<https://en.wikipedia.org/wiki/Spectral_density>

Spectral density: the statistical average of the energy or power of any type of signal as analyzed in terms of its frequency content

For energy signals, we may compute the energy spectral density (ESD). Power spectral density (PSD or power spectrum) is more commonly used and applies to signals existing over all time (or over a large enough time period).

Energy signal: total energy is finite, or is a square-integrable function

PSD units: or (ignore impedance)

**ESD**

ESD units: or or (ignore impedance)

If , energy, is finite, then we can apply Parseval’s theorem (which also applies to discrete-time signals):

The left side is the total energy of the signal, so we can interpret the right side as a density function multiplied by an infinitesimally small frequency interval, describing the energy contained in the signal in . Therefore **energy spectral density** is defined as

Fourier transform of autocorrelation of x is equal to ESD

Generalizes to discrete-time signal:

**PSD**

ESD is suitable for transients (pulse-like signals) since FT of these generally exist.

For continuous signals over all time, like stationary processes, we have to define the PSD, which describes how the power of a signal is distributed over frequency.

Power of a signal is given by

Or, if we define where is a rectangular pulse of duration ,

When , the integral grows to infinity at least as quickly as , which means .

Under suitable conditions, certain generalizations of the Fourier transform still adhere to Parseval’s theorem, so

As in ESD, is a density function, the **power spectral density**:

Power spectral density has another definition: the Fourier transform of the autocorrelation function.

1. (by definition of FT)

<https://en.wikipedia.org/wiki/Spectral_density_estimation>

# Overview

Spectral density estimation (SDE), aka spectrum analysis or frequency domain analysis, is the process of estimating the PSD of a random signal. It breaks down a time-domain signal into frequency components.

SDE may be performed on the entire signal or on short segments of the signal (sometimes called frames). Segmenting the signal is particularly useful for periodic functions.

The Fourier transform converts from time domain to frequency domain. Each frequency component consists of a magnitude and a phase. When the magnitude is squared and the phase discarded, the resulting function (of frequency) is known as a power spectrum.

DFT – via the FFT algorithm – is used to approximate the Fourier transform. The squared-magnitude frequency components are a type of power spectrum known as a periodogram.

A periodogram does not provide any processing gain when applied to noiselike signals: The variance of the power spectrum at a given frequency does not decrease as the number of samples used in the DFT increases. This can be mitigated by averaging over time (Welch’s method) or over frequency (smoothing).

# Periodogram

1. Let your signal be length
2. Compute the -point DFT of your signal to get , where is the discrete index for frequency components
3. Compute . This is your periodogram.

# Bartlett’s method / method of averaged periodograms

Reduces variance of the periodogram but also reduces resolution.

1. Let your signal be length
2. Split your signal into non-overlapping segments, each of length
3. For each segment, compute the -point periodogram
4. Average the periodograms to obtain the power spectrum

Variance is reduced by , and resolution is reduced by .

# Welch’s method

Builds on Bartlett’s method: Further reduces resolution but improves spectral leakage.

1. Let your signal be length
2. Split your signal into segments of length , with each segment overlapping adjacent segments by points
   1. : overlap is 50%
   2. : overlap is 0%, as in Bartlett’s method
3. Apply a time-domain window to each segment. Since the windows roll off at either end, overlapping segments helps mitigate the loss of information.
4. For each windowed segment, compute the -point periodogram
5. Average the periodograms to obtain the power spectrum

Compared to the rectangular windowing in Bartlett’s method, proper windowing using Hanning/Hamming/Blackman-Harris/Kaiser reduces resolution but improves spectral leakage.

# PSD units and integrated power

All SDE methods use the magnitude of the DFT squared to estimate PSD. For example,

If the units of are (Volts), then the units of are . **This is not** . That is, if you double your DFT size and integrate over the signal bandwidth, you will not get the same power. Roughly speaking,

This is because the DFT is simply a sampled version of the DTFT: Unlike a spectrum analyzer, it is not estimating the power of the signal per frequency bin / resolution bandwidth.

# Signal.welch – difference between ‘density’ and ‘spectrum’ argument