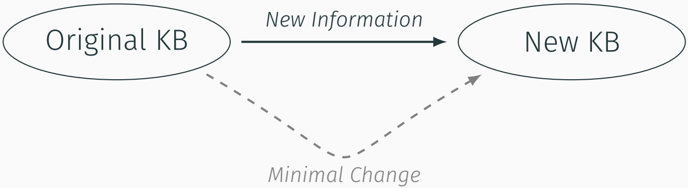
Finite Base Change via Models

Ricardo Guimarães - University of Bergen
Jandson S. Ribeiro - FernUniversität in Hagen
Ana Ozaki - University of Bergen

SLSS 2022

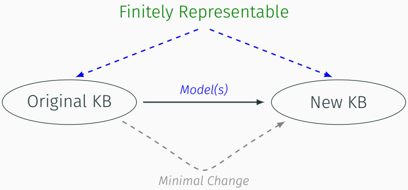
Belief Change [AGM85; Han99]



KBs: sets of formulas

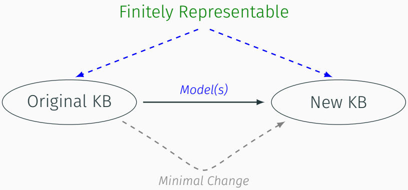
Input: formula(s) to be accepted/rejected

Proposal



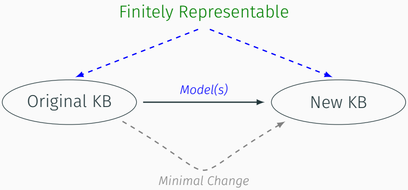
- Minimise loss of information

Proposal



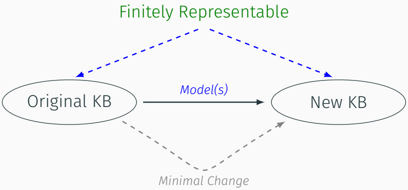
- Minimise loss of information
- Remove or add ONLY the necessary **models**

Proposal



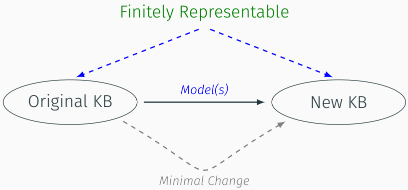
- Minimise loss of information
- Remove or add ONLY the necessary **models**
- Indifferent about syntactical structure

Proposal



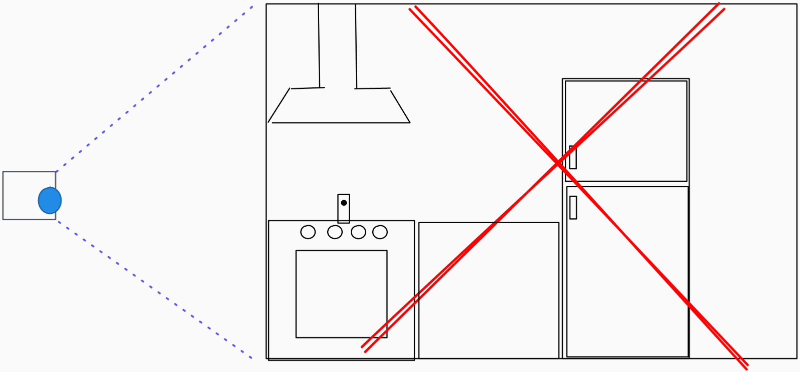
- Minimise loss of information
- Remove or add ONLY the necessary **models**
- Indifferent about syntactical structure
- Preserve finite representability

Proposal

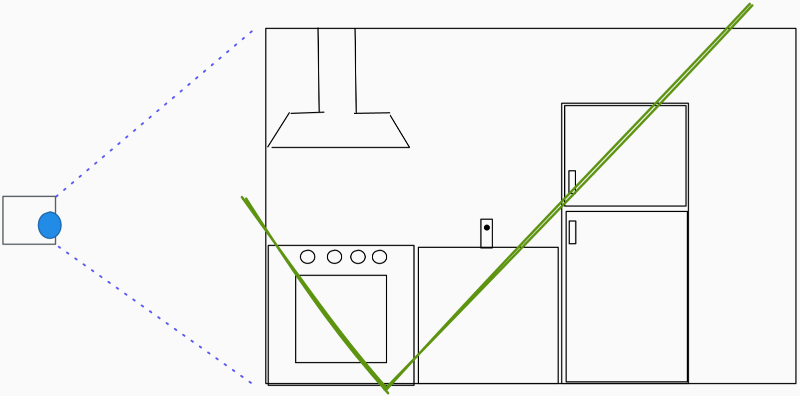


- Minimise loss of information
- Remove or add ONLY the necessary **models**
- Indifferent about syntactical structure
- Preserve finite representability
- Few assumptions about the logic

Motivation



Motivation

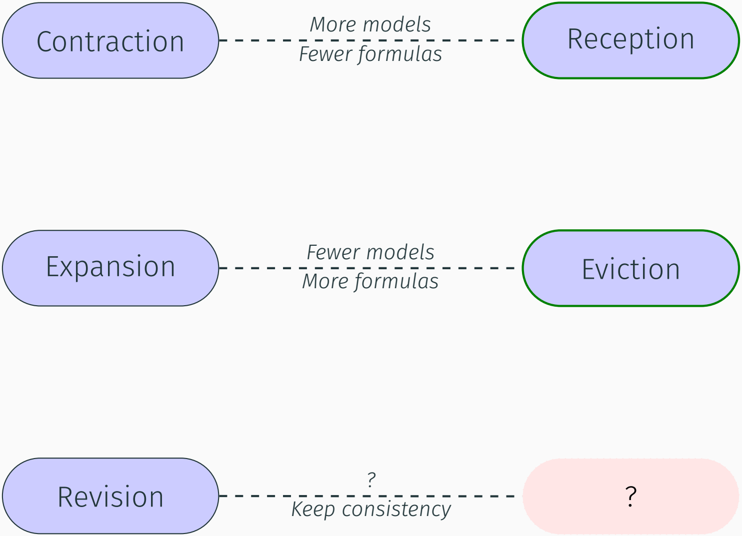


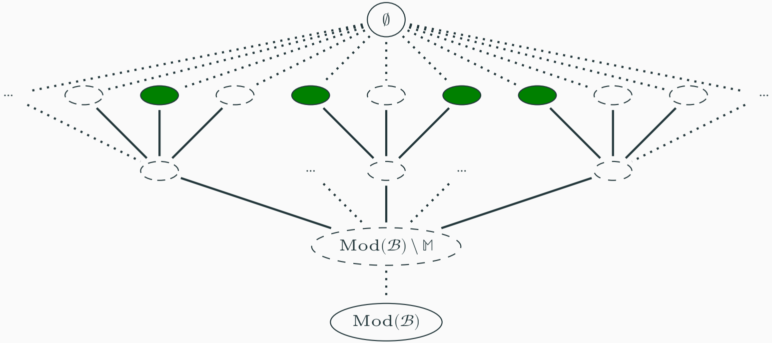
Logic as a Satisfaction System

$$\Big(\overbrace{\mathcal{L}}^{\text{language}}, \underbrace{\mathfrak{M}}_{\text{universe of models}}, \overbrace{\models}^{\text{satisfaction relation}}\Big)$$

The satisfaction relation just maps sets of formulas to sets of models

Model Change Operations





$$\mathrm{evc}(\mathcal{B}, M) = \mathrm{selKB}(\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus M))$$

Eviction (Ideal): Characterisation and Postulates

Eviction (Ideal): Characterisation and Postulates

The input model must be removed

(success) $\mathbb{M} \cap \mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \emptyset.$

Eviction (Ideal): Characterisation and Postulates

Do not add new models

(success) $\mathbb{M} \cap \mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \emptyset$.

(inclusion) $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \subseteq \mathrm{Mod}(\mathcal{B})$.

Eviction (Ideal): Characterisation and Postulates

Only lose other models to ensure finite representability

(success) $\mathbb{M} \cap \mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \emptyset$.

(inclusion) $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \subseteq \mathrm{Mod}(\mathcal{B})$.

(finite retainment) If $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \subset \mathbb{M}' \subseteq \mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}$ then $\mathbb{M}' \notin \mathrm{FRSets}(\Lambda)$.

Eviction (Ideal): Characterisation and Postulates

The result is determined by $\mathrm{MaxFRSubs}$ (regardless of actual inputs)

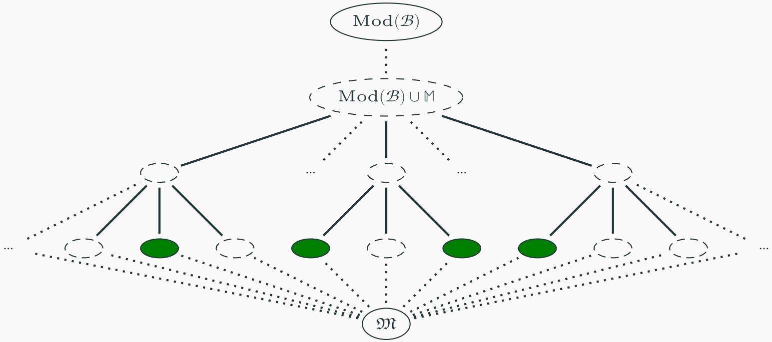
(success) $\mathbb{M} \cap \mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \emptyset$.

(inclusion) $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \subseteq \mathrm{Mod}(\mathcal{B})$.

(finite retainment) If $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \subset \mathbb{M}' \subseteq \mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}$ then $\mathbb{M}' \notin \mathrm{FRSets}(\Lambda)$.

(uniformity) If $\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B} \setminus \mathbb{M}), \Lambda) = \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}') \setminus \mathbb{M}', \Lambda)$ then $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{evc}(\mathcal{B}', \mathbb{M}'))$.

FBCvM: Reception (Ideal)



$$\mathrm{rcp}(\mathcal{B}, M) = \mathrm{selKB}(\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup M))$$

Reception (Ideal): Characterisation and Postulates

Reception (Ideal): Characterisation and Postulates

The input model must satisfy the result

(success) $\mathbb{M} \subseteq \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$.

Reception (Ideal): Characterisation and Postulates

Do not lose existing models

(success) $\mathbb{M} \subseteq \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$.

(persistence) $\mathrm{Mod}(\mathcal{B}) \subseteq \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$.

Reception (Ideal): Characterisation and Postulates

Only add other models to ensure finite representability

(success) $\mathbb{M} \subseteq \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$.

(persistence) $\mathrm{Mod}(\mathcal{B}) \subseteq \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$.

(finite temperance) If $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M} \subseteq \mathbb{M}' \subset \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$ then $\mathbb{M}' \notin \mathrm{FRSets}(\Lambda)$.

Reception (Ideal): Characterisation and Postulates

The result is determined by $\mathrm{MinFRSups}$ (regardless of actual inputs)

(success) $\mathbb{M} \subseteq \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$.

(persistence) $\mathrm{Mod}(\mathcal{B}) \subseteq \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$.

(finite temperance) If $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M} \subseteq \mathbb{M}' \subset \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$ then $\mathbb{M}' \notin \mathrm{FRSets}(\Lambda)$.

(uniformity) If $\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda) = \mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}') \cup \mathbb{M}', \Lambda)$ then $\mathrm{Mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{rcp}(\mathcal{B}', \mathbb{M}'))$.

Problem: Continuous Chains

$\mathcal{ALC}$ concepts

$$C ::= A \mid \neg C \mid (C \sqcap C) \mid \exists r.C$$

$\mathcal{ALC}_{bool}$ Formulae

$$\varphi ::= \alpha \mid \neg(\varphi) \mid (\varphi \wedge \varphi) \qquad \alpha ::= C(a) \mid r(a,b) \mid (C = \top)$$

where $A \in$ NC, $r \in$ NR and $a, b \in$ NI

Examples of Formulae in $\mathcal{ALC}_{bool}$

- $\neg Worker \sqcup Person = \top$ (same as $\forall x(Worker(x) \rightarrow Person(x))$)

- $\neg(Professor(alice)) \wedge worksAt(alice, uib)$

- $\neg(Dev(bob) \wedge worksAt(bob, google))$

How get a Finite Representation?

- Propositional Logic: Easy (with finite signature)
- $\mathcal{ALC}_{bool}$ formula: infinite sets of models even with finite signature

How get a Finite Representation?

- Propositional Logic: Easy (with finite signature)
- $\mathcal{ALC}_{bool}$ formula: infinite sets of models even with finite signature
- Solution: quasimodels!

Quasimodels [Agi+03]

- Each model for an $\mathcal{ALC}_{bool}$ formula φ can be converted into a quasimodel for φ
- There are finitely many quasimodels for any $\mathcal{ALC}_{bool}$ formula
- They are finite because they are restricted to the terms and concept types in φ and its subformulas
- A qm $(T, o, \mathbf{f})$ for φ: T is a set of types, o a function from individual names to types and $\mathbf{f}$ formula type

Formula Type

- A subset of the subformulas of φ and their negations
- $\psi \wedge \xi \in \mathbf{f}$ iff $\psi, \xi \in \mathbf{f}$
- $\psi \in \mathbf{f}$ iff $\neg\psi \notin \mathbf{f}$
- Example: one formula type for $\neg(Professor(alice)) \wedge worksAt(alice, uib)$:

$$\{\neg(Professor(alice)) \wedge worksAt(alice, uib),\\ \neg(Professor(alice)), worksAt(alice, uib)\}$$

$\mathcal{ALC}_{bool}$ formulae in DNF

How far can we push the propositional approach?

Let $\mathsf{S}(\varphi)$ be the set of all quasimodels for φ. We define $\varphi^\dagger$ as

$$\bigvee_{(T,o,\mathbf{f})\in\mathsf{S}(\varphi)} \Big(\bigwedge_{\alpha\in\mathbf{f}} \alpha \wedge \bigwedge_{\neg\alpha\in\mathbf{f}} \neg\alpha\Big),$$

where α is of the form $(C = \top), C(a), r(a,b)$ (i.e. $\alpha, \neg\alpha \in lit(\mathbf{f})$)

We proved that $\varphi \equiv \varphi^\dagger$

Eviction in $\mathcal{ALC}_{bool}$: Definition

Idea: Remove quasimodels (disjuncts) using a filter (μ)

$\mathrm{evc} : \mathcal{L} \times \mathfrak{M} \mapsto \mathcal{L}$ such that

$$\mathrm{evc}(\varphi, \mathbb{M}) = \begin{cases} \bot & \text{if } \mu(\varphi, \mathbb{M}) = \emptyset \\ \bigvee_{\mathbf{f} \in \mu(\varphi, \mathbb{M})} \bigwedge lit(\mathbf{f}), & \text{otherwise,} \end{cases}$$

where $\mu(\varphi, \mathbb{M}) =$

$$\underbrace{\mathrm{ftypes}(\varphi)}_{\text{all formula types of all quasimodels of } \varphi} \setminus \overbrace{\mathrm{ft}(\varphi, \mathbb{M})}^{\text{formula types of the quasimodels of } \varphi \text{ corresponding t}}$$

Eviction in $\mathcal{ALC}_{bool}$: Example

Remove $M = (\Delta^M, \cdot^M)$, where:

- $p, s \in \Delta^M$,
- enabled$^M = \{(p, s)\}$,
- Stove$^M = \{s\}$,
- Peter$^M = p$, and
- stv1315$^M = s$

If the formula φ is of the form

$$\varphi := (\text{Stove}(\text{stv1315}) \wedge \text{enabled}(\text{Peter}, \text{stv1315})) \vee$$
$$(\text{Stove}(\text{stv1315}) \wedge \neg\text{enabled}(\text{Peter}, \text{stv1315}))$$

Then

$$\mathrm{evc}(\varphi, \{M\}) = (\text{Stove}(\text{stv1315}) \wedge \neg\text{enabled}(\text{Peter}, \text{s})).$$

Side-effects of Propositionalisation

If M and M' satisfy the same 'literals' (atomic subformulas) of φ (in symbols $M \equiv^{\varphi} M'$, or $M' \in [M]^{\varphi}$) …

Side-effects of Propositionalisation

If M and M' satisfy the same ‘literals’ (atomic subformulas) of φ (in symbols $M \equiv^{\varphi} M'$, or $M' \in [M]^{\varphi}$) …

…then operations based on quasimodels will treat them as the same model

Eviction in $\mathcal{ALC}_{bool}$: Postulates

(success) $\mathbb{M} \cap \mathrm{Mod}(\mathrm{evc}(\varphi, \mathbb{M})) = \emptyset$.

(inclusion) $\mathrm{Mod}(\mathrm{evc}(\varphi, \mathbb{M})) \subseteq \mathrm{Mod}(\varphi)$.

(atomic retainment) If
$\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \subset \mathbb{M}' \subseteq \mathrm{Mod}(\mathcal{B}) \setminus (\cup_{M \in \mathbb{M}} [M]^{\varphi})$ then
$\mathbb{M}' \notin \mathrm{FRSets}(\Lambda(\mathcal{ALC}_{bool}))$.

(atomic extensionality) If
$\{[M']^{\varphi} \mid M' \in \mathbb{M}'\} = \{[M]^{\varphi} \mid M \in \mathbb{M}\}$ then
$\mathrm{Mod}(\mathrm{evc}(\varphi, \mathbb{M})) = \mathrm{Mod}(\mathrm{evc}(\varphi, \mathbb{M}'))$.

Reception in $\mathcal{ALC}_{bool}$: Definition

Idea: add disjuncts corresponding to M's quasimodel w.r.t. φ

$\mathrm{rcp} : \mathcal{L} \times \mathcal{P}(\mathfrak{M}) \to \mathcal{L}$ such that

$$\mathrm{rcp}(\varphi, \mathbb{M}) = \begin{cases} \varphi & \text{if } \mathbb{M} \subseteq \mathrm{Mod}(\varphi) \\ \varphi \vee \bigwedge_{\mathbf{f} \in \nu(\neg\varphi, \mathbb{M})} lit(\mathbf{f}) & \text{otherwise,} \end{cases}$$

where $\nu(\neg\varphi, \mathbb{M}) = \underbrace{\{\mathrm{ft}(\neg\varphi, M) \mid M \in \mathbb{M} \setminus \mathrm{Mod}(\varphi)\}}_{\text{formula types of quasimodels corresponding to **new** models}}$.

Reception in $\mathcal{ALC}_{bool}$: Postulates

(success) $\mathbb{M} \in \mathrm{Mod}(\mathrm{rcp}(\varphi, \mathbb{M}))$.

(persistence) $\mathrm{Mod}(\varphi) \subseteq \mathrm{Mod}(\mathrm{rcp}(\varphi, \mathbb{M}))$.

(atomic temperance) If
$\mathrm{Mod}(\varphi) \cup (\cup_{M \in \mathbb{M}} [M]^{\varphi}) \subseteq \mathbb{M}' \subset \mathrm{Mod}(\mathrm{rcp}(\varphi, \mathbb{M}))$ then
$\mathbb{M}' \notin \mathrm{FRSets}(\Lambda(\mathcal{ALC}_{bool}))$.

(atomic extensionality) If
$\{[M']^{\varphi} \mid M' \in \mathbb{M}'\} = \{[M]^{\varphi} \mid M \in \mathbb{M}\}$ then
$\mathrm{Mod}(\mathrm{rcp}(\varphi, \mathbb{M})) = \mathrm{Mod}(\mathrm{rcp}(\varphi, \mathbb{M}'))$.

Concluding Remarks

- Proposal of Ontology Repair (Belief Change) via Models
- Propositionalisation strategy for $\mathcal{ALC}_{bool}$ with characterisation via postulates
- Constraints imposed by finite representability

Future Works

- Relation with Learning from Interpretations paradigm
- Different strategies (finer-grained control)
- Syntactic relevance
- Connect with pseudo-contraction, axiom weakening and gentle repairs

Thank you!

Questions?

E-mail: ricardo dot guimaraes at uib dot no

Website: `https://rfguimaraes.github.io`

References

References

[Agi+03] Kurucz Agi et al. *Many-dimensional modal logics : theory and applications*. Amsterdam Boston: Elsevier North Holland, 2003. ISBN: 0444508260.

[AGM85] Carlos E. Alchourrón, Peter Gärdenfors and David Makinson. 'On the Logic of Theory Change: Partial Meet Contraction and Revision Functions'. In: *Journal of Symbolic Logic* 50.2 (1985), pp. 510–530.

[Han99] Sven Ove Hansson. *A Textbook of Belief Dynamics: Theory Change and Database Updating*. Applied Logic Series. Kluwer Academic Publishers, 1999.

[SGO21] Jandson S. Ribeiro, Ricardo Guimarães and Ana Ozaki. 'Revising Ontologies via Models: The ALC-formula Case'. In: *Proceedings of the 34th International Workshop on Description Logics (DL 2021) part of Bratislava Knowledge September (BAKS 2021), Bratislava, Slovakia, September 19th to 22nd, 2021*. Ed. by Martin Homola, Vladislav Ryzhikov and Renate A. Schmidt. Vol. 2954. CEUR Workshop Proceedings. CEUR-WS.org, 2021. URL: `http://ceur-ws.org/Vol-2954/paper-26.pdf`.

Reception in $\mathcal{ALC}_{bool}$: Example

Add $M = (\Delta^M, \cdot^M)$, where:

- $c, s \in \Delta^M$,
- $\mathsf{AirControl}^M = \{c\}$,
- $\mathsf{on}^M = \{(c, s)\}$,
- $\mathsf{Stove}^M = \{s\}$,
- $\mathsf{ac7182}^M = c$, and
- $\mathsf{stv1314}^M = s$

Let φ be:

$$(\mathsf{AirControl} \sqcap \exists \mathsf{in}.\mathsf{Fridge} \sqsubseteq \bot) \wedge$$
$$(\mathsf{AirControl} \sqcap \exists \mathsf{on}.\mathsf{Stove} \sqsubseteq \bot)$$

Then $\mathrm{rcp}(\varphi, \{M\}) =$

$$\varphi \wedge ((\mathsf{AirControl} \sqcap \exists \mathsf{in}.\mathsf{Fridge} \sqsubseteq \bot) \wedge$$
$$\neg(\mathsf{AirControl} \sqcap \exists \mathsf{on}.\mathsf{Stove} \sqsubseteq \bot))$$