

# **DYNAMIC PROGRAMMING**

# Dynamic Programming

**Dynamic Programming (DP)** is an algorithm technique used to solve problems that can be broken down into **simpler, overlapping subproblems**.

## Key Concepts of Dynamic Programming

- **Overlapping subproblems:** a problem has overlapping subproblems if it can be broken down into subproblems.
- **Memoization (Top-Down Approach):** store the results in a cache (typically a dictionary or array) to avoid recalculation – recursion and caching approach.
- **Tabulation (Bottom-Up Approach):** first solve all possible subproblems iteratively, and store them in a table.

# Common Patterns in Dynamic Programming

- **Toy example (Fibonacci):** Climbing Stairs, N-th Tribonacci Number, Perfect Squares
- **Constant Transition:** Min Cost Climbing Stairs, House Robber, Decode Ways, Minimum Cost For Tickets, Solving Questions With Brainpower
- **Grid:** Unique Paths, Unique Paths II, Minimum Path Sum, Count Square Submatrices with All Ones, Maximal Square, Dungeon Game
- **Dual-Sequence:** Longest Common Subsequence, Uncrossed Lines, Minimum ASCII Delete Sum for Two Strings, Edit Distance, Distinct Subsequences, Shortest Common Supersequence
- **Interval:** Longest Palindromic Subsequence, Stone Game VII, Palindromic Substrings, Minimum Cost Tree From Leaf Values, Burst Balloons, Strange Printer
- **Longest Increasing Subsequence:** Count Number of Teams, Longest Increasing Subsequence, Partition Array for Maximum Sum, Largest Sum of Averages, Filling Bookcase Shelves
- **Knapsack:** Partition Equal Subset Sum, Number of Dice Rolls With Target Sum, Combination Sum IV, Ones and Zeroes, Coin Change, Coin Change II, Target Sum, Last Stone Weight II, Profitable Schemes
- **Topological Sort on Graphs:** Longest Increasing Path in a Matrix, Longest String Chain, Course Schedule III
- **DP on Trees:** House Robber III, Binary Tree Cameras
- **Other problems:** 2 Keys Keyboard, Word Break, Minimum Number of Removals to Make Mountain Array, Out of Boundary Paths

## Credits

[https://www.youtube.com/watch?v=9k31KcQmS\\_U](https://www.youtube.com/watch?v=9k31KcQmS_U)

<https://algo.monster/problems/dp-list>

# Dynamic Programming – Example – Fibonacci Sequence

## Naive Recursive Approach

$O(2^n)$

```
int fib(int n) {  
    if (n <= 1) {  
        return n;  
    }  
    return fib(n - 1) + fib(n - 2);  
}
```

## Memoization (Top-Down DP)

$O(n)$

```
std::unordered_map<int, int> memo;  
  
int fib(int n) {  
    if (n <= 1) {  
        return n;  
    }  
    if (memo.find(n) != memo.end()) {  
        return memo[n];  
    }  
    memo[n] = fib(n - 1) + fib(n - 2);  
    return memo[n];  
}
```

## Tabulation (Bottom-up DP)

$O(n)$

```
int fib(int n) {  
    if (n <= 1) {  
        return n;  
    }  
    int dp[n + 1];  
    dp[0] = 0;  
    dp[1] = 1;  
    for (int i = 2; i <= n; i++) {  
        dp[i] = dp[i - 1] + dp[i - 2];  
    }  
    return dp[n];  
}
```

# Problem – Climbing Stairs

Easy



LeetCode

[leetcode.com/problems/climbing-stairs](https://leetcode.com/problems/climbing-stairs)

## Problem Statement

You need to climb a staircase with  $n$  steps to get to the top. Each time you can choose to climb either **1 step** or **2 steps** at a time. Find out how many different ways you can climb to the top of the staircase.

### Example 1

**Input:**  $n = 2$

**Output:** 2

**Explanation:** There are two ways to get to the top

1. Climb 1 step at a time, twice
2. Climb 2 steps in one go

### Example 2:

**Input:**  $n = 3$

**Output:** 3

**Explanation:** There are three ways to get to the top:

1. Climb 1 step at a time, three times
2. Climb 1 step, then 2 steps
3. Climb 2 steps, then 1 ste.

# Solution – Climbing Stairs

Easy



LeetCode

[leetcode.com/problems/climbing-stairs](https://leetcode.com/problems/climbing-stairs)

```
std::unordered_map<int, int> memo;
```

```
int climbStairs(int n) {  
    // Identify the sequence, when:  
    // n = 0 (0 way), there is no way to get up  
    // n = 1 (1 way): only one way : 1-step  
    // n = 2 (2 ways): 1s + 1s | 2s  
    // n = 3 (3 ways): 1s + 1s + 1s | 1s + 2s | 2s + 1s  
    // n = 4 (5 ways): 1s + 1s + 1s + 1s | 1s + 1s + 2s | 1s + 2s + 1s | 2s + 1s + 1s | 2s + 2s |  
  
    if (n <= 2) {  
        return n;  
    }  
  
    if (memo.find(n) != memo.end()) {  
        return memo[n];  
    }  
  
    memo[n] = climbStairs(n - 1) + climbStairs(n - 2);  
    return memo[n];  
}
```

# Problem – 1143. Longest Common Subsequence

Medium

 <https://leetcode.com/problems/longest-common-subsequence>

**Problem Statement / Solution / Code**   Time:  $O(-)$    Space:  $O(-)$

■ ...

# Problem – 62. Unique Paths

Medium



LeetCode

<https://leetcode.com/problems/unique-paths>

## Problem

- The robot is placed in a  $m \times n$  grid
- It starts at the top-left cell  $(0,0)$  and must reach the bottom-right  $(m - 1, n - 1)$
- The robot can only move right or down at any point
- Return the number of unique paths the robot can take to reach the destination





# Solution – 62. Unique Paths

Medium



LeetCode

<https://leetcode.com/problems/unique-paths>

## Solution 1 (recursive)

- Define a recursive function `countPaths(m, n)`

- **Base case**

If  $m == 1$  or  $n == 1$ , there's only one way to reach that cell (either all downs or all rights).

- **Recursive case**

To reach cell  $(m, n)$  the robot must come from:

Cell  $(m - 1, n) \rightarrow$  from above

Cell  $(m, n - 1) \rightarrow$  from left

So the number of of paths to  $(m, n)$  is the **sum** of the paths to those two cells

- **Memoization**

Use a 2D vector  $[m + 1][n + 1]$

# Code – 62. Unique Paths

Medium



LeetCode

<https://leetcode.com/problems/unique-paths>

**Code** Time:  $O(m * n)$  Space:  $O(m * n)$

```
int countPaths(int m, int n, vector<vector<int>>& memo) {
    if (m == 1 || n == 1) return 1;
    if (memo[m][n] != -1) return memo[m][n];
    memo[m][n] = countPaths(m, n - 1, memo) + countPaths(m - 1, n, memo);
    return memo[m][n];
}
int uniquePaths(int m, int n) {
    /*
        count(m, n) = count(m, n + 1) + count(m + 1, n)
        same as (imagine robot going from m,n to 0,0 up and left)
        count(m, n) = count(m, n - 1) + count(m - 1, n)
    */
    vector<vector<int>> memo(m + 1, vector<int>(n + 1, -1));
    return countPaths(m, n, memo);
}
```

# Solution – 62. Unique Paths

Medium



LeetCode

<https://leetcode.com/problems/unique-paths>

## Solution 2 (iterative)

- Create a 2D vector `dp` of size  $(m+1) \times (n+1)$  to store intermediate results
  - Set `dp[1][1] = 1` because there is exactly one way to stand on the starting cell
  - Iterate through each cell `(row, col)` from `(1, 1)` to `(m, n)`:
    - Skip `(1, 1)` since it's already initialized
    - For every other cell, the number of unique paths to it is the sum of:
      - Paths from the cell above: `dp[row-1][col]`
      - Paths from the cell to the left: `dp[row][col-1]`
- `dp[row][col] = dp[row - 1][col] + dp[row][col - 1]`**

# Code – 62. Unique Paths

Medium



LeetCode

<https://leetcode.com/problems/unique-paths>

**Code** Time:  $O(m * n)$  Space:  $O(m * n)$

```
int uniquePaths(int m, int n) {
    vector<vector<int>> dp(m + 1, vector<int>(n + 1));
    dp[1][1] = 1;
    for (int row = 1; row <= m; ++row) {
        for (int col = 1; col <= n; ++col) {
            if (row == 1 && col == 1) continue;
            dp[row][col] = dp[row-1][col] + dp[row][col-1];
        }
    }
    return dp[m][n];
}
```

```
// Optimized 1DP
int uniquePaths(int m, int n) {
    vector<int> dp(n, 1); // base case: first row is all 1s
    for (int row = 1; row < m; ++row) {
        for (int col = 1; col < n; ++col) {
            dp[col] = dp[col] + dp[col - 1];
        }
    }
    return dp[n - 1];
}
```

# Solution – 62. Unique Paths

Medium



LeetCode

<https://leetcode.com/problems/unique-paths>

## Solution 3 (combinatorics)

- **Grid size:**  $m \times n$
- **Start:** top-left cell  $(0, 0)$
- **End:** bottom-right cell  $(m - 1, n - 1)$
- To get from the top-left to the bottom-right:

You must move exactly  $m - 1$  times down

And exactly  $n - 1$  times right

These two types of moves must be made in some order, with a total of:

$$(m - 1) + (n - 1) = m + n - 2 \text{ moves}$$

- **Hence**, from a sequence of  $m + n - 2$  moves, choose  $m - 1$  of them to be down moves (the rest will be right), or vice versa

$$\text{Number of unique paths} = \binom{m + n - 2}{m - 1} = \frac{(m + n - 2)!}{(m - 1)! (n - 1)!}$$

# Code – 62. Unique Paths

Medium



LeetCode

<https://leetcode.com/problems/unique-paths>

**Code** Time:  $O(\min(m,n))$  Space:  $O(1)$

```
int uniquePaths(int m, int n) {
    // we will compute the binomial coefficient:
    // (m + n - 2) choose (m - 1) => total moves choose down moves
    // = (m + n - 2)! / ((m - 1)! * (n - 1)!)

    long long res = 1;

    // we compute the result iteratively to avoid large factorials
    // res = (n) * (n+1) * ... * (m+n-2) / (1 * 2 * ... * (m - 1))

    for (int i = 1; i <= m - 1; ++i) {
        // multiply numerator: (n - 1 + i)
        // divide by denominator: i
        res = res * (n - 1 + i) / i;
    }

    return (int)res;
}
```

# Problem – 983. Minimum Cost For Tickets

Medium



LeetCode

[leetcode.com/problems/minimum-cost-for-tickets](https://leetcode.com/problems/minimum-cost-for-tickets)

## Problem

- You are given two arrays of integers, days and costs
- Days represent

# Problem – 983. Minimum Cost For Tickets

Medium



LeetCode

[leetcode.com/problems/minimum-cost-for-tickets](https://leetcode.com/problems/minimum-cost-for-tickets)

## Solution

- ...



# Problem – 983. Minimum Cost For Tickets

Medium



LeetCode

[leetcode.com/problems/minimum-cost-for-tickets](https://leetcode.com/problems/minimum-cost-for-tickets)

**Code** Time:  $O(-)$  Space:  $O(-)$

■ ...

# Problem – 198. House Robber

Medium



LeetCode

[leetcode.com/problems/house-robber](https://leetcode.com/problems/house-robber)

## Problem

- You are robbing houses lined up in a row
- Each house has a cash value **nums[i]**
- You **cannot rob two adjacent houses**
- Goal: maximize total money robbed without triggering alarms



LeetCode

[leetcode.com/problems/house-robber](https://leetcode.com/problems/house-robber)

## Solution

- Solve as a DP problem
- `dp` represents the maximum value you can get once rob only `i` or the previous houses
- At each “house” `i`, you can either rob or skip

- **Example**

`nums = [1,2,3,1]`

at position `i = 0`, you can have two options:

**rob**: you end up with 1; or **skip**: you end up with 0

at position `i = 1`

**rob**: you get 2 + total of two houses before; or **skip**: total of robbed before

- Therefore:

**rob** = `nums[0] + dp[i - 2]`

**skip** = `dp[i - 1]`

# Problem – 198. House Robber

Medium



LeetCode

[leetcode.com/problems/house-robber](https://leetcode.com/problems/house-robber)

**Code** Time:  $O(n)$  Space:  $O(n)$

```
int rob(vector<int>& nums) {
    int n = nums.size() + 1;
    // base case: start dp[0] = 0 (no rob)
    // dp[1] = nums[0]
    vector<int> dp(n, 0);
    dp[1] = nums[0];
    for (int i = 2; i < n; ++i) {
        int numPos = i - 1;
        // if I rob
        int rob = nums[numPos] + dp[i - 2];
        // if I skip
        int skip = dp[i - 1];
        dp[i] = max(rob, skip);
    }
    return dp[n - 1];
}
```

# Problem – 198. House Robber

Medium



LeetCode

[leetcode.com/problems/house-robber](https://leetcode.com/problems/house-robber)

**Code (space optimized)** Time:  $O(n)$  Space:  $O(1)$

```
int rob(vector<int>& nums) {
    int prev = nums[0];
    int prev2 = 0;

    for (int i = 2; i <= nums.size(); ++i) {
        int numPos = i - 1;
        // if I rob
        int rob = nums[numPos] + prev2;
        // if I skip
        int skip = prev;
        prev2 = prev;
        prev = max(rob, skip);
    }
    return prev;
}
```

# Problem – 213. House Robber II

Medium



LeetCode

[leetcode.com/problems/house-robber-ii](https://leetcode.com/problems/house-robber-ii)

## Problem

- Similar to House Robber I but with a new constraint:
- You are robbing houses **lined up in a circle**
- Each house has a cash value **nums[i]**
- You **cannot rob two adjacent houses**
- You **cannot rob the last and first houses at the same time**
- Goal: maximize total money robbed without triggering alarms

# Solution – 213. House Robber II

Medium



LeetCode

[leetcode.com/problems/house-robber-ii](https://leetcode.com/problems/house-robber-ii)

## Solution

- 1
- 2

# Code – 213. House Robber II

Medium



LeetCode

[leetcode.com/problems/house-robber-ii](https://leetcode.com/problems/house-robber-ii)

**Code** Time:  $O(-)$  Space:  $O(-)$

- 1
- 2



**EOF**

**Problem Statement / Solution / Code** Time:  $O(n)$  Space:  $O(n)$

- ...

# Problem – number. name

Easy

Hard

Medium



LeetCode

[leetcode.com/problems/...](https://leetcode.com/problems/...)

**Problem / Solution / Code** Time:  $O(-)$  Space:  $O(-)$

- 1
- 2