Belief Change with Models as Input

Adapting Expansion and Contraction

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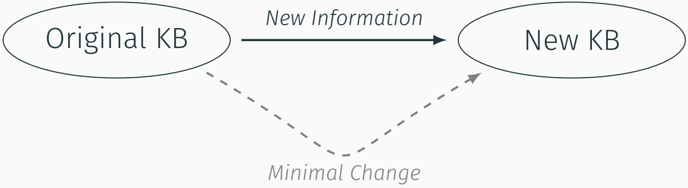
Outline

Belief Change

Belief Change [AGM85; Han99]



KBs: sets of formulas

Input: formula(s) to be accepted/rejected

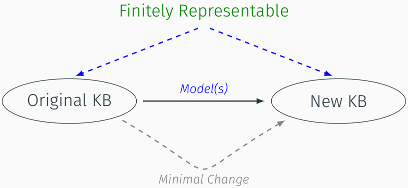
Belief Change Triad

Operations: types of rational change

Postulates: properties that must be satisfied

Constructions: syntactic or semantic methods to implement the operations

Proposal



- Minimise loss of information

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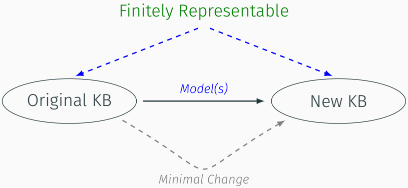

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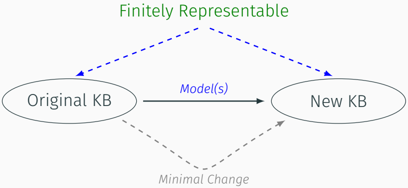

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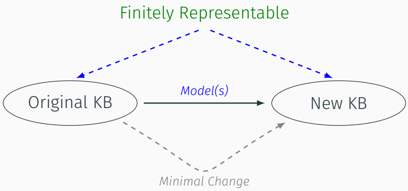

- Minimise loss of information
- Remove or add ONLY the necessary **models**
- Indifferent about syntactical structure
- Preserve finite representability

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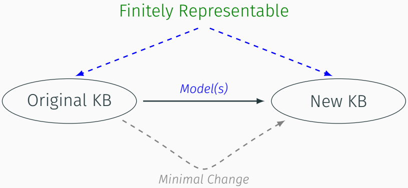
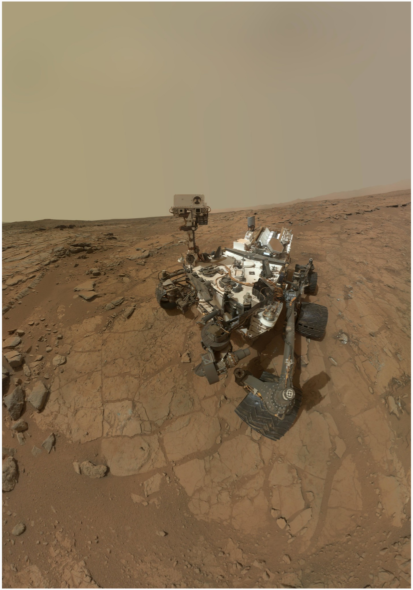


- Minimise loss of information
- Remove or add ONLY the necessary **models**
- Indifferent about syntactical structure
- Preserve finite representability
- Few assumptions about the logic

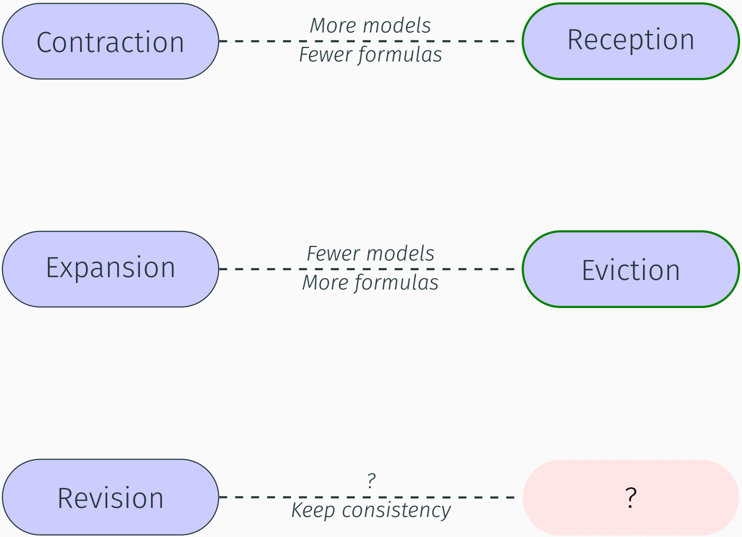
Motivation

- Finiteness: algorithms, limited storage capacity
- Models as input: flexibility
- Settings: learning from interpreations ([De 97]), Horn theories from graphs ([AKM07]), Learning Theory and Iterated Belief Change [BGS18]



[NAS13]

Model Change Operations



Finite Based Change via Models

Finite Based Contraction and Expansion via Models [GOS23]

- To appear in AAAI 2023
- Joint work with Ana Ozaki and Jandson S. Ribeiro

Logic as a Satisfaction System

Language

Universe of Models

$$\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$$

Satisfaction System

Satisfaction Relation

Logic as a Satisfaction System

$$\underbrace{\Lambda}_{\text{Satisfaction System}} = (\overbrace{\mathcal{L}}^{\text{Language}}, \overbrace{\mathfrak{M}}^{\text{Universe of Models}}, \underbrace{\models}_{\text{Satisfaction Relation}})$$

Example: propositional logic with signature $\{p, q\}$

$\mathcal{L}_{\mathsf{Prop}}$: p, q, $\neg p$, $p \wedge q$, $p \wedge p$, ...

$\mathfrak{M}_{\mathsf{Prop}}$: $\{\bar{p}\bar{q}, \bar{p}q, p\bar{q}, pq\}$

$\models_{\mathsf{Prop}}$: the usual satisfaction relation

Finitely Representable Sets of Models

$X \in \mathrm{FR}(\Lambda)$ iff

- $X \subseteq \mathfrak{M}$
- There is $\mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\mathcal{L})$ with $\mathrm{Mod}(\mathcal{B}) = X$

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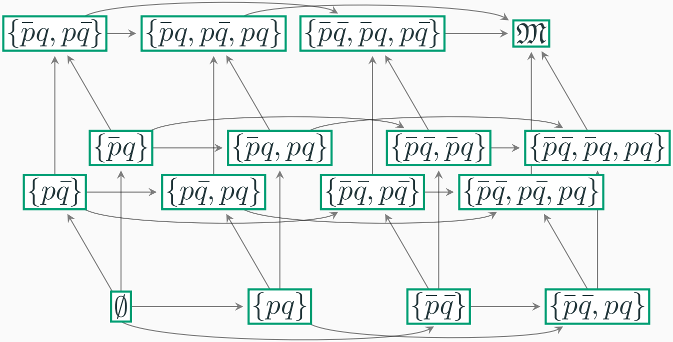

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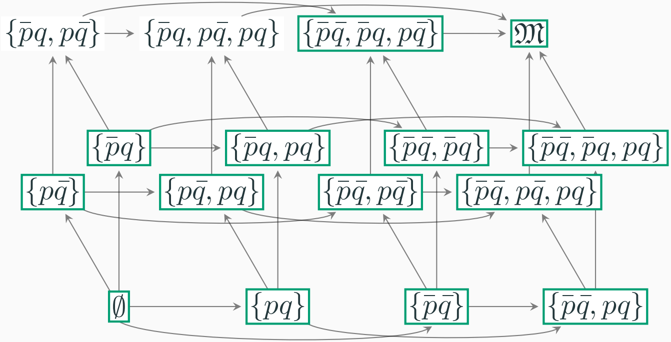
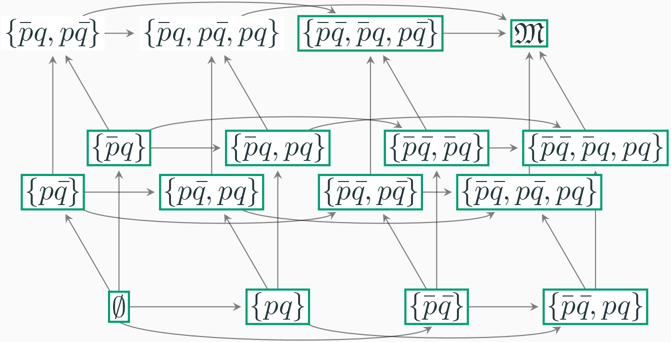


Selecting Sets of Models

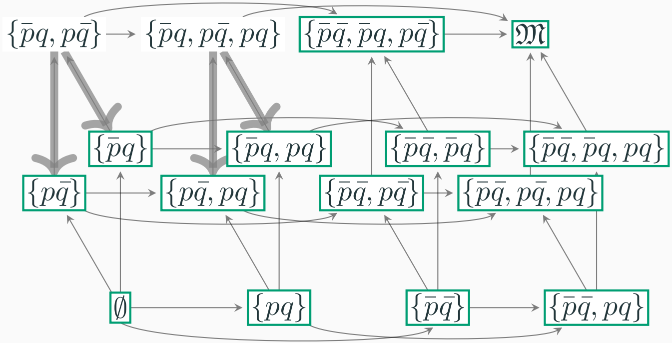
FR selection function: $\mathrm{sel} : \mathcal{P}^*(\mathrm{FR}(\Lambda)) \to \mathrm{FR}(\Lambda)$

$$f(\{\{\bar{p}\bar{q}\}, \{\bar{p}q, p\bar{q}\}, \mathfrak{M}\}) = \{\bar{p}q, p\bar{q}\}$$



Maximal, finitely representable subsets

$$\mathrm{MaxFRSubs}(\{\bar{p}\bar{q}, \bar{p}q, p\bar{q}\}, \Lambda(\mathsf{Horn})) = \{\ \overbrace{\{\bar{p}\bar{q}, \bar{p}q, p\bar{q}\}}^{\mathrm{Mod}(p \wedge q \to \bot)}\ \}$$



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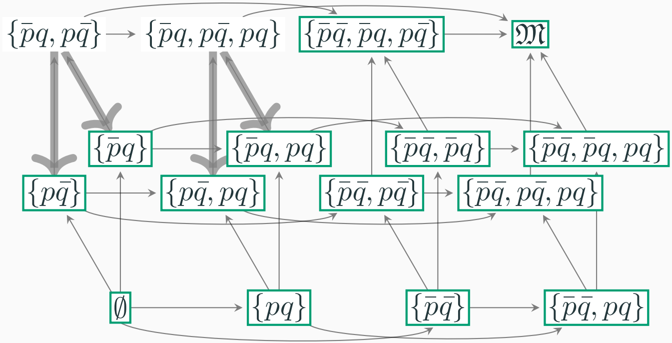
$$\mathrm{MaxFRSubs}(\{\bar{p}q, p\bar{q}, pq\}, \Lambda(\mathsf{Horn})) = \{ \underbrace{\{\bar{p}q, pq\}}_{\mathrm{Mod}(\bot \to q)}, \underbrace{\{p\bar{q}, pq\}}_{\mathrm{Mod}(\bot \to p)} \}$$



Eviction

sel: selects a finitely representable set of models

$\mathcal{B}$: current base

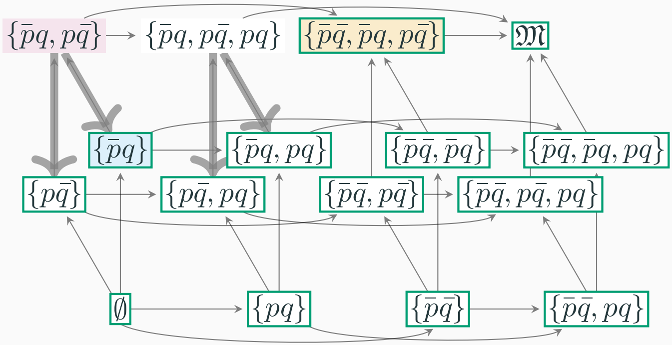
$\mathbb{M}$: models to remove

$$\mathrm{Mod}(\mathrm{evc}_{\mathrm{sel}}(\mathcal{B}, \mathbb{M})) = \mathrm{sel}(\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda)).$$

Eviction: Example with $\Lambda(\text{Horn})$

$$\mathrm{Mod}(\mathrm{evc}(\{p \land q \to \bot\}, \{\bar{p}\bar{q}\})) = \mathrm{sel}(\mathrm{MaxFRSubs}(\{\bar{p}q, p\bar{q}\})) =$$

$$\mathrm{sel}(\{\{\bar{p}q\}, \{p\bar{q}\}\}) = \mathrm{Mod}(\{\neg p, q\})$$



Eviction: Postulates

$$\mathrm{Mod}(\mathrm{evc}_{\mathrm{sel}}(\mathcal{B}, \mathbb{M})) = \mathrm{sel}(\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda)).$$

(success) all the required models are removed

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(uniformity) if $\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda) = \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}') \setminus \mathbb{M}', \Lambda)$, then the final result is the same

MinFRSups

Minimal, finitely representable supersets

$\mathrm{Mod}(p \wedge q \to \bot)$

$$\mathrm{MinFRSups}(\{\bar{p}\bar{q}, \bar{p}q, p\bar{q}\}, \Lambda(\textsf{Horn})) = \{ \{\bar{p}\bar{q}, \bar{p}q, p\bar{q}\} \}$$

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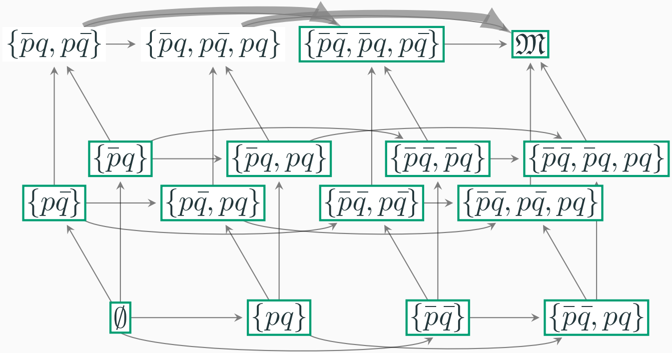

sel: selects a finitely representable set of models
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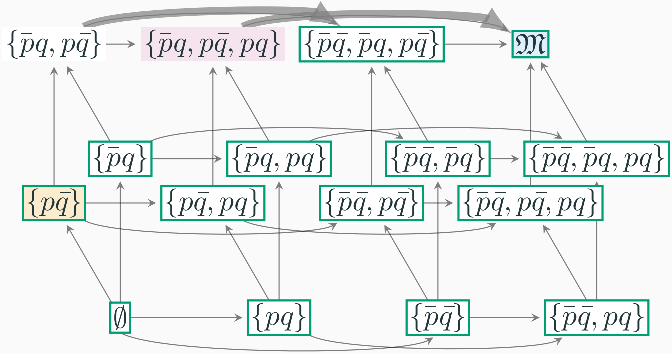
$$\mathrm{Mod}(\mathrm{rcp}_{\mathrm{sel}}(\mathcal{B}, \mathbb{M})) = \mathrm{sel}(\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda)).$$

Reception: Example with $\Lambda(\mathrm{Horn})$

$$\mathrm{Mod}(\mathrm{rcp}(\{p, \neg q\}, \{\bar{p}q, pq\})) = \mathrm{sel}(\mathrm{MaxFRSubs}(\{\bar{p}q, p\bar{q}, pq\})) = \mathrm{sel}(\{\mathfrak{M}\}) = \mathrm{Mod}(\{\})$$



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Eviction & Reception: Propositional Case

$$\mathrm{evc}_{\mathsf{Prop}}(\mathcal{B}, \mathbb{M}) = \bigvee_{v \in \mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}} \left(\bigwedge_{v(a)=\mathsf{T}} a \wedge \bigwedge_{v(a)=\mathsf{F}} \neg a \right)$$

$$\mathrm{rcp}_{\mathsf{Prop}}(\mathcal{B}, \mathbb{M}) = \bigvee_{v \in \mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}} \left(\bigwedge_{v(a)=\mathsf{T}} a \wedge \bigwedge_{v(a)=\mathsf{F}} \neg a \right)$$

Is it always this easy?

Compatibility

Compatible Satisfaction Systems

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Compatible Satisfaction Systems

- Can we always define eviction and reception?
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- Example: Horn logic without $\perp$
- Corollary [GOS23]: if $\mathrm{FR}(\Lambda)$ is finite then:
  - Λ is eviction-compatible iff $\emptyset \in \mathrm{FR}(\Lambda)$
  - Λ is reception-compatible iff $\mathfrak{M} \in \mathrm{FR}(\Lambda)$

Truth values: $\mathsf{F} < \mathsf{U} < \mathsf{T}$

$\mathcal{L}_{\text{Prop}}$: propositional formulae

$\mathfrak{M}_3$: $v : \mathcal{L} \to \{\mathsf{F}, \mathsf{U}, \mathsf{T}\}$ s.t.

- $$v(\neg\varphi) = \begin{cases} \mathsf{T}, & \text{if } v(\varphi) = \mathsf{F} \\ \mathsf{U}, & \text{if } v(\varphi) = \mathsf{U} \\ \mathsf{F}, & \text{if } v(\varphi) = \mathsf{T} \end{cases}$$
- $v(\varphi \wedge \psi) = \min_<(\{v(\varphi), v(\psi)\})$
- $v(\varphi \vee \psi) = \max_<(\{v(\varphi), v(\psi)\})$

Kleene's

$\Lambda(\mathsf{K3}) = (\mathscr{L}_{\mathsf{Prop}}, \mathfrak{M}_3, \vDash_{K3})$

$$v \vDash_{K3} \varphi \text{ iff } v(\varphi) = \mathsf{T}$$

- $\emptyset \in \mathrm{FR}(\Lambda(\mathsf{K3}))$
- $\mathfrak{M}_3 \in \mathrm{FR}(\Lambda(\mathsf{K3}))$

Priest's

$\Lambda(\mathsf{P3}) = (\mathscr{L}_{\mathsf{Prop}}, \mathfrak{M}_3, \vDash_{P3})$

$$v \vDash_{P3} \varphi \text{ iff } v(\varphi) \neq \mathsf{F}$$

- $\emptyset \notin \mathrm{FR}(\Lambda(\mathsf{K3}))$
- $\mathfrak{M}_3 \in \mathrm{FR}(\Lambda(\mathsf{K3}))$

$v_{\mathsf{U}}(\varphi) = \mathsf{U}, \forall\varphi$ v_{U} is a model of every base in $\Lambda(\mathsf{P3})$

Propositional Gödel Logic

$\Lambda(\text{Gödel}, \theta) = (\mathcal{L}_{\mathsf{G}}, \mathfrak{M}_G, \models^\theta_G)$ with $\theta \in (0, 1]$

$\mathcal{L}_{\mathsf{G}}$: propositional formulas over `At` with $\wedge$, $\vee$, $\neg$, and $\rightarrow$;

$\mathfrak{M}_G$: $v : \mathcal{L} \to [0, 1]$ s.t.

$$v(\neg\varphi) = \begin{cases} 1 & \text{if } v(\varphi) = 0, \\ 0 & \text{otherwise;} \end{cases}$$

$$v(\varphi \wedge \psi) = \min(v(\varphi), v(\psi));$$

$$v(\varphi \vee \psi) = \max(v(\varphi), v(\psi));$$

$$v(\varphi \to \psi) = \begin{cases} 1 & \text{if } v(\varphi) \leq v(\psi), \\ v(\psi) & \text{otherwise; and} \end{cases}$$

$\models^\theta_G$: $v \models^\theta_G \mathcal{B}$ iff $\mathcal{B} = \emptyset$ or $v(\bigwedge_{\varphi \in B} \varphi) \geq \theta$

Compatibilities for Gödel Logic

- $\mathfrak{M}_G$ is infinite now!

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- However, $v \models^{\theta}_{G} \mathcal{B}$ depends only on the total preorder defined over finitely many symbols
- $\mathrm{FR}(\Lambda(\text{Gödel}, \theta))$ is finite
- $\mathrm{Mod}(\{\neg a \wedge a\}) = \emptyset$ and $\mathrm{Mod}(\{\}) = \mathfrak{M}_G$
- Therefore, $\Lambda(\text{Gödel}, \theta)$ is eviction- and reception-compatible

Infinite $\mathrm{FR}(\Lambda)$

$\Lambda_q = (\mathcal{L}_q, \mathfrak{M}_q, \vDash_q)$

- $\mathcal{L}_q = \{[x, y] \mid x, y \in \mathbb{Q}$ and $x \leq y\}$
- $\mathfrak{M}_q = \mathbb{Q}$
- $z \vDash_q \mathcal{B}$ iff $x \leq z \leq y$ for every $[x, y] \in \mathcal{B}$.

Example:

$$0.3 \vDash_q \{[0, 0.25], [0.12, 0.80]\}$$

Too Many Candidates

We can have incompatibility even if $\{\emptyset, \mathfrak{M}\} \in \mathrm{FR}(\Lambda)$

$$\Lambda_q = (\mathcal{L}_q, \mathfrak{M}_q, \models_q)$$

$$\mathrm{evc}(\{[0,1]\}, \{1\}) = [0, 0.9]$$

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Description Logics

- Fragments of guarded dyadic FOL (usually)
- Good computational properties (usually decidable)
- Each fragment has a different expressivity and complexities
- Formalisation of OWL ontologies (used in data integration, Medical fields, Biology)

The Description Logic $\mathcal{ALC}$: Signature and Constructors

Concept names ($\mathsf{N_C}$): $Person, Student, University$
Role names ($\mathsf{N_R}$): $studiesAt, supervisorOf$
Individual names ($\mathsf{N_I}$): `Ann`, `UiB`

$$C ::= \top \mid A \mid \neg C \mid (C \sqcap C) \mid \exists r.C$$

Complex concepts

- $Person \sqcap \neg Student$
- $\exists worksAt.\top$
- $Lawyer \sqcap \neg \exists hasPet.Dog$

Concept Inclusions: $Student \sqsubseteq Person \sqcap \exists studies.\top$,
$Professor \sqsubseteq \exists worksAt.University$

Concept Assertions: $Student(\mathsf{Ann})$, $University(\mathtt{UiB})$

Role Assertions: $worksAt(\mathsf{Bob}, \mathtt{UiB})$, $studies(\mathtt{Carol}, \mathtt{Logic})$

Models are interpretations over the signature:

$$\mathcal{J} = (\underbrace{\Delta^{\mathcal{J}}}_{\text{Domain}} , \underbrace{\cdot^{\mathcal{J}}}_{\text{Mapping function}})$$

What do we know?

Satisfaction System	Compatible	
	Eviction	Reception
$\Lambda(\mathrm{Prop})$	Yes	Yes
$\Lambda(\mathrm{Horn})$	Yes	Yes
$\Lambda(\mathrm{K3})$	Yes	Yes
$\Lambda(\mathrm{P3})$	No	Yes
$\Lambda(\text{Gödel}, \theta)$	Yes	Yes
$\Lambda(\mathrm{LTL}_X)$	No	Yes
$\Lambda(\mathrm{ABox})$	Yes	No
$\Lambda(\text{DL-Lite}_{\mathcal{R}})$[1]	Yes	Yes
$\Lambda(\mathcal{ALC})$	No	No

[1]with finite signature

Eviction

- $\mathbb{M} \in \mathbf{FR}(\Lambda)$
- $\mathbb{M}$ has a FR immediate predecessor
- There is no superset of $\mathbb{M}$ in $\mathbf{FR}(\Lambda)$

Reception

- $\mathbb{M} \in \mathbf{FR}(\Lambda)$
- $\mathbb{M}$ has a FR immediate successor
- There is no subset of $\mathbb{M}$ in $\mathbf{FR}(\Lambda)$

Concluding Remarks

- Belief Change: modifying knowledge bases rationally
- New setting: finite representation and models as input
- Definition and characterisation of eviction and reception
- Compatibility in different logics
- Future work: what to do when the system is incompatible?

Thank you!

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- `https://rfguimaraes.github.io`

License

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