# A Proofs for Section 3

**Proposition 4.** Given a satisfaction system $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ and set of models $\mathbb{M} \subseteq \mathfrak{M}$, if $\mathcal{M} \subseteq \mathrm{MaxFRSubs}(\mathbb{M}, \Lambda)$ and $|\mathcal{M}| \geq 2$ then **not necessarily** $(\bigcap_{\mathbb{M} \in \mathcal{M}} \mathbb{M}) \in \mathrm{FR}(\Lambda)$.

*Proof.* As an example, we consider the satisfaction $\Lambda_t = (\mathcal{L}_t, \mathfrak{M}_t, \models_t)$ where: $\mathcal{L}_t = \{a, b\}$ with $a, b$ being propositional atoms; $\mathfrak{M}_t$ the boolean valuations (T for 'true' and F for 'false') to the pair $(a, b)$, and the satisfaction relation $\models_t$ defined as usual.

We have that

$$\mathrm{MaxFRSubs}(\{(\mathrm{T},\mathrm{T}), (\mathrm{T},\mathrm{F}), (\mathrm{F},\mathrm{T})\}, \Lambda_\wedge) = \{\{(\mathrm{F},\mathrm{T}), (\mathrm{T},\mathrm{T})\}, \{(\mathrm{T},\mathrm{F}), (\mathrm{T},\mathrm{T})\}\}.$$

The intersection of the resulting subsets is $\{(\mathrm{T},\mathrm{T})\}$, which cannot be represented in $\Lambda_t$. $\square$

**Proposition 9.** Given a satisfaction system $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ and set of models $\mathbb{M} \subseteq \mathfrak{M}$, if $\mathcal{M} \subseteq \mathrm{MinFRSups}(\mathbb{M}, \Lambda)$ and $|\mathcal{M}| \geq 2$ then **not necessarily** $(\bigcup_{\mathbb{M} \in \mathcal{M}} \mathbb{M}) \in \mathrm{FR}(\Lambda)$.

*Proof.* As an example, we consider the satisfaction $\Lambda_p = (\mathcal{L}_p, \mathfrak{M}_p, \models_p)$ where: $\mathcal{L}_p = \{\bot, a, b\}$ with $a, b$ being propositional atoms and $\bot$ *falsum*; $\mathfrak{M}_p$ the boolean valuations (T for 'true' and F for 'false') to the pair $(a, b)$, and the satisfaction relation $\models_p$ defined as usual.

We have that

$$\mathrm{MinFRSups}(\{(\mathrm{T},\mathrm{T})\}, \Lambda_p) = \{\{(\mathrm{T},\mathrm{T}), (\mathrm{T},\mathrm{F})\}, \{(\mathrm{T},\mathrm{T}), (\mathrm{F},\mathrm{T})\}\}.$$

The union of the resulting supersets is the set $\{\{(\mathrm{T},\mathrm{T})(\mathrm{T},\mathrm{F}), (\mathrm{F},\mathrm{T})\}\}$ , which cannot be represented in $\Lambda_p$, as we cannot express disjunction. $\square$

To prove the representation theorem for eviction, we will need some auxiliary tools. Recall that we write $\mathcal{P}^*(A)$ as a shorthand for $\mathcal{P}(A) \setminus \{\emptyset\}$, that is, the power set of $A$ without the empty set $\emptyset$. Given an eviction function $\mathrm{evc}$ on an eviction-compatible satisfaction system $\Lambda$, we define the function $\xi^- : \mathcal{P}^*(\mathrm{FR}(\Lambda)) \to \mathcal{P}_{\mathrm{f}}(\mathcal{L}) \times \mathcal{P}(\mathfrak{M})$, such that

$$\xi^-(X) = \{(\mathcal{B}, \mathbb{M}) \mid \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda) = X\}.$$

Intuitively, $\xi^-(X)$ holds all the pairs $(\mathcal{B}, \mathbb{M})$ such that $X$ contains exactly all finite representable sets of models closest to $\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}$. We also define the function $\mathcal{C}^- : \mathcal{P}^*(\mathrm{FR}(\Lambda)) \to \mathcal{P}(\mathfrak{M})$ such that

$$\mathcal{C}^-(X) = \{\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \mid (\mathcal{B}, \mathbb{M}) \in \xi^-(X)\}.$$

**Lemma A.1.** Let $\Lambda$ be an eviction-compatible satisfaction system. If a model change operation $\mathrm{evc}$ satisfies *uniformity* then for all $X \in \mathcal{P}(\mathrm{FR}(\Lambda))$:

(i) $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{evc}(\mathcal{B}', \mathbb{M}'))$ for all $(\mathcal{B}, \mathbb{M}), (\mathcal{B}', \mathbb{M}') \in \xi^-(X)$;

(ii) $\mathcal{C}^-(X)$ is a singleton, if $\xi^-(X) \neq \emptyset$.

*Proof.* Let $\mathrm{evc}$ be a model change operation satisfying *uniformity*, and $X \in \mathrm{FR}(\Lambda)$, where $\Lambda$ is an eviction-compatible satisfaction system.

(i) Let $(\mathcal{B}, \mathbb{M}), (\mathcal{B}', \mathbb{M}') \in \xi^-(X)$. Thus, by definition of $\xi^-$, we have that:

$$\begin{aligned} X &= \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda) \\ &= \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}') \setminus \mathbb{M}', \Lambda). \end{aligned}$$

Hence, from *uniformity*, we get

$$\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{evc}(\mathcal{B}', \mathbb{M}')).$$

(ii) Let us suppose that $\xi^-(X) \neq \emptyset$. Let us fix such a $(\mathcal{B}, \mathbb{M}) \in \xi^-(X)$. By definition of $\xi^-$, we have that

$$X = \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda).$$

By definition of $\mathcal{C}^-$:

$$\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \in \mathcal{C}^-(X).$$

Hence, to show that $\mathcal{C}^-(X)$ is a singleton, we need to prove that: $Y = \mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M}))$, for all $Y \in \mathcal{C}^-(X)$. Let $Y \in \mathcal{C}^-(X)$. By definition of $\mathcal{C}^-$, we have that for some $(\mathcal{B}', \mathbb{M}') \in \xi^-(X)$ it holds that $Y = \mathrm{Mod}(\mathrm{evc}(\mathcal{B}', \mathbb{M}'))$. Thus, as both pairs $(\mathcal{B}, \mathbb{M}), (\mathcal{B}', \mathbb{M}') \in \xi^-(X)$, we get from item (i) above that $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{evc}(\mathcal{B}', \mathbb{M}'))$. Therefore, $Y = \mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M}))$. This concludes the proof. $\square$

**Proposition A.2.** Let $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ be an eviction-compatible satisfaction system. If a model change operation $\mathrm{evc}$ satisfies *success, inclusion* and *finite retainment*, then $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \in \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda)$ for all $\mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\mathcal{L})$ and $\mathbb{M} \subseteq \mathfrak{M}$.

*Proof.* Let us suppose for contradiction that there is a model change operation that satisfies *success, inclusion* and *finite retainment*, but $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \notin \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda)$, for some finite base $\mathcal{B}$ and set of models $\mathbb{M}$. Let us fix such a base $\mathcal{B}$ and set $\mathbb{M}$.

From *success* and *inclusion*, we have that

$$\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \subseteq \mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}.$$

By construction, $\mathrm{evc}(\mathcal{B}, \mathbb{M})$ is a finite base, which means

$$\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \in \mathrm{FR}(\Lambda). \tag{1}$$

We know that $\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda) \neq \emptyset$ as $\Lambda$ is eviction-compatible. Let

$$Y = \{X \in \mathrm{FR}(\Lambda) \mid X \subseteq (\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M})\}.$$

$\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \notin \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda)$ from hypothesis, which means that either $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \notin \mathrm{FR}(\Lambda)$ or $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M}))$ is not $\subseteq$-maximal within $Y$. This fact combined with Equation (1) implies that $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M}))$ is not $\subseteq$-maximal within $Y$. Therefore, there is some $\mathbb{M}' \in \mathrm{FR}(\Lambda)$ such that $\mathbb{M}' \subseteq (\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M})$ and $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \subset \mathbb{M}'$. But *finite retainment* states that $\mathbb{M}' \notin \mathrm{FR}(\Lambda)$, which is a contradiction. $\square$

**Theorem 5.** A model change operation $\mathrm{evc}$, defined on an eviction-compatible satisfaction system $\Lambda$, is a maxichoice eviction function iff it satisfies the following postulates:

**(success)** $\mathbb{M} \cap \mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \emptyset$.
**(inclusion)** $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \subseteq \mathrm{Mod}(\mathcal{B})$.
**(vacuity)** If $\mathbb{M} \cap \mathrm{Mod}(\mathcal{B}) = \emptyset$, then $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathcal{B})$.
**(finite retainment)** If $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \subset \mathbb{M}' \subseteq \mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}$ then $\mathbb{M}' \notin \mathrm{FR}(\Lambda)$.
**(uniformity)** If $\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda) = \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}') \setminus \mathbb{M}', \Lambda)$ then $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{evc}(\mathcal{B}', \mathbb{M}'))$.

*Proof.* "$\Rightarrow$" Let $\mathrm{evc}_{\mathrm{sel}}$ be a maxichoice eviction function over $\Lambda$ based on a FR selection function sel, and $\mathbb{M}$ be a set of models.

The function $\mathrm{evc}_{\mathrm{sel}}$ satisfies success and inclusion since $\mathrm{Mod}(\mathrm{evc}_{\mathrm{sel}}(\mathcal{B}, \mathbb{M})) \subseteq \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda)$.

**(vacuity)** Assume that $\mathbb{M} \cap \mathrm{Mod}(\mathcal{B}) = \emptyset$. Then, $\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda) = \{\{\mathrm{Mod}(\mathcal{B})\}\}$, which implies that

$$\mathrm{evc}(\mathcal{B}, \mathbb{M}) = \mathrm{sel}(\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda)) = \mathcal{B}',$$

such that $\mathcal{B}' = \mathrm{Mod}(\mathcal{B})$. Therefore, $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathcal{B})$.
**(finite retainment)** Suppose that $M' \in \mathrm{Mod}(\mathcal{B}) \setminus \mathrm{Mod}(\mathrm{evc}_{\mathrm{sel}}(\mathcal{B}, \mathbb{M}))$, then, by construction, there is no $\mathbb{M}' \in \mathrm{FR}(\Lambda)$ which contains $M'$ and $\mathrm{Mod}(\mathrm{evc}_{\mathrm{sel}}(\mathcal{B}, \mathbb{M})) \subset \mathbb{M}'$.
**(uniformity)** Let $\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda) = \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}') \setminus \mathbb{M}', \Lambda)$. By definition,

$$\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \mathrm{sel}(\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda))$$

and

$$\mathrm{Mod}(\mathrm{evc}(\mathcal{B}', \mathbb{M}')) = \mathrm{sel}(\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}') \setminus \mathbb{M}', \Lambda)).$$

Therefore, as $\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda) = \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}') \setminus \mathbb{M}', \Lambda)$, we can conclude that $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{evc}(\mathcal{B}', \mathbb{M}'))$.

Hence, every maxichoice eviction function based on a FR selection function satisfies all postulates stated.

"$\Leftarrow$" Let $\mathrm{evc} : \mathcal{P}_{\mathrm{f}}(\mathcal{L}) \times \mathcal{P}(\mathfrak{M}) \to \mathcal{P}_{\mathrm{f}}(\mathcal{L})$ be a function satisfying the postulates stated. As evc satisfies uniformity, we known from Lemma A.1 that $\mathcal{C}^-(X)$ is a singleton for every $X \in \mathcal{P}(\mathrm{FR}(\Lambda))$. Thus, we can construct the function $\mathrm{sel} : \mathcal{P}^*(\mathrm{FR}(\Lambda)) \to \mathrm{FR}(\Lambda)$ such that

$$\mathrm{sel}(X) = \begin{cases} Z \text{ s.t. } \mathcal{C}^-(X) = \{Z\} & \text{if } \xi^-(X) \neq \emptyset, \\ Y \text{ s.t. } Y \in X & \text{otherwise.} \end{cases}$$

We will prove that: (i) sel is indeed a FR selection function, and (ii) that $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \mathrm{sel}(\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda))$.

(i) sel *is indeed a selection function.* Let $X \in \mathcal{P}^*(\mathrm{FR}(\Lambda))$. We only need to show that $\mathrm{sel}(X) \in X$. The case that $\xi^-(X) = \emptyset$ is trivial, as sel chooses an arbitrary $Y \in X$ (by the axiom of choice). Let us focus on the case $\xi^-(X) \neq \emptyset$. From above, we have that $\mathrm{sel}(X) = Z$, where $\mathcal{C}^-(X) = \{Z\}$. By definition of $\mathcal{C}^-$, we have that there is a pair $(\mathcal{B}, \mathbb{M}) \in \xi^-(X)$ such that

$$Z = \mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})).$$

Let us fix such a $(\mathcal{B}, \mathbb{M}) \in \xi^-(X)$. Thus, by definition of $\xi^-$, we get

$$X = \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda).$$

Additionally, we know that $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \in \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda)$ as a consequence of Proposition A.2. Thus, from the identities above we get that $Z \in X$, which means $\mathrm{sel}(X) \in X$.

(ii) $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \mathrm{sel}(\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda))$. Let $X = \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda)$. We know that $X \neq \emptyset$ due to eviction-compatibility. By definition of $\xi$, we get that

$$(\mathcal{B}, \mathbb{M}) \in \xi^-(X).$$

By construction, we have that $\mathrm{sel}(X) = Z$ such that $\mathcal{C}^-(X) = \{Z\}$, which implies from definition of $\mathcal{C}^-$ that $Z = \mathrm{Mod}(\mathrm{evc}(\mathcal{B}', \mathbb{M}'))$, for some $(\mathcal{B}', \mathbb{M}') \in \xi^-(X)$. Therefore, as $(\mathcal{B}, \mathbb{M}) \in \xi^-(X)$, we get from Lemma A.1, that for all $(\mathcal{B}', \mathbb{M}') \in \xi^-(X)$,

$$\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{evc}(\mathcal{B}', \mathbb{M}')).$$

Thus, $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = Z$. As $\mathrm{sel}(X) = Z$ and $X = \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda)$, we have that $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M}))$ is equal to $\mathrm{sel}(\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda))$. $\square$

**Proposition 6.** If a model change operation evc satisfies *inclusion* and *finite retainment*, then it satisfies *vacuity*.

*Proof.* Assume that evc satisfies finite retainment and inclusion, and that $\mathrm{Mod}(\mathcal{B}) \cap \mathbb{M} = \emptyset$. This means that $\mathrm{Mod}(\mathcal{B})$ is the closest finite representable set of models disjoint with $\mathbb{M}$. From inclusion, $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \subseteq \mathrm{Mod}(\mathcal{B})$. Thus, from finite retainment, we get that $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathcal{B})$. $\square$

To prove the representation theorem for maxichoice reception, we will need some auxiliary tools. The auxiliary tools are analagous to the ones defined for the representation theorem of eviction. Given a reception function rcp on a reception-compatible satisfaction system $\Lambda$, we define the function $\xi^+ : \mathcal{P}^*(\mathrm{FR}(\Lambda)) \to \mathcal{P}_{\mathrm{f}}(\mathcal{L}) \times \mathcal{P}(\mathfrak{M})$ as

$$\xi^+(X) = \{(\mathcal{B}, \mathbb{M}) \mid \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda) = X\}.$$

Intuitively, $\xi^+(X)$ holds all the pairs $(\mathcal{B}, \mathbb{M})$ such that $X$ contains exactly all finite representable sets of models closest to $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}$.

We also define the function $\mathcal{C}^+ : \mathcal{P}^*(\mathrm{FR}(\Lambda)) \to \mathcal{P}(\mathfrak{M})$ as

$$\mathcal{C}^+(X) = \{\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) \mid (\mathcal{B}, \mathbb{M}) \in \xi^+(X)\}.$$

**Lemma A.3.** Let $\Lambda$ be a reception-compatible satisfaction system. If a model change operation rcp satisfies *uniformity* then for all $X \in \mathcal{P}(\mathrm{FR}(\Lambda))$:

(i) $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}', \mathbb{M}'))$ for all $(\mathcal{B}, \mathbb{M}), (\mathcal{B}', \mathbb{M}') \in \xi^+(X)$; and
(ii) $\mathcal{C}^+(X)$ is a singleton, if $\xi^+(X) \neq \emptyset$.

*Proof.* Let $\mathrm{rcp}$ be a model change operation satisfying *uniformity*, and $X \in \mathrm{FR}(\Lambda)$, where $\Lambda$ is a reception compatible satisfaction system.

(i) Let $(\mathcal{B}, \mathbb{M}), (\mathcal{B}', \mathbb{M}') \in \xi^+(X)$. Thus by definition of $\xi^+$, we have that

$$\begin{aligned} X &= \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda) \\ &= \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}') \cup \mathbb{M}', \Lambda). \end{aligned}$$

Thus, from *uniformity*, we get

$$\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}', \mathbb{M}')).$$

(ii) Let us suppose that $\xi^+(X) \neq \emptyset$. Then, there is some $(\mathcal{B}, \mathbb{M}) \in \xi^+(X)$. Let us fix such a $(\mathcal{B}, \mathbb{M})$. By definition of $\xi^+$, we have that

$$X = \mathrm{MaxFRSubs}(\mathcal{B} \cup \mathbb{M}, \Lambda)$$

By definition of $\mathcal{C}^+$,

$$\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) \in \mathcal{C}^+(X)$$

Thus, to show $\mathcal{C}^+(X)$ is a singleton, we need to show that for all $Y \in \mathcal{C}^+(X)$, $Y = \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$. Let $Y \in \mathcal{C}^+(X)$. By definition of $\mathcal{C}^+$, we have that

$$Y = \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}', \mathbb{M}')), \text{ for some } (\mathcal{B}', \mathbb{M}') \in \xi^+(X)$$

Thus, as both pairs $(\mathcal{B}, \mathbb{M}), (\mathcal{B}', \mathbb{M}') \in \xi^+(X)$, we get from item (i) above that $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}', \mathbb{M}'))$. Thus, $Y = \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$. This concludes the proof. $\square$

**Proposition A.4.** Given a reception-compatible satisfaction system $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$. If a model change operation $\mathrm{rcp}$ satisfies *success, persistence* and *finite temperance*, then $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) \in \mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda)$ for all $\mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\Lambda)$ and $\mathbb{M} \subseteq \mathfrak{M}$.

*Proof.* Let us suppose for contradiction that there is a model change operation that satisfies *success, inclusion* and *finite temperance*, but $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) \notin \mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda)$, for some finite base $\mathcal{B}$ and set of models $\mathbb{M}$. Let us fix such a base $\mathcal{B}$ and set $\mathbb{M}$.

From *success* and *inclusion*, we have that

$$\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M} \subseteq \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$$

From construction, $\mathrm{rcp}(\mathcal{B}, \mathbb{M})$ is a finite base, which means

$$\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) \in \mathrm{FR}(\Lambda). \tag{2}$$

As $\Lambda$ is eviction compatible, $\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda) \neq \emptyset$. Let

$$Y = \{X \in \mathrm{FR}(\Lambda) \mid (\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}) \subseteq X\}.$$

We have $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) \notin \mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda)$ from hypothesis, which means that either $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) \notin \mathrm{FR}(\Lambda)$ or $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$ is not $\subseteq$-minimal within $Y$. This fact taken together with Equation (2) implies that $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$ is not $\subseteq$-minimal within $Y$. Therefore, there is some $\mathbb{M}' \in \mathrm{FR}(\Lambda)$ such that

$$(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}) \subseteq \mathbb{M}' \text{ and } \mathbb{M}' \subset \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) \tag{3}$$

Note that $\mathbb{M} \subseteq \mathbb{M}'$ and $\mathbb{M} \subseteq \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, M))$ which implies from Equation (3) above that $\mathbb{M}' \subset \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) \cup \mathbb{M}$. This implies from *finite temperance* that $\mathbb{M}' \notin \mathrm{FR}(\Lambda)$, which is a contradiction. $\square$

**Theorem 10.** A model change operation $\mathrm{rcp}$, defined on a reception-compatible satisfaction system $\Lambda$, is a maxichoice reception function iff it satisfies the following postulates:

**(success)** $\mathbb{M} \subseteq \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$.
**(persistence)** $\mathrm{Mod}(\mathcal{B}) \subseteq \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$.
**(vacuity)** $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathcal{B})$, if $\mathbb{M} \subseteq \mathrm{Mod}(\mathcal{B})$.
**(finite temperance)** If $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M} \subseteq \mathbb{M}' \subset \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$ then $\mathbb{M}' \notin \mathrm{FR}(\Lambda)$.
**(uniformity)** If $\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda) = \mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}') \cup \mathbb{M}', \Lambda)$ then $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}', \mathbb{M}'))$.

*Proof.* "$\Rightarrow$" Let $\mathrm{rcp}_{\mathrm{sel}}$ be a maxichoice reception function over $\Lambda$ based on a FR selection function $\mathrm{sel}$. Success follows directly from the construction of $\mathrm{rcp}$. For persistence, note that $\mathrm{Mod}(\mathrm{rcp}_{\mathrm{sel}}(\mathcal{B}, \mathbb{M})) \in \mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda)$, which implies that $\mathrm{Mod}(\mathcal{B}) \subseteq \mathrm{Mod}(\mathrm{rcp}_{\mathrm{sel}}(\mathcal{B}, \mathbb{M}))$.

**(vacuity)** Assume that $\mathbb{M} \subseteq \mathrm{Mod}(\mathcal{B})$. Thus, $\mathrm{Mod}(\mathcal{B}) = \mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}$. Thus, $\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda) = \{\mathrm{Mod}(\mathcal{B})\}$ which implies that

$$\mathrm{rcp}(\mathcal{B}, \mathbb{M}) = \mathrm{sel}(\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda)) = \mathcal{B}',$$

such that $\mathrm{Mod}(\mathcal{B}') = \mathrm{Mod}(\mathcal{B})$. Thus,

$$\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathcal{B})$$

**(finite temperance)** Suppose that $\mathbb{M}' \notin \mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}$ but $\mathbb{M}' \in \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$, then, by construction, there is no $\mathbb{M} \in \mathrm{FR}(\Lambda)$ which contains $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}$ and $\mathbb{M} \subset \mathrm{Mod}(\mathrm{rcp}_{\mathrm{sel}}(\mathcal{B}, \mathbb{M}))$.

**(uniformity)** Let $\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda) = \mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}') \cup \mathbb{M}', \Lambda)$. By definition,

$$\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = \mathrm{sel}(\mathrm{MinFRSups}(\mathbb{Y}, \Lambda))$$
and
$$\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}', \mathbb{M}')) = \mathrm{sel}(\mathrm{MinFRSups}(\mathbb{Y}', \Lambda)),$$

where $\mathbb{Y} = \mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}$ and $\mathbb{Y}' = \mathrm{Mod}(\mathcal{B}') \cup \mathbb{M}'$. Thus, as $\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda) = \mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}') \cup \mathbb{M}', \Lambda)$, we get that

$$\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}', \mathbb{M}')).$$

Hence, every maxichoice reception function based on a FR selection function satisfies all postulates stated above.

"$\Leftarrow$" Let $\mathrm{rcp} : \mathcal{P}_{\mathrm{f}}(\mathcal{L}) \times \mathcal{P}(\mathfrak{M}) \to \mathcal{P}_{\mathrm{f}}(\mathcal{L})$ be a function satisfying the postulates stated. As $\mathrm{rcp}$ satisfies uniformity, we known from Lemma A.3 that $\mathcal{C}^+(X)$ is a singleton for every $X \in \mathcal{P}(\mathrm{FR}(\Lambda))$. Thus, we can construct the function $\mathrm{sel} : \mathcal{P}^*(\mathrm{FR}(\Lambda)) \to \mathrm{FR}(\Lambda)$ such that

$$\mathrm{sel}(X) = \begin{cases} Z \text{ s.t. } \mathcal{C}^+(X) = \{Z\} & \text{if } \xi^+(X) \neq \emptyset, \\ Y \text{ s.t. } Y \in X & \text{otherwise.} \end{cases}$$

We will prove that: (i) sel is indeed a selection function, and (ii) that $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = \mathrm{sel}(\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda))$.

(i) sel *is indeed a selection function.* Let $X \in \mathcal{P}^*(\mathrm{FR}(\Lambda))$. We only need to show that $\mathrm{sel}(X) \in X$. The case that $\xi^+(X) = \emptyset$ is trivial, as sel chooses an arbitrary $Y \in X$ (by the axiom of choice). Let us focus on the case $\xi^+(X) \neq \emptyset$. From above, we have that $\mathrm{sel}(X) = Z$, where $\mathcal{C}^+(X) = \{Z\}$. By definition of $\mathcal{C}^+$, we have that there is a pair $(\mathcal{B}, \mathbb{M}) \in \xi^+(X)$ such that

$$Z = \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})).$$

Let us fix such a $(\mathcal{B}, \mathbb{M}) \in \xi^+(X)$. Thus, by definition of $\xi^+$, we get

$$X = \mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda).$$

From Proposition A.4, we get that $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) \in \mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda)$. Thus, from the identities above we get that $Z \in X$, which means $\mathrm{sel}(X) \in X$.

(ii) $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = \mathrm{sel}(\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda))$. Let $X = \mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda)$. We know that $X \neq \emptyset$ due to reception-compatibility. By definition of $\xi^+$, we get that

$$(\mathcal{B}, \mathbb{M}) \in \xi^+(X).$$

By construction, we have that $\mathrm{sel}(X) = Z$ such that $\mathcal{C}^+(X) = \{Z\}$, which implies from definition of $\mathcal{C}^+$ that $Z = \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}', \mathbb{M}'))$, for some $(\mathcal{B}', \mathbb{M}') \in \xi^+(X)$. Therefore, as $(\mathcal{B}, \mathbb{M}) \in \xi^+(X)$, we get from Lemma A.3, that for all $(\mathcal{B}', \mathbb{M}') \in \xi^+(X)$,

$$\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}', \mathbb{M}')).$$

Thus, $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = Z$. Thus as $\mathrm{sel}(X) = Z$ and $X = \mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda)$, we have that

$$\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = \mathrm{sel}(\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda)).$$

□

**Proposition 11.** If a model change operation rcp satisfies *persistence* and *finite temperance*, then it satisfies *vacuity*.

*Proof.* Assume that rcp satisfies finite temperance and persistence, and that $\mathbb{M} \subseteq \mathrm{Mod}(\mathcal{B})$. This means that $\mathrm{Mod}(\mathcal{B})$ is the closest finite representable superset of $\mathcal{B}$ containing $\mathbb{M}$. From persistence, $\mathrm{Mod}(\mathcal{B}) \subseteq \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$. Thus, from finite temperance, we get that $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathcal{B})$. □

**Proposition 13.** Let $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ be a satisfaction system with the RMBP. Then $|\mathrm{MinFRSups}(\mathbb{M}, \Lambda)| \leq 1$ and $|\mathrm{MaxFRSubs}(\mathbb{M}, \Lambda)| \leq 1$ for all $\mathbb{M} \subseteq \mathfrak{M}$.

*Proof.* We only prove the result for MinFRSups, as the case for MaxFRSubs is analogous. Assume that $\mathbb{M}_1, \mathbb{M}_2 \in \mathrm{MinFRSups}(\mathbb{M}, \Lambda)$ with $\mathbb{M}_1 \neq \mathbb{M}_2$. Since $\mathbb{M}_1$ and $\mathbb{M}_2$ are finitely representable in $\mathcal{L}$, there are two finite bases $\mathcal{B}_1$, $\mathcal{B}_2$ such that $\mathrm{Mod}(\mathcal{B}_1) = \mathbb{M}_1$ and $\mathrm{Mod}(\mathcal{B}_2) = \mathbb{M}_2$.

First, let $\mathbb{M}' = \mathrm{Mod}(\mathcal{B}_1 \cup \mathcal{B}_2)$ and $M \in \mathbb{M}$. We know that $M \in \mathbb{M}_1 \cap \mathbb{M}_2$ because both $\mathbb{M}_1$ and $\mathbb{M}_2$ are supersets of $\mathbb{M}$. From the RMBP, it holds that $M \in \mathrm{Mod}(\mathcal{B}_1 \cup \mathcal{B}_2)$. Since the choice of $M$ was arbitrary, we can conclude that $\mathbb{M} \subseteq \mathbb{M}'$.

Now, let $M \in \mathbb{M}'$. Due to the RMBP we have that $M \in \mathrm{Mod}(\mathcal{B}_1) = \mathbb{M}_1$ and $M \in \mathrm{Mod}(\mathcal{B}_2) = \mathbb{M}_2$. That is, $\mathbb{M}' \subseteq \mathbb{M}_1 \cap \mathbb{M}_2$. Therefore, $\mathbb{M}'$ is a finitely representable (just take $\mathcal{B}_1 \cup \mathcal{B}_2$ as the base) superset of $\mathbb{M}$. However, since we assume that $\mathbb{M}_1, \mathbb{M}_2 \in \mathrm{MinFRSups}(\mathbb{M}, \Lambda)$, by minimality we get that $\mathbb{M}_1 = \mathbb{M}' \subseteq \mathbb{M}_1 \cap \mathbb{M}_2$ which implies $\mathbb{M}_1 = \mathbb{M}_2$, a contradiction. Hence, there can be at most one set of models in $\mathrm{MinFRSups}(\mathbb{M}, \Lambda)$. □

# B Proofs for Section 4

First, we prove in our claim about eviction- and reception-compatibility of $\Lambda_q$ from Example 15 with Proposition B.5.

**Proposition B.5.** Let $\Lambda_q = (\mathcal{L}_q, \mathfrak{M}_q, \models_q)$ be such that $\mathcal{L}_q = \{[x, y] \mid x, y \in \mathbb{Q} \text{ and } x \leq y\}$, $\mathfrak{M}_q = \mathbb{Q}$ and $Q \models_q \mathcal{B}$ (with $Q \subseteq \mathbb{Q}$) iff for all $z \in Q$, $x \leq z \leq y$ for every $[x, y] \in \mathcal{B}$.

*Proof.* We will show that this system is not eviction-compatible. Consider the base $\{[0, 1]\}$ and the set of models $\{1\}$. Since the language only admits closed intervals and by definition of $\models_q$, any finite base in $\mathcal{L}_q$ is either inconsistent or equivalent to a single continuous interval. Therefore, for any $\mathcal{B}' \in \mathcal{P}_{\mathrm{f}}(\mathcal{L}_q)$ that does not include $\{1\}$ there will always be a finite base that has more models. More precisely, let $[x', y']$ be the interval corresponding to a candidate finite base $\mathcal{B}'$. We can assume without loss of generality that $y' < 1$ and we know that there are infinitively many rational numbers between $y'$ and 1. Thus, we can always extend the interval to a new rational, capturing more models than before, without losing finite representability or including 1 in the models of the base. Therefore $\mathrm{MaxFRSubs}([0, 1), \Lambda_q) = \emptyset$, that is, $\Lambda_q$ is not eviction-compatible.

Now, we will prove that $\Lambda_q$ is not reception-compatible. Consider the base $\{[0.5, 1]\}$ and the set of models $(0, 1]$. Using the same argument as before, we can conclude that $\mathrm{MinFRSups}(\mathbb{M}, \Lambda_q)$ corresponds to either the smallest closed interval containing $(0, 1]$. Since $\{[0, 1]\}$ is finitely representable, any candidate must be equivalent to a closed interval $[x', y']$ such that $0 < x' < y' = 1$. Otherwise, either it would not be a superset of $(0, 1]$, or would include too many models, losing minimality. However, for any $x' \in \mathbb{Q}$ with $0 < x' < 1$ there is a $x''$ with $0 < x'' < x'$. This means that we can always find a candidate finite base that has fewer models. Therefore, $\mathrm{MinFRSups}((0, 1], \Lambda_q) = \emptyset$, that is, $\Lambda_q$ is not reception-compatible. □

**Proposition B.6.** Let $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ and $\mathbb{M} \subseteq \mathfrak{M}$. Then, there are $\mathcal{B} \in \mathcal{P}_f(\mathcal{L})$ and $\mathbb{M}' \subseteq \mathfrak{M}$ such that $\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}' = \mathbb{M}$, iff there is a $\mathbb{M}'' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M} \subseteq \mathbb{M}''$.

*Proof.* $\Rightarrow$: If we suppose that there are $\mathcal{B} \in \mathcal{P}_f(\mathcal{L})$ and $\mathbb{M}' \subseteq \mathfrak{M}$ such that $\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}' = \mathbb{M}$, then we can take $\mathbb{M}'' = \mathrm{Mod}(\mathcal{B})$.

$\Leftarrow$: Assuming that there is a $\mathbb{M}'' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M} \subseteq \mathbb{M}''$, we can take $\mathcal{B} \in \mathcal{P}_f(\mathcal{L})$ such that $\mathrm{Mod}(\mathcal{B}) = \mathbb{M}''$ and $\mathbb{M}' = (\mathfrak{M} \setminus \mathbb{M})$. Then $\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}' = \mathbb{M}'' \setminus \mathfrak{M} \setminus \mathbb{M}$, and as $\mathbb{M} \subseteq \mathbb{M}'' \subseteq \mathfrak{M}$, we get $\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}' = \mathbb{M}$. □

**Theorem 16.** A satisfaction system $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ is

- eviction-compatible iff for every $\mathbb{M} \subseteq \mathfrak{M}$ either (i) $\mathbb{M} \in \mathrm{FR}(\Lambda)$, (ii) $\mathbb{M}$ has an immediate predecessor in $(\mathrm{FR}(\Lambda) \cup \{\mathbb{M}\}, \subset)$, or (iii) there is no $\mathbb{M}' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M} \subseteq \mathbb{M}'$; and
- reception-compatible iff for every $\mathbb{M} \subseteq \mathfrak{M}$ either (i) $\mathbb{M} \in \mathrm{FR}(\Lambda)$, (ii) $\mathbb{M}$ has an immediate successor in $(\mathrm{FR}(\Lambda) \cup \{\mathbb{M}\}, \subset)$, or (iii) there is no $\mathbb{M}' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M}' \subseteq \mathbb{M}$.

*Proof.* We split the statement of the theorem into the following two claims, which directly imply the theorem.

**Claim 7.** A satisfaction system $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ is eviction-compatible iff for every $\mathbb{M} \subseteq \mathfrak{M}$ either (i) $\mathbb{M} \in \mathrm{FR}(\Lambda)$, (ii) $\mathbb{M}$ has an immediate predecessor in $(\mathrm{FR}(\Lambda) \cup \{\mathbb{M}\}, \subset)$, or (iii) there is no $\mathbb{M}' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M} \subseteq \mathbb{M}'$.

*Proof.* $\Rightarrow$: Suppose that $\Lambda$ is eviction-compatible, that is, $\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda) \neq \emptyset$ for all $\mathcal{B} \in \mathcal{P}_f(\mathcal{L})$ and $\mathbb{M} \subseteq \mathfrak{M}$.

Let $\mathbb{M}_1 \subseteq \mathfrak{M}$. If $\mathbb{M}_1 \in \mathrm{FR}(\Lambda)$ then the theorem holds trivially.

Now, we consider two cases $\mathrm{MaxFRSubs}(\mathbb{M}_1, \Lambda) \neq \emptyset$ and $\mathrm{MaxFRSubs}(\mathbb{M}_1, \Lambda) = \emptyset$.

In the first case, we know there is a $\mathbb{M}_2 \in \mathrm{MaxFRSubs}(\mathbb{M}_1, \Lambda)$. We will show that $\mathbb{M}_2$ is an immediate predecessor of $\mathbb{M}_1$. Since $\mathbb{M}_2 \in \mathrm{MaxFRSubs}(\mathbb{M}_1, \Lambda)$, $\mathbb{M}_2 \subseteq \mathbb{M}_1$ and by Definition 1 there is no $\mathbb{M}_2' \in \mathrm{FR}(\Lambda)$ such that $\mathbb{M}_2 \subset \mathbb{M}_2' \subset \mathbb{M}_1$. Consequently, $\mathbb{M}_2$ is an immediate predecessor of $\mathbb{M}_1$ in $(\mathrm{FR}(\Lambda) \cup \{\mathbb{M}_1\}, \subset)$.

In the second case, due to eviction-compatibility, we know that there is no $\mathcal{B} \in \mathcal{P}_f(\mathcal{L})$ and $\mathbb{M}_3 \subseteq \mathfrak{M}$ such that $\mathbb{M}_1 = \mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}_3$. Therefore, we can use Proposition B.6 to conclude that there is no $\mathbb{M}' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M}_1 \subseteq \mathbb{M}'$.

$\Leftarrow$: Assume that for all $\mathbb{M} \subseteq \mathfrak{M}$, $\mathbb{M} \in \mathrm{FR}(\Lambda)$, $\mathbb{M}$ has an immediate predecessor in $(\mathrm{FR}(\Lambda) \cup \{\mathbb{M}\}, \subset)$, or there is no $\mathbb{M}' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M} \subseteq \mathbb{M}'$. Let $\mathbb{M}_1 \subseteq \mathfrak{M}$. We consider following cases.

(i) $\mathbb{M}_1 \in \mathrm{FR}(\Lambda)$: by Definition 1 we have that $\mathrm{MaxFRSubs}(\mathbb{M}_1, \Lambda) = \{\mathbb{M}_1\} \neq \emptyset$.

(ii) $\mathbb{M}_1$ has an immediate predecessor in $(\mathrm{FR}(\Lambda) \cup \{\mathbb{M}_1\}, \subset)$: then there is a $\mathbb{M}_2 \in \mathrm{FR}(\Lambda)$ such that $\mathbb{M}_2 \subset \mathbb{M}_1$ and there is no $\mathbb{M}_2' \in \mathrm{FR}(\Lambda)$ such that $\mathbb{M}_2 \subset \mathbb{M}_2' \subset \mathbb{M}_1$. In other words, $\mathbb{M}_2 \in \mathrm{MaxFRSubs}(\mathbb{M}_1, \Lambda)$.

(iii) There is no $\mathbb{M}' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M}_1 \subseteq \mathbb{M}'$: then, we know from Proposition B.6 that there is no $\mathcal{B} \in \mathcal{P}_f(\mathcal{L})$ and $\mathbb{M}'' \subseteq \mathfrak{M}$ such that $\mathbb{M}_1 = \mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}''$.

Hence, if there are $\mathcal{B} \in \mathcal{P}_f(\Lambda)$ and $\mathbb{M} \in \mathfrak{M}$ such that $\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M} = \mathbb{M}_1$, then $\mathrm{MaxFRSubs}(\mathbb{M}_1, \Lambda) \neq \emptyset$. Since the choice of $\mathbb{M}_1$ was arbitrary, we can conclude that $\Lambda$ is eviction-compatible. □

**Proposition B.8.** Let $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ and $\mathbb{M} \subseteq \mathfrak{M}$. There are $\mathcal{B} \in \mathcal{P}_f(\mathcal{L})$ and $\mathbb{M}' \subseteq \mathfrak{M}$ such that $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}' = \mathbb{M}$, iff there is a $\mathbb{M}'' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M}'' \subseteq \mathbb{M}$.

*Proof.* $\Rightarrow$: If we suppose that there are $\mathcal{B} \in \mathcal{P}_f(\mathcal{L})$ and $\mathbb{M}' \subseteq \mathfrak{M}$ such that $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}' = \mathbb{M}$, then we can take $\mathbb{M}'' = \mathrm{Mod}(\mathcal{B})$.

$\Leftarrow$: Assuming that there is a $\mathbb{M}'' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M}'' \subseteq \mathbb{M}$, we can take $\mathcal{B} \in \mathcal{P}_f(\mathcal{L})$ such that $\mathrm{Mod}(\mathcal{B}) = \mathbb{M}''$ and $\mathbb{M}' = \mathbb{M}$. Then $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}' = \mathbb{M}'' \cup \mathbb{M}$, and as $\mathbb{M}'' \subseteq \mathbb{M}$, we get $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}' = \mathbb{M}$. □

**Claim 9.** A satisfaction system $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ is reception-compatible iff for every $\mathbb{M} \subseteq \mathfrak{M}$ either (i) $\mathbb{M} \in \mathrm{FR}(\Lambda)$, (ii) $\mathbb{M}$ has an immediate successor in $(\mathrm{FR}(\Lambda) \cup \{\mathbb{M}\}, \subset)$, or (iii) there is no $\mathbb{M}' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M}' \subseteq \mathbb{M}$.

*Proof.* $\Rightarrow$: Suppose that $\Lambda$ is reception-compatible, that is, $\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda) \neq \emptyset$ for all $\mathcal{B} \in \mathcal{P}_f(\mathcal{L})$ and $\mathbb{M} \subseteq \mathfrak{M}$.

Let $\mathbb{M}_1 \subseteq \mathfrak{M}$. If $\mathbb{M}_1 \in \mathrm{FR}(\Lambda)$ then the theorem holds trivially.

Now, we consider two cases $\mathrm{MinFRSups}(\mathbb{M}_1, \Lambda) \neq \emptyset$ and $\mathrm{MinFRSups}(\mathbb{M}_1, \Lambda) = \emptyset$.

In the first case, we know there is a $\mathbb{M}_2 \in \mathrm{MinFRSups}(\mathbb{M}_1, \Lambda)$. We will show that $\mathbb{M}_2$ is an immediate successor of $\mathbb{M}_1$. Since $\mathbb{M}_2 \in \mathrm{MinFRSups}(\mathbb{M}_1, \Lambda)$, $\mathbb{M}_1 \subseteq \mathbb{M}_2$ and by Definition 7 there is no $\mathbb{M}_2' \in \mathrm{FR}(\Lambda)$ such that $\mathbb{M}_1 \subset \mathbb{M}_2' \subset \mathbb{M}_2$. Consequently, $\mathbb{M}_2$ is an immediate successor of $\mathbb{M}_1$ in $(\mathrm{FR}(\Lambda) \cup \{\mathbb{M}_1\}, \subset)$.

In the second case, due to reception-compatibility, we know that there is no $\mathcal{B} \in \mathcal{P}_f(\mathcal{L})$ and $\mathbb{M}_3 \subseteq \mathfrak{M}$ such that $\mathbb{M}_1 = \mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}_3$. Therefore, we can use Proposition B.8 to conclude that there is no $\mathbb{M}' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M}' \subseteq \mathbb{M}_1$.

$\Leftarrow$: Assume that for all $\mathbb{M} \subseteq \mathfrak{M}$, $\mathbb{M} \in \mathrm{FR}(\Lambda)$, $\mathbb{M}$ has an immediate successor in $(\mathrm{FR}(\Lambda) \cup \{\mathbb{M}\}, \subset)$, or there is no $\mathbb{M}' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M}' \subseteq \mathbb{M}$. Let $\mathbb{M}_1 \subseteq \mathfrak{M}$. We consider following cases.

(i) $\mathbb{M}_1 \in \mathrm{FR}(\Lambda)$: by Definition 7 we have that $\mathrm{MinFRSups}(\mathbb{M}_1, \Lambda) = \{\mathbb{M}_1\} \neq \emptyset$.

(ii) $\mathbb{M}_1$ has an immediate successor in the poset $(\mathrm{FR}(\Lambda) \cup \{\mathbb{M}_1\}, \subset)$: then there is a $\mathbb{M}_2 \in \mathrm{FR}(\Lambda)$ such that $\mathbb{M}_1 \subset \mathbb{M}_2$ and there is no $\mathbb{M}_2' \in \mathrm{FR}(\Lambda)$ such that $\mathbb{M}_1 \subset \mathbb{M}_2' \subset \mathbb{M}_2$. In other words, $\mathbb{M}_2 \in \mathrm{MinFRSups}(\mathbb{M}_1, \Lambda)$.

(iii) There is no $\mathbb{M}' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M}' \subseteq \mathbb{M}_1$: then, we know from Proposition B.8 that there is no $\mathcal{B} \in \mathcal{P}_f(\mathcal{L})$ and $\mathbb{M}'' \subseteq \mathfrak{M}$ such that $\mathbb{M}_1 = \mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}''$.

Hence, if there are $\mathcal{B} \in \mathcal{P}_f(\Lambda)$ and $\mathbb{M} \in \mathfrak{M}$ such that $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M} = \mathbb{M}_1$, then $\mathrm{MinFRSups}(\mathbb{M}_1, \Lambda) \neq \emptyset$. By the arbitrariety of $\mathbb{M}_1$ we can conclude that $\Lambda$ is reception-compatible. □

□

**Corollary 17.** Let $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ be satisfaction system in which $\mathrm{FR}(\Lambda)$ is finite. Then:

- $\Lambda$ is eviction-compatible iff $\emptyset \in \mathrm{FR}(\Lambda)$.
- $\Lambda$ is reception-compatible iff $\mathfrak{M} \in \mathrm{FR}(\Lambda)$.

*Proof.* Since $\mathrm{FR}(\Lambda)$ is finite, the existence of an immediate predecessor is guaranteed for all $\emptyset \neq \mathbb{M} \subseteq \mathfrak{M}$ and so is ensured the existence of an immediate successor for all $\mathbb{M} \subset \mathfrak{M}$. Therefore, this result is a direct consequence of Theorem 16 (Item 1) for the first point and of Theorem 16 (Item 2) for the second point. □

# C Proofs for Section 5

## C.1 Proofs for Subsection 5.1

**Theorem 18.** $\Lambda(\text{Prop})$ is reception-compatible and eviction-compatible.

*Proof.* Since we need only to consider finitely many symbols, there are finitely many possible valuations. If there are $n$ propositional atoms, there are at most $2^n$ distinct models, meaning that there are at most $2^m$ distinct sets of valuations where $m = 2^n$. Consequently, $\mathrm{FR}(\Lambda(\text{Prop}))$ is finite. Additionally, since both the empty set and the set of all valuations are representable in this satisfaction system, we obtain as a consequence of Corollary 17 that $\Lambda(\text{Prop})$ is both eviction- and reception-compatible. □

**Proposition C.10.** Let $\Lambda(\text{Prop})$ be the satisfaction system with the entailment relation given by the standard semantics of propositional logic with finite signature. The function $\mathrm{evc}_{\text{Prop}}$ defined next is a maxichoice eviction on $\Lambda(\text{Prop})$.

$$\mathrm{evc}_{\text{Prop}}(\mathcal{B}, \mathbb{M}) = \bigvee_{v \in \mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}} \left( \bigwedge_{v(a)=\mathrm{T}} a \wedge \bigwedge_{v(a)=\mathrm{F}} \neg a \right)$$

*Proof.* We will use Theorem 5 to prove this result, by showing that $\mathrm{evc}_{\text{Prop}}$ satisfies each of the postulates stated. Recall that each model is a valuation over a finite number of propositional atoms, and therefore, the set of all models is finite.

**(success)** Let $v \in \mathbb{M}$. Clearly, $v \notin \mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}$. We know that $v$ does not satisfy any of the disjuncts that compose $\mathrm{evc}_{\text{Prop}}$, as each is satisfied by exactly one valuation. It follows from the standard semantics of proposition logic with finite signature that $v \notin \mathrm{Mod}(\mathrm{evc}_{\text{Prop}}(\mathcal{B}, \mathbb{M}))$. As we only assumed that $v \in \mathbb{M}$, we can conclude that $\mathbb{M} \cap \mathrm{Mod}(\mathrm{evc}_{\text{Prop}}(\mathcal{B}, \mathbb{M})) = \emptyset$.
**(inclusion)** Let $v \notin \mathrm{Mod}(\mathcal{B})$. Consequently, $v$ does not satisfy any of the disjuncts that compose $\mathrm{evc}_{\text{Prop}}$, as each is satisfied by exactly one valuation. It follows from the standard semantics of proposition logic with finite signature that $v \notin \mathrm{Mod}(\mathrm{evc}_{\text{Prop}}(\mathcal{B}, \mathbb{M}))$. Since $v$ was arbitrarily chosen, we obtain $\mathrm{Mod}(\mathrm{evc}_{\text{Prop}}(\mathcal{B}, \mathbb{M})) \subseteq \mathrm{Mod}(\mathcal{B})$.
**(vacuity)** If $\mathbb{M} \cap \mathrm{Mod}(\mathcal{B}) = \emptyset$ then

$$\mathrm{evc}_{\text{Prop}}(\mathcal{B}, \mathbb{M}) = \bigvee_{v \in \mathrm{Mod}(\mathcal{B})} \left( \bigwedge_{v(a)=\mathrm{T}} a \wedge \bigwedge_{v(a)=\mathrm{F}} \neg a \right).$$

Since each disjunct is associated to exactly one model, every model of $\mathcal{B}$ will also be a model of $\mathrm{evc}_{\text{Prop}}(\mathcal{B}, \mathbb{M})$, and exactly those, i.e., $\mathrm{Mod}(\mathrm{evc}_{\text{Prop}}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathcal{B})$.
**(finite retainment)** Each disjunct of $\mathrm{evc}_{\text{Prop}}(\mathrm{Mod}(\mathcal{B}), \mathbb{M})$ is associated to exactly one model in $\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}$, hence $\mathrm{Mod}(\mathrm{evc}_{\text{Prop}}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}$. Therefore, there is no $\mathbb{M}' \in \mathrm{FR}(\Lambda(\text{Prop}))$ such that $\mathrm{Mod}(\mathrm{evc}_{\text{Prop}}(\mathcal{B}, \mathbb{M})) \subset \mathbb{M}' \subseteq \mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}$.
**(uniformity)** In $\Lambda(\text{Prop})$ every set of models is finitely representable, thus, $\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda(\text{Prop})) = \mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}$. Therefore, if $\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda(\text{Prop})) = \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}') \setminus \mathbb{M}', \Lambda(\text{Prop}))$ then $\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M} = \mathrm{Mod}(\mathcal{B}') \setminus \mathbb{M}'$. In this case, we have that $\mathrm{evc}_{\text{Prop}}(\mathcal{B}, \mathbb{M}) = \mathrm{evc}_{\text{Prop}}(\mathcal{B}', \mathbb{M}')$ which implies $\mathrm{Mod}(\mathrm{evc}_{\text{Prop}}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{evc}_{\text{Prop}}(\mathcal{B}', \mathbb{M}'))$.

Since $\mathrm{evc}_{\text{Prop}}$ satisfies all the postulates from Theorem 5, it follows that it is a maxichoice eviction function over $\Lambda(\text{Prop})$. □

**Proposition C.11.** Let $\Lambda(\text{Prop})$ be the satisfaction system with the entailment relation given by the standard semantics of propositional logic with finite signature. The function $\mathrm{rcp}_{\text{Prop}}$ defined next is a maxichoice reception on $\Lambda(\text{Prop})$.

$$\mathrm{rcp}_{\text{Prop}}(\mathcal{B}, \mathbb{M}) = \bigvee_{v \in \mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}} \left( \bigwedge_{v(a)=\mathrm{T}} a \wedge \bigwedge_{v(a)=\mathrm{F}} \neg a \right)$$

*Proof.* We will use Theorem 10 to prove this result by showing that $\mathrm{rcp}_{\text{Prop}}$ satisfies each of the postulates stated. Recall that each model is a valuation over a finite number of propositional atoms, and therefore, the set of all models is finite.

**(success)** Let $v \in \mathbb{M}$. Clearly, $v \in \mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}$. Consequently, $v$ satisfies one of the disjuncts that compose $\mathrm{rcp}_{\text{Prop}}$, as each is satisfied by exactly one valuation. It follows from the standard semantics of proposition logic with finite signature that $v \in \mathrm{Mod}(\mathrm{rcp}_{\text{Prop}}(\mathcal{B}, \mathbb{M}))$. As we only assumed that $v \in \mathbb{M}$, we can conclude that $\mathbb{M} \subseteq \mathrm{Mod}(\mathrm{rcp}_{\text{Prop}}(\mathcal{B}, \mathbb{M}))$.
**(persistence)** Let $v \in \mathrm{Mod}(\mathcal{B})$. We know that $v$ satisfies one of the disjuncts that compose $\mathrm{rcp}_{\text{Prop}}$, as each is satisfied by exactly one valuation. It follows from the standard semantics of proposition logic with finite signature that $v \in \mathrm{Mod}(\mathrm{rcp}_{\text{Prop}}(\mathcal{B}, \mathbb{M}))$. Since $v$ was arbitrarily chosen, we obtain $\mathrm{Mod}(\mathcal{B}) \subseteq \mathrm{Mod}(\mathrm{rcp}_{\text{Prop}}(\mathcal{B}, \mathbb{M}))$.
**(vacuity)** If $\mathbb{M} \subseteq \mathrm{Mod}(\mathcal{B}) = \emptyset$ then

$$\mathrm{rcp}_{\text{Prop}}(\mathcal{B}, \mathbb{M}) = \bigvee_{v \in \mathrm{Mod}(\mathcal{B})} \left( \bigwedge_{v(a)=\mathrm{T}} a \wedge \bigwedge_{v(a)=\mathrm{F}} \neg a \right).$$

Since each disjunct is associated to exactly one model, only models of $\mathcal{B}$ will be a models of $\mathrm{rcp}_{\text{Prop}}(\mathcal{B}, \mathbb{M})$, that is, $\mathrm{Mod}(\mathrm{rcp}_{\text{Prop}}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathcal{B})$.
**(finite temperance)** Each disjunct of $\mathrm{rcp}_{\text{Prop}}(\mathrm{Mod}(\mathcal{B}), \mathbb{M})$ is associated to exactly one model in $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}$, hence $\mathrm{Mod}(\mathrm{rcp}_{\text{Prop}}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}$. Therefore, there is no $\mathbb{M}' \in \mathrm{FR}(\Lambda(\text{Prop}))$ such that $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M} \subset \mathbb{M}' \subseteq \mathrm{Mod}(\mathrm{rcp}_{\text{Prop}}(\mathcal{B}, \mathbb{M}))$.

**(uniformity)** In $\Lambda(\text{Prop})$ every set of models is finitely representable, thus, $\text{MinFRSups}(\text{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda(\text{Prop})) = \text{Mod}(\mathcal{B}) \cup \mathbb{M}$. Therefore, if $\text{MinFRSups}(\text{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda(\text{Prop})) = \text{MaxFRSubs}(\text{Mod}(\mathcal{B}') \cup \mathbb{M}', \Lambda(\text{Prop}))$ then $\text{Mod}(\mathcal{B}) \cup \mathbb{M} = \text{Mod}(\mathcal{B}') \cup \mathbb{M}'$. In this case, we have that $\text{rcp}_{\text{Prop}}(\mathcal{B}, \mathbb{M}) = \text{rcp}_{\text{Prop}}(\mathcal{B}', \mathbb{M}')$ which implies $\text{Mod}(\text{rcp}_{\text{Prop}}(\mathcal{B}, \mathbb{M})) = \text{Mod}(\text{evc}(\text{rcp}'_{\text{Prop}}, \mathbb{M}'))$.

Since $\text{rcp}_{\text{Prop}}$ satisfies all the postulates from Theorem 10, it follows that it is a maxichoice reception function over $\Lambda(\text{Prop})$. $\square$

**Proposition 19.** The functions $\text{evc}_{\text{Prop}}$ and $\text{rcp}_{\text{Prop}}$ defined next are, respectively, maxichoice eviction and reception functions on $\Lambda(\text{Prop})$.

$$\text{evc}_{\text{Prop}}(\mathcal{B}, \mathbb{M}) = \bigvee_{v \in \text{Mod}(\mathcal{B}) \setminus \mathbb{M}} \left( \bigwedge_{v(a)=\text{T}} a \wedge \bigwedge_{v(a)=\text{F}} \neg a \right)$$

$$\text{rcp}_{\text{Prop}}(\mathcal{B}, \mathbb{M}) = \bigvee_{v \in \text{Mod}(\mathcal{B}) \cup \mathbb{M}} \left( \bigwedge_{v(a)=\text{T}} a \wedge \bigwedge_{v(a)=\text{F}} \neg a \right).$$

As usual, F stands for 'false' and T stands for 'true'.

*Proof.* Direct consequence of Propositions C.10 and C.11. $\square$

**Theorem 20.** $\Lambda(\text{Horn}) = (\mathcal{L}_{\text{H}}, \mathfrak{M}_{\text{Prop}}, \models_{\text{Prop}})$ $\Lambda(\text{Horn})$, is both eviction- and reception-compatible.

*Proof.* As for classical propositional logics, we have that $\text{FR}(\Lambda(\text{Horn}))$ is finite. Observe that $\text{Mod}(\{a \to a\}) = \mathfrak{M}$, as $a \to a$ is tautological. Moreover, the set $\text{Mod}(\{\bot\}) = \emptyset$. Thus, both $\emptyset$ and $\mathfrak{M}$ are finitely representable. Therefore, according to Corollary 17, $\Lambda(\text{Horn})$ is both eviction and reception compatible. $\square$

## C.2 Proofs for Subsection 5.2

**Theorem 21.** $\Lambda(\text{K3})$ and $\Lambda(\text{P3})$ are reception-compatible but $\Lambda(\text{K3})$ is reception-compatible, while $\Lambda(\text{P3})$ is not.

*Sketch.* As in the propositional case, $\mathfrak{M}_3$ is finite and $\mathfrak{M}_3$ are finitely representable in both systems. However, $\emptyset$ is finitely representable in $\Lambda(\text{K3})$ but not in $\Lambda(\text{P3})$. Hence, the theorem is a consequence of Corollary 17. $\square$

*Proof.* In both systems, we have exactly the same set of models which is finite, precisely we have $3^{|\texttt{At}|}$ models, where $\texttt{At}$ is the set of propositional symbols (which is assumed to be finite). Thus, we have $2^m$ classes of equivalences of formulae, where $m = 3^{|\texttt{At}|}$. Thus, for every $K \subseteq \mathcal{L}_{\text{Prop}}$, there is a finite base $\mathcal{B} \in \mathcal{P}_{\text{f}}(\mathcal{L}_{\text{Prop}})$ such that $\text{Mod}(K) = \text{Mod}(\mathcal{B})$. Observe that in both systems $\text{Mod}(\emptyset) = \mathfrak{M}_3$, which means that $\mathfrak{M}_3$ is finitely representable in both $\Lambda(\text{K3})$ and $\Lambda(\text{P3})$. Also, $\text{Mod}(\mathcal{L}) = \emptyset$, in $\Lambda(\text{K3})$. Thus, as every set of formulae has a finite base, we get that $\mathcal{L}_{\text{Prop}}$ also has a finite base in $\Lambda(\text{K3})$. However, the model that assigns U to every propositional formula will satisfy any base according to $\models_{P3}$. Thus, $\emptyset$ is finitely representable in $\Lambda(\text{K3})$ but not in $\Lambda(\text{P3})$. Therefore, it follows directly from Corollary 17 that both systems are and reception-compatible but $\Lambda(\text{K3})$ is eviction-compatible, while $\Lambda(\text{P3})$ is not. $\square$

## C.3 Proofs for Subsection 5.3

**Definition C.12.** Let $\theta \in (0, 1]$. The satisfaction system of the propositional Gödel logic, in symbols, $\Lambda(\text{Gödel}, \theta)$ is defined as $\Lambda(\text{Gödel}, \theta) = (\mathcal{L}_{\text{G}}, \mathfrak{M}_G, \models^{\theta}_{G})$ in which

- $\mathcal{L}_{\text{G}}$ consists of propositional formulas defined over a non-empty finite set of propositional atoms $\texttt{At}$ and the connectives $\wedge$, $\vee$, $\neg$, and $\to$;
- $\mathfrak{M}_G$ is the set of all functions $v : \mathcal{L} \to [0, 1]$ respecting the standard Gödel semantics for the boolean connectives given below

$$v(\neg\varphi) = \begin{cases} 1 & \text{if } v(\varphi) = 0, \\ 0 & \text{otherwise;} \end{cases}$$
$$v(\varphi \wedge \psi) = \min(v(\varphi), v(\psi));$$
$$v(\varphi \vee \psi) = \max(v(\varphi), v(\psi));$$
$$v(\varphi \to \psi) = \begin{cases} 1 & \text{if } v(\varphi) \leq v(\psi), \\ v(\psi) & \text{otherwise; and} \end{cases}$$

- $v \models^{\theta}_{G} B$ iff $v(\bigwedge_{\varphi \in B \cup \{(\neg a \vee a)\}} \varphi) \geq \theta$, with some $a \in \texttt{At}$.

Henceforth, given $v \in \mathfrak{M}_G$ and $\varphi \in \mathcal{L}_{\text{G}}$, we will abuse the notation and write $v \models^{\theta}_{G} \varphi$ as a shorthand for $v \models^{\theta}_{G} \{\varphi\}$.

**Definition C.13.** Let $\theta \in (0, 1]$, $\texttt{At}$ be a non-empty finite set of propositional atoms, $\mathcal{L}_{\text{G}}$ defined over $\texttt{At}$ and $v : \mathcal{L}_{\text{G}} \to [0, 1]$. Also let $s_\theta \notin \texttt{At}$. We define the $\theta$-extension of $v$ as: $v^* : \mathcal{L} \cup \{s_\theta\} \to [0, 1]$ defined as

$$v^*(\varphi) = \begin{cases} v(\varphi) & \text{if } \varphi \in \mathcal{L}_{\text{G}}, \\ \theta & \text{if } \varphi \text{ is } s_\theta. \end{cases}$$

**Definition C.14.** Let $\theta \in (0, 1]$, $\texttt{At}$ be a non-empty finite set of propositional atoms, $\mathcal{L}_{\text{G}}$ defined over $\texttt{At}$ and $v : \mathcal{L}_{\text{G}} \to [0, 1]$. From the $\theta$-extension of $v$ we define the following total preorders[2]

- $\preceq_v \subseteq (\mathcal{L}_{\text{G}} \cup \{s_\theta\}) \times (\mathcal{L}_{\text{G}} \cup \{s_\theta\})$ such that $\varphi \preceq_v \psi$ iff $v^*(\varphi) \leq v^*(\psi)$; and
- $\preceq'_v \subseteq (\texttt{At} \cup \{s_\theta\}) \times (\texttt{At} \cup \{s_\theta\})$ such that $\varphi \preceq'_v \psi$ iff $v^*(\varphi) \leq v^*(\psi)$.

**Lemma C.15.** Let $v, w \in \mathfrak{M}_G$ with $\preceq_v = \preceq_w$, then, for all $\varphi \in \mathcal{L}_{\text{G}}$, $v \models^{\theta}_{G} \varphi$ iff $w \models^{\theta}_{G} \varphi$.

*Proof.* We prove this lemma by induction on the structure of the formula $\varphi$.

**Base case:** if $\varphi \in \texttt{At}$ then $v \models^{\theta}_{G} \varphi$ iff $v(\varphi) \geq \theta$. And by Definition C.14 $v(\varphi) \geq \theta$ iff $\varphi \preceq_v s_\theta$. As we assume $\preceq_v = \preceq_w$, we have that $\varphi \preceq_w s_\theta$. Using again Definition C.14 and the definition of $\models^{\theta}_{G}$, we can conclude that $\varphi \preceq_w s_\theta$ iff $v \models^{\theta}_{G} \varphi$. Therefore, if $\varphi \in \texttt{At}$ then $v \models^{\theta}_{G} \varphi$ iff $w \models^{\theta}_{G} \varphi$.

[2]A preorder is a binary relation that is reflexive and transitive.

**Induction step:** Now, we assume that for all formulas $\psi \in \mathcal{L}_{\mathrm{G}}$ with length (number of connectives) at most $n$, it holds that whenever $\preceq_v = \preceq_w$ then $v \models^{\theta}_{G} \psi$ iff $w \models^{\theta}_{G} \psi$. We will consider now a formula $\varphi \in \mathcal{L}_{\mathrm{G}}$ that has length $n+1$, and treat each of the following cases separately.

$\varphi = \neg\psi$**:** First, we remark that as a consequence Definition C.12, every valuation in $\mathfrak{M}_G$ must assign 0 to $a \wedge \neg a$ for $a \in \mathtt{At}$ ($\mathtt{At} \neq \emptyset$). Consequently, every minimal element in the induced total preorder must be assigned 0 by the corresponding valuation. On the other hand, every formula assigned 0 by a valuation will be a minimal element in the induced total preorder. Thus, due to the semantics of negation in $\Lambda(\text{Gödel}, \theta)$, $v \models^{\theta}_{G} \neg\psi$ iff $\psi$ is a minimal element in $\preceq_v$. By our assumption that $\preceq_v = \preceq_w$, we can use the same argument to conclude that $w \models^{\theta}_{G} \neg\psi$ iff $\psi$ is a minimal element in $\preceq_w$. Hence, $v \models^{\theta}_{G} \neg\psi$ iff $w \models^{\theta}_{G} \neg\psi$.

$\varphi = \psi_1 \wedge \psi_2$**:** We know that $v \models^{\theta}_{G} \psi_1 \wedge \psi_2$ iff $v(\psi_1) \geq \theta$ and $v(\psi_2) \geq \theta$. In other words, $v \models^{\theta}_{G} \psi_1 \wedge \psi_2$ iff $\psi_1 \preceq_v s_\theta$ and $\psi_2 \preceq_v s_\theta$. Using the assumption that $\preceq_v = \preceq_w$ and the induction hypothesis, we get that $v \models^{\theta}_{G} \psi_1 \wedge \psi_2$ iff $w \models^{\theta}_{G} \psi_1 \wedge \psi_2$.

$\varphi = \psi_1 \vee \psi_2$**:** We know that $v \models^{\theta}_{G} \psi_1 \vee \psi_2$ iff $v(\psi_1) \geq \theta$ or $v(\psi_2) \geq \theta$. In other words, $v \models^{\theta}_{G} \psi_1 \vee \psi_2$ iff $s_\theta \preceq_v \psi_1$ or $s_\theta \preceq_v \psi_2$. Using the assumption that $\preceq_v = \preceq_w$ and the induction hypothesis, we get that $v \models^{\theta}_{G} \psi_1 \vee \psi_2$ iff $w \models^{\theta}_{G} \psi_1 \vee \psi_2$.

$\varphi = \psi_1 \rightarrow \psi_2$**:** $v \models^{\theta}_{G} \psi_1 \rightarrow \psi_2$ iff (i) $v(\psi_1) \leq v(\psi_2)$ or (ii) $v(\psi_2) \geq \theta$. In other words, $v \models^{\theta}_{G} \psi_1 \rightarrow \psi_2$ iff $\psi_1 \preceq_v \psi_2$ or $s_\theta \preceq_v \psi_2$. As in the case of $\wedge$ and $\vee$, we can employ the assumption that $\preceq_v = \preceq_w$ together with the induction hypothesis to conclude $v \models^{\theta}_{G} \psi_1 \rightarrow \psi_2$ iff $w \models^{\theta}_{G} \psi_1 \rightarrow \psi_2$.

Hence, if $\preceq_v = \preceq_w$ then $v \models^{\theta}_{G} \varphi$ iff $w \models^{\theta}_{G} \varphi$. □

**Proposition C.16.** Let $\theta \in (0, 1]$, $\mathtt{At}$ be a non-empty finite set of propositional atoms, $\Lambda(\text{Gödel}, \theta) = (\mathcal{L}_{\mathrm{G}}, \mathfrak{M}_G, \models^{\theta}_{G})$ as in Definition C.12. Then, for any $v \in \mathfrak{M}_G$ and $\varphi \in \mathcal{L}_{\mathrm{G}}$, $v(\varphi) \in \{0, 1\} \cup \{v(a) \mid a \in \mathtt{At}\}$.

*Proof.* This clearly holds for $\varphi \in \mathtt{At}$. For complex formulas we just need to consider the possible valuations defined in the semantics of the connectives in Definition C.12. For all of the connectives, the valuation is either one of the values of the subformulas, 0 or 1. □

**Lemma C.17.** Let $v, w \in \mathfrak{M}_G$ with $\preceq'_v = \preceq'_w$, then $\preceq_v = \preceq_w$.

*Proof.* From Definition C.12, the values assigned to all formulas in $\mathcal{L}_{\mathrm{G}}$ depend only on the valuations on $\mathtt{At}$. This means that there is only one possible way to extend a valuation on $\mathtt{At}$ to $\mathcal{L}_{\mathrm{G}}$. Moreover, it follows from Proposition C.16 that every formula in $\mathcal{L}_{\mathrm{G}}$ can only assume values in $\in \{0, 1\} \cup \{v(a) \mid a \in \mathtt{At}\}$. Furthermore, as a consequence of the semantics of the connectives, for every $\varphi \in \mathcal{L}_{\mathrm{G}}$, if $\varphi = \neg\psi$, then $v(\varphi)$ depends on whether $\psi$ is a minimal element in $\preceq_v$, otherwise, if $\varphi = \psi_1 \circ \psi_2$ with $\circ \in \{\wedge, \vee, \rightarrow\}$ then $v(\varphi)$ depends only on the relation between $\psi_1$ and $\psi_2$ according to $\preceq_v$. As each formula will receive values from a finite set depending only on the total preorder induced on the propositional atoms, for any valuation $v$, $\preceq'_v$ determines $\preceq_v$. □

**Theorem C.18.** $\mathrm{FR}(\Lambda(\text{Gödel}, \theta))$ is finite.

*Proof.* From Lemmas C.15 and C.17 and the definition of $\models^{\theta}_{G}$, we can conclude that whether $v \models^{\theta}_{G} B$, for $v \in \mathfrak{M}_G$ and $\mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\mathcal{L}_{\mathrm{G}})$, depends only on $\preceq_v$. However, as the induced total preorders over $\mathtt{At} \cup \{s_\theta\}$ are defined over a finite set, there is a finite amount of distinct ones. In fact, there are at most $\sum_{i=0}^{|\mathtt{At}|+1} k! S(n, i)$ such preorders, where $S(n, k)$ denotes the Stirling partition number. This implies that while there infinitely many valuations in $\mathfrak{M}_G$, there is only a finite number subsets of $\mathfrak{M}_G$ that can be represented via a base in $\mathcal{L}_{\mathrm{G}}$. Therefore, $\mathrm{FR}(\Lambda(\text{Gödel}, \theta))$ must be finite. □

**Theorem 22.** The satisfaction system $\Lambda(\text{Gödel}, \theta)$ is eviction- and reception-compatible.

*Proof.* It follows from Corollary 17 and Theorem C.18 that we only need to prove that $\emptyset, \mathfrak{M}_G \in \mathrm{FR}(\Lambda(\text{Gödel}, \theta))$. Let $\mathcal{B}_\perp = \{\neg a \wedge a\}$ for some $a \in \mathtt{At}$ and also let $\mathcal{B}_\top = \emptyset$. As a consequence of Definition C.12, for any $\theta \in (0, 1]$ and valuation $v \in \mathfrak{M}_G$: $v(\mathcal{B}_\perp) = 0 < \theta$ and $v(\mathcal{B}_\top) = 1 \geq \theta$. Therefore, $\Lambda(\text{Gödel}, \theta)$ is eviction-compatible and reception-compatible. □

## C.4 Proofs for Subsection 5.4

**Proposition C.19.** In $\Lambda(\mathrm{LTL_X})$, every finite set of formulae is a theory, that is, for every $\mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\mathcal{L}_X)$ and $\varphi \in \mathcal{L}_X$, if $M \in \mathrm{Mod}(\mathcal{B})$, then $M \models \{\varphi\}$ iff $\varphi \in \mathcal{B}$.

*Proof.* It suffices to show that for every finite set $\mathcal{B}$, and formula $\varphi \notin \mathcal{B}$, there is a model $(M, s)$ such that $(M, s) \models \mathcal{B}$ but $(M, s) \not\models \{\varphi\}$. By definition, every formula in this logics is of the form $X^n p$, where $p$ is an atomic propositional formula and $n \in \mathbb{N}$. Let $m = \max(\{k \in \mathbb{N} \mid X^k p \in \mathcal{B} \cup \{\varphi\}\})$. The value $m$ contains the highest value of $X^k$ of the formulae in $\mathcal{B} \cup \{\varphi\}$. This works as an upper bound on the size of the model $M$, we will construct. Let us construct the model $M = (S, R, \lambda)$ where

- $S = \{s_1, \ldots, s_m\}$,
- $R = \{(s_i, s_{i+1}) \mid i < m\} \cup \{(s_m, s_m)\}$
- $\lambda(s_i) = \{p \in \mathtt{At} \mid X^i p \in \mathcal{B}\}$

Observe that $M$ is indeed a Kripke structure. We only need to show that (1) $(M, s_1) \models_X \mathcal{B}$ and (2) $(M, s_1) \not\models_X \{\varphi\}$.

(1) let $\psi \in \mathcal{B}$. Thus, $\psi = X^i p$, for some $i \geq 0$. By definition of $M$, $p \in \lambda(s_i)$, which means that, $(M, s_1) \models_X X^i p$. Therefore, $(M, s_1) \models_X \psi$, for all $\psi \in \mathcal{B}$, that is, $(M, s_1) \models_X \mathcal{B}$.

(2) $\varphi = X^i q$, for some $i \geq 0$. By hypothesis, $\varphi \notin X$. Thus, by definition of $\lambda$, we get $q \notin \lambda(i)$. Thus, $(M, s_1) \not\models_X X^i q$, that is, $(M, s_1) \not\models_X \varphi$.

□

**Proposition 23.** Let $\mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\mathcal{L}_X)$, $\mathbb{M} \subseteq \mathfrak{M}_X$ and $\mathrm{rcp}_X : \mathcal{P}_{\mathrm{f}}(\mathcal{L}_X) \times \mathcal{P}(\mathfrak{M}_X) \to \mathcal{P}_{\mathrm{f}}(\mathcal{L}_X)$ defined as

$$\mathrm{rcp}_X(\mathcal{B}, \mathbb{M}) = \{\varphi \in \mathcal{B} \mid \mathbb{M} \models \varphi\}.$$

It holds that $\mathrm{rcp}_X(\mathcal{B}, \mathbb{M}) \in \mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M})$.

*Proof.* Let us suppose for contradiction that $\mathrm{rcp}_X(\mathcal{B}, \mathbb{M}) \notin \mathrm{MinFRSups}(\mathcal{B} \cup \mathbb{M}, \Lambda(\mathrm{LTL_X}))$. Thus, there is some finite representable $Y \subseteq \mathfrak{M}$ such that

$$\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M} \subseteq Y \subset \mathrm{Mod}(\mathrm{rcp}_X(\mathcal{B}, \mathbb{M})), \qquad (4)$$

and $Y = \mathrm{Mod}(\mathcal{B}_Y)$, for some finite $\mathcal{B}_Y$. From Proposition C.19, every finite set is a theory, which implies that both $\mathrm{rcp}_X(\mathcal{B}, \mathbb{M})$ and $\mathcal{B}_Y$ are finite theories. Thus, as the logic is monotonic, we get that $\mathrm{rcp}_X(\mathcal{B}, \mathbb{M}) \subset \mathcal{B}_Y$. Thus there is some $\varphi \in \mathcal{B}_Y$ such that $\varphi \notin \mathrm{rcp}_X(\mathcal{B}, \mathbb{M})$. By definition,

$$\psi \in \mathrm{rcp}_X(\mathcal{B}, \mathbb{M}) \text{ iff } \mathrm{Mod}(\mathcal{B}) \cup \mathbb{M} \models_X \psi$$

Thus, as $\varphi \notin \mathrm{rcp}_X(\mathcal{B}, \mathbb{M})$, we get that $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M} \not\models_X \varphi$. However, from (4), we have that $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M} \subseteq Y$. Thus, as $\varphi \in \mathcal{B}_Y$, we have that $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M} \models_X \varphi$, which is a contradiction. □

**Proposition C.20.** $\Lambda(\mathrm{LTL_X})$ is reception-compatible.

*Proof.* It follows from Proposition 23 that $\mathrm{rcp}_X(\mathcal{B}, \mathbb{M}) \in \mathrm{MinFRSups}(\mathcal{B} \cup \mathbb{M}, \Lambda(\mathrm{LTL_X}))$, which means that $\mathrm{rcp}_X$ is a maxichoice reception function. Therefore, according to Theorem 10, $\mathrm{rcp}_X$ satisfies all rationality postulates for reception. Thus, $\Lambda(\mathrm{LTL_X})$ is reception-compatible. □

**Proposition C.21.** $\Lambda(\mathrm{LTL_X})$ is not eviction-compatible.

*Proof.* Let $\mathcal{B} = \{p\}$ and $\mathbb{M} = \mathrm{Mod}(\mathcal{B})$. Thus, eviction of $\mathcal{B}$ by $\mathbb{M}$ must result in a finite base $\mathcal{B}'$ such that $\mathrm{Mod}(\mathcal{B}') = \emptyset$. However, the empty set of models is not finitely representable in this logic. To prove this, it is enough to show that every finite base $\mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\Lambda(\mathrm{LTL_X}))$ has a model. In fact, for any $\mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\mathcal{L})$, the model $M = (S, R, \lambda)$ such that $S = \{s\}$, $R = \{(s, s)\}$ and $\lambda(s) = \{p\}$ will satisfy any finite set of formulae of the form $X^k p$ with $k \in \mathbb{N}$. □

**Theorem 24.** $\Lambda(\mathrm{LTL_X})$ is reception-compatible but it is not eviction-compatible.

*Proof.* Follows directly from Propositions C.20 and C.21. □

## C.5 Proofs for Subsection 5.5

In the following proofs, we will consider in this work standard abbreviations for concept constructors in $\mathcal{ALC}$ that were not describe in Subsection 5.5. For example $\bot$ is interpreted as the empty set and $\top$ is interpreted as the whole domain. In some of the proofs, we will also employ the fact that usual concept inclusions $C \sqsubseteq D$ can be expressed equivalently as $\top \sqsubseteq \neg C \sqcup D$ and $\neg C \sqcup D \sqsubseteq \top$. We will also write $\exists r^m.C$ to denote the nesting of the existential restriction $\exists r$ $m$ times over the concept $C$. We establish in Theorem C.22 that $\Lambda(\mathcal{ALC})$ is not eviction-compatible. Our proof holds both in the case in which the disjoint sets $\mathsf{N_C}, \mathsf{N_R}, \mathsf{N_I}$ are assumed to be finite or (countably) infinite.

**Theorem C.22.** $\Lambda(\mathcal{ALC})$ is not eviction-compatible.

*Proof.* Let $\Lambda(\mathcal{ALC}) = (\mathcal{L}_{\mathcal{ALC}}, \mathfrak{M}_{\mathcal{ALC}}, \models_{\mathcal{ALC}})$ be the usual satisfaction system for $\mathcal{ALC}$. For conciseness, we will write $\models$ instead of $\models_{\mathcal{ALC}}$ within this proof. Let $\mathcal{B}_\top = \{\bot \sqsubseteq \top\}$, that is, $\mathrm{Mod}(\mathcal{B}_\top) = \mathfrak{M}$. Also, given a fixed but arbitrary $a \in \mathsf{N_I}$ and $r \in \mathsf{N_R}$, we define models of the form $M^n = (\mathbb{N}, \cdot^{M^n})$ where

$$r^{M^n} = \{(i, i+1) \mid i \in \mathbb{N}, 0 \leq i < n\}$$

and $a^{M_n} = 0$, and similarly $M^\infty = (\mathbb{N}, \cdot^{M^\infty})$ where

$$r^{M^\infty} = \{(i, i+1) \mid i \in \mathbb{N}\}$$

and $a^{M^\infty} = 0$. Let $\mathbb{M}$ be the set of all models $M$ such that for some $n \in \mathbb{N}$ we have that $a^M \in (\forall r^n.\bot)^M$. That is, there is no loop or infinite chain of elements connected via the role $r$ starting from $a^M$. By definition of $\mathbb{M}$, we have that $M^\infty \notin \mathbb{M}$ since this model has an infinite chain of elements connected via the role $r$ starting from $a^M$, while $M^n \in \mathbb{M}$ for all $n \in \mathbb{N}$.

To prove that $\Lambda(\mathcal{ALC})$ is not eviction-compatible, we need to prove that there is no $\mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\mathcal{L}_{\mathcal{ALC}})$ such that $\mathrm{Mod}(\mathcal{B}) \in \mathrm{MaxFRSubs}(\mathbb{M}, \Lambda(\mathcal{ALC}))$, that is, $\mathrm{MaxFRSubs}(\mathbb{M}, \Lambda(\mathcal{ALC})) = \emptyset$. Intuitively, we want to show that we cannot find a maximal $\mathcal{ALC}$ ontology that finitely represents the result of removing the models in $\mathfrak{M} \setminus \mathbb{M}$ from $\mathcal{B}_\top$. First, we show the following claims.

**Claim 23.** For every $\mathcal{ALC}$ concept $C$ if $M^\infty \models C(a)$ then there is $n \in \mathbb{N}$ such that for all $m \geq n$, with $m \in \mathbb{N}$, we have that $M^m \models C(a)$.

The proof is by structural induction. We assume w.l.o.g. that $C$ is in negation normal form, which means that we need to deal with expressions of the form $\exists r.D_1$, $\forall r.D_1$, $D_1 \sqcap D_2$, $D_1 \sqcup D_2$ (but we can disregard $\neg D_1$). In the base case we have $C = \exists r.\top$ and $C = \forall r.\top$. The claim holds in the base case since, by definition of $M^n$, we have that $M^n \models \exists r.\top(a)$, for all $n \in \mathbb{N}$, and the premisse is violated for $\forall r.\bot$ (that is, $M^\infty \not\models \forall r.\bot(a)$). Suppose that the claim holds for $D \in \{D_1, D_2\}$, that is, if $M^\infty \models D(a)$ then there is $n \in \mathbb{N}$ such that for all $m \geq n$, we have that $M^m \models D(a)$. We now consider the following cases.

- $\exists r.D_1$: Suppose that $M^\infty \models \exists r.D_1(a)$. By definition of $M^\infty$, we have that $M^\infty \models D_1(a)$ and so, by the inductive hypothesis, there is $n \in \mathbb{N}$ such that for all $m \geq n$, we have that $M^m \models D_1(a)$. By definition of $M^m$, for all $m \geq n$, with $m \in \mathbb{N}$, we have that $M^{m+1} \models \exists r.D_1(a)$.
- $\forall r.D_1$: Suppose that $M^\infty \models \forall r.D_1(a)$. By definition of $M^\infty$, we have that $M^\infty \models D_1(a)$ and so, by the inductive hypothesis, there is $n \in \mathbb{N}$ such that for all $m \geq n$, we have that $M^m \models D_1(a)$. By definition of $M^m$, for all $m \geq n$, with $m \in \mathbb{N}$, we have that $M^{m+1} \models \forall r.D_1(a)$.
- $D_1 \sqcap D_2$: Suppose that $M^\infty \models D_1 \sqcap D_2(a)$. Then, $M^\infty \models D_1(a)$ and $M^\infty \models D_2(a)$. By the inductive hypothesis, there are $n_1, n_2$ such that for all $m_1 \geq n_1$ and all $m_2 \geq n_2$, we have that $M^{m_1} \models D_1(a)$ and $M^{m_2} \models D_2(a)$. Assume w.l.o.g. that $n_1 \geq n_2$. Then, for all $m \geq n_1$, we have that $M^m \models D_1 \sqcap D_2(a)$.

- $D_1 \sqcup D_2$: Suppose that $M^\infty \models D_1 \sqcup D_2(a)$. Then, $M^\infty \models D_1(a)$ or $M^\infty \models D_2(a)$. Assume w.l.o.g. that $M^\infty \models D_1(a)$. By the inductive hypothesis, there is $n$ such that for all $m \geq n$, we have that $M^m \models D_1(a)$. Then, for all $m \geq n$, we have that $M^m \models D_1 \sqcup D_2(a)$.

**Claim 24.** For every $\mathcal{ALC}$ concept $C$ if there is $n \in \mathbb{N}$ such that for all $m \geq n$, with $m \in \mathbb{N}$, we have that $M^m \models C(a)$ then $M^\infty \models C(a)$.

The proof is by structural induction but we do not use negation normal form in this proof. In the base case we have $C = \exists r.\top$. The claim holds in the base case for all $n \in \mathbb{N}$ and all $m \geq n$, with $m \in \mathbb{N}$, since, by definition of $M^\infty$, we have that $M^\infty \models \exists r.\top(a)$. Suppose that the claim holds for $D \in \{D_1, D_2\}$, that is, if there is $n \in \mathbb{N}$ such that for all $m \geq n$, we have that $M^m \models D(a)$ then $M^\infty \models D(a)$. We now consider the following cases.

- $\exists r.D_1$: Suppose that there is $n \in \mathbb{N}$ such that for all $m \geq n$, we have that $M^m \models \exists r.D_1(a)$. By definition of $M^m$, for all $m \geq n > 0$, we have that $M^{m-1} \models D_1(a)$ (note that we can assume w.l.o.g. that there is such $n > 0$ because if there is $n$ satisfying the claim then $n+1$ also satisfies the claim). Then, by the inductive hypothesis, $M^\infty \models D_1(a)$. Finally, by definition of $M^\infty$, if $M^\infty \models D_1(a)$ then $M^\infty \models \exists r.D_1(a)$.
- $\neg D_1$: In this case, we use the contrapositive. Suppose that $M^\infty \not\models \neg D_1(a)$. We want to show that there is no $n \in \mathbb{N}$ such that for all $m \geq n$, with $m \in \mathbb{N}$, we have that $M^m \models \neg D_1(a)$. If $M^\infty \not\models \neg D_1(a)$ then $M^\infty \models D_1(a)$ and so, by Claim 23, there is $n \in \mathbb{N}$ such that for all $m \geq n$, with $m \in \mathbb{N}$, we have that $M^m \models D_1(a)$. Then, there can be no $n \in \mathbb{N}$ such that for all $m \geq n$, with $m \in \mathbb{N}$, we have that $M^m \models \neg D_1(a)$.
- $D_1 \sqcap D_2$: Suppose that there is $n \in \mathbb{N}$ such that for all $m \geq n$, we have that $M^m \models D_1 \sqcap D_2(a)$. Then, for all $m \geq n$, we have that $M^m \models D_1(a)$ and $M^m \models D_2(a)$. By the inductive hypothesis, $M^\infty \models D_1(a)$ and $M^\infty \models D_2(a)$. So $M^\infty \models D_1 \sqcap D_2(a)$.

**Claim 25.** For every $\mathcal{ALC}$ concept $C$ if there is $n \in \mathbb{N}$ such that for all $m \geq n$, with $m \in \mathbb{N}$, we have that $M^m \models \top \sqsubseteq C$ then $M^\infty \models \top \sqsubseteq C$.

Suppose to the contrary that, for some $\mathcal{ALC}$ concept $C$, there is $n \in \mathbb{N}$ such that, for all $m \geq n$, with $m \in \mathbb{N}$, we have that $M^m \models \top \sqsubseteq C$ but $M^\infty \not\models \top \sqsubseteq C$. If $M^\infty \not\models \top \sqsubseteq C$ then there is $k \in \mathbb{N}$ such that $k \notin C^{M^\infty}$. By definition of $k \in \mathbb{N}$ and the models of the form $M^n$ (recall that the domain of such models is $\mathbb{N}$), for all $m \in \mathbb{N}$, there is a bisimulation between $M^{m-k}$ and $M^m$ containing $(a^{M^{m-k}}, k)$. Since $\mathcal{ALC}$ is invariant under bisimulations, for all $m' \geq m-k$, we have that $a^{M^{m'}} \in C^{M^{m'}}$. Then, by Claim 24, $a^{M^\infty} \in C^{M^\infty}$. By definition of $k \in \mathbb{N}$ and $M^\infty$, there is a bisimulation between $M^\infty$ and itself (that is, $M^\infty$) containing $(a^{M^\infty}, k)$. Therefore, $k \in C^{M^\infty}$.

We are now ready to show that $\Lambda(\mathcal{ALC})$ is not eviction-compatible. Suppose to the contrary that there is $\mathcal{B} \in \mathcal{P}_f(\mathcal{L}_{\mathcal{ALC}})$ such that $\mathrm{Mod}(\mathcal{B}) \in \mathrm{MaxFRSubs}(\mathbb{M}, \Lambda(\mathcal{ALC}))$. We can assume w.l.o.g. that $\mathcal{B}$ is of the form $\{\top \sqsubseteq D, C(a)\}$. Indeed, if it contains e.g. $C_1(a), \ldots, C_k(a)$ then this is equivalent to $C_1 \sqcap \ldots \sqcap C_k(a)$. Also, concept inclusions $C_1 \sqsubseteq D_1, \ldots, C_k \sqsubseteq D_k$ can be equivalently rewritten as $\top \sqsubseteq ((\neg C_1 \sqcup D_1) \sqcap \ldots \sqcap (\neg C_k \sqcup D_k))$.

If there is $n \in \mathbb{N}$ such that $M^n \not\models \mathcal{B}$ then[3]

$$\mathcal{B}' := \{\top \sqsubseteq D \sqcup (\exists r^{n+1}.\top \sqcap \neg \exists r^{n+2}.\top),$$

$$C \sqcup (\exists r^{n+1}.\top \sqcap \neg \exists r^{n+2}.\top)(a)\}$$

is such that $M^n \models \mathcal{B}'$. Moreover, $\mathrm{Mod}(\mathcal{B}) \subset \mathrm{Mod}(\mathcal{B}')$. By definition of $\mathcal{B}'$ and $\mathbb{M}$, we also have that $\mathrm{Mod}(\mathcal{B}') \in \mathrm{MaxFRSubs}(\mathbb{M}, \Lambda(\mathcal{ALC}))$. This contradicts the assumption that $\mathrm{Mod}(\mathcal{B}) \in \mathrm{MaxFRSubs}(\mathbb{M}, \Lambda(\mathcal{ALC}))$. So, for all $n \in \mathbb{N}$, we have that $M^n \models \mathcal{B}$.

Then, by Claims 24 and 25, it follows that $M^\infty \models \mathcal{B}$. Since, as already mentioned, $M^\infty \notin \mathbb{M}$, this contradicts the assumption that $\mathrm{Mod}(\mathcal{B}) \in \mathrm{MaxFRSubs}(\mathbb{M}, \Lambda(\mathcal{ALC}))$. Thus, $\mathrm{MaxFRSubs}(\mathbb{M}, \Lambda(\mathcal{ALC})) = \emptyset$. □

We now prove Theorem C.26 (the signature is infinite).

**Theorem C.26.** $\Lambda(\mathcal{ALC})$ is not reception-compatible.

*Proof.* Let $\Lambda(\mathcal{ALC}) = (\mathcal{L}_{\mathcal{ALC}}, \mathfrak{M}_{\mathcal{ALC}}, \models_{\mathcal{ALC}})$ be the usual satisfaction system for $\mathcal{ALC}$. For conciseness, we will write $\models$ instead of $\models_{\mathcal{ALC}}$ within this proof. Assume for contradiction that $\Lambda(\mathcal{ALC})$ is reception-compatible.

Consider the signature $\mathsf{N_C} = \{C_i \mid i \in \mathbb{N}\}$, $\mathsf{N_I} = \{a_i \mid i \in \mathbb{N}\}$, and $\mathsf{N_R}$ an arbitrary countably infinite set disjoint with $\mathsf{N_C} \cup \mathsf{N_R}$. Also, consider the model $M = (\Delta^M, \cdot^M)\}$ where $\Delta^M = \mathbb{N}$, and $\cdot^M$ is such that $r^M = \emptyset$ for all $r \in \mathsf{N_R}$ and $A_i^M = \{a_i\}$ and $a_i^M = i$ for all $i \in \mathbb{N}$. Now, let $\mathcal{B}_\perp = \{\top \sqsubseteq \perp\}$, $\mathcal{B}_\perp$ is inconsistent (it has no models).

By hypothesis, $\Lambda(\mathcal{ALC})$ is reception-compatible which means that there is a $\mathcal{B} \in \mathcal{P}_f(\mathcal{L}_{\mathcal{ALC}})$ such that $\mathrm{Mod}(\mathcal{B}) = \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}_\perp, \{M\}))$, that is, $\mathrm{Mod}(\mathcal{B}) \in \mathrm{MinFRSups}(\{M\}, \Lambda(\mathcal{ALC}))$. Let

$$J = \{i \in \mathbb{N} \mid \forall M', M'' \in \mathrm{Mod}(\mathcal{B}),$$
$$M' \models A_i(a_i) \text{ iff } M'' \models A_i(a_i)\}$$

We have two cases: either (i) $J \neq \mathbb{N}$, or (ii) $J = \mathbb{N}$. In all cases, we will reach a contradiction, and therefore we conclude that $\Lambda(\mathcal{ALC})$ is not reception-compatible.

(i) $J \neq \mathbb{N}$. Then $\mathcal{B}$ does not specify whether some $A_k(a_k)$ with $k \in \mathbb{N} \setminus J$ holds or not, that is, it will have both models in which $A_k(a_k)$ holds and models in which $\neg A_k(a_k)$ holds. We can build a base $\mathcal{B}' = \mathcal{B} \cup \{A_k(a_k)\}$. The base $\mathcal{B}'$ is finite, $M \in \mathrm{Mod}(\mathcal{B}')$, and $\mathrm{Mod}(\mathcal{B}') \subset \mathrm{Mod}(\mathcal{B})$. Hence, $\mathrm{Mod}(\mathcal{B}) \notin \mathrm{MinFRSups}(\{M\}, \Lambda(\mathcal{ALC}))$, a contradiction.

[3]Recall that $M^n$ has a chain of $n+1$ elements connected via the role $r$.

(ii) $J = \mathbb{N}$. In this case, $M \models A_i(a_i)$ for all $M \in \mathrm{Mod}(\mathcal{B})$ and all $i \in \mathbb{N}$. Without loss of generality, we can assume that $\mathcal{B}$ has a single concept inclusion $\top \sqsubseteq P$, with $P$ an $\mathcal{ALC}$ concept. Moreover, as $\mathcal{B}$ is finite, it can only have finitely many assertions. Thus, we write $\mathcal{B}$ as

$$\mathcal{B} = \{\top \sqsubseteq P\} \cup \{C_k(a_k) \mid k \in K\},$$

where $K$ is a finite subset of $\mathbb{N}$ and $P$ and $C_k$ are $\mathcal{ALC}$ concepts for all $k \in K$. Let $j \in \mathbb{N} \setminus K$. As the assertions cannot enforce $A_j(a_j)$, we have, by the monotonicity of $\mathcal{ALC}$, that $\models \top \sqsubseteq P$ must entail $A_j(a_j)$, in other words, $P \sqsubseteq A_j$ must be a tautology. But this also implies that $\top \sqsubseteq A_j$ must hold in every model of $\{\top \sqsubseteq P\}$. However, $M \not\models A_j(a_i)$ for all $i \neq j$, therefore $M$ is not a model of $\{\top \sqsubseteq P\}$ ($M \notin \mathrm{Mod}(\{\top \sqsubseteq P\})$). Additionally, by semantics and monotonicity of $\mathcal{ALC}$, we have that $\mathrm{Mod}(\mathcal{B}) \subseteq \mathrm{Mod}(\{\top \sqsubseteq P\})$, thus $M \notin \mathrm{Mod}(\mathcal{B})$, a contradiction.

Therefore, there is no finite base $\mathcal{B}$ such that: $\mathrm{Mod}(\mathcal{B}) \in \mathrm{FR}(\Lambda(\mathcal{ALC}))$, $M \in \mathrm{Mod}(\mathcal{B})$ and $\mathrm{Mod}(\mathcal{B})$ is $\subseteq$-minimal. Consequently, $\mathrm{MinFRSups}(\{M\}, \Lambda(\mathcal{ALC})) = \emptyset$. Hence, $\Lambda(\mathcal{ALC})$ is not reception-compatible. □

**Theorem 25.** $\Lambda(\mathcal{ALC})$ is neither reception-compatible nor eviction-compatible.

*Proof.* Direct consequence of Theorems C.22 and C.26 □

We now consider a simpler satisfaction system that we called $\Lambda(\text{ABox})$. We point out that we need negative assertions to be eviction-compatible since we cannot express contradiction with only positive assertions (logics that cannot express contradiction are not eviction-compatible).

**Theorem 26.** $\Lambda(\text{ABox})$ is not reception-compatible but it is eviction-compatible.

*Proof.* The proof that $\Lambda(\text{ABox})$ is not reception-compatible is similar to the proof of Theorem C.26, but simpler since we only need to consider assertions (not concept inclusions).

We now show that $\Lambda(\text{ABox})$ is eviction-compatible. For this we need to prove that, for every set of (positive and negative) assertions $\mathcal{O}$—we call it an ABox ontology—and every $\mathbb{M} \subseteq \mathfrak{M}$, we have that $\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{O}) \setminus \mathbb{M}, \Lambda(\text{ABox})) \neq \emptyset$.

Suppose to the contrary that there exists an ABox ontology $\hat{\mathcal{O}}$ and a set $\mathbb{M} \subseteq \mathfrak{M}$ such that $\mathrm{MaxFRSubs}(\mathrm{Mod}(\hat{\mathcal{O}}) \setminus \mathbb{M}, \Lambda(\text{ABox})) = \emptyset$. By definition of $\mathrm{MaxFRSubs}(\mathrm{Mod}(\hat{\mathcal{O}}) \setminus \mathbb{M}, \Lambda(\text{ABox}))$, this can only happen if either

- there is no ABox ontology $\mathcal{O}$ such that $\mathrm{Mod}(\mathcal{O}) \subseteq (\mathrm{Mod}(\hat{\mathcal{O}}) \setminus \mathbb{M})$, or
- for all $i \in \mathbb{N}$, there are ABox ontologies $\mathcal{O}_i, \mathcal{O}_{i+1}$ such that $\mathrm{Mod}(\mathcal{O}_i) \subset \mathrm{Mod}(\mathcal{O}_{i+1}) \subseteq (\mathrm{Mod}(\hat{\mathcal{O}}) \setminus \mathbb{M})$.

The former cannot happen since we can express contradiction in our restricted language (so there is always an ABox ontology $\mathcal{O}$, e.g. $A(a), \neg A(a)$, such that $\mathrm{Mod}(\mathcal{O}) = \emptyset$ and this is for sure a subset of $(\mathrm{Mod}(\hat{\mathcal{O}}) \setminus \mathbb{M})$). The fact that the latter also cannot happen is because given two ontologies $\mathcal{O}, \mathcal{O}'$ in this restricted language we have that $\mathrm{Mod}(\mathcal{O}) \subset \mathrm{Mod}(\mathcal{O}')$ iff $\mathcal{O}' \subset \mathcal{O}$. So there cannot be an infinite sequence of ABox ontologies $\mathcal{O}_i$ such that, for all $i \in \mathbb{N}$, $\mathrm{Mod}(\mathcal{O}_i) \subset \mathrm{Mod}(\mathcal{O}_{i+1})$ because this implies $\mathcal{O}_i \supset \mathcal{O}_{i+1}$, for all $i \in \mathbb{N}$, and $\mathcal{O}_i, \mathcal{O}_{i+1}$ are finite. □

**Theorem 27.** $\Lambda(\text{DL-Lite}_{\mathcal{R}})$ (with finite signature) is reception-compatible and eviction-compatible.

*Proof.* We start proving that $\Lambda(\text{DL-Lite}_{\mathcal{R}})$ is eviction-compatible. For conciseness, we will write $\mathfrak{M}$ and $\models$ to represent, respectively, the universe of models and the satisfaction relation in $\Lambda(\text{DL-Lite}_{\mathcal{R}})$ in this proof. For this we need to prove that, for every DL-Lite$_{\mathcal{R}}$ ontology $\mathcal{O}$ and every $\mathbb{M} \subseteq \mathfrak{M}$, we have that $\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{O}) \setminus \mathbb{M}, \Lambda(\text{DL-Lite}_{\mathcal{R}})) \neq \emptyset$.

Suppose to the contrary that there exists a DL-Lite$_{\mathcal{R}}$ ontology $\hat{\mathcal{O}}$ and a set $\mathbb{M} \subseteq \mathfrak{M}$ such that $\mathrm{MaxFRSubs}(\mathrm{Mod}(\hat{\mathcal{O}}) \setminus \mathbb{M}, \Lambda(\text{DL-Lite}_{\mathcal{R}})) = \emptyset$. By definition of $\mathrm{MaxFRSubs}(\mathrm{Mod}(\hat{\mathcal{O}}) \setminus \mathbb{M}, \Lambda(\text{DL-Lite}_{\mathcal{R}}))$, this can only happen if either

- there is no DL-Lite$_{\mathcal{R}}$ ontology $\mathcal{O}$ such that $\mathrm{Mod}(\mathcal{O}) \subseteq (\mathrm{Mod}(\hat{\mathcal{O}}) \setminus \mathbb{M})$, or
- for all $i \in \mathbb{N}$, there are DL-Lite$_{\mathcal{R}}$ ontologies $\mathcal{O}_i, \mathcal{O}_{i+1}$ such that $\mathrm{Mod}(\mathcal{O}_i) \subset \mathrm{Mod}(\mathcal{O}_{i+1}) \subseteq (\mathrm{Mod}(\hat{\mathcal{O}}) \setminus \mathbb{M})$.

The former cannot happen since we can express contradiction in DL-Lite$_{\mathcal{R}}$ (so there is always a DL-Lite$_{\mathcal{R}}$ ontology $\mathcal{O}$ such that $\mathrm{Mod}(\mathcal{O}) = \emptyset$ and this is for sure a subset of $(\mathrm{Mod}(\hat{\mathcal{O}}) \setminus \mathbb{M})$). The fact that the latter also cannot happen is a consequence of the following two claims.

**Claim 27.** Given a satisfiable DL-Lite$_{\mathcal{R}}$ ontology $\mathcal{O}$ (over a finite signature $\mathsf{N_C} \cup \mathsf{N_R} \cup \mathsf{N_I}$), we have that the DL-Lite$_{\mathcal{R}}$ ontology $\mathcal{O}^t = \{\alpha \mid \mathcal{O} \models \alpha\}$ is finite.

We first argue that the number of possible DL-Lite$_{\mathcal{R}}$ concept and role inclusions that can be formulated with a finite signature $\mathsf{N_C} \cup \mathsf{N_R} \cup \mathsf{N_I}$ is finite. Indeed, concepts are of the form $A, \neg A, \exists r, \neg\exists r, \exists r^-, \neg\exists r^-$ and role expressions are of the form $R, R^-, \neg R, \neg R^-$. So if the number of concept names plus the number of role names is $n$, then there are at most $(6n)^2$ possible concept inclusions and at most $(4n)^2$ possible role inclusions (with concept and role names occurring in $\mathcal{O}$). This finishes the proof of the claim.

**Claim 28.** Let $\mathcal{O}, \mathcal{O}'$ be a satisfiable DL-Lite$_{\mathcal{R}}$ ontologies. If $\mathrm{Mod}(\mathcal{O}) \subset \mathrm{Mod}(\mathcal{O}')$ then $\mathcal{O}^t \supset \mathcal{O}'^t$.

If $\mathrm{Mod}(\mathcal{O}) \subset \mathrm{Mod}(\mathcal{O}')$ then $\mathcal{O} \models \mathcal{O}'$. This means that if $\mathcal{O}' \models \alpha$ then $\mathcal{O} \models \alpha$. So if $\alpha$ is in $\mathcal{O}'^t$ then it is in $\mathcal{O}^t$. In other words, $\mathcal{O}^t \supset \mathcal{O}'^t$. This finishes the proof of the claim.

By Claims 27 and 28 there cannot be an infinite sequence of DL-Lite$_{\mathcal{R}}$ ontologies $\mathcal{O}_i$ such that, for all $i \in \mathbb{N}$,

$\mathrm{Mod}(\mathcal{O}_i) \subset \mathrm{Mod}(\mathcal{O}_{i+1})$ because this implies $\mathcal{O}_i^t \supset \mathcal{O}_{i+1}^t$, for all $i \in \mathbb{N}$, and $\mathcal{O}_i^t, \mathcal{O}_{i+1}^t$ are finite.

The proof that $\Lambda(\text{DL-Lite}_{\mathcal{R}})$ is reception-compatible is similar. For this we need to prove that, for every DL-Lite$_{\mathcal{R}}$ ontology $\mathcal{O}$ and every $\mathbb{M} \subseteq \mathfrak{M}$, we have that $\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{O}) \cup \mathbb{M}, \Lambda(\text{DL-Lite}_{\mathcal{R}})) \neq \emptyset$.

Suppose to the contrary that there exists a DL-Lite$_{\mathcal{R}}$ ontology $\hat{\mathcal{O}}$ and a set $\mathbb{M} \subseteq \mathfrak{M}$ such that $\mathrm{MinFRSups}(\mathrm{Mod}(\hat{\mathcal{O}}) \cup \mathbb{M}, \Lambda(\text{DL-Lite}_{\mathcal{R}})) = \emptyset$. By definition of $\mathrm{MinFRSups}(\mathrm{Mod}(\hat{\mathcal{O}}) \cup \mathbb{M}, \Lambda(\text{DL-Lite}_{\mathcal{R}}))$, this can only happen if either

- there is no DL-Lite$_{\mathcal{R}}$ ontology $\mathcal{O}$ such that $(\mathrm{Mod}(\hat{\mathcal{O}}) \cup \mathbb{M}) \subseteq \mathrm{Mod}(\mathcal{O})$, or
- for all $i \in \mathbb{N}$, there are DL-Lite$_{\mathcal{R}}$ ontologies $\mathcal{O}_i, \mathcal{O}_{i+1}$ such that $(\mathrm{Mod}(\hat{\mathcal{O}}) \cup \mathbb{M}) \subseteq \mathrm{Mod}(\mathcal{O}_{i+1}) \subset \mathrm{Mod}(\mathcal{O}_i)$.

The former cannot happen since we can express tautologies in DL-Lite$_{\mathcal{R}}$ (so there is always a DL-Lite$_{\mathcal{R}}$ ontology $\mathcal{O}$ such that $\mathrm{Mod}(\mathcal{O}) = \mathfrak{M}$ and this is for sure a superset of $(\mathrm{Mod}(\hat{\mathcal{O}}) \cup \mathbb{M})$). The latter is a consequence of Claims 27 and 28. There cannot be an infinite sequence of DL-Lite$_{\mathcal{R}}$ ontologies $\mathcal{O}_i$ such that, for all $i \in \mathbb{N}$, $\mathrm{Mod}(\mathcal{O}_{i+1}) \subset \mathrm{Mod}(\mathcal{O}_i)$ because this implies $\mathcal{O}_i^t \subset \mathcal{O}_{i+1}^t$, for all $i \in \mathbb{N}$, and $\mathcal{O}_i^t, \mathcal{O}_{i+1}^t$ are bounded by a polynomial in the size of the finite signature (see proof of Claim 27). $\square$