San Francisco State University

School of Engineering

ENGR 305 – Linear Systems Analysis

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Professor Azadi

Homework Assignment # 3

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4.2-2

a. $\mathcal{L}\{u(t) - u(t-1)\} = \mathcal{L}\{u(t)\} - \mathcal{L}\{u(t-1)\}$

$= \frac{1}{s} - \frac{1}{s}e^{-1s}$

$$\boxed{\frac{1}{s} - \frac{1}{s}e^{-s}}$$

e. $\mathcal{L}\{te^{-t}u(t-\tau)\} = \frac{1}{(s+1)^2}e^{-s\tau}$

$$\boxed{\frac{1}{(s+1)^2}e^{-s\tau}}$$

4.2-7

a. $\mathcal{L}^{-1}\left\{\frac{(2s+5)e^{-2s}}{s^2+5s+6}\right\}$

$$\frac{(2s+5)e^{-2s}}{(s+2)(s+3)} = \frac{K_1}{s+2} + \frac{K_2}{s+3}$$

$$\frac{(2s+5)e^{-2s}}{s+3} = K_1 + \frac{K_2(s+2)}{s+3}$$

LET $s = -2$

$$K_1 = e^4$$

$$\frac{(2s+5)e^{-2s}}{s+2} = \frac{K_1(s+3)}{s+2} + K_2$$

LET $s = -3$

$$K_2 = e^6$$

$$\mathcal{L}^{-1}\left\{\frac{e^4}{s+2} + \frac{e^6}{s+3}\right\} = \left[e^{-2(t-2)} + e^{-3(t-2)}\right]u(t-2)$$

$$\boxed{\left[e^{-2(t-2)} + e^{-3(t-2)}\right]u(t-2)}$$

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4.2-8 $\mathcal{L}^{-1}\left\{ s^{-1}\frac{d}{ds}\left(\frac{e^{-2s}}{s}\right)\right\}$

$$\frac{1}{s}\frac{d}{ds}\left(\frac{e^{-2s}}{s}\right) = \frac{1}{s}\left(-\frac{2e^{-2s}}{s} - \frac{e^{-2s}}{s^2}\right) = \frac{-2e^{-2s}}{s^2} - \frac{e^{-2s}}{s^3}$$

$$X(s) = \frac{-2e^{-2s}}{s^2} - \frac{e^{-2s}}{s^3}$$

$$\boxed{\mathcal{L}^{-1}\{X(s)\} = -2e^{-(t-2)}t\,u(t-2) - e^{-(t-2)}t^2u(t-2)}$$

4.2-13

$X(s) = \frac{1}{(s+1)^{13}}$ ROC $\sigma > -1$

$$\mathcal{L}^{-1}\{X(s)\} = \mathcal{L}^{-1}\left\{\frac{n!}{(s-\lambda)^{n+1}}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{12!}\frac{12!}{(s+1)^{13}}\right\} = t^n e^{\lambda t}u(t)$$

$n = 12$ $\lambda = -1$ $\boxed{\frac{1}{12!}t^{12}e^{-t}u(t)}$

4.3-1

a. $(D^2+3D+2)\,y(t) = Dx(t)$ $\qquad y(0^-)=\dot{y}(0^-)=0$

$x(t) = u(t)$

$\mathcal{L}\{(D^2+3D+2)\,y(t)\} = \mathcal{L}\{D\,x(t)\}$

$D\,y(t) = sY(s) - y(0^-) = sY(s)$

$D^2 y(t) = s^2Y(s) - y(0^-) - \dot{y}(0^-) = s^2 Y(s)$

$Dx(t) = D\,u(t) = sX(s) = s\,\frac{1}{s} = 1$

$$Y(s) = \frac{1}{s^2+3s+2} = \frac{1}{(s+1)(s+2)} = \frac{K_1}{s+1} + \frac{K_2}{s+2}$$

$$\frac{1}{s+2} = K_1 + \frac{K_2(s+1)}{s+2}$$

LET $s=-1$

$\rightarrow K_1 = 1$

$$\frac{1}{s+1} = \frac{K_1(s+2)}{s+1} + K_2$$

LET $s=-2$

$K_2 = -1$

$$Y(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$$\mathcal{L}\{Y(s)\} = e^{-t}u(t) - e^{-2t}u(t)$$

$$\boxed{(e^{-t}-e^{-2t})u(t)}$$

4.3-3

$$2\dot{y}(t) + 6y(t) = \dot{x}(t) - 4x(t) \qquad y(0^-) = -3$$

a. $\mathcal{L}\{(2D+6)y(t)\} = \mathcal{L}\{(D-4)x(t)\}$

$$Dy(t) = sY(s) - y(0^-) = sY(s) + 3$$

$$\mathcal{L}\{(2D+6)y(t)\} = \mathcal{L}\{0\}$$

$$\boxed{Y_{ZIR}(s) = -\frac{3}{2s+6}}$$

$$2(sY(s)+3) + 6Y(s) = 0$$

$$(2s+6)Y(s) = -3$$

$$Y(s) = \frac{-3}{2s+6}$$

4.3-3

b. $\mathcal{L}\{2\dot{y}(t) + 6y(t) = \dot{x}(t) - 4x(t)\}$

$$x(t) = e\delta(t-\pi)$$

$$X(s) = e^{1-\pi s}$$

$$\mathcal{L}\{(2D+6)y(t)\} = \mathcal{L}\{(D-4)x(t)\}$$

$$2sY(s) + 6 = sX(s) - 4$$

$$Y(s) = \frac{sX(s)-10}{2} = \frac{se^{1-\pi s}}{2} - 5$$

$$\boxed{Y(s)_{ZSR} = \frac{se^{1-\pi s}}{2} - 5}$$

4.3-4

$$\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = 2\dot{x}(t) - x(t)$$

a. $\dot{y}(0^-) = 2$, $y(t) = -3$

$$\mathcal{L}\{(D^2 + 3D + 2)\, y(t)\} = \mathcal{L}\{0\}$$

$$s^2 Y(s) - s\,y(0^-) - \dot{y}(0^-) + 3\left(s\,Y(s) - y(0^-)\right) + 2Y(s) = 0$$

$$s^2 Y(s) + 3s - 2 + 3s\,Y(s) + 9 + 2Y(s) = 0$$

$$\boxed{Y(s)_{ZIR} = \frac{-7 - 3s}{s^2 + 3s + 2}}$$

4.3-4

b. $\mathcal{L}\{(D^2+3D+2)y(t)\} = \mathcal{L}\{(2D-1)x(t)\}$

$x(t) = u(t)$

$X(s) = \frac{1}{s}$

$s^2Y(s) - 3sY(s) + 2Y(s) = 2sX(s) - X(s)$

$s^2Y(s) - 3sY(s) + 2Y(s) = 2 - \frac{1}{s}$

$\boxed{Y(s)_{ZSR} = \frac{2s-1}{(s^2-3s+2)s}}$

4.3-5

a. $(D+3)y_1(t) - 2y_2(t) = x(t)$

$-2y_1(t) + (2D+4)y_2(t) = 0$

$x(t) = u(t)$

$X(s) = \frac{1}{s}$

(1) $(s+3)Y_1(s) - 2Y_2(s) = \frac{1}{s}$

(2) $-2Y_1(s) + (2s+4)Y_2(s) = 0$

(2) $Y_1(s) = (s+2)Y_2(s)$

(1) $(s+3)(s+2)Y_2(s) - 2Y_2(s) = \frac{1}{s}$

$Y_2(s) = \frac{1}{s(s^2+5s+4)}$

$-2Y_1(s) + \frac{2s+4}{s(s^2+5s+4)} = 0$

$Y_1(s) = \frac{s+2}{s(s^2+5s+4}$

$\boxed{Y_1(s) = \frac{s+2}{s(s^2+5s+4)}}$

$\boxed{Y_2(s) = \frac{1}{s(s^2+5s+4)}}$

4.3-5

b. $(D+2)\,y_1(t) - (D+1)\,y_2(t) = 0$

$-(D+1)\,y_1(t) + (2D+1)\,y_2(t) = x(t)$ $\qquad X(s) = \frac{1}{s}$

1) $sY_1(s) + 2Y_1(s) - sY_2(s) - Y_2(s) = 0$

2) $-sY_1(s) - Y_1(s) + 2sY_2(s) + Y_2(s) = \frac{1}{s}$

$$Y_1(s) = \frac{s+1}{s+2}\,Y_2(s)$$

$$-\frac{s(s+1)}{s+2}Y_2(s) - \frac{s+1}{s+2}Y_2(s) + 2sY_2(s) + Y_2(s) = \frac{1}{s}$$

$$Y_2(s)\left[\frac{-s(s+1)}{s+2} - \frac{s+1}{s+2} + 2s + 1\right] = \frac{1}{s}$$

$$Y_2(s)\left[-s(s+1) - (s+1) + 2s(s+2) + (s+2)\right] = \frac{s+2}{s}$$

$$Y_2(s)\left[(-s-1)(s+1) + (2s+1)(s+2)\right] = \frac{s+2}{s}$$

$$Y_2(s)\left[(-s^2-2s-1) + (2s^2+5s+2)\right] = \frac{s+2}{s}$$

$$Y_2(s)\left[s^2+3s+1\right] = \frac{s+2}{s}$$

$$Y_2(s) = \frac{s+2}{s(s^2+3s+1)}$$

$$Y_1(s) = \frac{s+1}{s(s^2+3s+1)}$$

$$\boxed{Y_1(s) = \frac{s+1}{s(s^2+3s+1)}}$$

$$\boxed{Y_2(s) = \frac{s+2}{s(s^2+3s+1)}}$$

4.3-5 CONT'D

$$\frac{s+2}{s(s+1)(s+4)} = \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+4}$$

$$\frac{s+2}{(s+1)(s+4)} = k_1 \qquad \text{LET } s = 0 \qquad k_1 = \frac{1}{2}$$

$$\frac{s+2}{s(s+4)} = k_2 \qquad \text{LET } s = -1 \qquad k_2 = -\frac{1}{3}$$

$$\frac{s+2}{s(s+1)} = k_3 \qquad \text{LET } s = -4 \qquad k_3 = -\frac{1}{6}$$

$$Y_1(s) = \frac{1}{2s} - \frac{1}{3(s+1)} - \frac{1}{6(s+4)}$$

INV. LAPLACE

$$y_1(t) = \left(\frac{1}{2} - \frac{1}{3}e^{-t} - \frac{1}{6}e^{-4t}\right) u(t)$$

$$\frac{y_1(t)}{x(t)} = \left(\frac{1}{2} - \frac{1}{3}e^{-t} - \frac{1}{6}e^{-4t}\right)$$

$$\boxed{\frac{y_1(t)}{x(t)} = \left(\frac{1}{2} - \frac{1}{3}e^{-t} - \frac{1}{6}e^{-4t}\right)}$$

4.3-5 CONT'D

a. $$Y_2(s) = \frac{1}{s(s+1)(s+4)} = \frac{K_1}{s} + \frac{K_2}{(s+1)} + \frac{K_3}{(s+4)}$$

$$\frac{1}{(s+1)(s+4)} = K_1 \qquad \text{LET } s = \emptyset$$

$$K_1 = \frac{1}{4}$$

$$\frac{1}{s(s+4)} = K_2 \qquad \text{LET } s = -1$$

$$K_2 = -\frac{1}{3}$$

$$\frac{1}{s(s+1)} = K_3 \qquad \text{LET } s = -4$$

$$K_3 = \frac{1}{12}$$

$$Y_2(s) = \frac{1}{4s} - \frac{1}{3(s+1)} + \frac{1}{12(s+4)}$$

(INV. LAPLACE)

$$y_2(t) = \frac{1}{4}u(t) + \left(-\frac{1}{3}e^{-t} + \frac{1}{12}e^{-4t}\right)u(t)$$

$$y_2(t) = \left(\frac{1}{4} - \frac{1}{3}e^{-t} + \frac{1}{12}e^{-4t}\right)u(t)$$

$$x(t) = u(t)$$

$$\boxed{\frac{y_2(t)}{x(t)} = \left(\frac{1}{4} - \frac{1}{3}e^{-t} + \frac{1}{12}e^{-4t}\right)}$$

4.3-5

b. $$Y_1(s) = \frac{s+1}{s(s^2+3s+1)} = \frac{K_1}{s} + \frac{K_2}{(s+0.38)} + \frac{K_3}{(s+2.62)}$$

$$\frac{s+1}{s(s+0.38)(s+2.62)} = \frac{K_1}{s} + \frac{K_2}{(s+0.38)} + \frac{K_3}{(s+2.62)}$$

$$\frac{s+1}{(s+0.38)(s+2.62)} = K_1$$ LET $s=0$

$\longrightarrow K_1 = 1$

$$\frac{(s+1)}{s(s+2.62)} = K_2$$ LET $s=-0.38$

$\longrightarrow K_2 = -0.72$

$$\frac{s+1}{s(s+0.38)} = K_3$$ LET $s=-2.62$

$\longrightarrow K_3 = -0.276$

$$Y_1(s) = \frac{1}{s} - \frac{0.72}{(s+0.38)} - \frac{0.276}{(s+2.62)}$$

INV. LAPLACE

$$y_1(t) = (1 - 0.72e^{-0.38t} - 0.276e^{-2.62t})u(t)$$

$$\boxed{\frac{y_1(t)}{x(t)} = (1-0.72e^{-0.38t} - 0.276e^{-2.62t})}$$

4.3-5

6. $$Y_2(s) = \frac{s+2}{s(s^2+3s+1)} = \frac{K_1}{s} + \frac{K_2}{(s+0.38)} + \frac{K_3}{(s+2.62)}$$

$$\frac{s+2}{(s+0.38)(s+2.62)} = K_1$$ LET $s=0$

$K_1 = 2$

$$\frac{s+2}{s(s+2.62)} = K_2$$ LET $s=-0.38$

$K_2 = -1.90$

$$\frac{s+2}{s(s+0.38)} = K_3$$ LET $s=-2.62$

$K_3 = -0.106$

$$Y_2(s) = \frac{2}{s} - \frac{1.90}{(s+0.38)} - \frac{0.106}{(s+2.62)}$$

INV. LAPLACE

$$y_2(t) = \left(2 - 1.90e^{-0.38t} - 0.106e^{-2.62t}\right)u(t)$$

$$\boxed{\frac{y_2(t)}{x(t)} = \left(2 - 1.90e^{-0.38t} - 0.106e^{-2.62t}\right)}$$

4.3-6

a. $\dot{y}(t) + 2y(t) = \dot{x}(t)$

$$\mathcal{L}\{(D+2)\,y(t)\} = \mathcal{L}\{Dx(t)\}$$

$$sY(s) + 2Y(s) = sX(s)$$

$$H(s) \quad \frac{Y(s)}{X(s)} = \frac{s}{s+2}$$

a) $\boxed{H(s) = \frac{s}{s+2}}$

b.

$$\frac{s}{s+2}\cdot\frac{1}{1} = \frac{K_1}{1} + \frac{K_2}{s+2}$$

$$s = (s+2)K_1 + K_2$$

LET $s = -2$

$K_2 = -2$

$$\frac{s}{s+2} = K_1 - \frac{2}{s+2}$$

LET $s = 1$

$$\frac{1}{3} = K_1 - \frac{2}{3}$$

$K_1 = 1$

$$H(s) = 1 - \frac{2}{s+2}$$

$$h(t) = (\delta(t) - 2e^{-2t})\,u(t)$$

$\boxed{h(t) = (\delta(t) - 2e^{-2t})\,u(t)}$

4.3-6

c. $\dot{y}(t) + 2y(t) = \dot{x}(t)$

$y(0^-) = \sqrt{2}$ $\qquad x(t) = e^{-t}u(t)$

$X(s) = \frac{1}{s+1}$

$$\mathcal{L}\{(D+2)y(t)\} = \mathcal{L}\{Dx(t)\}$$

$$sY(s) - y(0^-) + 2Y(s) = sX(s)$$

$$Y(s)(s+2) - \sqrt{2} = \frac{s}{s+1}$$

$$Y(s)(s+2) = \frac{s}{s+1} + \sqrt{2}$$

$$Y(s) = \frac{s}{(s+1)(s+2)} + \frac{\sqrt{2}}{s+2}$$

$$Y(s) = \frac{s + \sqrt{2}(s+1)}{(s+1)(s+2)} = \frac{K_1}{s+1} + \frac{K_2}{s+2}$$

$$\frac{s + \sqrt{2}(s+1)}{s+2} = K_1 \qquad \text{LET } s = -1$$

$$K_1 = -\frac{1}{2}$$

$$\frac{s + \sqrt{2}(s+1)}{s+1} = K_2 \qquad \text{LET } s = -2$$

$$K_2 = 2 + \sqrt{2}$$

$$K_2 = 3.414$$

$$Y(s) = -\frac{1}{2(s+1)} + \frac{3.414}{(s+2)}$$

$$\boxed{y(t) = \left(-\frac{1}{2}e^{-t} + 3.414e^{-2t}\right)u(t)}$$

4.3-9

a. $\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 24y(t) = 5\frac{x(t)}{dt} + 3x(t)$

$\mathcal{L}\{(D^2 + 11D + 24)\,y(t)\} = \mathcal{L}\{(5D+3)\,x(t)\}$

$(s^2 + 11s + 24)\,Y(s) = (5s+3)\,X(s)$

$$\boxed{H(s) = \frac{Y(s)}{X(s)} = \frac{5s+3}{(s+8)(s+3)}} = \frac{K_1}{s+8} + \frac{K_2}{s+3}$$

$\frac{5s+3}{s+3} = K_1$ LET $s = -8$

$K_1 = 7.4$

$\frac{5s+3}{s+8} = K_2$ LET $s = -3$

$K_2 = -7$

<u>OR</u>

$$\boxed{H(s) = \frac{7.4}{s+8} - \frac{7}{s+3}}$$

4.3-10

a. $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{s+5}{s^2+3s+8}$

$$(s^2+3s+8)\,Y(s) = (s+5)\,X(s)$$

$$s^2Y(s) + 3sY(s) + 8Y(s) = sX(s) + 5X(s)$$

$$(D^2 + 3D + 8)\,y(t) = (D + 5)\,x(t)$$

$$\boxed{\frac{d^2y(t)}{dt^2} + \frac{3\,dy(t)}{dt} + 8y(t) = \frac{dx(t)}{dt} + 5x(t)}$$