

DYNAMIC PROGRAMMING

Dynamic Programming

Dynamic Programming (DP) is an algorithm technique used to solve problems that can be broken down into **simpler, overlapping subproblems**.

Key Concepts of Dynamic Programming

- **Overlapping subproblems:** a problem has overlapping subproblems if it can be broken down into subproblems.
- **Memoization (Top-Down Approach):** store the results in a cache (typically a dictionary or array) to avoid recalculation – recursion and caching approach.
- **Tabulation (Bottom-Up Approach):** first solve all possible subproblems iteratively, and store them in a table.

Dynamic Programming – Example – Fibonacci Sequence

Naive Recursive Approach

$O(2^n)$

```
int fib(int n) {  
    if (n <= 1) {  
        return n;  
    }  
    return fib(n - 1) + fib(n - 2);  
}
```

Memoization (Top-Down DP)

$O(n)$

```
std::unordered_map<int, int> memo;  
int fib(int n) {  
    if (n <= 1) {  
        return n;  
    }  
    if (memo.find(n) != memo.end()) {  
        return memo[n];  
    }  
    memo[n] = fib(n - 1) + fib(n - 2);  
    return memo[n];  
}
```

Tabulation (Bottom-up DP)

$O(n)$

```
int fib(int n) {  
    if (n <= 1) {  
        return n;  
    }  
    int dp[n + 1];  
    dp[0] = 0;  
    dp[1] = 1;  
    for (int i = 2; i <= n; i++) {  
        dp[i] = dp[i - 1] + dp[i - 2];  
    }  
    return dp[n];  
}
```

Problem – Climbing Stairs

Blind
75

Easy



LeetCode

leetcode.com/problems/climbing-stairs

Problem Statement

You need to climb a staircase with n steps to get to the top. Each time you can choose to climb either **1 step** or **2 steps** at a time. Find out how many different ways you can climb to the top of the staircase.

Example 1

Input: $n = 2$

Output: 2

Explanation: There are two ways to get to the top

1. Climb 1 step at a time, twice
2. Climb 2 steps in one go

Example 2:

Input: $n = 3$

Output: 3

Explanation: There are three ways to get to the top:

1. Climb 1 step at a time, three times
2. Climb 1 step, then 2 steps
3. Climb 2 steps, then 1 ste.

Solution – Climbing Stairs

Blind
75

Easy



LeetCode

```
std::unordered_map<int, int> memo;

int climbStairs(int n) {
    // Identify the sequence, when:
    // n = 0 (0 way), there is no way to get up
    // n = 1 (1 way): only one way : 1-step
    // n = 2 (2 ways): 1s + 1s | 2s
    // n = 3 (3 ways): 1s + 1s + 1s | 1s + 2s | 2s + 1s
    // n = 4 (5 ways): 1s + 1s + 1s + 1s | 1s + 1s + 2s | 1s + 2s + 1s | 2s + 1s + 1s | 2s + 2s |

    if (n <= 2) {
        return n;
    }

    if (memo.find(n) != memo.end()) {
        return memo[n];
    }

    memo[n] = climbStairs(n - 1) + climbStairs(n - 2);
    return memo[n];
}
```