THE CIRCLET TRANSFORM

The Circlet Transform (CT) proposed by Chauris et al [1] is a state of the art and robust tool for detecting objects with circular patterns in which binary image segmentation is no longer needed. The following descriptions for the CT are very close to the one presented by Chauris et al [1]. The CT decomposes an image into circles called “circlets” with different radii and a certain width, via a series of Fast Fourier Transforms (FFTs). The decomposition of the CT is formulated in the Fourier domain using special filters.

The circlet components are described by a central position (x_0, y_0), radius r_0 and central frequency content f_0. All circlet components $c_\mu(x, y)$ could be created by a basic circlet $c_{ref}(x, y)$ which can be shifted or be changed in radius or the central frequency. The circlet function can be written as (1) [1]:

$$c_\mu(x, y) = \Omega\left[2\pi f_0 (r - r_0)\right] \tag{1}$$

where $r = \sqrt{(x - x_0)^2 + (y - y_0)^2}$. Ω is considered as a fluctuating function such as a wavelet function which is formulated to reveal discontinuities. From a practical point of view, c_μ is defined in the 2D Fourier domain [1].

The definition of the CT is very similar to Curvelet transform [1]. Indeed an image $f(x, y)$ is decomposed into a sum of basic functions c_μ:

$$f(x, y) = \sum_\mu A_\mu \cdot c_\mu(x, y) \tag{2}$$

In fact, in the CT, basic functions (tight frames) have a circular pattern and the associated amplitudes A_μ can be achieved by (3):

$$A_\mu = \langle f, c_\mu \rangle = \iint f(x, y) \cdot c_\mu(x, y)\, dx\, dy \tag{3}$$

As it is mentioned, from a practical point of view, the circlet coefficients are defined in the Fourier domain, using Parseval’s theorem:

$$A_\mu = \langle \hat{f}, \hat{c}_\mu \rangle = \iint \hat{f}(\omega_1, \omega_2) \cdot \hat{c}_\mu^*(\omega_1, \omega_2)\, d\omega_1 d\omega_2 \tag{4}$$

where $\hat{f}$ denotes the 2D Fourier transform of f, and f^* is the conjugate of f. Since the CT is defined in the 2D Fourier domain, proper filters must be defined for $\hat{c}_\mu^*(\omega_1, \omega_2)$, the Fourier transform of c_μ, such that circular shapes could be achieve for basic functions $c_\mu(x, y)$ [1].

The filters are defined in the Fourier domain and 2D filters G_k are constructed by the 1D filters F_k. The F_k filters are defined as (5):

$$F_k(\omega) = \begin{cases} \cos(\omega \pm \omega_k) & , \ |\omega \pm \omega_k| \le \pi/(N-1) \\ 0 & othgerwise \end{cases} \tag{5}$$

where N is the number of filters and $\omega_k = \pi(k-1)/(N-1)$. By considering a phase delay in order to have circular shape in the spatial domain, the G_k filters are defined as (6):

$$G_k(\omega_1, \omega_2) = e^{i|\omega| r_0} \cdot F_k(|\omega|) \tag{6}$$

where $\omega = (\omega_1, \omega_2)$ and $|\omega| = \sqrt{\omega_1^2 + \omega_2^2}$. By the definition of the filters G_k, the formulation of a circlet in the Fourier domain will be as (7):

$$\hat{c}_\mu(\omega) = e^{i \langle \omega, x_c \rangle} \cdot G_k(\omega) \tag{7}$$

where $x_c = (x_0, y_0)$ is the central position and r_0 is the radius of the circlet. With utilizing polar coordinates, it is shown that the 2D inverse Fourier Transform of G_k is circular which indicates that the basic functions $c_\mu(x, y)$ have circular shapes. More details about the CT, the application of CT and provided formulations in this note can be found in [2].

[1] H. Chauris, I. Karoui, P. Garreau, H. Wackernagel, P. Craneguy, and L. Bertino, "The circlet transform: A robust tool for detecting features with circular shapes," *Computers & Geosciences*, vol. 37, pp. 331-342, 2011.
[2] O. Sarrafzadeh, A. Mehri, H. Rabbani, N. Ghane, A. Talebi, "Circlet based framework for red blood cells segmentation and counting", in Proc. IEEE Workshop on Signal Processing Systems**,** China, Oct. 14-16, 2015.