# Thermal noise

## AWGN (additive white Gaussian noise)

A passive resistor generates an RMS noise voltage equal to

is Boltzmann’s constant, is temperature in kelvin, is resistance in Ohms, and is the integration/measurement BW in Hz.

Room temperature is ~300 K.

Model the resistor as the Thevenin equivalent: a voltage source in series with a noise-free resistance. If the resistor is the source impedance (antenna) of a receiver or a load (antenna) connected to a transmitter, then maximum power is delivered when source and load impedances are matched. Then the power delivered is

is thermal noise density in Watts/Hz.

**Noise figure and sensitivity:**

**Cascaded noise voltage in IC design:**

**AWGN in simulations:**

AWGN can be generated using a function that returns normally distributed pseudorandom numbers.

The RMS level of the noise sequence controls the noise power, which will usually be specified relative to the signal power (as SNR).

Given and SNR, we can calculate

In simulation, the noise is spread out over , but SNR is defined within the signal bandwidth. This means that the previous equation specifies RMS noise within the signal bandwidth, so the RMS level of the noise in the simulation must be scaled:

## Flicker noise and AWGN modeling

Flicker noise = 1/f noise.

1/f noise because flicker noise PSD scales as 1/f, i.e. -10 dB/decade.

The corner frequency is defined as where the flicker noise is equal to the thermal noise floor.

The total noise PSD is

This is a one-sided PSD. To create , we need to convert this to a two-sided PSD, define a phase spectrum to go with it, and apply IDFT.

First, define the phase spectrum as a uniformly distributed random process over , i.e.

# Oscillator phase noise

For complex baseband signal , the ideal RF signal is

With LO phase noise, this becomes

Like flicker noise, phase noise is commonly described in the frequency domain by its PSD in dBc/Hz (or radians squared/Hz in linear). It is the ratio between the noise power measured in a 1Hz bandwidth at frequency offset and the carrier power.

By the convolution theorem of Fourier transforms,

The signal power is “smeared” by the phase noise, which affects signal quality.

In the receiver, in the presence of blockers, the phase noise introduces an additional effect called reciprocal mixing where the LO phase noise mixes with a strong interferer.

Another effect is that the interferer spectrum is also smeared by the phase noise, and the skirt can fall directly within the the receive signal bandwidth.

## Phase noise modeling in the frequency domain

In transceivers, the LOs are usually made with PLLs that are widely utilized for frequency synthesis in ASIC designs.

The PFD (phase-frequency discriminator or phase comparator) compares the phase and frequency of the VCO output to the phase and frequency of the reference oscillator. The output of the PFD, or the error signal, is low-pass filtered for controlling the VCO. A frequency divider is commonly placed in the feedback path to control the output frequency.

Definitions:

* is the frequency of the reference oscillator
* is the (VCO) output frequency
* is the feedback divider,
* is the mean period
* is the natural pulsation (related to PLL bandwidth)
* is the damping coefficient

If the PLL loop filter is first order, then the closed-loop transfer function from input to output is a second-order LPF:

The closed-loop transfer function from the VCO to the output is a second-order HPF:

PLL in-band phase noise is dominated by the reference, while out-of-band noise is mainly due to the VCO.

The PLL phase noise is defined by the 1/f noise generated by the PFD, the noise of the VCO, and two noise floors: inband and out-of-band.

is frequency offset from the carrier, is the 3dB bandwidth of the PLL, and is the inband phase noise level in .

Flicker noise is

At ,

Then

The complete phase noise model:

The flicker noise is filtered by the 2nd-order closed-loop filter.

Rearranging, we have the PSD

### Time-domain simulation

As with the flicker noise simulation, we define the FT of the phase noise and take the IDFT.

is a random process with a uniform distribution in .

Then

Then if the lowpass equivalent signal is , the signal with the phase noise is

If you plot the PDF of , the PDF is Gaussian because of the central limit theorem.

### SNR

The error vector is

Since is uncorrelated with and ,

Then EVM and SNR are

is equal to the integrated phase noise (Parseval’s theorem).

### Impact on OFDM