# LTI systems

## CT causal LTI systems described by differential equations

LCC differential equations model an extremely important class of CT systems.

Example.

DE provide an implicit specification of the system – they describe a relationship between input and output, rather than an explicit expression for the system output as a function of the input.

To obtain an explicit expression, we need to solve the DE.

To solve the DE, we need auxiliary conditions.

For example, if we apply a constant voltage of 1V to an RC circuit for 10 seconds, we cannot determine the final capacitor voltage without knowing the initial capacitor voltage.

To completely characterize a system, we need both the DE and the auxiliary conditions.

Going back to the example, let , where is a real number.

The complete solution to the DE is the sum of a particular solution and a homogeneous solution.

satisfies , while is a solution of the homogeneous DE

A common method for finding is to look for a “forced response” – i.e. a signal of the same form as the input. Since for , we guess a solution for of the form

is a number we must determine.

Substitute into the DE:

To find , we guess a solution of the form

Substitute into homogeneous DE:

Therefore is a solution to the homogeneous DE for any value of .

The complete solution of the DE for is

As noted earlier, the DE by itself does not uniquely specify in response to . There are infinitely many solutions for infinitely many values of . To determine , we need to specify an auxiliary condition – different choices for this auxiliary condition lead to different and different relationships between the input and output.

For the most part, we want our systems to be LTI and causal, and in this case, the auxiliary condition is the condition of initial rest.

Initial rest means that if for , then must also equal 0 for .

For our example, for . Initial rest implies that for .

Then for , . For , . Then the full solution is

**Important points concerning LCC differential equations and the systems they represent:**

* The response to input will generally consist of the sum of a particular solution to the DE and a homogeneous solution, which is the solution to the DE with the input set to 0. The homogeneous solution is often called the natural response of the system.
* To completely determine the input-output relationship of a systsem described by a DE, we must specify auxiliary conditions. Different auxiliary conditions lead to different input-output relationships. For the most part, we use the condition of initial rest, which means that the output of the system is zero until the input becomes nonzero. Under this condition, the system is LTI and causal.
* Mathematically, initial rest means that if for , then for . To solve the DE, we evaluate at to solve for the output for . For example, when charging an RC circuit, initial rest means that the capacitor is initially discharged. This also explains time invariance: when charging a capacitor, if you apply the same input voltage at different times, the capacitor voltage should have the same response except time-shifted.
* If the initial condition is nonzero, the resulting system is incrementally linear. The overall response is the superposition of the response to the initial conditions alone (input set to 0) and the response to the input with an initial condition of 0 (i.e., the response of the causal LTI system).

General th-order LCC differential equation:

When , reduces to an explicit function of .

When , the input-output relationship is implicitly specified. The solution to the DE consists of the particular solution and the homogeneous solution. The homogeneous solution is the solution to the homogeneous DE:

The solutions to this equation are also called the natural responses of the system.

Auxiliary conditions are needed to determine the explicit input-output relationship. We mostly use the at rest condition, which means if for , we assume for , and therefore, the response for can be calculated from the DE with the initial conditions

Under the condition of initial rest, the system is causal and LTI.

## DT causal LTI systems described by difference equations

LCC difference equations are an extremely important class of DT systems.