Recurrence Relations

1. Find Pattern by computing from $f(0) \rightarrow f(n)$ where n is enough to establish a pattern.

2. Make a general formula

3. Solve w/ summation formulas or telescoping.

$$3^2 \qquad 3^4 \qquad 3^6$$

$$S = 2^{m-1}(3^1)^2 + 2^{m-2}(3^2)^2 + 2^{(m-3)}(3^2)^2 + 2^1(3^{m-1})^2 + 2^0(3^m)^2$$

$$\frac{9}{2} S = \cancel{2^{m-2}(3^2)^2} \ldots\ldots 2^{-1}(3^{m+1})^2$$

$$\frac{9}{2} - \frac{2}{2} = \boxed{\frac{7}{2}} S = \left(\cancel{2^{-1}}(3^{m+1})^2 - 2^{m-1}(3^1)^2\right)$$

$$\frac{1}{7}\left(\left(3^{(m+1)}\right)^2 - 2^m\left(3^1\right)^2\right)$$

$$9^{(m+1)} - 9\, 2^m$$

$$\frac{9}{7}\left(9^m - 2^m\right)$$

Telescoping:

1. What would you need to multiply the 1st term by to get the second?
2. Multiply both sides by this number.
3. Subtract the previous equation (or the other way) to telescope terms.
4. Simplify.

Logs

$$a^{\log_x b} = b^{\log_x a}$$

$$(a^b)^c = a^{b \cdot c}$$

$$a^c \cdot b^c = (ab)^c$$

Summation

$$1+2+3+4\ldots\ldots = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{2n^3+3n^2+n}{6} = \frac{n(2n+1)(n+1)}{6}$$

$$\sum_{i=1}^{\log n} n = n \log n$$

$$\sum_{i=0}^{\infty} a^i = \frac{1}{1-a} \text{ for } 0 < a < 1$$

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1} \text{ for } a \neq 1$$

$$\sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}$$

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

$$\sum_{i=0}^{\log n} 2^i = 2^{\log n+1} - 1 = 2n - 1$$

$$\sum_{i=1}^{n} \frac{i}{2^i} = 2 - \frac{n+2}{2^n}$$

a) let $n = 2^4$ let $n = 2^m \to m = \log_2 n$

$T(2^4) = 2T(15) + c2^{(2^4)}$

$T(2^3) = 2T(7) + c2^{2^3}$

$T(2^2) = 2T(3) + c2^{(2^2)}$

$T(2^1) = 2T(1) + c2^{(2)}$

$T(0) = 2T + 2c = k$

$T(n) = 2T(2^m - 1) + c2^m$

$T(n) = 2T(2^{\log_2 n} - 1) + c2^{\log_2 n}$

$T(n) = 2T(n-1) + cn$

$T(n) = 2kn - 2k + cn$

$\lim_{n \to \infty} = 2n \quad T(n) \in O(n)$

$$T(n) = 5T(n/4) + \sqrt{n}$$

$T(4^m) = \boxed{5^m k} + 5^{m-1}\sqrt{4^1} + 5^{m-2}\sqrt{4^2} + 5^{m-3}\sqrt{4^3} + \ldots$
$5^1\sqrt{4^{m-1}} + 5^0\sqrt{4^m}$

d)

let $n = 4^5$

$T(4^5) = 5T(4^4) + \sqrt{4^5} = 5^4 k + 5^3\sqrt{4^1} + 5^2\sqrt{4^2} + 5^1\sqrt{4^3} + 5^0\sqrt{4^4}$

$T(4^4) = 5T(4^3) + \sqrt{4^4} = 5^4 k + 5^3\sqrt{4^1} + 5^2\sqrt{4^2} + \sqrt{4^3}$

$T(4^3) = 5T(4^2) + \sqrt{4^3} = 5^3 k + 5^2\sqrt{4^1} + 5^1\sqrt{4^2} + \sqrt{4^3}$

$T(4^2) = 5T(4^1) + \sqrt{4^2} = 5^2 k + 5^1\sqrt{4^1} + \sqrt{4^2}$

$T(4^1) = 5T(4^0) + \sqrt{4^1} = 5^1 k + \sqrt{4^1}$

$T(4^0) = T^1 = k$

- Build up from 0.
- like dynamic programming.

Simplify:

$S =$ →

$\boxed{T(4^m) = 5^m k + 5^{m-1} 2 + 5^{m-2} 2^2 + 5^{m-3} 2^3 + \ldots 5^1 2^{m-1} + 5^0 2^m}$

$\frac{2}{5}S = 5^{m-2} 2^2 + 5^{m-3} 2^3 + 5^{m-4} 2^4 + \ldots + 5^0 2^m + 5^{-1} 2^{m+1}$

$\boxed{S - \frac{2}{5}S = 5^{m-1} 2^1 - 5^{-1} 2^{m+1}}$

$S = \frac{5}{3}\left(5^{m-1} 2^1 - 5^{-1} 2^{m+1}\right)$

$S = \frac{1}{3}\left(5^m 2 - 2^{m+1}\right)$

$S = \frac{2}{3}\left(5^m - 2^m\right)$

$T(4^m) = 5^m k + \boxed{\frac{2}{3}\left(5^m - 2^m\right)}$

$T(n) = n^{\log_4 5} + \frac{2}{3} n^{\log_4 5} - \frac{2}{3}\sqrt{n}$

$T(n) \in O(n^{\log_4 5})$

$n = 4^m$

$\log_4(n) = \log_4(4^m)$

$\log_4(n) = m \log_4(4)$

$\log_4 n = m$

$2^m = (\sqrt{4})^m = \sqrt{4^m}$

$= \sqrt{n}$