

# KRUSKALS MINIMAL SPANNING TREE

**Graph:** vertices and edges

**cyclical:** it's possible to get back to an origin through other vertices

undirected: no arrows

complete: Every vertex is connected to every other vertex

**a loop:** both endpoints are the same vertex

**complete graph** on 5 vertices can be shorthand to  $K_5$ . This is the most dense graph and the most dense simple graph

**a simple graph** - no loops or double edges

**tree:** connected acyclical graph

tree: number of edges is one less than the number of vertices  $\rightarrow e = v - 1$  and  $v = e + 1$

**A minimal spanning tree** tries to find the lowest "cost" path to connect all vertices of a simple weighted graph.

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## Algorithm: kruskals minimum spanning tree algorithm

Imagine a graph where each edge has a weight.

Minimal Spanning TREE Array: Index 1: Shortest, Index 2: second shortest etc.

1. sort the edges by weight from smallest to largest

A  $\rightarrow$  B Cost: 1, B  $\rightarrow$  C Cost: 3, etc etc  $\rightarrow$  6,8,10

2. Be greedy: Add each next edge with the least weight, but never add an edge that creates a cycle.

3. We're done when there is one less edge than vertices

To Detect CYCLES:

ones represent that B and C are a part of tree 1. 2s represent that E and F are a part of tree 2

-1 represents no connection.

Index	0	1	2	3	4
Weight	1	2	4	5	5
Vertex 1	B	E	A	B	C
Vertex 2	C	F	B	E	D

Undirected Connections.	A	B	C	D	E	F	
None	-1	-1	-1	-1	-1	-1	
B < - > C	-1	1	1	-1	-1	-1	Add B → C from index 0 since it's cheapest. Both values become a 1.
E < - > F	-1	1	1	-1	2	2	Add E → F as its own tree. We know this doesn't connect with tree 1 because neither vertex has a 1 in it. Represent new tree with a 2.
Add A < - > B	1	1	1	-1	2	2	Add A → B. One of these is a 1 and the other is a -1, so we are joining a tree.
Add B < - > E	1	1	1	-1	1	1	Add B → E. This connects the two trees together, so convert 2s to 1s.
Add C < - > D	1	1	1	1	1	1	Add C → E. Connects the last point to the tree.

Efficiency:  $O(V^2)$  for the connections +  $E \log E$  for sorting the edges)

$[v * (v-1)]/2 \rightarrow$  the number of edges in worst case is  $v^2$  in a complete graph.  $\rightarrow O(v^2 + v^2 * \log(v^2)) = O(V^2 + V^2 * \log V) \rightarrow$  polynomial work

PRIMS algorithm is next: far better if things are very very dense.