Measuring the speed of light

John P. Cummings

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In this lab you'll see how Ole Rømer used variations in the observed times of Io solar eclipses by Jupiter to determine the speed of light in 1676. You will use data from recent observations to reproduce Rømers measurement and determine the value of c yourself.

History

Attempts to measure the speed of light go back hundreds of years. Galileo Galelei describes an attempt to measure the speed of light in 1638. In Dialogues Concerning Two New Sciences[1], Salviati describes to Sagredo and Simplicio an experiment that we assume was performed by Galileo: two men, equipped with shuttered lanterns, climb to the tops of hills about one mile apart. The first man opens the shutter of his lantern and the second man, as soon as he sees the light from the first, opens the shutter on his lantern. The first man then notes the time that passes between his opening the shutter and seeing the light from the second lantern. With this time and the distance between lanterns, he can then calculate the speed of light.

[1] http://oll.libertyfund.org/?option=com_staticxt&staticfile=show.php%3Ftitle=753

Galileo's report of his conclusion is interesting. He realizes that the error inherent in his experiment, dominated by the reaction time of the lantern operators, limits the maximum speed he can measure, and thus he can only set a lower bound:

> In fact I have tried the experiment only at a short distance, less than a mile, from which I have not been able to ascertain with certainty whether the appearance of the opposite light was instantaneous or not; but if not instantaneous it is extraordinarily rapid.

Figure 1: Jupiter and its Galilean moons

THE FIRST FINITE MEASUREMENT was made (as often happens in science) by accident. in the late 1600's, Ole Rømer was studying the eclipses of one of Jupiters largest moons, Io.

At the time, it was difficult for navigators to determine their longitude, since this required an accurate clock, which did not exist at the time. Galileo had suggested that the timing of Io's eclipses could be used as a celestial clock, and in fact this method was the best until improvements in mechanical clocks surpassed it in the 1800's. Rømer was making careful measurements of the times of Io's eclipses for this purpose when he noticed that there were irregularities in the times. He noticed that the period between eclipses was slightly longer when the earth was moving away from Jupiter, and slightly shorter when moving towards it. His great insight was that this was due to the finte speed of light. He presented this argument to the French Academy of Sciences in 1676.

Rømer's measurement

To aid the discussion of Rømer's measurement, let's familiarize ourselves with some astronomical terms and concepts.

conjunction occurs when Jupiter and the Sun have the same right ascension, as viewed from earth. Basically Jupiter and the Sun pass each other in the sky.

opposition occurs when Jupiter and the Sun are at their largest angular separation in the sky, as viewed from earth: Jupiter and the Sun are opposite each other in the sky.

Examine Figure 2. The Earth is between a conjunction and an opposition. The radius of the orbit of Io is equal to six times the radius of Jupiter. The angle $SJT = \theta$ is always smaller than 11 deg. On this figure, indicate the points where the beginning and the end of the following four phenomena take place:

Eclipse The satellite enters the shadow of Jupiter.

Occultation The satellite, as seen from Earth, goes behind Jupiter.

Shadow The shadow of the satellite is seen on the planet.

Passage The satellite, as seen from Earth, passes in front of the planet.

- Now explain why only the beginnings of the eclipses can be observed from Earth.
- What happens when the Earth is close to a conjunction?

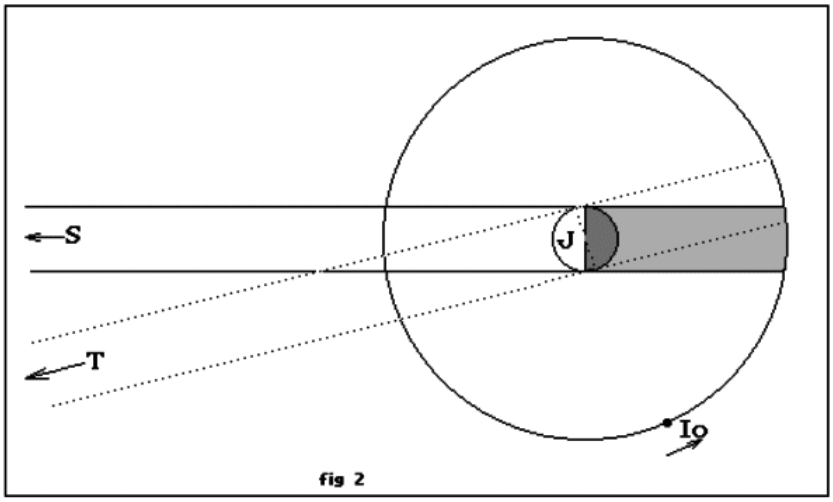

Figure 2: Jupiter (J) , Io (Io) and the directions of the Sun (S) and the Earth (T) at a point between a conjunction and an opposition.

- Draw another figure that shows what happens when the Earth is between an opposition and a conjunction.
- Explain why then only the ends of the eclipses can be observed from Earth.

We can now look at how Rømer measured the speed of light.

Looking at figure 3 he noticed that the end of eclipses at K or E seemed to happen late if he started reckoning at point L or H. He reasoned that this delay was caused by the extra time it took light to propagate this extra distance.

Although Rømer never calculated a value for the speed of light, Christian Huygens quickly got Rømer's data and did so, getting a value of $16\frac{2}{3}$ Earth diameters per second (or 2.125×10^8 m/s). Pretty good for 1676!

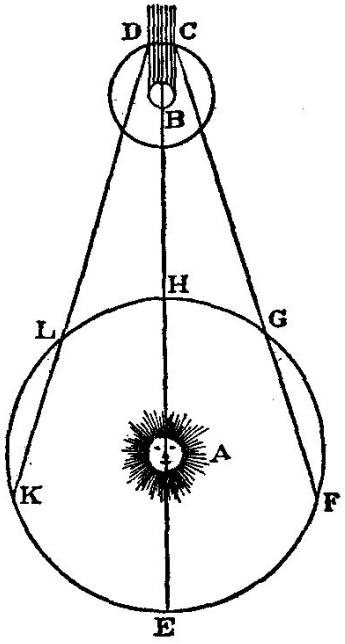

Figure 3: From the 1676 article describing Rømer's method.

A modern recalculation

The exercise for this lab is to reconstruct Rømers argumant and recalculate using modern observations. Below is a table of eclipse timings from the mid 1990's. This table, which I will also provide as a text file also so you can read it into MATLAB, and the radius of the Earth's orbit is all you should need to calculate c.

You should first use the data to determine the period of Io. I would suggest using data from a region of Earths orbit that is not affected significantly by the finite speed of light. What part of the Earths orbit meets this criterion?

Once you have determined a period, use it to predict the time of a later eclipse (either beginning or end of eclipse, depending on

the data you are using). Once you have the predicted time, find the difference in time with the corresponding eclipse observation from the tables. By using this time difference and the distance the earth has moved between the two eclipse observations, you should be able to calculate the speed of light.

If you are reading the data files into MATLAB, I would suggest spending a few minutes building some tools to help your analysis.

Write a function which takes a filename argument and reads the data into an array. It should be called something like

```
eclipse_array = read_dat('eclipses.dat');
```

You will probably need to use the `fscanf` function, as in

```
a = fscanf(fp,'%d %d/%d/%d %dh%d');
```

if you were to use a line like this in your function, `a` would be array with six values for each eclipse: the number, day, month, year, hour, and minute of the eclipse. It is a one dimensional array, so the hour (fifth number) of the third eclipse, for instance is in `a(3*6+5)`.

Next write a function that takes an array like above and an eclipse number, and returns the time in a decimal format. It could be days, hours, or minutes, but it is important to make it decimal to facilitate finding time differences. It is much easier to calculate 516928-501637 than it is 24/12/94 23h28 minus 14/12/94 08h37. Its call should look something like

```
time=get_time(3, eclipse_array);
```

to get the time of the third eclipse in the file.

Once you have these, you should be able to write a little MATLAB script that will calculate the speed of light.

THOSE OF YOU doing the exercise "by hand" might find Wolfram Alpha useful for such things as finding the difference between two dates. Visit the Alpha website and experiment with various date formats to make sure you understand the answers it gives you.

References

Some sources used to prepare this document, and which may be useful to you as you work on this lab:

1. `http://en.wikipedia.org/wiki/Ole_R%C3%B8mer`
2. `http://www.eaae-astronomy.org/WG3-SS/WorkShops/Romer.html`
3. `http://www.mathpages.com/home/kmath203/kmath203.htm`
4. `http://www.is.wayne.edu/mnissani/a&s/light.htm`

#	date	time	#	date	time	#	date	time
00	14/12/94	08h37	34	12/02/95	12h41	68	13/04/95	16h41
01	16/12/94	03h06	35	14/02/95	07h09	69	15/04/95	11h09
02	17/12/94	21h34	36	16/02/95	01h37	70	17/04/95	05h37
03	19/12/94	16h03	37	17/02/95	20h05	71	19/04/95	00h05
04	21/12/94	10h31	38	19/02/95	14h34	72	20/04/95	18h34
05	23/12/94	04h59	39	21/02/95	09h02	73	22/04/95	13h02
06	24/12/94	23h28	40	23/02/95	03h30	74	24/04/95	07h30
07	26/12/94	17h56	41	24/02/95	21h58	75	26/04/95	01h59
08	28/12/94	12h25	42	26/02/95	16h27	76	27/04/95	20h27
09	30/12/94	06h53	43	28/02/95	10h55	77	29/04/95	14h55
10	01/01/95	01h21	44	02/03/95	05h23	78	01/05/95	09h23
11	02/01/95	19h50	45	03/03/95	23h51	79	03/05/95	03h52
12	04/01/95	14h18	46	05/03/95	18h19	80	04/05/95	22h20
13	06/01/95	08h46	47	07/03/95	12h48	81	06/05/95	16h48
14	08/01/95	03h15	48	09/03/95	07h16	82	08/05/95	11h17
15	09/01/95	21h43	49	11/03/95	01h44	83	10/05/95	05h45
16	11/01/95	16h11	50	12/03/95	20h12	84	12/05/95	00h14
17	13/01/95	10h40	51	14/03/95	14h41	85	13/05/95	18h42
18	15/01/95	05h08	52	16/03/95	09h09	86	15/05/95	13h10
19	16/01/95	23h36	53	18/03/95	03h37	87	17/05/95	07h39
20	18/01/96	18h05	54	19/03/95	22h05	88	19/05/95	02h07
21	20/01/95	12h33	55	21/03/95	16h33	89	19/05/95	20h36
22	22/01/95	07h01	56	23/03/95	11h02	90	22/05/95	15h04
23	24/01/95	01h30	57	25/03/95	05h30	91	24/05/95	09h32
24	25/01/95	19h56	58	26/03/95	23h58	92	26/05/95	04h01
25	27/01/95	14h26	59	28/03/95	18h26	93	27/05/95	22h29
26	29/01/95	08h55	60	30/03/95	12h55	94	29/05/95	16h58
27	31/01/95	03h23	61	01/04/95	07h23	95	31/05/95	11h26
28	01/02/95	21h51	62	03/04/95	01h51			
29	03/02/95	16h19	63	04/04/95	20h19			
30	05/02/95	10h48	64	06/04/95	14h48			
31	07/02/95	05h16	65	08/04/95	09h16			
32	08/02/95	23h44	66	10/04/95	03h44			
33	10/02/95	18h12	67	11/04/95	22h12			

Table 1: Begin of Eclipse between the conjunction on November 17, 1994 (20h00) and the opposition on June 1, 1995 (11h00).

#	date	time	#	date	time	#	date	time
00	02/06/95	08h06	34	01/08/95	12h20	68	30/09/95	16h40
01	04/06/95	02h34	35	03/08/95	06h48	69	02/10/95	11h09
02	05/06/95	21h03	36	05/08/95	01h17	70	04/10/95	05h37
03	07/06/95	15h31	37	06/08/95	09h56	71	06/10/95	00h06
04	09/06/95	10h00	38	08/08/95	14h15	72	07/10/95	18h35
05	11/06/95	04h28	39	10/08/95	08h44	73	09/10/95	13h04
06	12/06/95	22h57	40	12/08/95	03h12	74	11/10/95	07h33
07	14/06/95	17h25	41	13/08/95	21h41	75	13/10/95	02h01
08	16/06/95	11h54	42	15/08/95	16h10	76	14/10/95	20h30
09	18/06/95	06h22	43	17/08/95	10h39	77	16/10/95	14h59
10	20/06/95	00h51	44	19/08/95	05h08	78	18/10/95	09h28
11	21/06/95	19h20	45	20/08/95	23h26	79	20/10/95	03h57
12	23/06/95	13h48	46	22/08/95	18h05	80	21/10/95	22h26
13	25/06/95	08h17	47	24/08/95	12h34	81	23/10/95	16h54
14	27/06/95	02h45	48	26/08/95	07h03	82	25/10/95	11h23
15	28/06/95	21h14	49	28/08/95	01h32	83	27/10/95	05h52
16	30/06/95	15h43	50	29/08/95	20h00	84	29/10/95	00h21
17	02/07/95	10h11	51	31/08/95	14h29	85	30/10/95	18h50
18	04/07/95	04h40	52	02/09/95	08h58	86	01/11/95	13h18
19	05/07/95	23h09	53	04/09/95	03h27	87	03/11/95	07h47
20	07/07/95	17h37	54	05/09/95	21h56	88	05/11/95	02h16
21	09/07/95	12h06	55	07/09/95	16h25	89	06/11/95	20h45
22	11/07/95	06h35	56	09/09/95	10h54	90	08/11/95	15h13
23	13/07/95	01h03	57	11/09/95	05h22	91	10/11/95	09h42
24	14/07/95	19h32	58	12/09/95	23h51	92	12/11/95	04h11
25	16/07/95	14h01	59	14/09/95	18h20	93	13/11/95	22h40
26	18/07/95	08h30	60	16/09/95	12h49	94	15/11/95	17h09
27	20/07/95	02h58	61	18/09/95	07h18	95	17/11/95	11h37
28	21/07/95	21h27	62	20/09/95	01h47	96	19/11/95	06h06
29	23/07/95	15h56	63	21/09/95	20h15	97	21/11/95	00h35
30	25/07/95	10h24	64	23/09/95	14h44	98	22/11/95	19h04
31	27/07/95	04h53	65	25/09/95	09h13			
32	28/07/95	23h22	66	27/09/95	03h42			
33	30/07/95	17h51	67	28/09/95	22h11			

Table 2: End of Eclipse between the opposition on June 1, 1995 (11:00) and the conjunction on December 18, 1995 (22:00)

#	date	time	#	date	time	#	date	time
00	14/01/96	19h09	34	14/03/96	23h15	68	14/05/96	03h18
01	16/01/96	13h38	35	16/03/96	17h44	69	15/05/96	21h46
02	18/01/96	08h06	36	18/03/96	12h12	70	17/05/96	16h15
03	20/01/96	02h35	37	20/03/96	06h40	71	19/05/96	10h43
04	21/01/96	21h03	38	22/03/96	01h29	72	21/05/96	05h11
05	23/01/96	21h03	39	23/03/96	19h37	73	22/05/96	23h40
06	25/01/96	10h00	40	25/03/96	14h05	74	24/05/96	18h08
07	27/01/96	04h29	41	27/03/96	08h34	75	26/05/96	12h36
08	28/01/96	22h57	42	29/03/96	03h02	76	28/05/96	07h05
09	30/01/96	17h25	43	30/03/96	21h30	77	30/05/96	01h33
10	01/02/96	11h54	44	01/04/96	15h59	78	31/05/96	20h01
11	03/02/96	06h22	45	03/04/96	10h27	79	02/06/96	14h30
12	05/02/96	00h51	46	05/04/96	04h55	80	04/06/96	08h58
13	06/02/96	19h19	47	06/04/96	23h24	81	06/06/96	03h26
14	08/02/96	13h48	48	08/04/96	17h52	82	07/06/96	21h55
15	10/02/96	08h16	49	10/04/96	12h20	83	09/06/96	16h23
16	12/02/96	02h45	50	12/04/96	06h48	84	11/06/96	10h52
17	13/02/96	21h13	51	14/04/96	01h17	85	13/06/96	05h20
18	15/02/96	15h41	52	15/04/96	19h45	86	17/06/96	23h48
19	17/02/96	10h10	53	17/04/96	14h13	87	16/06/96	18h17
20	19/02/96	04h38	54	19/04/96	08h42	88	18/06/96	12h45
21	20/02/96	23h07	55	21/04/96	03h10	89	20/06/96	07h14
22	22/02/96	17h35	56	22/04/96	21h38	90	22/06/96	01h42
23	24/02/96	12h03	57	24/04/96	16h07	91	23/06/96	20h11
24	26/02/96	06h32	58	26/04/96	10h35	92	25/06/96	14h39
25	28/02/96	01h00	59	28/04/96	05h03	93	27/06/96	09h08
26	29/02/96	19h29	60	29/04/96	23h31	94	29/06/96	03h36
27	02/03/96	13h57	61	01/05/96	18h00	95	30/06/96	22h04
28	04/03/96	08h25	62	03/05/96	12h28	96	02/07/96	16h33
29	06/03/96	02h54	63	05/05/96	06h56			
30	07/03/96	21h22	64	07/05/96	01h25			
31	09/03/96	15h50	65	08/05/96	19h53			
32	11/03/96	10h19	66	10/05/96	14h21			
33	13/03/96	04h47	67	12/05/96	08h50			

Table 3: Begin of Eclipse between the conjunction on December 18, 1995 (22h00) and the opposition on July 4, 1996 (12h00).

#	date	time	#	date	time	#	date	time
00	04/07/96	13h16	34	02/09/96	17h31	68	01/11/96	21h51
01	06/07/96	07h45	35	04/09/96	12h00	69	03/11/96	16h20
02	08/07/96	02h13	36	06/09/96	06h28	70	05/11/96	10h49
03	09/07/96	20h42	37	08/09/96	00h57	71	07/11/96	05h18
04	11/07/96	15h10	38	09/09/96	19h26	72	08/11/96	23h47
05	13/07/96	09h39	39	11/09/96	13h55	73	10/11/96	18h16
06	15/07/96	04h08	40	13/09/96	08h24	74	12/11/96	12h44
07	16/07/96	22h36	41	15/09/96	02h52	75	14/11/96	07h13
08	18/07/96	17h05	42	16/09/96	21h21	76	16/11/96	01h42
09	20/07/96	11h33	43	18/09/96	15h50	77	17/11/96	20h11
10	22/07/96	06h02	44	20/09/96	10h19	78	19/11/96	14h40
11	24/07/96	00h31	45	22/09/96	04h48	79	21/11/96	09h09
12	25/07/96	18h59	46	23/09/96	23h17	80	23/11/96	03h37
13	27/07/96	13h28	47	25/09/96	17h45	81	24/11/96	22h06
14	29/07/96	07h56	48	27/09/96	12h14	82	26/11/96	16h35
15	31/07/96	02h25	49	29/09/96	06h43	83	28/11/96	11h04
16	01/08/96	20h54	50	01/10/96	01h12	84	30/11/96	05h33
17	03/08/96	15h22	51	02/10/96	19h41	85	02/12/96	00h02
18	05/08/96	09h51	52	04/10/96	14h10	86	03/12/96	18h31
19	07/08/96	04h20	53	06/10/96	08h38			
20	08/08/96	22h48	54	08/10/96	03h07			
21	10/08/96	17h17	55	09/10/96	21h36			
22	12/08/96	11h46	56	11/10/96	16h05			
23	14/08/96	06h15	57	13/10/96	10h34			
24	16/08/96	00h43	58	15/10/96	05h03			
25	17/08/96	19h12	59	16/10/96	23h32			
26	19/08/96	13h41	60	18/10/96	18h00			
27	21/08/96	08h09	61	20/10/96	12h29			
28	23/08/96	02h38	62	22/10/96	06h58			
29	24/08/96	21h07	63	24/10/96	01h27			
30	26/08/96	15h36	64	25/10/96	16h56			
31	28/08/96	10h04	65	27/10/96	14h25			
32	30/08/96	04h33	66	29/10/96	08h54			
33	31/08/96	23h02	67	31/10/96	03h22			

Table 4: End of Eclipse after the opposition on July 4, 1996 (12h00)