Currently on lecture 10

Determinant of a triangular matrix is the product of the diagonal elements. When a matrix is singular, its determinant is 0. When a matrix is invertible, its determinant is non-zero.

Rank

Projection

Different factorization/decomposition algos

Nullspace, pivots, rank, invertibility

Different ways to invert a matrix? Cramer’s rule

Cofactor expansion – write custom function

Currently on LU factorization

# Dot product of complex vectors

# Signal power and dot product

Starting from standard continuous-time RF signal definition:

is a real-valued, nonnegative function. It’s the envelope of .

Average power is given by , but we can ignore :

Now, let’s translate to discrete-time complex baseband signal:

is the envelope of . is also the magnitude of . The instantaneous power of is given by .

Important difference between real and complex signals:

1. To calculate the power of a real signal, you need to compute the product of the signal multiplied with itself, e.g.
2. To calculate the power of a complex signal, you need to compute the product of the signal multiplied with its complex conjugate, e.g.

Therefore, for complex baseband signals, we may define a slightly different average power:

For a time-limited signal of length , this becomes

We represent discrete-time signals in Matlab as vectors. So let be the vector that corresponds to . Then translates to

In linear algebra, is the length of , so roughly speaking, the length of a vector is equal to its rms voltage.

Roughly speaking, the dot product of with itself is equal to its power.

# LSE equalizer and dot product

## Projection of real and complex vectors

From Zwick’s notes (lecture 22), we know the projection of vector onto vector is given by

He derives this for real vectors using the geometric interpretation of dot product. The dot product for real vectors is symmetric, so

What about for complex vectors, where dot product is not symmetric?

For complex vectors, the projection of vector onto vector is given by

If and are orthogonal, the projection is .

## Error vector and LSE

is the vector in the direction of that is “closest” to , where “closest” minimizes . is the error vector,

The error vector is perpendicular to .

## Different perspective on projection

Let’s rewrite for a different perspective:

So the scalar projection or scalar component of in the direction of is . is the unit vector in the direction of .

## Projection onto a column space

Starting with projection onto a vector,

Projection onto a matrix:

Let denote the columns of :

Why is invertible?

1. If , then That is, .
2. If , then . can be rewritten as . If , then it must be true that That is, .

From #1 and #2, we conclude that and have the same nullspace, that is .

If the columns of are linearly independent, then , so is invertible.

## Signal equalizer

Let be the reference signal and be the impaired signal or signal under test. Equalizing based on is equivalent to projecting onto .

For a single-tap equalizer, this is

For a multi-tap equalizer, construct from delayed copies of :

## 3GPP FD equalizer (TBD)

Then

Thereforemust be a real number.

We can reduce the denominator using the associative property of matrix multiplication:

When , the terms become

When , each term is

When and swap values, you have

# Inverting a matrix (TBD)

Solving a matrix equation for the LSE solution requires inverting an square matrix (call it ). How do we know if a matrix is invertible? These are all equivalent:

1. The columns of are linearly independent: They span
2. The rows of are linearly independent: They span
3. is full rank:
4. There are pivots after elimination, that is, all pivots are nonzero
5. The determinant of is nonzero, that is,

A singular (aka degenerate) matrix has no inverse.

Nullspace, pivots, rank, invertibility, linear independence of columns, condition number, determinant

For to be invertible, and need to be individually invertible. Related,

## Determinant (TBD)

Determinant is product of pivots

## Using Gaussian elimination to invert a matrix

Let where is unknown and is invertible. To solve for using elimination,

is the elimination matrix s.t. , an upper triangular matrix. It’s trivial to then convert to . Let be the matrix s.t. . Then

, in equation form, is , which from the original equation is equal to , that is .

Let . Then .

## Properties of the elimination matrix

The elimination matrix takes higher rows, scales them, and adds them to lower rows so the resultant matrix is upper triangular. This means the elimination matrix is lower triangular:

A triangular matrix is always invertible.

By inspection, using Gaussian elimination, the inverse of a lower triangular matrix is also a lower triangular matrix.

The inverse of an elimination matrix will always have 1 on the diagonal.

## LU factorization

Continuing from the previous section on Gaussian elimination,

We proved that is invertible and is lower triangular. Let :

Where is lower triangular and is upper triangular. This is LU factorization.

has 1 on its diagonal, but may be further factored into , where the new has 1 on its diagonal and is a diagonal matrix of the old ’s pivots.

## Pivot strength / covariance matrix / kernel pruning (TBD)

## Matrix conditioning (TBD)