# Basic concepts

## Big picture

How have we been able to make cell phones more affordable while their functionality has exploded (cellular, WiFi, Bluetooth, GPS, computing, storage, digital camera, user-friendly interface)?

* Integration: how much functionality can be placed on a single chip (or how few components are left off-chip)
* Integration is due to (1) scaling of VLSI processes – CMOS technology – (2) innovations in RF architectures, circuits, devices.

These disciplines are all required, to some degree, for an RF designer:

* Communication theory
* Random signals
* Transceiver architectures
* IC design
* CAD tools
* Wireless standards
* Multiple access
* Signal propagation
* Microwave theory

RF design hexagon:

* Noise
* Power
* Frequency
* Gain
* Supply voltage
* Linearity
* … back to noise

Each metric trades off with its two adjacent metrics. For example, to lower the noise of a front-end amplifier, we must consume more power or sacrifice linearity.

Generic RF transceiver architecture:

* Receiver: antenna 🡪 LNA 🡪 downconverter (driven by oscillator, generated by frequency synthesizer) 🡪 ADC 🡪 digital baseband processor
* Transmitter: digital baseband processor 🡪 DAC 🡪 upconverter 🡪 PA 🡪 antenna

## Units in RF design

Voltage gain:

Power gain:

**Voltage and power gain are equal in dB if and only if the input and output voltages appear across equal impedances.** For example, the gain of an amplifier with an input resistance of and driving a load of is

and are rms values.

Powers are expressed in dBm:

For example, if we deliver a power of 0dBm across a 50Ohm load for a sinusoidal signal, what is the peak-to-peak voltage swing?

Another example, a GSM receiver senses a narrowband modulated signal having a level of -100dBm. If the front-end amplifier has a voltage gain of 15dB, what is the peak-to-peak voltage swing at the amplifier output?

We assumed that

* The input impedance of the front-end amplifier is 50Ohm
* A narrowband signal has roughly the same peak-to-average power relationship as a sinusoid

In most integrated RF systems, we prefer voltage quantities to power quantities since

* Input and output impedances of cascade stages may be unequal, so voltage gain and power gain are not equal
* Impedances may be largely capacitive or inductive, in which case there is no “real” (active) power

However, we still sometimes use dBm at interfaces that do not necessarily entail power transfer. If we drive a purely-capacitive load, the delivered average power is zero, but we can still calculate dBm as if we were driving a 50Ohm load with our voltage signal.

## Time variance

A system is linear if and only if it satisfies the principle of superposition.

Systems with nonzero initial conditions or DC offsets are technically nonlinear, but we often relax the rule to accommodate these two effects (in this case, the system is incrementally linear).

A system is time invariant if a time-shift in the input causes the same time-shift in the output.

A system that changes with time is time variant.

Take the ideal switch for example. Let drive the control s.t. the switch is on if , and let drive the input.

If we look at as the input and as part of the system, then the system is both nonlinear and time variant. The output is independent of the amplitude of , and the system varies over time with .

If we look at as the input and as part of the system, then the system is linear but time variant. The output scales with , but the system varies over time with . In this case, the input-output relationship is

is a square wave toggling between 0 and 1 with frequency .

The output spectrum consists of copies of at

**A linear system can generate frequency components that don’t exist in the input signal if the system is time variant.**

## Nonlinearity

A system is “memoryless” or “static” if its output does not depend on past values of the input and/or output.

A memoryless linear system is given by

A memoryless nonlinear system is given by

are functions of time if the system is time variant.

When for even , the nonlinear system has odd symmetry, which means that is an odd function of , i.e.

This kind of system is also called balanced. One example: a differential pair.

A system is “memory” or “dynamic” if its output depends on past values of input and/or output.

An LTI dynamic system is represented by

This is the convolution integral.

If a dynamic system is linear but time variant, its impulse response depends on the time origin. Then

If a system is both nonlinear and dynamic (what about time variant?), then its impulse response can be approximated by a Volterra series.

**Example: square-law MOS transistor operating in saturation**

For a differential pair, the transfer curve is

Factoring out from the square root and assuming , and using , we get

The first term is the small signal gain . Due to symmetry, there is no even-order nonlinearity. Square-law devices generate a third-order term.

## Effects of memoryless nonlinearity

Model:

is the small signal gain of the system.

We will analyze the effect of nonlinearity on sinusoidal inputs.

### Harmonic distortion (signal only)

Let the input be a single real sinusoid.

Even-order nonlinearity introduces the DC offset. Ideally, even-order nonlinearity vanishes in balanced circuits, but random mismatches corrupt the symmetry, yielding finite even-order harmonics.

th-order harmonic amplitude is proportional to .

**RF harmonics are typically less critical because they fall way outside the frequencies of interest.**

**However, you need to be careful if mixing occurs.**

For example, let’s say are the two inputs to an analog multiplier (mixer). The ideal multiplier is modeled as

is a constant.

Let .

The ideal output has frequency components .

What happens if experiences third-order nonlinearity at the input?

Then the output will have spurious frequency components . Let . Let’s say the desired output frequency component is . , which can fall inband – this is a problem.

### Gain compression (signal + 1 inteferer)

Nonlinearity compresses small signal gain, as seen in the fundamental component

Where and have opposite signs for compressive behavior.

The 1dB compression point – the point at which gain drops by 1dB – is one standard way of characterizing nonlinearity.

In general,

Note that is the peak value of the sinusoidal input.

**Often, gain compression is not dominated by your desired signal but by a large interferer.**

Let

If you multiply out , you will get a term .

Then the fundamental is

Since ,

As interferer amplitude increases, gain decreases and can even become 0.

### Cross modulation (signal + 1 inteferer)

What if the interferer has amplitude modulation, i.e. ?

is a constant. Then

The desired signal suffers from amplitude modulation at and .

Cross modulation commonly occurs in systems that must simultaneously process multiple independent channels.

**In a memoryless nonlinear system, cross modulation does not occur if the interferer has phase modulation, e.g.**.

### Intermodulation (signal + 2 interferers)

The two interferers mix inside the nonlinear system.

If are close in frequency, then the 3rd-order products at are close to .

Let’s say we have a signal at , and it happens that . The IM3 product falls directly on the desired signal and corrupts it.

**For IM3, we typically model narrowband signals by condensing them into unmodulated tones.**

**Even if gain is not significantly compressed, IM3 products may still severely corrupt the desired signal.**

Example: we have a LNA with a gain of 10 and input impedance of 50Ohms. The LNA senses a desired signal level of -80dBm @ 2.41GHz and two interferers of equal power at 2.42 and 2.43GHz. Assume the LNA drives a 50Ohm load.

What value of yields a 1dB compression point of -30dBm? -30dBm is average power.

If each interferer is 10dB below , what is the corruption of the desired signal?

The IM3 at the LNA output is given by

The signal at the LNA output is , so the IM product is as large as the signal even though the LNA does not experience significant compression.

#### IP3 and two-tone test

The two-tone test is very useful for characterizing the nonlinearity of the system since the frequency difference can be made arbitrarily small to ensure the IM3 product lands inband (unlike HD).

We apply two sinusoids of equal amplitude, representing the interferers, and normalize the IM3 amplitude to the fundamental amplitude at the system output. This gives us relative IM:

When increases by 6dB (2x), IM3 increases by 18dB and relative IM3 increases by 12dB.

IP3 is a measure of IM3 nonlinearity that is independent of . The idea is that as increases, at a (theoretical) point, relative IM3 becomes 0dBc. In terms of amplitude, fundamental increases 20dB/decade, while IM3 increases 60dB/decade.

This point is

is peak input amplitude. At this (theoretical) point, the output amplitude of one IM3 tone is equal to the output amplitude of one fundamental tone.

Note: is a theoretical quantity – it cannot be directly measured. It is 9.6dB higher than , which means we cannot say that the output fundamental is because of gain compression. Furthermore, may be higher than the supply voltage. When measuring , we must ensure we measure in the region where 1dB increase in results in 3dB increase in IM3.

Example: LNA senses -80dBm signal at 2.41GHz and two -20dBm interferers at 2.42 and 2.43GHz. What is the value of IP3 that results in IM3 being 20dB below the desired signal? Assume 50Ohm interfaces.