There are many questions still left to be answered:

* Simple question: why is it ? Why is it imaginary? Where does that come from? Is it because you decompose into complex sinusoids (that may add up to be real)?
* Are LCCDE inherently LTI?
* Is LTI system required for all of these properties?
* Can all LTI systems be described by LCCDE?
* LTI systems not described by LCCDE
* When can you use LCCDE?
* Why do we use unilateral LT but bilateral ZT?
* I think we typically talk about PSD, which is applicable to stationary processes. Even if a signal is time-limited, we assume it extends to infinity.
* DTFT in the limit
* I think all impulses – certainly for continuous time – need to be taken in the limit. E.g. to take the inverse CTFT of 1 to get dirac delta in time, or to take the CTFT of a constant or a sinusoid. It’s duality.

Random notes:

* PSD vs. ESD – notice that Fourier series is averaged over the period. Fourier transform is not because it is energy.
* Every integration/summation over one period is averaged
* Adding powers, orthogonality in freq domain
* Integrating powers/energy – Parseval’s or Plancherel’s theorem
* System must be initially at rest because . If there is no input, then output must be zero.

Summarize the most important points from memory.

System properties

Linearity

Time invariance

Causality

Stability

In the frequency domain, each frequency component is defined by frequency, amplitude, and phase. When analyzing signals all at the same frequency, you can use phasor analysis, which is vector arithmetic using amplitude and phase. That is, signals are represented as vectors, and you can add/subtract them accordingly.

Continuous time

Fourier series

Fourier transform

Laplace transform

Typically, we use unilateral LT, not bilateral, so we analyze causal signals and systems. I think this is also a requirement for a system described by LCCDE to be LTI.

For causal signals and systems, the ROC is of the form , where is the most positive pole. For FT to exist, the ROC must contain the axis, which means that all poles must be in the LHP. This is the same condition as being absolutely integrable (aka being in L1 space). For a system, this also means the system is BIBO stable. When this occurs, the FT will converge pointwise.

The FT also exists for signals that have mean square convergence (aka being in L2 space). This is a weaker condition than being absolutely integrable. In both cases, these signals are energy signals. For the L2 case, the FT converges in the mean square sense, meaning as the integration limits go to infinity, the mean-square error of the FT goes to zero. This is the reason for the Gibbs phenomenon.

* Time-domain sinc, freq-domain brick wall – does this have a LT? I don’t think this is realizable. Does LT even exist? I think it should. Perhaps the pole is on the axis. But we use unilateral LT – can we use bilateral LT?

Formally, the FT exists only for energy signals.

For energy signals, the square of the FT is the energy spectral density (ESD), which has units of or or . Integrating over the ESD yields the total energy of the signal, which is equal to integrating over the square of the signal over all time (Parseval’s theorem).

If there are poles on the axis, the FT may still exist – in the limit, which means we take the FT of the signal over a time period . This allows us to take the FT of periodic power signals or a constant signal, which in the time domain is impulses at harmonics of the fundamental frequency scaled by the FS coefficients (alternatively, it is a sampled version of the FT of one period of the signal).

For power signals, the square of the FT is the power spectral density (PSD), which has units of or . Integrating over the PSD yields the total power of the signal. The PSD is equal to the ESD taken to the limit (I think).

If a stable, causal, LTI causal can be represented as a LCCDE, then its pole-zero geometry determines the frequency response. Assume the transfer function is defined by a rational fraction of polynomials in and a real gain. If all poles and zeros are either real or occur in complex conjugate pairs, then

* The polynomial coefficients are real
* The frequency response is conjugate symmetric
* The impulse response is real
* The complex conjugate pairs have complex conjugate residues when using PFE

LCC differential equations

With initial conditions

Sampling

causes to be mod , aka periodic with period .

Discrete time

Fourier transform

z-transform

Discrete Fourier series

Discrete Fourier transform

Many of the properties observed for CT also hold for DT.

For causal signals and systems, the ROC is of the form , where is largest pole magnitude. For FT to exist, the ROC must contain the unit circle, which means that all poles must be inside the unit circle. This is the same condition as being absolutely summable (aka being in L1 space). For a system, this also means the system is BIBO stable. When this occurs, the FT will converge pointwise.

The FT also exists for signals that have mean square convergence (aka being in L2 space). This is a weaker condition than being absolutely integrable. In both cases, these signals are energy signals. For the L2 case, the FT converges in the mean square sense, meaning as the integration limits go to infinity, the mean-square error of the FT goes to zero. This is the reason for the Gibbs phenomenon.

* Time-domain sinc, freq-domain brick wall – what is the ZT?

Formally, the FT exists only for energy signals.

For energy signals, the square of the FT is the energy spectral density (ESD), which has units of or or . Integrating over the ESD yields the total energy of the signal, which is equal to integrating over the square of the signal over all time (Parseval’s theorem).

If there are poles on the unit circle, the FT may still exist – in the limit.

Periodic signals are periodic only if , where and are integers. is the fundamental period. Then can use DFS. DFS consists of frequencies at harmonics of .

For power signals, the square of the FT is the power spectral density (PSD), which has units of or . Integrating over the PSD yields the total power of the signal. The PSD is equal to the ESD taken to the limit (I think).

If a stable, causal, LTI causal can be represented as a LCCDE, then its pole-zero geometry determines the frequency response. Assume the transfer function is defined by a rational fraction of polynomials in and a real gain. If all poles and zeros are either real or occur in complex conjugate pairs, then

* The polynomial coefficients are real
* The frequency response is conjugate symmetric
* The impulse response is real
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DFT:

Remember, the “underlying” signal is periodic with period , so that’s why it’s circular time shift, frequency shift, and convolution.

You can implement freq domain filtering. Make sure circular convolution is equal to linear convolution. FFT/IFFT.

LCCDE: number of poles equals number of zeros; poles and zeros can be at 0 or infinity to fill this out

Important transform properties, continuous time and discrete time:

Linearity

Time shift

Important transform pairs, continuous time and discrete time, poles, zeros, etc.

Time domain impulse:

Delayed impulse (assume

Cosine:

Sine

Sinc

Decaying exponential

Unit step

Rectangular pulse

Triangular pulse

<https://en.wikipedia.org/wiki/Laplace_transform#Table_of_selected_Laplace_transforms>

<https://en.wikipedia.org/wiki/Z-transform#Table_of_common_Z-transform_pairs>