Polynomial Division

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1 Introduction

Instead of long division, we can take advantage of a pattern in division. The goal is to eliminate the highest term in the numerator.

$$\frac{3x^2 + 7x - 11}{x - 5}$$

Instead of guessing a value that eliminates $3x^2$, we know that getting rid of it involves the first term in the denominator. We name the things we are focusing on, which are the first terms in numerator and denominator:

$$A = 3x^2$$

$$B = x$$

$$\frac{A + 7x - 11}{B - 5}$$

We subtract and re-add A/B, which doesnt change the value of our polynomial fraction:

$$(\frac{A}{B} - \frac{A}{B}) + \frac{A + 7x - 11}{B - 5}$$

Multiplying and dividing by itself by a value doesn't change anything either. So we do this by the denominator the value that we subtracted out.

$$(\frac{A}{B} + -\frac{A}{B} * \frac{(B-5)}{(B-5)}) + \frac{A + 7x - 11}{B - 5}$$

Notice that this will cause the first A in numerator to cancel out when we simplify. First, just substitute in for A and B

$$\frac{3x^2}{x} + -\frac{3x^2}{x} * \frac{(x-5)}{(x-5)} + \frac{3x^2 + 7x - 11}{x - 5}$$

$$3x + -3x\frac{(x-5)}{(x-5)} + \frac{3x^2+7x-11}{x-5}$$

$$3x + \frac{-3x^2+15}{x-5} + \frac{3x^2+7x-11}{x-5}$$

$$3x + \frac{15}{x-5} + \frac{7x-11}{x-5}$$

$$3x + \frac{7x+4}{x-5}$$

And we do another round to get rid of the $7x$, by adding and subtracting $\frac{7x}{x}$, which is just 7:

$$3x + (7 - 7 * \frac{x-5}{x-5}) + \frac{7x+4}{x-5}$$

$$3x + (7 + \frac{-7x+35}{x-5}) + \frac{7x+4}{x-5}$$

$$3x + 7 + \frac{35}{x-5} + \frac{4}{x-5}$$

So, we get $3x+7$ remainder 39:

$$3x + 7 + \frac{39}{x-5}$$

Check by multiplying it by denominator

$$(3x + 7 + \frac{39}{x-5})(x-5)$$

$$3x(x-5) + 7(x-5) + 39$$

$$3x^2 - 15 + 7x - 35 + 39$$

$$3x^2 - 15 + 7x + 4$$

$$3x^2 + 7x - 11$$