There are many questions still left to be answered:

* Are LCCDE inherently LTI?
* Is LTI system required for all of these properties?
* Can all LTI systems be described by LCCDE?
* LTI systems not described by LCCDE
* When can you use LCCDE?
* Why do we use unilateral LT but bilateral ZT?

Summarize the most important points from memory.

In the frequency domain, each frequency component is defined by frequency, amplitude, and phase. When analyzing signals all at the same frequency, you can use phasor analysis, which is vector arithmetic using amplitude and phase. That is, signals are represented as vectors, and you can add/subtract them accordingly.

PSD vs. ESD – notice that Fourier series is averaged over the period. Fourier transform is not because it is energy.

Every integration/summation over one period is averaged

Adding powers, orthogonality in freq domain

Integrating powers/energy – Parseval’s or Plancherel’s theorem

System must be initially at rest because . If there is no input, then output must be zero.

Continuous time

Fourier series

Fourier transform

Laplace transform

Typically, we use unilateral LT, not bilateral, so we analyze causal signals and systems. I think this is also a requirement for a system described by LCCDE to be LTI.

For causal signals and systems, the ROC is of the form , where is the most positive pole. For FT to exist, the ROC must contain the axis, which means that all poles must be in the LHP. This is the same condition as being absolutely integrable (aka being in L1 space). For a system, this also means the system is BIBO stable. When this occurs, the FT will converge pointwise.

The FT also exists for signals that have mean square convergence (aka being in L2 space). This is a weaker condition than being absolutely integrable. In both cases, these signals are energy signals. For the L2 case, the FT converges in the mean square sense, meaning as the integration limits go to infinity, the mean-square error of the FT goes to zero. This is the reason for the Gibbs phenomenon.

* Time-domain sinc, freq-domain brick wall – does this have a LT? I don’t think this is realizable. Does LT even exist?

Formally, the FT exists only for energy signals.

For energy signals, the square of the FT is the energy spectral density (ESD), which has units of or or . Integrating over the ESD yields the total energy of the signal, which is equal to integrating over the square of the signal over all time (Parseval’s theorem).

If there are poles on the axis, the FT may still exist – in the limit. This allows us to take the FT of periodic power signals or a constant signal, which in the time domain is impulses at harmonics of the fundamental frequency scaled by the FS coefficients (alternatively, it is a sampled version of the FT of one period of the signal).

For power signals, the square of the FT is the power spectral density (PSD), which has units of or . Integrating over the PSD yields the total power of the signal.

Periodic power signals, PSD

Energy signals, ESD

FT in the limit

Poles and zeros and frequency response

Real signal/system = only real poles or complex conjugate poles

For real coefficients, poles and zeros are either real or occur in complex conjugate pairs.

Residues for complex conjugate poles must be complex conjugates.

LCC differential equations

With initial conditions

Sampling

Discrete time

Fourier transform

Discrete Fourier series

Discrete Fourier transform

Remember, the “underlying” signal is periodic with period , so that’s why it’s circular time shift, frequency shift, and convolution.

You can implement freq domain filtering. Make sure circular convolution is equal to linear convolution. FFT/IFFT.