01_CS 350 Math Reference - Exam 1

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> Complexity notes: ‘n’ almost always means size of input. Input ≠ bigO.

> —- decreasing work is log(n), not log2(n) -—little O means less than

> types of time: constant, linear, log, quadratic, exponential, factorial

> Manipulation Options:
>
> - Multiply top and bottom by 1
> - add or subtract forms of 1
> - divide bottom by n to find similar terms
> - re-define variable in terms of another lesser variable
> - pull a term out and evaluate it as its limit

> - l’hospitals rule: indeterminate: derivative of the top / derivative of the bottom.
> - To prove: big O: Take the ratio of the limits.
> - Get exponentials to similar exponents and cancel
> - split up the equation and take the sum of the limits
> - the same log on both sides can be canceled
> - all logs are the same order of growth, substitute (hand wavy)

Logarithm rules

Rule name	Rule
Logarithm product rule	$\log_b(x \cdot y) = \log_b(x) + \log_b(y)$
Logarithm quotient rule	$\log_b(x / y) = \log_b(x) - \log_b(y)$
Logarithm power rule	$\log_b(x^y) = y \cdot \log_b(x)$
Logarithm base switch rule	$\log_b(c) = 1 / \log_c(b)$
Logarithm base change rule	$\log_b(x) = \log_c(x) / \log_c(b)$
Derivative of logarithm	$f(x) = \log_b(x) \Rightarrow f'(x) = 1 / (x \ln(b))$
Integral of logarithm	$\int \log_b(x)\, dx = x \cdot (\log_b(x) - 1 / \ln(b)) + C$
Logarithm of negative number	$\log_b(x)$ is undefined when $x \leq 0$
Logarithm of 0	$\log_b(0)$ is undefined
	$\lim_{x \to 0^+} \log_b(x) = -\infty$
Logarithm of 1	$\log_b(1) = 0$
Logarithm of the base	$\log_b(b) = 1$
Logarithm of infinity	$\lim \log_b(x) = \infty$, when $x \to \infty$

Derivative rules

Derivative sum rule	$(\,af(x) + bg(x)\,)' = af'(x) + bg'(x)$
Derivative product rule	$(\,f(x) \cdot g(x)\,)' = f'(x)\,g(x) + f(x)\,g'(x)$
Derivative quotient rule	$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$
Derivative chain rule	$f(\,g(x)\,)' = f'(\,g(x)\,) \cdot g'(x)$

$$\frac{d}{dx}\left[\sqrt{x}\right] = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$$

^^Square Root Derivative

$$\log_b(x) = \log_c(x) \,/\, \log_c(b)$$

^^Change of Base Rule

If *n* is a positive integer greater than 1 and both a and b are positive real numbers then,

1. Inverse Property	n√ an = a if n is odd or
	n√ an = | a | if n is even
2. Product Rule	n√ ab = n√ a · n√ b
3. Quotient Rule	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

^^Radicals (square roots)

Exponents rules and properties

Rule name	Rule	Example
Product rules	$a^n \cdot a^m = a^{n+m}$	$2^3 \cdot 2^4 = 2^{3+4} = 128$
	$a^n \cdot b^n = (a \cdot b)^n$	$3^2 \cdot 4^2 = (3 \cdot 4)^2 = 144$
Quotient rules	$a^n / a^m = a^{n-m}$	$2^5 / 2^3 = 2^{5-3} = 4$
	$a^n / b^n = (a / b)^n$	$4^3 / 2^3 = (4/2)^3 = 8$
Power rules	$(b^n)^m = b^{n \cdot m}$	$(2^3)^2 = 2^{3 \cdot 2} = 64$
	$b^{n^m} = b^{(n^m)}$	$2^{3^2} = 2^{(3^2)} = 512$
	${}^m\sqrt{(b^n)} = b^{n/m}$	${}^2\sqrt{(2^6)} = 2^{6/2} = 8$
	$b^{1/n} = {}^n\sqrt{b}$	$8^{1/3} = {}^3\sqrt{8} = 2$
Negative exponents	$b^{-n} = 1 / b^n$	$2^{-3} = 1/2^3 = 0.125$
Zero rules	$b^0 = 1$	$5^0 = 1$
	$0^n = 0$, for $n>0$	$0^5 = 0$
One rules	$b^1 = b$	$5^1 = 5$
	$1^n = 1$	$1^5 = 1$
Minus one rule	$(-1)^n = \begin{cases} 1 & ,n\ even \\ -1 & ,n\ odd \end{cases}$	$(-1)^5 = -1$
Derivative rule	$(x^n)' = n \cdot x^{n-1}$	$(x^3)' = 3 \cdot x^{3-1}$
Integral rule	$\int x^n dx = x^{n+1}/(n+1)+C$	$\int x^2 dx = x^{2+1}/(2+1)+C$

List of Derivative Rules

Below is a list of all the derivative rules we went over in class.

- **Constant Rule:** $f(x) = c$ then $f'(x) = 0$
- **Constant Multiple Rule:** $g(x) = c \cdot f(x)$ then $g'(x) = c \cdot f'(x)$
- **Power Rule:** $f(x) = x^n$ then $f'(x) = nx^{n-1}$
- **Sum and Difference Rule:** $h(x) = f(x) \pm g(x)$ then $h'(x) = f'(x) \pm g'(x)$
- **Product Rule:** $h(x) = f(x)g(x)$ then $h'(x) = f'(x)g(x) + f(x)g'(x)$
- **Quotient Rule:** $h(x) = \dfrac{f(x)}{g(x)}$ then $h'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
- **Chain Rule:** $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$
- **Trig Derivatives:**
  - $f(x) = \sin(x)$ then $f'(x) = \cos(x)$
  - $f(x) = \cos(x)$ then $f'(x) = -\sin(x)$
  - $f(x) = \tan(x)$ then $f'(x) = \sec^2(x)$
  - $f(x) = \sec(x)$ then $f'(x) = \sec(x)\tan(x)$
  - $f(x) = \cot(x)$ then $f'(x) = -\csc^2(x)$
  - $f(x) = \csc(x)$ then $f'(x) = -\csc(x)\cot(x)$
- **Exponential Derivatives**
  - $f(x) = a^x$ then $f'(x) = \ln(a)a^x$
  - $f(x) = e^x$ then $f'(x) = e^x$
  - $f(x) = a^{g(x)}$ then $f'(x) = \ln(a)a^{g(x)}g'(x)$
  - $f(x) = e^{g(x)}$ then $f'(x) = e^{g(x)}g'(x)$
- **Logarithm Derivatives**
  - $f(x) = \log_a(x)$ then $f'(x) = \dfrac{1}{\ln(a)x}$
  - $f(x) = \ln(x)$ then $f'(x) = \dfrac{1}{x}$
  - $f(x) = \log_a(g(x))$ then $f'(x) = \dfrac{g'(x)}{\ln(a)g(x)}$
  - $f(x) = \ln(g(x))$ then $f'(x) = \dfrac{g'(x)}{g(x)}$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

^^ summation of 1 + 2 + 3.... + n-1 + n

4^log_6(n) = n^(log_6(4))

8^(log_2(n)) = n^(log_2(8)) = n^3 —>

DONT FORGET TO SIMPLIFY!!!!!!!!

Derivatives	Function	Derivative
	f (x)	f '(x)
Constant	const	0
Linear	x	1
Power	x ^a	a x ^(a-1)
Exponential	e ^x	e^x
Exponential	a ^x	a^x ln a
Natural logarithm	ln(x)	1/x
Logarithm	logb(x)	1/[x ln(b)]
Sine	sin x	cos x
Cosine	cos x	-sin x
Tangent	tan x	1/cos^2(x)
Hyperbolic sine	sinh x	cosh x
Hyperbolic cosine	cosh x	sinh x
Hyperbolic tangent	tanh x	1/(cosh^2)(x)