

MatricesWorksheet.R

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```
# Problem 1
```

```
# a. Check if  $A^3 = 0$ 
```

```
A <- matrix(data = c(1,5,-2,1,2,-1,3,6,-3), nrow = 3, ncol = 3)
```

```
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    3
## [2,]    5    2    6
## [3,]   -2   -1   -3
```

```
A%*%A%*%A
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    0
## [2,]    0    0    0
## [3,]    0    0    0
```

```
# b. Replace the third column of A by the sum of the second and third columns.
```

```
A[,3] <- A[,2]+A[,3]
```

```
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    4
## [2,]    5    2    8
## [3,]   -2   -1   -4
```

```
# Problem 2. Create Matrix B and calculate  $B^TB$ 
```

```
B <- matrix(data=c(10,-10,10), byrow = TRUE, nrow = 15, ncol = 3)
```

```
Prod <- t(B)%*%B
```

```
Prod
```

```
##      [,1] [,2] [,3]
## [1,] 1500 -1500 1500
## [2,] -1500 1500 -1500
## [3,] 1500 -1500 1500
```

```
# Problem 3. Create Matrix matE with all entries equal to 0
```

```
matE <- matrix(0, nrow = 6, ncol = 6)
```

```
row(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    1    1    1    1    1
## [2,]    2    2    2    2    2    2
## [3,]    3    3    3    3    3    3
## [4,]    4    4    4    4    4    4
## [5,]    5    5    5    5    5    5
## [6,]    6    6    6    6    6    6
```

```
col(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    2    3    4    5    6
```

```
## [2,] 1 2 3 4 5 6
## [3,] 1 2 3 4 5 6
## [4,] 1 2 3 4 5 6
## [5,] 1 2 3 4 5 6
## [6,] 1 2 3 4 5 6
```

```
(abs(row(matE)-col(matE))==1)*1
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,] 0 1 0 0 0 0
## [2,] 1 0 1 0 0 0
## [3,] 0 1 0 1 0 0
## [4,] 0 0 1 0 1 0
## [5,] 0 0 0 1 0 1
## [6,] 0 0 0 0 1 0
```

Problem 4. Use function outer to create a patterned matrix

```
s <- 0:4
outer(s,s,"+")
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,] 0 1 2 3 4
## [2,] 1 2 3 4 5
## [3,] 2 3 4 5 6
## [4,] 3 4 5 6 7
## [5,] 4 5 6 7 8
```

Problem 5. Create large patterned matrices

#a.

```
s <- 0:4
outer(s,s,"+")%%5
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,] 0 1 2 3 4
## [2,] 1 2 3 4 0
## [3,] 2 3 4 0 1
## [4,] 3 4 0 1 2
## [5,] 4 0 1 2 3
```

#b.

```
a <- 0:9
outer(a,a,"+")%%10
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## [1,] 0 1 2 3 4 5 6 7 8 9
## [2,] 1 2 3 4 5 6 7 8 9 0
## [3,] 2 3 4 5 6 7 8 9 0 1
## [4,] 3 4 5 6 7 8 9 0 1 2
## [5,] 4 5 6 7 8 9 0 1 2 3
## [6,] 5 6 7 8 9 0 1 2 3 4
## [7,] 6 7 8 9 0 1 2 3 4 5
## [8,] 7 8 9 0 1 2 3 4 5 6
## [9,] 8 9 0 1 2 3 4 5 6 7
## [10,] 9 0 1 2 3 4 5 6 7 8
```

#c.

```
outer(0:8,0:8,"-")%%9
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
## [1,]    0    8    7    6    5    4    3    2    1
## [2,]    1    0    8    7    6    5    4    3    2
## [3,]    2    1    0    8    7    6    5    4    3
## [4,]    3    2    1    0    8    7    6    5    4
## [5,]    4    3    2    1    0    8    7    6    5
## [6,]    5    4    3    2    1    0    8    7    6
## [7,]    6    5    4    3    2    1    0    8    7
## [8,]    7    6    5    4    3    2    1    0    8
## [9,]    8    7    6    5    4    3    2    1    0
```

```
# Problem 6. Solve system of linear equations
y <- matrix(c(7,-1,-3,5,17),c(5,1), byrow = TRUE)
A <- matrix(0,nrow = 5,ncol = 5)
A <- abs(row(A)-col(A))+1
x <- solve(A,y)
x
```

```
##      [,1]
## [1,]  -2
## [2,]   3
## [3,]   5
## [4,]   2
## [5,]  -4
```

```
# Problem 7. Create Matrix of random intergers from from 1,2,...,10. Matrix is 6 x 10.
set.seed(75)
aMat <- matrix( sample(10, size=60, replace=T), nr=6)
aMat
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## [1,]    3    6    7    7    2    4    3    7    1    4
## [2,]    1    9    8    7    2    6   10    9    5    2
## [3,]    7   10    8    4   10    5    4    8    4    4
## [4,]    4    3    1    1    3    3    9    7    4    2
## [5,]    1    8    1    9    9    8    1    3    7    7
## [6,]    2    6    7    5    6   10    4    6   10    1
```

```
# a. Find numbers of entities in each row which are greater than 4.
cond <- (aMat>4) *1
rowSums(cond)
```

```
## [1] 4 7 6 2 6 7
```

```
# b. Which rows contain exactly two occurrences of the number seven?
cond <- (aMat==7) *1
x <- rowSums(cond)
which(x==2)
```

```
## [1] 5
```

```
# c. Find those pairs of columns whose total (over both columns) is greater than 75.
ColSumsaMat <- colSums(aMat)
ColSumsaMat
```

```
## [1] 18 42 32 33 32 36 31 40 31 20
```

```
t <- outer(ColSumsaMat,ColSumsaMat, '+') #forms matrix of two column sums
t1 <- t>75 # filter for pairs whose sum >75 (Logicals)
```

```
t1 <- t1*1 # filtered as Binaries
t1 <- upper.tri(t1,diag=TRUE)*1*t1 #eliminate the repeated answers in the lower triangle
t1 <- upper.tri(t1,diag=TRUE)&t1 #eliminates repeated answers in logical matrix using logical AND operation
t1
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## [1,] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [2,] FALSE  TRUE FALSE FALSE FALSE  TRUE FALSE  TRUE FALSE FALSE
## [3,] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [4,] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [5,] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [6,] FALSE FALSE FALSE FALSE FALSE FALSE FALSE  TRUE FALSE FALSE
## [7,] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [8,] FALSE FALSE FALSE FALSE FALSE FALSE FALSE  TRUE FALSE FALSE
## [9,] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## [10,] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
```

```
which(t1,arr.ind=TRUE, useNames = TRUE)
```

```
##      row col
## [1,]    2  2
## [2,]    2  6
## [3,]    2  8
## [4,]    6  8
## [5,]    8  8
```

```
# Problem 8. Calculate
```

```
options(digits = 2)
# a. Need to separate the sums: sum[(i=1:20) i^4 * sum[(i=1:5) (i+3)^-1]] #calculate the inner sum first
s1 <- sum(1/4:8)
s1 * sum((1:20)^4) #multiple previous sum by the outer sum
```

```
## [1] 639215
```

```
# b. Need to make a matrix by forming 2 vectors and using outer(). Make the denominator first.
outer(1:20,1:5,"*") # matrix of ij products
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    2    3    4    5
## [2,]    2    4    6    8   10
## [3,]    3    6    9   12   15
## [4,]    4    8   12   16   20
## [5,]    5   10   15   20   25
## [6,]    6   12   18   24   30
## [7,]    7   14   21   28   35
## [8,]    8   16   24   32   40
## [9,]    9   18   27   36   45
## [10,]   10   20   30   40   50
## [11,]   11   22   33   44   55
## [12,]   12   24   36   48   60
## [13,]   13   26   39   52   65
## [14,]   14   28   42   56   70
## [15,]   15   30   45   60   75
## [16,]   16   32   48   64   80
## [17,]   17   34   51   68   85
```

```
## [18,] 18 36 54 72 90
## [19,] 19 38 57 76 95
## [20,] 20 40 60 80 100
```

```
denom <- outer(1:20,1:5,"*") +3
denom
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,] 4    5    6    7    8
## [2,] 5    7    9   11   13
## [3,] 6    9   12   15   18
## [4,] 7   11   15   19   23
## [5,] 8   13   18   23   28
## [6,] 9   15   21   27   33
## [7,] 10  17   24   31   38
## [8,] 11  19   27   35   43
## [9,] 12  21   30   39   48
## [10,] 13 23   33   43   53
## [11,] 14 25   36   47   58
## [12,] 15 27   39   51   63
## [13,] 16 29   42   55   68
## [14,] 17 31   45   59   73
## [15,] 18 33   48   63   78
## [16,] 19 35   51   67   83
## [17,] 20 37   54   71   88
## [18,] 21 39   57   75   93
## [19,] 22 41   60   79   98
## [20,] 23 43   63   83  103
```

```
numer <- (1:20)^4
numer
```

```
## [1] 1 16 81 256 625 1296 2401 4096 6561 10000
## [11] 14641 20736 28561 38416 50625 65536 83521 104976 130321 160000
```

```
sum(numer/denom)
```

```
## [1] 89912
```

```
# c. Similiar to b, need a 10x10 denom matrix and a length=10 numerator
```

```
denom <- outer(1:10, 1:10,"*") +3
```

```
numer <- (1:10)^4
```

```
t <- numer/denom
```

```
t
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
## [1,] 0.25 0.2 0.17 0.14 0.12 0.11 0.10 0.091 0.083
## [2,] 3.20 2.3 1.78 1.45 1.23 1.07 0.94 0.842 0.762
## [3,] 13.50 9.0 6.75 5.40 4.50 3.86 3.38 3.000 2.700
## [4,] 36.57 23.3 17.07 13.47 11.13 9.48 8.26 7.314 6.564
## [5,] 78.12 48.1 34.72 27.17 22.32 18.94 16.45 14.535 13.021
## [6,] 144.00 86.4 61.71 48.00 39.27 33.23 28.80 25.412 22.737
## [7,] 240.10 141.2 100.04 77.45 63.18 53.36 46.17 40.695 36.379
## [8,] 372.36 215.6 151.70 117.03 95.26 80.31 69.42 61.134 54.613
## [9,] 546.75 312.4 218.70 168.23 136.69 115.11 99.41 87.480 78.107
## [10,] 769.23 434.8 303.03 232.56 188.68 158.73 136.99 120.482 107.527
##      [,10]
## [1,] 0.077
```

```
## [2,] 0.696
## [3,] 2.455
## [4,] 5.953
## [5,] 11.792
## [6,] 20.571
## [7,] 32.890
## [8,] 49.349
## [9,] 70.548
## [10,] 97.087
```

```
# sum either upper or lower triangle, try out both, use lower
sum(lower.tri(t,dia=TRUE)*t)
```

```
## [1] 6945
```