There are many questions still left to be answered:

* Simple question: why is it ? Why is it imaginary? Where does that come from? Is it because you decompose into complex sinusoids (that may add up to be real)?
* Are LCCDE inherently LTI?
* Is LTI system required for all of these properties?
* Can all LTI systems be described by LCCDE?
* LTI systems not described by LCCDE
* When can you use LCCDE?
* We like to analyze LTI systems. Realizable LTI systems are represented by LCCDE, which are LTI if initial conditions are zero – system is at rest. 0 input must generate 0 output.
  + Homogenous response
  + Particular response
* Why do we use unilateral LT but bilateral ZT? I think in practical terms, we use one-sided. We’re talking about realizable systems, which are always causal. This is also why LT/ZT can handle initial conditions – there is an implicit understanding that the signal is right-sided, and it makes sense to conventionally make them causal. I guess technically you can use inverse LT/ZT to calculate system response, but at least for DT systems, it’s very simple to simulate in time domain – you did this in a notebook. Fourier transforms are for steady-state analysis, which is why we use infinity limits. Well, that’s not exactly true. FT are for energy signals, which are finite duration. FT are explicitly about translating between time and frequency representations, LT and ZT are also for translating between time and s/z domains, but with the added benefit of easily incorporating initial conditions for LCCDE. LT and ZT can also transform signals that aren’t L1 or L2, without having to go “in the limit”.
* As poles get closer to axis or unit circle, the system becomes less stable. For signals, this means the signal does not decay as quickly (same goes for impulse response).
* All of these transforms and pairs are for developing intuition about real life signals and systems. In practice, we use computer tools to analyze real life signals and systems, and we use that intuition from these transforms to explain what we see.
* I think we typically talk about PSD, which is applicable to stationary processes. Even if a signal is time-limited, we assume it extends to infinity.
* DTFT in the limit
* I think all impulses – certainly for continuous time – need to be taken in the limit. E.g. to take the inverse CTFT of 1 to get dirac delta in time, or to take the CTFT of a constant or a sinusoid. It’s duality.

Random notes:

* PSD vs. ESD – notice that Fourier series is averaged over the period. Fourier transform is not because it is energy.
* Every integration/summation over one period is averaged
* Adding powers, orthogonality in freq domain
* Integrating powers/energy – Parseval’s or Plancherel’s theorem
* System must be initially at rest because . If there is no input, then output must be zero.

Summarize the most important points from memory.

# System properties

**Linearity**

**Time invariance**

**Causality**

**Stability**

# Systems defined by LCCDE

If system is at rest, LCCDE system is LTI.

If a stable, causal, LTI causal can be represented as a LCCDE, then its pole-zero geometry determines the frequency response. Assume the transfer function is defined by a rational fraction of polynomials in and a real gain. If all poles and zeros are either real or occur in complex conjugate pairs, then

* The polynomial coefficients are real
* The frequency response is conjugate symmetric
* The impulse response is real
* The complex conjugate pairs have complex conjugate residues when using PFE

LCC differential equations

With initial conditions

# Transforms

Transforms are used for both signals and LTI systems.

Transforms exhibit duality because forward and inverse transforms are similar.

CT and DT Fourier transforms and Fourier series switch between time domain and frequency domain representations. Each frequency component in the frequency domain is defined by frequency, amplitude, and phase.

* FS map periodic power signals to harmonic coefficients.
* FT map energy signals (L1 or L2) to a continuous spectrum. The limits of the forward transform are double infinity, but formally, FT only exists for signals that decay quickly enough. For L1 signals, the FT converges pointwise, and for L2 signals, the FT exhibits mean-square convergence.
* We can calculate the FT of periodic power signals (including a DC signal) using FT in the limit. The FT is composed of dirac deltas with area equal to the FS coefficients.
* FT also map the impulse response of stable LTI systems to their frequency response. FT is used for steady-state analysis.

Laplace and z-transforms switch between time domain and / domain.

* LT/ZT are a generalization of FT. They are typically used to solve for the complete solution – transient and steady state – of systems that are represented by LCCDE.
* LT/ZT consists of both the transform and the ROC.
* Typically, we use the one-sided transforms (integration and summation limits from 0 to infinity) since realizable signals and systems are causal.
* LT/ZT exist for signals that do not have a formal FT, like unit step and sinusoids, which makes analysis easier.
* LT/ZT allow us to solve for the complete solution of LCCDE using simple algebraic manipulation, and they allow us to easily include initial conditions of the system.
  + The poles and zeros of the transfer function allow us to easily determine whether the system is stable and if so, the shape of the frequency response.
  + For causal signals and systems, the poles must be in the LHP of the plane and inside the unit circle of the plane for the FT to formally exist, which also means the system is stable. When this condition is satisfied, the FT is equal to the LT/ZT evaluated on the axis or unit circle.

**Continuous time transforms:**

CT Fourier series

CT Fourier transform

Laplace transform

Sampling

causes to be mod , aka periodic with period .

**Discrete time transforms:**

DT Fourier transform

z-transform

Discrete Fourier series

Discrete Fourier transform

Many of the properties observed for CT also hold for DT.

For causal signals and systems, the ROC is of the form , where is largest pole magnitude. For FT to exist, the ROC must contain the unit circle, which means that all poles must be inside the unit circle. This is the same condition as being absolutely summable (aka being in L1 space). For a system, this also means the system is BIBO stable. When this occurs, the FT will converge pointwise.

The FT also exists for signals that have mean square convergence (aka being in L2 space). This is a weaker condition than being absolutely integrable. In both cases, these signals are energy signals. For the L2 case, the FT converges in the mean square sense, meaning as the integration limits go to infinity, the mean-square error of the FT goes to zero. This is the reason for the Gibbs phenomenon.

* Time-domain sinc, freq-domain brick wall – what is the ZT?

Formally, the FT exists only for energy signals.

For energy signals, the square of the FT is the energy spectral density (ESD), which has units of or or . Integrating over the ESD yields the total energy of the signal, which is equal to integrating over the square of the signal over all time (Parseval’s theorem).

If there are poles on the unit circle, the FT may still exist – in the limit.

Periodic signals are periodic only if , where and are integers. is the fundamental period. Then can use DFS. DFS consists of frequencies at harmonics of .

For power signals, the square of the FT is the power spectral density (PSD), which has units of or . Integrating over the PSD yields the total power of the signal. The PSD is equal to the ESD taken to the limit (I think).

If a stable, causal, LTI causal can be represented as a LCCDE, then its pole-zero geometry determines the frequency response. Assume the transfer function is defined by a rational fraction of polynomials in and a real gain. If all poles and zeros are either real or occur in complex conjugate pairs, then

* The polynomial coefficients are real
* The frequency response is conjugate symmetric
* The impulse response is real
* The complex conjugate pairs have complex conjugate residues when using PFE

DFT:

Remember, the “underlying” signal is periodic with period , so that’s why it’s circular time shift, frequency shift, and convolution.

You can implement freq domain filtering. Make sure circular convolution is equal to linear convolution. FFT/IFFT.

LCCDE: number of poles equals number of zeros; poles and zeros can be at 0 or infinity to fill this out

# Transform properties

Important transform properties, continuous time and discrete time:

Linearity

Time shift

Symmetry

**Parseval’s theorem:**

For energy signals, the square of the FT is the energy spectral density (ESD), which has units of or or . Integrating over the ESD yields the total energy of the signal, which is equal to integrating over the square of the signal over all time (Parseval’s theorem).

For power signals, the square of the FT is the power spectral density (PSD), which has units of or . Integrating over the PSD yields the total power of the signal. The PSD is equal to the ESD taken to the limit.

# Transform pairs

Important transform pairs, continuous time and discrete time, poles, zeros, etc.

**Time domain impulse:**

**Delayed time domain impulse (assume :**

**Complex sinusoid:**

Sine:

Time domain impulse train:

Time domain sinc:

Rectangular pulse

Triangular pulse

Decaying exponential

Unit step

<https://en.wikipedia.org/wiki/Laplace_transform#Table_of_selected_Laplace_transforms>

<https://en.wikipedia.org/wiki/Z-transform#Table_of_common_Z-transform_pairs>