22 Electric Interactions

Electricity is a familiar term—outlets, batteries, light bulbs, computers all involve electricity. It is no understatement to say that modern life depends on electricity, but what exactly *is* electricity? We all know what electricity does, but it's not that easy to explain what electricity *is*.

Electricity manifests itself in many ways: from the sparks that fly when you scuff your feet across a carpet on a dry winter day to the electricity we use in our homes to the transmission of radio and television programs. Even the attraction between magnets has to do with electricity. In this chapter, we begin our treatment of electricity with a discussion of static electricity.

22.1 Static electricity

When you tear off some plastic wrap from its roll, the wrap is attracted to anything that gets close: your hand, the countertop, a dish. This interaction between the plastic wrap and other objects doesn't have to involve any physical contact. For example, you can feel the presence of a piece of freshly torn-off plastic wrap with your cheek or the back of your hand even when your face or hand is held some distance away from the piece. You may have experienced many similar interactions: Styrofoam peanuts are attracted to your arms when you unpack a box full of them (**Figure 22.1**). Running a comb through your hair on a dry day causes the comb to attract your hair. After rubbing a balloon against a woolen sweater, you can hold the balloon close to a wall and *see* the attraction as the balloon moves toward the wall. In all these instances, the mass of the objects is too small for the interactions to be gravitational. What, then, is this interaction?

You may never have thought of these interactions as being particularly strong, but consider this: If you rub a comb through your hair and then pass the comb over some small bits of paper, the bits of paper jump up to your comb and stick to it. In other words, the bits of paper accelerate upward, which means the force exerted by your comb on them must be *greater* than the gravitational force exerted on them by Earth!

Now try this: Quickly pull a 20-cm strip of transparent tape* out of a dispenser and suspend it from the edge of a table (just be sure the table is not metal). Notice how the tape is attracted to anything brought nearby. It might even take some practice to prevent the tape from curling up and sticking to the underside of the table or to your hand. Bring a few objects near the suspended tape and notice the attractive interaction between them.† Go ahead—experiment!

Figure 22.1 Styrofoam peanuts cling to the cat's fur because of static electricity.

22.1 Suspend a freshly pulled piece of transparent tape from the edge of your desk. (*a*) What happens when you hold a battery near the tape? Does it matter whether you point the + side or the − side of the battery toward the tape? Does a spent battery yield a different result? Does a wooden object yield a different result? (*b*) What happens when you hold a strip of freshly pulled tape near the power cord of a lamp? Does it make any difference if the lamp is on or off?

All these interactions involving static electricity are examples of **electric interactions.** The experiment you just did tells you there is no obvious connection between electric interactions and the electricity we think of as "flowing" in electric circuits and batteries. In Chapter 31 we shall see, however, that the two are connected.

Objects that participate in electric interactions exert an **electric force** on each other. The electric force is a field force (see Section 8.3): Objects exerting electric forces on each other need not be physically touching. As you may have noticed from the interaction between the strips of tape and various nearby objects, the magnitude of the electric force depends on distance: It decreases as you increase the separation.

22.2 Suspend a freshly pulled strip of transparent tape from the edge of your desk. (*a*) Pull a second strip of tape out of the dispenser and hold it near the first strip. What do you notice? (*b*) Does it matter which sides of the strips you orient toward each other?

As Checkpoint 22.2 makes clear, not all electric interactions are attractive. Even if you increase the mass of the strips by suspending paper clips from them, the repulsion between the strips is great enough to keep the paper clips apart (**Figure 22.2**). Now place your hand between two repelling strips and notice how both strips fly toward your hand! Then run each strip of tape several times between your fingers and notice how the electric interaction diminishes or even disappears.

22.3 Suspend two freshly pulled 20-cm strips of transparent tape from the edge of your desk. Cut two 20-cm strips of paper, making each strip the same width as the tape, and investigate the interactions between the paper strips and the tape by bringing them near each other. Which of the following combinations display an electric interaction: paper-paper, tape-paper, tape-tape?

*For best results, use the type called "magic" tape.

†If you find something that *repels* the tape, wipe the entire surface of the object with your hand and see if it still repels—it shouldn't. Mystified? Hang on! We'll soon be able to resolve your questions.

CONCEPTS

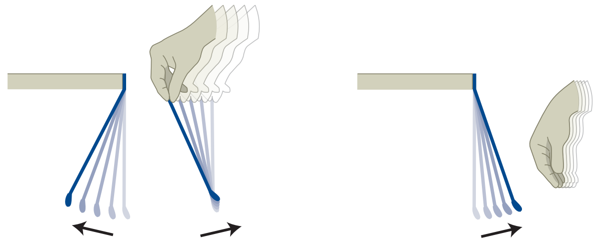
Figure 22.2 Strips of tape just pulled out of a dispenser repel each other. The repulsive force is great enough to keep the strips apart even when they are weighted down by paper clips.

22.2 Electrical charge

As we saw in the preceding section, electric interactions are sometimes attractive and sometimes repulsive. In addition, the experiment you performed in Checkpoint 22.3 demonstrates that paper strips, which do not interact electrically with each other, do interact electrically with transparent tape. What causes these interactions? To answer this question, we need to carry out a systematic sequence of experiments.

Figure 22.3 illustrates a simple procedure for reproducibly creating strips of tape that interact electrically. A suspended strip created according to this procedure interacts in the following ways: It repels another strip created in the same manner, and it attracts any other object that does not itself interact electrically with other objects (**Figure 22.4**).

Figure 22.4



Let us call the attribute responsible for the electric interaction **electrical charge,** or simply **charge.** Saying that something carries an electrical charge is just another way of saying that that object interacts electrically with other objects that carry electrical charge. Freshly pulled strips of tape carry electrical charge, and two such strips interact because each possesses an electrical charge, just as your body and Earth interact because each possesses mass. The general term for any microscopic object that carries an electrical charge, such as an electron or ion, is **charge carrier.**

It is not immediately clear what attributes to assign to objects that do not interact electrically with each other but do interact with a charged tape strip—a strip of paper, your hand, an eraser, you name it. All we know for now is that the interaction between these objects and a charged tape is attractive rather than repulsive.

The electric charge on an object is not a permanent property; if you let a charged strip of tape hang for a while, it loses its ability to interact electrically. In other words, the strip is no longer charged—it is *discharged*. Depending on the humidity of the air, the discharging can take minutes or hours, but you can speed up the discharging by rubbing your fingers a few times over the entire length of a suspended charged strip of tape.* (The rubbing allows the charge to “leak away” from the tape by distributing itself over your body.)

*If rubbing your fingers along the tape doesn’t do the job, try licking them before rubbing them over the tape.

Figure 22.3 Procedure for making strips of transparent tape that interact electrically. The purpose of the foundation strip is simply to provide a standard surface.

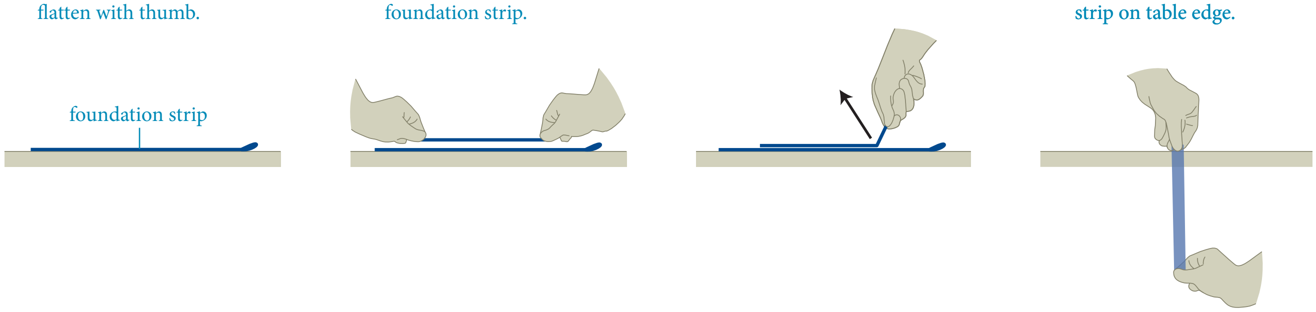
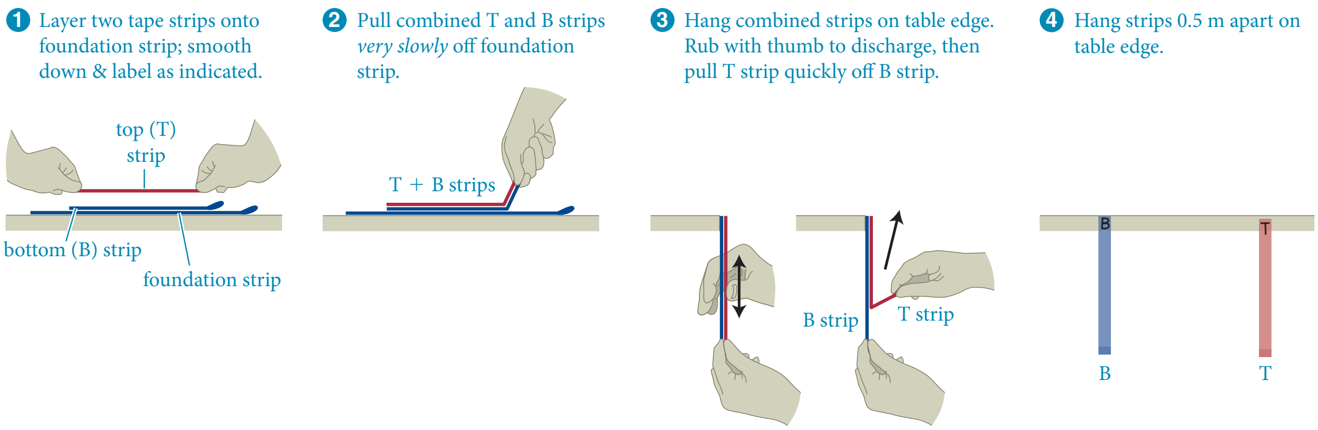


Figure 22.5 Procedure for making strips of transparent tape that carry opposite charges.



22.4 (*a*) Prepare a charged strip of transparent tape as described in Figure 22.3 and then suspend the strip from the edge of your desk. Verify that the tape interacts as you would expect with your hand, with a strip of paper, and with another charged strip of tape. (*b*) Rub your fingers along the hanging strip to remove all the charge from it, and then verify that it no longer interacts with your hand. If it does interact, rub again until it no longer interacts. (*c*) Predict and then verify experimentally how the uncharged suspended strip interacts with a strip of paper and with a charged strip of tape.

To restore the charge on a discharged strip, stick the strip on top of the foundation strip from which you pulled it off (step 1 in Figure 22.3), smooth it out, and then quickly pull it off again. You can recharge a strip quite a few times before it loses its adhesive properties. Once the tape does lose its adhesiveness, however, recharging it becomes impossible. It is generally a good idea to rub your finger over the foundation strip before you reuse it to make sure that it, too, is uncharged.

22.5 Recharge the discharged strip from Checkpoint 22.4 and verify that it interacts as before with your hand, with a strip of paper, and with another charged strip of tape.

A discharged tape strip interacts in the same way as objects that carry no charge. Such objects are said to be electrically **neutral.** They do not interact electrically with other neutral objects, but they do interact electrically with charged objects. We shall examine this surprising fact in more detail in Section 22.4.

Where does the electrical charge on a charged tape strip come from? Is charge *created* when two strips are pulled apart as in Figure 22.3? This is something we can check by sticking two strips of tape together, rubbing with our fingers to remove all charge from the combination, and then quickly separating the two strips (**Figure 22.5**).

22.6 Follow the procedure illustrated in Figure 22.5 to separate a pair of charged strips. (*a*) How does strip B interact with a neutral object? How does strip T interact with a neutral object? (*b*) Create a third charged strip and see how it interacts with strip B and with strip T. (*c*) Is strip T charged? (*d*) Is strip B charged? (*e*) Check what happens to the interactions with B and T strips when you discharge a B or a T strip by rubbing your fingers along its length.

As Checkpoint 22.6 shows, separating an uncharged pair of strips produces two charged ones, but the behavior of strip B is different from that of the other strips we have encountered so far!

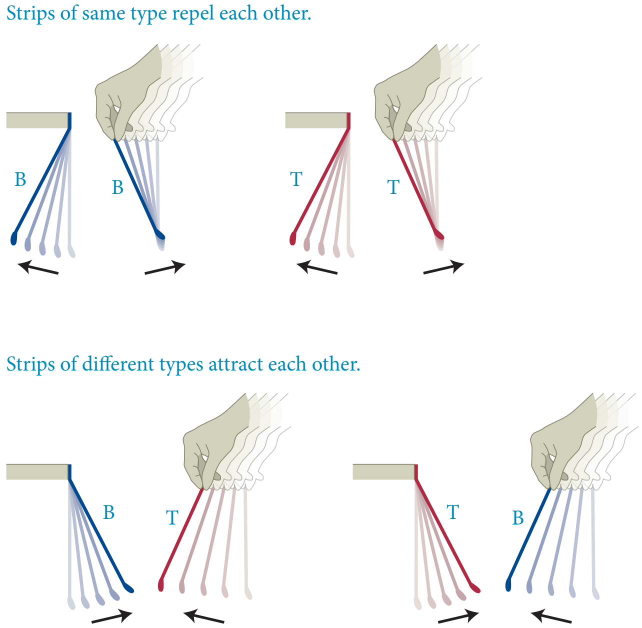
22.7 Make two charged pairs of strips (B and T) following the procedure illustrated in Figure 22.5. Investigate the interaction of B with T, T with T, and B with B.

The interactions between the B and T strips are illustrated in **Figure 22.6**: Strips of the same type repel each other, while strips of different types attract each other. This series of experiments leads us to conclude that there are two types of charge on the tapes, one type on B strips and another type on T strips. Strips that carry the same type of charge, called *like charges,* exert repulsive forces on each other; strips that carry different types of charge, called *opposite charges,* exert attractive forces on each other.

Having determined that two types of electrical charge exist, a logical next question is: Are there even more types?

22.8 (*a*) Prepare one charged strip of tape according to Figure 22.3 and hang it from the edge of your desk. Hang a narrow strip of paper from the desk edge also, about 0.5 m away from the tape strip. Pass a *plastic* comb six times quickly through your hair and then show that the comb is charged. Be sure to use a plastic comb; combs made from other materials do not acquire a charge when passed through hair. The cheapest type of comb usually works best. (*b*) Make a pair of oppositely charged B and T strips (Figure 22.5) and investigate how they interact with a charged comb. (*c*) Does your comb behave like a B strip, a T strip, or neither?

Figure 22.6 Interactions of B and T charged strips.



Experiments show that *any* charged object—obtained by rubbing objects together or otherwise—always attracts either a B strip or a T strip and repels the other. No one has ever found a charged object that repels or attracts *both* types of strips. In other words:

> **There are two and only two types of charge. Objects that carry like charges repel each other; objects that carry opposite charges attract each other.**

The two types of charge never appear independently of each other: Whenever two neutral objects are either rubbed together and then separated or, if an adhesive surface is involved, stuck together and then separated and one of them acquires a charge of one type, the other object always acquires a charge of the other type. The generation of opposite charges is obvious when you separate a neutral pair of tape strips. When you pass a comb through your hair, the comb acquires a charge of one type and your hair acquires a charge of the other type. On a dry day, you may have noticed that some hair strands stand up away from your head. Each charged strand is being repelled by the other charged strands, and so they are all getting as far away from one another as possible.

It can be shown that when two tape strips are separated, the forces exerted by the B strip and the T strip on a third charged strip are equal in magnitude, although one is attractive and the other repulsive. Furthermore, when the B and T strips are recombined, the combination is neutral again. These observations suggest that after you rub and then separate a pair of objects, the objects carry equal amounts of opposite charge. Combining these equal amounts of opposite charge produces zero charge. These observations indicate that all neutral matter contains equal amounts of positive and negative charge. The two types of charge are called **positive** and **negative charges.** The definition of negative charge is as follows:*

> **Negative charge is the type of charge acquired by a plastic comb that has been passed through hair a few times.**

22.9 Does the B strip you created in Checkpoint 22.8 carry a positive charge or a negative charge?

When two neutral objects touch, some charge can be transferred from one object to the other, with the result that one object ends up with a surplus of one type of charge and the other object ends up with an equal surplus of the other type of charge. For example, when a neutral piece of styrofoam is rubbed with a neutral piece of plastic wrap, the styrofoam acquires a positive charge (meaning it contains more positive than negative charge) and the plastic wrap acquires a negative charge (it has a surplus of negative charge). Without further information, however, we cannot tell whether positive charge has been transferred from the wrap to the styrofoam, or negative charge has been transferred from the styrofoam to the plastic wrap, or a combination of these two. (See **Figure 22.7** on the next page.) Summarizing:

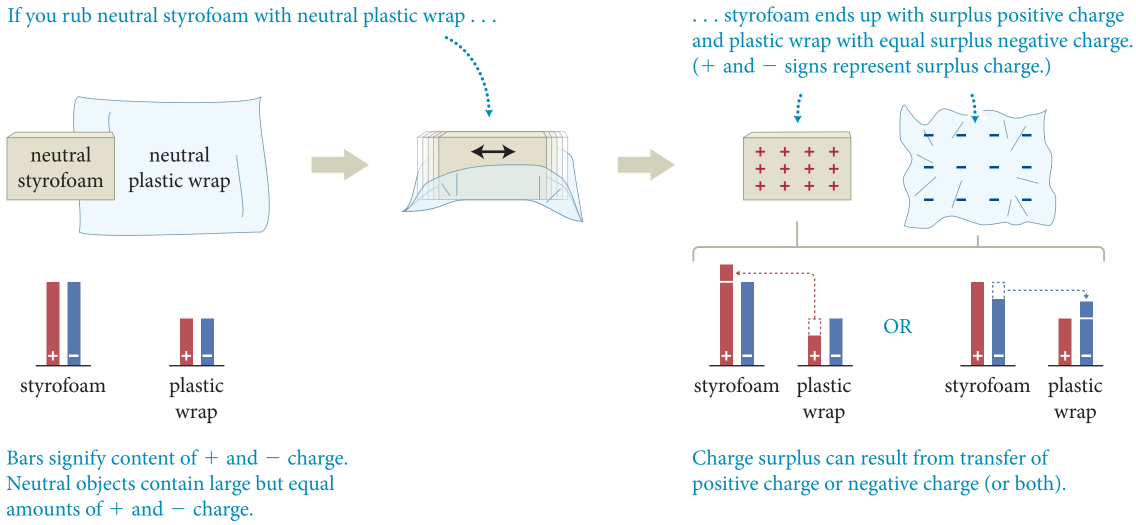
> **All neutral matter contains equal amounts of positive and negative charge; charged objects contain unequal amounts of positive and negative charge.**

In illustrations, surplus charge is represented by plus or minus signs. Keep in mind, however, that these signs never represent the only type of charge in an object. The plus signs on the positively charged styrofoam in Figure 22.7, for example, mean only that the styrofoam contains more positive than negative charge, either because some of its negative charge has been removed or because some positive charge has been added. In addition to the 12 positive charge carriers shown in Figure 22.7, the styrofoam contains millions and millions of positive charge carriers paired with millions and millions of negative charge carriers. A drawing such as Figure 22.7, shows only *unpaired* charge carriers (usually referred to as *surplus charge*).

As our observations in Figure 22.6 show, oppositely charged B and T strips attract each other. The interaction between positive and negative charge tends to bring positive and negative charge carriers as close together as possible. Because combining equal amounts of positive and negative charge results in zero charge, we can say that charge carriers always tend to arrange themselves in such a way as to produce uncharged objects—indeed, all matter around us tends to be neutral.

*Historically, negative charge was (arbitrarily) defined by Benjamin Franklin (1706–1790) as the charge acquired by a rubber rod rubbed with cat fur. Because plastic combs and hair are more easily accessible than rubber rods and cat fur, the definition of negative charge given here is more convenient.

Figure 22.7 Rubbing neutral styrofoam with neutral plastic wrap leaves the two objects with equal charges of opposite types.



22.10 Imagine having a collection of charged marbles that retain their charge even when they touch other objects. Red marbles are positively charged, and blue marbles are negatively charged. (*a*) What happens if you place a bunch of red marbles close together on a flat horizontal surface? (*b*) What happens if you do the same with a bunch of blue marbles? (*c*) What happens if you do the same with an equal mixture of red and blue marbles? (*d*) What happens in part *c* if you have a few more red marbles than blue ones? (*e*) As a whole, is the collection of marbles in part *d* positively charged, negatively charged, or neither?

22.3 Mobility of charge carriers

To gain a better understanding of electrical charge, many additional experiments are required, most of which require items not easily found at home. A rubber rod rubbed with a piece of cat fur acquires—by Benjamin Franklin's original definition—a negative charge (and the fur acquires a positive charge). A glass rod rubbed with silk acquires a positive charge (and the silk a negative charge). Other materials also acquire a charge upon contact or rubbing, but these two combinations of rubber/fur and glass/silk provide the most convenient means of generating relatively large amounts of charge.

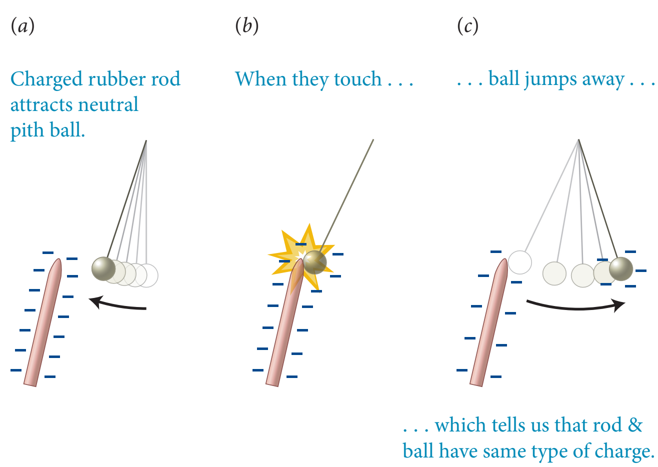
Interesting things happen when a charged rubber rod is brought into contact with an uncharged pith ball.* As the rod is brought near the ball, the ball moves toward the rod because of the attraction between the charged rod and the neutral ball (**Figure 22.8*a***). As the ball touches the rod, however (Figure 22.8*b*), the crackling sounds of tiny sparks may be heard. The ball suddenly jumps away from the rod (Figure 22.8*c*), indicating that the interaction between rod and ball has become repulsive. This repulsive interaction indicates that the ball has acquired the same type of charge as the rod (negative). In other words, some of the surplus negative charge on the rod has been transferred to the ball.

Charge can be transferred from one object to another by bringing the two into contact.

We can use this phenomenon to investigate the electrical behavior of different kinds of materials. For example, if we transfer some charge to one end of an uncharged rubber rod and then extend the charged end toward an uncharged pith ball, the two interact electrically, as shown in **Figure 22.9*a***. If we hold the *uncharged* end near the pith ball, as in Figure 22.9*b*, however, no interaction occurs. This tells us that the charge does not flow from one end of the rubber rod to the other; instead, it remains near the spot where it has been deposited. Any material in which charge doesn't flow (or moves only with great difficulty) is called an **electrical insulator.**

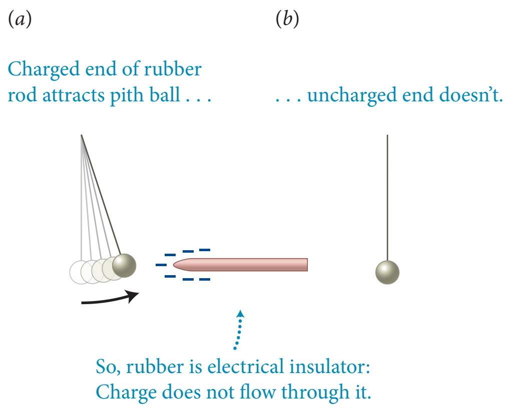
Electrical insulators are materials through which charge carriers cannot flow easily. Any charge transferred to an insulator remains near the spot at which it was deposited.

Figure 22.8 A charged rubber rod can transfer charge to a neutral pith ball.



*Pith is the soft, lightweight, spongelike material that makes up the interior of the stems of flowering plants.

Figure 22.9 A rubber rod is an example of an electrical insulator.



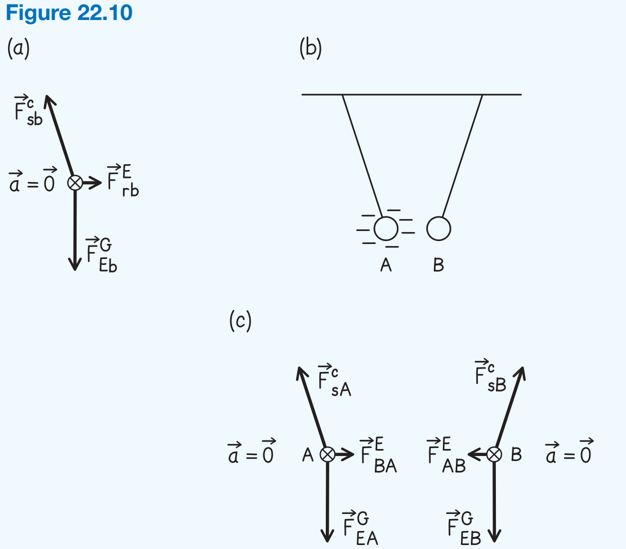
Glass, rubber, wood, and plastic are examples of electrical insulators. Air, particularly dry air, is also an insulator, although the presence of large amounts of charge can cause charge carriers to "jump" from one object to another, causing sparks.

Exercise 22.1 Electric forces

(*a*) Draw a free-body diagram for the pith ball in Figure 22.9*a*. (*b*) Two identical neutral pith balls A and B are suspended side by side from two vertical strings. After some charge is transferred from a charged rod to A, A and B interact. (B remains neutral because the two balls never come into contact with each other.) Sketch the orientation of A and B after the charge has been transferred to A. (*c*) Draw a free-body diagram for each ball.

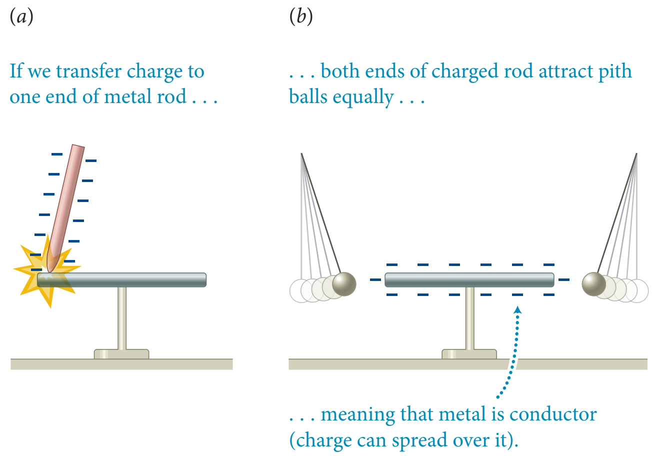
SOLUTION (*a*) The ball is subject to three forces: a gravitational force, a contact force exerted by the string, and an attractive electric force exerted by the charged particles in the rod. This last force is directed horizontally toward the rod. Because the ball is at rest, I know that the vector sum of these three forces is zero. So the horizontal component of the force exerted by the string on the ball must be equal in magnitude to the electric force exerted by the rod on the ball, and the vertical component of the force exerted by the string on the ball must be equal in magnitude to the gravitational force exerted by Earth on the ball (**Figure 22.10*a***).

Figure 22.10



(*b*) As we saw in Section 22.2, a neutral object interacts electrically with a charged object. The electric force exerted by A on B and that exerted by B on A form an interaction pair and so their magnitudes are equal. Because the masses of the pith balls are the same, each is pulled in by the same distance. Thus my sketch is as shown in Figure 22.10*b*. (*c*) See Figure 22.10*c*.

Figure 22.11 A metal rod is an electrical conductor.



In **Figure 22.11**, a charged rod is brought into contact with an uncharged metal rod supported on an electrically insulating stand. Once the charged rod has touched the metal rod, *all* points on the surface of the metal rod interact electrically with other objects, indicating that the charge spreads out over the metal rod. The tendency of charge to spread out over metal objects can be demonstrated with an *electroscope* (**Figure 22.12*a***). Two strips made of metal foil are suspended from a small metal rod in an electrically insulating enclosure; the rod is connected to a metal ball on top of the enclosure. When the metal ball is charged by an exterior source, the strips move away from each other. The explanation for this movement is that the added charge quickly moves from the metal ball through the metal rod and onto the two metal strips. Once the strips carry the same type of charge, they repel each other (Figure 22.12*b*).

Figure 22.12 An electroscope depends on electrical conduction.

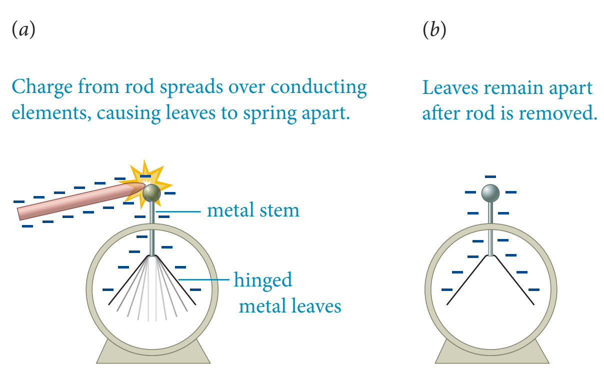
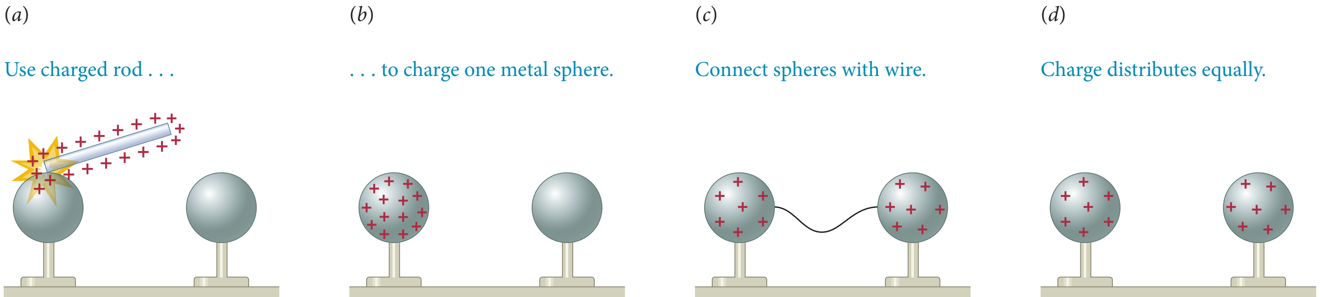


Figure 22.13 A conducting wire distributes charge between two conducting spheres.



Another demonstration of the free motion of charges through metals is shown in **Figure 22.13**: When a long wire is used to connect a charged metal sphere to an uncharged metal sphere, charged particles flow from the charged sphere to the uncharged one. Because wires are made of metal, this experiment shows that, in contrast to what happens with electrical insulators, charge moves easily through a metal and across a metal-to-metal contact. Materials through which charge carriers can flow are called electrical **conductors,** and the flow of charge through conductors is called **conduction.**

Electrical conductors are materials through which charge carriers can flow easily. Any charge transferred to a conductor spreads out over the conductor and over any other conductors in contact with it.

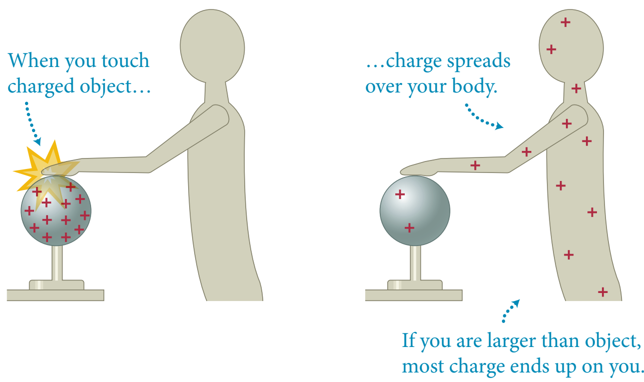
Metals are the only solid materials that are conductors at room temperature. (As noted earlier, glass, plastic, and most other solids are electrical insulators.) Although charge does not flow easily through pure water, minute amounts of impurities turn water into a fairly good conductor. Because most water contains some impurities, water is therefore usually considered a conductor.

Except for the outer layer of soil, Earth is also a good conductor. Consequently, when a charged, conducting object is connected to Earth by a wire, a process called **grounding,** charge carriers can flow between Earth ("ground") and the object. Because Earth is so large, it can supply or absorb a nearly unlimited number of charge carriers. In the absence of other nearby electrical influences, the grounded object is left with no surplus of either type of charge.

Because of its high water content, the human body is a conductor. Consequently, any time you touch a charged object, as in **Figure 22.14**, some of the charge moves into you—you act like a grounding agent just the way Earth does. As long as you keep touching the object, charge flows into your body, reducing the charge on the object (Figure 22.14). If the charge on the object is large, the charge that accumulates on your hair makes your hair stand up and separate as far as possible, like the leaves of an electroscope (**Figure 22.15**).

22.11 (*a*) Why is it impossible to charge a metal rod held in your hand by rubbing the rod with other materials? (*b*) Why can you charge a rubber rod even when you hold it in your hand?

Figure 22.14 Because of its water content, the human body is a conductor.

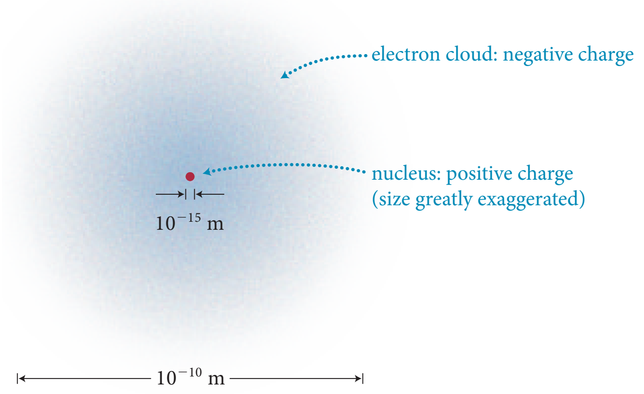


Is charge some sort of fluid that flows from one object to another, or is it composed of small particles that can be peeled off or stuck onto objects? To answer this question, we must look at the atomic structure of matter. All matter consists of atoms (see Section 1.3), the structure of which is schematically illustrated in **Figure 22.16**. Nearly all the atom's mass is concentrated in the extremely small nucleus at the center. The nucleus is composed of protons and neutrons. The region surrounding the nucleus, representing most of the atom's volume, is a cloud of electrons.

Figure 22.15 Charge spreads over the human body, so a large charge will cause your hairs to repel one another and stand on end.

CONCEPTS

Figure 22.16 Structure of the atom (not to scale).



Experiments show that electrons have a negative electrical charge—they are repelled by a charged comb* and by other electrons. Protons carry a positive charge, and neutrons carry no charge. The protons and neutrons in the nucleus are held together tightly by the strong interaction (see Chapter 7), which is great enough to overcome the electrical repulsion between the positively charged protons. The electrical attraction between the positively charged nucleus and the negatively charged electrons is responsible for keeping the electrons bound to the nucleus. The electron cloud does not collapse on the nucleus because of additional constraints imposed on the electrons by the laws of quantum mechanics.

Charge is an inherent property of the electron, which means it is impossible to remove the charge from an electron—there is no such thing as a discharged electron. Experiments show that

> **All electrical charge comes in whole-number multiples of the electrical charge on the electron.**

For this reason, the magnitude of the charge on the electron, designated by the letter e, is called the **elementary charge.**

Every atom contains equal numbers of electrons and protons. Because atoms are neutral, the fact that they contain equal numbers of electrons and protons tells us that the magnitude of the positive charge on the proton is also e. The charge on an electron is $-e$, and that on a proton $+e$. As with electrons, the charge cannot be removed from a proton.

Given that macroscopic objects contain an immense number of atoms and that each atom can contain dozens of electrons and protons, we see that ordinary objects contain an immense number of positively charged protons, exactly balanced by an equal number of negatively charged electrons. A surplus of just a minute fraction of these numbers is sufficient to give rise to a noticeable macroscopic charge. For example, when you pull apart two strips of transparent tape, the separation causes a surplus of less than one in a trillion (10^{12}) electrons. (Because there are about 10^{22} electrons in the strip, that fraction represents some 10^{10}, or ten billion electrons.)

When two atoms are brought close together, they may form a chemical bond by transferring one or more electrons from one atom to the other. Once such an electron transfer takes place, both atoms contain unequal numbers of electrons and protons and are now called **ions** instead of atoms. One of the two ions has gained one or more electrons, meaning it contains more electrons than protons and therefore carries a negative charge. The other ion, the one that lost electrons, contains more protons than electrons and so carries a positive charge.

Ions in solids are always immobile, but ions in liquids can move freely. For instance, in table salt, a compound made of pairs of sodium (Na^+) and chloride (Cl^-) ions, the charged ions hardly move at all, meaning that solid table salt is an electrical insulator. Dissolve table salt in water, however, and the solution contains large quantities of positively charged sodium ions and negatively charged chloride ions. Because these ions can move freely, the solution is an electrical conductor.

Some solids are made not of paired ions the way sodium chloride is but rather of individual atoms. In atomic solids that are electrical insulators, the electrons in the atoms are unable to move because each electron is bound to a specific atom. Diamond (made of the element carbon) and glass are two familiar examples. Metals are also atomic solids rather than ionic solids, but in metals, each atom gives up one or more electrons to a shared "gas" of electrons that spreads throughout the volume of the metal. The metal as a whole is still neutral: The negative charge of the electron gas is exactly balanced by the positive charge of the ions. The electrons in the gas are called *free electrons* because they can move freely inside the metal; these electrons are responsible for the easy flow of charge through a metal.

Nearly all electrical phenomena are due to the transfer of electrons—and therefore charge—from one atom to another. For example, when the sticky side of one strip of transparent tape is applied on top of the nonsticky side of a second strip, atoms in the adhesive from the top strip form chemical bonds with atoms in the nonsticky surface of the bottom strip by transferring electrons, as shown in **Figure 22.17** on the next page. These bonds are responsible for the adhesion of one strip to the other. When the strips are pulled apart quickly, the bonds are broken, but not all electrons manage to get back to the top strip. The bottom strip thus ends up with a surplus of electrons, making it negatively charged, and the top one with a deficit of electrons, making it positively charged.*

*Recall from our discussion of positive and negative charge carriers in Section 22.2 that a plastic comb carries a negative charge.

*Depending on the type of adhesive and the material of the backing, the transfer of electrons can also be in the other direction.

Figure 22.17 How strips of tape can acquire opposite charges when pulled apart.

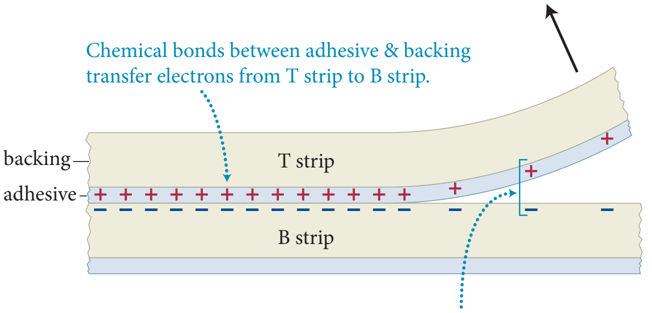
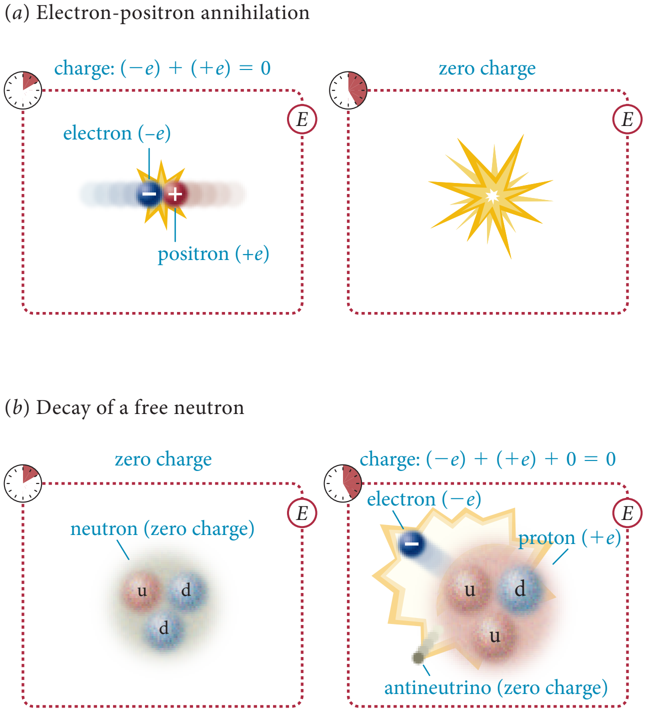


Figure 22.18 Because charge is conserved, the charge of a closed system does not change even when particles are created or destroyed.



As we saw in Section 10.4, friction between two surfaces also involves the breaking of chemical bonds. As with the separation of the tape strips, this bond breaking sometimes leaves surplus charge on the surfaces. When you touch a piece of plastic food wrap to a piece of styrofoam, for instance, chemical bonds form between atoms on the two surfaces. In these bonds, electrons from the styrofoam move to the wrap. If you then rub the surfaces against each other, these bonds are broken, and some of the electrons originally on the styrofoam stay on the wrap. If the breaking of the chemical bonds occurs slowly, the electrons migrate back and no surplus charge builds up. For that reason it is necessary to rub vigorously or to separate strips of tape quickly. The key point is:

Any two dissimilar materials become charged when brought into contact with each other. When they are separated rapidly, small amounts of opposite charge may be left behind on each material.

Because charging by breaking of chemical bonds is due to a transfer of charge, we now see that for every surplus of negative charge that appears in one place, an equal surplus of positive charge appears somewhere else. After the two strips in Figure 22.17 are separated, the sum of the positive charge on the T strip and the negative charge on the B strip is still zero. No creation or destruction of charge is involved, suggesting that electrical charge—like momentum, energy, and angular momentum—is a conserved quantity. The principle of **conservation of charge** states:

Electrical charge can be created or destroyed only in identical positive-negative pairs such that the charge of a closed system always remains constant.

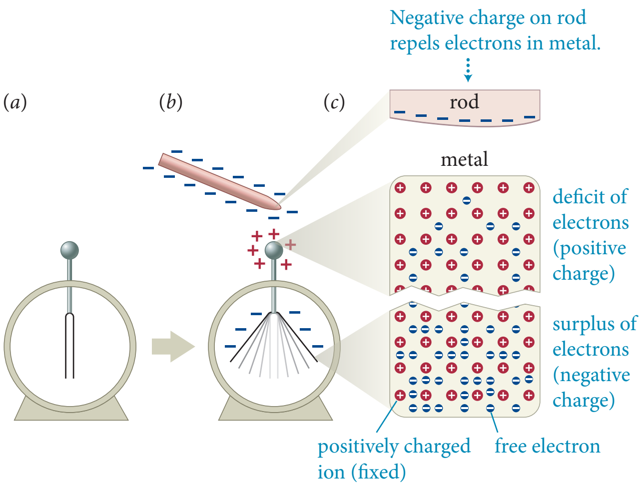
No process has ever been found to violate this principle. Even when charged subatomic particles, such as electrons and protons, are created or destroyed—a process that can be observed in high-energy particle accelerators—charge is conserved. For example, when an electron (charge $-e$) collides with a subatomic particle called the *positron* (charge $+e$), both particles are destroyed, leaving nothing but a flash of highly energetic radiation (**Figure 22.18a**). The charge of the electron-positron system before the collision is $(-e) + (+e) = 0$, and it is still zero after the collision. Likewise, in a process called beta decay, when a free neutron (carrying zero charge and made up of two down quarks and one up quark) decays into a proton (charge $+e$, one down and two up quarks), an electron (charge $-e$), and a neutral subatomic particle called the antineutrino (zero charge), the charge of the system comprising the neutron and the particles into which it decays remains zero (Figure 22.18*b* and Section 7.6).

22.12 When two objects made of the same material are rubbed together, friction occurs but neither material acquires surplus charge. Why?

22.4 Charge polarization

Let us now reexamine the interaction between a charged object and a neutral one. **Figure 22.19** shows the interaction between a charged rubber rod and an uncharged electroscope. With the rod far away (Figure 22.19*a*), the leaves of the electroscope hang straight down. When the rod is brought near the ball of the electroscope (Figure 22.19*b*), the leaves separate even without any contact between the rod and the electroscope. As the distance between the rod and the electroscope is increased again, the leaves drop down, showing that no charge has been transferred from the rod to the electroscope.

Figure 22.19 In (*a*) and (*b*), a charged rod induces polarization in an electroscope. (*c*) A schematic atom-level view.



Why do the leaves separate even though the electroscope remains neutral? They separate because the negative charge on the rod repels the free electrons in the metallic parts of the electroscope: The free electrons are pushed as far away as possible from the rod (Figure 22.19*c*) and pile up in the leaves. This redistribution of charge is nearly instantaneous. The top of the electroscope thus ends up with a deficit of electrons—a positive charge—and the leaves end up with a surplus of electrons—a negative charge. The negative charge on the leaves is responsible for the repulsion between them. When the rod is removed, the electrons, being repelled by one another and attracted to the positive charge on the electroscope ball, immediately flow back to their normal positions, evening out the distribution of positive and negative charge.

22.13 (*a*) In Figure 22.19*b*, is the electroscope as a whole positively charged, negatively charged, or neutral? (*b*) How does the magnitude of the positive charge on the electroscope ball compare with the magnitude of the negative charge on the leaves? (*c*) Is the force exerted by the rod on the electroscope ball attractive or repulsive? Is the force exerted by the rod on the leaves attractive or repulsive? (*d*) How do you expect the magnitude of the force the rod exerts on the ball to compare with the magnitude of the force the rod exerts on the leaves?

Any separation of charge carriers in an object is called **charge polarization,** or simply **polarization,** and an object in which charge polarization occurs is said to be *polarized*. The electroscope of Figure 22.19*b*, for instance, is polarized by the nearby charged rod. In any object in which charge is polarized, there are two charged *poles*, one positive and the other negative. In the electroscope of Figure 22.19*b*, the positive pole is at the ball and the negative pole is in the foil strips.

In metals, the polarization induced by the presence of a nearby charged object is very great because the free electrons in the metal move easily in response to the presence of the charged object. Even in electrical insulators, however, where there are no free electrons moving about, a nearby charged object induces some polarization. The basic reason for the polarization of insulators is illustrated in **Figure 22.20**: In the presence of an external charge, the center of the electron cloud and the nucleus of an atom shift away from each other, causing the atom to become polarized. So, when a negatively charged comb is brought near a small piece of paper, each atom in the paper becomes polarized—the electron clouds are pushed away from the comb, and the nuclei are pulled toward the comb. If we consider the paper as consisting of two overlapping parts that have the same shape but carry opposite charges, the positively charged part is pulled a bit toward the comb and the negatively charged part is pushed away, as shown in **Figure 22.21*a*** on the next page. This leaves the central part of the paper neutral but creates a sliver of surplus positive charge on the side facing the comb and an equal amount of surplus negative charge on the opposite side, and so the paper is polarized.

22.14 In an atom, what limits the separation between the electron cloud and the nucleus in the presence of an external charge? Why, for example, isn't the electron cloud in Figure 22.20*b* pulled all the way to the location of the external positive charge?

The polarization of atoms is responsible for the attraction between charged and neutral objects. In Figure 22.21, for example, the positively charged side of the paper is closer to the comb than the negatively charged side. Because the electric force decreases with increasing distance, the magnitude of the attractive force exerted on the positive side is greater than the magnitude of the repulsive force exerted on the negatively charged side (Figure 22.21*b*). Consequently, the vector sum of the electric forces exerted by the comb on the neutral piece of paper points toward the comb and the paper is pulled toward the comb.

Figure 22.20 Polarization of a neutral atom.

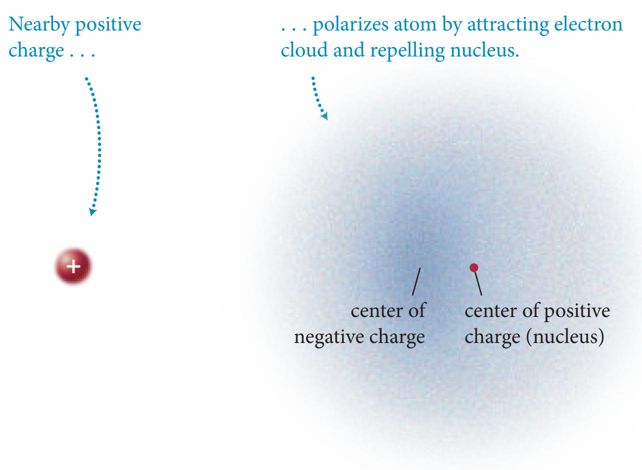
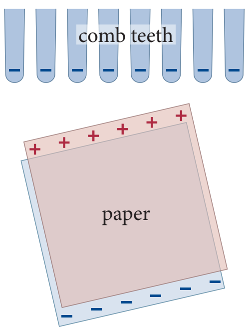


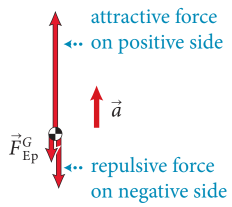
Figure 22.21 Polarization of a neutral insulator (bits of paper) by a charged comb. In (*a*), a single bit of paper is modeled as two offset sheets with opposite charges.

Charged comb picks up neutral paper

(*a*) Schematic model of interaction between comb and paper



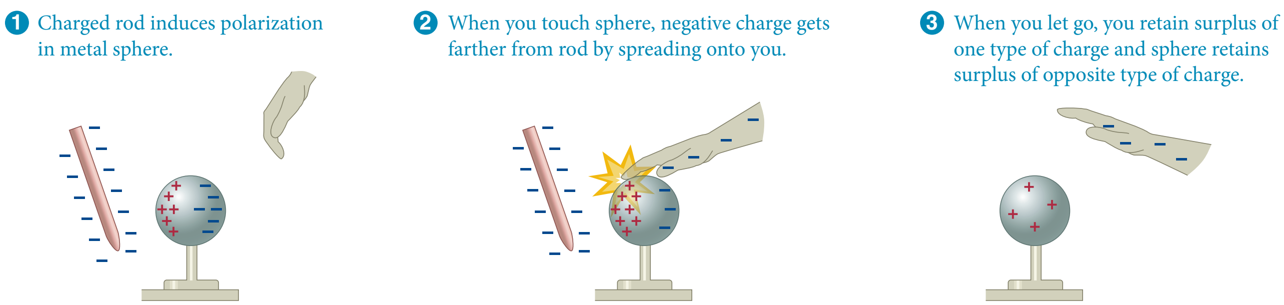
(*b*) Free-body diagram for paper



22.15 (*a*) When a positively charged object is brought near a neutral piece of paper, is the vector sum of the forces exerted by the charged object on the paper attractive or repulsive? (*b*) Describe what would happen when a negatively charged comb is brought near an electroscope if protons, not electrons, were mobile in metals. (*c*) Can you deduce from the experiment illustrated in Figure 22.19 which charge carriers—electrons or protons—are mobile in metals?

Figure 22.22 illustrates how you can exploit polarization to charge neutral conducting objects. A negatively charged rod brought near a metal sphere induces polarization in the sphere. To get as far away as possible from the negative charge on the rod, the electrons in the metal of the sphere move to the right surface. Thus the surface of the sphere nearer the rod becomes the positive pole, and the surface farther from the rod becomes the negative pole. When you touch the metal, the electrons can move into your body, thereby getting even farther away from the negative charge on the rod. In essence, you have become the negative pole, while the sphere is the positive pole. If you remove your hand from the sphere, the sphere is left with a deficit of electrons (they stayed inside you!) and so carries a positive charge—it has become charged without ever touching a charged object. (Likewise, you now have a surplus of negative charge and carry a negative charge, which is spread so thin that it is hardly noticeable.) This process is called **charging by induction.**

Figure 22.22 Polarization can be exploited to charge neutral conducting objects.



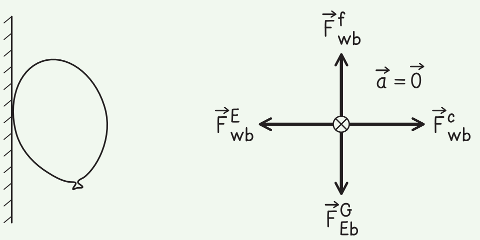
Self-quiz

1. You can use a positively charged object to charge a neutral object (*i*) by conduction or (*ii*) by induction. For each process, which type of charge (positive, negative, or neither) does the neutral object acquire?
2. Because we observe two types of electric interactions, attractive and repulsive, we postulate two types of charge. Do you think there are also two types of mass? Why do you think this? Do you think there are two types of magnetic pole? Why do you think this?
3. Is the statement *A plastic comb that has been passed through the hair a few times carries a negative charge* a physical law or a definition? What are some of the differences between a law and a definition?
4. A balloon rubbed in your hair or on your clothes sticks to a wall. If you place the rubbed side of the balloon against the wall, it sticks to the wall immediately. Try this, however: After rubbing the balloon in your hair, place the side of the balloon you did not rub against the wall and notice how the balloon turns until the rubbed side is touching the wall (*a*) Draw a free-body diagram for the balloon sticking to the wall. (*b*) Given that the balloon rotates so that the rubbed area is against the wall, do you think the balloon is an electrical insulator or conductor? (*c*) Was charge created in either the balloon or the wall in order for the sticking to occur? (*d*) Is any charge transferred from the balloon to the wall? Why or why not?
5. Air can act as both an insulator and a conductor. Consider reaching for a metal doorknob after scuffing your feet over a carpet. As your hand approaches the knob, a spark jumps between your hand and the knob. Explain how air acts as both insulator and conductor in this situation.

Answers

1. (*i*) Positive. When the two objects are brought into contact with each other (a necessary condition for conduction), surplus positive charge moves from the charged object to the neutral one. Thus the neutral object acquires the same type of charge as the charged object. (*ii*) Negative. Because the objects don't touch during charging by induction, charge carriers of the same type as the charged object escape from the neutral object during grounding.
2. Mass is the quantity responsible for gravitational interactions. Because all gravitational interactions are attractive, we can assume there is only one type of mass. Magnetic poles are responsible for magnetic interactions, which can be attractive or repulsive. Therefore there must be two types of magnetic pole (called north and south).
3. Definition. A law arises from observable phenomena and is found to be true in all cases that have been tested or observed (see Section 1.1). A definition cannot be tested. There is no test that allows us to observe that the charge on a comb passed through hair is negative. All we can do is show that the charged comb behaves in a fashion similar to other objects whose charge we call negative.
4. (*a*) See **Figure 22.23**. (*b*) Insulator. Because the only portion of the balloon that is attracted to the wall is the rubbed area, we know that the charge created by the rubbing does not spread out over the balloon surface. (*c*) No. Electrical charge can never be created. The surplus charge on the balloon was transferred from your hair or clothing. (*d*) No. If change were transferred, the balloon and the wall would repel each other.

Figure 22.23



5. Scuffing your feet on the carpet transfers charge from the carpet to you. Before you get near the knob, the air insulates your charged body from the knob. As you move nearer and nearer to the knob, the magnitude of the electric force between the charge carriers in your hand and those in the knob increases until the forces are so great that the air molecules are ionized, thereby producing a conducting pathway between your hand and the knob. Now the ionized air acts as a conductor for the jumping charge.

22.5 Coulomb's law

Quantitative experiments with electrical charge are difficult to carry out because objects lose their charge and because charge carriers on objects tend to rearrange themselves in the presence of other charged objects. In the 18th century, however, the English clergyman and scientist Joseph Priestley carried out a remarkable experiment: He charged a hollow sphere and showed that no electric force was exerted on a small piece of charged cork placed inside the sphere. Remembering that Newton had proven that no gravitational force exists inside a hollow sphere (see the box "Zero gravitational force inside a spherical shell" and **Figure 22.24**, below) because the gravitational force decreases with the square of distance, Priestley proposed that the electric force, too, decreases as $1/r^2$.

In 1785, Charles Coulomb, a French physicist, provided direct evidence for an inverse-square law by measuring how the electric force between two charged spheres changes as the distance between the spheres changes. The basic apparatus for Coulomb's experiment is shown in **Figure 22.25**. A small dumbbell is suspended from a long fiber. When spheres A and B are charged, the electric

Zero gravitational force inside a spherical shell

A consequence of the inverse-square dependence on distance of the gravitational force is that a uniform spherical shell exerts *no force at all* on a mass placed anywhere inside it. This result is very important in electrostatics because it provides a strong test of the law describing the electric force between charged objects.

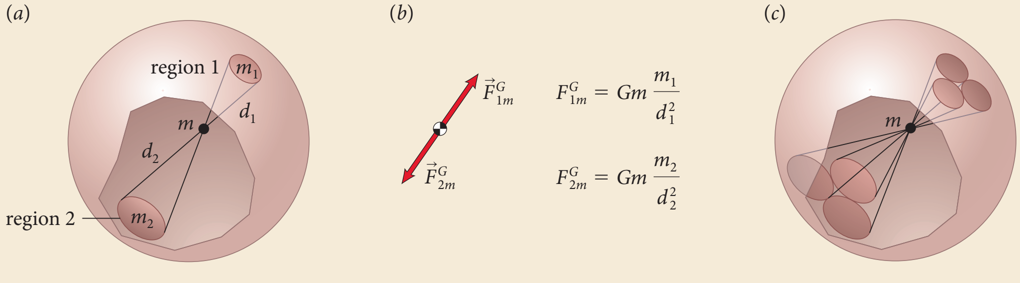
Consider the uniform spherical shell in Figure 22.24*a*. A particle of mass m is placed off-center inside the shell. To determine the force exerted by the shell on the particle, consider first the force exerted by region 1, defined as a very small region of the shell surface. Let the mass of this region be m_1 and its distance from the particle be d_1, which means the magnitude of the gravitational force exerted by this region on the particle is Gm_1m/d_1^2. Extending the cone defined by the particle and region 1 to the opposite side of the shell gives us a small region 2, of mass m_2 and at distance d_2 from the particle. The magnitude of the gravitational force exerted by this region on the particle is Gm_2m/d_2^2. If the particle is closer to region 1 than to region 2, the area of region 2 must be greater than the area of region 1 (because of how we defined region 2 as being an extension of the cone formed by region 1 and the particle), which means region 2 contains more mass: $m_2 > m_1$.

Because the mass of the shell is distributed uniformly over the shell, the mass of each of our two regions is proportional to its area: $m_1/m_2 = a_1/a_2$. Because they are marked out by similar cones, the areas of the two regions are proportional to the squares of the distances to the particle: $a_1/a_2 = d_1^2/d_2^2$, which means that

$$\frac{m_1}{m_2} = \frac{d_1^2}{d_2^2}.$$

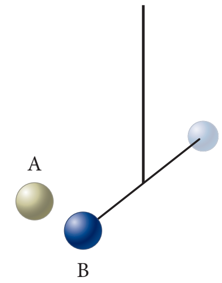
Rearranging terms, we get $m_1/d_1^2 = m_2/d_2^2$ *regardless* of the position of the particle, and so we see that the forces cancel: The two regions exert forces of equal magnitude in opposite directions on the particle (Figure 22.24*b*). We can now apply the same arguments to other pairs of small regions on either side of the particle, each yielding equal and opposite forces on the particle (Figure 22.24*c*). The vector sum of all the forces exerted on the particle by all the small regions making up the spherical shell is thus zero.

Figure 22.24



force between them twists the fiber. The amount of twist is a measure of the magnitude of the force between the two spheres (see also Section 15.7). A similar arrangement was used a few years later by Cavendish to study gravitational interactions (see Section 13.5).

Figure 22.25 Schematic diagram of Coulomb's apparatus for measuring the electric force between two charged spheres.



Coulomb also devised a method for systematically varying the "quantity of charge" q on a metal sphere. He found that when a charged metal sphere is brought into contact with an identical uncharged metal sphere, the final charge is the same on each sphere—both exert a force of equal magnitude on a third charged object. In other words, each sphere gets half the original charge. By sharing charge among several identical metal spheres, Coulomb could produce spheres whose charge was one-half, one-quarter, one-eighth, and so on of the original charge (Figure 22.26).

By thus varying the charges on spheres A and B of his apparatus, Coulomb found that the **electric force** is proportional to the charge on each sphere. We can summarize these findings in one equation, called **Coulomb's law,** which gives the magnitude of the electric force exerted by two charged particles separated by a distance r_{12} and carrying charges q_1 and q_2:

$$F^E_{12} = k\frac{|q_1|\ |q_2|}{r^2_{12}}. \tag{22.1}$$

As we shall see in Chapter 27, the interaction between charged particles becomes more complicated when the particles are not at rest. For this reason the force in Coulomb's law is sometimes called the *electrostatic* force and the branch of physics that deals with stationary distributions of charge is called *electrostatics*.

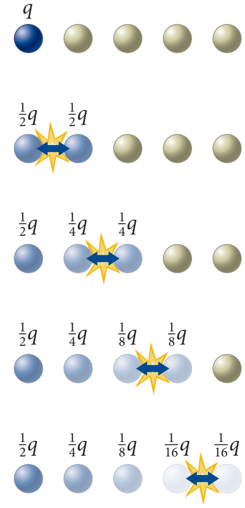
If the positions of the two charged particles are given by the vectors $\vec{r}_1$ and $\vec{r}_2$, respectively, then the distance between them is $r_{12} = |\vec{r}_2 - \vec{r}_1|$. The value of the constant of proportionality k depends on the units used for charge, force, and length. The absolute-value signs around the charges in Eq. 22.1 are necessary because q_1 and q_2 can be negative but the *magnitude* F^E_{12} of the electric force must always be positive.

Coulomb's law bears a striking resemblance to Newton's law of gravity (Eq. 13.1):

$$F^G_{12} = G\frac{m_1 m_2}{r^2_{12}}. \tag{22.2}$$

Why these two laws have the same mathematical form remains a mystery. The main differences between the two are that mass is always positive but electrical charge can be positive or negative, which means that the gravitational force is always attractive but the electric force can be attractive or repulsive.

Figure 22.26 By successively allowing a charged sphere to touch an initially uncharged neighbor, we can distribute an amount of charge in ever-lessening amounts over a number of spheres.



The derived SI unit of charge, called the **coulomb** (C), is defined as the quantity of electrical charge transported in 1 s by a current of 1 ampere, a quantity and unit we shall define in Chapter 27. One coulomb is equal to the magnitude of the charge on about 6.24×10^{18} electrons. Conversely, the magnitude of the charge of the electron is

$$e = 1/6.24 \times 10^{18}\ \mathrm{C} = 1.60 \times 10^{-19}\ \mathrm{C}. \tag{22.3}$$

The charge on any object comes in only whole-number multiples of this elementary charge:

$$q = ne, \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots. \tag{22.6}$$

This means that an object can have charge $q = 0$, $q = +7e$, $q = -4e$, and so forth, but not, for instance, $+1.2e$. Because the elementary charge is very small, the fact that charge exists only as whole-number multiples of the elementary charge isn't noticeable under ordinary circumstances. For example, running a comb through your hair easily gives the comb a surplus of about 10^{12} electrons, and the quantity of electrons flowing through a 100-W light bulb each second is about 10^{19}. These are such large numbers that the fact that charge comes in only whole-number multiples of the elementary charge normally remains unnoticed.

Using the coulomb as the unit of charge, we can determine the value of k in Eq. 22.1 experimentally by measuring the force between two known charged particles separated by a known distance:

$$k = 9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}. \tag{22.5}$$

The value of this constant shows how large a unit the coulomb is: Two particles, each carrying a charge of 1 C, separated by 1 m exert on each other a force of 9 billion newtons—equal to the gravitational force exerted by Earth on several dozen loaded supertankers! It is very difficult to build up a charge of this magnitude on all but very large objects because things get ripped apart by the enormous forces. The largest accumulations of charge we know of occur in the atmosphere: Large clouds that accumulate a charge of about 50 C discharge through the air to Earth, causing lightning.

Example 22.2 Gravity versus electricity

Compare the magnitudes of the gravitational and electric forces exerted by the nucleus of a hydrogen atom—a single proton ($m_\mathrm{p} = 1.7 \times 10^{-27}$ kg)—on an electron ($m_\mathrm{e} = 9.1 \times 10^{-31}$ kg) when the two are 0.50×10^{-10} m apart.

❶ **GETTING STARTED** For simplicity, I assume I can treat the proton and electron as particles. I also assume they are at rest so that I can use the principles of electrostatics.

❷ **DEVISE PLAN** I can use Eq. 22.2 to calculate the magnitude of the gravitational force and Eq. 22.1 to calculate the magnitude of the electric force.

❸ **EXECUTE PLAN**

$$F^G_{\mathrm{pe}} = G\frac{m_\mathrm{p} m_\mathrm{e}}{r^2_{\mathrm{pe}}}$$

$$= (6.7 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})\frac{(9.1 \times 10^{-31}\ \mathrm{kg})(1.7 \times 10^{-27}\ \mathrm{kg})}{(0.50 \times 10^{-10}\ \mathrm{m})^2}$$

$$= 4.1 \times 10^{-47}\ \mathrm{N}$$

and

$$F^E_{\mathrm{pe}} = k\frac{|q_\mathrm{p}|\ |q_\mathrm{e}|}{r^2_{\mathrm{pe}}}$$

$$= (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(1.6 \times 10^{-19}\ \mathrm{C})(1.6 \times 10^{-19}\ \mathrm{C})}{(0.50 \times 10^{-10}\ \mathrm{m})^2}$$

$$= 9.2 \times 10^{-8}\ \mathrm{N}.$$

The electric force exerted by the proton on the electron is $(9.2 \times 10^{-8}\ \mathrm{N})/(4.1 \times 10^{-47}\ \mathrm{N}) \approx 10^{39}$ times greater than the gravitational force exerted by the proton on the electron. ✔

❹ **EVALUATE RESULT** The difference in magnitudes is in agreement with the information given in Table 7.1.

Example 22.3 Comb electricity

(*a*) A 0.020-kg plastic comb acquires a charge of about -1.0×10^{-8} C when passed through your hair. What is the magnitude of the electric force between two such combs held 1.0 m apart after being passed through your hair? (*b*) If two identical 0.020-kg combs carry one surplus electron for every 10^{11} electrons in the combs, what is the magnitude of the electric force between these combs held 1.0 m apart?

❶ **GETTING STARTED** Both parts of the problem require me to calculate the magnitude of the electric force between the combs. If I treat the combs as particles, I can use Eq. 22.1 to calculate this force.

❷ **DEVISE PLAN** To calculate the magnitude of the electric force between two charged objects, I need to know the charge on each object and their separation distance. I know these data for part *a*: $q1 = q2 = -1.0 \times 10^{-8}$ C and $r_{12} = 1.0$ m, where the subscripts 1 and 2 denote the two combs. For part *b* I am given only the separation distance, and so I need to determine the charge on each comb.

I am given the fraction of electrons added, and I know the charge on one electron. So, to determine the charge on each comb, I need to determine how many electrons each comb contains. The number of electrons in each comb is equal to the number of protons in the comb: $N_e = N_p$. I am given the mass of the comb and I know that the mass is determined by the protons and neutrons in all the atoms making up the comb (the electrons contribute very little). Given that the protons and neutrons have almost identical mass ($m_p = m_n = 1.7 \times 10^{-27}$ kg), I can determine the number N of protons and neutrons by dividing the mass of the comb by m_p. Given that most atoms contain roughly equal numbers of protons and neutrons, I can say that the number of protons is $N_p \approx N/2$.

❸ **EXECUTE PLAN** (*a*) Substituting the values given into Eq. 22.1, I get

$$F^E_{12} = k\frac{|q_1|\,|q_2|}{r^2_{12}}$$

$$= (9.0 \times 10^9\ \text{N} \cdot \text{m}^2/\ \text{C}^2)\frac{(1.0 \times 10^{-8}\ \text{C})(1.0 \times 10^{-8}\ \text{C})}{(1.0\ \text{m})^2}$$

$$= 9.0 \times 10^{-7}\ \text{N}. ✔$$

(*b*) The number of protons plus neutrons in the comb is

$$N = \frac{0.020\ \text{kg}}{1.7 \times 10^{-27}\ \text{kg}} = 1.2 \times 10^{25},$$

and so $N_p \approx N/2 = 6 \times 10^{24}$. The number of electrons is equal to the number of protons, and so there are 6×10^{24} electrons in each comb to begin with. Adding one surplus electron for every 10^{11} electrons means adding $(6 \times 10^{24})/(1 \times 10^{11}) = 6 \times 10^{13}$ electrons to each comb; these electrons carry a combined charge of $(6 \times 10^{13})(-1.6 \times 10^{-19}\ \text{C}) = -9.6 \times 10^{-6}$ C. The magnitude of the repulsive electric force between the combs is then

$$F^E_{12} = k\frac{|q_1|\,|q_2|}{r^2_{12}}$$

$$= (9.0 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)\frac{(9.6 \times 10^{-6}\ \text{C})(9.6 \times 10^{-6}\ \text{C})}{(1.0\ \text{m})^2}$$

$$\approx 1\ \text{N}. ✔$$

❹ **EVALUATE RESULT** My answer to part *a* is a force too small to be felt, which is what I expect based on experience (two combs passed through hair don't exert an appreciable force on each other). In contrast, my answer to part *b* is phenomenally large for an electric force. The magnitude of the initial acceleration acquired by the combs would be $a_1 = F^E_{21}/m_1 = (1\ \text{N})/(0.020\ \text{kg}) = 50\ \text{m/s}^2$, or about five times the acceleration due to gravity! Even though the fraction of electrons removed—one in 100 billion—is very small, the factor k in Eq. 22.1 is so great that the resulting force is also great. Indeed, I learned in Table 7.1 that the electromagnetic interaction is 36 orders of magnitude stronger than the gravitational interaction, so my answer is not unreasonable.

22.16 Two identical conducting spheres, one carrying charge $+q$ and the other carrying charge $+3q$, are initially held a distance d apart. The spheres are allowed to touch briefly and then returned to separation distance d. Is the magnitude of the force they exert on each other after the touching greater than, smaller than, or the same as the magnitude of the force they exerted on each other before the touching?

Like the gravitational force, the electric force is *central;* that is, its line of action is along the line connecting the two interacting charged particles. Consider, for example, the two particles carrying charges q_1 and q_2 shown in **Figure 22.27*a***. The vector $\vec{r}_{12} \equiv \vec{r}_2 - \vec{r}_1$ gives the position of particle 2 relative to particle 1; this vector points from particle 1 to particle 2. We can define a unit vector pointing in this direction by dividing the vector $\vec{r}_{12}$ by its magnitude:

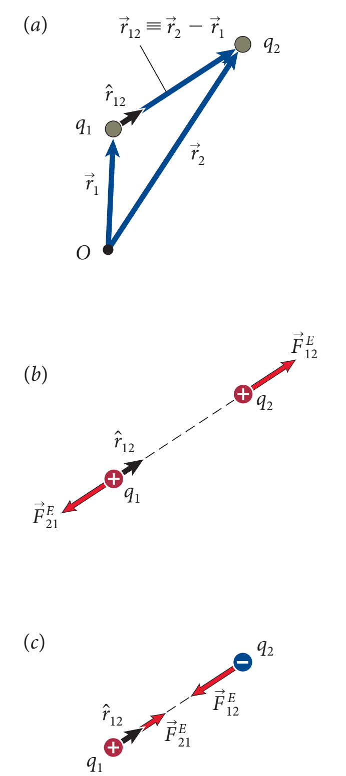
$$\hat{r}_{12} \equiv \frac{\vec{r}_2 - \vec{r}_1}{r_{12}}. \quad (22.6)$$

Depending on the algebraic sign of the charges, the electric force can be attractive or repulsive. For like charges ($q_1q_2 > 0$), the force is repulsive. In this case, the force $\vec{F}^E_{12}$ exerted by particle 1 on particle 2 points in the same direction as the unit vector $\hat{r}_{12}$ (Figure 22.27*b*). For opposite charges ($q_1q_2 < 0$), the force is attractive, and so $\vec{F}^E_{12}$ points in the direction opposite the direction of $\hat{r}_{12}$ (Figure 22.27*c*). In either case, $\vec{F}^E_{12}$ can be written in the form

$$\vec{F}^E_{12} = k\frac{q_1q_2}{r^2_{12}}\hat{r}_{12}. \quad (22.7)$$

Because $r_{12} = r_{21}$ and because $\hat{r}_{21}$ points in the direction opposite the direction of $\hat{r}_{12}$, the force $\vec{F}^E_{21}$ exerted *by* particle 2 *on* particle 1, which points in the direction

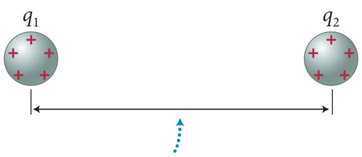
Figure 22.27 (*a*) Position vectors for two charged particles. (*b*) Repulsive forces exerted on each other by two particles carrying like charges. (*c*) Attractive forces exerted on each other by two particles carrying opposite charges.



QUANTITATIVE TOOLS

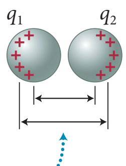
Figure 22.28 The reason Coulomb's law does not apply in a strict sense to macroscopic charged objects. The law is approximately correct if the objects are far apart relative to their radii.

(*a*) Charged spheres separated by a distance large compared to the sphere radii



Distance between centers of charge distributions same as distance between sphere centers

(*b*) Charged spheres separated by a distance small compared to the sphere radii



Charge repulsion causes distance between centers of charge distributions to differ from distance between sphere centers.

opposite the direction of $\vec{F}^E_{12}$, is obtained by simply switching the indices 1 and 2 in Eq. 22.7:

$$\vec{F}^E_{21} = k\frac{q_2 q_1}{r^2_{21}}\hat{r}_{21} = k\frac{q_1 q_2}{r^2_{12}}(-\hat{r}_{12}) = -\vec{F}^E_{12}, \tag{22.8}$$

as we would expect for an interaction pair (see Eq. 8.15). Equation 22.7 is the vectorial form of Eq. 22.1.

22.17 Using your knowledge about work and potential energy, determine whether the potential energy of a closed system of two charged particles carrying like charge increases, decreases, or stays the same when the distance between the two is increased. Repeat for two particles carrying opposite charge.

Before going on, I should mention a limitation to Coulomb's law. Strictly speaking, it is applicable only to charged particles. This is so because the distance r_{12} is well defined only when the size of the charged objects is negligibly small compared with their separation distance. When the charged objects are not particles, the distance r_{12} is not equal to the center-to-center distance. You can see why with the help of Figure 22.28. In **Figure 22.28a**, the charge is distributed uniformly over the surface of each of two widely separated metal spheres. The way in which a collection of charge carriers is spread out over a macroscopic object is called a **charge distribution.** Because the charge distributions over the metal spheres in Figure 22.28*a* are uniform, the center of each charge distribution coincides with the center of the sphere, and so r_{12} is well defined. When we bring the spheres close together, as in Figure 22.28*b*, the like charge carriers repel one another and move to the far side of each sphere. Now the centers of the charge distributions no longer coincide with the spheres' centers, so r_{12} (the center-to-center distance of the two charge distributions) is not simply the distance separating the centers of the two conductors.

22.18 (*a*) Is the magnitude of the electric force between the two conducting spheres in Figure 22.28*b* greater or smaller than that obtained from Coulomb's law, which assumes the charge is concentrated at the center of each sphere? (*b*) Is the answer to part *a* the same if the charge on one of the conductors is negative instead of positive?

22.6 Forces exerted by distributions of charge carriers

Coulomb's law deals only with *pairs* of charged objects. To calculate the force exerted by an assembly of objects carrying charges $q_2, q_3, q_4, \ldots$ on an object 1 carrying a charge q_1, we take the vector sum of all the forces exerted on object 1 by each of the other charged objects independently:

$$\Sigma\vec{F}^E_1 = \vec{F}^E_{21} + \vec{F}^E_{31} + \vec{F}^E_{41} + \cdots, \tag{22.9}$$

where each term is given by Coulomb's law:

$$\Sigma\vec{F}^E_1 = k\frac{q_2 q_1}{r^2_{21}}\hat{r}_{21} + k\frac{q_3 q_1}{r^2_{31}}\hat{r}_{31} + k\frac{q_4 q_1}{r^2_{41}}\hat{r}_{41} + \cdots. \tag{22.10}$$

In other words, we calculate the force exerted by object 2 on object 1, then calculate the force exerted by object 3 on object 1, and so forth, and then add the forces. This means that if we know the details of some distribution of charged objects, we can calculate the force exerted by this distribution of charged objects on a single charged particle. For distributions that contain large numbers of charged objects, the summation can be accomplished via an integration. We will limit our discussion here to simple cases involving only a few charged objects.

22.19 **Figure 22.29** shows how a charged particle 1 interacts with two other charged particles 2 and 3. Determine the direction of the vector sum of the electric forces exerted on particle 2.

The basic limitation of Coulomb's law continues to apply when we are analyzing collections of charged objects: Eq. 22.10 is valid only for charged *particles,* not for charged extended bodies. Suppose, for example, that we replace each charged particle in Figure 22.29 by a conducting sphere carrying the same charge as each particle. Let us first consider just the interaction between spheres 1 and 2. When these oppositely charged spheres are placed near each other, the charge carriers on the two spheres rearrange themselves to be as close as possible to each other (**Figure 22.30*a***). A similar type of rearrangement takes place when just spheres 1 and 3 are placed near each other (Figure 22.30*b*). When all three spheres are placed near one another, the positive charge on sphere 3 pushes the positive charge on sphere 2 up and pulls the negative charge on sphere 1 down (Figure 22.30*c*). Consequently, the forces that the spheres exert on one another are not the same as the forces exerted by the individual pairs (compare the forces in Figures 22.29 and 22.30).

Figure 22.29 Forces exerted by two charged particles 2 and 3 on charged particle 1.

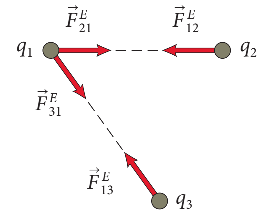
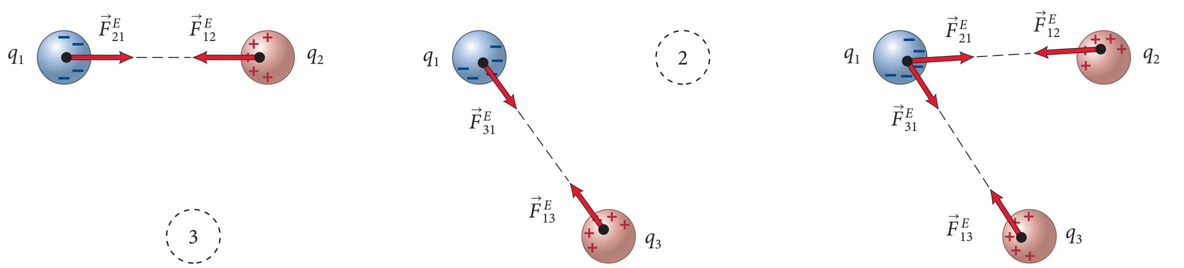


Figure 22.30

(*a*) Sphere 1 interacts with just sphere 2 (*b*) Sphere 1 interacts with just sphere 3 (*c*) Sphere 1 interacts with both spheres



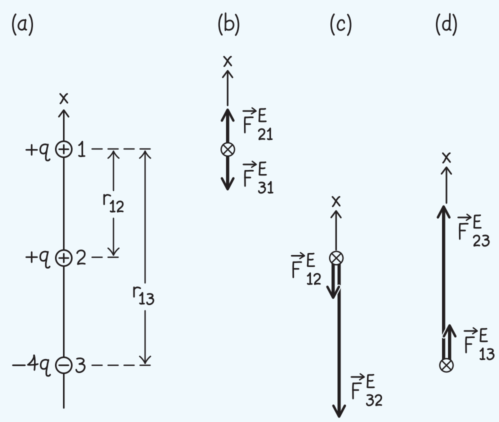
Example 22.4 Electric tug of war

You are given three charged particles. Particles 1 and 2 carry charge $+q$ and particle 3 carries charge $-4q$. (*a*) Determine the relative values of the separation distances r_{12} and r_{13} when the three particles are arranged along a straight line in such a way that the vector sum of the forces exerted on particle 1 is zero. (*b*) With the particles arranged this way, are the vector sums of the forces exerted on particles 2 and 3 also zero?

❶ GETTING STARTED Particle 2 exerts a repulsive force on particle 1; particle 3 exerts an attractive force on particle 1. In order for the two forces exerted on particle 1 to cancel, particles 2 and 3 must be on the same side of particle 1. Because the magnitude of the charge on particle 2 is smaller than that of the charge on particle 3, the distance r_{12} must be shorter than the distance r_{13}.

❷ DEVISE PLAN I choose my x axis vertically up along the line defined by the particles (**Figure 22.31*a***). To determine the relative values of the separation distances r_{12} and r_{13} when the vector sum of the forces exerted on particle 1 is zero, I draw a free-body diagram for particle 1 (Figure 22.31*b*). The magnitude of each force exerted on this particle is given by Eq. 22.1, and so by setting the sum of the x components of the forces equal to zero, I have an expression containing r_{12} and r_{13} and I can manipulate the expression to get the relative values of r_{12} and r_{13}.

Figure 22.31



(Continued)

❸ **EXECUTE PLAN** (*a*) The x components of the two forces in Figure 22.31*b* must add to zero, so

$$\begin{aligned}\Sigma F_{1x} &= F^E_{21x} + F^E_{31x} \\ &= +k\frac{|q_1||q_2|}{r^2_{12}} - k\frac{|q_1||q_3|}{r^2_{13}} = k\frac{qq}{r^2_{12}} - k\frac{(q)(4q)}{r^2_{13}} \\ &= +k\frac{q^2}{r^2_{12}} - k\frac{4q^2}{r^2_{13}} = 0.\end{aligned}$$

I therefore must have $\dfrac{q^2}{r^2_{12}} = \dfrac{4q^2}{r^2_{13}}$,

or $r^2_{13} = 4r^2_{12}$, and so $r_{13} = 2r_{12}$. ✔

(*b*) The forces exerted by particles 1 and 3 on particle 2 both point in the negative x direction (Figure 22.31*c*) and so cannot sum to zero. The forces exerted by particles 1 and 2 on particle 3 both point in the positive x direction (Figure 22.31*d*) and so cannot sum to zero. ✔

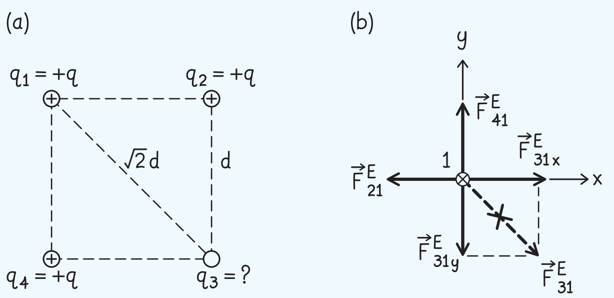
❹ **EVALUATE RESULT** My answer for part *a* makes sense because the force exerted by each particle is inversely proportional to the square of the separation distance, and this force varies directly with the quantity of charge. Because the charge on particle 3 is four times greater than that on particle 2, the square of the separation distance r_{13} must be four times r_{12}, and so $r_{13} = 2r_{12}$, as I found.

Example 22.5 Electrostatic equilibrium

Consider four charged particles placed at the corners of a square whose sides have length d. Particles 1, 2, and 4 carry identical positive charges. In order for the vector sum of the forces exerted on particle 1 to be zero, what charge must be given to particle 3, which is in the corner diametrically opposite particle 1?

❶ **GETTING STARTED** I begin by making a sketch of the situation (**Figure 22.32*a***). Because particles 1, 2, and 4 all carry identical positive charge, I write $q_1 = q_2 = q_4 = +q$. The separation between neighboring particles on the square is d. The separation between particles 1 and 3 is $\sqrt{2}d$.

Figure 22.32



❷ **DEVISE PLAN** To determine the charge needed on particle 3, I must determine the electric force magnitude F^E_{31} needed to yield a zero vector sum of forces exerted on particle 1. I should therefore draw a free-body diagram for particle 1 and work out the vector sum.

❸ **EXECUTE PLAN** In my free-body diagram (Figure 22.32*b*), I choose my y axis pointing up and x axis pointing to the right. The force $\vec{F}^E_{21}$ is repulsive and so points in the negative x direction; the force $\vec{F}^E_{41}$ is repulsive and points in the positive y direction. To make the vector sum of the forces exerted on 1 zero, $\vec{F}^E_{31}$ must be such that its components cancel $\vec{F}^E_{21}$ and $\vec{F}^E_{41}$. Along the x axis, I therefore have

$$\Sigma F_{1x} = F^E_{21x} + F^E_{31x} + F^E_{41x} = -F^E_{21} + F^E_{31}\cos 45° + 0 = 0,$$

so $F^E_{21} = F^E_{31}\cos 45°$. Substituting the Coulomb's law expressions for these two force magnitudes, I have

$$\begin{aligned}k\frac{|q_2||q_1|}{d^2} &= k\frac{|q_3||q_1|}{(d\sqrt{2})^2}\cos 45° \\ \frac{q^2}{d^2} &= \frac{|q_3|q}{2\sqrt{2}d^2} \\ |q_3| &= 2\sqrt{2}\,q.\end{aligned}$$

From Figure 22.32*b*, I also know that $\vec{F}^E_{31}$ must be an attractive force, so

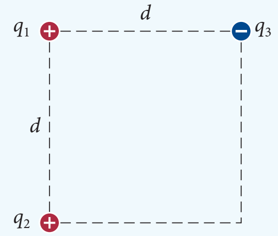
$$q_3 = -2\sqrt{2}q. ✔$$

❹ **EVALUATE RESULT** My result indicates that the charge on particle 3 is greater than q. This makes sense because the attractive force exerted by this particle must balance the forces exerted by particles 2 and 4, which are both closer to particle 1. To obtain my answer I solved only for the x component of $\vec{F}^E_{31}$ and did not consider the y component. However, my free-body diagram shows that the magnitudes of the components along the y axis are the same as those along the x axis, so analyzing the y components would have given me the same result.

Example 22.6 Electric trajectory

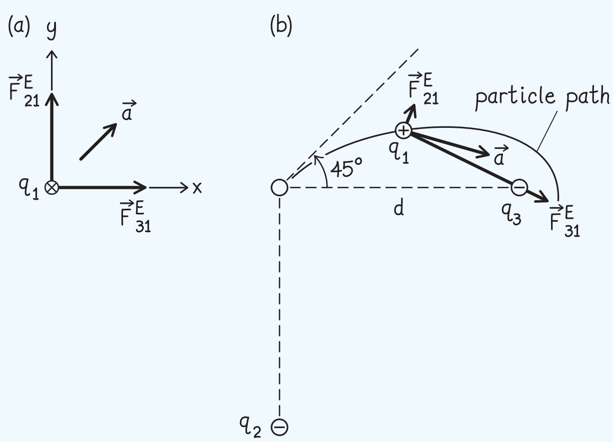
Consider the arrangement of charged particles shown in **Figure 22.33**. The charge magnitudes are the same in all three cases, but q_1 and q_2 are positive and q_3 is negative. Sketch the trajectory of particle 1 if it is released while particles 2 and 3 are held fixed. Ignore any gravitational force exerted by Earth on the particle.

Figure 22.33 Example 22.6



❶ GETTING STARTED I begin by drawing a free-body diagram for particle 1, choosing the x axis to the right and the y axis up (**Figure 22.34*a***). Because the charges on the two particles have the same magnitude and because the separation distances are the same, the magnitudes F^E_{21} and F^E_{31} are the same.

Figure 22.34



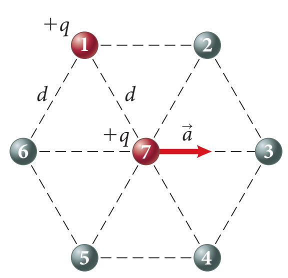
❷ DEVISE PLAN The direction in which particle 1 accelerates is determined by the vector sum of the forces exerted on it, and I can determine this direction from my free-body diagram. As the particle moves along its trajectory, however, the forces exerted on it change in both direction and magnitude, and so I must consider these changes in formulating my answer.

❸ EXECUTE PLAN Because the magnitudes F^E_{21} and F^E_{31} are the same, the vector sum of the two forces exerted on particle 1 bisects the angle between the two forces, and so the initial acceleration of 1 (which points in the same direction as the vector sum of forces) points up and to the right at an angle of 45°, as illustrated in Figure 22.34*a*. As particle 1 moves in the direction indicated in Figure 22.34*a*, $\vec{F}^E_{21}$ and $\vec{F}^E_{31}$ change in both direction and magnitude. The magnitude of $\vec{F}^E_{21}$ decreases because the distance between 1 and 2 increases, and the magnitude of $\vec{F}^E_{31}$ increases because the distance between 1 and 3 decreases. The direction of the acceleration of particle 1 is the same as the direction of the vector sum of these two forces. The resultant motion is qualitatively illustrated in Figure 22.34*b*. ✔

❹ EVALUATE RESULT My sketch makes sense: Particle 1 first moves up and to the right because it is repelled by particle 2 and attracted by particle 3. As it moves away from 2 and approaches 3, the effect of the attraction increases and so the trajectory curves and heads toward 3.

22.20 Seven small metal spheres are arranged in a hexagonal pattern as illustrated in **Figure 22.35**. Spheres 1 and 7 carry equal amounts of positive charge; the other spheres are uncharged. (*a*) To give sphere 7 an acceleration $\vec{a}$ that points to the right, what (single) other sphere must be charged? There may be more than one possibility. (*b*) What are the sign and magnitude of that charge?

Figure 22.35 Checkpoint 22.20.



QUANTITATIVE TOOLS

Chapter Glossary

SI units of physical quantities are given in parentheses.

Charge (electrical) q (C) A scalar that represents the attribute responsible for electromagnetic interactions, including electric interactions. There are two types of charge: *positive* ($q > 0$) and *negative* ($q < 0$). Two objects that carry the same type of charge exert repulsive forces on each other; objects that carry different types of charge exert attractive forces on each other.

Charge carrier Any microscopic object that carries an electrical charge.

Charge distribution The way in which a collection of charge carriers is distributed in space.

Charge polarization A spatial separation of the positive and negative charge carriers in an object. The polarization of neutral objects induced by the presence of external charged objects is responsible for the electric interaction between charged and neutral objects.

Charging by induction A method of charging a neutral object using a charged object, with no physical contact between them.

Conduction The flow of charge carriers through a material.

Conductor (electrical) Any material or object through which charge carriers can flow easily.

Conservation of charge The principle that the charge of a closed system cannot change. Thus charge can be transferred from one object to another and can be created or destroyed only in identical positive-negative pairs.

Coulomb (C) The derived SI unit of charge equal to the magnitude of the charge on about 6.24×10^{18} electrons. (The coulomb is defined as the quantity of electrical charge transported in 1 s by a current of 1 ampere, a unit we shall define in Chapter 27).

Coulomb's law The force law that gives the direction and magnitude of the electric force between two particles at rest carrying charges q_1 and q_2 separated by a distance r_{12}:

$$\vec{F}^E_{12} = k\frac{q_1 q_2}{r^2_{12}}\hat{r}_{12}. \tag{22.7}$$

The constant k has the value $k = 9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}$.

Electric force $\vec{F}^E$ (N) The force that charge carriers (and macroscopic objects that carry a surplus electrical charge) exert on each other. The magnitude and direction of this force are given by Coulomb's law.

Electric interaction A long-range interaction between charged particles or objects that carry a surplus electrical charge and that are at rest relative to the observer.

Elementary charge The smallest observed quantity of charge, corresponding to the magnitude of the charge of the electron: $e = 1.60 \times 10^{-19}$ C. See also *Coulomb*.

Grounding The process of electrically connecting an object to Earth ("ground"). Grounding permits the exchange of charge carriers with Earth, a huge reservoir of charge carriers. A charged, conducting object that is grounded will retain no surplus of either type of charge, assuming no other nearby electrical influences.

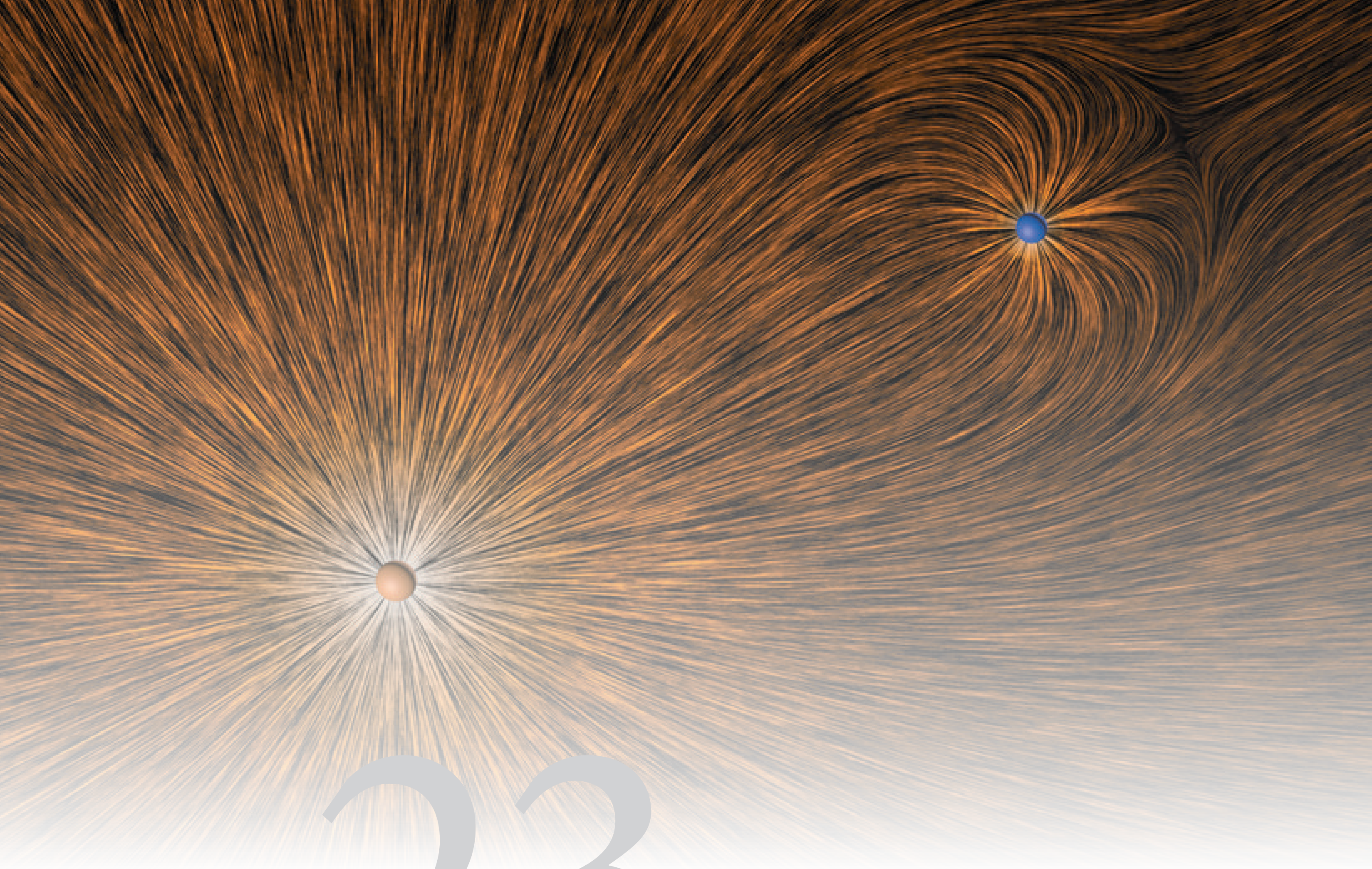
Insulator (electrical) Any material or object through which charge cannot flow easily.

Ion An atom or molecule that contains unequal numbers of electrons and protons and therefore carries a surplus charge.

Negative charge The type of charge acquired by a plastic comb that has been passed through hair a few times.

Neutral The electrical state of objects whose charge is zero. Electrically neutral macroscopic objects contain the same number of positively and negatively charged particles (protons and electrons).

Positive charge The type of charge acquired by hair after a plastic comb has been passed through it a few times.



23 The Electric Field

CONCEPTS

QUANTITATIVE TOOLS

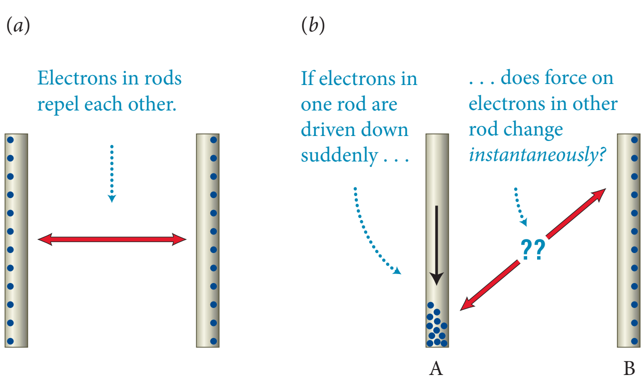
In this chapter we revisit an issue we discussed briefly in Chapter 7: the long-range nature of electric and gravitational interactions. How does one charged object "reach out" and affect another charged object? What are the invisible "springs" that pull us—and everything around us—toward Earth's surface? You can describe these long-range interactions by saying that every charged object and every object that has mass has a "sphere of influence" surrounding it. The modern word for this sphere of influence is *field.* Fields are not imaginary. That sensation you felt when, while reading the beginning of Chapter 22, you held a piece of plastic food wrap near your face was the sensation of a field created by the charged particles in the wrap. The closer the wrap is to your skin, the stronger the sensation.

The concept of field is important for two reasons. First, it is impossible to describe the interaction between moving charged particles without it. Second, as you will soon see, it is often easier to deal with fields than with distributions of charge because frequently more is known about fields than about the way charge is distributed.

23.1 The field model

Newton's law of gravity and Coulomb's law describing the electric force between charged particles successfully account for the magnitudes of the gravitational and electric forces between stationary objects. However, they do not address the fundamental puzzle of how objects separated in space can interact without any mediator of the interaction (such an interaction is called *action at a distance*). Worse, they share a fundamental flaw: Both imply that the action of one object on another is instantaneous everywhere throughout space. Consider, for example, the two metal rods in **Figure 23.1**. Even if both are electrically neutral, their electrons interact. Suppose you quickly drive the electrons in rod A down to the bottom, as in Figure 23.1*b*. According to Coulomb's law, doing this will instantly change the force exerted by the electrons in A on those in B, regardless of how far apart the rods are. This means that it would be possible—in principle—to be standing at one position in space and instantly detect a change that occurs at some far distant position.

Figure 23.1 Newton's and Coulomb's laws imply that forces are exerted instantaneously across a distance—but experiments show that they do not.



The idea that an object can directly and instantly influence another object regardless of their separation was troubling in the 19th century but became untenable in the early 20th century when it was demonstrated experimentally that the interaction between charged objects is not instantaneous. The principle illustrated in Figure 23.1, for example, is what makes possible the transmission of radio signals from a transmitting antenna (rod A) to a receiving antenna (rod B). We know from many experiments that such transmission is not instantaneous. As just one example, a radio signal takes about 0.1 s to travel from Earth's surface to an orbiting communications satellite. In other words, the picture conveyed by Newton's law of gravity and Coulomb's law—that one object directly and instantly affects other objects regardless of the distance between them—cannot be correct.

Instead we must adopt another model of long-range interactions, a model in which interactions take place through the intermediary of an **interaction field** (or simply a **field**). In the field model, an interacting object fills the space around itself with a field. When an object A is placed in the field of an object B, A can "feel" the presence of B's field. Instead of the two objects interacting directly as illustrated in **Figure 23.2*a***, it is the field created by each object that acts on the other object (Figure 23.2*b*). The stronger the field, the greater the magnitude of the force resulting from the interaction.

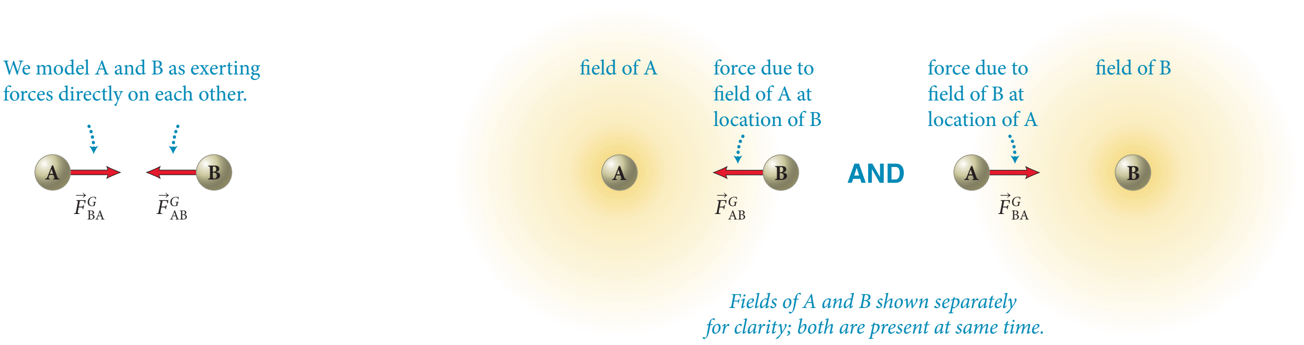
In Figure 23.1, the electrons in rod B feel the field set up by the electrons in rod A. When the electrons in A accelerate, their motion causes a disturbance in A's field, and this disturbance propagates outward through space like the ripples on the surface of a pond. Only when these ripples in the field reach rod B can the motion of A's electrons be detected by those in rod B. In Chapter 30 we shall study the propagation of disturbances in fields due to accelerating charge carriers. For now, we shall concentrate on fields created by stationary objects.

The field model applies equally well to gravitational and electric interactions, with each interaction having its own type of field. The space around any object that has mass is filled with a *gravitational field,* and the space around any electrically charged object is filled with an *electric field.* Gravitational fields exert forces on objects that have mass, and electric fields exert forces on objects that either carry a charge or can be polarized. Let's begin by developing the concept of gravitational field.

Before we attempt to obtain a physical quantity we can use to describe any gravitational field, we should note a number of things. First, for any object A located in a gravitational field created by an object S (S is called the *source* of the field), the magnitude of the field felt by A depends only on the properties of S and on the position of A relative to S; the field magnitude does not depend in any way on the properties of A. Second, the field of an object is always there, even when the object is not interacting with anything else. A field therefore must be represented by a set of numerical

Figure 23.2 The field model for interaction at a distance.

(*a*) Model of direct interaction at a distance (*b*) Field model of interaction at a distance



values that cover all the space outside the field source, with every point of this space having a different numerical value. One field representation you are already familiar with is that of a "temperature field," where the temperature across the surface of a region has a specific value at each location (**Figure 23.3**). Third, for stationary objects the field model must give the same forces as Newton's law of gravity and Coulomb's law. In particular, the field model must still yield forces between the two objects that are equal in magnitude and opposite in direction. It is not immediately obvious that the field model preserves this symmetry in the forces because a field is not something shared by two interacting objects—each object has its own field.

For example, in the gravitational interaction between a ball and Earth, the gravitational force exerted by Earth on the ball is due to Earth's gravitational field and the gravitational force exerted by the ball on Earth is due to the ball's field. These two gravitational fields are very different from each other: Earth's field pulls strongly on, say, a paper clip, while the effect of the ball's field on that paper clip is unmeasurably small. As you will see shortly, however, the symmetry of the interaction is preserved despite the asymmetry in the fields (see Checkpoint 23.3).

What physical quantity can we use to describe the gravitational field of an object? How about the gravitational force exerted by the object? Let's examine this possibility using Earth as our object. As we saw in Section 8.8, the magnitude of the gravitational force exerted by Earth on an object of mass m near Earth's surface is $F^G_{Eo} = mg$. This force is not a good quantity for describing Earth's field because the force depends not only on the source—Earth (which determines g)—but also on the mass m of the object placed in the field. As illustrated in **Figure 23.4**, two objects that have different masses m_1 and m_2 but are placed at the same height above Earth's surface are subject to different gravitational forces m_1g and m_2g. The quantity $g = F^G_E/m$, however—the gravitational force per unit of mass—is the same for any object.* This quantity is determined solely by the properties of Earth and is independent of those of any object that experiences a gravitational force exerted by Earth.

23.1 Two objects 1 and 2, of mass m_1 and m_2, are released from rest far from Earth, at a location where the magnitude of the acceleration due to gravity is much less than $g = 9.8\ \text{m/s}^2$. (*a*) What is the ratio F^G_{E1}/F^G_{E2}? (*b*) For these two objects, is the magnitude of the gravitational force exerted by Earth per unit mass independent of the properties of the objects?

Figure 23.3 The temperature across a region is specified by a set of values, with a specific temperature value for every position in that region. Such a set of values is called a *field*.

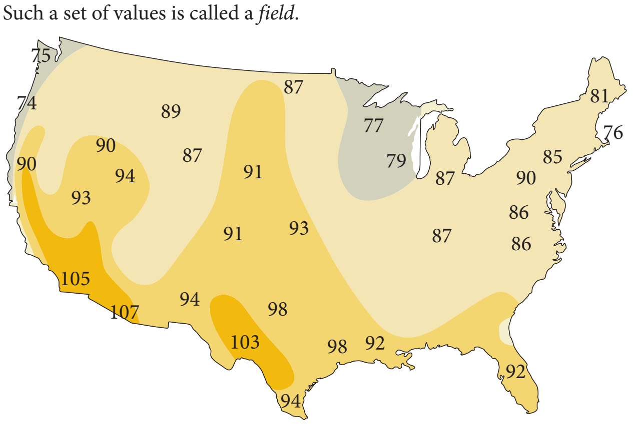
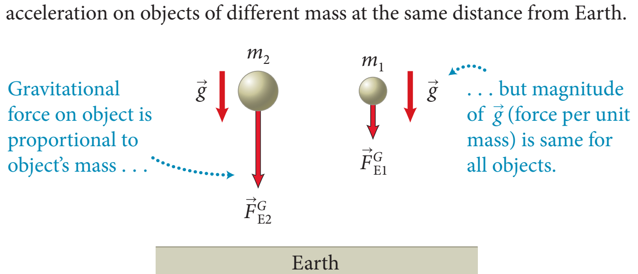
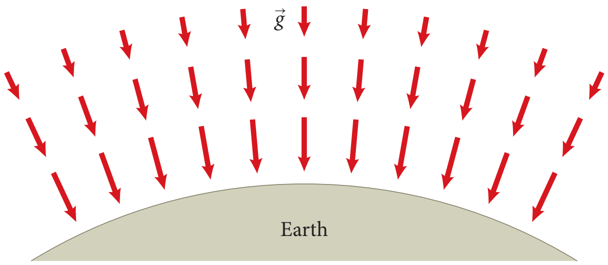


Figure 23.4 Comparison between gravitational force and gravitational acceleration on objects of different mass at the same distance from Earth.



*This is so because mass and inertia are equivalent (see Section 13.1), and so the force per unit of mass is equal to force divided by inertia, which is acceleration.

Figure 23.5 Vector field diagram for the gravitational field in a region near Earth.



We can use the gravitational force per unit mass exerted by an object as a measure of the magnitude of the object's gravitational field. For example, near Earth's surface, because $g = 9.8\ \text{m/s}^2$, the magnitude of Earth's gravitational field is 9.8 N/kg; near the surface of the Moon, where $g_{\text{moon}} = 1.6\ \text{m/s}^2$, the magnitude of the Moon's gravitational field is 1.6 N/kg. (Remember that $1\ \text{N/kg} = 1\ (\text{kg}\cdot\text{m/s}^2)/\text{kg} = 1\ \text{m/s}^2$.)

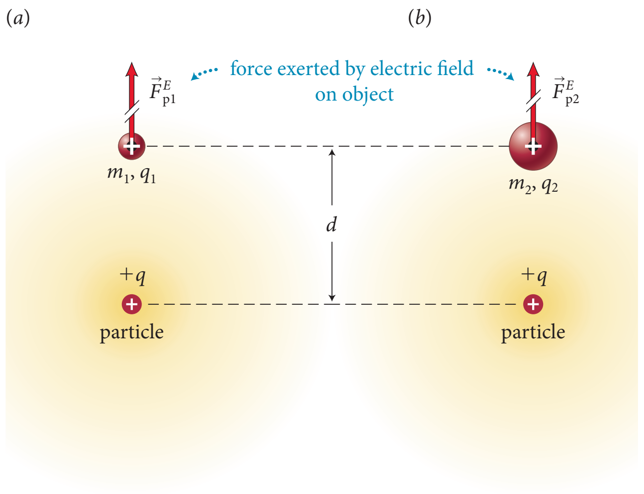
At any given location in the space surrounding a source object S, the magnitude of the gravitational field created by S is the magnitude of the gravitational force exerted on an object B placed at that location divided by the mass of B.

Unlike the temperature field in Figure 23.3, which is a *scalar field,* the gravitational field is a *vector field:* At every position, it has both a magnitude and a direction. **Figure 23.5**, for example, shows a **vector field diagram** representing the gravitational field near Earth. You can determine the magnitude and direction of this field in the space surrounding Earth using a **test particle** (an idealized particle whose mass is small enough that its presence does not perturb the object whose gravitational field we are measuring). Measure, at each location, the gravitational force exerted by Earth on the test particle, and then divide that force by the mass of the test particle to obtain the direction and the magnitude of the gravitational field at that location. As you can see from Figure 23.5, Earth's gravitational field, which can be represented at each position by a vector $\vec{g}$, always points toward the center of Earth, and its magnitude decreases with increasing distance away from Earth. Near Earth's surface, the magnitude of these vectors is $g = 9.8$ N/kg.

23.2 A communications satellite orbits 1.4×10^7 m from Earth's center, at a location where the magnitude of Earth's gravitational field is 2.0 N/kg. (*a*) If the mass of the satellite is $m_s = 2000$ kg, what is the magnitude of $\vec{F}^G_{\text{Es}}$? (*b*) If you place a 0.20-kg ball at the satellite's location, what is the magnitude of $\vec{F}^G_{\text{Eb}}$?

Checkpoint 23.2 illustrates that if you know the gravitational field at a certain position, you can easily calculate the gravitational force exerted by the source of that field on any object at that position by taking the product of the magnitude of the gravitational field at the location of the object and the mass of the object.

Figure 23.6 Electric force exerted on two objects of different inertia m and charge q by the electric fields created by two identical charged particles.



23.3 (*a*) Is the magnitude of the gravitational force exerted by Earth on a ball greater than, equal to, or smaller than the magnitude of the gravitational force exerted by the ball on Earth? (*b*) Is the magnitude of Earth's gravitational field at the position of the ball greater than, equal to, or smaller than the magnitude of the gravitational field of the ball at a distance equal to Earth's radius? (*c*) Explain how the answers to parts *a* and *b* can both be correct.

23.2 Electric field diagrams

Let us now apply the same ideas to electric interactions. **Figure 23.6a** shows object 1 of mass m_1 and charge q_1 a distance d from a particle that carries a charge $+q$. What is the electric field $\vec{E}$ created by the particle at the position of object 1? Before answering this question, answer the next checkpoint, which concerns the interactions of the particle with object 1 and with an object 2 of mass m_2 and charge q_2 (Figure 23.6*b*; $m_2 \neq m_1$ and $q_2 \neq q_1$).

23.4 (*a*) Are the electric forces $\vec{F}^E_{\text{p1}}$ and $\vec{F}^E_{\text{p2}}$ in Figure 23.6 equal? (*b*) What does the quantity $\vec{F}^E_{\text{p}i}/m_i$ represent? (*c*) Is this quantity the same for objects 1 and 2? If not, what quantity is the same for both of these objects?

As Checkpoint 23.4 makes clear, the quantity $\vec{F}^E_{\text{p}i}/q_i$—the electric force per unit charge—is determined entirely by the source of the electric field and is independent of the object on which the field exerts a force. So, in analogy to the gravitational field, we can say:

At any given location in the space surrounding a source object S, the electric field created by S is the electric force exerted on a charged test particle placed at that location divided by the charge of the test particle: $\vec{E}_S \equiv \vec{F}^E_{\text{St}}/q_t$.

CONCEPTS

Like gravitational fields, electric fields are vector fields. There is one difference between the two types of fields, however. Electric interactions can be either repulsive or attractive, and so the direction of the electric force—and hence the direction of the electric field—depends on the sign of the charge. Our rule is that:

The direction of the electric field at a given location is the same as the direction of the electric force exerted on a positively charged object at that location.

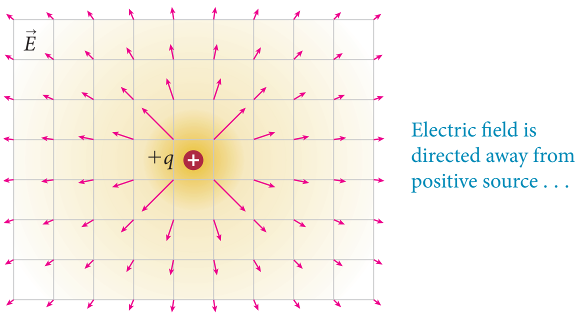
23.5 (*a*) If the particle in Figure 23.6 carries a negative charge $q < 0$ and q_1 and q_2 are positive, what are the directions of $\vec{F}^E_{p1}$ and $\vec{F}^E_{p2}$? (*b*) Does the electric field created by the particle point toward or away from the particle? (*c*) If q and q_2 are negative, what are the direction of $\vec{F}^E_{p2}$ and the direction of the electric field created by the particle at the location of object 2? (*d*) If q is positive, does the electric field created by the particle point toward or away from the particle? (*e*) How does the magnitude of the electric field created by a particle that carries a charge $+q$ ($q > 0$) compare with the magnitude of the electric field created by a particle that carries a charge $-q$ of identical magnitude at a distance d from each particle?

Figure 23.7 shows the vector field diagrams for the electric fields of particles that carry positive and negative charges. Because it is impossible to draw electric field vectors at *all* locations, the diagrams show vectors at only certain positions; from these representative vectors you can get an idea of how the electric field looks as a whole. In addition, the drawing is limited to two dimensions, but you should visualize the electric field as spreading out in all three dimensions.

Electric fields can be made visible by putting charged objects in a (nonconducting) liquid that contains small uncharged plastic fibers or grass seed. Each fiber aligns itself in the direction of the electric field at the fiber's location (**Figure 23.8**).

Figure 23.7 Vector field diagrams for positively and negatively charged particles. The lengths of the vectors show that the electric field magnitude decreases with increasing distance from the source.

(*a*) Electric field of positively charged particle



(*b*) Electric field of negatively charged particle

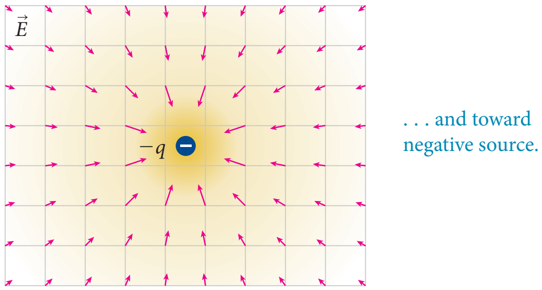
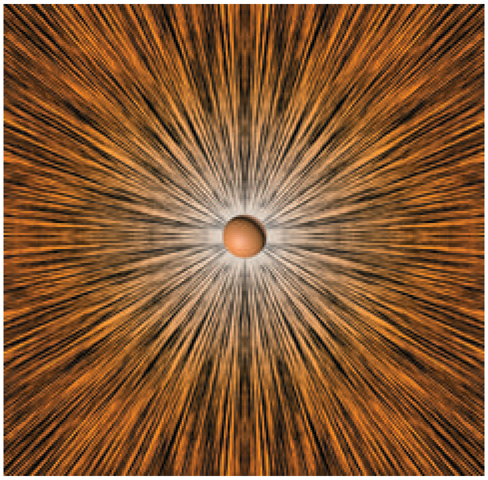


Figure 23.8 Electric field pattern created by a small charged object in a solution that contains plastic fibers. The fibers align with the direction of the electric field created by the charged object.



23.6 If you know the electric field $\vec{E}$ at some location, how can you determine the magnitude and direction of the electric force exerted by that field on an object carrying a charge q and placed at that location?

23.3 Superposition of electric fields

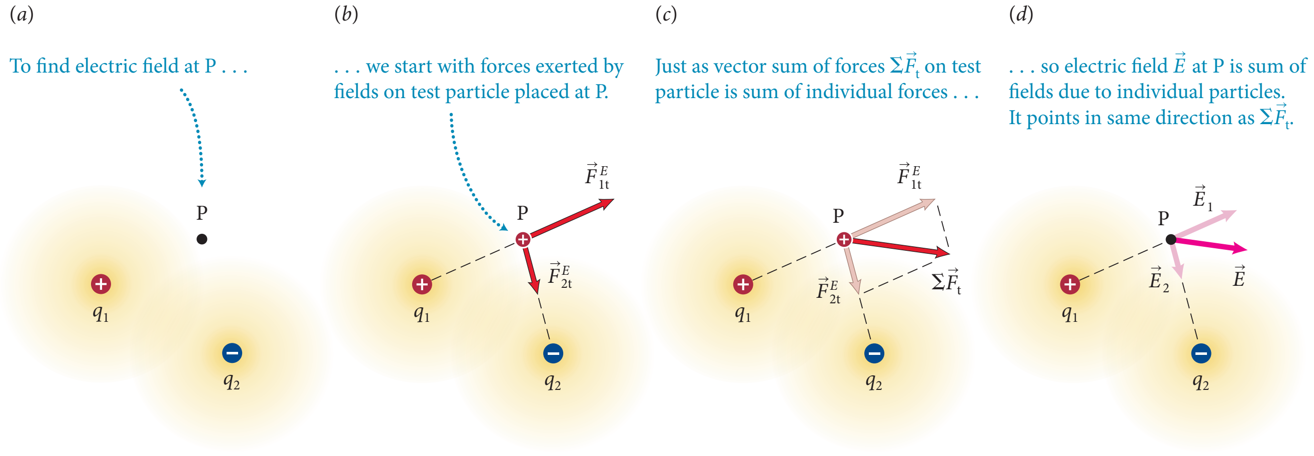
The concept of electric field becomes especially useful when we consider the combined electric field that results from more than one charged object. Suppose we are interested in the electric field created by two particles that carry charges of equal magnitude but opposite sign. To determine the electric field created by the particles at a point P, we place a test particle* carrying a positive charge q_t at P and measure the vector sum of the forces exerted on it by the two charged source particles (**Figure 23.9** on the next page). The electric field at P is then equal to this vector sum divided by q_t.

Figure 23.9 illustrates the **superposition of electric fields:**

The combined electric field created by a collection of charged objects is equal to the vector sum of the electric fields created by the individual objects.

*When measuring electric fields, we assume that the charge q_t on the test particle is so small that the particle does not perturb the particles or objects that generate the electric field we are measuring.

Figure 23.9 The electric field due to multiple charged objects (here, a pair of charged particles) is the vector sum of the fields created by the individual objects.



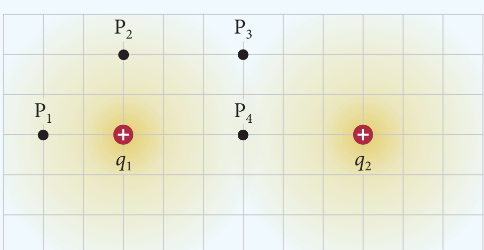
The superposition principle holds regardless of the number of sources. Because of the vectorial nature of the electric interaction, electric forces add vectorially (Eq. 22.9). Consequently the electric field at P is equal to $(\vec{F}^E_{1t} + \vec{F}^E_{2t})/q_t = \vec{F}^E_{1t}/q_t + \vec{F}^E_{2t}/q_t$, which is the vector sum of the electric fields created by the two sources individually.

The only caveat is the one I pointed out in Figure 22.30. When we deal with conductors, the distribution of charge on the individual conductors in isolation might be different from what it is when the conductors are placed close together.

Exercise 23.1 Electric field of two positively charged particles

Consider two identical particles 1 and 2 carrying charges $q_1 = q_2 > 0$ (**Figure 23.10**). What is the direction of the combined electric field at points P_1 through P_4?

Figure 23.10 Exercise 23.1.



SOLUTION I place a positively charged test particle at each location and determine the vector sum of the (repulsive) forces exerted by 1 and 2 on each test particle. Because $\vec{E} = \Sigma\vec{F}/q_{\text{test}}$, the direction of $\vec{E}$ is the same as the direction of $\Sigma\vec{F}$ (**Figure 23.11**).

Figure 23.11

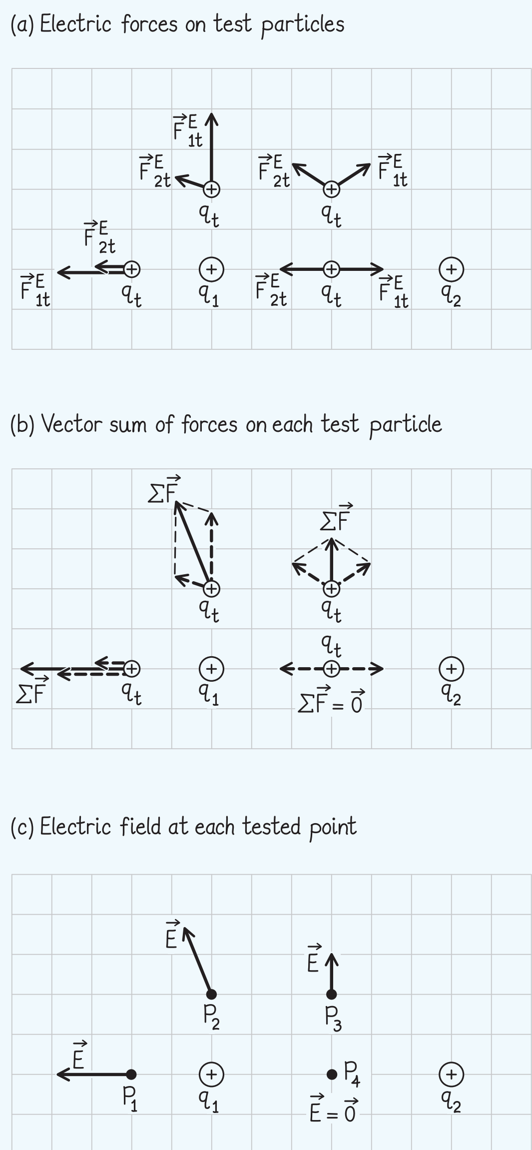
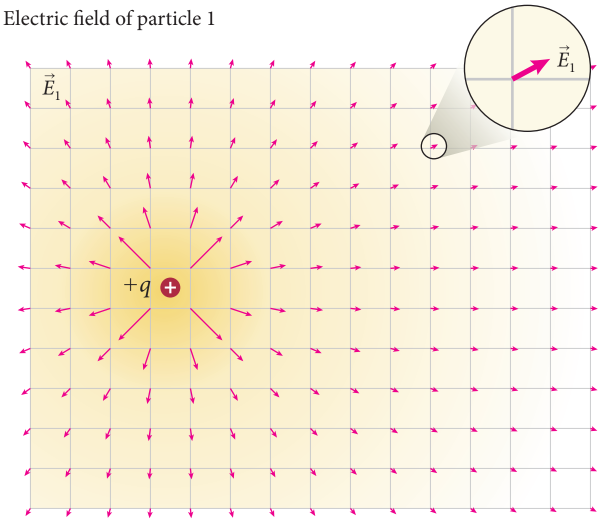
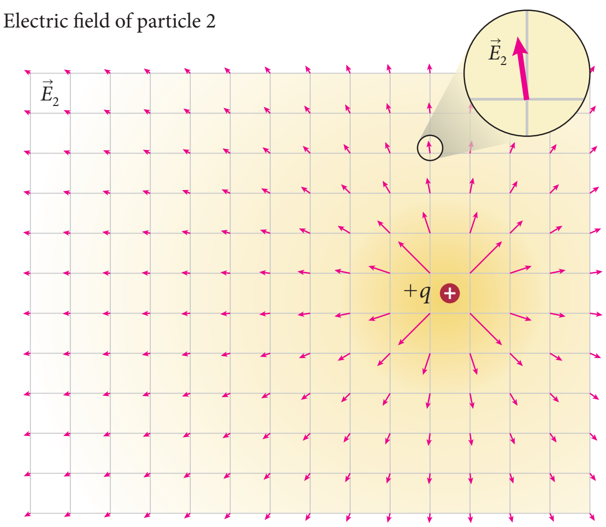


Figure 23.12 Vector field diagrams showing the superposition of the electric fields of the two charged particles of Figure 23.10.

(*a*) Electric field of particle 1



(*b*) Electric field of particle 2



(*c*) Electric field of both particles

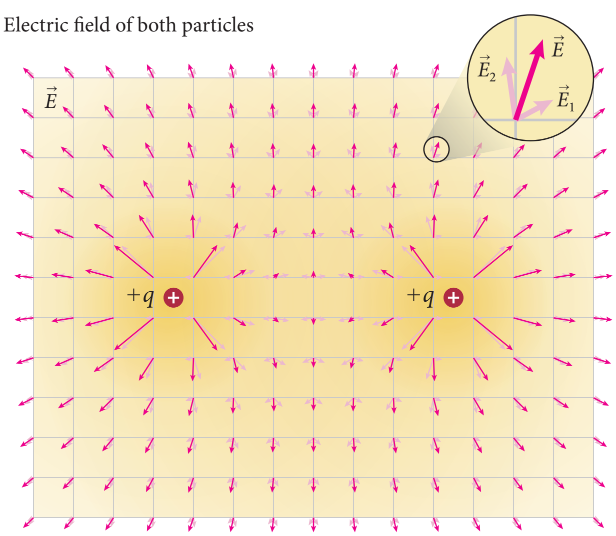
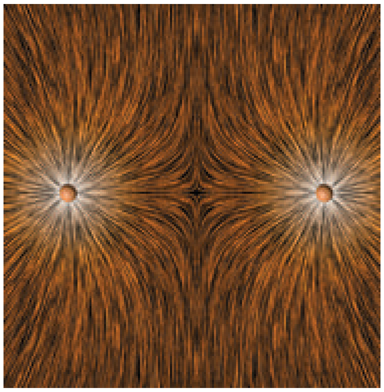


Figure 23.13 Pattern created by two identical charged particles in a liquid containing plastic fibers. Compare with the vector field diagram in Figure 23.12*c*.

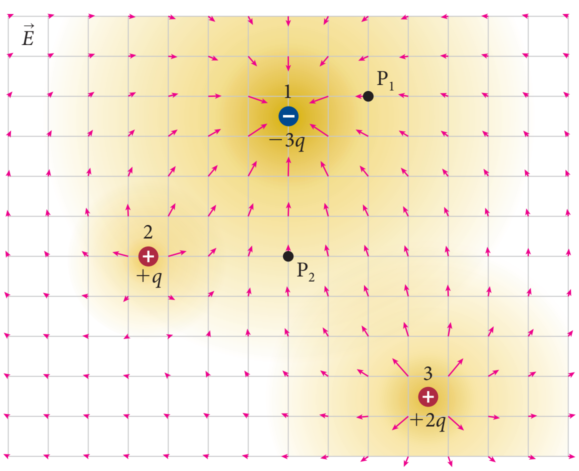


23.7 (*a*) If the charge on particle 2 in Exercise 23.1 is doubled so that $q_2 = 2_{q1}$, what happens to the direction of the electric field at points P_1 through P_4? (*b*) If the charge on particle 2 is negative so that $q_2 = -q_1$, what is the direction of the electric field at points P_1 through P_4?

Figure 23.11 provides a limited view of the electric field created by the two particles. A more complete view is given in **Figure 23.12**. This diagram is obtained by vectorially adding, for each grid point, the electric field vectors for the individual particles. Note how the pattern of vectors resembles the pattern created by two identically charged particles in a solution of plastic fibers (**Figure 23.13**).

Using the superposition principle, we can determine the electric field produced by any system of charged particles. **Figure 23.14**, for example, shows a vector diagram for the electric field generated by three charged particles. Because every charged object is made up of charged particles—electrons and protons—we can determine the electric field of any object at any position in space. For a real object, the calculation might be very tedious or even intractable because of the large number of charged particles, but the basic principle is as given above.

Figure 23.14 Vector field diagram of the electric field created by three charged objects.



23.8 (*a*) In Figure 23.14, what is the direction of the force $\sum \vec{F}^E$ exerted on a particle carrying a charge $+q$ and placed at P_1? (*b*) How does $\sum \vec{F}^E$ change when the particle at P_1 carries a charge $+3q$? (*c*) Do the magnitude and direction of $\sum \vec{F}^E$ on a charged particle at P_1 change if the $+2q$ charge on object 3 is halved? (*d*) What is the direction of $\sum \vec{F}^E$ exerted on a particle carrying a charge $-q$ and placed at point P_2?

23.4 Electric fields and forces

Before developing additional techniques for determining the electric fields created by systems of charged particles, let us consider this question: What are the forces exerted by an electric field on charged or polarized objects? One of the advantages of working with electric fields is that, for any system of charged particles, once we know the electric field the system creates at some point P in space, we can determine the force exerted by the system on any other charged particle placed at P without worrying about any of the individual source particles in the system anymore.* In Chapter 22 we used the action-at-a-distance model to discuss the forces exerted by charged objects either on other charged objects or on polarized objects. Now we can use the field model to do the same thing. For stationary charged particles, both methods must yield the same result.

When we study the forces exerted by electric fields, it is useful to distinguish between uniform and nonuniform. In a *uniform electric field,* the direction and magnitude of the electric field are the same everywhere. No electric field is ever uniform throughout all space, but as we shall see in Section 23.7 it is possible to create regions of space where the electric field is uniform. In a *nonuniform electric field,* the direction and magnitude of the electric field vary from position to position. (All the electric fields we have considered so far are nonuniform.)

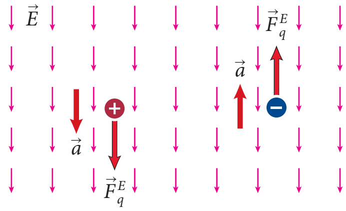
Let us first consider what happens to a charged particle placed in a uniform electric field. Because the electric field is defined as the electric force per unit of charge, the force $\vec{F}^E_p$ exerted by an electric field $\vec{E}$ on a particle carrying a charge q is $\vec{F}^E_p = q\vec{E}$.† Because $\vec{E}$ is the same everywhere, the force $\vec{F}^E_p$ exerted on the particle is constant and so it undergoes a constant acceleration $\vec{a} = \vec{F}^E_p/m = q\vec{E}/m = (q/m)\vec{E}$, where m is the particle's mass.

A charged particle placed in a uniform electric field undergoes constant acceleration.

*This procedure is valid only if the presence of the charged particle at P does not alter the way charge is distributed over the system. We shall refer to the particle or system of particles that creates the electric field at P as being "fixed."

†When dealing with forces exerted by fields, we drop the subscript representing the object that exerts the force (the "by" subscript) because the field is due to *all* objects other than the object on which the force is exerted. The superscript E reminds us that we are dealing with the force exerted by an electric field.

Figure 23.15 Forces exerted by a uniform electric field on a positively and a negatively charged particle.



If the particle carries a positive charge, $q > 0$, $\vec{F}^E_p$ and $\vec{a}$ point in the same direction as $\vec{E}$. If $q < 0$, $\vec{F}^E_p$ and $\vec{a}$ point in the direction opposite the direction of the electric field (**Figure 23.15**).

Note from $\vec{a} = (q/m)\vec{E}$ that the magnitude of the acceleration depends on the magnitude of the electric field and on the charge-to-mass ratio q/m of the particle. A large charge q causes a greater force to be exerted on the particle and therefore a greater acceleration; a larger mass m means the particle has greater inertia and therefore the acceleration is smaller.

Because we have already studied motion with constant acceleration, we can apply our knowledge to the motion of charged particles in a uniform electric field. In general, the trajectory of these particles is parabolic, like the trajectory of a projectile fired near Earth's surface, where the gravitational field can be considered uniform over a limited area. In the special case where the initial velocity of a charged particle is parallel to the direction of the electric field, the trajectory is a straight line, like the vertical fall of an object released from rest. The main difference between the motion of projectiles near Earth's surface and the motion of charged particles in an electric field is that Earth's gravitational field is always directed vertically downward, whereas the electric field can be in any direction.

Example 23.2 Charged particle trajectories

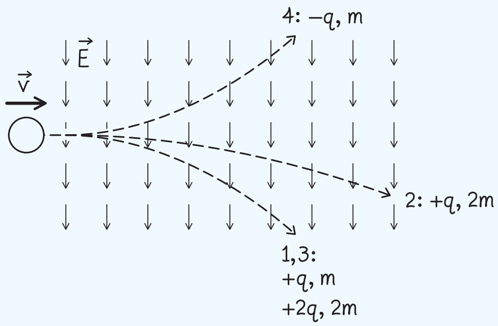
Four charged particles are fired with a horizontal initial velocity $\vec{v}$ into a uniform electric field that is directed vertically downward. The effect of gravity is negligible. The particles have the following charges and masses: particle 1 $(+q, m)$; 2 $(+q, 2m)$; 3 $(+2q, 2m)$; 4 $(-q, m)$. Sketch the four trajectories.

❶ **GETTING STARTED** Because the electric field direction is vertically down, the three positively charged particles experience a downward force and the negatively charged particle experiences an upward force. The magnitude of this force doesn't change as the particles move through the electric field because both the field magnitude and the charges on the particles are constant.

❷ **DEVISE PLAN** Because the force exerted on each particle is constant, the particles experience constant accelerations. Because the direction of the force is perpendicular to the direction of the particles' initial motion, they all have a parabolic trajectory. The positively charged particles have a constant downward acceleration; the negatively charged particle has a constant upward acceleration. Because $\vec{a} = (q/m)\vec{E}$, the acceleration magnitude is greatest when q is large and/or m is small.

3 EXECUTE PLAN I draw trajectories that curve down for 1, 2, and 3 and up for 4 (**Figure 23.16**). The magnitude of the electric force exerted on particle 2 is the same as that exerted on particle 1, but the acceleration of particle 2 is smaller because this particle has the greater mass. I indicate this difference in acceleration by making trajectory 1 more curved than trajectory 2. The magnitude of the electric force exerted on particle 3 is twice as great as that exerted on particle 1, but 3's mass is also twice as great, and so the two particles have the same charge-to-mass ratio and therefore the same acceleration and trajectory. The magnitude of the electric force exerted on particle 4 is the same as that exerted on particle 1 but points in the opposite direction, and so trajectories 1 and 4 are identical in shape but curve in opposite directions. ✔

Figure 23.16



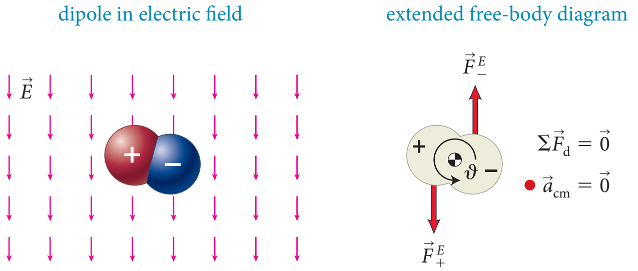
4 EVALUATE RESULT My sketch indicates that particles with increasingly positive charge-to-mass ratios curve increasingly downward. Conversely, particles with increasingly negative charge-to-mass ratios curve increasingly upward. This is what I expect because a particle's deflection is a function of both its charge, which determines the magnitude of the force exerted by the electric field on it (greater charge, greater deflection), and its mass, which relates the particle's acceleration to the force exerted on it (greater mass, smaller deflection).

23.9 A water droplet carrying a positive charge is released from rest in a uniform horizontal electric field near Earth's surface. The horizontal electric force is comparable in magnitude to the gravitational force exerted by Earth. Describe the droplet's trajectory.

In a nonuniform electric field, the force exerted on a charged particle varies from one position to another, so we cannot easily specify the particle's trajectory without knowing more about the electric field. As in a uniform electric field, however:

A positively charged particle placed in a nonuniform electric field has an acceleration in the same direction as the electric field; a negatively charged particle placed in such a field has an acceleration in the opposite direction.

Figure 23.17 Extended free-body diagram for a permanent dipole placed in a uniform electric field.



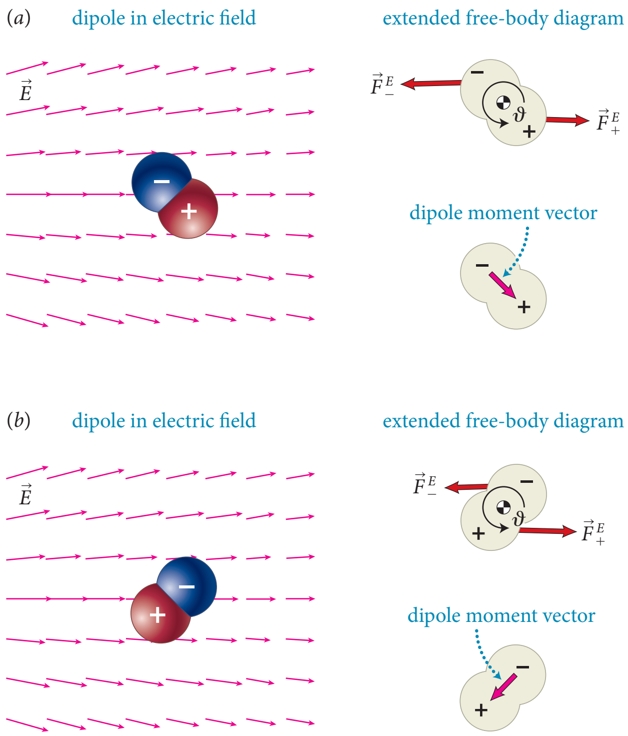
In Chapter 22 we found that charged objects can polarize electrically neutral objects by separating the centers of positive and negative charge in the latter. The resulting configuration of charge—equal amounts of positive and negative charge separated by a small distance—is called an **electric dipole** or simply **dipole.** Many molecules, such as water molecules, are *permanent dipoles;* that is to say, the centers of positive and negative charge are kept separated by some internal mechanism. **Figure 23.17** illustrates the forces exerted on a permanent electric dipole in a uniform electric field. Because the electric field is uniform and the magnitude of the charge on the positive end of the dipole is equal to the magnitude of the charge on the negative end, the forces exerted on the two ends are equal in magnitude but opposite in direction, making their vector sum zero. However, the forces exerted on the two ends cause a torque (see Chapter 12).

23.10 (*a*) What effect does the torque caused by the electric field have on the electric dipole in Figure 23.17? (*b*) Is the torque the same for every orientation of the molecule?

The orientation of an electric dipole can be characterized by a vector, the **dipole moment,** that, by definition, points from the center of negative charge to the center of positive charge, as shown in **Figure 23.18** on the next page. As Checkpoint 23.10 illustrates, the electric forces create a torque on the dipole that tends to align the dipole moment with the electric field.

In a nonuniform electric field, the situation is more complicated because the two ends of the dipole are now subject to forces that have different magnitudes as well as different directions. Consider, for example, the nonuniform electric field in Figure 23.18*a*, which is due to a positively charged particle to the left side of the figure. The magnitude of $\vec{F}^E_-$ is greater than the magnitude of $\vec{F}^E_+$ because the negative end of the dipole is closer to the positively charged particle. Thus the vector sum of the forces exerted on the

Figure 23.18 Extended free-body diagrams for permanent dipoles in nonuniform electric fields. The electric field shown is due to a positively charged particle to the left of the figure.



two ends is nonzero, and so the dipole experiences an acceleration whose magnitude and direction depend on its orientation with respect to the electric field. In addition, the forces create a torque about the dipole's center of mass. As in a uniform electric field:

> **A permanent electric dipole placed in an electric field is subject to a torque that tends to align the dipole moment with the direction of the electric field. If the field is uniform, the dipole has zero acceleration; if the electric field is nonuniform, the dipole has a nonzero acceleration.**

23.11 (*a*) Draw a free-body diagram for the dipole in Figure 23.18*a* and determine the direction of the dipole's center-of-mass acceleration. (*b*) Draw a free-body diagram for the dipole in Figure 23.18*b* and qualitatively describe the dipole's motion.

Self-quiz

1. Suppose someone discovers that blue and yellow objects attract each other, that two blue objects repel each other, and that two yellow objects repel each other. The strength of this "chromatic interaction" is found to depend on color depth: The deeper the color, the greater the magnitude of the interaction. How would you define the magnitude and direction of the "chromatic field" of an object?
2. (*a*) Does an electrically neutral particle that has mass interact with an electric field? (*b*) Does a charged particle interact with a gravitational field?
3. The two particles in **Figure 23.19** have the same mass, carry charges of the same magnitude ($q_1 = -q_2 > 0$), and are equidistant from point P. (*a*) What is the electric field direction at P? (*b*) At P, what is the direction of the gravitational field due to the two particles? Ignore Earth's gravitational field.
4. Can electric and gravitational fields exist in the same place at the same time?
5. What are the directions of the acceleration of each particle in **Figure 23.20**? Describe the resulting motions.

Figure 23.19

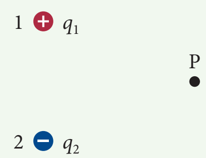
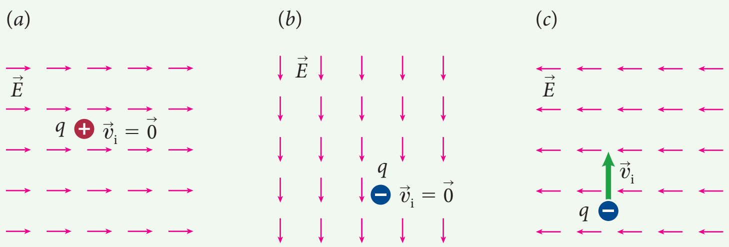


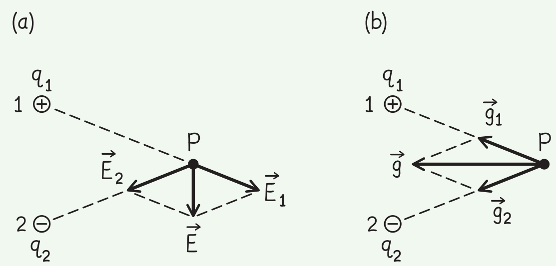
Figure 23.20



Answers

1. The gravitational field is defined as the gravitational force per unit of mass, with the field direction the same as the direction of the force. The electric field is defined as the electric force per unit of charge, with the field direction parallel to that of the force exerted on a positively charged particle. Therefore, the chromatic field can be defined as the chromatic force per unit of color, with the field direction parallel to that of the force exerted on a particle carrying some chosen color.
2. (*a*) No. Uncharged particles don't interact with electric fields. (Remember that a particle has no extent and therefore cannot be polarized.) (*b*) Yes, because any particle or object, charged or uncharged, interacts with a gravitational field.
3. (*a*) The electric field of particle 1 points away from the particle, which means that at P it points to the right and down. The electric field of particle 2 points toward the particle, meaning to the left and down at P. The vector sum of the electric fields at P therefore points straight down (**Figure 23.21*a***). (*b*) The gravitational fields of the two particles point toward them from P, and so their vector sum points to the left (Figure 23.21*b*).

Figure 23.21



4. Yes. Consider a charged object near Earth's surface. This object is surrounded by an electric field, but it is also surrounded by Earth's (and to a lesser extent its own) gravitational field. These electric and gravitational fields exist in the same place at the same time.
5. (*a*) Recall from the discussion following Checkpoint 23.4 that the direction of the electric field at a given location is the same as the direction of the electric force exerted on a positively charged particle at that location. In Figure 23.20*a*, therefore, the positively charged particle experiences a force directed to the right. Because its initial velocity is zero, the particle moves in a straight line in the direction of the electric field. (*b*) The negatively charged particle in Figure 23.20*b* experiences an acceleration up the page, opposite the direction of the electric field. This particle moves in a straight line up the page. (*c*) The negatively charged particle in Figure 23.20*c* experiences an acceleration to the right, in the direction opposite the direction of the electric field. Because its initial velocity is perpendicular to the direction of the electric field, the particle travels in a parabolic trajectory up the page and curving to the right.

23.5 Electric field of a charged particle

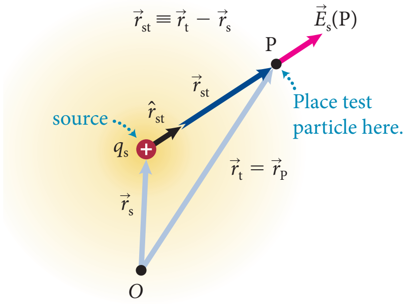
In Section 23.2 we defined the **electric field** at a certain point P in space as the electric force experienced at P by a test particle carrying a charge q_t divided by the charge of the test particle:

$$\vec{E} \equiv \frac{\vec{F}_t^E}{q_t}. \tag{23.1}$$

The SI unit of electric field is the newton per coulomb (N/C).

Equation 23.1 requires no knowledge of the charge distribution that causes the electric field: It gives a prescription for determining the electric field at a given position in space. We can use Coulomb's law, however, to derive an expression for the electric field created at some point P due to a source particle carrying a charge q_s at position $\vec{r}_s$ (**Figure 23.22**). If we place a test particle carrying a charge q_t at P, Coulomb's law (Eq. 22.7) tells us that the force exerted on the test particle is

Figure 23.22 To determine the electric field at P generated by a charged source particle, we place a test particle at P.



$$\vec{F}_{st}^E = k\frac{q_s q_t}{r_{st}^2}\hat{r}_{st}, \tag{23.2}$$

where $k = 9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}$ is the proportionality constant that appears in Coulomb's law (Eq. 22.5), r_{st} is the distance between the two particles, and $\hat{r}_{st}$ is a unit vector pointing from the source particle to the test particle. If we divide the electric force exerted by the source particle on the test particle by the charge q_t on the test particle, we obtain an expression for the electric field created by the source particle at P:

$$\vec{E}_s = \frac{\vec{F}_{st}^E}{q_t} = k\frac{q_s}{r_{st}^2}\hat{r}_{st}. \tag{23.3}$$

Because the test particle has nothing to do with this electric field, we can omit any reference to it by writing $\vec{r}_{st} = \vec{r}_{sP}$ and referring only to the position of point P:

$$\vec{E}_s(\mathrm{P}) = k\frac{q_s}{r_{sP}^2}\hat{r}_{sP}. \tag{23.4}$$

This expression represents the electric field at P due to a source particle carrying a charge q_s at position $\vec{r}_s$.

As expected, the magnitude of the electric field at P is proportional to q_s, is independent of q_t, and decreases as the inverse square of the distance r_{sP} from the source particle. The direction of the electric field is outward (that is to say, in the direction given by $\hat{r}_{sP}$) when q_s is positive and inward (antiparallel to $\hat{r}_{sP}$) when q_s is negative.

Using the superposition principle, we now can determine the electric field due to a system of particles 1, 2, . . . carrying charges $q_1, q_2, \ldots$. The combined electric field is the vector sum of the individual electric fields:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots = \sum k\frac{q_i\hat{r}_{iP}}{r_{iP}^2}. \tag{23.5}$$

Once the electric field at a certain position is known, the force exerted on any particle carrying charge q placed at that position can be found from

$$\vec{F}_p^E = q\vec{E}. \tag{23.6}$$

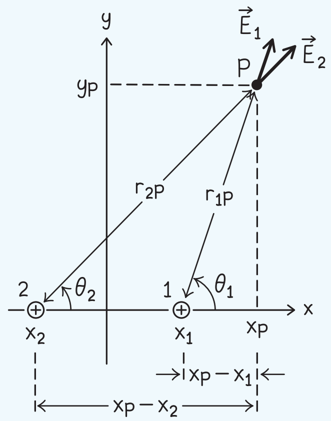
(Remember that we omit the "by" subscript on the force when the force is exerted by a field.) If q is positive, the force exerted on the particle is in the same direction as the electric field; if q is negative, the force exerted on the particle is in the direction opposite the direction of the electric field.

Example 23.3 Electric field due to two charged particles

A point P is located at $x_P = 2.0$ m, $y_P = 3.0$ m. What are the magnitude and direction of the electric field at P due to a particle 1 carrying charge $q_1 = +10\ \mu$C and located at $x_1 = 1.0$ m, $y_1 = 0$ and a particle 2 carrying charge $q_2 = +20\ \mu$C and located at $x_2 = -1.0$ m, $y_2 = 0$?

❶ **GETTING STARTED** I begin by making a sketch of the situation (**Figure 23.23**). Each particle carries a positive charge, so the electric field due to each particle points away from the particle.

Figure 23.23



❷ **DEVISE PLAN** To determine the electric field $\vec{E}_P$ at P, I must take the vector sum of $\vec{E}_1$ and $\vec{E}_2$ at P. I can use Eq. 23.4 to calculate the magnitudes E_1 and E_2. To obtain the vector sum of the two fields, I add their x and y components.

❸ **EXECUTE PLAN** The distances from the particles to P are $r_{1P} = \sqrt{(x_P - x_1)^2 + y_P^2} = \sqrt{10\text{ m}^2} = 3.2$ m and $r_{2P} = \sqrt{(x_P - x_2)^2 + y_P^2} = \sqrt{18\text{ m}^2} = 4.2$ m. The magnitudes of the electric fields created by the particles at P are thus

$$E_1 = k\frac{|q_1|}{r_{1P}^2} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.0 \times 10^{-5}\ \text{C})}{10\ \text{m}^2}$$
$$= 0.90 \times 10^4\ \text{N/C}$$

$$E_2 = k\frac{|q_2|}{r_{2P}^2} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2.0 \times 10^{-5}\ \text{C})}{18\ \text{m}^2}$$
$$= 1.0 \times 10^4\ \text{N/C}.$$

To calculate $\vec{E}_P$, I take the vector sum of $\vec{E}_1$ and $\vec{E}_2$ at P. In component form, I have

$$E_{Px} = E_{1x} + E_{2x} = E_1 \cos\theta_1 + E_2 \cos\theta_2$$
$$= E_1\frac{(x_P - x_1)}{r_{1P}} + E_2\frac{(x_P - x_2)}{r_{2P}}$$

$$E_{Py} = E_{1y} + E_{2y} = E_1 \sin\theta_1 + E_2 \sin\theta_2$$
$$= E_1\frac{y_P}{r_{1P}} + E_2\frac{y_P}{r_{2P}}.$$

Substituting the values given, I have

$$E_{Px} = (0.9 \times 10^4\ \text{N/C})\frac{1.0\ \text{m}}{3.2\ \text{m}} + (1.0 \times 10^4\ \text{N/C})\frac{3.0\ \text{m}}{4.2\ \text{m}}$$
$$= +1.0 \times 10^4\ \text{N/C}$$

$$E_{Py} = (0.9 \times 10^4\ \text{N/C})\frac{3.0\ \text{m}}{3.2\ \text{m}} + (1.0 \times 10^4\ \text{N/C})\frac{3.0\ \text{m}}{4.2\ \text{m}}$$
$$= +1.6 \times 10^4\ \text{N/C}.$$

Finally, I write this in vector form as

$$\vec{E}_P = (+1.0 \times 10^4\ \text{N/C})\hat{\imath} + (+1.6 \times 10^4\ \text{N/C})\hat{\jmath}. ✔$$

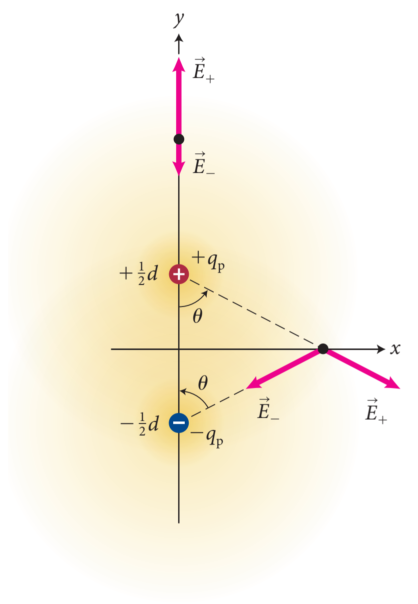
❹ **EVALUATE RESULT** Both E_{Px} and E_{Py} are positive, as I expect based on my sketch. The magnitudes of $\vec{E}_1$ and $\vec{E}_2$ are comparable, which is what I would expect: Particle 2 carries twice the charge of particle 1, but the square of its distance to P is greater by a factor of 1.8.

23.12 What is the magnitude of the electric force exerted by the electric field on an electron placed at point P in Figure 23.23? What is the initial acceleration of the electron if it is released from rest from that point? [$e = 1.6 \times 10^{-19}$ C; $m_e = 9.1 \times 10^{-31}$ kg]

23.6 Dipole field

Next we examine the electric field due to a permanent electric dipole. **Figure 23.24** on the next page shows a dipole that consists of a particle carrying a charge $+q_p$ at $x = 0$, $y = +\frac{1}{2}d$, and another particle carrying a charge $-q_p$ at $x = 0$, $y = -\frac{1}{2}d$, where d is the distance between the two particles. The charge q_p of the positively charged pole is called the *dipole charge*, and the distance d is called the *dipole separation*. Each particle creates an electric field at all positions in space, so the two fields overlap everywhere. We can determine the combined electric field at any position by adding the two fields vectorially. Let us do this for two general locations: anywhere along the x axis and anywhere along the y axis.

Figure 23.24 Calculating the electric field due to a dipole.



Along the x axis, which bisects the dipole, the magnitudes of the electric fields due to the two ends of the dipole are equal:

$$E_+ = E_- = k\frac{q_p}{x^2 + (d/2)^2}. \tag{23.7}$$

The x components of these two electric fields point in opposite directions and so add to zero. The magnitude of the combined electric field is thus equal to the sum of the y components:

$$E_y = E_{+y} + E_{-y} = -(E_+ + E_-)\cos\theta$$

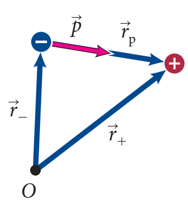
$$= -\left(2k\frac{q_p}{x^2 + (d/2)^2}\right)\left(\frac{d/2}{[x^2 + (d/2)^2]^{1/2}}\right) = -k\frac{q_p d}{[x^2 + (d/2)^2]^{3/2}}. \tag{23.8}$$

The product $q_p d$ is a measure of the strength of the dipole and is the magnitude of the dipole moment introduced in Section 23.4. To specify both the strength and the orientation of the dipole we can write this quantity as a vector, called the **dipole moment:**

$$\vec{p} \equiv q_p \vec{r}_p, \tag{23.9}$$

where $\vec{r}_p \equiv \vec{r}_{-+} = \vec{r}_+ - \vec{r}_-$ is the position of the positively charged particle relative to the negatively charged particle (and so $d = |\vec{r}_p|$). Because q_p is always taken to be positive, the dipole moment $\vec{p}$ points in the same direction as $\vec{r}_p$: along the axis of the dipole (the line that passes through the center of each particle), in the direction from the negative to the positive pole (**Figure 23.25**). Large permanent dipole moments can be caused either by a large dipole separation d or by a large dipole charge q_p. Conceptually you can think of the magnitude of the dipole moment as a measure of how strongly the dipole wants to align itself in the direction of an electric field. The SI unit of dipole moment is the C · m.

Figure 23.25 The dipole moment $\vec{p}$ points along the axis of the dipole from the negative to the positive pole.



For distances far from the dipole ($x \gg d/2$), we may ignore $d/2$, and so $[x^2 + (d/2)^2]^{3/2} \to x^3$. Equation 23.8 thus becomes

$$E_y \approx -k\frac{p}{|x^3|} \quad \text{(far from dipole along the positive } x \text{ axis).} \tag{23.10}$$

The right side of this equation is negative for both positive and negative x, and so anywhere along the x axis the dipole's electric field $\vec{E}$ points in the negative y direction, opposite the direction of the dipole moment. Equation 23.10 also shows that the magnitude of the electric field is inversely proportional to x^3, in contrast to the electric field of a charged particle, which is inversely proportional to x^2 (Eq. 23.4). The reason the electric field of a dipole approaches zero faster as x increases is that the angle between $\vec{E}_+$ and $\vec{E}_-$ in Figure 23.24 approaches 180° as x increases, and so the electric fields of the two poles tend to cancel each other more and more.

Along the y axis, the electric field created by either end of the dipole is directed along the y axis. Thus to determine the y component of the electric field of the dipole at any position along the y axis, we must add the y components of the fields from each particle. For $y > +d/2$:

$$E_y = E_{+y} + E_{-y} = k\frac{q_p}{[y - (d/2)]^2} - k\frac{q_p}{[y + (d/2)]^2}. \tag{23.11}$$

After some algebra, this can be rewritten in the form

$$E_y = k\frac{q_p}{y^2}\left[\left(1 - \frac{d}{2y}\right)^{-2} - \left(1 + \frac{d}{2y}\right)^{-2}\right] \quad (y > +d/2). \tag{23.12}$$

For distances far from the dipole, $y \gg +d/2$, so we can use the binomial series expansion, which states that for $x \ll 1$, $(1 + x)^n \approx 1 + nx$ (see Appendix B). Applying this expansion to the two terms inside the square brackets in Eq. 23.12, we get

$$E_y \approx k\frac{q_p}{y^2}\left[\left(1 + 2\frac{d}{2y}\right) - \left(1 - 2\frac{d}{2y}\right)\right]$$

$$= k\frac{q_p}{y^2}\left[\frac{2d}{y}\right] = 2k\frac{q_p d}{y^3} = 2k\frac{p}{y^3} \qquad (y \gg d/2). \tag{23.13}$$

The right side of this equation has the same algebraic sign as y, so the dipole's electric field $\vec{E}$ points in the positive y direction. (Carrying out the same calculation for $y < -d/2$, you can show that the electric field still points in the positive y direction. In between the two charged particles, the electric field points in the negative y direction.) The magnitude of the electric field is inversely proportional to y^3—just as along the x axis, the electric fields of each of the two poles tend to cancel each other more and more as the distance from the dipole increases. One can show that the electric field of the dipole depends on $1/r^3$ for all positions far from the dipole (where r is the distance between the point under consideration and the center of the dipole). The reason is that the electric fields of the positive and negative ends of the dipole partially cancel each other, and this cancellation becomes more complete far from the dipole: The farther you are from the dipole, the smaller the separation between the charged particles appears to be.

23.13 The magnitude of the electric field created by dipole A at a certain point P is E_A. If the dipole is replaced with another dipole B that has its dipole moment oriented in the same direction, the magnitude of the electric field at point P is found to be greater: $E_B > E_A$. Which dipole has the greater dipole moment? For which of these two dipoles is the dipole charge q_p greater?

23.7 Electric fields of continuous charge distributions

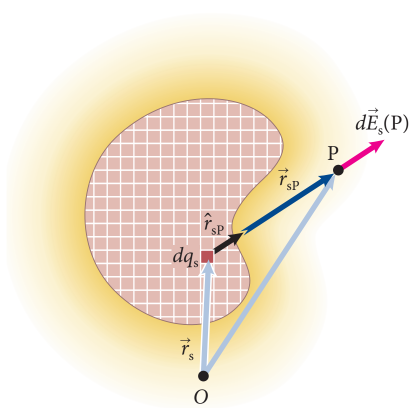
So far we have dealt with only charged particles because Coulomb's law applies only to charged particles. However, most charged objects of interest—from charged combs to electrical components—are not particles. Instead, they are extended bodies. Although every macroscopic object consists of very large numbers of charged particles—protons and electrons—it is not practical to calculate the individual field of each of these particles and then add them vectorially. Instead, we shall treat any macroscopic charged object as having a continuous charge distribution and calculate the electric field created by the object by dividing the charge distribution on the object into infinitesimally small segments that may be considered charged source particles carrying a charge dq_s. For the charged macroscopic object shown in Figure 23.26, for example, we can use Coulomb's law to obtain the infinitesimal portion of the electric field at point P contributed by a segment:

$$d\vec{E}_s(\mathrm{P}) = k\frac{dq_s}{r_{sP}^2}\hat{r}_{sP}. \tag{23.14}$$

Using the principle of superposition, we can then sum the contributions of all the segments that make up the object. Because the segments are infinitesimally small, this sum corresponds to an integral:

$$\vec{E} = \int d\vec{E}_s = k\int \frac{dq_s}{r_{sP}^2}\hat{r}_{sP}. \tag{23.15}$$

Figure 23.26 To calculate the electric field created at P by a continuous charge distribution, we divide the distribution into infinitesimally small segments that can be treated as charged source particles carrying charge dq_s.



QUANTITATIVE TOOLS

In order to evaluate this integral, we must express dq_s, $1/r_{sP}^2$, and $\hat{r}_{sP}$ in terms of the same coordinate(s). To do so, it is necessary to express the charge on the object in terms of a **charge density**—the amount of charge per unit of length, per unit of surface area, or per unit of volume. For a one-dimensional object, such as a thin charged wire of length ℓ carrying a charge q uniformly distributed along the wire, the *linear charge density*—the amount of charge per unit of length (in coulombs per meter)—is given by

$$\lambda \equiv \frac{q}{\ell} \quad \text{(uniform charge distribution).} \tag{23.16}$$

For uniformly charged two-dimensional objects, we use the *surface charge density*—the amount of charge per unit of area (in coulombs per square meter). For example, the surface charge density of a flat plate of area A carrying a uniformly distributed charge q is

$$\sigma \equiv \frac{q}{A} \quad \text{(uniform charge distribution).} \tag{23.17}$$

For a uniformly charged three-dimensional object, we use the *volume charge density:*

$$\rho \equiv \frac{q}{V} \quad \text{(uniform charge distribution),} \tag{23.18}$$

which gives the amount of charge per cubic meter.

The procedure on this page provides some helpful steps for carrying out the integral in Eq. 23.15, and the next four examples show how to put the procedure into practice.

Procedure: Calculating the electric field of continuous charge distributions by integration

To calculate the electric field of a continuous charge distribution, you need to evaluate the integral in Eq. 23.15. The following steps will help you evaluate the integral.

1. Begin by making a sketch of the charge distribution. Mentally divide the distribution into small segments. Indicate one such segment that carries a charge dq_s in your drawing.
2. Choose a coordinate system that allows you to express the position of the segment in terms of a minimum number of coordinates (x, y, z, r, or θ). These coordinates are the integration variables. For example, use a radial coordinate system for a charge distribution with radial symmetry. Unless the problem specifies otherwise, let the origin be at the center of the object.
3. Draw a vector showing the electric field caused by the segment at the point of interest. Examine how the components of this vector change as you vary the position of the segment along the charge distribution. Some components may cancel, which greatly simplifies the calculation. If you can determine the direction of the resulting electric field, you may need to calculate only one component. Otherwise express $\hat{r}_{sP}$ in terms of your integration variable(s) and evaluate the integrals for each component of the field separately.
4. Determine whether the charge distribution is one-dimensional (a straight or curved wire), two-dimensional (a flat or curved surface), or three-dimensional (any bulk object). Express dq_s in terms of the corresponding charge density of the object and the integration variable(s).
5. Express the factor $1/r_{sP}^2$, where r_{sP} is the distance between dq_s and the point of interest, in terms of the integration variable(s).

At this point you can substitute your expressions for dq_s and $1/r_{sP}^2$ into Eq. 23.15 and carry out the integral (or component integrals), using what you determined about the direction of the electric field (or substituting your expression for $\hat{r}_{sP}$).

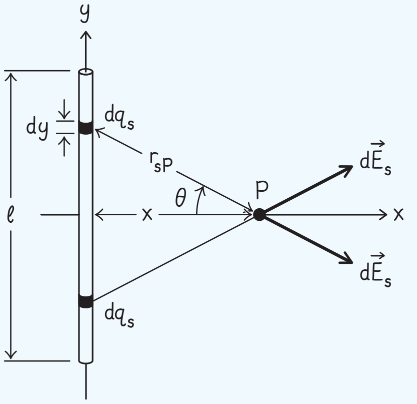
QUANTITATIVE TOOLS

Example 23.4 Electric field created by a uniformly charged thin rod

A thin rod of length ℓ carries a uniformly distributed charge q. What is the electric field at a point P along a line that is perpendicular to the long axis of the rod and passes through the rod's midpoint?

❶ **GETTING STARTED** I begin by making a sketch of the situation. After drawing a set of axes, I place the rod along the y axis, with the origin at the rod center and point P on the positive x axis (**Figure 23.27**).

Figure 23.27



❷ **DEVISE PLAN** The word *thin* implies that I can treat the rod as a one-dimensional object. Because the rod is uniformly charged, I can thus use Eq. 23.16 to determine the linear charge density along the rod. To determine the electric field at P, I divide the rod lengthwise into a large number of infinitesimally small segments, each of length dy. Each segment contributes to the electric field at P an amount given by Eq. 23.14. For each segment above the x axis there is a corresponding segment below the axis at the same distance from P. The y components of the electric fields $d\vec{E}_s$ due to these two segments add up to zero, so I need to calculate only the x component dE_{sx}. To get the electric field created by the entire rod, I use Eq. 23.15 to integrate my result over the length of the rod.

❸ **EXECUTE PLAN** The charge dq_s on each segment dy is $dq_s = \lambda\, dy = (q/\ell)\, dy$. The x component of the electric field created by each segment at P is thus

$$dE_{sx} = k\frac{dq_s}{r_{sP}^2}\cos\theta = k\frac{q}{\ell r_{sP}^2}\cos\theta\, dy, \tag{1}$$

where θ is the angle between the x axis and the line that connects the segment dy with P. Both θ and r_{sP} depend on the position y of the segment, so I must choose one integration variable and express the others in terms of that variable. I choose θ as the integration variable, which means I must express the factor dy/r_{sP}^2 in Eq. 1 in terms of θ. Using trigonometry, I have

$$\cos\theta = \frac{x}{r_{sP}} \tag{2}$$

and

$$\tan\theta = \frac{y}{x}, \tag{3}$$

where x is the x coordinate of point P. Differentiating Eq. 3 yields

$$dy = x\, d(\tan\theta) = \frac{x}{\cos^2\theta}d\theta. \tag{4}$$

Next I divide Eq. 4 by r_{sP}^2 to obtain the factor dy/r_{sP}^2 I need. I use r_{sP}^2 on the left, but on the right I use Eq. 2 to write r_{sP}^2 in the form $x^2/\cos^2\theta$, yielding

$$\frac{dy}{r_{sP}^2} = \left(\frac{\cos^2\theta}{x^2}\right)\left(\frac{x}{\cos^2\theta}d\theta\right) = \frac{1}{x}d\theta.$$

Substituting this result into Eq. 1 and integrating over the entire rod yield

$$E_x = k\frac{q}{\ell}\int_{-\theta_{\max}}^{+\theta_{\max}}\frac{\cos\theta}{x}d\theta = \frac{kq}{\ell x}\int_{-\theta_{\max}}^{+\theta_{\max}}\cos\theta\, d\theta$$

$$= \frac{kq}{\ell x}\sin\theta\Big|_{-\theta_{\max}}^{+\theta_{\max}} = \frac{2kq}{\ell x}\sin\theta_{\max},$$

where $\theta_{\max}$, the maximum value of θ, is the angle between the x axis and the line that connects the top end of the rod with P. Substituting $\sin\theta_{\max} = y/r_{sP} = \frac{1}{2}\ell/\sqrt{(\ell/2)^2 + x^2}$ finally yields

$$E_x = \frac{kq}{x\sqrt{\ell^2/4 + x^2}};\ E_y = 0;\ E_z = 0. ✔$$

❹ **EVALUATE RESULT** Very far from the rod along the positive x axis, $x \ll \ell$, so the rod looks like a particle. In this case, I can ignore the ℓ^2 term in the denominator and my result becomes identical to Eq. 23.4, the equation for a particle that carries a charge q ($E = kq/r^2$, or using the symbols from this problem, $E_x = kq/x^2$).

When P is very close to the rod, $x \ll \ell$, I can ignore the x^2 term in the denominator and write

$$E_x = \frac{2k(q/\ell)}{x} = \frac{2k\lambda}{x}. \tag{5}$$

In this $x \ll \ell$ case, the distance from P to either end of the rod is much greater than the distance from P to the closest point on the rod (which is the rod midpoint, located at the origin), and thus the rod essentially looks "infinitely long" to an observer at P. Indeed, Eq. 5 shows that my result no longer depends on ℓ.

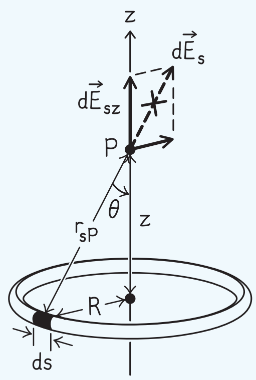
I also note that the rod's electric field is now inversely proportional to x rather than x^2. I saw in Chapter 17 that the amplitudes of waves that spread out in three dimensions are inversely proportional to x^2, whereas the amplitudes of waves that spread out in two dimensions are inversely proportional to x. My result therefore make sense because the electric field that emanates from a charged particle "spreads out" in three dimensions, but the field that emanates from an infinitely long charged rod spreads out in just two dimensions.

Example 23.5 Electric field created by a uniformly charged thin ring

A thin ring of radius R carries a uniformly distributed charge q. What is the electric field at point P along an axis that is perpendicular to the plane of the ring and passes through its center?

❶ **GETTING STARTED** I begin by making a sketch of the situation. I let the ring be in the xy plane, with the origin at the center of the ring, and I place P on the positive z axis (**Figure 23.28**).

Figure 23.28



❷ **DEVISE PLAN** Because the ring is thin, I can use Eq. 23.16 to determine its linear charge density. To determine the electric field at P, I divide the ring into a large number of infinitesimally small segments, each of arc length ds. Each segment contributes to the electric field an amount given by Eq. 23.14. Because all segments are at the same distance from P, all contribute an electric field $d\vec{E}_s$ of the same magnitude. As shown in my sketch, the contribution $d\vec{E}_s$ makes an angle θ with the z axis, and so each segment ds produces a component parallel to the z axis and a component perpendicular to it. For each pair of segments on opposite sides of the ring, the components of $d\vec{E}_s$ perpendicular to the z axis add up to zero, and so I am concerned only with the z components.

To get the electric field created by the ring, I can use Eq. 23.15 to integrate my $d\vec{E}_s$ result over the circumference of the ring.

❸ **EXECUTE PLAN** Each segment carries a charge $dq_s = \lambda\, ds$, where $\lambda = q/2\pi R$ is the linear charge density along the ring. The magnitude of each segment's contribution to the electric field is, from Eq. 23.14,

$$dE_s = k\frac{dq_s}{r_{sP}^2} = k\frac{\lambda\, ds}{r_{sP}^2} = k\frac{\lambda\, ds}{z^2 + R^2}.$$

For the z component of $d\vec{E}_s$, I see from Figure 23.28 that the angle between the vectors $d\vec{E}_{sz}$ and $d\vec{E}_s$ is also θ, and so $\cos\theta = dE_{sz}/dE_s$. Then, combining this expression with the relationship

$$\cos\theta = \frac{z}{r_{sP}} = \frac{z}{\sqrt{z^2 + R^2}},$$

I have

$$dE_{sz} = \cos\theta\, dE_s = \left[\frac{z}{\sqrt{z^2 + R^2}}\right]\left[k\frac{\lambda\, ds}{z^2 + R^2}\right] = k\frac{z\lambda}{[z^2 + R^2]^{3/2}}\, ds.$$

To determine the electric field created by the ring, I must integrate the contributions around the ring, from $s = 0$ to $s = 2\pi R$. Because k, z, R, and λ are all independent of s, I can move everything out of the integral except ds:

$$E_z = \int dE_{sz} = k\frac{z\lambda}{[z^2 + R^2]^{3/2}}\int_0^{2\pi R} ds = k\frac{z\lambda(2\pi R)}{[z^2 + R^2]^{3/2}}.$$

Because $\lambda = q/2\pi R$, the term $\lambda(2\pi R)$ is equal to the charge q on the ring, so I get for the z component of the electric field along the axis perpendicular to the plane of the ring and passing through the ring center

$$E_x = 0;\ E_y = 0;\ E_z = k\frac{qz}{[z^2 + R^2]^{3/2}}. \checkmark$$

❹ **EVALUATE RESULT** Very far from the ring my expression for E_z should become the same as that for a charged particle. Indeed, when $z \gg R$, I can ignore the R^2 term in my result, and so

$$E_z \approx k\frac{qz}{[z^2]^{3/2}} = k\frac{qz}{z^3} = k\frac{q}{z^2},$$

as I expect.

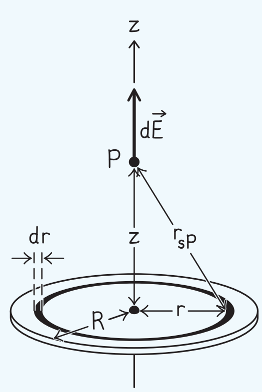
At the center of the ring, $z = 0$ and so my expression yields $E_z = 0$. This result is reasonable because at the center of the ring the electric forces exerted by segments on opposite sides of the ring on a charged test particle add to zero. When the vector sum of these forces is zero, the electric field must be zero also.

Example 23.6 Electric field created by a uniformly charged disk

A thin disk of radius R carries a uniformly distributed charge. The surface charge density on the disk is σ. What is the electric field at a point P along the perpendicular axis through the disk center?

❶ **GETTING STARTED** I begin with a sketch, placing the disk in the xy plane, with the disk center at the origin. I let point P lie on the positive z axis (**Figure 23.29**).

Figure 23.29



❷ **DEVISE PLAN** Because of the circular symmetry of the disk, I divide it into a large number of ring-shaped segments, each of radius r and width dr. The charge on each ring is the product of the ring surface area (circumference times width) and the surface charge density: $dq_s = (2\pi r)dr\,\sigma$. The contribution of each ring to the electric field at P is given by the expression for E_z I obtained for a uniformly charged thin ring in Example 23.5 with $2\pi r\,\sigma\,dr$ substituted for q and r substituted for R. So all I need to do is integrate over the entire disk.

❸ **EXECUTE PLAN** Substituting $q = 2\pi r\,\sigma\,dr$ and $R = r$ into the Example 23.5 expression for E_z and integrating the result over the disk from $r = 0$ to $r = R$, I have

$$E_z = \int dE_z = k\int_0^R \frac{2\pi r\,\sigma z}{(z^2 + r^2)^{3/2}}dr.$$

Because σ and z are independent of r, I can move them out of the integral:

$$E_x = 0;\ E_y = 0;\ E_z = k\pi\sigma z\int_0^R \frac{2r\,dr}{(z^2 + r^2)^{3/2}}$$

$$= k\pi\sigma z\int_0^R \frac{d(r^2)}{(z^2 + r^2)^{3/2}} = k\pi\sigma z \left.\frac{-2}{(z^2 + r^2)^{1/2}}\right|_0^R$$

$$= 2k\pi\sigma z\left[\frac{1}{(z^2)^{1/2}} - \frac{1}{(z^2 + R^2)^{1/2}}\right]. ✔ \tag{1}$$

❹ **EVALUATE RESULT** Let me evaluate my result for $z \gg R$, where, to an observer at P, the disk looks like a particle. For positive z, I can write the E_z in Eq. 1 as

$$E_z = 2k\pi\sigma\left[1 - \frac{z}{(z^2 + R^2)^{1/2}}\right]. \tag{2}$$

From the binomial series expansion (see Appendix B), I get in the case that $z \gg R$,

$$\frac{z}{(z^2 + R^2)^{1/2}} = \left(1 + \frac{R^2}{z^2}\right)^{-1/2} \approx 1 - \tfrac{1}{2}\frac{R^2}{z^2}.$$

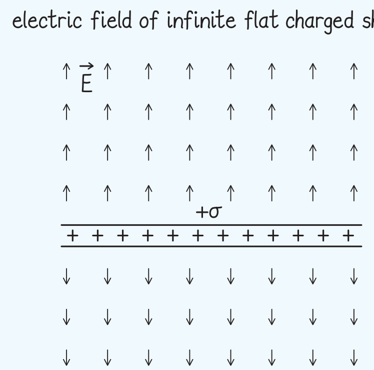
Substituting this result into Eq. 2 and writing $q = \sigma(\pi R^2)$ for the charge on the disk, I get

$$E_z = 2k\pi\sigma\left(\tfrac{1}{2}\frac{R^2}{z^2}\right) = k\frac{\sigma\pi R^2}{z^2} = k\frac{q}{z^2},$$

which is the result for a charged particle, as I expect.

I can also evaluate my result for $z \approx 0$, where, to an observer at that location, the disk looks like it has an infinite radius (that is to say, it looks like an infinite flat sheet). In that case, the second term inside the brackets in Eq. 2 vanishes and $E_z = 2k\pi\sigma$. This tells me that the electric field of an infinite flat charged sheet is independent of z and constant throughout space, as I have sketched in **Figure 23.30**. (In other words, it is uniform.) While this lack of dependence on z is somewhat counterintuitive, it agrees with what I concluded earlier: The electric field that emanates from a charged particle "spreads out" in three dimensions and its amplitude is inversely proportional to the square of the distance from the particle, whereas the electric field that emanates from an infinitely long charged rod spreads out in just two dimensions and its amplitude is inversely proportional to the distance from the rod. As my sketch shows, the electric field that emanates from an infinite plane can't spread out at all (if the plane is truly infinite), and therefore its amplitude is independent of distance.

Figure 23.30



23.14 (*a*) Describe the electric field between two infinitely large parallel charged sheets if the charge density of one sheet is $+\sigma$ and that of the other is $-\sigma$. (*b*) Describe the electric field outside the sheets.

Example 23.7 Electric field created by a uniformly charged sphere

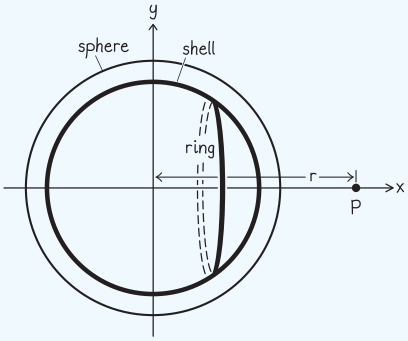
A solid sphere of radius R carries a fixed, uniformly distributed charge q. Exploiting the analogy between Newton's law of gravity and Coulomb's law, use the result obtained in Section 13.8 to obtain an expression for the magnitude of the electric field created by the sphere at a point P outside the sphere.

❶ **GETTING STARTED** To determine the electric field magnitude at point P, I need to determine the magnitude of the electric force exerted by the sphere on a test particle carrying a charge q_t at P and then divide that force magnitude by q_t.

❷ **DEVISE PLAN** I can follow the same procedure as in Section 13.8 to calculate the gravitational force of a spherical object: I first divide the sphere into a series of thin concentric shells that resemble the layers in an onion and then divide each shell into a series of vertical rings (**Figure 23.31**). I then calculate the contribution of each ring to the electric field at P and integrate first over each shell and then over the sphere. The expression I get for F^E_{sphere} must be of the same form as that for the gravitational sphere, F^G_{sphere} (Eq. 13.37), because the gravitational force and the electric force are both inversely proportional to the square of the distance between the interacting particles. So all I need to do is replace G in Eq. 13.37 by k, M_{sphere} by q, and m by q_t to obtain the magnitude of the electric force exerted by the sphere on the test particle. To obtain an expression for the electric field, I then divide the result by q_t.

(Continued)

Figure 23.31



❸ **EXECUTE PLAN** In analogy to Eq. 13.37, I write

$$F^G_{\mathrm{sp}} = G\frac{mM_{\mathrm{sphere}}}{r^2} \rightarrow F^E_{\mathrm{st}} = k\frac{q_t q}{r^2},$$

where r is the distance from the center of the sphere to P. To obtain the magnitude of the electric field, I divide the electric force by q_t:

$$E_{\mathrm{sphere}} = k\frac{q}{r^2}. ✔$$

❹ **EVALUATE RESULT** Comparing my result with Eq. 23.3 (in scalar form), $E_s = kq_s/r^2_{\mathrm{st}}$, shows that outside a uniformly charged solid sphere the magnitude of the electric field is the same as that surrounding a particle carrying the same charge and located at the center of the sphere. The result is independent of the radius R of the sphere and similar to the result obtained in Section 13.8: A solid sphere exerts a gravitational force as if the entire mass of the sphere were concentrated at the center. It makes sense that I obtain a similar result for the charged sphere because the gravitational force and the electric force are both proportional to $1/r^2$.

23.15 How does the electric field inside a uniformly charged sphere vary with distance from the sphere center? [Hint: What is the electric field inside a hollow uniformly charged sphere?]

23.8 Dipoles in electric fields

Let us end this chapter by considering the forces exerted by electric fields on dipoles. **Figure 23.32*a*** shows a dipole consisting of two particles that carry charges of equal magnitude but opposite sign connected by a rod of length d; the dipole makes an angle θ with a uniform electric field $\vec{E}$ created by some unseen distant source. As we saw in Section 23.4, the forces exerted by the electric field on the charged ends of the dipole are equal in magnitude but opposite in direction, and so the vector sum of the forces exerted on the dipole is zero. Consequently the acceleration of the center of mass of the dipole is zero. Because the forces are exerted on opposite ends of the dipole, however, they create torques that cause the dipole to rotate counterclockwise about its center of mass. Figure 23.32*b* shows that the force exerted on the positive end causes a counterclockwise torque of magnitude

$$\tau_+ = r_\perp F^E_+ = (\tfrac{1}{2}d\sin\theta)(q_p E), \tag{23.19}$$

where $r_\perp$ is the lever arm of the force. The force exerted on the negative end causes an identical torque because the lever arm and the magnitude of the force are the same. The electric field thus causes a torque on the dipole equal to

$$\Sigma\tau_\vartheta = 2(\tfrac{1}{2}d\sin\theta)(q_p E) = (q_p d)E\sin\theta \equiv pE\sin\theta. \tag{23.20}$$

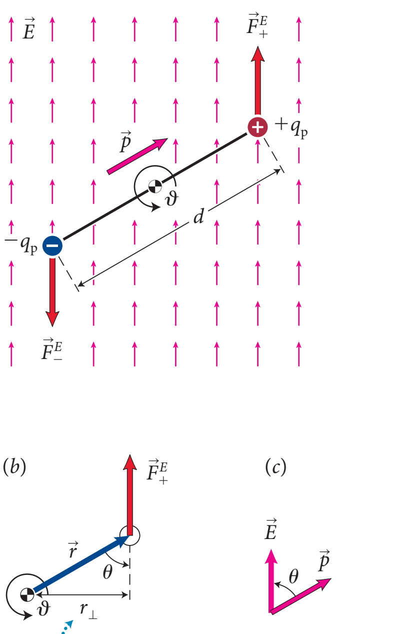
This can be written in vectorial form as

$$\Sigma\vec{\tau} = \vec{p}\times\vec{E}, \tag{23.21}$$

where $\vec{p}$ is the dipole moment, which by definition points from the negative end of the dipole to the positive end and whose magnitude is given by Eq. 23.9. According to the right-hand rule (see Section 12.4), the vector product $\vec{p}\times\vec{E}$ in Eq. 23.21 gives a torque that points out of the plane of the drawing in

Figure 23.32 The torque on an electric dipole caused by an electric field tends to align the dipole moment $\vec{p}$ with the direction of the electric field.

(*a*) Electric dipole in electric field



Lever arm $\vec{r}_\perp$ depends on angle θ of dipole with respect to electric field.

Torque on dipole is vector product $\Sigma\vec{\tau} = \vec{p}\times\vec{E}$.

Figure 23.33 A dipole interacts with a charged particle.

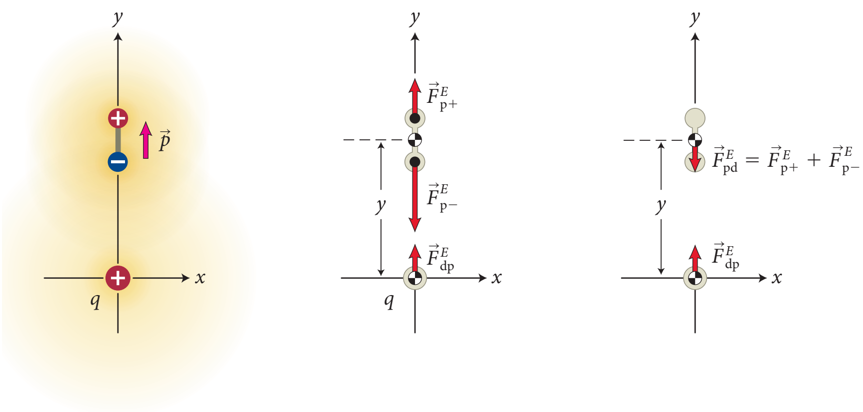


Figure 23.32*c*. As we saw in Section 12.8, such a torque indeed causes a counterclockwise rotation. The torque on the dipole is maximum when the dipole moment is perpendicular to the electric field and zero when it is parallel or antiparallel to the electric field.

23.16 Is Eq. 23.20 valid if the center of mass is not in the middle of the dipole?

As we saw in Section 23.4, the vector sum of the forces exerted on dipoles in nonuniform electric fields is not zero. Consider, for example, the situation illustrated in **Figure 23.33*a***. A dipole with its dipole moment $\vec{p}$ aligned along the y axis is placed in the nonuniform electric field generated by a particle carrying a charge q and located at the origin. Because the distance between the negative end of the dipole and the particle is smaller than the distance between the positive end and the particle, the magnitude of the attractive force $\vec{F}^E_{p-}$ on the negative end is greater than the repulsive force $\vec{F}^E_{p+}$ on the positive end. Consequently the vector sum of the forces exerted by the nonuniform field on the dipole is nonzero, and the dipole is attracted to the particle. How does this attraction vary with the position y of the dipole?

To answer this question, we can write an expression for the vector sum of the forces $\sum \vec{F}^E_d$ exerted on the two ends of the dipole and examine how this sum varies with y. Alternatively, we can calculate the force $\vec{F}^E_{dp}$ exerted by the dipole on the particle, using our results from Section 23.6. This force and the vector sum of the forces exerted by the particle on the dipole form an interaction pair, and so their magnitudes are the same. Equation 23.13 tells us that, along the dipole axis, the magnitude of the electric field created by the dipole is $2k(p/y^3)$, and so the magnitude of the force exerted by the dipole on the particle is

$$F^E_{dp} = qE_d = 2k\frac{pq}{y^3}. \tag{23.22}$$

The magnitude of the force exerted by the particle on the dipole, being equal in magnitude to F^E_{dp}, is thus

$$F^E_{pd} = F^E_{p-} - F^E_{p+} = 2k\frac{pq}{y^3}. \tag{23.23}$$

QUANTITATIVE TOOLS

Like the electric field of a dipole, the forces between a charged object carrying charge q and a dipole is inversely proportional to the cube of the distance between them.

23.17 How does doubling each of the following quantities affect the force between a dipole and a particle placed near the dipole and carrying charge q? (*a*) the charge q, (*b*) the dipole separation d of the dipole, (*c*) the dipole charge q_p, (*d*) the distance between the dipole and the charged particle

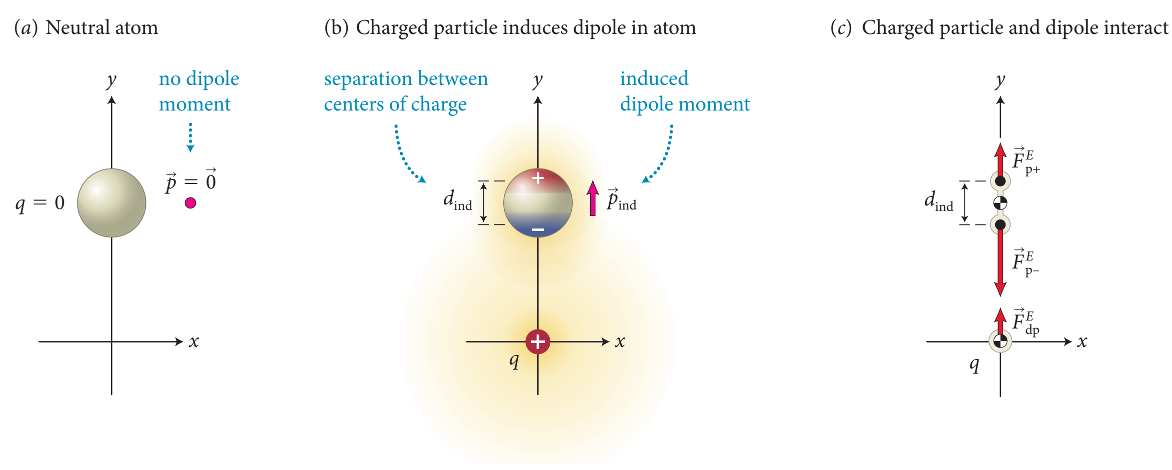
As we saw in Chapter 22, electrically neutral objects interact with a charged object because they become polarized in the presence of the charged object. Consider an isolated neutral atom. The centers of the atom's positive and negative charge distributions coincide, and therefore the atom's dipole moment is zero: $d = 0$ and so $\vec{p} = \vec{0}$ (**Figure 23.34*a***).The presence of an external electric field—that is, an electric field created by some other charged object—causes a separation between the positive and negative charge centers and so induces a dipole moment (Figure 23.34*b*). To understand the interaction between charged objects and neutral ones, we must therefore study the interaction between a charged particle and what is called an **induced dipole**. The first question to ask is: How does the magnitude of the induced dipole moment depend on the presence of a charged particle?

When a neutral atom is placed in an electric field $\vec{E}$, it is found that, as long as the electric forces exerted by that field on the charged particles in the atom are not too large, the induced dipole separation d_{ind} in the atom obeys Hooke's law. In other words, the induced dipole separation is proportional to the magnitude of the applied electric force, $F^E_{\text{d}} = cd_{\text{ind}}$, with c being the "spring constant" of the atom. We can rewrite this as $d_{\text{ind}} = (1/c)F^E_{\text{d}}$, and because d_{ind} is proportional to the magnitude of the induced dipole moment p_{ind} and the magnitude of the force exerted on the dipole is proportional to the magnitude E of the electric field at the position of the dipole, the **induced dipole moment** is proportional to the field at the position of the dipole:

$$\vec{p}_{\text{ind}} = \alpha\vec{E} \quad (\vec{E} \text{ not too large}), \tag{23.24}$$

where α, the **polarizability** of the atom, is a constant that expresses how easily the charge distributions in the atom are displaced from each other. The SI unit of polarizability is $\text{C}^2 \cdot \text{m/N}$.

Figure 23.34 A charged particle induces a dipole in an electrically neutral atom.



QUANTITATIVE TOOLS

23.18 Given that the induced dipole moment $\vec{p}_{\text{ind}}$ points from the negative to the positive end of an induced dipole and the electric field $\vec{E}$ displaces the positive charge center in the direction of the electric field and the negative charge center in the opposite direction, do you expect the polarizability α to be positive or negative?

The electric field of a charged particle is given by Eq. 23.4, so the magnitude of the induced dipole moment is proportional to the inverse square of the distance between the particle and the dipole:

$$p_{\text{ind}} = \alpha E = \alpha\, k\frac{q}{y^2}. \tag{23.25}$$

In contrast, the dipole moment of a permanent dipole is constant.

We can now substitute the induced-dipole result of Eq. 23.25 into Eq. 23.23 to determine the force exerted by a charged particle on an induced dipole:

$$F^E_{\text{pd}} = 2k\frac{p_{\text{ind}}q}{y^3} = \alpha\frac{2k^2q^2}{y^5}. \tag{23.26}$$

This result shows that the interaction between a charged particle and a polarized object depends much more strongly on the distance between them ($1/y^5$) than does the interaction between two charged objects ($1/y^2$). You may have noticed this in Chapter 22 when comparing the attraction between two charged strips of tape with the attraction between a charged strip and a neutral object.* As the neutral object approaches the charged strip, the force varies so fast with distance that it is often difficult to prevent the tape from sticking to the neutral object.

23.19 (*a*) How does doubling the charge q_A carried by an object A affect the force exerted by A on another charged particle? (*b*) How does doubling q_A affect the force exerted by A on an induced dipole? (*c*) Explain why your answers to parts *a* and *b* are the same or different. (*d*) Can the force exerted by a charged particle cause a torque on an induced dipole?

*Try it! Pull two strips of transparent tape out of a dispenser, suspend one from the edge of a table and then move the other slowly toward it. Notice how the interaction between the strips varies relatively smoothly as a function of separation. Next, move your hand slowly toward the suspended strip and note how the force increases rapidly.

Chapter Glossary

SI units of physical quantities are given in parentheses.

Charge density, linear λ (C/m), surface σ (C/m^2), or volume ρ (C/m^3): A scalar that is a measure of the amount of charge per unit of length, area, or volume on a one-, two-, or three-dimensional object, respectively.

Dipole (electric) A neutral charge configuration in which the center of positive charge is separated from the center of negative charge by a small distance. Dipoles can be *permanent,* or they can be *induced* by an external electric field.

Dipole moment (electric) $\vec{p}$ (C · m) A vector defined as the product of the *dipole charge* q_p (the positive charge of the dipole) and the vector $\vec{r}_p$ that points from the center of negative charge to the center of positive charge:

$$\vec{p} \equiv q_p \vec{r}_p. \qquad (23.9)$$

Electric field $\vec{E}$ (N/C) A vector equal to the electric force exerted on a charged test particle divided by the charge on the test particle:

$$\vec{E} \equiv \frac{\vec{F}^E_t}{q_t}. \qquad (23.1)$$

Induced dipole A separation of the positive and negative charge centers in an electrically neutral object caused by an external electric field.

Induced dipole moment $\vec{p}_{ind}$ (C · m) A dipole moment induced by an external electric field in an electrically neutral object. For small electric fields, the induced dipole moment in an atom is proportional to the applied electric field:

$$\vec{p}_{ind} = \alpha \vec{E}, \qquad (23.24)$$

where α is the *polarizability* of the atom.

Interaction field or **field** A physical quantity surrounding objects that mediates an interaction. Objects that have mass are surrounded by a *gravitational field*; those that carry an electrical charge are surrounded by an *electric field*. Both are *vector fields* specified by a direction and a magnitude at each position in space.

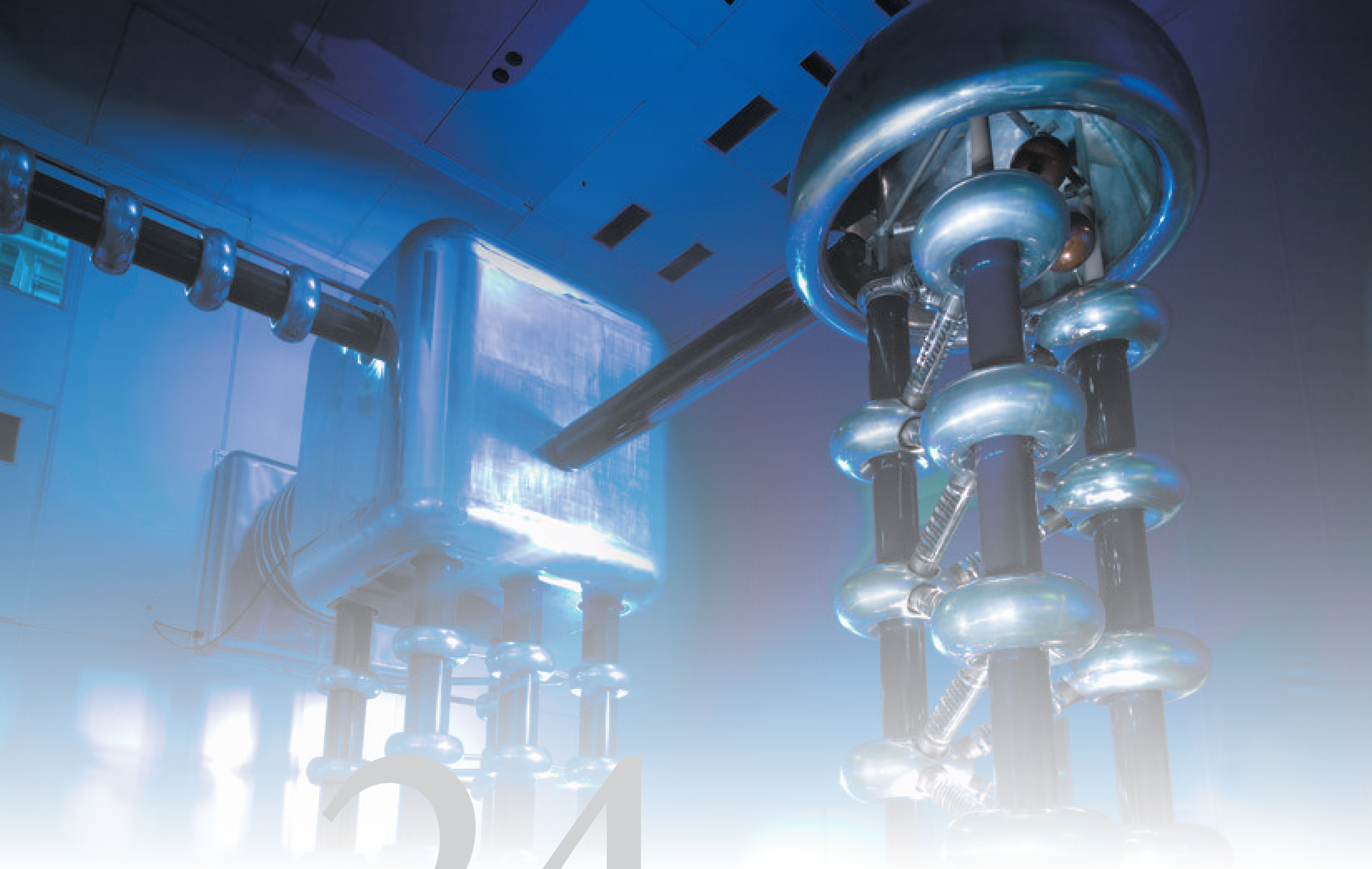
Polarizability α (C^2 · m/N) A scalar measure of the amount of charge separation that occurs in an atom or molecule in the presence of an externally applied electric field.

Superposition of electric fields The electric field of a collection of charged particles is equal to the vector sum of the electric fields created by the individual charged particles:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots. \qquad (23.5)$$

Test particle An idealized particle whose physical properties (mass or charge) are so small that the particle does not perturb the particles or objects generating the field we are measuring.

Vector field diagram A diagram that represents a vector field, obtained by plotting field vectors at a series of locations.



24 Gauss's Law

In principle, Coulomb's law allows us to calculate the electric field produced by any discrete or continuous distribution of charged objects. In practice, however, the calculation is often so complicated that the sums or integrals that arise might require numerical evaluation on a computer. For this reason, it pays to search for additional methods to determine the electric field produced by a charge distribution. In this chapter we develop a relationship between an electric field and its source, known as *Gauss's law,* that can be used to determine the electric fields due to charge distributions that exhibit certain simple symmetries. These symmetries appear in many common applications, which makes Gauss's law an important tool in calculating electric fields. As we shall see in Chapter 30, Gauss's law is one of the fundamental equations of *electromagnetism*—the theory that describes electromagnetic interactions and electromagnetic waves.

24.1 Electric field lines

In Chapter 23 we used vector field diagrams to visualize electric fields. Another way to visualize electric fields, which will help us reach some new insights, is to draw **electric field lines.** These lines are drawn so that at any location the electric field $\vec{E}$ is tangent to them. Because the electric field is a vector, we assign to field lines a direction that corresponds with the direction of the electric field.

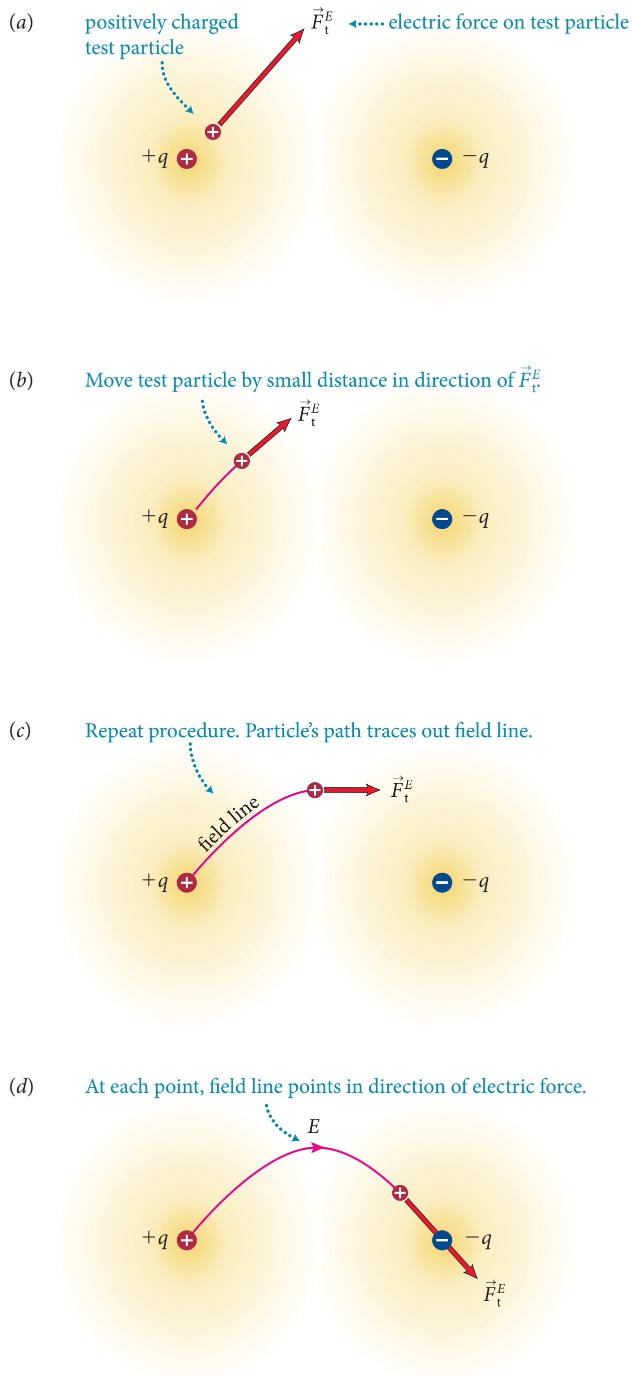
To draw an electric field line, imagine placing a test particle carrying a positive charge q_t somewhere near a charge distribution. Then move the test particle a small distance in the direction of the electric force exerted on it. (Remember from Chapter 23 that the electric field points in the same direction as the electric force exerted on a positively charged test particle.) Repeat the procedure to trace out a line (**Figure 24.1**). We label the field lines with the symbol E to remind us that they represent an electric field.

24.1 Draw several field lines representing the electric field of an isolated positively charged particle. Repeat for a negatively charged particle.

As Checkpoint 24.1 illustrates, the field line diagrams for a positive and for a negative isolated charged particle are similar, even though the electric fields point in opposite directions. They point radially outward from a positively charged particle and point radially inward toward a negatively charged particle. This direction means that electric field lines always start from a positively charged object and always end on a negatively charged object, never the other way around.

Because an electric field is present everywhere around a charged object, a field line passes through every location in space. In practice, we draw only a finite number of field lines to represent the entire field. **Figure 24.2**, for example, shows the pattern of field lines created by a pair of oppositely charged particles (that is, a dipole). Sixteen field lines emanate from the positively charged particle on the

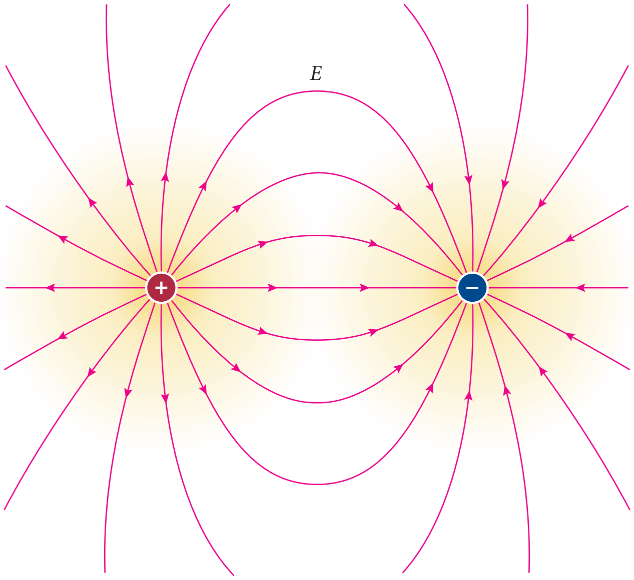
Figure 24.1 Using a positively charged test particle to trace out an electric field line.



left, and 16 field lines terminate on the negatively charged particle on the right. Notice the correspondence between this pattern and the corresponding vector field diagram in **Figure 24.3*a*** and the pattern created by the fibers in Figure 24.3*b*.

The number of field lines that emanate from a positively charged object is arbitrary; we could have chosen some number other than 16 for Figure 24.2. However, in a given field line diagram, the number of field lines is always proportional to the magnitude of the charge carried by the

Figure 24.2 Electric field line diagram for an electric dipole.



object. If, for example, 16 field lines emanate from an object that carries a charge $+q$, then 32 lines emanate from an object that carries a charge $+2q$ and eight lines terminate on an object that carries a charge $-q/2$.

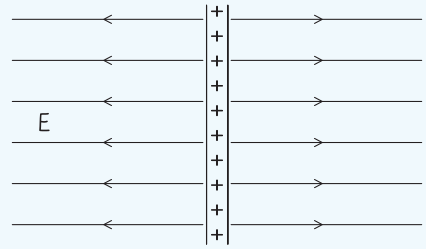
The number of field lines that emanate from a positively charged object or terminate on a negatively charged object is proportional to the charge carried by the object.

Exercise 24.1 Field lines of infinite charged plate

Draw a field line diagram for an infinite plate that carries a uniform positive charge distribution.

SOLUTION I know from Chapter 23 that the electric field produced by a charged plate of infinite area is always perpendicular to the plate (see Figure 23.30). Thus, the field lines must be straight lines perpendicular to the plate. Because the plate is positively charged, I draw the field lines perpendicular to and away from the plate on either side (**Figure 24.4**). ✔

Figure 24.4



Like vector field diagrams, field line diagrams provide an incomplete view of the electric field and are awkward to draw for all but the simplest charge distributions. Both types of diagram are limited by the two-dimensional nature of illustrations. In particular, you should keep in mind that field lines emanate in three dimensions (**Figure 24.5**), not just in the plane of the drawing.

Figure 24.5 Although we generally use two-dimensional representations of field line diagrams, field lines emanate in three dimensions.

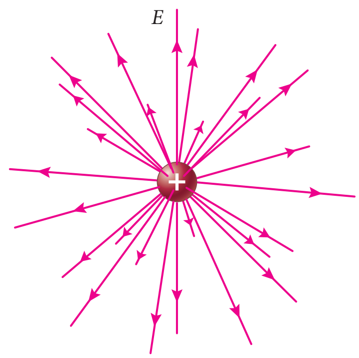
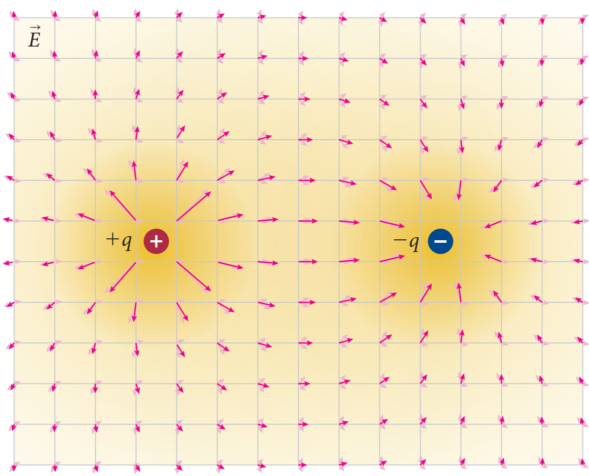
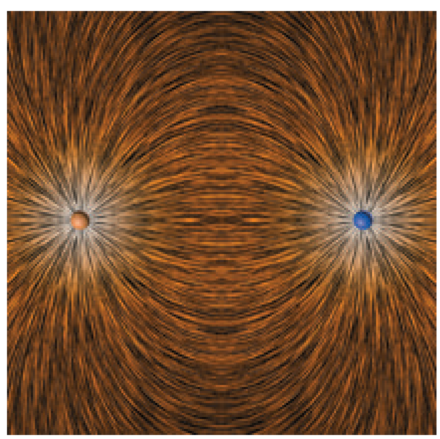


Figure 24.3 Two representations of the electric field of an electric dipole.

(*a*) Vector field diagram of an electric dipole



(*b*) Pattern created by electric dipole in a suspension of plastic fibers that align with the electric field



24.2 (*a*) Is it possible for two electric field lines to cross? (Hint: What is the direction of the electric field at the point of intersection?) (*b*) Can two electric field lines touch?

24.2 Field line density

The most remarkable feature of field lines is this: Even though we take into account only the *direction* of the electric field when drawing field lines, they also contain information about the *magnitude* of the electric field. Figure 24.5 shows that as the distance from the charged object increases, the field lines are spaced farther apart from one another. To see whether there is a quantitative correspondence between field line spacing and electric field magnitude, complete the next checkpoint.

24.3 Imagine a hollow sphere enclosing the charged object in Figure 24.5, centered on the object. (*a*) Given that 26 field lines emanate from the charged object, how many field lines cross the surface of the hollow sphere? (*b*) If the radius of the hollow sphere is R, what is the number of field line crossings per unit surface area? (*c*) Now consider a second sphere with radius $2R$, also centered on the charged object. How many field lines cross the surface of this second sphere? (*d*) How does the number of field line crossings per unit area on the second sphere compare with that on the first sphere? (*e*) How does the electric field at a location on the second sphere compare with the field at a location on the first sphere?

From Checkpoint 24.3, we see that the electric field and the number of field line crossings per unit area both decrease as $1/r^2$. To express this correspondence quantitatively, we define a new quantity, the **field line density:**

The field line density at a given position is the number of field lines per unit area that cross a surface perpendicular to the field lines at that position.

Figure 24.6 illustrates why the surface through which the field lines pass must be perpendicular to the field lines. The field represented by the field lines in the figure is uniform—its magnitude and direction are the same everywhere. As you can see in the figure, the number of field lines that cross the surface depends on the orientation of the surface. The number of field lines that cross the surface is maximum when the surface is perpendicular to the field lines and decreases for any other orientation. We shall see later in this chapter how to account for the orientation of a surface when calculating field line density.

Because the number of field lines in a field line diagram is arbitrary, the field line density is also an arbitrary number, and so you may be wondering why field line density is a useful quantity. As you will see shortly, however, the field line density allows us to draw conclusions about electric field magnitudes. The only condition we make is that, in a given field line diagram, the number of field lines emanating from or terminating on charged objects is proportional to the magnitude of the charge carried by these objects.

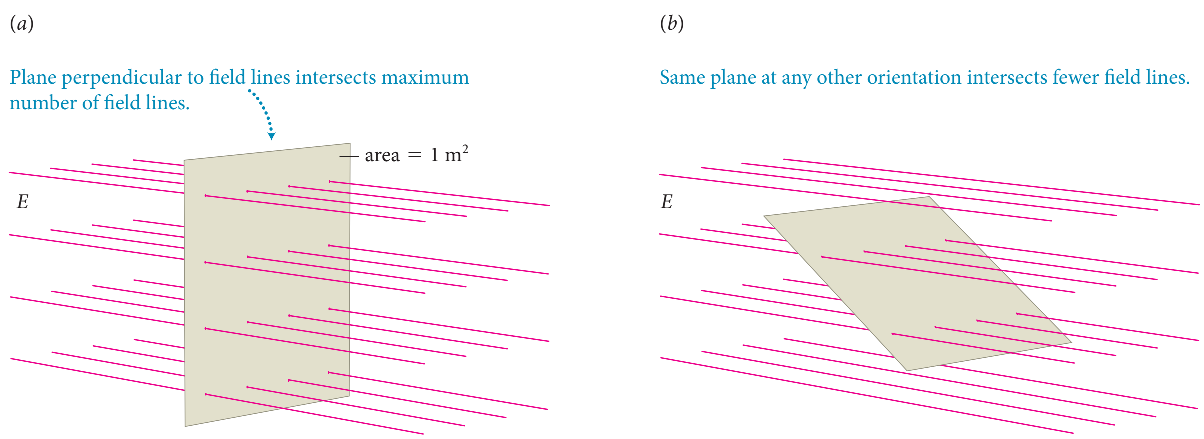
24.4 (*a*) In Figure 24.6, for what orientation is the number of field lines that cross the surface a minimum? (*b*) How many field lines cross a plane surface of area 0.5 m^2 placed perpendicular to the field lines in Figure 24.6? (*c*) Using your answer to part *b*, what is the number of field line crossings *per unit area* through the 0.5-m^2 surface? (*d*) How does this compare to the number of field line crossings per unit area for the 1-m^2 surface in Figure 24.6*a*?

For the spherical surfaces of Checkpoint 24.3, the field lines are all perpendicular to the surface because the field lines are radial. The number of field line crossings you calculated per unit area *is* the field line density. These results lead us to conclude:

At every position in a field line diagram, the magnitude of the electric field is proportional to the field line density at that position.

The box "Properties of electric field lines" on page 643 summarizes the properties of electric field lines.

Figure 24.6 The number of field lines that cross a given surface depends on the orientation of the surface relative to the field lines.



CONCEPTS

Properties of electric field lines

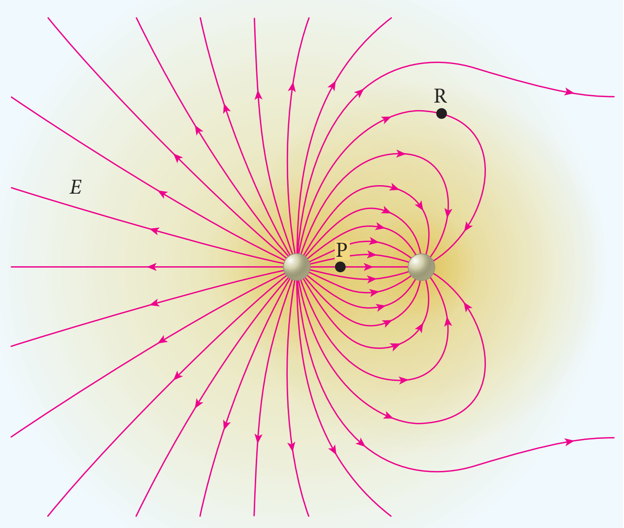
When working with electric field lines, keep the following points in mind:

1. Field lines emanate from positively charged objects and terminate on negatively charged objects.
2. At every position, the direction of the electric field is given by the direction of the tangent to the electric field line through that position.
3. Field lines never intersect or touch.
4. The number of field lines emanating from or terminating on a charged object is proportional to the magnitude of the charge on the object.
5. At every position, the magnitude of the electric field is proportional to the field line density.

Exercise 24.2 Field strength from field lines

Consider the field line diagram shown in **Figure 24.7**. (*a*) What are the signs of the charges on the two small spherical objects? (*b*) What are the relative magnitudes of these charges? (*c*) What is the ratio of the magnitudes of the electric fields at points P and R? (*d*) Is the electric field zero anywhere in the region shown?

Figure 24.7 Exercise 24.2.



SOLUTION (*a*) Because field lines leave the left object and terminate on the right one, the left object carries a positive charge and the right object carries a negative charge. ✔

(*b*) From Figure 24.7 I see that about twice as many field lines leave the left object as end on the right one. Thus, the charge on the left object is about twice that on the right object.* ✔

(*c*) The magnitude of the electric field at each position is proportional to the field line density at that position. The field line density is equal to the number of field lines per unit length, which is proportional to the inverse of the distance between adjacent field lines. Measuring with a ruler, I see that the distances between adjacent field lines at points P and R are $d_P \approx 1$ mm and $d_R \approx 6$ mm, so

$$\frac{E_P}{E_R} = \frac{d_R}{d_P} \approx \frac{6\text{ mm}}{1\text{ mm}} = 6. ✔$$

(*d*) The absence of field lines on the right suggests that the electric field is small (or even zero). Indeed, if a test particle carrying a positive charge is placed in that region, it is subject to a repulsive force exerted by the positively charged object on the left and an attractive force exerted by the negatively charged object on the right. If the test particle is to the right of the particles and $\sqrt{2}$ as far from the positively charged particle as it is from the negatively charged particle, the vector sum of the two forces is zero and so the electric field at that position is zero. ✔

*If you answered that the magnitude of the charge on the left is four times that on the right because in three dimensions there must be four times as many field lines radiating outward from the left object, don't worry—we've hit on one of the shortcomings of field line representations. In general we shall go by the number of dimensions represented in the drawing (in this case, two).

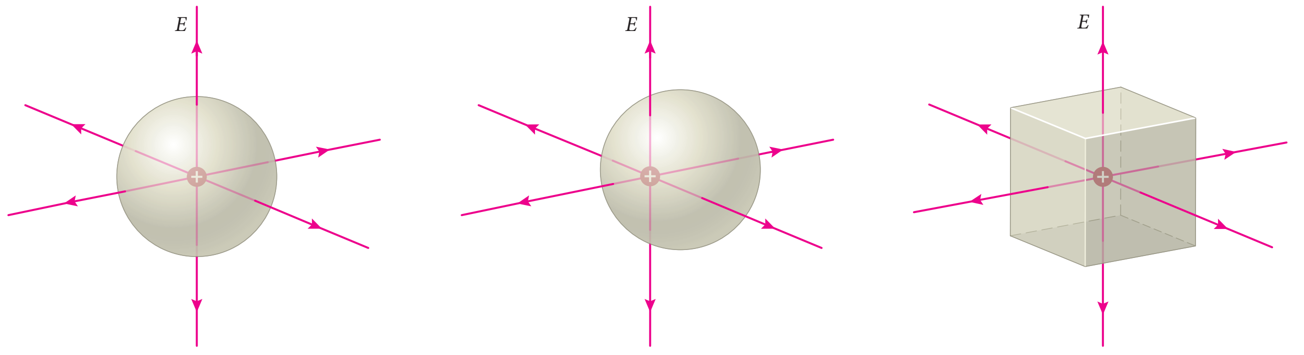
Note that in Exercise 24.2, half of the field lines leave the area of interest. These field lines either eventually terminate on a negatively charged object (not shown) or continue out to "infinity."

24.5 Imagine moving the hollow sphere of radius R of Checkpoint 24.3*a* sideways so that the charged object is no longer at the center of the sphere (but still within it). (*a*) How does the number of field line crossings through the surface of the sphere change as it is moved? (*b*) How does the average number of field line crossings per unit surface area of the sphere change? (*c*) Does the electric field at a fixed position on the surface of the sphere change or remain the same as the sphere is moved? (*d*) Are your answers to parts *b* and *c* in contradiction, given the relationship between the electric field magnitude and field line crossings per unit area?

24.3 Closed surfaces

Checkpoint 24.5 leads us to another result that will be important in deriving Gauss's law: Whenever a charged particle is placed inside a hollow spherical surface, the number of field lines that pierce the surface is the same *regardless of where inside the surface the particle is placed.* This is true simply because so long as the charged particle is inside the surface, all the field lines emanating from the particle must go through the spherical surface. In fact, we don't even need to use a spherical surface—a

Figure 24.8 Any surface that encloses a positively charged particle is pierced by all the field lines that emanate from that particle, regardless of the shape of the surface and the position of the particle within the surface.



cube-shaped surface or any other surface enclosing the charged particle will do (**Figure 24.8**). In each case, the number of field lines that pierce the surface is equal to the number of field lines that emanate from or terminate on the charged particle enclosed by the surface.

> **24.6** Suppose eight field lines emanate from an object carrying a charge $+q$. How many field lines pierce the surface of a hollow sphere if the sphere contains (*a*) a single object carrying a charge $+2q$ and (*b*) two separate objects, each carrying a charge $+q$? (*c*) If the sphere is pierced by 20 field lines, what can you deduce about the combined charge on objects inside the sphere?

A surface that completely encloses a volume is called a **closed surface**. Checkpoint 24.6 suggests that a direct relationship exists between the number of field lines that cross a closed surface and the **enclosed charge**—the sum of all charge enclosed by that surface. However, what happens if a field line reenters the closed surface, as illustrated in **Figure 24.9*a***? Field line 4 now crosses the closed surface *three* times, so the number of field line crossings is not six but eight. If you look closely at the figure, however, you will discover that not all the crossings are the same. For seven of the crossings the field line goes outward (from the inside of the closed surface to the outside), while for the eighth crossing the field line goes inward. If we assign a value of $+1$ to each outward crossing and a value of -1 to each inward crossing, we obtain $(+7) + (-1) = 6$.

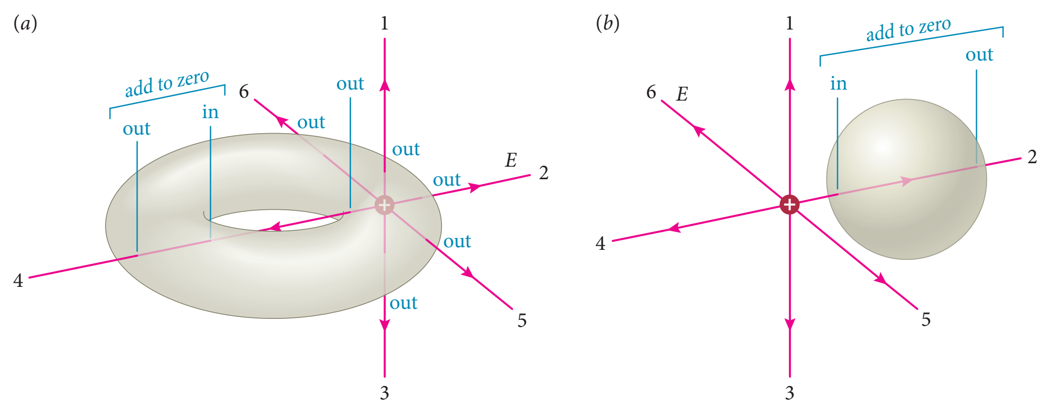
To keep track of the number of inward and outward field crossings, we define a new quantity called the *field line flux*:

For any closed surface, the *field line flux* is the number of outward field lines crossing the surface minus the number of inward field lines crossing the surface.

In calculating the field line flux for any closed surface, we assign a value of $+1$ to each outward field line crossing the surface and a value of -1 to each inward field line crossing the surface

> **24.7** (*a*) If more than one field line reenters the donut in Figure 24.9*a*, what happens to the field line flux? (*b*) Are there any closed surfaces enclosing a charged particle through which the field line flux is different from that through a simple sphere around that particle?

Figure 24.9 The number of field lines exiting a closed surface minus the number entering it is always equal to the number of field lines generated inside the surface.



The field line flux through any closed surface is always equal to the number of field lines that originate from within that surface minus the number of field lines that terminate on charged objects within that surface. So far, we have drawn an arbitrary number of field lines, but we can make our statement more precise:

The *field line flux* through a closed surface is equal to the charge enclosed by the surface multiplied by the number of field lines per unit charge.

What about charged objects outside the closed surface? To see what effect such charged objects have, complete the following checkpoint.

24.8 (*a*) What is the field line flux through the closed spherical surface in Figure 24.9*b* due to a charged particle outside the sphere? (*b*) Does your answer to part *a* change if we move the particle around (but keep it outside the volume enclosed by the surface)?

Checkpoint 24.8 demonstrates a very important point:

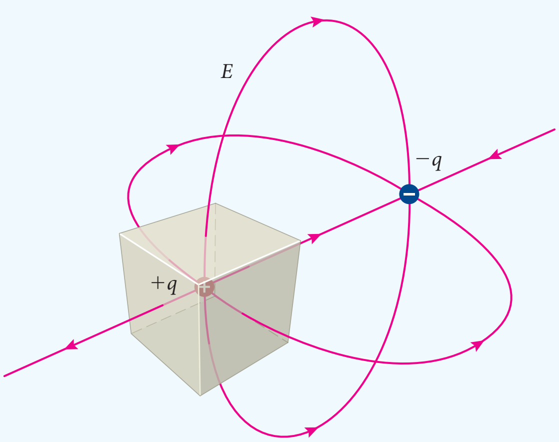
The field line flux through a closed surface due to charged objects outside the volume enclosed by that surface is always zero.

This means that if we know the field line flux through a closed surface, then we can determine the charge enclosed by that surface, regardless of the distribution of charge outside the surface. This statement is a form of Gauss's law, which we shall describe mathematically in Section 24.7.

Example 24.3 Flux of an electric dipole

Consider the three-dimensional dipole field line diagram shown in **Figure 24.10**. Six field lines emanate from the positively charged end, and six terminate on the negatively charged end. (*a*) What is the field line flux through the surface of the cube that encloses the positively charged end shown in the figure? (*b*) What is the field line flux through the surface of a similar cube that encloses the negatively charged end?

Figure 24.10 Example 24.3.



SOLUTION (*a*) Six field lines emanate from the positively charged particle. Each line crosses the surface of the cube in the outward direction and thus contributes a value of +1 to the field line flux. The field line flux is +6. ✔

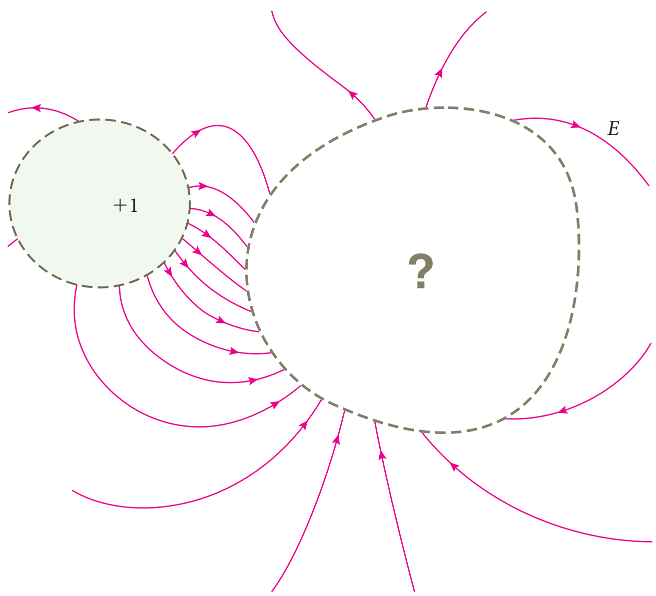
(*b*) Six field lines terminate on the negatively charged particle, so there are again six field line crossings. However, these field lines are directed inward and so the field line flux is −6. ✔

24.9 What is the field line flux through the surface of a rectangular box that encloses *both* ends of an electric dipole?

The relationship between the field line flux through a closed surface and the enclosed charge is important because it can help us determine one from a knowledge of the other. For example, in the next section we shall use this relationship to derive two important theorems about isolated conducting objects.

24.10 Consider the two-dimensional field line diagram in **Figure 24.11**, part of which is hidden from view. (*a*) If the object in the top left carries a charge of +1 C, what is the charge enclosed in the region that is hidden? (*b*) What is the field line flux through a surface that encloses the entire area represented by the diagram?

Figure 24.11 Checkpoint 24.10: What is the charge inside the dashed region?

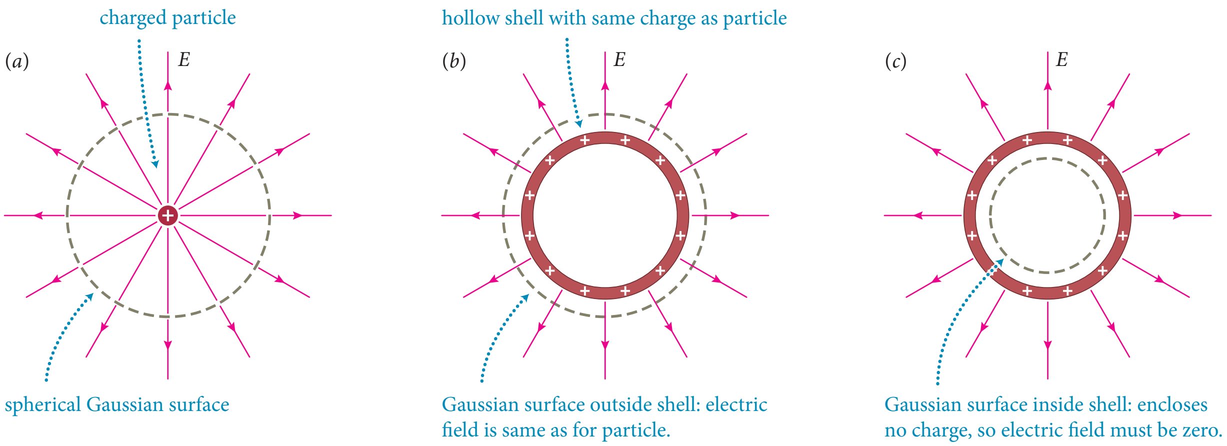


24.4 Symmetry and Gaussian surfaces

The relationship between field line flux and enclosed charge allows us to reach several important conclusions about charged objects and their electric fields without having to do

CONCEPTS

Figure 24.12 Using spherical Gaussian surfaces to examine the electric fields of a charged particle and a uniformly charged spherical shell. The electric fields, Gaussian surfaces, and charged shell are spherical and are shown here in cross section.



any calculations. To apply this relationship in a given situation, we first need to select a closed surface. This surface need not correspond to a real object—any surface, real or imagined, will do. We'll refer to these closed surfaces as **Gaussian surfaces.** The choice of surface is dictated by the symmetry of the situation at hand. As a rule of thumb, we choose a surface such that the electric field is the same (and possibly zero) everywhere along as many regions of the surface as possible, because such a choice makes it easy to determine the field line flux through the surface.

Consider, for example, the charged particle shown in **Figure 24.12*a***. As we have seen, the field lines for the particle radiate outward from it. (The figure shows only a two-dimensional cross section of the three-dimensional situation.) The field is symmetrical in all three dimensions—it has the same magnitude at the same distance from the center in any direction. Therefore, if we draw a spherical Gaussian surface that is concentric with the particle, the magnitude of the electric field is the same at all locations on the sphere. In other words, the field line density is the same all over the surface of the sphere. As we have seen in the preceding section, the field line flux through the Gaussian surface is proportional to the charge enclosed by the sphere.

Now suppose we replace the charged particle by a spherical shell that carries the same charge as the particle and still fits within our Gaussian surface (Figure 24.12*b*). If the charge is uniformly distributed over the shell, then the electric field should still be the same in all directions. Like the charged particle, the charged shell has *spherical symmetry* (see the box "Symmetry and Gauss's law" on page 647): Reorienting the spherical shell by rotating it over an angle about any axis does not change the charge configuration and so should not change the electric field at a given location. This means that the field lines should again be straight lines radiating uniformly outward. Also, the field line flux through the Gaussian surface should still be the same because the surface encloses the same amount of charge. The only way that the field line fluxes through the Gaussian surfaces in Figures 24.12*a* and *b* can be uniform *and* equal in magnitude is if the electric fields are the same at every position on the spherical Gaussian surface. Because this argument holds for a spherical Gaussian surface of any radius, we can conclude:

The electric field outside a uniformly charged spherical shell is the same as the electric field due to a particle that carries an equal charge located at the center of the shell.

This means that a uniformly charged shell exerts a electric force on a charged particle outside the shell as if all the shell's charge were concentrated at the center of the shell. Because a sphere may be viewed as a collection of shells, we can extend this statement to uniformly charged spheres.

Let us now turn our attention to the electric field in the space enclosed by the shell. We draw a spherical Gaussian surface that fits within the shell (Figure 24.12*c*). This Gaussian surface encloses no charge, so the field line flux through the Gaussian surface is zero. Because the electric field can only be radially outward by symmetry, the electric field must be zero everywhere on the Gaussian surface. Because we can vary the radius of the Gaussian surface from zero to the inner radius of the shell without changing this argument, we can conclude:

In the absence of other charged objects, the electric field in the space enclosed by a uniformly charged spherical shell is zero everywhere in the enclosed space.

Physically this means that a uniformly charged shell exerts no electric force on a charged particle located inside the shell.

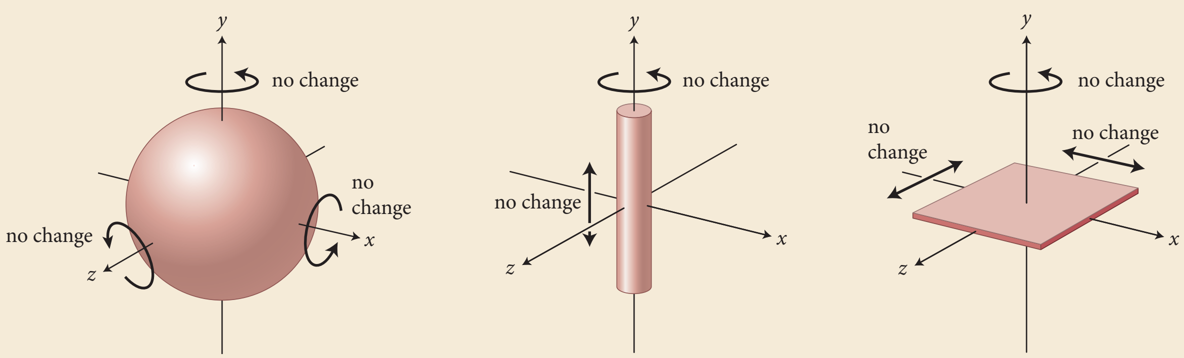
Symmetry and Gauss's law

The symmetry of an object is determined by its *symmetry operations*—manipulations that leave its appearance unchanged (see Section 1.2). A sphere, for example, looks the same if we reorient it by rotating it about any axis (**Figure 24.13*a***). This type of symmetry is called **spherical symmetry.** An infinitely long, cylindrical rod does not look any different if we rotate it, reverse it, or translate it about its long axis (Figure 24.13*b*). The rod is said to have **cylindrical symmetry.** An infinite flat sheet has **planar symmetry:** It remains unchanged if it is rotated about an axis perpendicular to the sheet or translated along either of the two axes perpendicular to this axis (Figure 24.13*c*).

Many other types of symmetry may occur, but these three types play an important role in electrostatics. For charge configurations that exhibit any of these three symmetries, we can calculate the electric field due to the charge distribution directly using Gauss's law.

Because objects are never infinite, they cannot exhibit true cylindrical or planar symmetry. However, for a long straight wire or a large flat sheet we can often obtain good results by assuming they have cylindrical or planar symmetry. When we work problems, the words *long* and *large* imply that you may assume the object has infinite dimensions compared to other length scales of interest.

Figure 24.13 Three symmetries important for applications of Gauss's law.

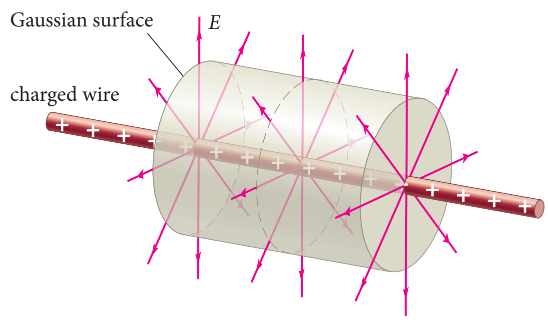


24.11 There are two reasons the field line flux through a closed surface may be zero: because the field is zero everywhere or because the outward flux is balanced by an equal inward flux. Why can't the latter situation be true for the Gaussian surface in Figure 24.12*c*?

Particles, shells, and spheres are the only objects that exhibit spherical symmetry. **Figure 24.14** illustrates a different type of symmetry: the *cylindrical symmetry* of an infinitely long, uniformly charged straight wire.* Because of this symmetry, rotating the wire about its axis or moving it along the axis should not have any effect on the electric field at any position in space. For this to be the case, the field lines must be arranged radially along planes that are perpendicular to the wire (Figure 24.14). We can take advantage of this symmetry by drawing a Gaussian surface in the shape of a cylinder that is concentric with the wire, as shown in Figure 24.14.

*For the wire to exhibit cylindrical symmetry, it has to be infinitely long. If the wire has finite length, you can tell when it is moved along its axis.

Figure 24.14 The electric field of an infinite uniformly charged wire exhibits cylindrical symmetry. We can examine this field by surrounding the wire with a concentric cylindrical Gaussian surface.

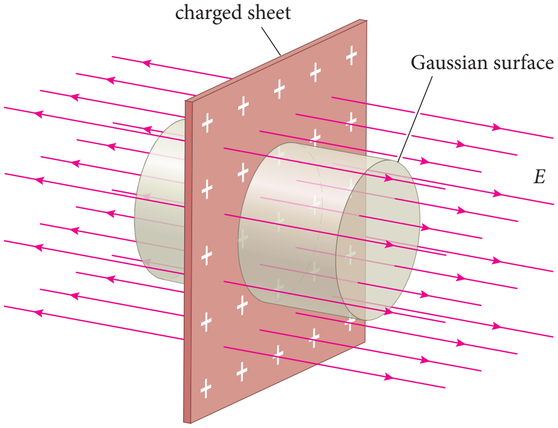


24.12 Consider a point on the curved part of the Gaussian surface in Figure 24.14. Does the magnitude of the electric field at that point increase, decrease, or stay the same if you (*a*) change the location of the point on the curved surface or (*b*) increase the radius of the Gaussian surface? (*c*) What is the field line flux through the left and right surfaces of the Gaussian surface?

We can use the cylindrical Gaussian surface to determine how the electric field due to the charged wire decreases with

CONCEPTS

Figure 24.15 The electric field of a uniformly charged sheet exhibits planar symmetry. We examine its field by drawing a cylindrical Gaussian surface that straddles the sheet.



distance from the wire. As you can see from the figure, the field line flux through the curved surface of the cylinder is independent of its radius r. No matter how large we make the radius of the cylinder, the same number of field lines pass through it. The area A of the curved surface is equal to the perimeter times the height h of the cylinder: $A = 2\pi rh$. As we increase the radius r of the cylinder, the surface area increases proportionally to r, and so the field line density must decrease as $1/r$ to maintain a constant number of field lines. Because the field line density is a measure of the electric field strength, this means that the electric field due to the wire decreases as $1/r$, as we established in Example 23.4. A quantitative expression for the electric field due to a charged rod is given in Section 24.8.

Figure 24.15 shows a situation with a different symmetry: a charged sheet. If the sheet is very large and the charge is uniformly distributed along it, then the electric field lines must be perpendicular to the sheet and also uniformly distributed along it. The one-dimensional symmetry exhibited by the electric field due to the charged sheet is an example of *planar symmetry*. To take advantage of this symmetry, we draw a cylindrical Gaussian surface that straddles the sheet, as illustrated in Figure 24.15.

CONCEPTS

24.13 Consider a point on the right surface of the Gaussian surface in Figure 24.15. Does the magnitude of the electric field at that point increase, decrease, or stay the same if you (*a*) change the location of the point on the right surface, or (*b*) increase the height h of the Gaussian surface? (*c*) Is the field line flux through the right surface of the Gaussian surface positive, negative, or zero? (*d*) How does the field line flux through the right surface compare to that through the left surface? (*e*) Is the field line flux through the curved surface of the Gaussian surface positive, negative, or zero?

Because the area of the right surface and the field line flux through that surface don't change as we change the height of the cylinder, we conclude that the electric field line density doesn't change with distance to the plane. Hence the magnitude of the electric field due to the charged sheet is the same everywhere (see also Example 23.6).

24.5 Charged conducting objects

Let as now apply the relationship between field line flux and enclosed charge to charged conducting objects. As we saw in Chapter 22, conducting materials permit the free flow of charge carriers within the bulk of the material. Conducting objects typically contain many charge carriers that are free to move, such as electrons (in a metal) or ions (in a liquid conductor). The material as a whole can still be electrically neutral; a neutral piece of metal, for example, contains as many positively charged protons as negatively charged electrons.

A consequence of this free motion of charged particles within a conducting object is that the particles always arrange themselves in such a way as to make the electric field inside the bulk of the object zero. To see how this comes about, consider a free electron in a slab of metal. If no field is present, no electric force is exerted on the electron. If we apply an external field, however, the free electron is subject to a force in a direction opposite the direction of the electric field (opposite because of the negative charge of the electron).

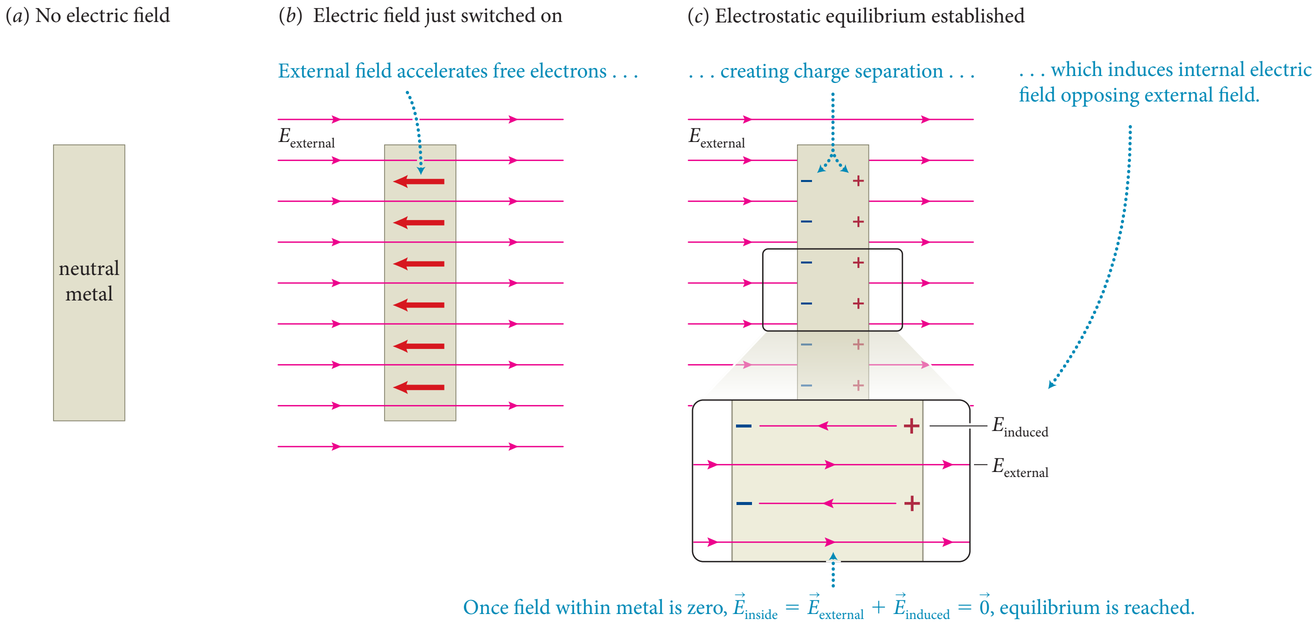
In a similar way, all the free electrons in a slab of metal initially accelerate in a direction opposite the direction of an applied field (**Figure 24.16**). This leaves behind a positive charge on one side of the slab and creates a negative charge on the opposite side. Because of this rearrangement of charge, an induced electric field builds up in a direction opposite the direction of the external field. As a result, the electric field inside the slab, which is the sum of the external electric field and the induced electric field, decreases. As this field decreases, so does the force exerted on the free electrons in the slab. When enough charge carriers have accumulated on each side of the slab to make the electric field inside the slab zero, the electric force exerted on the free electrons in the metal becomes zero and the material reaches **electrostatic equilibrium**—the condition in which the distribution of charge in a system does not change. The time interval it takes for a metal to reach electrostatic equilibrium is very short (about 10^{-16} s), so the rearrangement of charge carriers is virtually instantaneous. The important point to remember is:

The electric field inside a conducting object that is in electrostatic equilibrium is zero.

Keep in mind that this statement holds *only* in electrostatic equilibrium. When charge carriers are made to flow through a conducting object—as in any electric or electronic device, like your stereo or a refrigerator—the electric field is *not* zero inside the object!

Suppose now we add charge to a conducting object. It makes sense to assume that the charged particles will

Figure 24.16 Why the electric field inside the bulk of a conducting object is zero when the object is in electrostatic equilibrium.



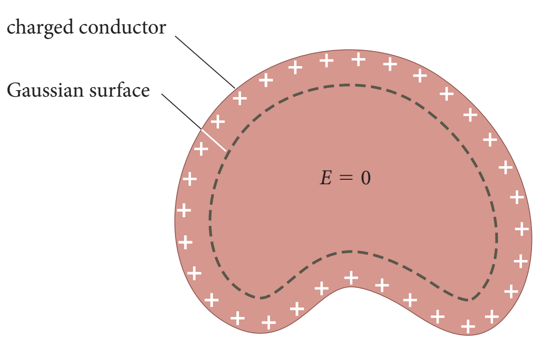
arrange themselves over the object in such a way as to spread out as far as possible from one another, given that particles carrying like charges repel. We can use a Gaussian surface to obtain a better understanding of where the charged particles go.

24.14 Consider a spherical Gaussian surface inside a positively charged conducting object that has reached electrostatic equilibrium. (*a*) Is the field line flux through the Gaussian surface positive, negative, or zero? (*b*) What can you conclude from your answer to part *a* about the charge enclosed by the Gaussian surface?

We can extend the result of Checkpoint 24.14 to conducting objects of any shape. Consider, for example, the irregularly shaped, charged conducting object shown in **Figure 24.17**. Draw a Gaussian surface of the same shape as the object, just below its surface. Given that the field is zero everywhere inside the conducting object, the field line flux through the Gaussian surface is zero and the charge enclosed by the Gaussian surface is also zero. Because we can choose the Gaussian surface arbitrarily close to the surface of the object, we conclude:

Any surplus charge placed on an isolated conducting object arranges itself at the surface of the object. No surplus charge remains in the body of the conducting object once it has reached electrostatic equilibrium.

Figure 24.17 Because the electric field inside a conducting object in electrostatic equilibrium is zero, we conclude that there cannot be any surplus charge inside the object.



24.15 Suppose the charged conducting object in Figure 24.17 contains an empty cavity. Does any surplus charge reside on the inner surface of the cavity?

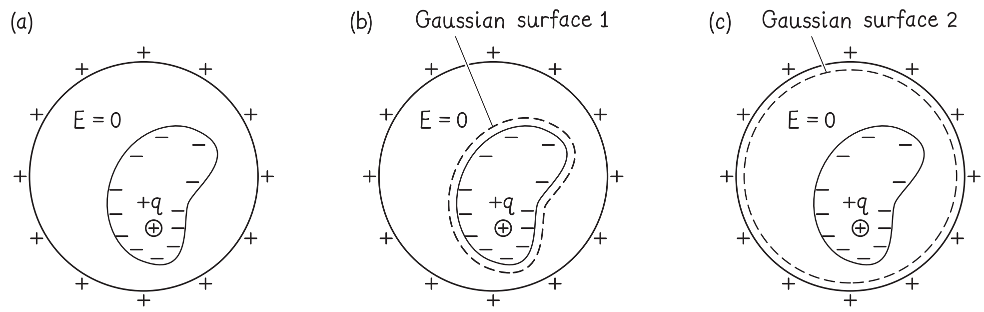
Example 24.4 Charged particle in a cavity

An electrically neutral, conducting sphere contains an irregularly shaped cavity. Inside the cavity is a particle carrying a positive charge $+q$. What are the sign and magnitude of the charge on the sphere's outer surface?

❶ GETTING STARTED I am told that a sphere made of material that is an electrical conductor has a cavity in its interior and that a particle in the cavity carries charge $+q$. My task is to determine the sign and magnitude of any charge residing on the sphere's outer surface. I begin by sketching a vertical cross section through the sphere showing the cavity and the charged particle inside it (**Figure 24.18*a*** on the next page). The problem states that the sphere is electrically neutral, but the question posed implies that some charge resides on its outer surface. Because $\vec{E} = \vec{0}$ inside the conducting material, I know that an equal quantity of the opposite charge must accumulate somewhere else on the sphere.

CONCEPTS

Figure 24.18



❷ **DEVISE PLAN** The sphere is conducting, so I know that once electrostatic equilibrium is reached, the electric field inside the bulk of the sphere must be zero: $\vec{E} = \vec{0}$. I can use this information to draw any Gaussian surface inside the sphere and use the following reasoning to determine the charge enclosed by my Gaussian surface: Because $\vec{E} = \vec{0}$ inside the bulk of the sphere and because I draw my Gaussian surface inside the sphere, $\vec{E} = \vec{0}$ everywhere on the Gaussian surface. Therefore the field line flux through the Gaussian surface is zero, which means the charge enclosed by this surface must be zero. Because I can draw my Gaussian surface anywhere inside the bulk of the conductor, I can use this information to determine the distribution of charge on the sphere.

❸ **EXECUTE PLAN** I begin by drawing a Gaussian surface 1 enclosing the cavity (Figure 24.18*b*). Because the field line flux through this surface is zero, the charge enclosed by the surface must be zero. There is charge $+q$ inside the surface (in the charged particle), however, and so in order for the charge enclosed by the surface to be zero, a quantity of charge $-q$ must have migrated from someplace in the region surrounding the cavity and accumulated on the inner cavity surface.

Because the sphere is electrically neutral, the charge $-q$ that migrated to the inner cavity surface must leave a charge $+q$ behind somewhere else on the sphere. If I now draw Gaussian surface 2 just inside the sphere's outer surface (Figure 24.18*c*), I see that, because the field line flux through this Gaussian surface has to be zero, the charge enclosed by this surface is also zero. The positive charge $+q$ that results from the migration of charge $-q$ to the cavity surface must therefore reside outside Gaussian surface 2. Because I can draw surface 2 arbitrarily close to the sphere's outer surface, I conclude that the sphere's outer surface carries a charge $+q$. ✔

❹ **EVALUATE RESULT** The negative charge that migrates from the region outside the cavity to the cavity surface arranges itself in such a way as to cancel the electric field that the charged particle creates in the region outside the cavity. Therefore all the field lines that start on the charged particle must end on the negative charge at the cavity surface. In order for all the field lines to end here, the quantity of negative charge on the cavity surface must be equal to the quantity of charge on the particle. The sphere is electrically neutral, and my choice for where I draw Gaussian surface 2 requires that all the positive charge resulting from the migration of negative charge to the cavity surface must accumulate outside Gaussian surface 2, which means right at the sphere's outer surface. Thus my answer makes sense.

That the electric field inside any conductor is zero in electrostatic equilibrium allows us to draw one additional important conclusion. Because the electric fields must be zero everywhere, including at the surface of a conducting object, there cannot be any component of the electric field parallel to the surface of the object, and therefore we can conclude:

In electrostatic equilibrium, the electric field at the surface of a conducting object is perpendicular to that surface.

If there were a component of the electric field parallel to the surface, that component would cause any free charge carrier to move along the surface, which means the conductor is not in electrostatic equilibrium.

24.16 In Example 24.4, is the electric field inside the cavity zero?

CONCEPTS

Self-quiz

(For this self-quiz assume all situations are two-dimensional.)

1. In **Figure 24.19**, which of the two charged spheres carries a charge of greater magnitude?
2. Consider Gaussian surfaces 1–3 in Figure 24.19. Determine the field line flux through each surface.
3. In Figure 24.19, is the field line density greater at point A or point B? At which of these locations is the magnitude of the electric field greater? Is the field line density at point C zero or nonzero?
4. The electric field lines in **Figure 24.20** tell you there must be one or more charged particles inside the Gaussian surface defined by the dashed line. Could the electric field shown be due to a single particle inside the Gaussian surface? What must the signs and relative magnitudes of the charged particle(s) be in order to create the electric field lines shown?
5. **Figure 24.21** shows a small ball that carries a charge of $+q$ inside a conducting metal shell that carries a charge of $+2q$. (*a*) What are the sign and magnitude of the charge on the inner surface of the shell? (*b*) What are the sign and magnitude of the charge on the outer surface of the shell?

Figure 24.19

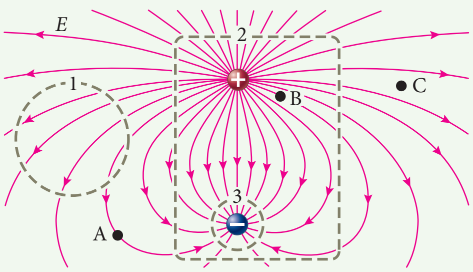


Figure 24.20

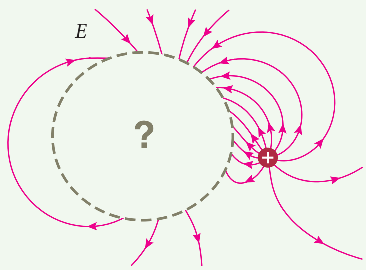
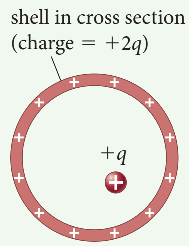


Figure 24.21



Answers

1. The number of field lines is proportional to the charge on the object. Because more lines emanate from the charged object on the top, that object must carry a greater charge.
2. For surface 1, all lines that enter the surface also exit the surface, so the field line flux is zero. For surface 2, 25 lines exit the surface and 6 lines enter the surface. The field line flux through surface 2 is thus $25 - 6 = 19$. Fifteen field lines cross surface 3, with all lines entering from the outside. The field line flux for surface 3 is -15.
3. Point B has the greater field line density because the lines are closer together at B than they are at A. The magnitude of the electric field is greater at point B because electric field strength is proportional to the field line density. Even though C is not on a field line, the field line density, which is represented by the spacing of the field lines *around* point C, is nonzero.
4. Because electric field lines converge on one point near the top of the area enclosed by the surface and diverge from a point near the bottom, there must be objects that carry both negative and positive charges inside the surface. Because more field lines enter the surface than exit the surface, the negatively charged object(s) must carry a charge of greater magnitude than the positively charged object(s).
5. (*a*) Because the electric field in the conducting shell is zero, the field line flux through a Gaussian surface drawn inside the material of the conducting shell must be zero. According to Gauss's law, the charge enclosed in the surface must also be zero. The charge on the inner surface of the shell must therefore be $-q$, which added to $+q$ gives zero. (*b*) For a neutral shell, a charge of $-q$ on the inside surface of the shell would leave a surplus of $+q$ on the outside surface of the shell. The surplus charge of $+2q$ that was placed on the shell also resides on the outside surface. The charge residing on the outside surface of the shell is thus $+3q$.

24.6 Electric flux

We introduced two important concepts in the first part of this chapter: the field line density, which is proportional to the strength of the electric field, and the field line flux, which represents the number of field lines going outward through a closed surface minus the number of field lines going inward. In this section we'll turn these concepts into quantities we can calculate.

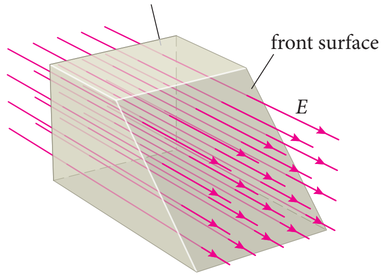
Consider, for example, a trapezoidal box in a uniform electric field $\vec{E}$ (**Figure 24.22*a***). The field line flux through the closed surface of the trapezoidal box is zero: Twenty field lines go into the back surface and 20 come out through the front surface. Another way of putting this is to say that the field line flux into the back surface is equal in magnitude to the field line flux out of the front surface. Instead of using field lines, however, whose number is chosen arbitrarily, we'll work with a quantity called the **electric flux,** represented by the symbol Φ_E (Φ is the Greek capital phi). The magnitude of the electric flux through a surface with area A in a uniform electric field of magnitude E is defined as

$$\Phi_E \equiv EA\cos\theta \quad \text{(uniform electric field)}, \tag{24.1}$$

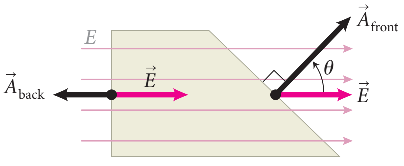
where θ is the angle between the electric field and the normal to the surface.

Figure 24.22 Determining the electric flux through a flat surface.

(*a*) Trapezoidal box in uniform electric field



(*b*) Vector areas of front and back surfaces



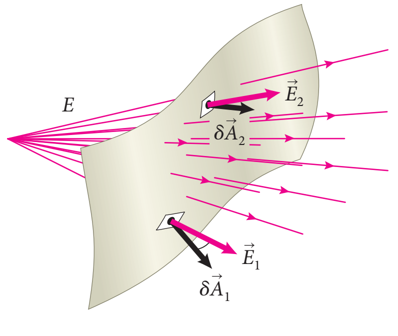
24.17 (*a*) Consider the front surface of the trapezoidal box in Figure 24.22*a*, detached from the rest of the trapezoidal box. Does the field line flux through that surface increase, decrease, or stay the same if any of the following quantities is increased: (*i*) the area of the front surface, (*ii*) the magnitude of the electric field (keeping the area constant), (*iii*) the slope of the front surface (that is to say, the angle between the surface and the direction of the electric field is increased)? (*b*) Does the *electric flux* through the front surface increase, decrease, or stay the same if any of these quantities is changed?

As Checkpoint 24.17 shows, electric flux, as we've defined it in Eq. 24.1, behaves like the field line flux. To make the correspondence more precise, we define an *area vector* $\vec{A}$ for a flat surface area as a vector whose magnitude is equal to the surface area A and whose direction is normal to the plane of the area. On closed surfaces we choose $\vec{A}$ to point outward (we'll deal with open surfaces later). Figure 24.22*b* shows the area vectors associated with the front and back surfaces of the (closed) trapezoidal box. With this definition, Eq. 24.1 can be written as a scalar product (Eq. 10.33):

$$\Phi_E \equiv EA\cos\theta = \vec{E}\cdot\vec{A} \quad \text{(uniform electric field)}, \tag{24.2}$$

where θ is the angle between $\vec{E}$ and $\vec{A}$. Electric flux is a scalar, and the SI unit of electric flux is $\text{N}\cdot\text{m}^2/\text{C}$.

Figure 24.23 To obtain the electric flux through an irregularly shaped, nonplanar surface and/or for a nonuniform electric field, we divide the surface into small segments. For very small segments, each segment is essentially flat and the field through each segment is essentially uniform. The flux through the entire surface is then given by the sum of all of the contributions through each segment.



24.18 Let the area of the back surface of the trapezoidal box in Figure 24.22 be $1.0\ \text{m}^2$, the magnitude of the electric field be $1.0\ \text{N/C}$, and $\theta = 30°$ for the front surface. (*a*) What are the magnitudes of the area vectors for the front and back surfaces of the trapezoidal box? (*b*) What are the electric fluxes through the front and back surfaces?

The above definition of electric flux applies only for uniform electric fields and flat surfaces. Let us therefore consider the more general case of an irregular surface in a nonuniform field (**Figure 24.23**). To calculate the electric flux through that surface, we divide the entire surface into small segments of surface area δA_i, with each segment being small enough that we can consider it to be essentially flat, and we can define an area vector $\delta\vec{A}_i$ whose magnitude is equal to the surface area δA_i of the segment and whose direction is normal to the segment. This allows us to apply Eq. 24.2 to each individual segment. The electric

QUANTITATIVE TOOLS

flux through a single segment is then $\Phi_{Ei} = \vec{E}_i \cdot \delta\vec{A}_i$, where $\vec{E}_i$ is the electric field vector at the location of the segment. To calculate the electric flux through the entire surface we must sum the electric flux through all the surface segments:

$$\Phi_E = \Sigma \vec{E}_i \cdot \delta\vec{A}_i. \quad (24.3)$$

If we let the area of each segment approach zero, then the number of segments approaches infinity and the sum is replaced by an integral:

$$\Phi_E = \lim_{\delta A_i \to 0} \Sigma \vec{E}_i \cdot \delta\vec{A}_i = \int \vec{E} \cdot d\vec{A}. \quad (24.4)$$

The integral in Eq. 24.4 is called a *surface integral* (see Appendix B); $d\vec{A}$ is the area vector of an infinitesimally small surface segment. If the surface is closed, this surface integral is written as

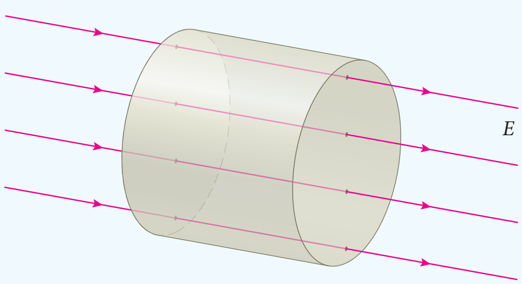
$$\Phi_E = \oint \vec{E} \cdot d\vec{A}, \quad (24.5)$$

where the circle through the integral sign indicates that the integration is to be taken over the entire closed surface and $d\vec{A}$ is chosen to point outward. Because evaluating a surface integral is mathematically more complicated than single-variable integration, it is important to exploit any symmetry to simplify the calculation.

Example 24.5 Cylindrical Gaussian surface in a uniform electric field

Consider a cylindrical Gaussian surface of radius r and length ℓ in a uniform electric field $\vec{E}$, with the length axis of the cylinder parallel to the electric field (**Figure 24.24**). What is the electric flux Φ_E through this Gaussian surface?

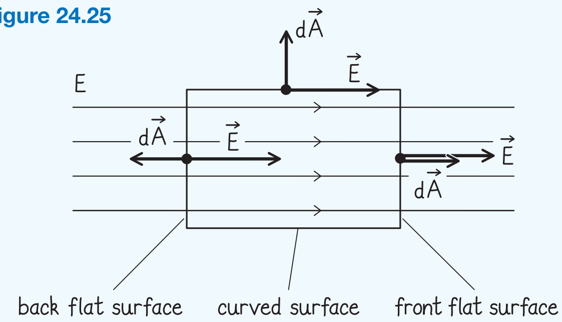
Figure 24.24 Example 24.5.



❶ GETTING STARTED From Figure 24.24 I see that a cylindrical Gaussian surface consists of three regions: front and back flat surfaces and a curved surface joining them. The electric field is perpendicular to the front and back flat surfaces and parallel to the curved surface.

❷ DEVISE PLAN The electric flux is given by Eq. 24.5, so I can calculate the electric flux through the Gaussian surface by applying Eq. 24.5 to each of the three regions and then summing the three contributions to the electric flux. In order to evaluate the scalar product $\vec{E} \cdot d\vec{A}$ for each region, I sketch a side view of the Gaussian surface showing the vectors $\vec{E}$ and $d\vec{A}$ for each of the three regions (**Figure 24.25**).

Figure 24.25



❸ EXECUTE PLAN Applying Eq. 24.5 to the three regions, I can write the flux through the Gaussian surface as the sum of three surface integrals: one over the back surface, one over the curved surface, and one over the front surface:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \int_{\substack{\text{back flat}\\\text{surface}}} \vec{E} \cdot d\vec{A} + \int_{\substack{\text{curved}\\\text{surface}}} \vec{E} \cdot d\vec{A} + \int_{\substack{\text{front flat}\\\text{surface}}} \vec{E} \cdot d\vec{A}. \quad (1)$$

From my sketch, I see that on the back flat surface the angle between $\vec{E}$ and $d\vec{A}$ is 180°, so $\vec{E} \cdot d\vec{A} = E(\cos 180°)\,dA = -E\,dA$. Because the magnitude of the electric field is the same everywhere, I can pull E out of the integral:

$$\int_{\substack{\text{back flat}\\\text{surface}}} \vec{E} \cdot d\vec{A} = \int (-E)\,dA = -E\int dA = -E(\pi r^2),$$

where πr^2 is the area of the back flat surface.

(Continued)

QUANTITATIVE TOOLS

The integral over the curved region of the Gaussian surface yields a value of zero because the angle between $\vec{E}$ and $d\vec{A}$ is 90° everywhere on the curved region, so $\vec{E} \cdot d\vec{A} = E(\cos 90°)\, dA = 0$.

Finally, for the front flat surface I have $\vec{E} \cdot d\vec{A} = E(\cos 0°)\, dA = E\, dA$, so

$$\int_{\text{front flat surface}} \vec{E} \cdot d\vec{A} = \int E\, dA = E \int dA = E(\pi r^2).$$

Adding up the three terms in Eq. 1 yields

$$\Phi_E = -E(\pi r^2) + 0 + E(\pi r^2) = 0. ✔$$

❹ **EVALUATE RESULT** Because there is no charge enclosed by the Gaussian surface, I know that the field line flux through the surface must be zero, a fact I confirm by looking at Figure 24.24: The four field lines shown contribute a flux of -4 on the back flat surface, $+4$ on the front flat surface, and 0 along the curved surface, for a total of $-4 + 4 + 0 = 0$. It therefore makes sense that the electric flux through this Gaussian surface also is zero.

24.19 Consider a spherical Gaussian surface of radius r with a particle that carries a charge $+q$ at its center. (*a*) What is the magnitude of the electric field due to the particle at the Gaussian surface? (*b*) What is the electric flux through the sphere due to the charged particle? (*c*) Combining your answers to parts *a* and *b*, what is the relationship between the electric flux through the sphere and the enclosed charge q_{enc}? (*d*) Would this relationship change if you doubled the radius r of the sphere?

24.7 Deriving Gauss's Law

Checkpoint 24.19 shows that the electric flux through a spherical Gaussian surface is equal to the charge q enclosed by the sphere times $4\pi k$, where $k = 9.0 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2$ is the proportionality constant that appears in Coulomb's law (see Eqs. 22.1 and 22.5). This relationship is usually written in the form

$$\Phi_E = 4\pi k q = \frac{q}{\epsilon_0}, \tag{24.6}$$

where ϵ_0 is called the **electric constant:**

$$\epsilon_0 \equiv \frac{1}{4\pi k} = 8.85418782 \times 10^{-12}\ \text{C}^2/(\text{N} \cdot \text{m}^2). \tag{24.7}$$

Equation 24.6 is a special case of **Gauss's law,** which states that the electric flux through the closed surface of an arbitrary volume is

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}, \tag{24.8}$$

where q_{enc} is the **enclosed charge**—the sum of all charge on an object or portion of an object enclosed by the closed surface. The formal proof of Gauss's law is an extension of the calculation you performed in Checkpoint 24.19 and is shown in the box "Electric flux though an arbitrary closed surface".

Gauss's law is a direct consequence of Coulomb's law with its $1/r^2$ dependence and the superposition of electric fields. In that respect, it contains nothing new. However, as the next section shows, Gauss's law greatly simplifies the calculation of electric fields due to charge distributions that exhibit one of the three symmetries we discussed in Section 24.4.

24.20 Suppose Coulomb's law showed a $1/r^{2.00001}$ dependence instead of a $1/r^2$ dependence. (*a*) Calculate the electric flux through a spherical Gaussian surface of radius R centered on a particle carrying a charge $+q$. (*b*) Substitute your result in Eq. 24.8. What do you notice?

QUANTITATIVE TOOLS

Electric flux through an arbitrary closed surface

Consider a particle that carries a positive charge $+q$, surrounded by the irregularly shaped closed surface shown in **Figure 24.26*a***.

1. To determine the electric flux through the irregular surface, we divide the volume enclosed by the surface into small square wedges that taper to a point at the charged particle, one of which is shown. We calculate the electric flux through the surface segment dA cut out by each wedge and then sum the contributions from all the wedges.
2. To determine the electric flux through dA in Figure 24.26*a*, we draw two spherical Gaussian surfaces around q: one with a radius r_1 equal to the distance between dA and q, the other with an arbitrary radius r_2. Our wedge from step 1 now defines two other small surface segments dA_1 and dA_2 on these two spheres.
3. If the segment dA is made very small, then the field lines in the wedge are nearly parallel to one another. Therefore, according to what we found in Section 24.6, the electric flux through dA is equal to that through dA_1 (Figure 24.26*b*). In addition, as you showed in Checkpoint 24.19, this flux is also equal to that through surface segment dA_2. In fact, the electric flux is the same through *any* surface that cuts through the wedge. Put differently, any surface that cuts through the wedge intercepts the same number of field lines.
4. We can repeat this procedure for each wedge. Each time, we see that the electric flux through a segment dA on the irregular surface is equal to that through a corresponding segment dA_2. As we add the contributions of all the wedges, we conclude that the electric flux through the closed irregular surface is equal to that through a sphere of arbitrary radius r centered on q.
5. If there is more than one charged particle inside the irregularly shaped surface, we can use the above arguments for each particle individually and then use the superposition of electric fields (Section 23.3):

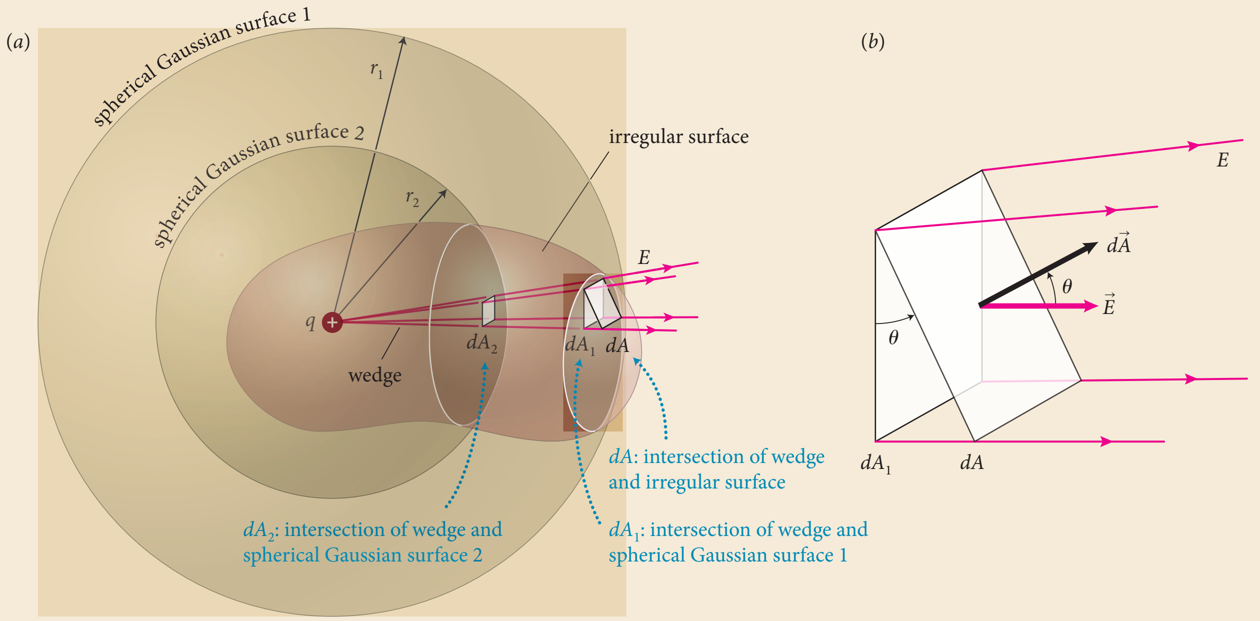
$$\vec{E} = \Sigma \vec{E}_i,$$

where $\vec{E}_i$ is the electric field due to particle i alone. The electric flux due to all charged particles is then the sum of the electric fluxes due to the individual electric fields:

$$\Phi_E = \Sigma \Phi_{Ei} = \Sigma \frac{q_i}{\epsilon_0} = \frac{q_{\text{enc}}}{\epsilon_0}.$$

24.21 Suppose q is *outside* the irregularly shaped surface in Figure 24.26. Show that the electric flux due to q through the closed surface is zero. (Hint: Draw a small wedge from q through the surface and determine the electric flux through the two intersections between the wedge and the surface.)

Figure 24.26 Formal proof of Gauss's law.



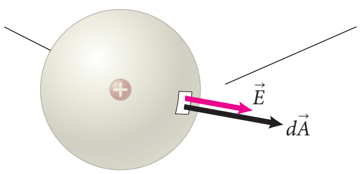
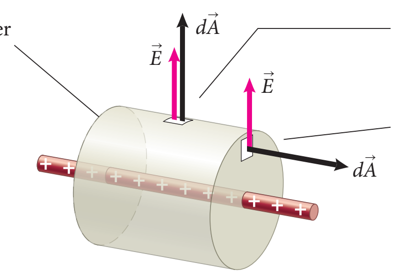
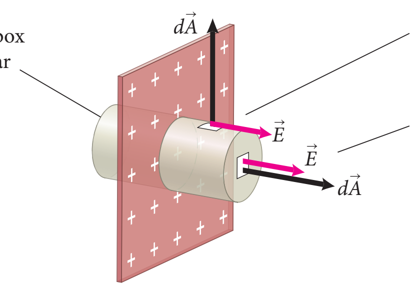
24.8 Applying Gauss's Law

Gauss's law relates the electric flux through a closed surface to the charge enclosed by it. In Section 24.5 we encountered one important application of Gauss's law: If we can choose a Gaussian surface such that the electric field is zero everywhere on that surface (inside a conducting material, for example), then we know that the charge enclosed by that surface must be zero. In this section we show how Gauss's law can be used to avoid having to carry out any integrations to calculate the electric field. In principle one can calculate the electric flux through any surface, but the calculation is not trivial in general. For the surfaces and charged objects shown in Section 24.4 and summarized in **Figure 24.27**, however, the electric flux is easy to calculate because the field lines are either parallel to the surface (in which case the electric flux is zero) or perpendicular to it and the magnitude of the electric field is constant (in which case the electric flux is simply the product of the magnitude of the electric field E and the surface area A).

To see the benefit of Gauss's law, consider a charged spherical shell of radius R that carries a uniformly distributed positive charge q. The electric field due to this charged shell can be calculated using the procedure outlined in Section 23.7: Divide the shell into infinitesimally small segments that carry a charge dq, apply Coulomb's law to each small segment, and integrate over the entire shell (Eq. 23.15):

$$\vec{E}_{sP} = k \int \frac{dq_s}{r_{sP}^2} \hat{r}_{sP}. \tag{24.9}$$

Figure 24.27 Applying Gauss's law to determine the electric fields of symmetrical charge distributions.

Symmetry of charge distribution	Electric field geometry	Gaussian surface		To find electric flux
Spherical (charged sphere)	$\vec{E}$ radiates uniformly outward in three dimensions.	Concentric sphere		At all points, $\vec{E}$ is perpendicular to surface and has same magnitude.
Cylindrical (infinite charged rod)	$\vec{E}$ radiates uniformly outward perpendicular to axis.	Coaxial cylinder		*Cylindrical surface:* At all points, $\vec{E}$ is perpendicular to surface and has same magnitude. *End faces:* $\vec{E}$ is parallel to face, so flux is zero.
Planar (infinite charged sheet)	$\vec{E}$ is uniform and perpendicular to plane.	Cylinder or box perpendicular to plane		*Surface perpendicular to plane:* $\vec{E}$ is parallel to face, so flux is zero. *Faces parallel to plane:* At all points, $\vec{E}$ is perpendicular to surface and has same magnitude.

As you may imagine, this so-called *direct integration* is no simple matter (see Section 13.8 for a similar integral), even though the result, which we derived qualitatively in Section 24.4, is surprisingly simple. Taking advantage of the symmetry of the problem, however, we can use Gauss's law to calculate the answer in just two steps.

We begin by drawing a concentric spherical Gaussian surface of radius $r > R$ around the shell (**Figure 24.28*a***). According to Gauss's law, the flux through the Gaussian surface is equal to the enclosed charge divided by ϵ_0:

$$\Phi_E = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{q}{\epsilon_0}. \tag{24.10}$$

In addition, we know that because of the spherical symmetry the electric field has the same magnitude E at each position on the Gaussian surface and the field is perpendicular to the surface. Because the electric field is perpendicular, we have $\vec{E} \cdot d\vec{A} = E\,dA$, and because E has the same value everywhere on the Gaussian surface, we can pull the electric field out of the integral in Eq. 24.5:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E\,dA = E \oint dA = EA, \tag{24.11}$$

where A is the area of the spherical Gaussian surface:

$$A = 4\pi r^2. \tag{24.12}$$

Substituting Eq. 24.12 into Eq. 24.11, we obtain

$$\Phi_E = 4\pi r^2 E. \tag{24.13}$$

Combining Eqs. 24.10 and 24.13, we obtain

$$4\pi r^2 E = \frac{q}{\epsilon_0} \tag{24.14}$$

or

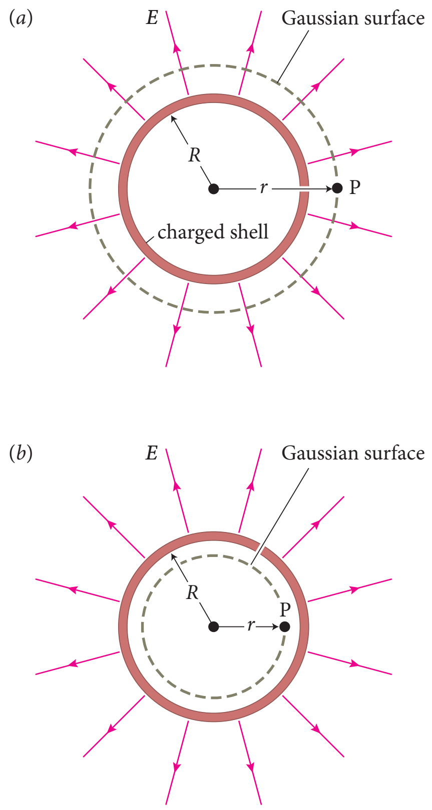
$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = k\frac{q}{r^2}. \tag{24.15}$$

This is exactly the magnitude of the electric field due to a particle carrying a charge q located at the center of the shell, as we concluded in Example 23.7 for a solid sphere.

We can also use Gauss's law to determine the electric field inside the shell by drawing a concentric spherical Gaussian surface with radius $r < R$ (Figure 24.28*b*). For this surface the enclosed charge is zero, $q_{\text{enc}} = 0$, so the right side of Eq. 24.14 becomes zero. Consequently, the electric field inside the uniformly charged spherical shell must be zero.

Note that our calculation did not involve working out any integrals, even though Eq. 24.5 does contain an integral—the symmetry of the problem allows us to bypass the integration. The procedure box on page 658 shows how to calculate the electric field using Gauss's law for charge distributions that exhibit one of the symmetries listed in Figure 24.27. In the next three exercises we apply this procedure to calculate the electric field of a number of different charge distributions.

Figure 24.28 Applying Gauss's law to a charged spherical shell.



QUANTITATIVE TOOLS

Procedure: Calculating the electric field using Gauss's Law

Gauss's law allows you to calculate the electric field for charge distributions that exhibit spherical, cylindrical, or planar symmetry without having to carry out any integrations.

1. Identify the symmetry of the charge distribution. This symmetry determines the general pattern of the electric field and the type of Gaussian surface you should use (see Figure 24.27).
2. Sketch the charge distribution and the electric field by drawing a number of field lines, remembering that the field lines start on positively charged objects and end on negatively charged ones. A two-dimensional drawing should suffice.
3. Draw a Gaussian surface such that the electric field is either parallel or perpendicular (and constant) to each face of the surface. If the charge distribution divides space into distinct regions, draw a Gaussian surface in each region where you wish to calculate the electric field.
4. For each Gaussian surface determine the charge q_{enc} enclosed by the surface.
5. For each Gaussian surface calculate the electric flux Φ_E through the surface. Express the electric flux in terms of the unknown electric field E.
6. Use Gauss's law (Eq. 24.8) to relate q_{enc} and Φ_E and solve for E.

You can use the same general approach to determine the charge carried by a charge distribution given the electric field of a charge distribution exhibiting one of the three symmetries in Figure 24.27. Follow the same procedure, but in steps 4–6, express q_{enc} in terms of the unknown charge q and solve for q.

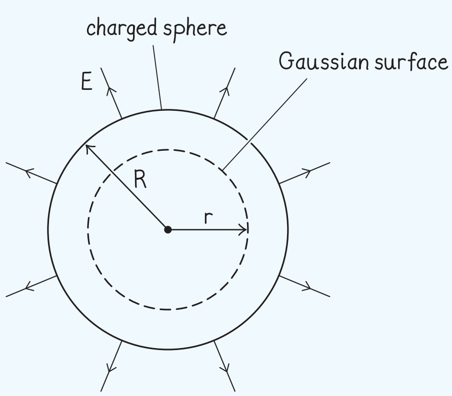
Exercise 24.6 Electric field inside uniformly charged sphere

Consider a charged sphere of radius R carrying a positive charge q that is uniformly distributed over the volume of the sphere. What is the magnitude of the electric field a radial distance $r < R$ from the center of the sphere?

SOLUTION The sphere has spherical symmetry, so I know that the field must point radially outward in all directions. I therefore draw a concentric spherical Gaussian surface with a radius $r < R$ (**Figure 24.29**). Because the sphere carries a uniformly distributed charge q, the amount of charge enclosed by the Gaussian surface is determined by the ratio of the volumes of the Gaussian surface and the charged sphere:

$$q_{enc} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} q = \frac{r^3}{R^3} q. \tag{1}$$

Figure 24.29



The electric flux is given by the product of the magnitude of the electric field $E(r)$ and the surface area A of the Gaussian surface (Eq. 24.13):

$$\Phi_E = 4\pi r^2 E. \tag{2}$$

Substituting Eqs. 1 and 2 into Eq. 24.8, I obtain

$$4\pi r^2 E = \frac{r^3}{R^3}\frac{q}{\epsilon_0}$$

or

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^3} r = k\frac{q}{R^3} r, ✔$$

the same result I obtained in Checkpoint 23.15.

24.22 What is the electric field outside a solid sphere carrying a charge $+q$ uniformly distributed throughout its volume?

QUANTITATIVE TOOLS

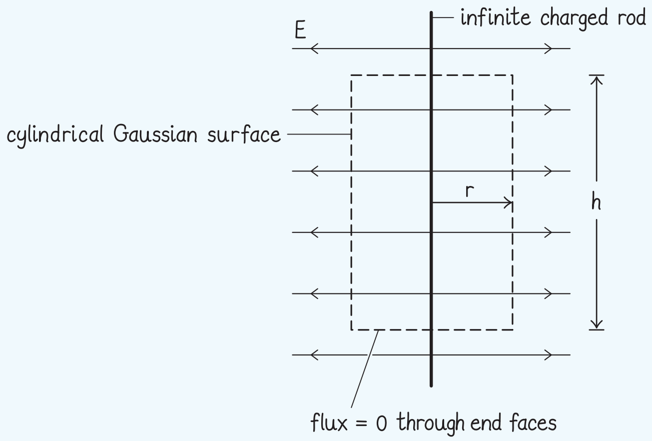
Exercise 24.7 Electric field of an infinitely long charged thin rod

What is the electric field magnitude a radial distance r from the central length axis of an infinitely long thin rod carrying a positive charge per unit length λ?

SOLUTION An infinitely long rod has cylindrical symmetry. I assume the rod's diameter is vanishingly small. From the symmetry, I know that the electric field points radially outward (see Section 24.4). I therefore make a sketch showing the rod and a few representative field lines. I then draw a cylindrical Gaussian surface of radius r and height h around the rod (**Figure 24.30**). The cylinder encloses a length h of the rod, so the enclosed charge is

$$q_{enc} = \lambda h. \tag{1}$$

Figure 24.30



The electric flux through the top and bottom faces of the Gaussian surface is zero because the electric field is parallel to those faces. I also know that symmetry requires the electric field to have the same magnitude E at each position on the cylindrical part of the surface. I can therefore pull the electric field out of the integral. The electric flux through that part of the surface is

$$\Phi_E = \int_{\text{cyl surface}} \vec{E} \cdot d\vec{A} = \int E\,dA = E\int dA. \tag{2}$$

The area of the cylindrical surface is equal to the circumference of the cylinder, $2\pi r$, times its height h, so Eq. 2 becomes

$$\Phi_E = E(2\pi r h). \tag{3}$$

Substituting Eqs. 1 and 3 into Eq. 24.8, I obtain

$$E(2\pi r h) = \frac{\lambda h}{\epsilon_0}$$

or

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r}. ✔$$

This is the same result I obtained by direct integration in Example 23.4 for a finite charged rod in the limit that I am close to the rod (which makes the rod appear infinitely long). The direct integration, however, took almost two pages of work!

24.23 The direct integration procedure also yields an expression for a rod of *finite* length (see Example 23.4). Can you use Gauss's law to derive this expression as well? Why or why not?

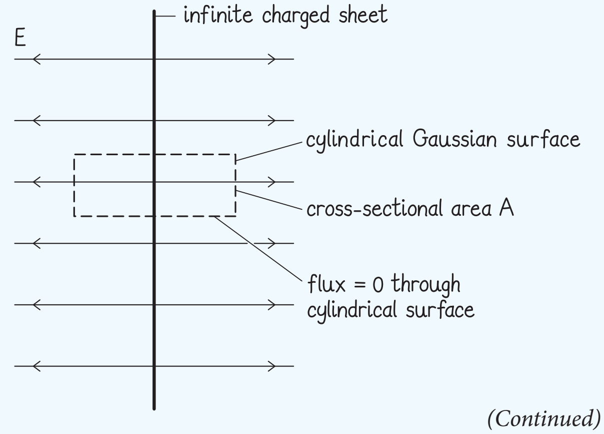
Exercise 24.8 Electric field of an infinite charged sheet

What is the electric field a distance d from a thin, infinite nonconducting sheet with a uniform positive surface charge density σ?

SOLUTION An infinite sheet has planar symmetry. From the symmetry I know that the electric field points away from the sheet and that the magnitude of the electric field is the same everywhere (see Section 24.4). I make a sketch of the sheet and the electric field and then draw a Gaussian surface in the form of a cylinder that straddles the sheet (**Figure 24.31**). If the cross section of the cylinder has area A, then the cylinder encloses a piece of the sheet of area A and the enclosed charge is

$$q_{enc} = \sigma A. \tag{1}$$

Figure 24.31



(Continued)

QUANTITATIVE TOOLS

The electric flux through the cylindrical part of the Gaussian surface is zero because the field lines are parallel to that surface. The field is perpendicular to the two ends and points outward, however, so the electric flux through those ends is the product of the area of each end, A, and the magnitude of the electric field:

$$\Phi_E = 2EA. \qquad (2)$$

Substituting Eqs. 1 and 2 into Eq. 24.8, I get

$$2EA = \frac{\sigma A}{\epsilon_0}$$

or

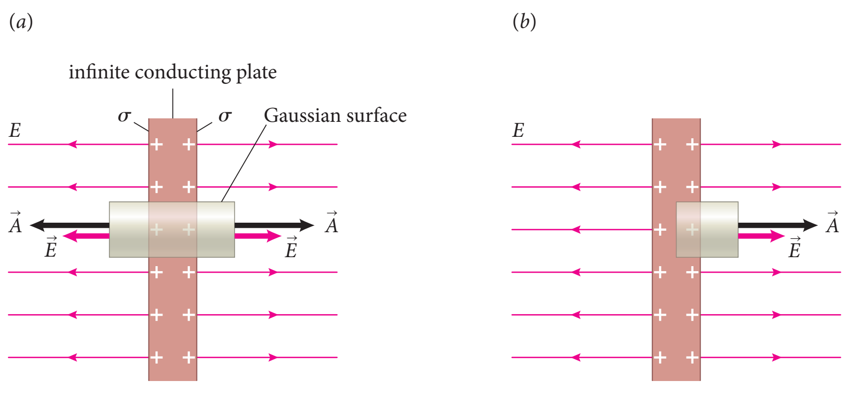
$$E = \frac{\sigma}{2\epsilon_0}. ✔$$

Because $1/(2\epsilon_0) = 2\pi k$, I can also write this as $E = 2\pi k\sigma$, which is the same result I obtained by direct integration in Example 23.6 for a uniformly charged disk in the limit that I am very close to the disk (which makes the disk appear like an infinite sheet).

The situation is a little different for a *conducting* plate. Consider, for example, the infinite charged conducting plate shown in **Figure 24.32*a***. As we saw in Section 24.5, any charge resides on the outside surfaces of the conducting object, so we have to consider the charge on *both* surfaces. If the surface charge density on the plate is σ and we use the same cylindrical Gaussian surface as we did for the nonconducting sheet, then the enclosed charge is not σA but $2\sigma A$ (σA for each of the two surfaces). Then Gauss's law yields

$$\Phi_E = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{2\sigma A}{\epsilon_0}. \qquad (24.16)$$

Figure 24.32 Applying Gauss's law to an infinite charged conducting plate.



Substituting Eq. 24.16 into Eq. 2 of Exercise 24.8 yields

$$E = \frac{\sigma}{\epsilon_0} \quad \text{(infinite conducting plate)}. \qquad (24.17)$$

Alternatively, you can choose a cylindrical Gaussian surface that has one end buried in the plate (Figure 24.32*b*). In that case only one of the two surfaces is enclosed and so the enclosed charge is σA. However, now the electric flux through the left end is zero because $E = 0$ inside the bulk of the plate, so the electric flux through the Gaussian surface is not $2EA$ but EA. Substituting these values into Gauss's law yields again Eq. 24.17.

24.24 (*a*) A very large metal plate of surface area A carries a positive charge q. What is the surface charge density of the plate? What is the magnitude of the field created by the plate? (*b*) A very large, thin nonconducting sheet of surface area A carries a fixed, uniformly distributed positive charge q. What is the surface charge density of the sheet? What is the magnitude of the field created by the sheet?

Chapter Glossary

SI units of physical quantities are given in parentheses.

Closed surface Any surface that completely encloses a volume.

Cylindrical symmetry A configuration that remains unchanged if rotated or translated about one axis exhibits cylindrical symmetry.

Electric constant ϵ_0 ($C^2/(N \cdot m^2)$) A constant that relates the electric flux to the enclosed charge in Gauss's law:

$$\epsilon_0 \equiv \frac{1}{4\pi k} = 8.85418782 \times 10^{-12}\ C^2/(N \cdot m^2). \quad (24.7)$$

Electric field lines A representation of electric fields using lines of which the tangent to the line at every position gives the direction of the electric field at that position.

Electric flux Φ_E ($N \cdot m^2/C$) A scalar that provides a quantitative measure of the number of electric field lines that pass through an area. The electric flux through a surface is given by the surface integral

$$\Phi_E \equiv \int \vec{E} \cdot d\vec{A}. \quad (24.4)$$

Electrostatic equilibrium The condition in which the distribution of charge in a system does not change.

Enclosed charge The sum of all the charge within a given closed surface.

Field line density The number of field lines per unit area that cross a surface perpendicular to the field lines at that position. The field line density at a given position in a field line diagram is proportional to the magnitude of the electric field.

Field line flux The number of outward field line crossings through a closed surface minus the number of inward field line crossings. The field line flux through a closed surface is equal to the charge enclosed by the surface multiplied by the number of field lines per unit charge.

Gauss's law The relationship between the electric flux through a closed surface and the charge enclosed by that surface:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}. \quad (24.8)$$

Gaussian surface Any closed surface used to apply Gauss's law.

Planar symmetry A configuration that remains unchanged if rotated about one axis or translated about any axis perpendicular to the axis of rotation exhibits planar symmetry.

Spherical symmetry A configuration that remains unchanged when rotated about any axis through its center exhibits spherical symmetry.