamma = 10$ dB. Each set of curves is obtained using three values of α.

simulating each state transition by drawing a random number. The analytical results coincide with the simulations as shown in Fig. 3, and any discrepancy between the curves can be attributed to the finite number of Monte Carlo trials (10^7 trials were executed per SNR point).

VI. Inter-CBR Interference

Let $\mathbf{P}_k$ represent the transition matrix of the k^{th} CBR. The transition probabilities in $\mathbf{P}_k$ are computed using (5), which depend on the $\{p_i^{(t)}\}$ associated with nodes in other CBRs. However, each $p_i^{(t)}$ represents the probability that node X_i transmits during time slot t, which can be found from the $\mathbf{P}_k$ of the corresponding CBR. Thus, interference causes a linkage between the transition probabilities in the given CBR and the transmission probabilities of adjacent CBRs. While this linkage could be handled by considering the overall BRN as one large Markov chain, this is a cumbersome solution as the number of states grows exponentially with the number of nodes. A more efficient approach is an iterative one, which alternates between computing the $\{p_i^{(t)}\}$ and the $\{\mathbf{P}_k\}$.

Let $\mathcal{S}_{i,t}$ be the set of states associated with time slot t and labeled with a '1' in the position corresponding to X_i. These are the states for which X_i transmits during time slot t. The probability $p_i^{(t)}$ can be found by adding the probabilities of being in each of the states in $\mathcal{S}_{i,t}$, given that the system starts from initial state $\mathbf{s}_1$. The probability of being in state $\mathbf{s}_j \in \mathcal{S}_{i,t}$ is found by multiplying the probabilities of all transitions in the Markov chain leading from state s_1 to state s_j

Let $\boldsymbol{P}_k[i_t]$ represent the state transition matrix for the k^{th} CBR corresponding to iteration i_t, and similarly let $p_i^{(t)}[i_t]$ represent the transmission probability for the i^{th} user during the t^{th} time slot corresponding to iteration i_t. The iterative method can be described as follows:

1) Initialization: Set $i_t = 0$ and initialize $p_i^{(t)}[0] = 0, \forall i, t$, which corresponds to neglecting interference. Then compute each $\boldsymbol{P}_k[0]$ using the initial $\{p_i^{(t)}[0]\}$.
2) Recursion: Increment i_t. Evaluate each $\boldsymbol{P}_k[i_t]$ using the $\{p_i^{(t)}[i_t - 1]\}$ from the previous iteration.
3) Decision: Halt the process if $||\boldsymbol{P}_k^{(i_t)} - \boldsymbol{P}_k^{(i_t-1)}||_F < \xi, \forall k$, where the operator $||\cdot||_F$ is the Frobenius norm and ξ is a tolerance. Otherwise, go back to step 2.

Example # 2 The approach is further simplified if it is assumed that the BRN is comprised of an infinite cascade of CBRs, all having identical topology. The $\mathbf{P}_k$ and the set of $\{p_i^{(t)}\}$ will be the same in all active zones. Because of this, performance can be described by a *typical* CBR. In this example, assume that each CBR is composed of four nodes which are configured as in Example #1. The typical CBR is analyzed assuming it receives CCI from just the two closest adjacent active zones (the interference from distant zones is neglected). Fig. 4 shows the CBR outage probability for a typical CBR as a function of the number of iterations used in the algorithm described above. Curves are shown for two values of Γ and three values of α. All six curves are obtained with $\beta = 6$ dB. Fig. 4 shows that the CBR outage probability increases as a function of the number of iterations, and that the iterative method converges rapidly after very few iterations.

VII. Network Optimization

The transport capacity [23] of an ad hoc network quantifies the rate that data can be reliably communicated over a unit of distance, and is typically expressed in units of meter-bits-per-second. In the context of a BRN, the TC of a typical CBR is

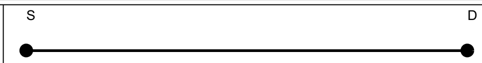
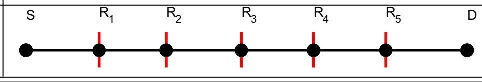
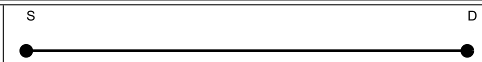
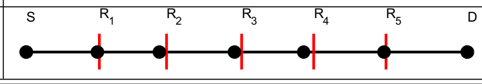
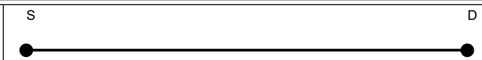
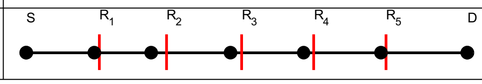
$$\Upsilon = Td \tag{8}$$

where T is the *throughput* of the CBR and d is the distance between the CBR's source and destination. From [24], the throughput can be written as

$$T = \frac{(1 - \epsilon_{CBR})}{2F} R \tag{9}$$

where R is the code rate expressed in units of bits per channel use (bpcu) and the factor 2 in the denominator is

TABLE I
OPTIMIZATION RESULTS.

Γ (in dB)	CCI	R	N	d	Optimal nodes' location for a line network	Υ_{opt}
0	✗	4.452	0	0.3		0.490
	✓	4.421	5	1.2		0.402
5	✗	4.611	0	0.4		0.683
	✓	4.452	5	1.7		0.559
10	✗	5.028	0	0.5		0.951
	✓	4.547	5	2.3		0.777

a consequence of the buffer nodes operating in half-duplex mode, or, equivalently, due to alternating between active and silent zones.

Substituting (9) into (8), and relating the rate and the SINR threshold by $R = \log_2(1+\beta)$, which is the Shannon capacity for complex discrete-time AWGN channels, yields

$$\Upsilon = d\frac{\hat{\epsilon}_{CBR}}{2(N+1)}\log_2(1+\beta). \tag{10}$$

Let $\boldsymbol{X}_k$ be a vector containing the position of the N relays in the k^{th} CBR. The goal of the network optimization is to determine the set $\boldsymbol{\Theta} = (\boldsymbol{X}_k, R, N, d)$ that maximizes the TC. Because the TC is nonlinear in $\boldsymbol{\Theta}$ and the search space is multi-dimensional, efficiently finding a solution is a challenge. However, by initially using an exhaustive search, we have confirmed that the optimization surface is convex over the set (R, N, d). Thus, the optimization can be solved through a convex optimization over (R, N, d) combined by a stochastic optimization over $\boldsymbol{X}_k$. It follows that an efficient approach to finding the optimal set $\boldsymbol{\Theta}$ is as follows:

1) For each of the parameters N, R and d, select a pair of endpoints and a midpoint ($3^3 = 27$ points). The endpoints should initially be far enough apart to guarantee that the optimal point lies within the endpoints.
2) Define a *reference vector* $\boldsymbol{X}_{\text{ref}}(N,d)$, which is a vector $\boldsymbol{X}_k$ for the pair (N,d) and a *reference set* $\{\boldsymbol{X}_{\text{ref}}(N,d)\}$ containing the reference vectors of all (N,d) considered by the algorithm. Create three vectors $\{\boldsymbol{X}_k\}$ for each pair of (N,d). When (N,d) is chosen for the first time, the first vector of $\{\boldsymbol{X}_k\}$ is obtained by placing the N relays on the length-d line connecting the source and the destination and this vector is added to the reference set, constituting $\boldsymbol{X}_{\text{ref}}(N,d)$ for the pair (N,d) chosen. The placement is done such that relays are separated by a minimum distance $r_{\text{s}} = \frac{d}{3N}$, but are otherwise uniformly distributed along the line. If the pair (N,d) has been already used, the first vector of $\{\boldsymbol{X}_k\}$ is set to be equal to $\boldsymbol{X}_{\text{ref}}(N,d)$. The other two vectors are obtained by *mutating* $\boldsymbol{X}_{\text{ref}}(N,d)$ as follows:
   a) For each relay in $\boldsymbol{X}_{\text{ref}}(N,d)$, draw a Bernoulli random variable indicating if the relay should be moved or if it should remain in its current location.
   b) If the node X_i is to be moved, the direction (right or left) of movement is selected with equal probability. The new position is obtained by moving the current position in the selected direction by a distance $\Delta = \frac{d}{n_\Delta N}$, where initially $n_\Delta = 2$.
3) Compute the TC for the endpoints and the midpoints of the interval of one of the parameters in the set (R, N, d) and for the different sets of locations of the relays. For each pair (N,d) chosen, determine the vector $\boldsymbol{X}_k$ that provides the higher TC and replace the $\boldsymbol{X}_{\text{ref}}(N,d)$ in the reference set with this vector.
4) For the same parameter of the set (R, N, d) used in step 3, move the midpoint of the interval towards the endpoint that gives higher TC. For each pair (N,d), generate the three sets of positions as in step 2.
5) Repeat step 3 and 4 recursively for all the parameters in the set (R, N, d) and gradually reduce for one parameter at the time the range between the endpoints and the midpoint until a certain tolerance is reached. If the optimal TC remains the same as in the previous iteration n_Δ is increased by one, until a certain value is achieved.
6) If the optimal TC remains the same as in the previous iteration, for all the parameters in the set (R, N, d) it is not possible to further reduce the range between the endpoints and the midpoint, since a given tolerance is achieved, and n_Δ reaches its maximum value, the algorithm stops.

Using the methodology described above, optimization results were obtained under the same assumptions that were used in Example #2. In particular, it is assumed that the BRN extends infinitely and CBRs are composed of line networks with the same number of relays which are all equally positioned. For each CBR the only CCI considered are the ones from the two closest adjacent active zones (the interference from farther active zones is neglected, since they are at least at $3d$ distance away). Although this assumptions can be relaxed using the analysis and methodology proposed along this paper, in this section they are used for simplicity of exposure.

Table I shows the optimal values of N, R, d and the optimal location for the N relays. Optimization results are provided for three values of Γ and when the CCI is neglected as well as when they are taken into account. Vertical red lines indicate the optimal position of the mobiles for $\Gamma = 0$ dB, and they are used to facilitate the graphical analysis and comparison of the optimal location of the mobiles for different scenarios. For all scenarios considered, the path loss exponent is fixed to $\alpha = 3.5$. Table I emphasizes that when there is no CCI, the optimal configuration for a cooperative BRN is to use short CBRs characterized by a point-to-point conventional communication, while five relays distributed over a larger CBR allow to achieve the optimal TC when there is CCI from the adjacent CBRs. As expected, the optimal TC increases by increasing Γ, since the links experience a more favorable channel. Furthermore, the optimal TC increases going from a scenario in which there is CCI to one in which CCI is neglected, and this is more prominent as Γ increases since the network becomes more interference sensitive. In agreement with [25], an increase in Γ produces a shift towards the source of the optimal position of the relays, which increases the likelihood to cooperate among themselves. A more favorable condition of the channel produces an increment in both the optimal code rate R and in the optimal dimension of the CBR.

VIII. Conclusion

This paper presents a new analysis and optimization for unicast in a BRN. A BRN is analyzed by describing the behavior of each constituent CBRs as a Markov process. The transition probabilities are computed by using a new closed form expression for the outage probability, which takes into account the path loss, Rayleigh fading, and interference. Interference causes an interdependence between the transmission and transition probabilities, which is taken into account by an iterative method, that updates the probability of collisions for each CBR at each iteration for each one of the time slots. The analysis is used to optimize a BRN and in particular to maximize the TC by finding the optimal number of relays N that need to be used in the CBRs that compose the BRN, their optimal placement $\boldsymbol{X}_k$, the optimal size of the CBRs d and the optimal code rate R at which the nodes transmit. While the analysis and the model have been used to study and optimize a BRN, this work could be extended to different types of cooperative ad hoc networks, for which multiple source transmissions are diversity combined at the receiver.

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