1- Find and sketch the frequency responses (magnitude not in db and phase versus frequency ω) for the following transfer functions: (similar to EXAMPLE 4.28)

a. An ideal delay of 2 seconds with transfer function of $H_1(s) = e^{-2s}$

$$H_1(s) = e^{-2s}$$

$$H_1(j\omega) = e^{-2j\omega} = \cos(-2\omega) + j\sin(-2\omega)$$

$$H_1(j\omega) = \cos(2\omega) - j\sin(2\omega)$$

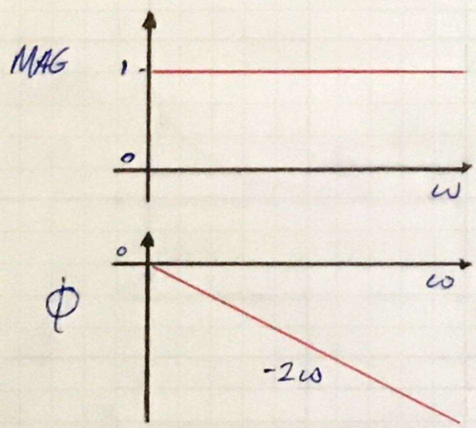
$$\|H_1(j\omega)\| = \sqrt{\cos^2(2\omega) + \sin^2(2\omega)} = 1$$

$$\boxed{\|H_1(s)\| = 1}$$

$$\angle H_1(s) = \tan^{-1}\left(\frac{-\sin(2\omega)}{\cos(2\omega)}\right)$$

$$\angle H_1(s) = \tan^{-1}(-\tan(2\omega)) = -2\omega$$

$$\boxed{\angle H_1(s) = -2\omega}$$



b. $H_2(s) = \frac{1}{s+1}$ $\quad H_2(j\omega) = \frac{1}{j\omega+1}$

$$\|H_2(j\omega)\| = \frac{\|1\|}{\|j\omega+1\|} = \frac{1}{\sqrt{1^2+\omega^2}}$$

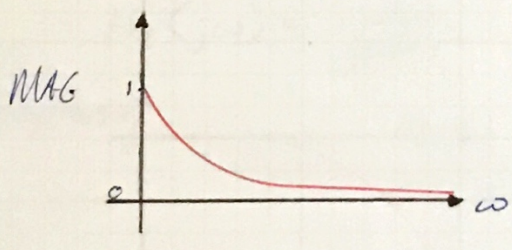
$$\boxed{\|H_2(j\omega)\| = \frac{1}{\sqrt{\omega^2+1}}}$$

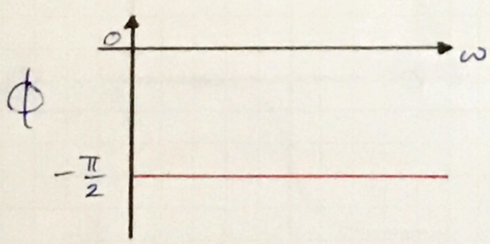
$$\angle H_2(j\omega) = \angle 1 - \angle j\omega - 1$$

$$\angle H_2(j\omega) = 1 - \tan^{-1}\left(\frac{-\omega}{1}\right)$$

$$\angle H_2(j\omega) = 1 - \frac{\pi}{2} = -\frac{\pi}{2}$$

$$\boxed{\angle H_2(s) = -\frac{\pi}{2}}$$

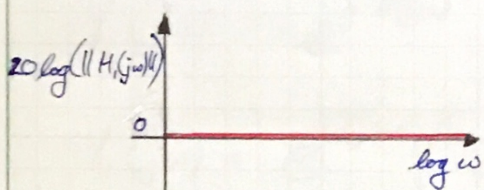


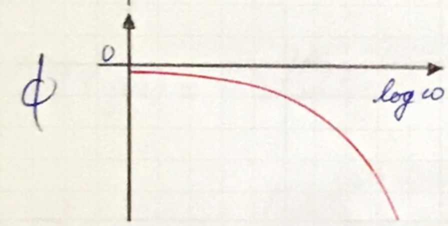


2- Find and sketch the bode plots (magnitude in db and phase versus frequency log (ω)) for the transfer functions mentioned above H_1 *and* H_2 in parts a and b in the first question. Based on the bode plot or frequency response curves from question one: what would be the magnitude (A) and phase (φ) response to a $10cos(\omega t)$. Hint: the response will be a sinusoidal function $Acos(\omega t + \varphi)$.

$$H_1(j\omega) = e^{-2j\omega} \qquad \|H_1(j\omega)\| = 1$$

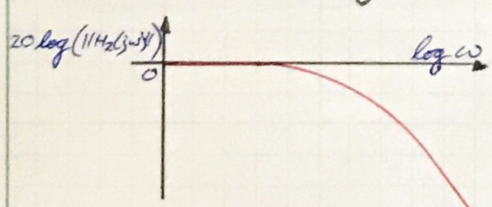
$$\angle H_1(j\omega) = -2\omega$$

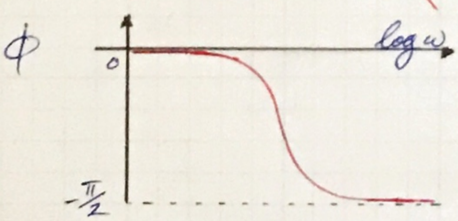




$$H_2(j\omega) = \frac{1}{j\omega+1} \qquad \|H_2(j\omega)\| = \frac{1}{\sqrt{\omega^2+1}}$$

$$\angle H_2(j\omega) = -\frac{\pi}{2}$$





2- (cont'd) Based on the bode plot or frequency response curves from question one: what would be the magnitude (A) and phase (φ) response to a $10cos(\omega t)$. Hint: the response will be a sinusoidal function $Acos(\omega t + \varphi)$.

$$H_1(s) = \frac{Y_1(s)}{X(s)} = \frac{Y_1(s)}{10\cos(\omega t)} = e^{-2s}$$

$$Y_1(j\omega) = e^{-2j\omega} \cdot 10\cos(\omega t)$$

$$Y_1(j\omega) = e^{-2j\omega}\left(\frac{10e^{j\omega t} + 10e^{-j\omega t}}{2}\right)$$

$$Y_1(j\omega) = \frac{10e^{-2j\omega + j\omega t} + 10e^{-2j\omega - j\omega t}}{2}$$

$$Y_1(j\omega) = \frac{10e^{(\omega t - 2\omega)j} + 10e^{(-\omega t - 2\omega)}}{2}$$

$$\boxed{Y_1(j\omega) = 10\cos(\omega t - 2\omega)}$$

$$H_2(s) = \frac{Y_2(s)}{X(s)} = \frac{Y_2(s)}{10\cos(\omega t)} = \frac{1}{s+1}$$

$$Y_2(j\omega) = \frac{10\cos(\omega t)}{j\omega + 1} = \frac{10\cos(\omega t)}{j\omega + 1}$$

$$\| Y_2(j\omega) \| = \frac{\| 10\cos(\omega t) \|}{\| j\omega + 1 \|} = \frac{10}{\sqrt{\omega^2 + 1}}$$

$$\boxed{\| Y_2(j\omega) \| = \frac{10}{\sqrt{\omega^2+1}}}$$

$$\angle Y_2(j\omega) = \angle [10\cos(\omega t)] - \angle (j\omega + 1)$$

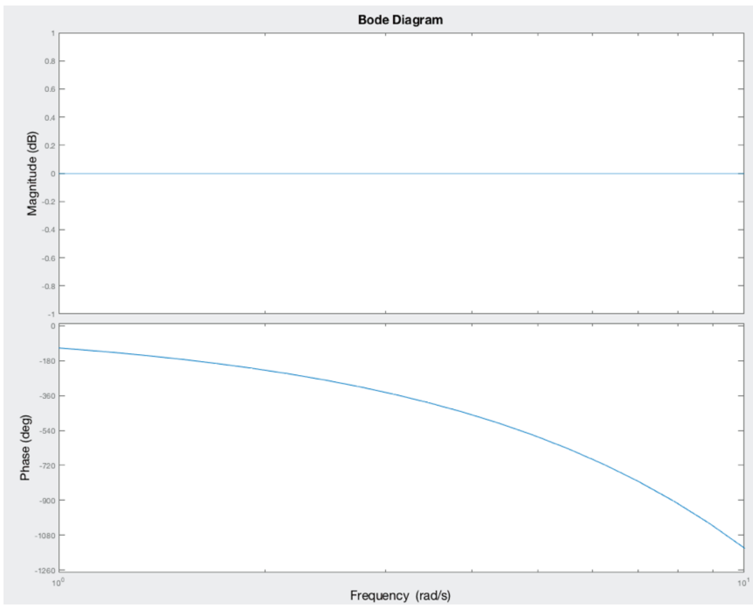
$$\angle Y_2(j\omega) = \tan^{-1}\left(\frac{0}{10\cos(\omega t)}\right) - \tan^{-1}\left(\frac{\omega}{1}\right)$$

$$\boxed{\angle Y_2(s) = -\tan^{-1}(\omega)}$$

MATLAB

```
s  = tf('s')
H1 = exp(-2*s)
H2 = 1/(s + 1)
bode(H1)
bode(H2)
```

H_1



H_2