4 Momentum

CONCEPTS

QUANTITATIVE TOOLS

In the preceding two chapters, we developed a mathematical framework for describing motion along a straight line. In this chapter, we continue our study of motion by investigating inertia, a property of objects that affects their motion. The experiments we carry out in studying inertia lead us to discover one of the most fundamental laws in physics—conservation of momentum.

4.1 Friction

Picture a block of wood sitting motionless on a smooth wooden surface. If you give the block a shove, it slides some distance but eventually comes to rest. Depending on the smoothness of the block and the smoothness of the wooden surface, this stopping may happen sooner or it may happen later. If the two surfaces in contact are very smooth and slippery, the block slides for a longer time interval than if the surfaces are rough or sticky. This you know from everyday experience: A hockey puck slides easily on ice but not on a rough road.

Figure 4.1 shows how the velocity of a wooden block decreases on three different surfaces. The slowing down is due to *friction*—the resistance to motion that one surface or object encounters when moving over another. Notice that, during the interval covered by the velocity-versus-time graph, the velocity decrease as the block slides over ice is hardly observable. The block slides easily over ice because there is very little friction between the two surfaces. The effect of friction is to bring two objects to rest with respect to each other—in this case the wooden block and the surface it is sliding on. The less friction there is, the longer it takes for the block to come to rest.

Figure 4.1 Velocity-versus-time graph for a wooden block sliding on three different surfaces. The rougher the surface, the more quickly the velocity decreases.

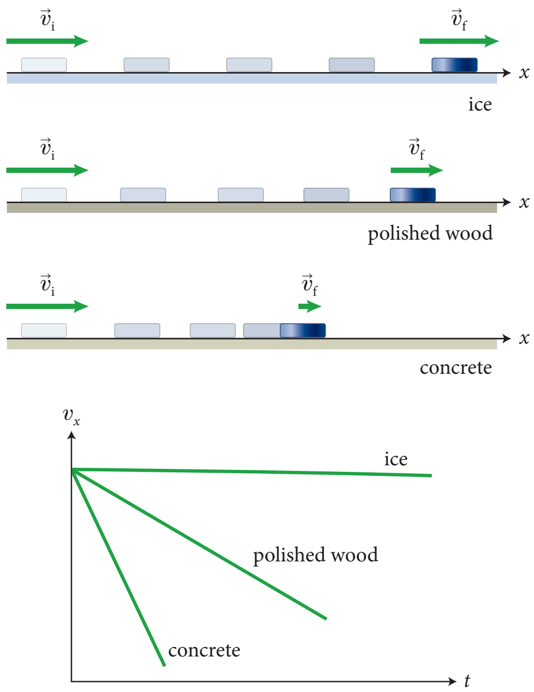
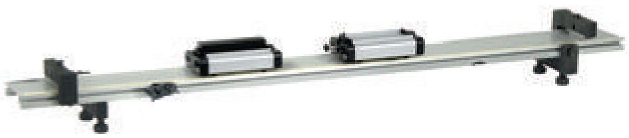


Figure 4.2 Low-friction track and carts used in the experiments described in this chapter.



You may wonder whether it is possible to make surfaces that have no friction at all, such that an object, once given a shove, continues to glide forever. There is no totally frictionless surface over which objects slide forever, but there are ways to minimize friction. You can, for instance, float an object on a cushion of air. This is most easily accomplished with a low-friction track—a track whose surface is dotted with little holes through which pressurized air blows. The air serves as a cushion on which a conveniently shaped object can float, with friction between the object and the track all but eliminated. Alternatively, one can use wheeled carts with low-friction bearings on an ordinary track. **Figure 4.2** shows low-friction carts you may have encountered in your lab or class. Although there is still some friction both for low-friction tracks and for the track shown in Figure 4.2, this friction is so small that it can be ignored during an experiment. For example, if the track in Figure 4.2 is horizontal, carts move along its length without slowing down appreciably. In other words:

> **In the absence of friction, objects moving along a horizontal track keep moving without slowing down.**

Another advantage of using such carts is that the track constrains the motion to being along a straight line. We can then use a high-speed camera to record the cart's position at various instants, and from that information determine its speed and acceleration.

4.1 (*a*) Are the accelerations of the motions shown in Figure 4.1 constant? (*b*) For which surface is the acceleration largest in magnitude?

4.2 Inertia

We can discover one of the most fundamental principles of physics by studying how the velocities of two low-friction carts change when the carts collide. Let's first see what happens with two identical carts. We call these *standard carts* because we'll use them as a standard against which to compare the motion of other carts. First we put one standard cart on the low-friction track and make sure it doesn't move. Next we place the second cart on the track some distance from the first one and give the second cart a shove toward the first. The two carts collide, and the collision alters the velocities of both.

Figure 4.3 shows a high-speed film sequence of such a collision. Cart 1 is initially at rest, and consequently its position doesn't change in the first seven frames. Cart 2 approaches cart 1 from the left and collides with it 60 ms after the beginning of the film sequence. The collision causes cart 1 to move to the right and cart 2 to stop dead in its tracks (or, rather, track).

Figure 4.3 (*a*) Two identical carts collide on a low-friction track. The length of track visible in each frame is 0.40 m. Cart 1 is initially at rest; cart 2 collides with it in the middle of the sequence. (*b*) Two curves, one marking the position of the rear of cart 1 and the other marking the position of the front of cart 2, superimposed on the film clip.

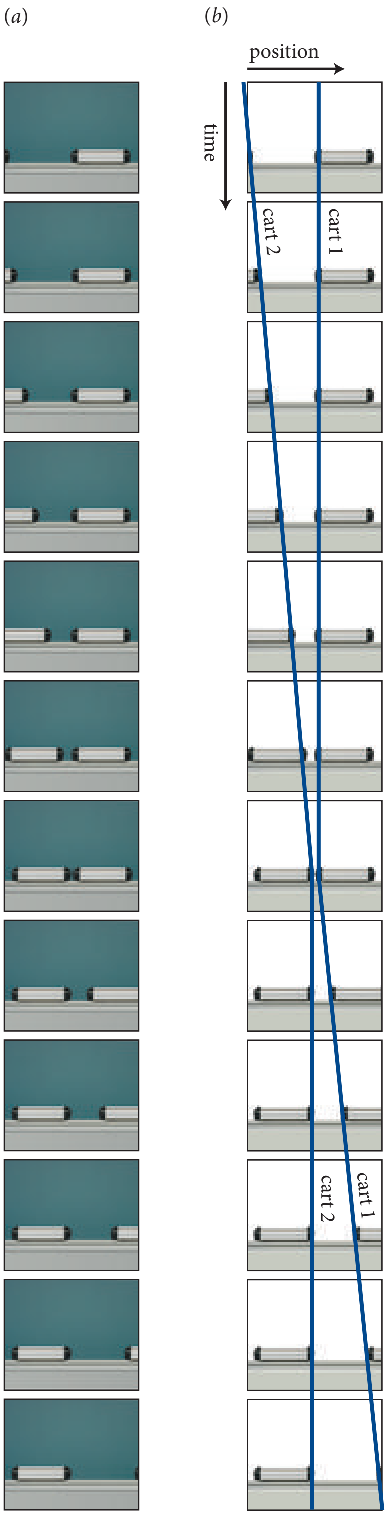
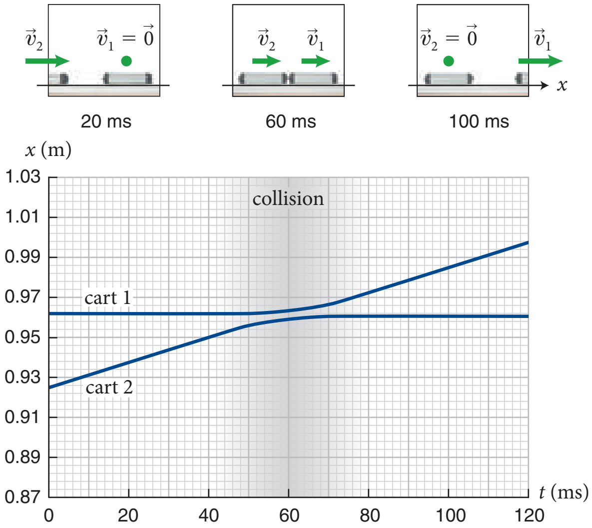


Figure 4.4 Position-versus-time graph for the two carts in Figure 4.3. The curves correspond to the curves of Figure 4.3*b*. All 120 original frames in the sequence were used to obtain this graph.



Measuring the positions of the two carts at various instants gives us the $x(t)$ curves shown in **Figure 4.4**, where the shaded region shows the time interval during which the collision took place. Although the collision appears "instantaneous" to any observer, it takes about 10 ms for the motion of the carts to adjust.

Figure 4.5, which shows the velocities of the two carts, tells us that initially the velocities are constant: 0 for cart 1 and about $+0.58$ m/s (the plus sign indicating motion to the right) for cart 2. After the collision, the velocities are

Figure 4.5 Velocity-versus-time graph for the carts of Figure 4.3 before and after the collision. Before the collision, cart 1 is at rest; after the collision, cart 2 is at rest and cart 1 moves at the initial velocity of cart 2.

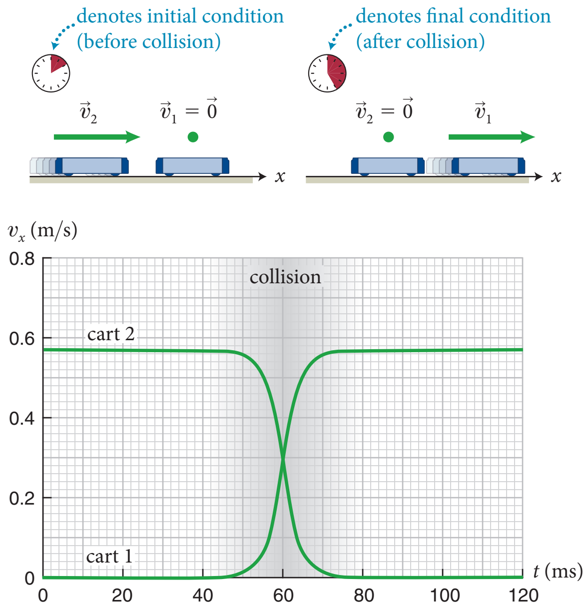


Figure 4.6 Velocity-versus-time graph for two identical carts before and after a collision on a low-friction track. Both carts are initially moving in the same direction.

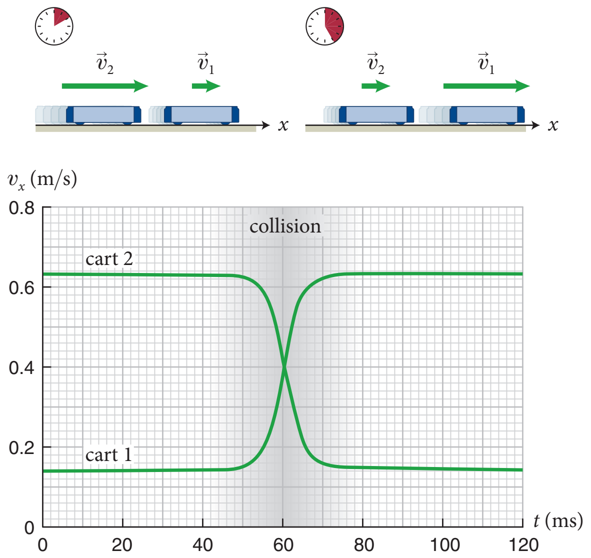
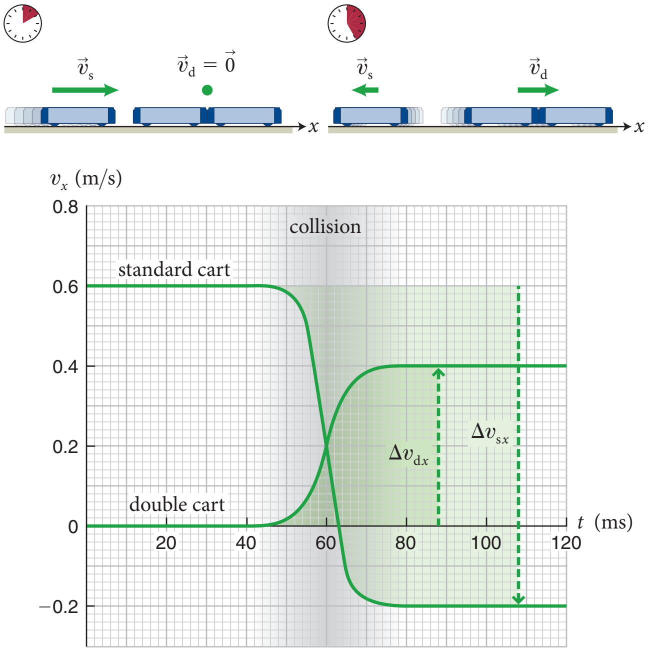


Figure 4.7 Velocity-versus-time graph for a standard cart and a double cart before and after the two collide on a low-friction track. The collision sets the double cart into motion and reverses the direction of travel of the standard cart.



interchanged: Cart 1 now moves to the right at $+0.58$ m/s, and cart 2 has come to a complete stop. We can also see this interchanging of velocities in Figures 4.3 and 4.4, where the slopes of the two $x(t)$ curves are interchanged after the collision.

We can repeat the experiment, this time giving cart 2 a harder shove, and then repeat it again, this third time giving cart 2 just a gentle nudge. What we discover is that, *no matter what the initial velocity of cart 2 is,* the collision always interchanges the two velocities.

Further experiments show that it is not necessary that cart 1 be initially at rest. **Figure 4.6** shows what happens when both carts are moving in the same direction at the instant of collision: Once again the collision interchanges the two velocities.

4.2 What is the change in velocity of (*a*) cart 1 and (*b*) cart 2 in Figure 4.6? (*c*) What do you notice about your two answers?

You can repeat this experiment with many different initial velocities, with the carts moving in the same direction or in opposite directions, and you will always observe that the collision interchanges the velocities of the carts. Moreover, Checkpoint 4.2 shows that, if the velocity of one cart increases by a certain amount as a result of the collision, then the velocity of the other cart decreases by exactly the same amount.

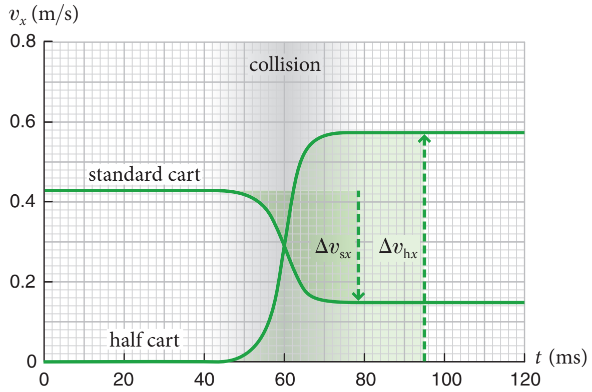
To determine whether the amount of material that makes up each cart affects the motion, let's fasten two standard carts together so that the size of this unit is twice the size of the other cart we are going to use, which we continue to call the standard cart. With this double cart at rest on the track, we shove the standard cart toward it at a speed of 0.60 m/s. This time the moving (standard) cart does not come to a complete stop; instead, the collision reverses its direction of travel, so that after the collision it moves to the left with a speed of 0.2 m/s, as **Figure 4.7** shows. After the collision, the initially stationary double cart, being more difficult to set in motion than the single standard cart, moves to the right, as before, but now its speed is lower than in the earlier experiments (0.40 m/s versus about 0.60 m/s). This result makes sense because we already know from experience that more massive objects are harder to set into motion than less massive ones—it's easier to throw a small stone than a large boulder.

The x component of the velocity of the double cart changed by $\Delta v_{dx} = +0.40\ \text{m/s} - 0 = +0.40\ \text{m/s}$, and the x component of the velocity of the standard cart changed by $\Delta v_{sx} = -0.20\ \text{m/s} - (+0.60\ \text{m/s}) = -0.80\ \text{m/s}$.* The magnitude of the double cart's velocity change, $|\Delta v_{dx}| = 0.40\ \text{m/s}$, is half that of the standard cart, $|\Delta v_{sx}| = 0.80\ \text{m/s}$.

We can repeat this experiment and vary the initial speeds and directions of motion, but we would continue to observe that *no matter how the carts move (or do not move) initially, the magnitude of the velocity change of the double cart is always half that of the standard cart.* Also, the two velocity

*Because we must keep track of the changes in velocity of two colliding objects, we need an additional subscript. For example, $\Delta v_{dx} = v_{dx,f} - v_{dx,i}$ is the change in velocity for the double cart. The final subscript, separated by a comma, designates the instant: f for final and i for initial. The middle subscript indicates that we are dealing with the x component. The first subscript designates the object (d for double cart).

Figure 4.8 Velocity-versus-time graph for a standard cart and a half cart before and after the two collide on a low-friction track.



changes are always in opposite directions, which means the x component of the velocity of one cart increases and that of the other cart decreases.

Now let us repeat the collision once again, this time with the original cart 1 cut in half so that it is half the size of the other cart we'll be using. With the half cart at rest on the track, we push a standard cart toward it. As **Figure 4.8** shows, the magnitude of the velocity change of the half cart is twice the magnitude of the velocity change of the standard cart: $|\Delta v_{\text{h}x}| = 0.58\ \text{m/s}$ and $|\Delta v_{\text{s}x}| = 0.29\ \text{m/s}$.

A pattern is developing here: For a group of objects all made of the same material, the motion of larger objects is harder to change than the motion of smaller objects. The larger objects put up more resistance when we try to change their velocity. This tendency of an object to resist a change in its velocity is called **inertia.**

Inertia is a measure of an object's tendency to resist any change in its velocity.

The results of our experiments are summarized in **Table 4.1**. As you can see, the *ratio of the inertias* of the two carts is equal to the inverse of the ratio of their velocity changes. In experiment 2, the double cart inertia is twice the standard cart inertia, and the ratio $|\Delta v_{\text{d}x}| : |\Delta v_{\text{s}x}|$ is 0.5. In experiment 3, the half cart inertia is half that of the standard cart, and the ratio $|\Delta v_{\text{h}x}| : |\Delta v_{\text{s}x}|$ is 2.

The data in Table 4.1 suggest that we could now take a new cart of unknown inertia, let a standard cart collide with it, and then *determine* from the changes in velocity the inertia of the new cart relative to that of the standard cart.

Table 4.1 **Ratio of Velocity Changes in Collisions Between Two Carts**

Experiment	Cart 1	Cart 2	$\lvert\Delta v_{1x}\rvert : \lvert\Delta v_{2x}\rvert$
1	standard	standard	1
2	double	standard	0.5
3	half	standard	2

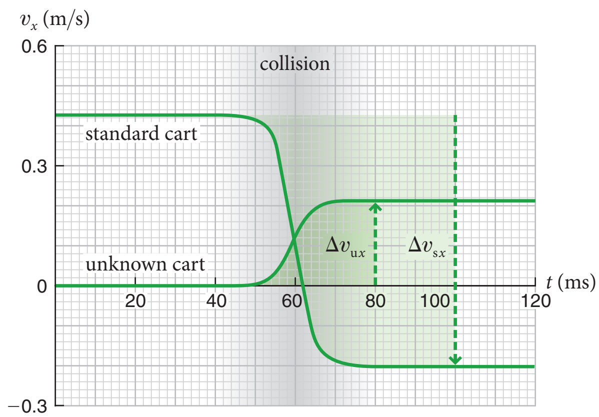
An oil supertanker has an enormous inertia. Even at full throttle it can take as long as 10 min to bring a large ship to a halt, which explains why ships like the *Titanic* and the *Exxon Valdez* could not avoid a collision in spite of seeing the danger well ahead of time.

4.3 The x component of the final velocity of the standard cart in Figure 4.8 is positive. Can you make it negative by adjusting this cart's initial speed while still keeping the half cart initially at rest?

Figure 4.9 is a graph of the collision between some unknown cart and a standard cart—both solid and made of the same material. The magnitude of the unknown cart's change in velocity is one-third that of the standard cart (Checkpoint 4.4). We conclude from this result that the unknown cart's inertia is three times that of the standard cart. In other words, the amount of material in the unknown cart is three times that in the standard cart.

4.4 Verify that $|\Delta v_{\text{u}x}|/|\Delta v_{\text{s}x}| \approx 1/3$ for the two carts in Figure 4.9.

Figure 4.9 Velocity-versus-time graph for a standard cart and a cart of unknown inertia before and after the two collide on a low-friction track. The unknown inertia can be determined from the ratio of the carts' velocity changes.



4.3 What determines inertia?

Having established that an object's inertia (its resistance to a change in velocity) is proportional to the amount of material in the object, we now come to the next important question: Is it only the amount of material that determines inertia? So far, all the carts we've been studying have been made of the same material. Would our results in experiment 1 be the same if we replaced one of the standard carts with a cart of the same volume of material but made of a different material—plastic instead of metal, say? As you probably know from experience, the answer is *no:* For example, a lead ball is more difficult to set in motion than a rubber ball of equal volume.

So let's make two carts of identical amounts of material, one of plastic and the other of metal, and see what happens when the two collide. The result, shown in **Figure 4.10**, is very different from the result for two identical carts (Figure 4.5).

4.5 What is the ratio of the x components of the change in velocity for the plastic and metal carts, $\Delta v_{px}/\Delta v_{mx}$, in Figure 4.10?

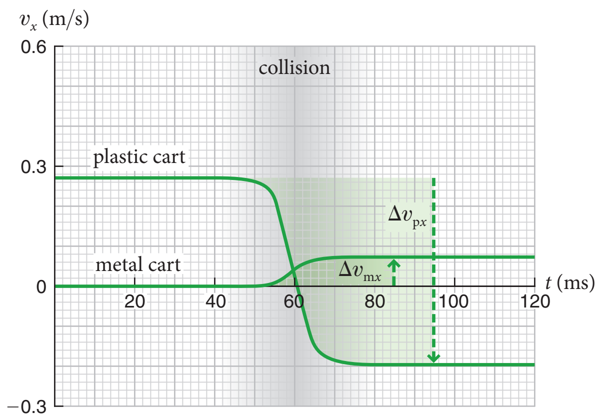
As we expect, the x component of the velocity of the plastic cart changes by a much larger amount than that of the metal cart (but it is still opposite in sign). In other words, it is easier to change the motion of the plastic cart than the motion of the metal cart. Given that inertia is a measure of an object's tendency to resist any change in its velocity, we conclude that an object's inertia is determined not just by the volume of material it contains but also by what that material is.

What about other physical properties—shape, say, or the smoothness of an object's surface? Experiments show that inertia does not depend on any of these other properties:

The inertia of an object is determined entirely by the type of material of which the object is made and by the amount of that material contained in the object.

CONCEPTS

Figure 4.10 Velocity-versus-time graph for two colliding carts that have the same volume but are made of different materials.



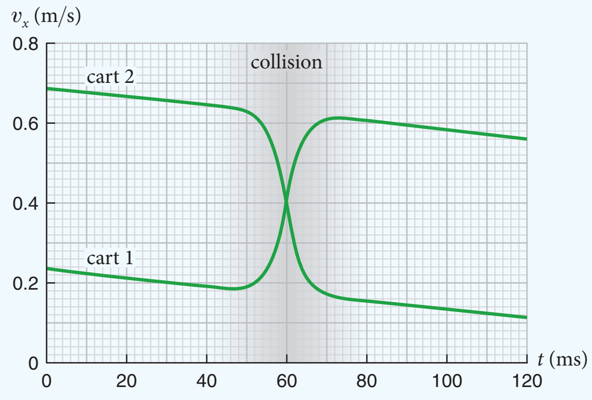
Once we have chosen one object as the standard for inertia, we can determine the inertia of any other object—whatever its size, shape, or composition—by letting it collide with our standard and measuring the ratio of the changes in the velocity of the two objects.

4.6 Is the inertia of the cart of unknown inertia in Figure 4.9 greater or less than that of the standard cart?

Example 4.1 Friction and collisions

Figure 4.11 shows the $v_x(t)$ curves for a collision between two identical carts moving not on a low-friction track but rather on a rough surface, so that friction affects their motion. Are the changes in the velocity of the carts caused by the collision still equal in magnitude?

Figure 4.11 Example 4.1.



❶ **GETTING STARTED** Looking at the graph, I'm tempted to say yes because if I tilt Figure 4.11 a bit, it looks very similar to Figure 4.6. Determining the change in velocity due to the collision, however, is not as straightforward because friction also affects the velocities. So, because the velocities of the carts before the collision decrease due to friction, it is not possible to read off a well-defined "initial" value of the velocity. The shape of the graph suggests that friction continues to decrease the velocities in the same way during and after the collision, so I have to determine a way to separate the change in velocity due to friction from the change due to the collision.

❷ **DEVISE PLAN** Because the effect of friction is a steady decrease in velocity, I can extrapolate the $v_x(t)$ curve for cart 1 to calculate what its velocity would have been at $t = 80$ ms if the collision had not taken place. To determine the change in velocity due to the collision, I then read off the actual value of the velocity of cart 1 at $t = 80$ ms and subtract the two velocities. Then I repeat the procedure for cart 2 and compare the changes in velocity.

❸ **EXECUTE PLAN** By placing a ruler along the first part of the $v_x(t)$ curve for cart 1, I observe that the curve lies in line with the second part of the $v_x(t)$ curve for cart 2. Reading off the value of the curve at $t = 80$ ms gives a value of $+0.15$ m/s. Repeating this procedure for cart 2, I obtain a value of $+0.60$ m/s. Now I can determine the velocities the carts would have had at $t = 80$ ms if the collision had not taken place. As the graph shows, however, the carts do collide and their velocities at $t = 80$ ms are interchanged. The magnitudes of the changes in velocity caused by the collision are thus 0.45 m/s and 0.45 m/s. These velocity changes are equal in magnitude. ✔

❹ **EVALUATE RESULT** Extrapolation of the velocity curves in Figure 4.11 shows that the collision interchanges the velocities of the two carts, just as in a collision between identical carts in the absence of friction. The interchanging means that the changes in velocity must be of equal magnitude, as I found numerically.

As this example shows, the magnitude of the changes in velocity in a collision is the same even in the presence of friction. This means that even in the presence of friction, we can determine an object's inertia from the magnitude of the velocity changes in a collision.

4.4 Systems

In the preceding chapters we were always concerned with just a single object—a puck, a car, and so on. Starting with this chapter, however, most situations we encounter deal with any number of objects that interact with one another. To analyze such situations, we shall usually focus on one or more principal objects—a person climbing a mountain, two carts colliding, gas atoms in a container. The first step in any analysis therefore is to separate the object(s) of interest from the rest of the universe:

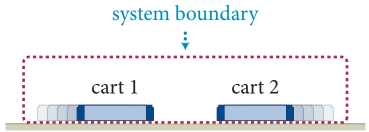
> **Any object or group of objects that we can separate, in our minds, from the surrounding environment is a system.**

For example, when considering a collision between two carts on a low-friction track, we might consider both carts together as the system. When someone throws a ball and we are interested in only the motion of the ball, a logical choice of system might be just the ball by itself. Once we have chosen the ball as our system, everything else—the thrower, the air around the ball, Earth—is outside the system and constitutes its **environment.** This imaginary separation of the universe into two parts—objects within the system and everything in the rest of the universe—makes it possible to develop simple but powerful accounting procedures.

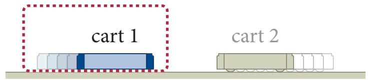
When solving problems, you should imagine a boundary enclosing the objects of interest. What is inside this well-defined boundary constitutes your system and what is outside is the environment. The region of space enclosed in the system boundary may be quite large and contain within it a great many processes or activities, or it may be very small and contain an inert piece of material undergoing no change. The precise shape of the system boundary

Figure 4.12 Two choices of system for carts colliding on a track.

(*a*) Choice 1: system consists of both carts



(*b*) Choice 2: system consists of one cart



is unimportant; what counts is that it clearly separates the objects inside the system from those outside. To make this separation explicit, it will generally be helpful to make a pictorial representation of the objects within the system. In your sketch, you should draw a boundary around the objects that constitute your system.

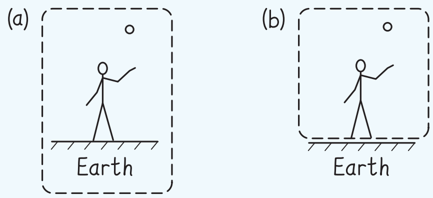
There may be more than one way to separate the system and the environment in any given situation. For two carts colliding on a low-friction track, as illustrated in **Figure 4.12**, we can define our system as containing both carts (in which case the track is part of the environment) or just one cart (in which case the track and the other cart are part of the environment). Deciding what to include in the system will be dictated by the information you wish to learn. Often the choice is obvious; sometimes you will need to rely on experience to make this decision. More important, once you decide to include a certain object in the system, *it must stay that way throughout the analysis.* Failing to make a consistent choice of system is a frequent source of error.

Exercise 4.2 Choosing a system

Indicate at least two possible choices of system in each of the following two situations. For each choice, make a sketch showing the system boundary and state which objects are inside the system and which are outside. (*a*) After you throw a ball upward, it accelerates downward toward Earth. (*b*) A battery is connected to a light bulb that illuminates a room.

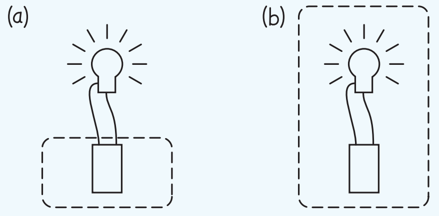
SOLUTION (*a*) The description of the situation mentions three objects: the ball, Earth, and you. One option is to include all three of them in the system (**Figure 4.13*a***). As a second choice, I include you and the ball in the system (Figure 4.13*b*). ✔

Figure 4.13



(*b*) Again I have three objects: the battery, the light bulb, and the room. I can choose just one of them—the battery—as my system (**Figure 4.14*a***) or two of them—the battery and the light bulb (Figure 4.14*b*). ✔

Figure 4.14



Note that my choices of system are arbitrary. Nothing in the problem prescribes the choice of system. If you tried this problem on your own before looking at my solution and you made different choices, then your answer is just as "correct" as mine!

Defining a system tells us nothing whatsoever about what is happening within it. It is simply a tool to help us set up an accounting scheme. Once we have chosen a system, we can study how certain quantities associated with the system change over time by determining the value of these quantities at the beginning and end of a time interval.

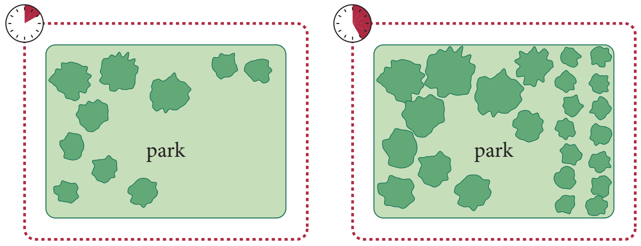
We shall in particular be interested in **extensive quantities**—that is, quantities whose value is proportional to the size or "extent" of the system. More specifically, if we divide the system into a number of pieces, then the sum of an extensive quantity for all the separate pieces is equal to the value of that quantity for the entire system. The number of trees in a park, for example, is extensive: If we divide the park into two parts and add the number of trees in each part, then we obtain the number of trees in the park. The price per gallon of gasoline is not an extensive quantity: If we divide a tankful of gas into two parts and add the price per gallon for the two parts, then we obtain twice the price per gallon for the entire tank. Quantities that do not depend on the extent of the system are **intensive quantities.**

4.7 Are the following quantities extensive or intensive: (*a*) inertia, (*b*) velocity, (*c*) the product of inertia and velocity?

Only four processes can change the value of an extensive quantity: input, output, creation, and destruction. To see this, consider **Figure 4.15**, which schematically represents a park's initial and final conditions over a certain time interval. The edge of the park is the system boundary, and the park itself—trees and all—is the system. Diagrams that show a system's initial and final conditions are called **system diagrams;** we shall use such diagrams throughout this book. The change in the number of trees over the time interval is given by

$$\text{change} = \text{final tree count} - \text{initial tree count}.$$

Figure 4.15 System diagram of a park. The number of trees in the park is an extensive quantity.



The number of trees can change because new trees grow (creation) and old trees die or are taken down (destruction). Alternatively, new trees can be brought into the park (input) and some trees can be moved out of the park (output):

$$\text{change} = \text{input} - \text{output} + \text{creation} - \text{destruction}.$$

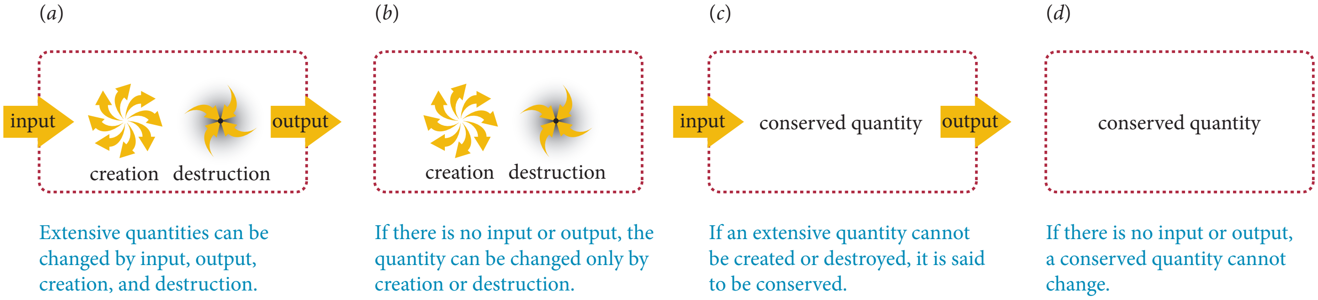
This equation is illustrated graphically in **Figure 4.16*a***. If we understand how these four processes affect the trees, we can explain (or predict) any change in their number.

The accounting is often simplified by constraints put on the system. For example, if we build a fence around the park so that no trees can be transported into or out of the park, then the number of trees can change only because of creation and destruction. If there is no transfer of the extensive quantity under consideration across the boundary of the system, the change in that property can be due to only creation and destruction (Figure 4.16*b*):

$$\text{change} = \text{creation} - \text{destruction}.$$

Under certain circumstances we can also exclude creation and destruction. Suppose, for example, that we are counting the number of indestructible benches in the park rather than trees. The number of these benches changes only when benches are transported into or out of the park across the park boundary (unless there is a factory making indestructible benches in the park—an unlikely assumption). Any extensive quantity that cannot be created or destroyed is said to be **conserved.** The value of a

Figure 4.16 The effect of input, output, creation, and destruction on the extensive quantities of a system.



conserved quantity can change only due to transfer of that quantity across the system boundary (Figure 4.16*c*):

$$\text{change} = \text{input} - \text{output}.$$

For a conserved quantity in a system for which there is no transfer, things are even more simple: Its value cannot change at all (Figure 4.16*d*). Conserved quantities play an important role in physics because their accounting is greatly simplified. In systems for which there is no transfer of a conserved quantity across the system boundary, this quantity does not change *regardless of the processes occurring inside or outside the system* (see the box "Conservation").

Exercise 4.3 Accounting principles

(*a*) Classify and give examples of the kinds of processes that can change (*i*) the number of loaves of bread in a bakery, (*ii*) the number of Lego pieces inside a house, and (*iii*) the number of coins in a safe that remains locked. (*b*) For each of these three cases, what is the system? Is the transfer of the quantity of interest across the system boundary possible? Is the quantity of interest conserved?

SOLUTION (*a*) (*i*) Input: a delivery of loaves of bread to the bakery; output: customers leaving the bakery with loaves they purchased; creation: the baking of a batch of loaves; destruction: the eating of loaves by patrons (inside the bakery; destruction outside the bakery falls under "output" because the loaves first leave the bakery). (*ii*) Input: Lego pieces are brought into the house;

Conservation

Imagine a sheet of paper of width a and length b. The circumference of this sheet is $2a + 2b$, and the surface area of one side is ab. Suppose we cut the sheet lengthwise into two equal parts. Are there any quantities that are unchanged by the cutting?

Circumference is not a conserved quantity: The sum of the circumferences of the two parts is not equal to the circumference of the single sheet. The circumference of each cut piece is $2(\frac{1}{2}a) + 2b$, and the combined circumference of the two pieces is $2(a + 2b)$, whereas the original circumference was $2(a + b)$.

Surface area is conserved: The surface area of each cut piece is $\frac{1}{2}ab$, and so the sum of the surface areas of the two pieces is ab, exactly as it was before we cut up the sheet.

Trivial? Well, suppose that instead of cutting the sheet in two, we cut it into 148 equal pieces. Can we say anything about the sum of the circumferences? Not without some brainwork! Conservation of surface area, on the other hand, allows us to state—with no calculations whatsoever—that the sum of the surface areas of all the pieces is still ab.

We have found a conserved quantity: surface area! No matter in how many pieces a sheet of paper is cut, the sum of the surface areas of the pieces is equal to the surface area of the original sheet.

If you are a skeptic (and in science you should always be skeptical), you could calculate the surface area of one of the 148 pieces and verify that the sum of the surface areas of all the pieces is indeed ab. To convince yourself that conservation of surface area is a general law, you might also want to repeat this procedure for a few more cases—say, 79 and 237 pieces.

Once you are convinced, however, it is easy to see the value of the conservation law. Suppose that instead of cutting the sheet neatly into equal pieces, someone tears it into ragged pieces, hands them to you, and asks, "What is the combined surface area of all of these pieces?" Even though an actual determination of the surface area would be difficult at best, you would answer, without hesitation, "ab"!

The idea that certain quantities are unaffected by events is appealing, not only because the notion of conservation intuitively makes sense (*some* things must remain unchanged) but also because it provides simple constraints on the behavior of a system.

Is surface area truly conserved? The answer is *no:* you can burn the sheet and make the entire surface area disappear. As we shall see, only a few quantities are truly conserved.

output: Lego pieces are taken out of the house; creation: assuming there is no manufacturing of Lego pieces in the house, there is no creation; destruction: for all practical purposes, I can consider the Lego pieces to be indestructible and so there is no destruction. (*iii*) Input: if the safe is locked it's impossible to add coins, so the input is zero; output: zero, too; creation: zero; destruction: zero. (Everything is zero, meaning the number of coins can't change inside a locked safe—which is precisely why people put valuables in safes!) ✔

(*b*) (*i*) The bakery and its contents make up the system. Loaves of bread can be taken into and out of the bakery, so transfer is possible. The number of loaves of bread is not conserved because loaves can be created and destroyed. (*ii*) The house and its contents are the system. Lego pieces can be taken into and out of it, so transfer is possible. Assuming that the Lego pieces cannot be fabricated or destroyed, I can consider their number to be conserved in this problem. (*iii*) The safe and its contents are the system. The safe remains locked so nothing can be put into it or taken out, and so transfer is not possible. Coins cannot be made or destroyed, so their number is conserved. ✔

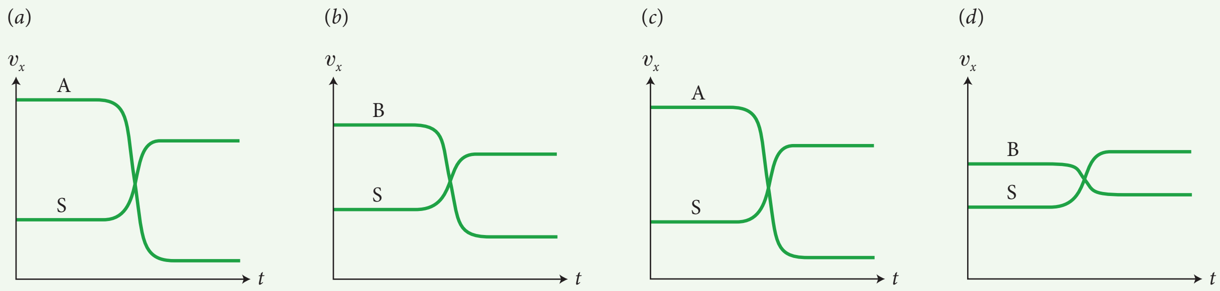
Strictly speaking, indestructible benches in a park, Lego pieces, and coins are not truly conserved—all of them are manufactured somewhere (creation) and all of them can be destroyed (destruction). You may be wondering, then, whether *anything* is conserved. The answer is yes: Several quantities we shall encounter are conserved, giving rise to some of the most fundamental principles in physics.

4.8 Which of these quantities is extensive? (*a*) money, (*b*) temperature, (*c*) humidity, (*d*) volume.

Self-quiz

1. Two carts give the same velocity-versus-time graphs when they collide with the same standard cart. Can you conclude from this information that the two carts are identical?
2. The graphs in Figures 4.17*a* and 4.17*b* show the effects of carts A and B colliding (separately) with a standard cart S. List the three carts in order of increasing inertia.

Figure 4.17



3. The graphs in Figures 4.17*c* and 4.17*d* show the effects of carts A and B colliding (separately) with a standard cart S. Which has greater inertia, A or B?
4. You do not know whether or not carts A and B of question 3 are made of the same material. Can you tell from the graphs which cart contains a larger quantity of material?
5. Imagine accounting for the number of cattle on a ranch. What do you choose as your system? What are the processes corresponding to input, output, creation, and destruction? Is transfer into and out of the system possible? Is the accountable quantity conserved?

Answers

1. No. Although the identical graphs tell you that the two carts have identical inertias, they need not be identical physically. A large plastic cart, for instance, can have the same inertia as a much smaller metal cart.
2. A and B (identical and less), S (greater). Because the magnitude of the velocity change for S is smaller than that for either A or B, cart S must have greater inertia. The *ratio* of the magnitudes of the velocity changes is about the same in both graphs, however, indicating that A and B have about the same inertias.
3. B. Don't be fooled by the bigger velocity change for S in part *c*; you must first consider each graph separately. Graph *c* tells you that A has less inertia than S because the magnitude of the velocity change for A is *larger* than that for S. Graph *d* tells you that B has greater inertia than S because the magnitude of the velocity change for B is *smaller* than that for S. Hence B must have greater inertia than A.
4. No. Although B has greater inertia, you cannot conclude anything about the quantity of material in it relative to A because you don't know from what materials the two carts are made.
5. The ranch is the system. Input: cattle transported into the ranch; output: cattle transported out of the ranch; creation: cattle born on the ranch; destruction: cattle expiring on the ranch. Transfer is possible because there is input and output, and the number of cattle is not conserved because cattle can be created and destroyed.

This replica of the standard kilogram is kept under two evacuated glass jars at the National Research Laboratory of Metrology in Japan. It is compared once every thirty years against the original international standard kept in France.

4.5 Inertial standard

The inertia of an object is represented by the symbol m (m is for *mass*, a concept that is related to inertia and that we shall examine later on). We shall use the term *inertia* rather than *mass* to underscore the physical meaning of this concept: An object's resistance to a change in its velocity. Inertia is a scalar, which means it does not depend on the orientation of any axis in space. By international agreement, the inertial standard is a platinum-iridium cylinder stored at the International Bureau of Weights and Measures in Sèvres, France. The inertia of this standard is defined as one **kilogram,** the basic SI unit of inertia ($m_s \equiv 1$ kg).*

Armed with a replica of this inertial standard and the knowledge gained in Sections 4.1 through 4.3, we can determine the inertia m of any other object. To do so, we make the object we are studying collide with the inertial standard and then measure the changes in velocity. In Section 4.2, we defined the ratio of the inertias of two colliding objects as the inverse of the ratio of the magnitude of their velocity changes, and so we can use this ratio for our inertial standard and our object of unknown inertia:

$$\frac{m_u}{m_s} \equiv -\frac{\Delta v_{s\,x}}{\Delta v_{u\,x}} \tag{4.1}$$

or

$$m_u \equiv -\frac{\Delta v_{s\,x}}{\Delta v_{u\,x}} m_s. \tag{4.2}$$

Why the minus sign? Well, suppose the object of unknown inertia is a replica of the standard. The left side of Eq. 4.1 then becomes $m_s/m_s = 1$. From the collision in Figure 4.5, we know, however, that the velocity of one of the carts increases (positive Δv_x) while that of the other decreases by the same amount (negative Δv_x). Without the minus sign, Eq. 4.1 would therefore become $1 = -1$. Notice that in all collisions we considered (Figures 4.5 through 4.9), the x components of the velocity change for the two colliding objects have opposite signs. Because this is true for *all* collisions, the minus sign in Eq. 4.1, the defining equation for inertia, means inertia is always a *positive* quantity.

In principle, this simple procedure makes it possible to determine the inertia of any object. Substituting the value $m_s \equiv 1$ kg in Eq. 4.2 allows us to determine the inertia of any other object in kilograms. In practice, it may not be a good idea to let objects collide in order to determine their inertia. (I don't recommend determining the inertia of a Ming vase in this way!) In Chapter 13 we shall discuss equivalent but more practical ways to determine the inertia of objects.

Example 4.4 Determining the inertia of a stone

A small stone is fastened to the top of a standard cart of inertia 1 kg to form a combination of unknown inertia m_u. A second standard cart is then launched with an initial velocity given by $v_{s\,x,i} = +0.46$ m/s toward the combination that is initially at rest. After the collision, the x component of the velocity of the cart with the stone is $v_{u\,x,f} = +0.38$ m/s and that of the standard cart is $v_{s\,x,f} = -0.08$ m/s. What is the inertia of the stone?

❶ GETTING STARTED This collision is much like the one depicted in Figure 4.7 with a standard cart moving toward a cart of greater inertia that is initially at rest. The ratio of the carts' inertias follows from the ratio of the magnitude of their velocity changes.

❷ DEVISE PLAN To calculate the unknown inertia m_u, I can use Eq. 4.2. I know the x components of the two carts' initial and final velocities, so all the quantities on the right side of the equation are known.

QUANTITATIVE TOOLS

*You may associate the unit kilogram with *weight,* a physical property that has to do with gravitation rather than inertia. More about the concept of weight in Chapter 13.

3 **EXECUTE PLAN** I begin by determining the changes in the x components of the velocity for each cart. For the cart with the stone, $\Delta v_{\text{u}x} = v_{\text{u}x,\text{f}} - v_{\text{u}x,\text{i}} = +0.38\text{ m/s} - 0 = +0.38\text{ m/s}$; for the standard cart $\Delta v_{\text{s}x} = v_{\text{s}x,\text{f}} - v_{\text{s}x,\text{i}} = -0.08\text{ m/s} - 0.46\text{ m/s} = -0.54\text{ m/s}$. Substituting these values into Eq. 4.2, I get

$$m_\text{u} = -\frac{\Delta v_{\text{s}x}}{\Delta v_{\text{u}x}} m_\text{s} = -\frac{-0.54\text{ m/s}}{+0.38\text{ m/s}}(1\text{ kg}) = 1.4\text{ kg}.$$

This represents the inertia of the stone and the cart as one unit. Given that the inertia of the standard cart is 1 kg, the inertia of the stone must be 0.4 kg. ✔

4 **EVALUATE RESULT** The inertia of the cart-and-stone combination is greater than 1 kg, as I would expect. If the magnitude of the velocity change of this combination were half that of the standard cart ($\frac{1}{2}(0.54\text{ m/s}) = 0.27\text{ m/s}$), then the inertia of the cart-and-stone combination would be twice that of a standard cart, and so the inertia of the stone would be 1.0 kg. The magnitude of the velocity change is more than half, so I should expect the stone's inertia to be less than 1.0 kg, which agrees with the answer I found.

4.9 (*a*) Suppose that instead of a replica of the platinum-iridium cylinder kept in France, we had chosen another object as our inertial standard and defined the inertia of that object as being exactly equal to 1 kg. Would the inertia of our unknown object as measured against the new standard be different from that object's inertia measured against the French standard? (*b*) Would the outcome of a collision between two arbitrary objects be different?

4.6 Momentum

Our definition of inertia leads to the definition of another important physical quantity: *momentum.* Let's rewrite Eq. 4.2 in a slightly different form by multiplying both sides by $\Delta v_{\text{u}x}$ and then bringing all terms to the left side:

$$m_\text{u}\Delta v_{\text{u}x} + m_\text{s}\Delta v_{\text{s}x} = 0. \tag{4.3}$$

Because the changes in velocity Δv_x can be written in the form $v_{x,\text{f}} - v_{x,\text{i}}$, we can rewrite this equation as

$$m_\text{u}(v_{\text{u}x,\text{f}} - v_{\text{u}x,\text{i}}) + m_\text{s}(v_{\text{s}x,\text{f}} - v_{\text{s}x,\text{i}}) = 0 \tag{4.4}$$

or

$$m_\text{u}v_{\text{u}x,\text{f}} - m_\text{u}v_{\text{u}x,\text{i}} + m_\text{s}v_{\text{s}x,\text{f}} - m_\text{s}v_{\text{s}x,\text{i}} = 0. \tag{4.5}$$

This form of Eq. 4.3, which contains nothing but products of inertias and velocities, is important because it suggests something new. The product of the inertia and the velocity of an object is called the **momentum** (plural: *momenta*) of that object. The conventional symbol for momentum is p, and so we can write

$$\vec{p} \equiv m\vec{v}. \tag{4.6}$$

As you learned in Section 2.6, the product of a vector and a scalar is a vector. Because inertia is a scalar and velocity is a vector, momentum is a vector. The direction of the momentum vector for any object is the same as the direction of the object's velocity vector. The x component of the momentum is the product of the inertia and the x component of the velocity:

$$p_x \equiv mv_x. \tag{4.7}$$

The SI units of momentum are the units of the product mv: kg · m/s. For example, the magnitude of the momentum of an object of inertia 1 kg moving at a speed of 1 m/s is 1 kg · m/s. Inertia is an intrinsic property of an object (you can't change it without changing the object), but the value of the momentum of that object, like its velocity, can change.

Example 4.5 Bullet and bowling ball

Compare the magnitude of the momenta of a 0.010-kg bullet fired from a rifle at 1300 m/s and a 6.5-kg bowling ball lumbering across the floor at 4.0 m/s.

❶ GETTING STARTED Momentum is the product of inertia and velocity. I have to calculate this quantity for both the bullet and the bowling ball and then compare the resulting values.

❷ DEVISE PLAN Equation 4.6 gives the momentum of an object. To determine the magnitude of the momentum of an object, I must take the product of the inertia m and the speed v: $p = mv$.

❸ EXECUTE PLAN Substituting the values given in the problem statement, I get

$$p_{\text{bullet}} = (0.010 \text{ kg})(1300 \text{ m/s}) = 13 \text{ kg}\cdot\text{m/s} \checkmark$$

$$p_{\text{bowling}} = (6.5 \text{ kg})(4.0 \text{ m/s}) = 26 \text{ kg}\cdot\text{m/s}. \checkmark$$

❹ EVALUATE RESULT Surprisingly, the magnitudes of the momenta are very close! I have no way of evaluating momenta because I don't have much experience yet with this quantity. However, the bullet has less inertia and a high speed and the bowling ball has greater inertia and a low speed, so it is not unreasonable that the product of these quantities is similar.

Momentum is a quantitative measure of "matter in motion" and depends on both the amount of matter in motion and how fast that matter is moving. Momentum is very different from inertia. A truck, for example, has greater inertia than a fly (it has a higher resistance to a change in its velocity), but if the truck is at rest and the fly is in motion, then the magnitude of the fly's momentum is larger than that of the truck, which is zero. In Example 4.5, the inertias of the bullet and the bowling ball are very different, yet their momenta are similar. Conceptually you can think of an object's momentum as its capacity to affect the motion of other objects in a collision.

With the definition of momentum, we can rewrite Eq. 4.5 in the form

$$p_{\text{u}x,\text{f}} - p_{\text{u}x,\text{i}} + p_{\text{s}x,\text{f}} - p_{\text{s}x,\text{i}} = 0. \tag{4.8}$$

If we write $\Delta p_{\text{u}x} \equiv p_{\text{u}x,\text{f}} - p_{\text{u}x,\text{i}}$ and $\Delta p_{\text{s}x} \equiv p_{\text{s}x,\text{f}} - p_{\text{s}x,\text{i}}$, Eq. 4.8 takes on the beautifully simple form

$$\Delta p_{\text{u}x} + \Delta p_{\text{s}x} = 0. \tag{4.9}$$

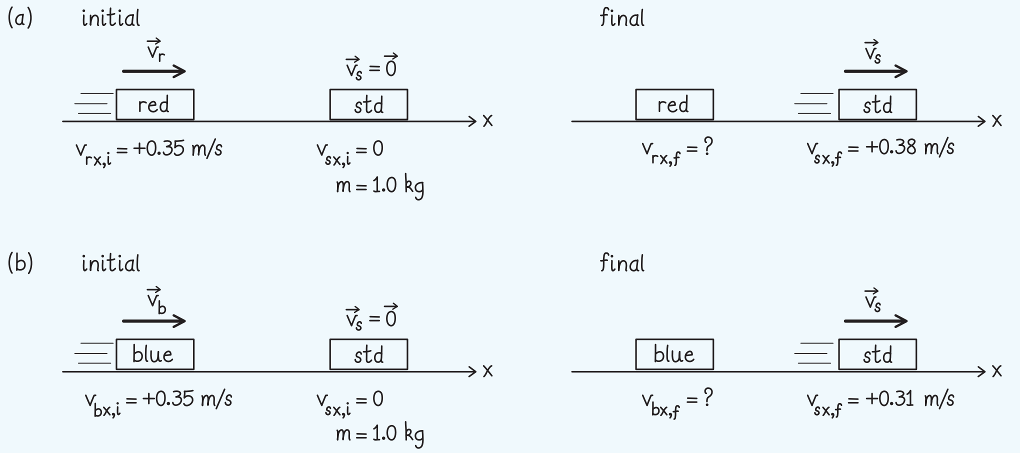
This equation means that, whenever an object of unknown inertia collides with the inertial standard, the changes in the x components of the momenta of the two objects add up to zero. In other words, the change in the x component of the momentum for one object is always the negative of the change for the other.

Example 4.6 Collisions and momentum changes

(*a*) A red cart with an initial speed of 0.35 m/s collides with a stationary standard cart ($m_\text{s} = 1.0$ kg). After the collision, the standard cart moves away at a speed of 0.38 m/s. What is the momentum change for each cart? (*b*) The experiment is repeated with a blue cart, and now the final speed of the standard cart is 0.31 m/s. What is the momentum change for each cart in this second collision? (*c*) If in the collisions $v_{\text{r}x,\text{f}} = +0.032$ m/s and $v_{\text{b}x,\text{f}} = -0.039$ m/s, what are the inertias of the red and the blue carts?

❶ GETTING STARTED I begin organizing the information given in the problem in a picture by showing the initial and final conditions for each of the two collisions (**Figure 4.18**).

Figure 4.18



❷ **DEVISE PLAN** In each collision, I know the inertia and the x components of the initial and final velocities of the standard cart, so I can readily determine the change in the x component of its momentum. To determine the change in momentum for the other carts, I can then use Eq. 4.9, which tells me that for each collision the changes in the x components of the momenta of the two carts must add up to zero. For part *c*, I can use Eq. 4.7 together with the values for the red and blue carts' initial and final velocities and their changes in momenta calculated in parts *a* and *b* to determine the inertias.

❸ **EXECUTE PLAN** (*a*) Because it initially is at rest, the standard cart's initial momentum is zero. The x component of its final momentum is $p_{\mathrm{s}x,\mathrm{f}} = (1.0\ \mathrm{kg})(+0.38\ \mathrm{m/s}) = +0.38\ \mathrm{kg \cdot m/s}$, and so the x component of its change in momentum is

$$\Delta p_{\mathrm{s}x} = +0.38\ \mathrm{kg \cdot m/s} - 0 = +0.38\ \mathrm{kg \cdot m/s}. ✔$$

Because the x components of the momentum changes of the two carts must add up to zero (Eq. 4.9), the corresponding x component of the red cart's momentum change is $\Delta p_{\mathrm{r}x} = -0.38\ \mathrm{kg \cdot m/s}$. ✔

(*b*) Applying the same procedure for this second collision, I get

$$\Delta p_{\mathrm{s}x} = (1.0\ \mathrm{kg})(+0.31\ \mathrm{m/s}) - 0 = +0.31\ \mathrm{kg \cdot m/s};$$

$$\Delta p_{\mathrm{b}x} = -0.31\ \mathrm{kg \cdot m/s}. ✔$$

(*c*) The change in momentum for the red cart is $\Delta p_{\mathrm{r}x} = p_{\mathrm{r}x,\mathrm{f}} - p_{\mathrm{r}x,\mathrm{i}}$. Using Eq. 4.7, I can write this change in momentum as

$$\Delta p_{\mathrm{r}x} = m_{\mathrm{r}} v_{\mathrm{r}x,\mathrm{f}} - m_{\mathrm{r}} v_{\mathrm{r}x,\mathrm{i}} = m_{\mathrm{r}}(v_{\mathrm{r}x,\mathrm{f}} - v_{\mathrm{r}x,\mathrm{i}})$$

and so

$$m_{\mathrm{r}} = \frac{\Delta p_{\mathrm{r}x}}{v_{\mathrm{r}x,\mathrm{f}} - v_{\mathrm{r}x,\mathrm{i}}} = \frac{-0.38\ \mathrm{kg \cdot m/s}}{+0.032\ \mathrm{m/s} - 0.35\ \mathrm{m/s}}$$
$$= 1.2\ \mathrm{kg}. ✔$$

Likewise, for the blue cart:

$$m_{\mathrm{b}} = \frac{\Delta p_{\mathrm{b},x}}{v_{\mathrm{b}x,\mathrm{f}} - v_{\mathrm{b}x,\mathrm{i}}} = \frac{-0.31\ \mathrm{kg \cdot m/s}}{-0.039\ \mathrm{m/s} - 0.35\ \mathrm{m/s}}$$
$$= 0.80\ \mathrm{kg}. ✔$$

❹ **EVALUATE RESULT** If either the red or the blue cart had an inertia of 1.0 kg, then the cart would come to rest in the collision. My work shows that the inertia of the red cart is greater than that of a standard cart. Indeed, after the collision it still moves in the same direction it initially moved, as it should for a cart of greater inertia colliding with one of lesser inertia (see, for example, Figure 4.8). In contrast, the inertia of the blue cart is less than that of the standard cart. After the collision it bounces back and travels in the opposite direction, as I would expect (see Figure 4.7).

Equation 4.9 can be written in vectorial form:

$$\Delta \vec{p}_{\mathrm{u}} + \Delta \vec{p}_{\mathrm{s}} = \vec{0} \tag{4.10}$$

and so, in a collision between an object of unknown inertia and the inertial standard, the changes in the momenta of the two objects add to zero.

4.10 In part *a* of Example 4.6, what are the directions of the changes in momentum, $\Delta\vec{p}_{\mathrm{s}}$ and $\Delta\vec{p}_{\mathrm{r}}$, of the two carts?

4.7 Isolated systems

As we saw in Checkpoint 4.7, momentum is an extensive property—you can add up the momenta of all the objects in a system to obtain the *momentum of the system*. In particular, the momentum of a system of two moving carts is the sum of the momenta of the two individual carts:

$$\vec{p} \equiv \vec{p}_1 + \vec{p}_2. \tag{4.11}$$

With this definition we can begin to develop an accounting scheme for the momentum of a system. In particular, we shall be interested in determining what causes the momentum of a system to change. Let us begin by reexamining a number of situations we have encountered. In **Figure 4.19*a*** a puck slides to a stop on a wood floor. In Figure 4.19*b* a cart moves at constant velocity on a low-friction track. In Figure 4.19*c* a standard cart collides with another cart on a low-friction track, and in Figure 4.19*d* someone gives a cart moving at constant

Figure 4.19 Velocity-versus-time and acceleration-versus-time graphs for four situations. The shading indicates nonzero acceleration.

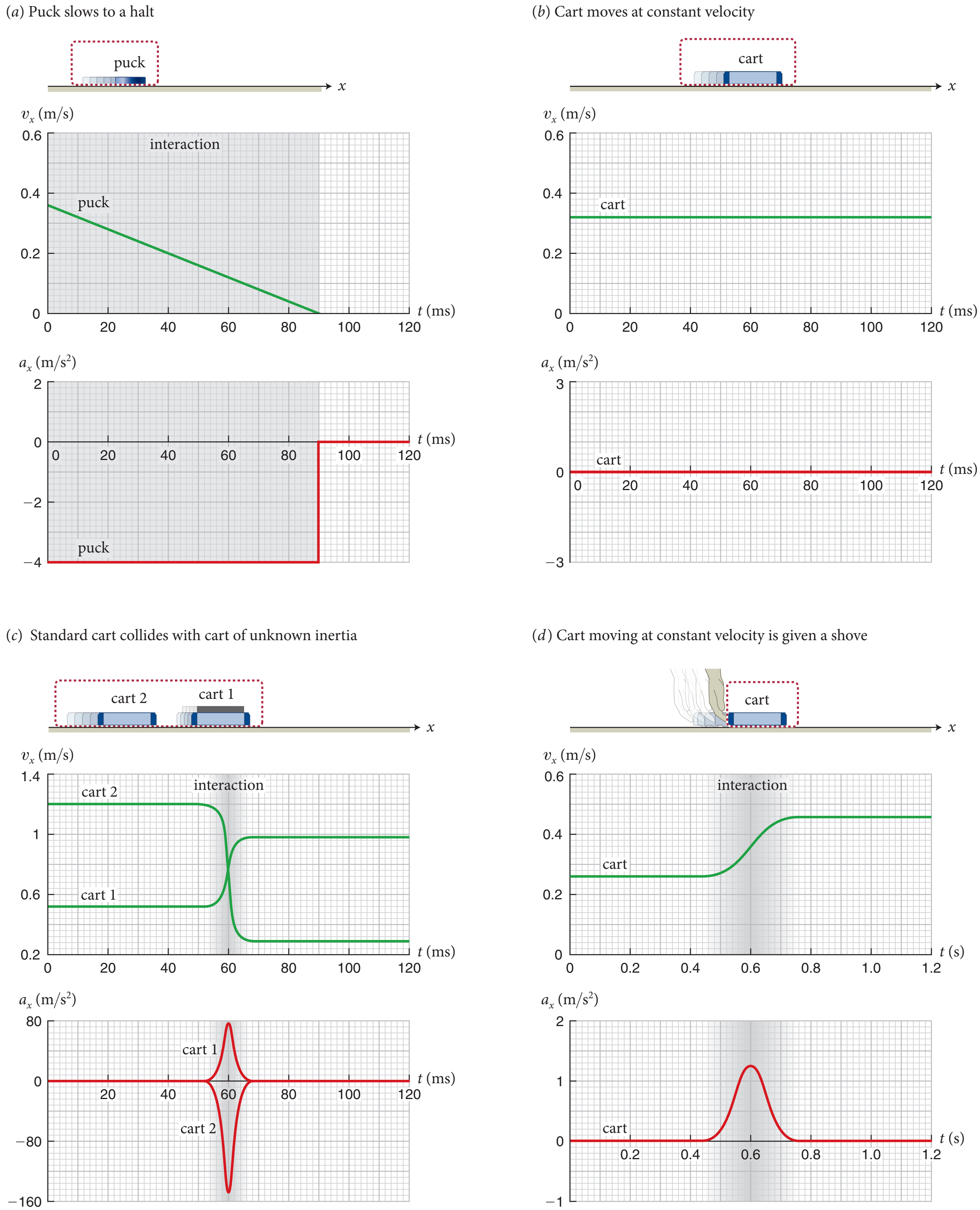


Table 4.2 Interactions and momentum changes in Checkpoint 4.11

Situation	Interacting objects	System	System interacting?	$\Delta\vec{p}$
a	floor ↔ puck	puck	yes	nonzero
b	none	cart	no	$\vec{0}$
c	cart 1 ↔ cart 2	cart 1	yes	nonzero
d	cart 1 ↔ cart 2	carts 1 & 2	no	$\vec{0}$
e	hand ↔ cart	cart	yes	nonzero

velocity on a low-friction track a shove. In some cases the velocity (and therefore the momentum) remains constant; in others it changes.

4.11 Is the change in the momentum $\Delta\vec{p}$ zero or nonzero for the following choices of system over the 120-ms time interval in Figure 4.19: (*a*) puck in Figure 4.19*a*, (*b*) cart in Figure 4.19*b*, (*c*) cart 1 in Figure 4.19*c*, (*d*) both carts in Figure 4.19*c*, (*e*) cart in Figure 4.19*d*?

In the graphs, I have highlighted the time intervals during which the acceleration of objects within the system is nonzero—that is, the time interval when the velocity and therefore the momentum of objects within the system change. Notice that this time interval always coincides with the time interval during which the object under consideration *interacts* with something else. In Figure 4.19*a* the puck slows down because the puck and the floor rub against (interact with) each other, in Figure 4.19*c* the two carts collide with (interact with) each other, and in Figure 4.19*d* the cart and the hand interact with each other. Conversely, in Figure 4.19*b* the interaction between the track and the cart has been virtually eliminated by reducing friction, and so the acceleration is zero.

In these examples, *interaction* refers to objects touching or rubbing against each other. Interactions are the subject in Chapter 7. For now, we'll use the word *interaction* to mean two objects acting on each other in such a way that one or both are accelerated.

Table 4.2 summarizes the observations from Checkpoint 4.11. Note that whenever the system interacts with something outside the system, the momentum of the system changes. Interactions across the boundary of a system are called *external interactions.* Interactions between two objects inside the system are called *internal interactions.* Looking at Figure 4.19 and Table 4.2, we see that external interactions transfer momentum into or out of the system.

A system for which there are no external interactions is said to be **isolated.** For such a system there is no input or output of momentum, and so the momentum of the system does not change: $\Delta\vec{p} = \vec{0}$. In Figure 4.19*b*, the cart's motion is not influenced by the track, and so the cart constitutes an isolated system. In Figure 4.19*c*, the two carts interact with each other but not with anything outside the system, and so the two carts together constitute a system that is isolated (but either cart by itself does not; see Table 4.3). In Figures 4.19*a* and 4.19*d*, where

Table 4.3 Two choices of system for carts colliding

	Choice 1	Choice 2
System:	carts 1 & 2	cart 1
Environment:	track	cart 2 & track
Interactions:	internal	external
System isolated?	yes	no
Momentum changing?	no	yes

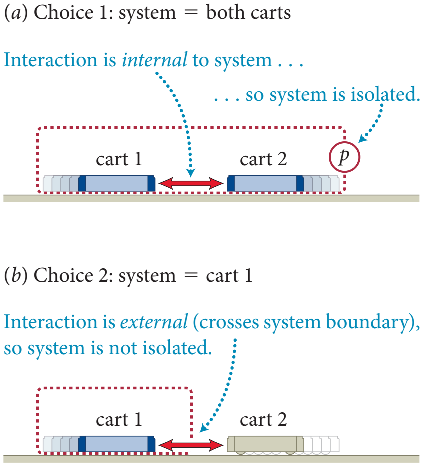


Figure 4.20 The choice of system determines whether the interactions in question are internal or external and hence whether the system is isolated or not. We use the symbol Ⓟ to denote an isolated system, indicating that no momentum crosses the system's boundary.

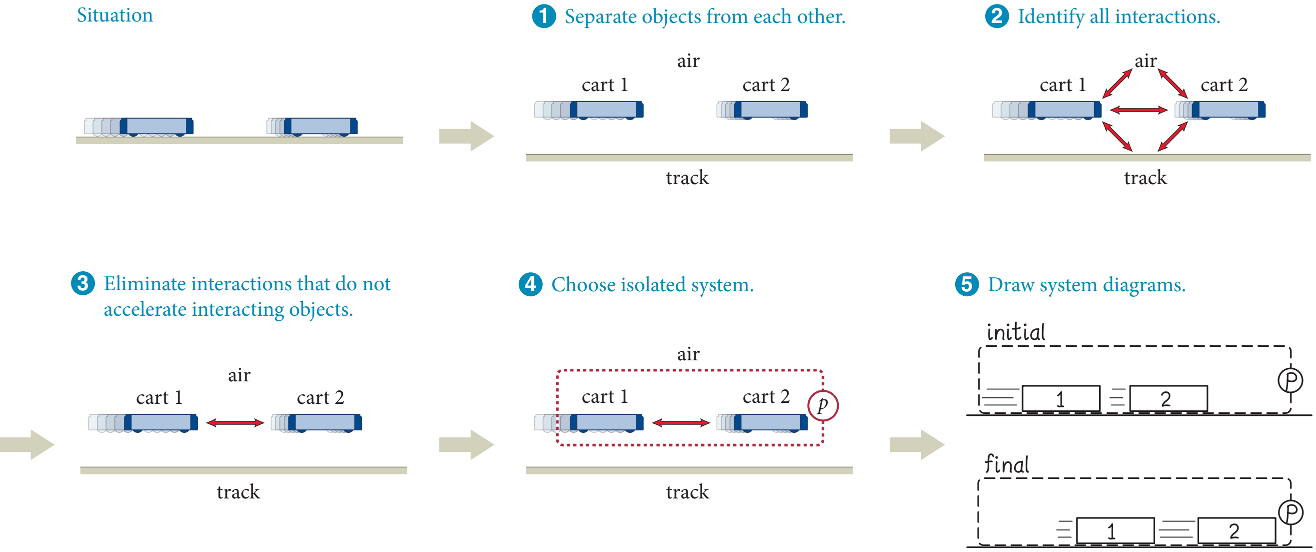
$\Delta\vec{p} \neq \vec{0}$, the system is not isolated. In Figure 4.19*a* the only object in the system (the puck) interacts with the floor, which is outside the system. In Figure 4.19*d*, the only object in the system (the cart) interacts with the hand, which is outside the system.

Figure 4.20 and Table 4.3 illustrate two system choices for studying a collision. The system containing both carts is isolated (the interaction between the two carts takes place inside the system; it is an internal interaction), and the momentum of the system is constant. The system containing just cart 1 is not isolated because it interacts with cart 2, which is now part of the environment, and the momentum of the system is not constant.

In isolated systems the analysis of momentum is simplified because we can concentrate on what happens inside the system. The procedure for choosing an isolated system is shown in the Procedure box on the next page and illustrated for a collision between two carts on a low-friction track in **Figure 4.21**. We begin by separating from one another all the objects that could play a role: the carts, the track, and the air. Next we identify all possible interactions: the collision between the two carts, the interaction between the air and the carts, and any possible interaction between the track and the carts. We know that each cart by itself rolls without slowing down on the track, so the interaction between the track and the carts and between the air and the carts does not have any significant effect on the carts' velocity. We therefore eliminate these and retain only the collision between the carts, which does change their velocities.*

Having eliminated all but the interaction between the carts, we can choose an isolated system by enclosing the two carts so that this interaction becomes internal. Finally, we complete the procedure by making a system diagram, showing the initial and final conditions of the isolated system (before and after the collision). In general, the interaction between two colliding objects affects their motion much more strongly than any other interaction, and so we can for all practical purposes always consider the system of two colliding objects to be isolated.

Figure 4.21 Applying the procedure for choosing an isolated system (see the Procedure box) to the collision between two carts on a low-friction track.



*Leaving out the air-cart and track-cart interactions is a simplification, because both *do* have a small effect on the carts' acceleration. However, you can ignore the effects of the these interactions because they are so much smaller than the effects of the collision. Never be afraid to make a simplifying assumption; you can always return to the beginning of a problem and refine your assumptions.

Procedure: Choosing an isolated system

When you analyze momentum changes in a problem, it is convenient to choose a system for which no momentum is transferred into or out of the system (an isolated system). To do so, follow these steps:

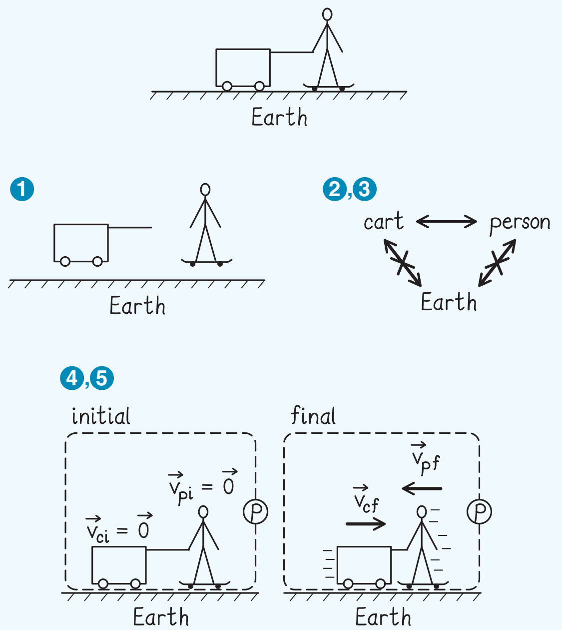
1. Separate all objects named in the problem from one another.
2. Identify all possible interactions among these objects and between these objects and their environment (the air, Earth, etc.).
3. Consider each interaction individually and determine whether it causes the interacting objects to accelerate. Eliminate any interaction that does not affect (or has only a negligible effect on) the objects' accelerations during the time interval of interest.
4. Choose a system that includes the object or objects that are the subject of the problem (for example, a cart whose momentum you are interested in) in such a way that none of the remaining interactions cross the system boundary. Draw a dashed line around the objects in your choice of system to represent the system boundary. None of the remaining interactions should cross this line.
5. Make a system diagram showing the initial and final states of the system and its environment.

Exercise 4.7 Who's pulling whom?

A person standing on a skateboard on horizontal ground pulls on a rope fastened to a cart. Both the person and the cart are initially at rest. Use the Procedure box to identify an isolated system and make a system diagram.

SOLUTION See **Figure 4.22**. I begin by separating the objects in the problem: the person, the cart, and Earth. (I could always go into more detail—include the air, the rope, and the skateboard—but it pays to keep things as simple as possible. For that reason, I consider the skateboard to be part of the person and the rope to be part of the cart.) The cart interacts with Earth and the person; the person interacts with the cart and Earth. Ignoring friction in the wheels of the cart, I know that the interaction between the cart and Earth has no effect on any motion, and so I can eliminate it from the analysis. The same holds for the interaction between the person (the skateboard) and Earth. I then draw a boundary around the person and the cart, making the interaction between the two internal. Because there are no external interactions, this system is isolated. Finally I draw a system diagram showing the initial and final conditions of the system with the person and cart initially at rest and then moving. ✔

Figure 4.22



4.12 Imagine sitting on a sled on the slippery surface of a frozen lake. To reposition yourself closer to the back of the sled, you push with your legs against the front end. (*a*) Do you constitute an isolated system? (*b*) How about you and the sled? Ignore the interaction between sled and ice.

4.8 Conservation of momentum

In Section 4.4 we saw that the change in an extensive property can be due to transport into or out of the system and to creation or destruction. In Section 4.7 we saw that whenever we consider a system that is isolated, the momentum of the system is not changing:

$$\Delta\vec{p} = \vec{0} \quad \text{(isolated system).} \tag{4.12}$$

You may be tempted to conclude from this observation that momentum is conserved. All we can conclude, however, is that there is no creation or destruction of momentum *inside the systems we considered*—not that it is impossible to create or destroy momentum under any circumstances.

Because it is a direct consequence of the definition of momentum, Eq. 4.10 must—by definition—hold for *any* object that collides with the inertial standard. So if we take any two objects 1 and 2 and let each collide with the inertial standard on a level, low-friction track, we *always* obtain—no matter how we vary the initial velocities and no matter what the values of inertias m_1 and m_2 are—that

$$\Delta\vec{p}_1 + \Delta\vec{p}_s = \vec{0} \quad \text{(by definition)} \tag{4.13}$$

and

$$\Delta\vec{p}_2 + \Delta\vec{p}_s = \vec{0} \quad \text{(by definition).} \tag{4.14}$$

Suppose now that, instead of making objects 1 and 2 collide with the inertial standard, we make them collide with each other. Should we expect to obtain

$$\Delta\vec{p}_1 + \Delta\vec{p}_2 = \vec{0}? \tag{4.15}$$

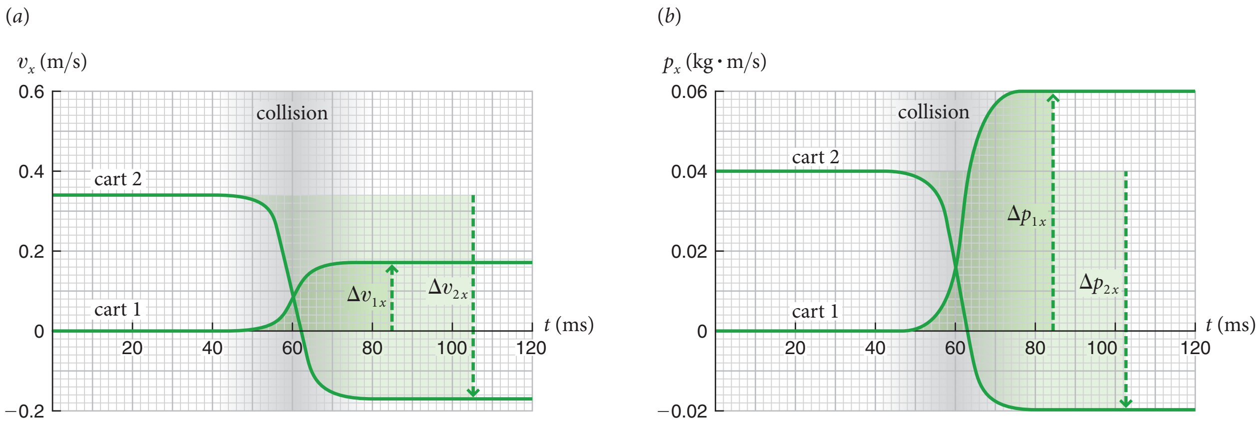
You may be tempted to think that this equation follows from our earlier observations, but that is not the case. The value of $\Delta\vec{p}_s$ in Eqs. 4.13 and 4.14 need not be the same because Δv_{sx}, and therefore $\Delta\vec{p}_s$, vary from collision to collision (compare, for instance, Figures 4.8 and 4.9). So each of these equations holds for a separate collision and therefore we cannot combine them to draw any conclusions about the relationship between $\Delta\vec{p}_1$ and $\Delta\vec{p}_2$. The only way to verify whether Eq. 4.15 holds is to do the measurement.

Figure 4.23 shows the graphs for a collision between two arbitrary carts. To get the momentum values shown in Figure 4.23*b*, I multiplied the *x* component of the velocity of each cart by the cart's inertia determined by *independently* letting each cart collide with the inertial standard. The change in the *x* component of the momentum of cart 1 can be obtained by multiplying its inertia ($m_1 = 0.36$ kg) with the *x* components of the initial and final velocities read off from Figure 4.23*a* ($v_{1x,i} = 0$ and $v_{1x,f} = +0.17$ m/s):

$$\begin{aligned}\Delta p_{1x} &= m_1(v_{1x,f} - v_{1x,i}) = (0.36\ \text{kg})(+0.17\ \text{m/s} - 0)\\ &= 0.061\ \text{kg}\cdot\text{m/s}.\end{aligned}$$

QUANTITATIVE TOOLS

Figure 4.23 (*a*) Velocity-versus-time graph and (*b*) momentum-versus-time graph of two carts before and after the two collide on a low-friction track. The sum of the momenta is the same before and after the collision. The inertias are $m_1 = 0.36$ kg and $m_2 = 0.12$ kg.



For cart 2 we have $v_{2x,\mathrm{i}} = +0.34$ m/s, $v_{2x,\mathrm{f}} = -0.17$ m/s, and $m_2 = 0.12$ kg, so

$$\Delta p_{2x} = m_2(v_{2x,\mathrm{f}} - v_{2x,\mathrm{i}}) = (0.12\ \mathrm{kg})(-0.17\ \mathrm{m/s} - 0.34\ \mathrm{m/s})$$
$$= -0.061\ \mathrm{kg \cdot m/s}.$$

Because the two changes in momentum are equal in magnitude and of opposite sign, they add up to zero: $\Delta p_{1x} + \Delta p_{2x} = 0$. Consequently the momentum of the system comprising the two carts does not change:

$$\Delta\vec{p} \equiv \Delta\vec{p}_1 + \Delta\vec{p}_2 = \vec{0}. \tag{4.16}$$

Repeating the experiment with another pair of carts or changing the initial velocities always yields the same result: The momentum of the system comprising any two colliding objects is not changed by the collision. The collision merely transfers some momentum from one cart to the other—one gains a certain amount of momentum, and the other loses the same amount.

4.13 (*a*) How much momentum is transferred in the collisions in Figure 4.23? (*b*) Is this momentum transferred from cart 1 to cart 2, or in the opposite direction?

Experiments show that *any* interaction between two objects—not just collisions—transfers momentum from one object to the other. The sum of the momenta of the interacting objects, however, never changes. No interactions or other phenomena have ever been observed where momentum is created out of nothing or destroyed. We must therefore conclude that momentum is conserved:

Momentum can be transferred from one object to another, but it cannot be created or destroyed.

This statement is one of the most fundamental principles of physics and is often referred to as the **conservation of momentum.**

For an isolated system, conservation of momentum means

$$\Delta\vec{p} = \vec{0} \quad \text{(isolated system).} \tag{4.17}$$

For a system that is not isolated, we have

$$\Delta\vec{p} = \vec{J}, \tag{4.18}$$

where $\vec{J}$ represents the transfer of momentum from the environment to the system. The quantity $\vec{J}$ is called the **impulse** delivered to the system. Like momentum, impulse is a vector and has SI units of kg · m/s. Depending on its direction relative to the momentum, the impulse can increase or decrease the magnitude of the momentum of the system. $\vec{J}$ therefore represents what we called "input" and "output" in Section 4.4.

4.14 (*a*) What is the magnitude of the impulse delivered to cart 1 in Figure 4.23? (*b*) Write the impulse delivered to cart 1 in vector form. (*c*) Does the fact that the change in the momentum of cart 1 is nonzero mean that momentum is not conserved?

Equation 4.18, which we shall use extensively throughout this book, is called the **momentum law.** This equation embodies conservation of momentum because it tells us that the momentum of a system can change only due to a transfer of momentum to the system ($\vec{J}$). If momentum were not conserved, then the right side of this equation would need to contain an additional term to account for changes in momentum due to the creation or destruction of momentum. For an isolated system the impulse is zero, $\vec{J} = \vec{0}$, and the momentum law takes on the form of Eq. 4.17.

Example 4.8 Bounce

A 0.20-kg rubber ball is dropped from a height of 1.0 m onto a hard floor and bounces straight up. Assuming the speed with which the ball rebounds from the floor is the same as the speed it has just before hitting the floor, determine the impulse delivered by the floor to the rubber ball.

❶ GETTING STARTED I define the ball to be my system in this problem. The impulse delivered to the ball is then given by the change in its momentum (Eq. 4.18). I need to develop a way to determine this change in momentum.

❷ DEVISE PLAN To solve this problem I need to first determine the velocity of the ball just before it hits the floor. I therefore break the problem into two parts: the downward fall of the ball and its collision with the floor. I can use Eq. 3.15 to determine the time interval it takes the ball to fall from its initial height (assuming the ball is initially at rest). As it falls, the ball undergoes constant acceleration, so I can use Eq. 3.10 to calculate its velocity just before it hits the floor and, because its speed is not changing as it rebounds, I also know its velocity after it bounces up. Knowing the velocities, I can calculate the ball's momentum before and after the bounce using Eq. 4.7. The change in momentum then gives the impulse according to Eq. 4.18.

❸ EXECUTE PLAN From Eq. 3.15 I see that it takes an object $t = \sqrt{2h/g} = \sqrt{2(1.0\text{ m})/(9.8\text{ m/s}^2)} = 0.45\text{ s}$ to fall from a height of 1.0 m. Choosing the positive x axis pointing upward and substituting this result into Eq. 3.10, I obtain for the x component of the velocity of the ball just before it hits the floor:

$$v_{x,\text{f}} = 0 + (-9.8\text{ m/s}^2)(0.45\text{ s})^2 = -2.0\text{ m/s}.$$

Now that I know the ball's velocity just before it hits the ground I can obtain the x component of the momentum of the ball just before it hits the ground by multiplying the ball's velocity by its inertia: $p_{x,\text{i}} = (0.20\text{ kg})(-2.0\text{ m/s}) = -0.40\text{ kg}\cdot\text{m/s}$. (I use the subscript i to indicate that this is the initial momentum of the ball before the collision with the floor.) If the ball rebounds with the same speed, then the x component of the momentum after the collision with the floor has the same magnitude but opposite sign: $p_{x,\text{f}} = +0.40\text{ kg}\cdot\text{m/s}$. The change in the ball's momentum is thus

$$\begin{aligned}\Delta p_x = p_{x,\text{f}} - p_{x,\text{i}} &= +0.40\text{ kg}\cdot\text{m/s} - (-0.40\text{ kg}\cdot\text{m/s})\\ &= +0.80\text{ kg}\cdot\text{m/s}.\end{aligned}$$

The interaction with the ground changes the momentum of the ball, making it rebound. The ball does not constitute an isolated system, and the change in its momentum is due to an impulse delivered by Earth to the ball. To determine the impulse, I substitute the change in momentum of the ball into Eq. 4.18:

$$\vec{J} = \Delta\vec{p} = \Delta p_x\,\hat{\imath} = (+0.80\text{ kg}\cdot\text{m/s})\hat{\imath}. \checkmark$$

❹ EVALUATE RESULT The x component of the velocity of the ball just before it hits the floor is negative because the ball moves downward, in the negative x direction. After the collision it moves in the opposite direction, and consequently the x components of the changes in velocity and momentum are both positive, as I calculated. Because the x component of the change in the ball's momentum is positive, the impulse is directed upward (in the positive x direction). This makes sense because this impulse changes the direction of travel of the ball from downward to upward.

Because the momentum law takes on the simplest form for an isolated system, we shall usually try to choose an isolated system (unlike the above example). In such a case, we can write out the $\Delta\vec{p}$ term in Eq. 4.17:

$$\vec{p}_{\mathrm{f}} = \vec{p}_{\mathrm{i}} \quad \text{(isolated system).} \tag{4.19}$$

In this form, the momentum law states that the initial and final values of the momentum of an isolated system are equal. For two colliding (or otherwise interacting) objects moving along the x axis, Eqs. 4.17 and 4.19 become

$$\Delta p_{1x} + \Delta p_{2x} = 0 \quad \text{(isolated system)} \tag{4.20}$$

and

$$p_{1x,\mathrm{f}} + p_{2x,\mathrm{f}} = p_{1x,\mathrm{i}} + p_{2x,\mathrm{i}} \quad \text{(isolated system).} \tag{4.21}$$

Equations 4.20 and 4.21 are equivalent: Eq. 4.20 states that any change in momentum in one object is made up by a change of the same magnitude but in the opposite direction in the other object; Eq. 4.21 states that the sum of the momenta is the same before and after the collision. Which of these two equations you use in solving a problem is a matter of convenience.

4.15 (*a*) Are the changes in velocity in Figure 4.23*a* equal in magnitude? Why or why not? (*b*) Determine the velocity changes of the carts and verify that $m_1/m_2 = -\Delta v_{2x}/\Delta v_{1x}$ in Figure 4.23*a*. (*c*) Determine the initial and final momenta of the two carts. (*d*) What is the momentum of the system before the collision? (*e*) After the collision? (*f*) Are the momentum changes equal in magnitude and opposite in direction? Why or why not?

To convince you of the fundamental importance of the conservation of momentum, let me show you something else. Suppose we let carts 1 and 2 of Figure 4.23 collide with the same initial velocities as before, but this time we put something sticky between them, so that they remain locked together once they hit each other, as in **Figure 4.24** (next page). The velocities of the two carts *after* the collision must now be identical.

4.16 (*a*) Do the two carts in Figure 4.24 still constitute an isolated system like the carts in Figure 4.23? (*b*) What does your answer to part *a* imply about the momentum of the system comprising the two carts after the collision?

As you saw in the preceding checkpoint, the system of the two carts is isolated and so the momentum of the system should not change. **Figure 4.25*a*** on page 99 shows graphically that the final velocity of the combined carts is somewhere between the values of the two initial velocities. Before the collision, cart 1 is at rest and the x component of the velocity of cart 2 is $+0.34$ m/s; after the collision, both carts move at $v_{x,\mathrm{f}} = +0.085$ m/s. The x component of the momentum of the system before the collision is thus $p_{1x,\mathrm{i}} + p_{2x,\mathrm{i}} = 0 + (0.12\ \mathrm{kg})(0.34\ \mathrm{m/s}) = 0.041\ \mathrm{kg \cdot m/s}$. After the collision, the x component of the momentum of the system is $p_{1x,\mathrm{f}} + p_{2x,\mathrm{f}} = (0.36\ \mathrm{kg})(0.085\ \mathrm{m/s}) + (0.12\ \mathrm{kg})(0.085\ \mathrm{m/s}) = 0.031\ \mathrm{kg \cdot m/s} + 0.010\ \mathrm{kg \cdot m/s} = 0.041\ \mathrm{kg \cdot m/s}$. The data indeed confirm that the momentum of the system does not change.

Note that the before-collision parts of Figures 4.23*a* and 4.25*a* are identical. The after-collision parts of these two figures are quite different from each other, however. Yet even with these two entirely different outcomes—the carts in Figure 4.23 flying off in opposite directions after colliding and those in Figure 4.25 sticking together—the momentum of the system does not change in both cases:

$$\vec{p}_{\mathrm{i}} = \vec{p}_{\mathrm{f}} \quad \text{(isolated system).} \tag{4.22}$$

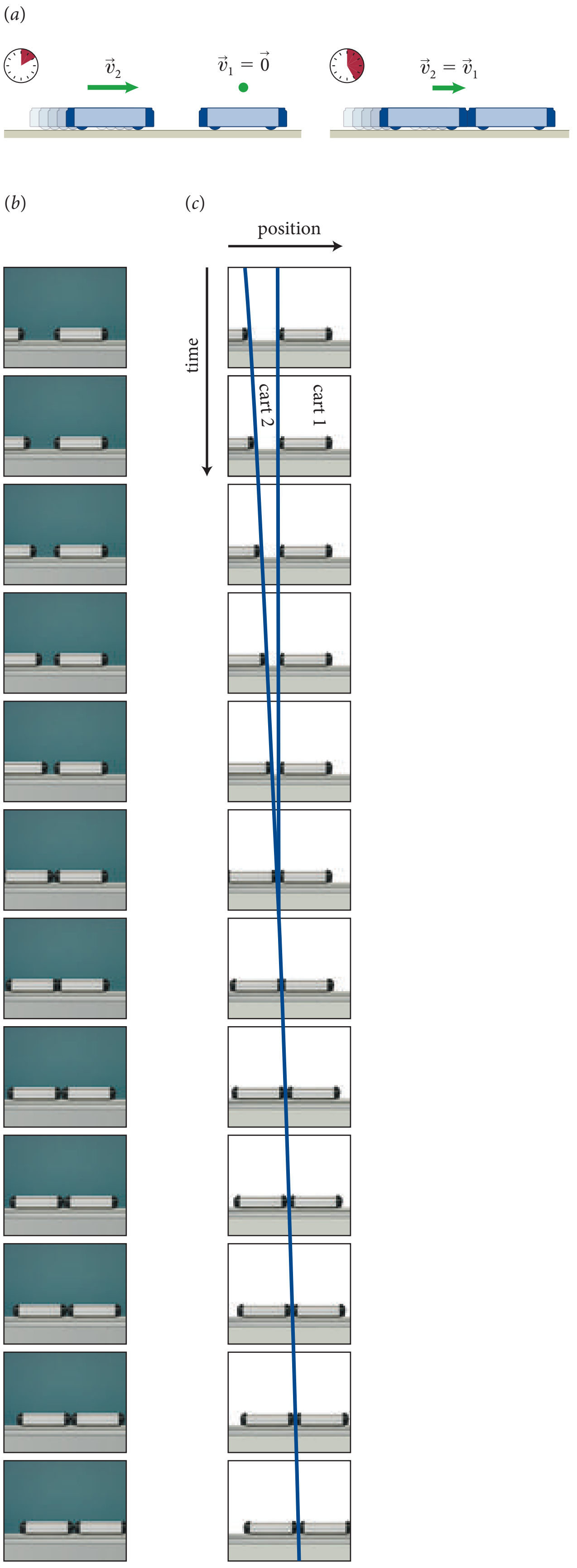
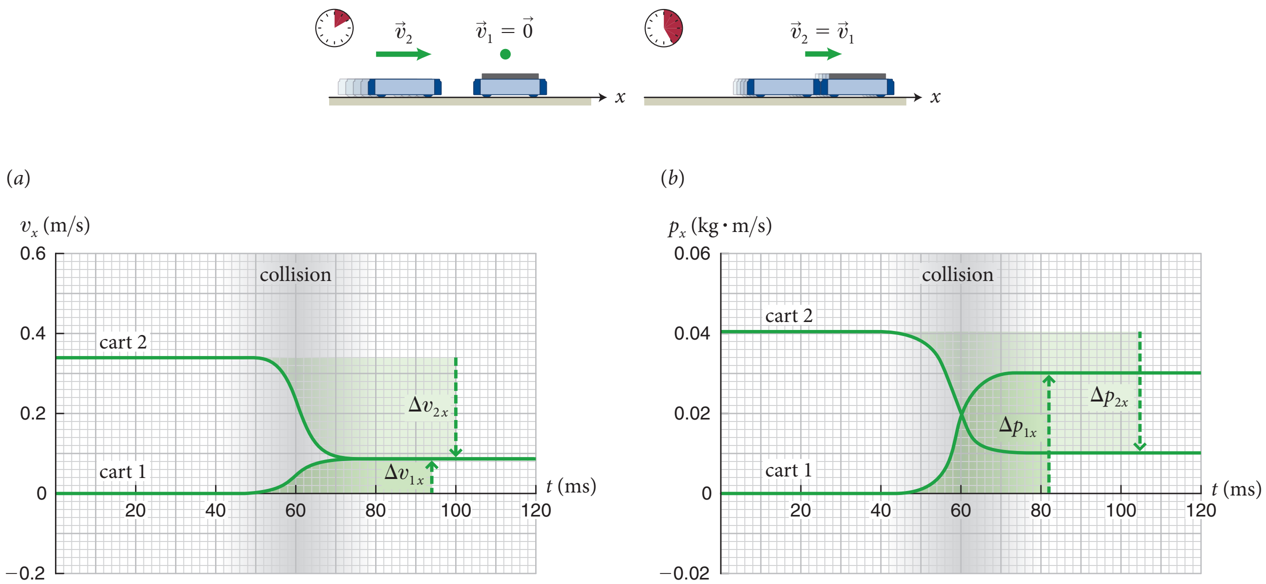


Figure 4.24 (*a*) Two identical carts collide and stick together on a low-friction track. (*b*) High-speed film sequence of such a collision. The length of track visible in each frame is 0.40 m. (*c*) Curves indicating the positions of the rear of cart 1 and the front of cart 2 superimposed on the sequence.

Figure 4.25 (*a*) Velocity-versus-time graph and (*b*) momentum-versus-time graph for the carts of Figure 4.24. Notice that the sum of the momenta is again the same before and after the collision. The inertia of cart 1 is 0.36 kg, and that of cart 2 is 0.12 kg.



The momentum law is not confined to two objects. Suppose we put *three* objects in motion on a low-friction track such that they all end up colliding with one another. Let 1 first collide with 2 and then 2 with 3 (you can pick whatever sequence you like). In the first collision, $\vec{p}_1$ and $\vec{p}_2$ change but their sum remains unchanged. In addition, the sum of all three momenta remains unchanged because nothing has happened to $\vec{p}_3$ yet. When 2 and 3 collide, $\vec{p}_2$ and $\vec{p}_3$ change but the sum $\vec{p}_2 + \vec{p}_3$ does not change. So again, the momentum of the system comprising all three remains constant. In other words, throughout the two collisions, the momentum of the system comprising all three objects, $\vec{p}$, remains unchanged:

$$\vec{p}_\mathrm{i} = \vec{p}_{1,\mathrm{i}} + \vec{p}_{2,\mathrm{i}} + \vec{p}_{3,\mathrm{i}} = \vec{p}_{1,\mathrm{f}} + \vec{p}_{2,\mathrm{f}} + \vec{p}_{3,\mathrm{f}} = \vec{p}_\mathrm{f} \quad \text{(isolated system).} \quad (4.23)$$

This argument can be extended to any number of objects and any number of collisions or interactions.

That we can apply conservation of momentum to predict the final momentum of carts colliding on a low-friction track may seem like nothing special because you are indeed unlikely to encounter this situation outside the physics classroom. Conservation of momentum has much broader applications, however, because it governs *everything that happens in the universe.* Conservation of momentum applies to atoms and elementary particles at the subatomic scale, to stars and galaxies at the cosmic scale, and to everything in between. Next time you fly in an airplane consider this: you are moving forward because of conservation of momentum. The aircraft engines "throw air backward" so the airplane and the passengers move forward. Conservation of momentum is used to solve many science and engineering problems: from calculating the forces of impact during vehicle collisions, to the trajectories of satellites, to the load-carrying capabilities of artificial limbs.

4.17 A railroad car moves at velocity v_i on a track toward three other railroad cars at rest on the track. All the cars are identical, and each stationary car is some distance from the others. The moving car hits the first stationary car and locks onto it. Then the two hit the second stationary car, lock onto that one, and so on. When all four cars have locked onto one another, what is the velocity v_f of the four-car train?

Chapter Glossary

SI units of physical quantities are given in parentheses.

Conservation of momentum Momentum can be transferred from one object to another, but it cannot be created or destroyed. The momentum of an isolated system therefore cannot change:

$$\Delta\vec{p} = \vec{0} \quad \text{(isolated system)}. \tag{4.17}$$

Conserved quantity A quantity that cannot be created or destroyed.

Environment Everything that is not part of the system.

Extensive quantity If the value of a quantity for a system is equal to the sum of the values of that quantity for each part of the system, then the quantity is extensive.

Impulse $\vec{J}$ (kg · m/s) A vector defined as the transfer of momentum from the environment to the system due to an interaction between the two. For an isolated system, the impulse is zero: $\vec{J} = \vec{0}$.

Inertia m (kg) A scalar that is a measure of an object's tendency to resist any change in its velocity. The inertia m_u of any object can be found by letting that object collide with the inertial standard $m_s = 1$ kg along the x axis and taking the negative of the ratio of the changes in the x components of the velocities:

$$m_u \equiv -\frac{\Delta v_{sx}}{\Delta v_{ux}} m_s. \tag{4.2}$$

Intensive quantity If the value of a quantity for a system is equal to the value of that quantity for any part of that system, then that quantity is intensive.

Isolated system A system for which there are no external interactions. For such a system there is no transfer of momentum into or out of the system. See the Procedure box on page 93.

Kilogram (kg) The SI base unit of inertia, defined as being equal to the inertia of a platinum-iridium cylinder stored in Sèvres, France.

Momentum $\vec{p}$ (kg · m/s) A vector that is the product of the inertia and the velocity of an object:

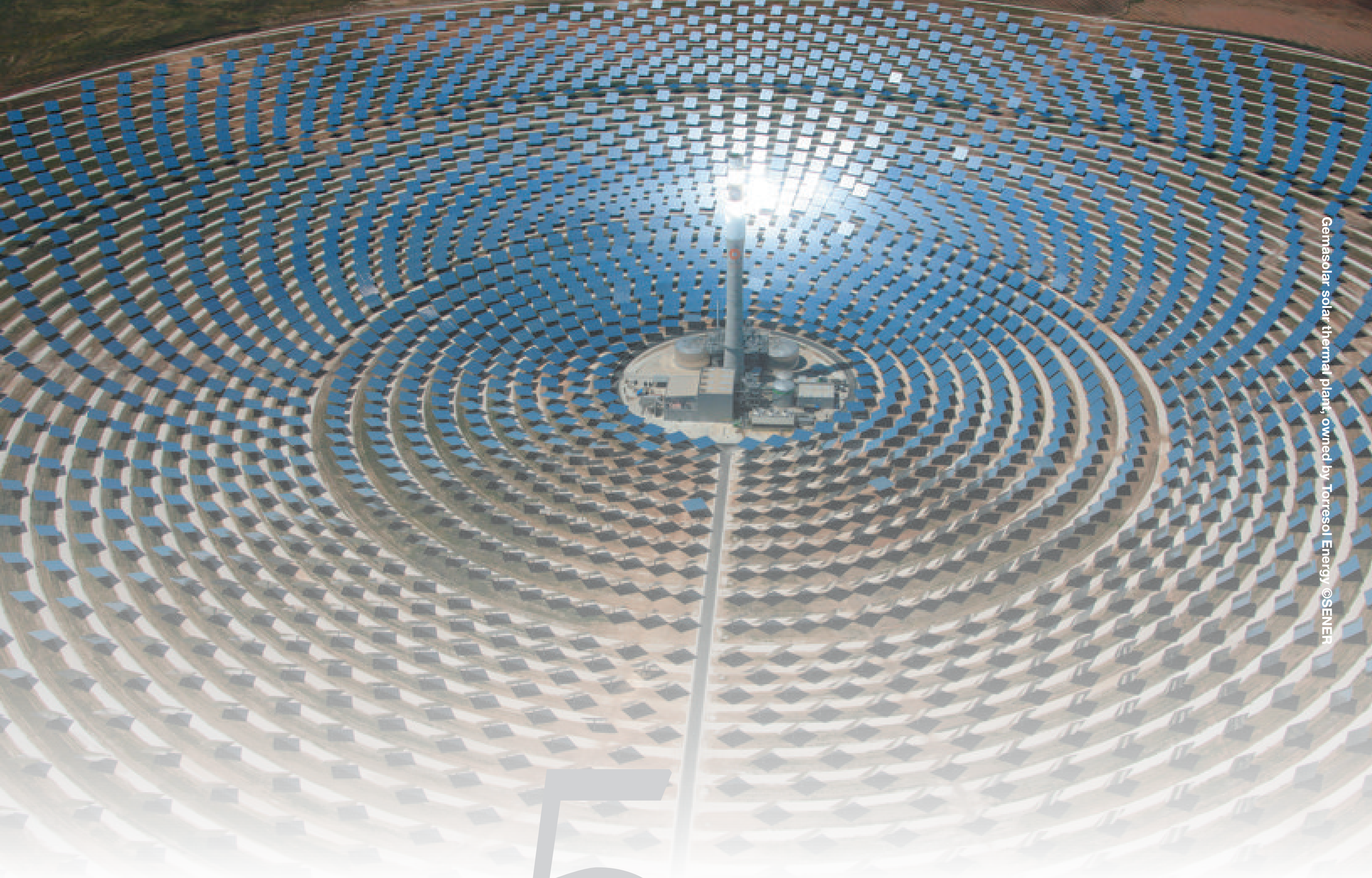
$$\vec{p} \equiv m\vec{v}. \tag{4.6}$$

Momentum law The law that accounts for the change in the momentum of a system or object. Because momentum is conserved, the momentum of a system can change only due to a transfer of momentum between the environment and the system:

$$\Delta\vec{p} = \vec{J}. \tag{4.18}$$

System Any object or group of objects that we can separate, in our minds, from the surrounding environment.

System diagram A schematic diagram that shows a system's initial and final conditions.



Gemasolar solar thermal plant, owned by Torresol Energy ©SENER

5 Energy

CONCEPTS

QUANTITATIVE TOOLS

Now that we know about conservation of momentum, can we determine the final velocities of two colliding objects if the only things we know are the initial velocities and the fact that the momentum is conserved? The answer is *no* if our only tool is the momentum law. In the collisions represented in Figures 4.23 and 4.25, for instance, momentum is unchanged in both cases, and yet the two outcomes are definitely not the same. So knowing only that momentum remains constant isn't enough. We need additional information in order to predict future positions and velocities. In the process of looking for this additional information, we shall develop another conservation law—the law of conservation of *energy*.

5.1 Classification of collisions

If you look at the velocity-versus-time graphs in Chapter 4, you will notice that the velocity difference in the two carts, $\vec{v}_2 - \vec{v}_1$, in most cases has the same magnitude before and after the collision. **Figure 5.1** shows two graphs of collisions we considered in Chapter 4 and highlights the velocity difference before and after the collision. This difference is the **relative velocity** of the carts: $\vec{v}_{12} \equiv \vec{v}_2 - \vec{v}_1$ is the velocity of cart 2 relative to cart 1 and $\vec{v}_{21} \equiv \vec{v}_1 - \vec{v}_2$ is the velocity of cart 1 relative to cart 2. The subscript in the relative velocity symbol is always shown with the object we are studying printed last: v_{12} is the velocity *of cart 2* and v_{21} is the velocity *of cart 1*. (We cannot use Δ here because we reserve that symbol to denote the change in a *single* quantity; now we are dealing with the difference between the same quantity for two different objects.)

The magnitude of the relative velocity is called the **relative speed.** Thus $v_{12} \equiv |\vec{v}_2 - \vec{v}_1|$ is the speed of cart 2 (last subscript in v_{12}) relative to cart 1. For motion along the x axis, the relative speed is the absolute value of the difference in the x components of the velocities: $v_{12} \equiv |v_{2x} - v_{1x}|$. Note that the sequence of the subscripts for relative speeds does not matter: $v_{12} \equiv |v_{2x} - v_{1x}| = |v_{1x} - v_{2x}| = v_{21}$ (the speed of cart 2 relative to cart 1 is equal to the speed of cart 1 relative to cart 2).

A collision in which the relative speed before the collision is the same as the relative speed after the collision is called an **elastic collision.** Collisions between hard objects are generally elastic collisions. For instance, a superball bouncing off a hard floor bounces up with nearly the same speed with which it came down. Thus the relative speed of floor and ball does not change, and the collision is elastic.

5.1 Two cars are moving along a highway with neither one accelerating. Is their relative speed equal to the difference between their speeds? Why or why not?

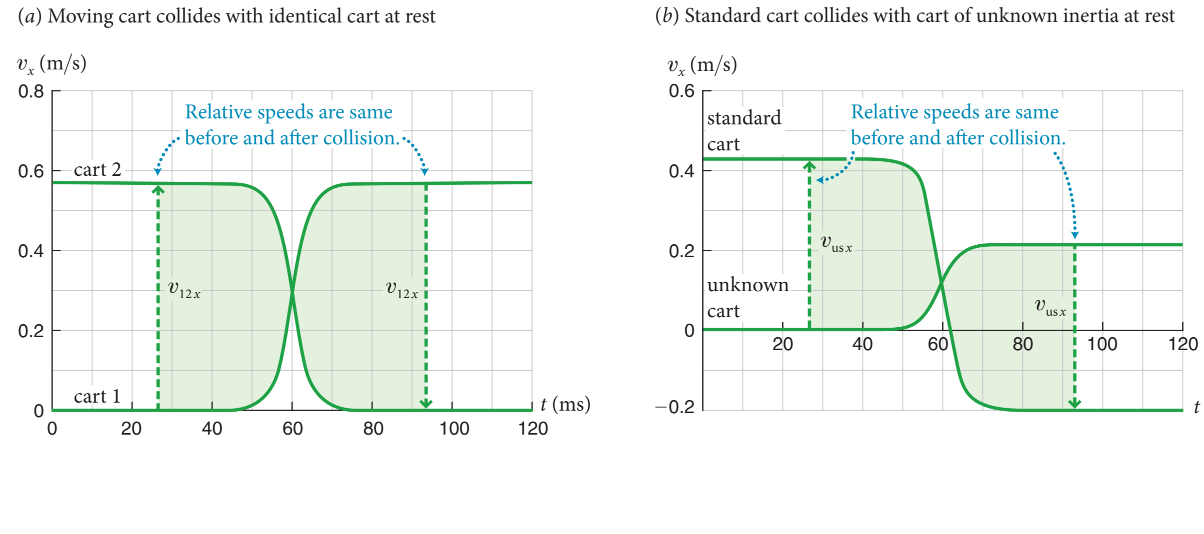
Any collision for which the relative speed after the collision is lower than that before the collision is called an **inelastic collision.** If you drop a tennis ball to the floor, it rebounds with a lower speed than it had when it hit the floor. The relative speed is reduced, and the collision is inelastic.

A special type of inelastic collision is one in which the two objects move together after the collision so that their relative speed is reduced to zero. We call this special case a **totally inelastic collision.** Imagine dropping a ball of dough to the floor. Splat! The dough sticks to the floor, and the relative speed of dough and floor is reduced to zero. The collision in Figure 4.25 is another example of an inelastic collision.

As you might imagine, whether a collision is elastic, inelastic, or totally inelastic depends on the properties of the objects involved. However, if we know what happens to the relative speed in a collision, we can use that knowledge together with conservation of momentum to determine the final velocities.

CONCEPTS

Figure 5.1 Velocity-versus-time graphs of (*a*) two identical carts colliding on a low-friction track, with one of the carts initially at rest, and (*b*) a standard cart colliding with a cart of unknown inertia that is initially at rest. Notice that for both collisions the relative speeds of the two carts are the same before and after the collision.



Exercise 5.1 Classifying collisions

Are the following collisions elastic, inelastic, or totally inelastic? (*a*) A red billiard ball moving at $v_{\text{r}x,\text{i}} = +2.2$ m/s hits a white billiard ball initially at rest. After the collision, the red ball is at rest and the white ball moves at $v_{\text{w}x,\text{f}} = +1.9$ m/s. (*b*) Cart 1 moving along a track at $v_{1x,\text{i}} = +1.2$ m/s hits cart 2 initially at rest. After the collision, the two carts move at $v_{1x,\text{f}} = +0.4$ m/s and $v_{2x,\text{f}} = +1.6$ m/s. (*c*) A piece of putty moving at $v_{\text{p}x,\text{i}} = +22$ m/s hits a wooden block moving at $v_{\text{b}x,\text{i}} = +1.0$ m/s. After the collision, the two move at $v_{x,\text{f}} = +1.7$ m/s.

SOLUTION (*a*) The initial relative speed is $v_{\text{wr,i}} = |v_{\text{r}x,\text{i}} - v_{\text{w}x,\text{i}}| = |+2.2\text{ m/s} - 0| = 2.2$ m/s; the final relative speed is $v_{\text{wr,f}} = |v_{\text{r}x,\text{f}} - v_{\text{w}x,\text{f}}| = |0 - 1.9\text{ m/s}| = 1.9$ m/s, lower than the initial relative speed, which means the collision is inelastic. ✔

(*b*) $v_{12\text{i}} = |v_{2x,\text{i}} - v_{1x,\text{i}}| = |0 - (+1.2\text{ m/s})| = 1.2$ m/s; $v_{12\text{f}} = |v_{2x,\text{f}} - v_{1x,\text{f}}| = |+1.6\text{ m/s} - (+0.4\text{ m/s})| = 1.2$ m/s. Because the relative speeds are the same, the collision is elastic. ✔

(*c*) After the collision, both the putty and the block travel at the same velocity, making their relative speed zero. The collision is totally inelastic. ✔

5.2 (*a*) An outfielder catches a baseball. Is the collision between ball and glove elastic, inelastic, or totally inelastic? (*b*) When a moving steel ball 1 collides head-on with a steel ball 2 at rest, ball 1 comes to rest and ball 2 moves away at the initial speed of ball 1. Which type of collision is this? (*c*) Is the sum of the momenta of the two colliding objects constant in part *a*? In part *b*?

5.2 Kinetic energy

Relative speed is not an extensive quantity, and so we cannot develop an accounting scheme for it as we did for momentum in Chapter 4. The trick in studying elastic collisions therefore is to obtain a quantity that allows us to express the fact that the relative speed doesn't change in the form: (something of object 1) + (something of object 2) doesn't change. As I shall show you in Section 5.5, for an object of inertia m moving at speed v this something is the quantity $K = \frac{1}{2}mv^2$, called the object's **kinetic energy** (literally "energy* associated with motion"). Unlike relative speed, which refers to two objects, kinetic energy is associated with a single object. Furthermore, kinetic energy is always positive and independent of the direction of motion (which means that it is a scalar). A ball moving at speed v to the left has exactly the same kinetic energy as the same ball moving at speed v to the right (or any other direction in space). Whenever the speed of an object changes, the kinetic energy changes.

5.3 Is kinetic energy an extensive quantity?

To develop some feel for this new quantity and its relationship to relative speed, let us calculate the kinetic energies of the carts before and after the collisions shown in **Figure 5.2.** The two collisions have identical initial conditions, but one is elastic (Figure 5.2*a*) and the other is totally inelastic (Figure 5.2*b*).

Table 5.1 gives the initial and final kinetic energies of the carts, obtained by reading off the x components of the velocities from the figure. For the elastic collision, where the relative speed doesn't change, the sum of the two kinetic energies before the collision is equal to the sum after the collision: $(0.12 + 0)\text{ kg}\cdot\text{m}^2/\text{s}^2 = (0.086 + 0.029)\text{ kg}\cdot\text{m}^2/\text{s}^2$. For the totally inelastic collision, both the relative speed and the sum of the two kinetic energies change.

In general we observe:

In an elastic collision, the sum of the kinetic energies of the objects before the collision is the same as the sum of the kinetic energies after the collision.

*For now think of "energy" as the capacity to do things like accelerating and heating objects. We shall develop a more complete picture of what energy is in later sections and chapters.

Figure 5.2 Velocities for elastic and inelastic collisions between two carts. Cart 1 has inertia 0.12 kg, and cart 2 has inertia 0.36 kg.

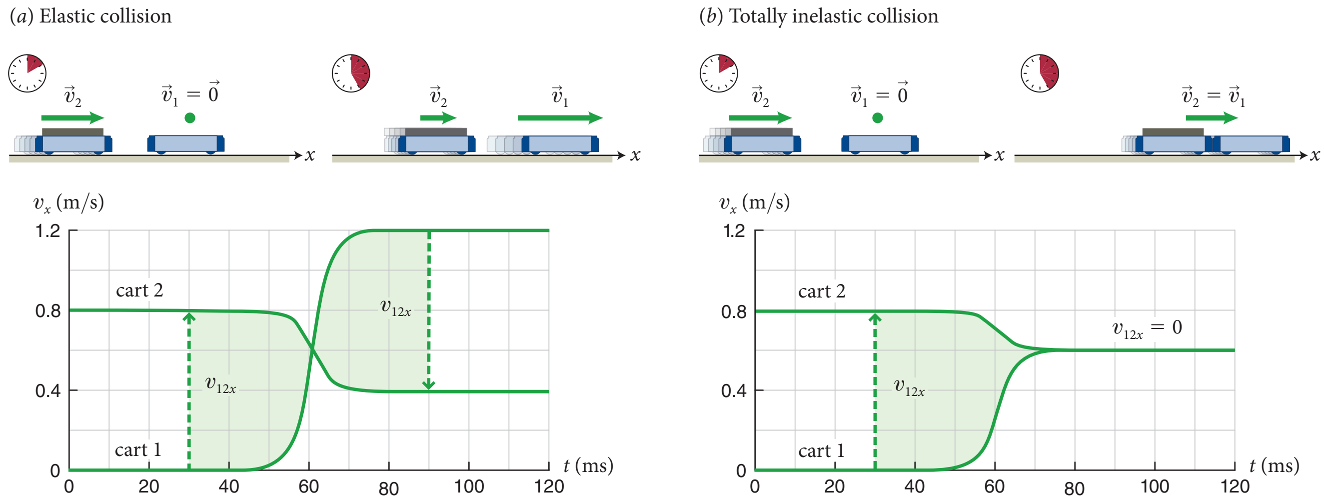


Table 5.1 Kinetic energy in elastic and totally inelastic collisions

		ELASTIC				TOTALLY INELASTIC			
	Inertia m (kg)	Velocity v_x (m/s)		Kinetic energy $\frac{1}{2}mv^2$ (kg · m²/s²)		Velocity v_x (m/s)		Kinetic energy $\frac{1}{2}mv^2$ (kg · m²/s²)	
		before	*after*	*before*	*after*	*before*	*after*	*before*	*after*
Cart 1	0.12	0	+1.2	0	0.086	0	+0.60	0	0.022
Cart 2	0.36	+0.80	+0.40	0.12	0.029	+0.80	+0.60	0.12	0.065
Relative speed		**0.80**	**0.80**			**0.80**	0		
Kinetic energy of system				**0.12**	**0.12**			**0.12**	**0.087**

In Section 5.5 we'll see that this statement is equivalent to the requirement that the relative speed remains the same.

For problems involving elastic collisions, we now have two tools: the momentum conservation law, which tells us that the momentum of an isolated system doesn't change, and the fact that the relative speed of the colliding objects doesn't change (or, alternatively, the fact that the kinetic energy of the system doesn't change). Let's now use both tools in an example.

Example 5.2 Carts colliding

(*a*) Is the collision in Figure 4.23 elastic, inelastic, or totally inelastic? How can you tell? (*b*) Verify your answer by comparing the initial kinetic energy of the two-cart system with the final kinetic energy.

❶ **GETTING STARTED** In Figure 4.23, it looks as if the initial and final relative speeds are the same, which makes the collision elastic. The problem asks me to confirm this fact by calculating the initial and final kinetic energies of the system.

❷ **DEVISE PLAN** To answer part *a*, I need to determine the initial and final relative speeds of the carts from the velocities, which I get from Figure 4.23*a*: $v_{1x,i} = 0$; $v_{2x,i} = +0.34$ m/s; $v_{1x,f} = +0.17$ m/s; $v_{2x,f} = -0.17$ m/s. To answer part *b*, I use $K = \frac{1}{2}mv^2$. The inertias of the carts are given in the figure caption: $m_1 = 0.36$ kg and $m_2 = 0.12$ kg.

❸ **EXECUTE PLAN** (*a*) $v_{12i} = |v_{2x,i} - v_{1x,i}| = |(+0.34\text{ m/s}) - 0| = 0.34$ m/s; $v_{12f} = |v_{2x,f} - v_{1x,f}| = |(-0.17\text{ m/s}) - (+0.17\text{ m/s})| = 0.34$ m/s. The relative speed is unchanged, and thus the collision is elastic. ✔

(*b*) The initial values are

$$K_{1i} = \tfrac{1}{2}m_1v_{1i}^2 = \tfrac{1}{2}(0.36\text{ kg})(0)^2 = 0$$

$$K_{2i} = \tfrac{1}{2}m_2v_{2i}^2 = \tfrac{1}{2}(0.12\text{ kg})(0.34\text{ m/s})^2 = 0.0069\text{ kg}\cdot\text{m}^2/\text{s}^2$$

so $\quad K_i = K_{1i} + K_{2i} = 0.0069\text{ kg}\cdot\text{m}^2/\text{s}^2.$

The final values are

$$K_{1f} = \tfrac{1}{2}m_1v_{1f}^2 = \tfrac{1}{2}(0.36\text{ kg})(0.17\text{ m/s})^2 = 0.0052\text{ kg}\cdot\text{m}^2/\text{s}^2$$

$$K_{2f} = \tfrac{1}{2}m_2v_{2f}^2 = \tfrac{1}{2}(0.12\text{ kg})(-0.17\text{ m/s})^2 = 0.0017\text{ kg}\cdot\text{m}^2/\text{s}^2$$

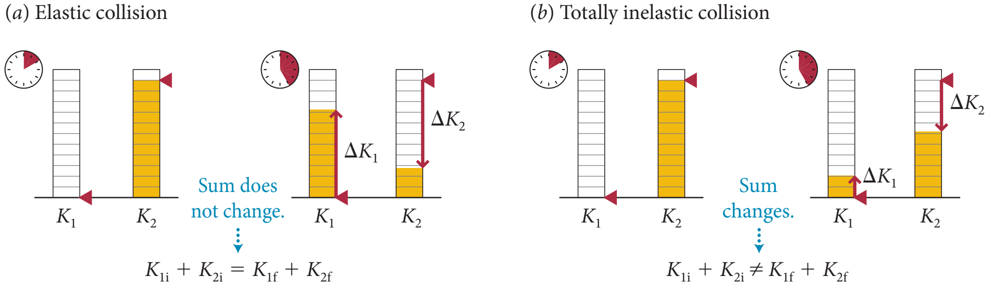
so
$$K_f = (0.0052\text{ kg}\cdot\text{m}^2/\text{s}^2) + (0.0017\text{ kg}\cdot\text{m}^2/\text{s}^2) = 0.0069\text{ kg}\cdot\text{m}^2/\text{s}^2,$$

which is the same as before the collision, as it should be for an elastic collision. ✔

❹ **EVALUATE RESULT** Because I've reached the same conclusion—the collision is elastic—using two approaches, I can be pretty confident that my solution is correct.

Because kinetic energy is a scalar extensive quantity, bar diagrams are a good way to visually represent changes in this quantity. **Figure 5.3**, for example, shows the initial and final kinetic energies of the carts in the two collisions of Figure 5.2. Before the collision, only cart 2 has kinetic energy; after the collision, both carts have kinetic energy. For the elastic collision (Figure 5.3*a*), the sum of the kinetic

Figure 5.3 Bar diagrams representing the initial and final kinetic energies of the carts for the collisions shown in Figure 5.2.
(*a*) The sum of the kinetic energies does not change for the elastic collision because the changes are equal and opposite.
(*b*) The sum of the kinetic energies changes for the totally inelastic collision: The change in K_1 is smaller than the change in K_2.



energies after the collision is the same as the sum before the collision because the change in the kinetic energy of cart 1 is the negative of that of cart 2. For the totally inelastic collision (Figure 5.3*b*), the changes in the kinetic energies do not cancel, and so the system's kinetic energy after the collision is not equal to the system's kinetic energy before the collision.

5.4 A moving cart collides with an identical cart initially at rest on a low-friction track, and the two lock together. What fraction of the initial kinetic energy of the system remains in this totally inelastic collision?

Figure 5.4 Before or after? (*a*) Inelastic collisions are an irreversible process. There is no question that the picture was taken *after* the accident because the state of the two cars has changed. (*b*) Elastic collisions are a reversible process. Can you tell whether this picture was taken before or after the ball collided with the racquet?

(*a*)

(*b*)

5.3 Internal energy

In both inelastic and totally inelastic collisions, the relative speed changes and therefore the kinetic energy of the system changes. For example, the decrease in the kinetic energy of cart 2 in Figure 5.2*b*, represented by the downward pointing arrow labeled ΔK_2 in Figure 5.3*b*, is larger than the increase in the kinetic energy K_1 of cart 1, resulting in a decrease in the kinetic energy of the stuck-together carts. Where does this kinetic energy go? Does it simply vanish, or does it go elsewhere?

We can determine the answer to this question by looking more closely at inelastic collisions between objects and noticing that changes occur in the **state** of one or both objects. What I mean by *state* is the condition of the object as specified by some complete set of physical parameters: shape, temperature, whatever—*every possible physical variable that defines the object*. In inelastic collisions, objects deform (their shape changes) and heat up (their temperature changes): A ball of dough changes shape as it hits the ground, two cars deform as they collide, hands vigorously clapped together heat up and make sound.

The transformation of a system from an initial state to a final state is called a **process.** Processes cause change (see Section 1.4), and so in physics we aim to understand processes. Collisions that change either the motion or the state of objects are an example of a process. Other examples are the melting of an ice cube, the burning of fuel, the flow of a liquid, and an explosive separation. Initially we focus on collisions because they are easily visualized and we can study what happens in collisions using carts on tracks. As we gain experience, we shall replace collisions with other processes.

In inelastic collisions, the state of the objects after the collision is different from the state before the collision. If I were to make a movie of the ball of dough hitting the ground and then play the movie in reverse, you'd have no trouble telling that the movie was being played in the wrong direction. This is because this inelastic collision involves changes that cannot undo themselves—the dough is flat and at rest after the collision and round and moving before (and never the other way around). The same is true for a collision between two cars: The cars are damaged after the collision and not before, and it is not possible to repair them simply by pulling them apart (Figure 5.4*a*). After the clapping, your hands are warmer than before, and you cannot cool your hands by "unclapping" them. All these inelastic collisions are **irreversible processes,** which means that the changes that occur in the state of the colliding objects cannot spontaneously undo themselves.

In contrast, viewing a movie of a superball bouncing off the floor or two carts colliding on a low-friction track (without sticking together), you'd be hard pressed to tell whether the movie is playing forward or in reverse. This is because elastic collisions are a **reversible process,** which means there are no permanent changes in the state of the colliding objects. The objects look the same before and after the collision (Figure 5.4*b*).

I have summarized these facts in Table 5.2. Notice how a change in the relative speed (and therefore a change in the sum of the kinetic energies) goes hand in hand with a change in the state: The sum of the kinetic energies of two colliding objects doesn't change unless their states change. Let us look at this connection with an eye to formulating a new conservation law.

Suppose we could associate some quantity having the same units as kinetic energy ($\mathrm{kg \cdot m^2/s^2}$) with the state of an object—let's call this quantity the object's **internal energy** (denoted by E_{int}). Suppose further that we could arrange things in such a way that in an inelastic collision the increase in the sum of the internal energies of the colliding objects is equal to the decrease in their kinetic energies. This would mean that in an inelastic collision one form of energy is converted to another form (kinetic to internal) but the sum of the kinetic and internal energies—collectively called the **energy** of the system—doesn't change.

Table 5.2 Elastic and inelastic collisions

Collision type	Relative speed	State
elastic	unchanged	unchanged
inelastic	changed	changed
totally inelastic	changed (becomes zero)	changed

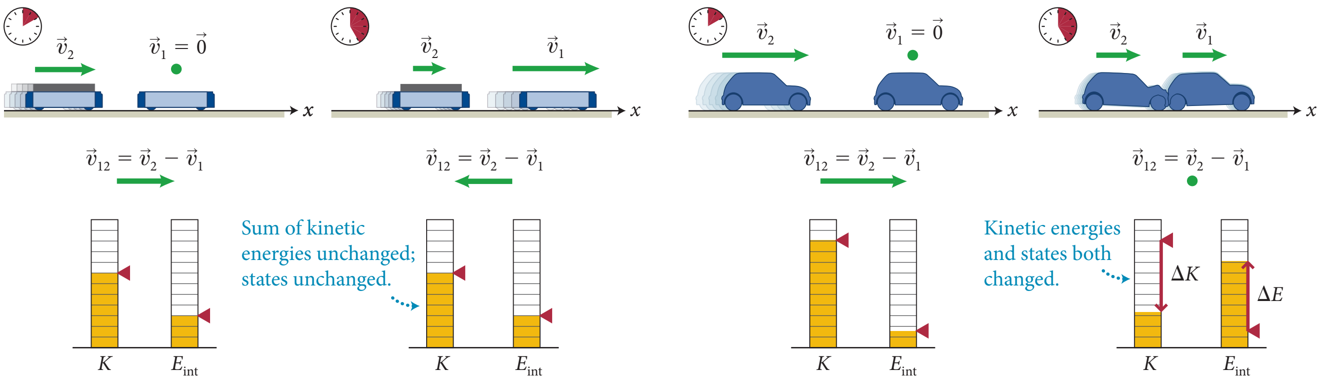


Figure 5.5 (*a*) Elastic collision. Because the relative speed of the carts does not change, the sum of the kinetic energies is also unchanged. The carts' states do not change, and so their internal energies are also unchanged. (*b*) Inelastic collision. Because the relative speed changes as a result of the collision, the kinetic energy changes. The cars' states also change, and so the internal energy changes too. For both collisions the sum of the kinetic and internal energies is constant.

In Chapter 7 we'll learn how to specify the state of an object and how to calculate the corresponding internal energy. For now, all we need to do is account for the missing kinetic energy in an inelastic collision by saying:

In any inelastic collision, the states of the colliding objects change, and the sum of their internal energies increases by an amount equal to the decrease in the sum of their kinetic energies.

Whenever some kinetic energy disappears in a collision, we can *always* determine changes in state to account for that loss. The big appeal of the relationship between state and internal energy is that we can now say:

The energy of a system of two colliding objects does not change during the collision.

This statement holds for all types of collisions: elastic, inelastic, and totally inelastic. In the elastic collision shown in **Figure 5.5*a***, the collision alters the velocities and thus the kinetic energies of both carts. However, the sum of the kinetic energies before and after the collision remains unchanged, as shown by the kinetic energy bars. Because the collision is elastic, there are no changes in the states of the carts, which means their internal energies also remain unchanged, as shown by the internal energy bars. So the energy of the two-cart system does not change.

In the totally inelastic collision of Figure 5.5*b*, the sum of the kinetic energies decreases. Because changes occur in the states—the cars change shape—the internal energies of the carts change as well. The changes in motion and state are such that the energy of the system is the same before and after the collision. We account for the lost kinetic energy by saying that it has been converted to internal energy.

Note that in making the bar diagrams in Figure 5.5, I had to choose some initial value for the internal energies of the colliding objects. As we have no way (yet) of calculating internal energies, the values I chose are arbitrary. At present we are interested only in *changes* in energy, however, and so the initial value is not important.

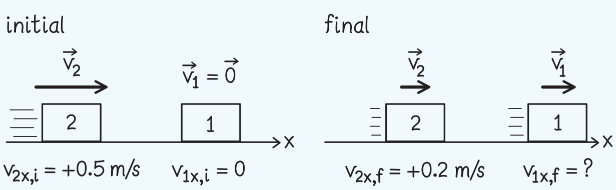
5.5 A piece of dough is thrown at a wall and sticks to it. Does the internal energy of the dough-wall system increase, decrease, or stay the same?

Example 5.3 Internal energy change

A 0.2-kg cart 1 initially at rest is struck by an identical cart 2 traveling at $v_{2x,i} = +0.5$ m/s along a low-friction track. After the collision, the velocity of cart 2 is reduced to $v_{2x,f} = +0.2$ m/s. (*a*) Is the collision elastic, inelastic, or totally inelastic? (*b*) By what amount does the internal energy of the two-cart system change? (*c*) Make a bar diagram showing the initial and final kinetic and internal energies of the two carts.

❶ GETTING STARTED I begin by organizing the information given in the problem in a sketch (**Figure 5.6**). To classify the collision, I need to determine the final relative speed, but the final velocity of cart 1 is not given.

Figure 5.6



❷ DEVISE PLAN The two-cart system is isolated, and so the momentum of the system does not change, regardless of the type of collision. I can use this information to determine the final velocity of cart 1 and the final relative speed of the carts. By comparing the final and initial relative speeds, I can determine the type of collision. Once I know the initial and final velocities, I can calculate the kinetic energies using $K = \frac{1}{2}mv^2$ and determine what fraction of the initial kinetic energy has been converted to internal energy.

CONCEPTS

3 **EXECUTE PLAN** (*a*) The initial relative speed is

$$|v_{2x,\mathrm{i}} - v_{1x,\mathrm{i}}| = |(+0.5\ \mathrm{m/s}) - 0| = 0.5\ \mathrm{m/s}.$$

To determine $v_{1x,\mathrm{f}}$, I apply conservation of momentum to the system. The initial momentum of the system is

$$(0.2\ \mathrm{kg})(+0.5\ \mathrm{m/s}) + (0.2\ \mathrm{kg})(0) = (0.2\ \mathrm{kg})(+0.5\ \mathrm{m/s})$$

and its final momentum is

$$(0.2\ \mathrm{kg})(+0.2\ \mathrm{m/s}) + (0.2\ \mathrm{kg})(v_{1x,\mathrm{f}}).$$

Conservation of momentum requires these two momenta to be equal:

$$(0.2\ \mathrm{kg})(+0.5\ \mathrm{m/s}) = (0.2\ \mathrm{kg})(+0.2\ \mathrm{m/s}) + (0.2\ \mathrm{kg})(v_{1x,\mathrm{f}})$$

$$(+0.5\ \mathrm{m/s}) = (+0.2\ \mathrm{m/s}) + v_{1x,\mathrm{f}}$$

$$v_{1x,\mathrm{f}} = +0.3\ \mathrm{m/s}.$$

The final relative speed is thus

$$|v_{2x,\mathrm{f}} - v_{1x,\mathrm{f}}| = |(+0.2\ \mathrm{m/s}) - (+0.3\ \mathrm{m/s})| = 0.1\ \mathrm{m/s},$$

which is different from the initial value. Thus the collision is inelastic. (I know that the collision is not *totally* inelastic because the relative speed has not been reduced to zero.) ✔

(*b*) The initial kinetic energies are

$$K_{1\mathrm{i}} = 0$$

$$K_{2\mathrm{i}} = \tfrac{1}{2}(0.2\ \mathrm{kg})(0.5\ \mathrm{m/s})^2 = 0.025\ \mathrm{kg\cdot m^2/s^2}$$

so $$K_\mathrm{i} = K_{1\mathrm{i}} + K_{2\mathrm{i}} = 0.025\ \mathrm{kg\cdot m^2/s^2}.$$

The final kinetic energies are

$$K_{1\mathrm{f}} = \tfrac{1}{2}(0.2\ \mathrm{kg})(0.3\ \mathrm{m/s})^2 = 0.009\ \mathrm{kg\cdot m^2/s^2}$$

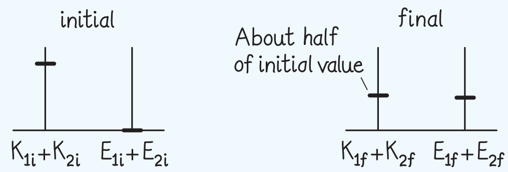
$$K_{2\mathrm{f}} = \tfrac{1}{2}(0.2\ \mathrm{kg})(0.2\ \mathrm{m/s})^2 = 0.004\ \mathrm{kg\cdot m^2/s^2}$$

so $$K_\mathrm{f} = K_{1\mathrm{f}} + K_{2\mathrm{f}} = 0.013\ \mathrm{kg\cdot m^2/s^2}.$$

The kinetic energy of the system has changed by an amount $(0.013\ \mathrm{kg\cdot m^2/s^2}) - (0.025\ \mathrm{kg\cdot m^2/s^2}) = -0.012\ \mathrm{kg\cdot m^2/s^2}$. To keep the energy of the system (the sum of its kinetic and internal energies) unchanged, the decrease in kinetic energy must be made up by an increase in internal energy. This tells me that the internal energy of the system increases by $0.012\ \mathrm{kg\cdot m^2/s^2}$. ✔

(*c*) My bar diagram is shown in **Figure 5.7**. The final kinetic energy bar is about half of the initial kinetic energy bar. Because I don't know the value of the initial internal energy, I set it to zero and make the final internal energy bar equal in height to the difference in the kinetic energy bars. ✔

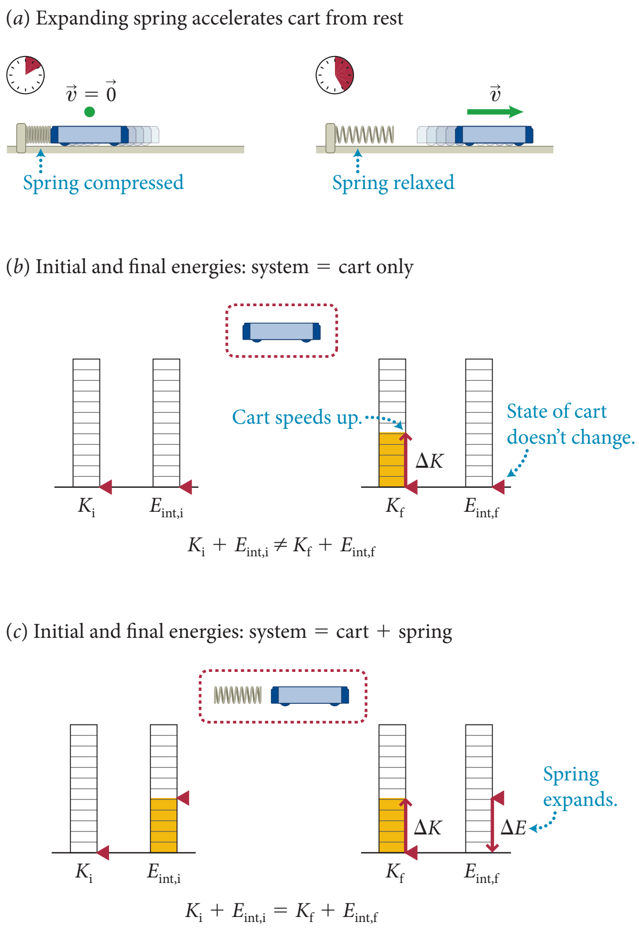
Figure 5.7



4 **EVALUATE RESULT** If the collision were elastic, the velocities of the carts would be interchanged and cart 2 would come to a stop (see Figure 4.5). So the collision must be inelastic. Indeed, I found that both the relative speed and the sum of the kinetic energies change in the collision, as expected for an inelastic collision.

We are now in a position to extend the idea of internal energy to other interactions. Consider, for example, a cart initially at rest on a low-friction track set in motion by an expanding spring that is held fixed at one end, as shown in **Figure 5.8*a***. As the cart is accelerated by the spring, the cart's kinetic energy increases but its state doesn't change, and so its energy increases (Figure 5.8*b*). Where did this energy come from? The spring sets the cart in motion, and so it makes sense to assume that the spring transfers energy to the cart. Indeed, the spring expands—its state changes—and so its internal energy changes. If we include the spring

Figure 5.8 Initial and final energies for two choices of system.



in the system and attribute the increase in the cart's kinetic energy to a decrease in the spring's internal energy, we can again arrange things in such a way that the energy of the cart-spring system remains unchanged (Figure 5.8*c*).

If we replace the cart in Figure 5.8 with a cart with a different inertia, compress the spring to the same initial state, and then let that cart go, we discover that its final kinetic energy is exactly the same as that of the first cart. So the spring always transfers the same amount of energy as it expands from a given compressed state to its relaxed state *regardless* of the object to which it transfers that energy.

5.6 Think of a few other ways to put the cart of Figure 5.8 in motion. In each case, can you account for the increase in the cart's kinetic energy by either a change in state or a change in motion of another object?

Checkpoint 5.6 suggests that a change in the cart's kinetic energy can always be attributed to either a change in state (and therefore a change in the internal energy) or a change in the motion of another object. Because the states of objects can change in many different ways, we associate different forms of internal energy with different kinds of state change. For example, the internal energy associated with a change in chemical state is called *chemical energy* and the internal energy associated with a change in the temperature of an object is called *thermal energy.* **Table 5.3** lists additional forms of internal energy.

Absolutely everything happening around us entails changes in state and therefore changes in internal energy. Rivers flowing, air masses moving, machines lifting things, people walking, and atoms emitting light can all be expressed in terms of changes in state (and therefore changes in internal energy). More important, we discover that any change in state or motion is always accompanied by a compensating change in state or motion, and we can always attribute energy to these changes in such a way that the energy of the system remains unchanged. In other words, energy cannot be destroyed or created, and energy, like momentum, is a conserved quantity. Indeed, no observation has ever been found to violate the law of **conservation of energy:**

Energy can be transferred from one object to another or converted from one form to another, but it cannot be destroyed or created.

CONCEPTS

Table 5.3 **Various state changes and their associated internal energy**

State change	Internal energy
temperature change	thermal energy
chemical change	chemical energy
reversible change in shape	elastic energy
phase transformation	transformation energy

Kinetic energy and all forms of internal energy are thus different manifestations of the same conserved quantity: *energy*.

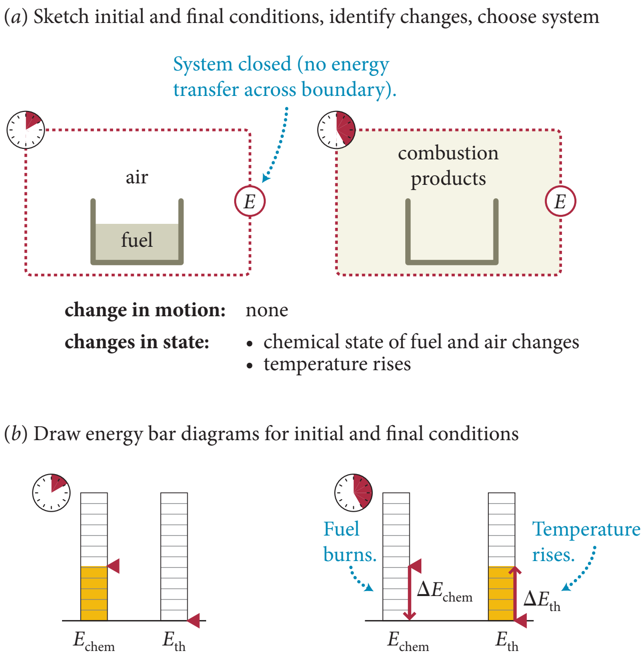
5.7 (*a*) Is the momentum of the cart-spring system in Figure 5.8 constant? (*b*) Is the system isolated?

5.4 Closed systems

As Checkpoint 5.7 shows, the system comprising the spring and cart in Figure 5.8 is not isolated. However, no energy is transferred to it, and therefore the energy of the system is constant.* Any system to or from which no energy is transferred is called a **closed system.** An important point to keep in mind is that a closed system need not be isolated (and likewise an isolated system is not necessarily closed). The procedure for choosing a closed system is described in the Procedure box. To see how this procedure works, let's look at the setup in **Figure 5.9**—some fuel (gasoline, say) being burned in a container open to the air.

I begin by making a sketch of the canister with the fuel before and after the combustion (Figure 5.9*a*). The combustion involves two changes in state: a change in the chemical state of the fuel and the air, and a change in the

Figure 5.9 (*a*) The burning of fuel involves a change in chemical composition and in temperature. (*b*) If we choose a closed system, the change in chemical energy must be compensated by a change in thermal energy.



*How do I know that no energy is transferred to the cart-spring system? The expanding spring and the accelerating cart do not cause any changes in the state or motion of the environment (the track, Earth, and so on). Consequently the energy of the environment doesn't change, which means that no energy has been transferred from the environment to the system.

Procedure: Choosing a closed system

When we analyze energy changes, it is convenient to choose a system for which no energy is transferred to or from the system (a closed system). To do so, follow this procedure:

1. Make a sketch showing the initial and final conditions of the objects under consideration.
2. Identify all the changes in state or motion that occur during the time interval of interest.
3. Choose a system that includes all the objects undergoing these changes in state or motion. Draw a dashed line around the objects in your chosen system to represent the system boundary. Write "closed" near the system boundary to remind yourself that no energy is transferred to or from the system.
4. Verify that nothing in the surroundings of the system undergoes a change in motion or state that is related to what happens inside the system.

Once you have selected a closed system, you know that its energy remains constant.

temperature. The fuel and the air immediately surrounding it are what undergo these changes in state, and so I include them in my system. Because there are no other related changes in state, I know that my system is closed and its energy remains constant.

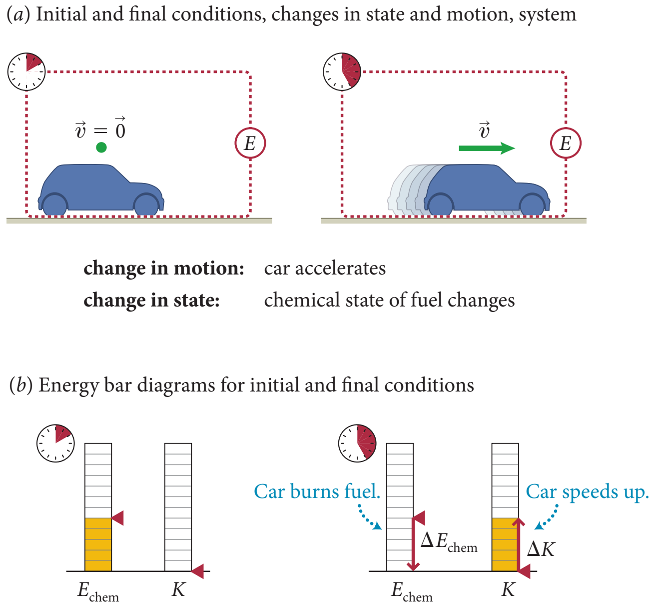
I can now make a bar diagram to represent the changes in energy that take place inside the system (Figure 5.9*b*). The change in the chemical state corresponds to a change in chemical energy, and the change in the temperature corresponds to a change in thermal energy. I therefore draw two bars. The combustion raises the temperature, and so the thermal energy increases. Because the system is closed and its energy remains constant, there must be a decrease in chemical energy that compensates for the increase in thermal energy.

Given that the energy of a closed system remains constant, we can focus on the energy conversions and transfers that happen inside the system. We speak of an *energy conversion* when energy is converted from one form to another. In the combustion illustrated in Figure 5.9, for example, an amount ΔE_{chem} of chemical energy is converted to an equal amount ΔE_{th} of thermal energy. When energy is transferred from one object to another, we speak of an *energy transfer*. An example is shown in Figure 5.3*a*, where energy is transferred from cart 2 to cart 1.

Because we are still lacking the quantitative tools, let me illustrate the transfer and conversion of energy qualitatively with a few examples. By converting some form of internal energy to kinetic energy, we can put objects in motion. For example, a car burns gasoline as it accelerates from rest. If we ignore the effects of air resistance, this situation represents a change in the chemical state of the gasoline and a change in the motion of the car. Chemical energy (a form of internal energy associated with the chemical state) stored in the gasoline is converted to kinetic energy of the car (**Figure 5.10**).

5.8 In describing what's going on in Figure 5.10, I ignored the change in temperature of the engine that accompanies the combustion of the fuel. Make a bar diagram that includes this change in temperature.

Figure 5.10 (*a*) An accelerating car constitutes a closed system because no changes occur in its environment. (*b*) The increase in the car's kinetic energy can be attributed to a decrease in chemical energy (due to the combustion of fuel).



The generation and consumption of electrical power provide many other examples of energy conversions and transfers. In a coal-burning electrical power plant, coal is burned in a boiler to heat water and convert it to steam. The two complementary changes in state (a change in the chemical state of the coal and a change in the temperature and state of the water) correspond to a conversion of chemical energy to thermal energy. The steam drives a turbine, where the steam cools and a turbine blade spins, and so thermal energy is converted to kinetic energy. The moving turbine drives a generator, which converts the kinetic energy to electrical energy. Power lines transfer energy from the plant to our homes, where it is converted to many forms. Lamps convert electrical energy to light (a form of energy) and thermal energy (lamps get hot), a stereo converts electrical energy to sound energy, and on and on.

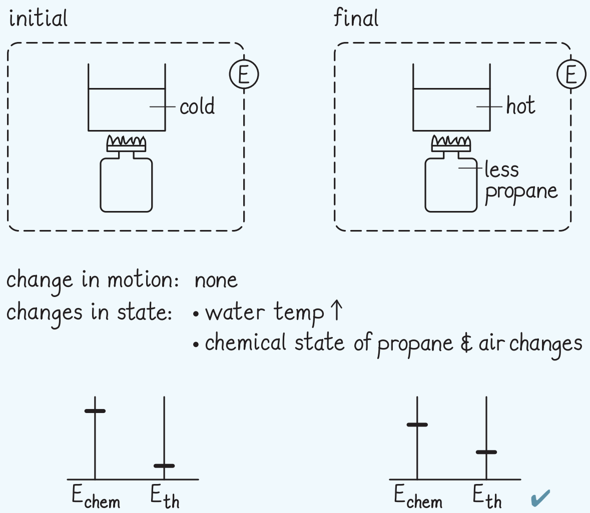
Exercise 5.4 State changes and internal energy

Choose an appropriate closed system and make a bar diagram representing the energy conversions and transfers that occur when (*a*) a pan of water is heated on a propane burner, (*b*) a cyclist accelerates from rest, and (*c*) a spring-loaded gun fires a ball of putty.

SOLUTION For each case, I apply the steps of the Procedure box on page 109. My sketches are shown in the figures.

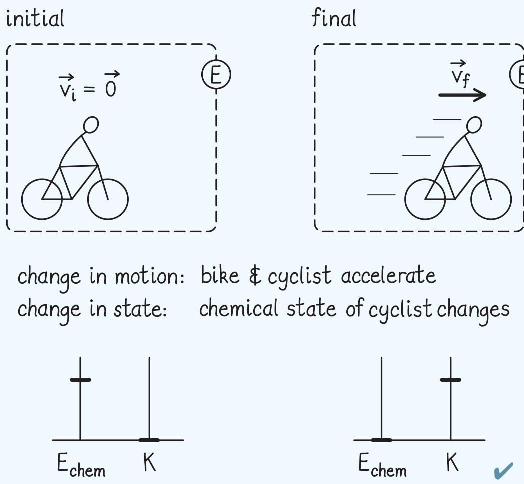
(*a*) Changes in motion: none. Changes in state: the temperature of the water increases, and the chemical states of the propane and the air change. The bar diagram shows an increase in thermal energy and an equal decrease in chemical energy (**Figure 5.11**). Chemical energy is converted to thermal energy, and in the process energy is transferred from the propane to the water.

Figure 5.11



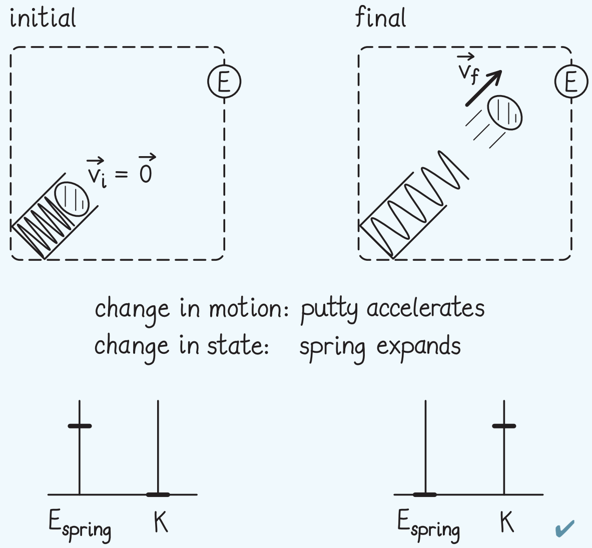
(*b*) Changes in motion: the bicycle and the cyclist accelerate. Changes in state: the chemical state of the cyclist changes because setting the bicycle in motion requires muscles to contract, a physiological process that involves a complex series of chemical reactions. The bar diagram shows an increase in kinetic energy and an equal decrease in chemical energy (**Figure 5.12**). Chemical energy is converted to kinetic energy of the bicycle and cyclist.

Figure 5.12



(*c*) Changes in motion: the putty is accelerated. Changes in state: the spring expands. The bar diagram shows a decrease in the elastic energy of the spring and an increase in the kinetic energy of the putty (**Figure 5.13**). As the spring expands, elastic energy is converted to kinetic energy of the putty.

Figure 5.13



We started our study of physics by describing motion in terms of velocities, accelerations, and inertias. Now we have two new quantities, *momentum* and *energy*, and we have found two fundamental conservation laws that allow us to use simple accounting principles to describe changes in motion.

5.9 (*a*) Can the magnitude of the momentum of an object change without a change in the object's kinetic energy? (*b*) Without a change in the object's energy? (*c*) What are your answers to parts *a* and *b* for a system consisting of more than one object?

CONCEPTS

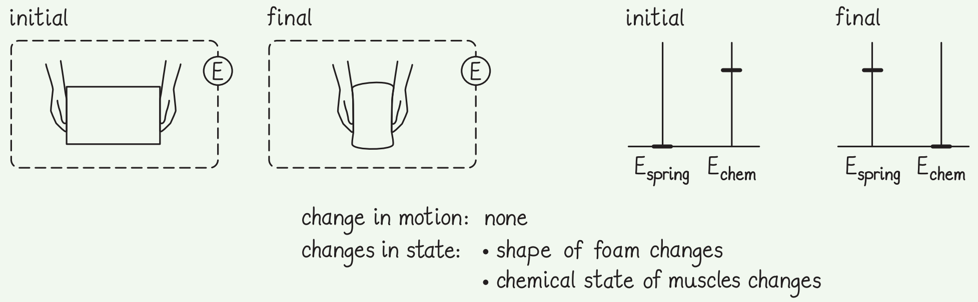
Self-quiz

1. Consider an isolated object at rest in space. The object contains internal energy in some form or another. Is it—in principle—possible to convert the internal energy to kinetic energy so that the object starts to move?
2. Imagine squeezing a piece of foam with your hands. Choose an appropriate closed system and make a bar diagram representing the energy conversions and transfers that occur during the squeezing.
3. When you heat a pot of water on a gas stove, the water temperature increases until the water begins to boil. This change in thermal state from cool water to hot water is due to chemical energy from the burning gas being converted to thermal energy of the water. Once boiling starts, the water temperature stays constant until all the water has turned to steam, even though the burning gas continues to transfer energy to the water. While the water is boiling off and becoming steam, what becomes of the energy released by the burning gas?
4. An electric fan turns electrical energy into wind energy (a form of kinetic energy because it involves moving air). Suppose a blowing fan is suddenly unplugged. Even though the fan no longer receives electrical energy, it continues to blow air while the blades slowly come to a stop. What type of energy is converted to wind energy after the fan is unplugged?

Answers

1. No. Getting the object to move would violate the law of conservation of momentum because the object would start with zero momentum ($\vec{p} = m\vec{v} = m\vec{0} = \vec{0}$) and end with nonzero momentum ($\vec{p} = m\vec{v}$).
2. Draw the foam before and after the squeezing. Two changes in state occur: The shape of the foam changes, and the chemical state of your muscles changes. These two changes in state correspond to changes in the foam's elastic energy and in chemical energy. In your closed system, include the foam and yourself (Figure 5.14).

Figure 5.14



3. It goes into changing the phase of the water from liquid to steam.
4. The kinetic energy of the fan blades is converted to wind energy.

5.5 Elastic collisions

As we saw in Section 5.1, we can classify collisions according to what happens to the relative velocity of the two colliding objects. The **relative velocity** of cart 2 relative to cart 1 is defined as

$$\vec{v}_{12} \equiv \vec{v}_2 - \vec{v}_1. \tag{5.1}$$

The relative velocity of cart 1 relative to cart 2 is the negative of this vector:

$$\vec{v}_{21} \equiv \vec{v}_1 - \vec{v}_2 = -\vec{v}_{12}. \tag{5.2}$$

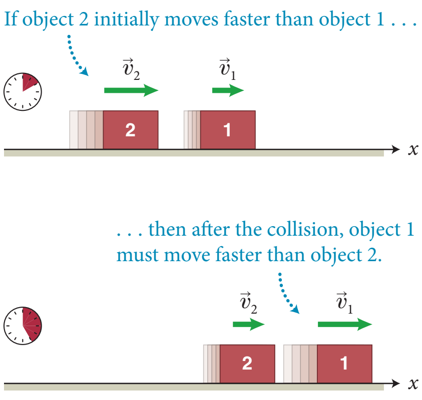
If the magnitude of this relative velocity (the objects' **relative speed** $v_{12} \equiv |\vec{v}_2 - \vec{v}_1|$) is the same before and after the collision, the collision is elastic:

$$v_{12\text{i}} = v_{12\text{f}} \quad \text{(elastic collision)}. \tag{5.3}$$

For two objects moving along the x axis, we can write this as

$$v_{2x,\text{i}} - v_{1x,\text{i}} = -(v_{2x,\text{f}} - v_{1x,\text{f}}) \quad \text{(elastic collision)}, \tag{5.4}$$

Figure 5.15 A collision between two objects moving in the same direction at different speeds.



where I have added a minus sign on the right because if object 2 initially moves faster than object 1, object 1 must move faster after the collision (**Figure 5.15**). In other words, in an elastic collision, if $v_{2x,\text{i}} > v_{1x,\text{i}}$, we must have $v_{2x,\text{f}} < v_{1x,\text{f}}$. The relative velocity in an elastic collision always changes sign after the collision: $v_{12x,\text{i}} = -v_{12x,\text{f}}$.

Because we can consider a system made up of two colliding objects to be isolated during the collision, conservation of momentum requires that the momentum of the colliding objects remains unchanged in the collision:

$$m_1 v_{1x,\text{i}} + m_2 v_{2x,\text{i}} = m_1 v_{1x,\text{f}} + m_2 v_{2x,\text{f}} \quad \text{(isolated system)}. \tag{5.5}$$

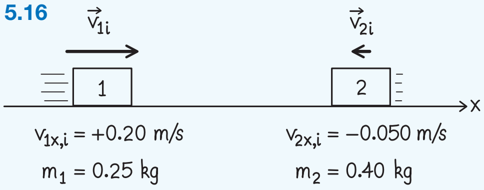
Equations 5.4 and 5.5 allow us to determine the final velocities of the objects, given their initial velocities.

Example 5.5 Elastic collision

Two carts, one of inertia $m_1 = 0.25$ kg and the other of inertia $m_2 = 0.40$ kg, travel along a straight horizontal track with velocities $v_{1x,\text{i}} = +0.20$ m/s and $v_{2x,\text{i}} = -0.050$ m/s. What are the carts' velocities after they collide elastically?

❶ GETTING STARTED I begin by organizing the information in a sketch to help visualize the situation (**Figure 5.16**). I need to determine the velocities after the collision from the information given, knowing that the collision is elastic.

Figure 5.16



❷ DEVISE PLAN To determine the two unknowns $v_{1x,\text{f}}$ and $v_{2x,\text{f}}$, I need two equations. Because the collision is elastic, I can use Eqs. 5.4 and 5.5.

❸ EXECUTE PLAN I begin by solving Eq. 5.4 for $v_{2x,\text{f}}$:

$$v_{2x,\text{f}} = v_{1x,\text{i}} - v_{2x,\text{i}} + v_{1x,\text{f}}$$

so that I can eliminate $v_{2x,\text{f}}$ from Eq. 5.5:

$$m_1 v_{1x,\text{i}} + m_2 v_{2x,\text{i}} = m_1 v_{1x,\text{f}} + m_2(v_{1x,\text{i}} - v_{2x,\text{i}} + v_{1x,\text{f}}).$$

Solving this expression for $v_{1x,\text{f}}$ yields

$$v_{1x,\text{f}} = \frac{m_1 - m_2}{m_1 + m_2} v_{1x,\text{i}} + \frac{2m_2}{m_1 + m_2} v_{2x,\text{i}}. \tag{1}$$

Repeating this procedure, I can eliminate $v_{1x,f}$ and solve for $v_{2x,f}$:

$$v_{2x,f} = \frac{2m_1}{m_1 + m_2} v_{1x,i} - \frac{m_1 - m_2}{m_1 + m_2} v_{2x,i}. \quad (2)$$

Substituting the given values into Eqs. 1 and 2 yields $v_{1x,f} = -0.11$ m/s and $v_{2x,f} = +0.14$ m/s. ✔

❹ **EVALUATE RESULT** The final velocities yield the final relative speed $v_{12f} = |(+0.14\ \mathrm{m/s}) - (-0.11\ \mathrm{m/s})| = 0.25\ \mathrm{m/s}$. This value is the same as the initial relative speed $v_{12i} = |(-0.05\ \mathrm{m/s}) - (+0.20\ \mathrm{m/s})| = 0.25\ \mathrm{m/s}$, as required for an elastic collision.

Let me now show how Eqs. 5.4 and 5.5 lead to an expression that shows that the kinetic energy of two colliding objects doesn't change in an elastic collision. Rearranging the terms in Eq. 5.5, I get

$$m_1(v_{1x,i} - v_{1x,f}) = m_2(v_{2x,f} - v_{2x,i}), \quad (5.6)$$

while Eq. 5.4 yields

$$v_{1x,i} + v_{1x,f} = v_{2x,i} + v_{2x,f}. \quad (5.7)$$

Now I multiply the left side of Eq. 5.6 by the left side of Eq. 5.7 and likewise for the right sides to get

$$m_1(v_{1x,i} - v_{1x,f})(v_{1x,i} + v_{1x,f}) = m_2(v_{2x,f} - v_{2x,i})(v_{2x,i} + v_{2x,f}), \quad (5.8)$$

which multiplies out to

$$m_1 v_{1x,i}^2 - m_1 v_{1x,f}^2 = m_2 v_{2x,f}^2 - m_2 v_{2x,i}^2. \quad (5.9)$$

For motion along the x axis, the square of the x component of a velocity is the same as the square of the speed, $v_x^2 = v^2$, and so I can drop all the subscripts x. If I put all the v_i terms on the left and all the v_f terms on the right, I get

$$m_1 v_{1i}^2 + m_2 v_{2i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2. \quad (5.10)$$

I now divide both sides by 2 to get

$$\tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2. \quad (5.11)$$

As you saw in Section 5.2, the quantity $\frac{1}{2}mv^2$ is the **kinetic energy**, which we denote by K:*

$$K \equiv \tfrac{1}{2} m v^2. \quad (5.12)$$

With this definition, Eq. 5.11 simplifies to

$$K_{1i} + K_{2i} = K_{1f} + K_{2f} \quad \text{(elastic collision)}, \quad (5.13)$$

where $K_{1i} = \frac{1}{2} m_1 v_{1i}^2$ is the initial kinetic energy of object 1, and so on. In words, in an elastic collision the sum of the initial kinetic energies is the same as the sum of the final kinetic energies.

*The factor $\frac{1}{2}$ in Eq. 5.12 is there for convenience. We could have defined kinetic energy as simply mv^2, but the definition given in Eq. 5.12 simplifies matters later on.

QUANTITATIVE TOOLS

Note that there is no new physics in Eqs. 5.11 and 5.13—both express the same fact as Eq. 5.3. The great advantage of writing this information in the form of Eq. 5.13 is that this form expresses Eq. 5.3 in terms of an extensive quantity. Because kinetic energy is an extensive quantity, the kinetic energy of a system comprising two objects is the sum of the kinetic energies of the individual objects: $K = K_1 + K_2$. Equation 5.13 thus tells us that the kinetic energy K of a system of objects undergoing an elastic collision does not change:

$$K_i = K_f \quad \text{(elastic collision)}. \tag{5.14}$$

This result can also be written in the form

$$\Delta K = 0 \quad \text{(elastic collision)}. \tag{5.15}$$

As you know from studying Table 5.1, the SI unit for kinetic energy is $\text{kg} \cdot \text{m}^2/\text{s}^2$. Because energy is such an important quantity, we give this combination of units the name **joule** (rhymes with *pool*), after the British physicist James Prescott Joule (1818–1889), who studied energy:

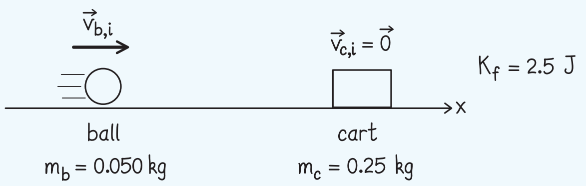
$$1\ \text{kg} \cdot \text{m}^2/\text{s}^2 \equiv 1\ \text{J}.$$

Example 5.6 Collision and kinetic energy

A rubber ball of inertia $m_b = 0.050$ kg is fired along a track toward a stationary cart of inertia $m_c = 0.25$ kg. The kinetic energy of the system after the two collide elastically is 2.5 J. (*a*) What is the initial velocity of the ball? (*b*) What are the final velocities of the ball and the cart?

❶ **GETTING STARTED** I organize the problem graphically (**Figure 5.17**). I choose my x axis in the direction of the incoming rubber ball. Only one initial velocity is given. I need to determine the other initial velocity and both final velocities.

Figure 5.17



❷ **DEVISE PLAN** Because the collision is elastic, I know that the kinetic energy of the system doesn't change, which means that the final value (2.5 J) is the same as the initial value. Because the cart is initially at rest, all of this kinetic energy belongs initially to the ball, and so I can use Eq. 5.12 to determine the initial velocity of the ball. Once I have this information, I know the initial velocities of both colliding objects and I can use Eqs. 1 and 2 from Example 5.5 to calculate the final velocities.

❸ **EXECUTE PLAN** (*a*) From Eq. 5.12, I obtain the initial speed of the ball:

$$v_{b,i} = \sqrt{\frac{2K_{b,i}}{m_b}} = \sqrt{\frac{2(2.5\ \text{J})}{0.05\ \text{kg}}} = 10\ \text{m/s}.$$

Because the ball is initially moving in the positive x direction, its initial velocity is given by $v_{bx,i} = +10$ m/s. ✔

(*b*) I can now substitute the two initial velocities and the inertias into Eqs. 1 and 2 of Example 5.5, which gives me $v_{bx,f} = -6.7$ m/s and $v_{cx,f} = +3.3$ m/s. ✔

❹ **EVALUATE RESULT** It makes sense that the velocity of the ball is reversed by the collision because the inertia of the cart is so much greater than that of the ball. Now that I know both the initial and final velocities, I can also check to make sure that the relative speed remains the same. Because the cart is initially at rest, the initial relative speed is 10 m/s; the final relative speed is $v_{bc,f} = |(+3.3\ \text{m/s}) - (-6.7\ \text{m/s})| = 10$ m/s, which is the same, as required for an elastic collision.

5.10 Conservation of momentum tells us that mv_x doesn't change for an isolated system. Now we see that in an elastic collision mv_x^2 also doesn't change. Does this mean that mv_x^3 remains unchanged, too?

5.6 Inelastic collisions

In a totally inelastic collision between two objects, the objects move together after the collision and so the two final velocities are identical. This means that the final relative speed is zero:

$$v_{12\mathrm{f}} = 0 \quad \text{(totally inelastic collision).} \tag{5.16}$$

Therefore if you know the inertias m_1 and m_2 of the objects and the x components of their initial velocities $v_{1x,\mathrm{i}}$ and $v_{2x,\mathrm{i}}$, you can obtain the final velocities from Eq. 5.5 by setting

$$v_{1x,\mathrm{f}} = v_{2x,\mathrm{f}} \quad \text{(totally inelastic collision).} \tag{5.17}$$

The majority of collisions, however, fall somewhere between the two extremes of elastic and totally inelastic. For these collisions, the final relative speed is between zero and the value of the initial relative speed. For these cases, it is convenient to define the ratio of relative speeds as

$$e \equiv \frac{v_{12\mathrm{f}}}{v_{12\mathrm{i}}}. \tag{5.18}$$

The unitless quantity e is called the **coefficient of restitution** of the collision. It tells us how much of the initial relative speed is restituted (restored) after the collision. Because it is a ratio of speeds (which are always positive), e is always positive.

In general, it is more convenient to write Eq. 5.18 in terms of components:

$$e = -\frac{v_{2x,\mathrm{f}} - v_{1x,\mathrm{f}}}{v_{2x,\mathrm{i}} - v_{1x,\mathrm{i}}} = -\frac{v_{12x,\mathrm{f}}}{v_{12x,\mathrm{i}}}, \tag{5.19}$$

where the minus sign appears because the relative velocity changes sign after the collision (see Figure 5.15).

When $e = 1$, Eq. 5.19 is equal to Eq. 5.4 and the collision is elastic (all of the initial relative speed is restored). A value of $e = 0$ means that $v_{12\mathrm{f}}$ must be zero, and so the collision is totally inelastic. For values of e between 0 and 1, the collision is inelastic. This information is summarized in **Table 5.4**.

In an inelastic collision, therefore, we again have two equations—Eqs. 5.1 and 5.19—but we need to know the value of e to be able to calculate the final velocities. The details of how the states of the objects change in the collision (these state changes depend on the material and structural properties of the objects) are hidden in this coefficient e.

Table 5.4 Coefficient of restitution for various processes

Process	Relative speed	Coefficient of restitution
totally inelastic collision	$v_{12\mathrm{f}} = 0$	$e = 0$
inelastic collision	$0 < v_{12\mathrm{f}} < v_{12\mathrm{i}}$	$0 < e < 1$
elastic collision	$v_{12\mathrm{f}} = v_{12\mathrm{i}}$	$e = 1$
explosive separation*	$v_{12\mathrm{f}} > v_{12\mathrm{i}}$	$e > 1$

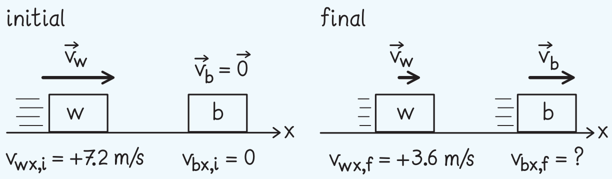
*See Section 5.8.

Example 5.7 Restitution

A white car of inertia 1200 kg that is moving at a speed of 7.2 m/s rear-ends a blue car of inertia 1000 kg that is initially at rest. Immediately after the collision, the white car has a speed of 3.6 m/s. What is the coefficient of restitution for this collision?

❶ GETTING STARTED By putting all the information in graphical form (**Figure 5.18**), I see that I do not have a value for the final velocity of the blue car, which I need in order to calculate the coefficient of restitution.

Figure 5.18



❷ DEVISE PLAN Because the system of the two colliding cars is isolated, I can use conservation of momentum (Eq. 5.5) to obtain the final velocity of the blue car, $v_{bx,f}$.

❸ EXECUTE PLAN I begin by writing Eq. 5.5 with subscripts appropriate to this particular collision:

$$m_w v_{wx,i} + m_b v_{bx,i} = m_w v_{wx,f} + m_b v_{bx,f}$$

or

$$m_w v_{wx,i} + 0 = m_w v_{wx,f} + m_b v_{bx,f}.$$

Solving for $v_{bx,f}$ and substituting the values given, I get

$$v_{bx,f} = \frac{m_w}{m_b}(v_{wx,i} - v_{wx,f})$$

$$= \frac{1200\text{ kg}}{1000\text{ kg}}(7.2\text{ m/s} - 3.6\text{ m/s}) = 4.3\text{ m/s}.$$

Now I can substitute the initial and final velocities into Eq. 5.19 to obtain the coefficient of restitution:

$$e = -\frac{v_{wx,f} - v_{bx,f}}{v_{wx,i} - v_{bx,i}}$$

$$= -\frac{(+3.6\text{ m/s}) - (+4.3\text{ m/s})}{(7.2\text{ m/s}) - 0} = 0.097. ✔$$

❹ EVALUATE RESULT The coefficient of restitution is rather small, but that is to be expected because cars don't bounce like superballs. Indeed, the blue car moves just a little bit faster than the white car after the collision (4.3 m/s versus 3.6 m/s), as I expect.

5.11 In a totally inelastic collision between two objects in an isolated system with one of the objects initially at rest, is it possible to lose *all* of the system's initial kinetic energy?

5.7 Conservation of energy

Because no energy is transferred to or from a closed system, conservation of energy requires that any change in the kinetic energy of a closed system be compensated by an equal change in the internal energy so that the sum of the kinetic and internal energies of the system does not change:

$$K_i + E_{int,i} = K_f + E_{int,f} \quad \text{(closed system).} \tag{5.20}$$

In contrast to Eq. 5.14, which holds for only an elastic collision, Eq. 5.20 holds for any closed system. If we write the sum of the kinetic and internal energies of an object or system as the **energy** of the object or system,

$$E \equiv K + E_{int}, \tag{5.21}$$

then we can rewrite Eq. 5.20 as

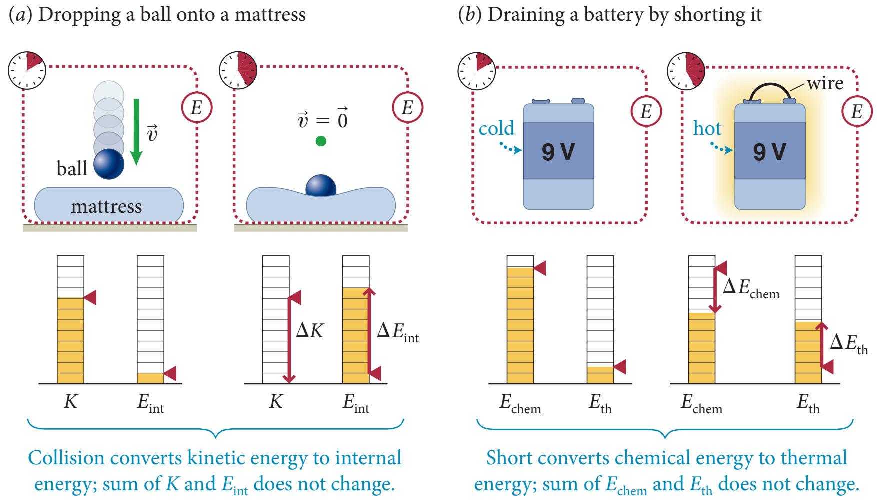
$$E_i = E_f \quad \text{(closed system)} \tag{5.22}$$

or

$$\Delta E = 0 \quad \text{(closed system).} \tag{5.23}$$

Note the parallel between Eq. 4.17, $\Delta\vec{p} = \vec{0}$, and Eq. 5.23. Equation 4.17 embodies conservation of momentum and states that the momentum of an isolated system cannot change. Equation 5.23 embodies conservation of energy: It states

Figure 5.19 Two examples of energy conservation in a closed system. (*a*) The ball's kinetic energy is converted to internal energy in the deformed mattress. (*b*) Chemical energy stored in the battery is converted to thermal energy (the battery gets hot). In both cases, as the bar graphs show, the system's energy remains the same.



that the energy of a closed system cannot change; that is, energy cannot be created or destroyed.

Even though we cannot yet calculate the internal energy E_{int}, Eq. 5.23 allows us to draw a number of important conclusions. First, if the kinetic energy of a closed system changes, then the state of the system must change in such a way that its internal energy changes by an amount

$$\Delta E_{int} = -\Delta K \quad \text{(closed system).} \tag{5.24}$$

As an example, consider **Figure 5.19*a***. A ball is dropped onto a mattress, where it comes to rest. During the time interval from the instant just before the ball hits the mattress to the instant at which the ball comes to rest, the motion of the ball changes and the shape of the mattress changes. The ball and mattress constitute a closed system, and so the decrease in kinetic energy must be accompanied by an increase in internal energy. Equation 5.24 requires the loss in kinetic energy to be equal to the gain in internal energy.

The second conclusion we can draw from Eq. 5.23 is that if the kinetic energy of a closed system does not change ($\Delta K = 0$), neither does the internal energy of the system ($\Delta E_{int} = 0$). Because E_{int} is the sum of the internal energies of all the parts that make up the system, however, $\Delta E_{int} = 0$ does *not* mean that no changes in state take place. For instance, E_{int} remains constant when internal energy is transferred from one part of a closed system to another:

$$\Delta E_{int,1} = -\Delta E_{int,2} \Rightarrow \Delta E_{int} = 0 \quad \text{(closed system, } \Delta K = 0\text{)} \tag{5.25}$$

or when one form of internal energy is converted to another:

$$\Delta E_{form\ 1} = -\Delta E_{form\ 2} \Rightarrow \Delta E_{int} = 0 \text{ (closed system, } \Delta K = 0\text{).} \tag{5.26}$$

As an example, consider Figure 5.19*b*. When the battery is drained rapidly, it becomes very hot. Because there is no motion before and after the draining, no change in kinetic energy occurs, but the chemical energy in the battery is converted to thermal energy. Equation 5.26 requires the loss in chemical energy to be equal to the gain in thermal energy:

$$\Delta E_{chem} + \Delta E_{th} = 0. \tag{5.27}$$

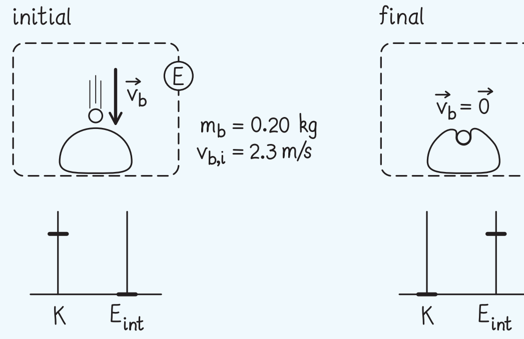
In practice there are often more than two simultaneous changes of state in a system (see Checkpoint 5.8, for instance), but regardless of how many energy conversions and transfers take place, the law of conservation of energy requires that the *amount of energy* in a closed system never changes.

Example 5.8 Making light

A 0.20-kg steel ball is dropped into a ball of dough, striking the dough at a speed of 2.3 m/s and coming to rest inside the dough. If it were possible to turn all of the energy converted in this totally inelastic collision into light, how long could you light a desk lamp? It takes 25 J to light a desk lamp for 1.0 s.

❶ **GETTING STARTED** I begin by applying the procedure for choosing a closed system. Although the problem doesn't specify it explicitly, I'm assuming the dough is at rest both before and after the steel ball is dropped in it; it could, for example, be at rest on a countertop. Only the steel ball has kinetic energy initially, and all of this energy is converted to internal energy as the ball comes to rest in the dough (**Figure 5.20**). So I have to calculate the initial kinetic energy of the ball and determine how long that amount of energy could light a lamp, given that 25 J lights a lamp for 1.0 s.

Figure 5.20



❷ **DEVISE PLAN** To determine the initial kinetic energy of the ball, I use Eq. 5.12. Then I divide this result by 25 J to determine how many seconds I can light a lamp.

❸ **EXECUTE PLAN** The initial kinetic energy of the ball is

$$K_{b,i} = \tfrac{1}{2}m_b v_{b,i}^2 = \tfrac{1}{2}(0.20\text{ kg})(2.3\text{ m/s})^2 = 0.53\text{ J}.$$

Given that a desk lamp requires 25 J per second, this 0.53 J lights a lamp for

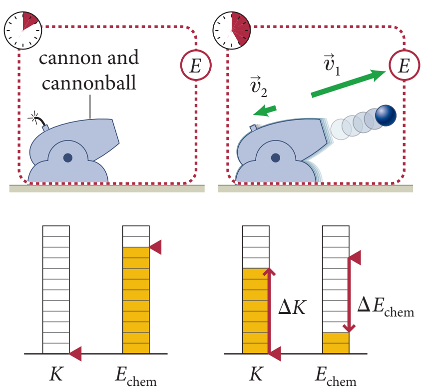
$$\frac{\text{energy available}}{\text{energy needed per second}} = \frac{0.53\text{ J}}{25\text{ J/s}} = 0.021\text{ s.}\ ✔$$

❹ **EVALUATE RESULT** The length of time I obtained, two hundreths of a second, is not very much! However, a 0.20-kg steel ball moving at 2.3 m/s does not have much kinetic energy: I know from experience that a small steel ball's ability to induce state changes—to crumple or deform objects, for example—is very limited. So it makes sense that one can't light a desk lamp for very long.

5.12 A gallon of gasoline contains approximately 1.2×10^8 J of energy. If all of this energy were converted to kinetic energy in a 1200-kg car, how fast would the car go?

5.8 Explosive separations

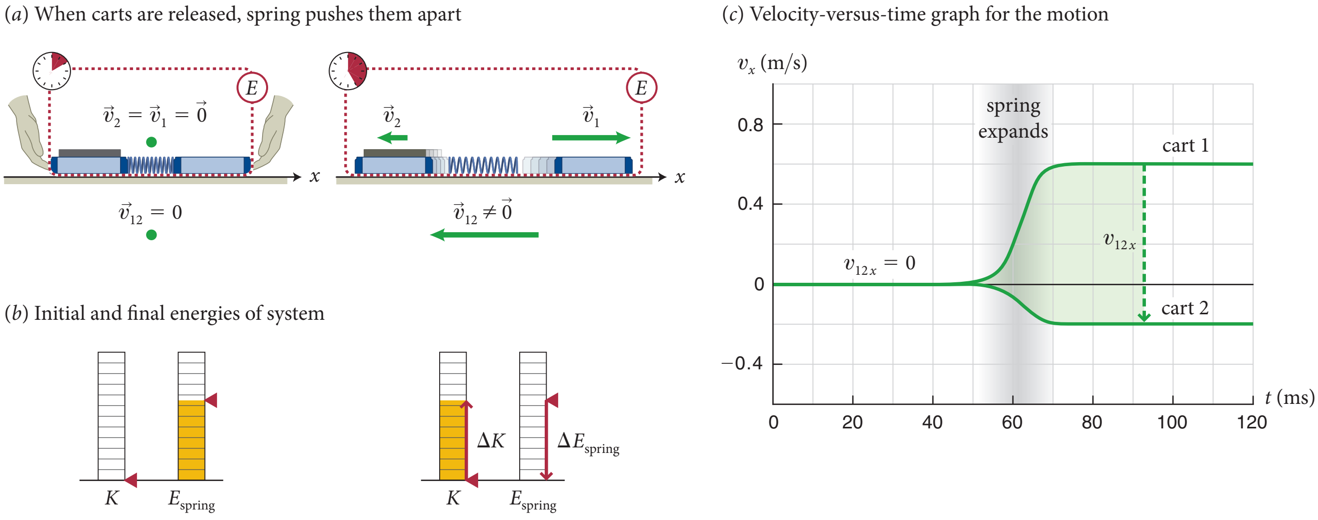
Figure 5.21 When a cannon is fired, internal (chemical) energy is converted to kinetic energy.



Is it possible to have a process in which kinetic energy is *gained* at the expense of internal energy? Yes, in an *explosion*, or any other type of **explosive separation,** where objects separate or break apart from each other, kinetic energy increases and internal energy decreases. The firing of a cannon is one example (**Figure 5.21**). Initially the cannon and cannonball are at rest. When the cannon is fired, the cannonball flies out of the barrel, and the cannon *recoils* in the opposite direction. (If it didn't recoil, the momentum of the system would not remain zero.) Both the cannon and cannonball therefore gain kinetic energy at the expense of chemical energy in the gun powder. This situation is the inverse of a totally inelastic collision: In this explosive separation, we start with the two objects together ($v_{12} = 0$) and end with them moving apart ($v_{12} > 0$).

Figure 5.22a shows an explosive separation that can be carried out on a low-friction track. Two carts, of inertias m and $3m$, are held against a compressed spring. When the carts are released, they move apart as the spring expands. As it expands, the spring's state changes and so does its internal energy; the decrease

Figure 5.22 Another example of an explosive separation.



in internal energy of the spring causes an increase in the kinetic energy of the carts (Figure 5.22*b*). Notice how the velocity-versus-time graph for this explosive separation (Figure 5.22*c*) is the inverse of the one in Figure 5.2*b*.

To determine the final speeds in Figure 5.22, we need to know how much energy E_{int} the spring releases, a topic not covered until Chapter 9. Once we know ΔE_{int}, we have two equations that allow us to obtain the two final velocities, one a consequence of conservation of momentum:

$$0 = m_1 v_{1x,f} + m_2 v_{2x,f} \tag{5.28}$$

(because $v_{1x,i} = v_{2x,i} = 0$) and the other a consequence of conservation of energy:

$$\Delta K + \Delta E_{int} = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 + \Delta E_{int} = 0. \tag{5.29}$$

Note that in this example the *initial* relative speed is zero, making e in Eq. 5.19 infinite. There is no restitution because there is no initial relative speed to be restituted. In the more general case where the two objects have a nonzero initial relative speed, $e > 1$.

Example 5.9 Spring energy

A 0.25-kg cart is held at rest against a compressed spring as in Figure 5.8*a* and then released. The cart's speed after it separates from the spring is 2.5 m/s. The spring is then compressed by the same amount between a 0.25-kg cart and a 0.50-kg cart, as shown in Figure 5.22*a*, and the carts are released from rest. What are the carts' speeds after separating from the spring?

❶ **GETTING STARTED** The key point in this problem is the identical compression of the spring in the two cases: The initial state of the spring is therefore the same before both releases. Because the spring ends in the same uncompressed state in both cases, the change in its internal energy must be the same in both cases. In the first case, all of this energy is transferred to the 0.25-kg cart. In the second case, the same amount of energy is distributed between the two carts.

❷ **DEVISE PLAN** To calculate the kinetic energy of the single cart in the first release, I use Eq. 5.12. This gives me the amount of energy stored in the compressed spring. The final velocities of the two carts in the second case are then given by Eqs. 5.28 and 5.29.

❸ **EXECUTE PLAN** From Eq. 5.12, I get

$$K = \tfrac{1}{2} m v^2 = \tfrac{1}{2}(0.25\ \text{kg})(2.5\ \text{m/s})^2 = 0.78\ \text{J}$$

and so the change in the spring's internal energy is $\Delta E_{int} = -0.78$ J. Next I rewrite Eq. 5.28 as $v_{1x,f} = -(m_2/m_1)v_{2x,f}$. Substituting this result in Eq. 5.29, I get

$$\tfrac{1}{2} m_1 \left(\frac{m_2}{m_1}\right)^2 v_{2x,f}^2 + \tfrac{1}{2} m_2 v_{2x,f}^2 = -\Delta E_{int}.$$

(Continued)

Solving for the final velocity of cart 2 gives

$$v_{2x\mathrm{f}} = \sqrt{\frac{-2m_1\Delta E_{\mathrm{int}}}{m_2(m_1 + m_2)}}$$

$$v_{2x\mathrm{f}} = \sqrt{\frac{-2(0.25\ \mathrm{kg})(-0.78\ \mathrm{J})}{(0.50\ \mathrm{kg})(0.25\ \mathrm{kg} + 0.50\ \mathrm{kg})}} = 1.0\ \mathrm{m/s}. ✔$$

Substituting this result into my rewritten Eq. 5.28, $v_{1x,\mathrm{f}} = -(m_2/m_1)v_{2x,\mathrm{f}}$, I get $v_{1x,\mathrm{f}} = -2.0\ \mathrm{m/s}$. ✔

4 **EVALUATE RESULT** The carts move in opposite directions, as expected. I also note that cart 1 moves at twice the speed of cart 2, as it should to keep the final momentum of the system zero. Finally, because my assignment of m_1 and m_2 is arbitrary, I verify that I get the same result when I substitute $m_1 = 0.50$ kg and $m_2 = 0.25$ kg. (You may want to check this yourself. When you reverse the inertias, why does the velocity of cart 1 reverse to positive and the velocity of cart 2 reverse to negative?)

5.13 Does each cart in Example 5.9 get half of the spring's energy? Why or why not?

Chapter Glossary

SI units of physical quantities are given in parentheses.

Closed system A system to or from which no energy is transferred. See the Procedure box on page 109.

Coefficient of restitution e (unitless) A scalar equal to the ratio of relative speeds after and before a collision of two objects:

$$e \equiv \frac{v_{12\mathrm{f}}}{v_{12\mathrm{i}}} \tag{5.18}$$

Conservation of energy Energy can be transferred from one object to another or converted from one form to another, but it cannot be created or destroyed. The energy of a closed system cannot change:

$$\Delta E = 0 \quad \text{(closed system)}. \tag{5.23}$$

Elastic, inelastic, totally inelastic collision Collisions between two objects are classified according to what happens to the relative speed $v_{12} = |\vec{v}_2 - \vec{v}_1|$ of the two objects, as summarized in Table 5.4.

Energy E (J) A scalar that provides a quantitative measure of the state or motion of an object or system. Energy appears in many different forms. The energy of an object or system always refers to the sum of all forms of energy in that object or system.

Explosive separation A process in which objects break apart from one another and the relative speed of the objects increases.

Internal energy E_{int} (J) Any energy not associated with the motion of an object or system. Internal energy is a quantitative measure of the state of the object or system.

Irreversible process A process involving changes that cannot undo themselves spontaneously.

Joule (J) The derived SI unit of energy, defined as $1\ \mathrm{J} \equiv 1\ \mathrm{kg \cdot m^2/s^2}$.

Kinetic energy K (J) Energy associated with the motion of an object. The kinetic energy K of an object of inertia m moving at speed v is

$$K \equiv \tfrac{1}{2}mv^2. \tag{5.12}$$

Process The transformation of a system from an initial state to a final state.

Relative velocity $\vec{v}_{12}$ (m/s) The velocity of one object relative to another:

$$\vec{v}_{12} \equiv \vec{v}_2 - \vec{v}_1. \tag{5.1}$$

The magnitude of this velocity is called the *relative speed* $v_{12} \equiv |\vec{v}_2 - \vec{v}_1|$.

Reversible process A process that can run backward so that the initial state is restored.

State The condition of an object (or a system) as specified by a complete set of variables.

6 Principle of Relativity

CONCEPTS

QUANTITATIVE TOOLS

Did you ever, sitting in a car at a red light and looking at the car next to you, slam on the brakes because you thought you were starting to roll but it turned out your car never moved? If you did, you experienced the relativity of motion, first described quantitatively by Galileo. You may have heard the term *relativity* in the context of Albert Einstein's famous theories of general relativity and special relativity. We'll touch upon one basic aspect of general relativity in Chapter 13 and discuss special relativity in Chapter 14. Underlying all of relativity is something we have so far ignored in our discussion of motion: the fact that the velocity measured for any object depends on the motion of the **observer** (the person doing the measuring). For example, to a person sitting in a moving train, a suitcase on the overhead rack is at rest, but to a person standing on a station platform watching the train speed by, the suitcase is not at rest. According to the person in the train, the suitcase has zero momentum ($m\vec{v}$) and zero kinetic energy ($\frac{1}{2}mv^2$). According to the person on the platform, the suitcase has nonzero momentum and nonzero kinetic energy. In this chapter we investigate whether or not the laws of conservation of momentum and conservation of energy depend on the velocity of the observer. In other words, if these laws are valid for one observer, are they also valid for an observer who is moving relative to the first observer?

6.1 Relativity of motion

Remember from Chapter 2 that whenever we talk about motion, we must specify a reference axis along which the motion occurs and an origin. Together, the axis and origin are called a **reference frame.** In our study of motion thus far, we have considered the motion of carts along a low-friction track, collecting our data using a ruler affixed to the track; that is to say, we collected our data while standing in a reference frame at rest relative to the surface of Earth (this is called an **Earth reference frame**).

What happens to the data if we are moving as we collect them—in other words, if we are in a reference frame that is moving relative to Earth? Imagine that we are moving alongside the track and want to describe a collision between two carts. Do the conservation laws hold in a reference frame that is moving relative to Earth? Or, to put the question another way, does the behavior of the carts change if we are in motion as we study that behavior?

Consider, for example, the situation depicted in **Figure 6.1*a***: two observers watching the motions of two cars. Car 1 is moving at constant velocity $\vec{v}_1$, and car 2 is at rest relative to Earth. Observer E is at rest relative to Earth, while observer M is inside car 1. The velocity $\vec{v}_M$ of observer M is the same as the velocity of car 1: $\vec{v}_M = \vec{v}_1$.

Relative to observer M, the position of car 1 is not changing; that is, car 1 is at rest relative to observer M. The distance to car 2, however, is steadily increasing, and so relative to observer M, car 2, observer E, and all the surroundings move to the left (Figure 6.1*b*). To distinguish quantities measured by different observers, we add a capital letter subscript to denote the observer. For instance, $\vec{v}_{M2}$ represents observer M's measurement of the velocity of car 2, also called the velocity of car 2 *relative to* observer M. When we are comparing quantities in various reference frames, quantities measured relative to Earth are given the subscript E. For example, relative to Earth the velocity of car 1 is represented by $\vec{v}_{E1}$ (When the Earth reference frame is the only reference frame, we drop the subscript E, recovering the notation we have used before: $\vec{v}_{E1} = \vec{v}_1$.)

To determine a relationship between the quantities measured by two observers moving relative to each other, let's start with a simple example. **Figure 6.2** shows five frames of a film of two carts on a low-friction track. In Figure 6.2*a* the positions of the two carts are measured with ruler A, which is affixed to the track (that is, in the Earth reference frame). Relative to ruler A, cart 1 is at rest and its position remains fixed at 22.5 mm. Cart 2 is moving to the right, and its position on ruler A changes from 0 to 14.4 mm.

In Figure 6.2*b* the positions of the two carts are measured with ruler B, which moves along with cart 2: The zero mark on ruler B remains aligned with the front of cart 2. The position of cart 2 therefore remains fixed at a ruler reading of 0, and so, relative to ruler B, cart 2 is at rest. On this ruler, the position of cart 1 does *not* remain fixed, however—the reading for this cart's position decreases from 22.5 mm to 8.1 mm over the five frames. That is to say, cart 1 moves 22.5 mm − 8.1 mm = 14.4 mm *to the left* along ruler B.

Figure 6.1 The motion you perceive depends on your reference frame.

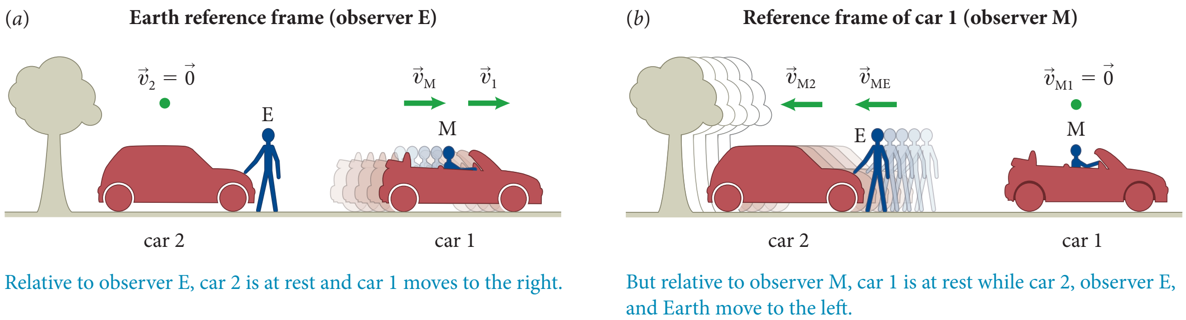


Figure 6.2 Two identical carts on a low-friction track. The positions of the carts are measured with (*a*) a ruler affixed to the track and (*b*) a ruler moving along with cart 2. In the Earth reference frame, cart 1 is at rest and cart 2 is moving to the right.

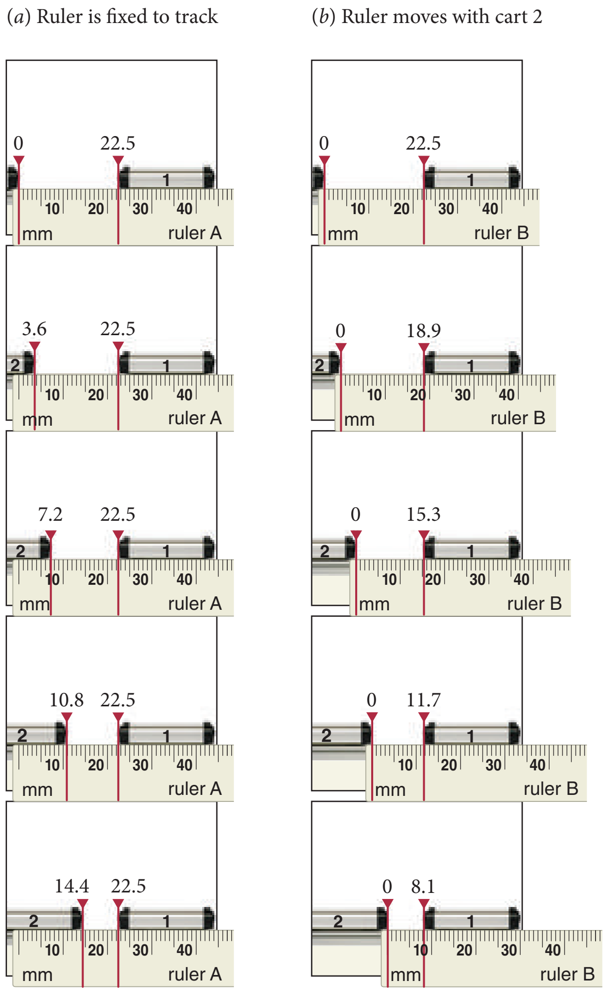


Figure 6.3a shows a position-versus-frame graph for the two carts drawn from the readings on ruler A in Figure 6.2*a*. What is shown in the graph corresponds to what an observer at rest alongside the track sees: cart 1 at rest and the *x* component of the velocity of cart 2 a constant +3.6 mm/frame. These are the observations taken in the Earth reference frame.

Figure 6.3*b* is the position-versus-frame graph for the two carts drawn from the readings on ruler B in Figure 6.2*b*. What is shown in the graph corresponds to what an observer M sitting on top of cart 2 sees: cart 2 at rest and the *x* component of the velocity of cart 1 a constant −3.6 mm/frame (the negative sign indicates that the cart moves to the left in the figure).

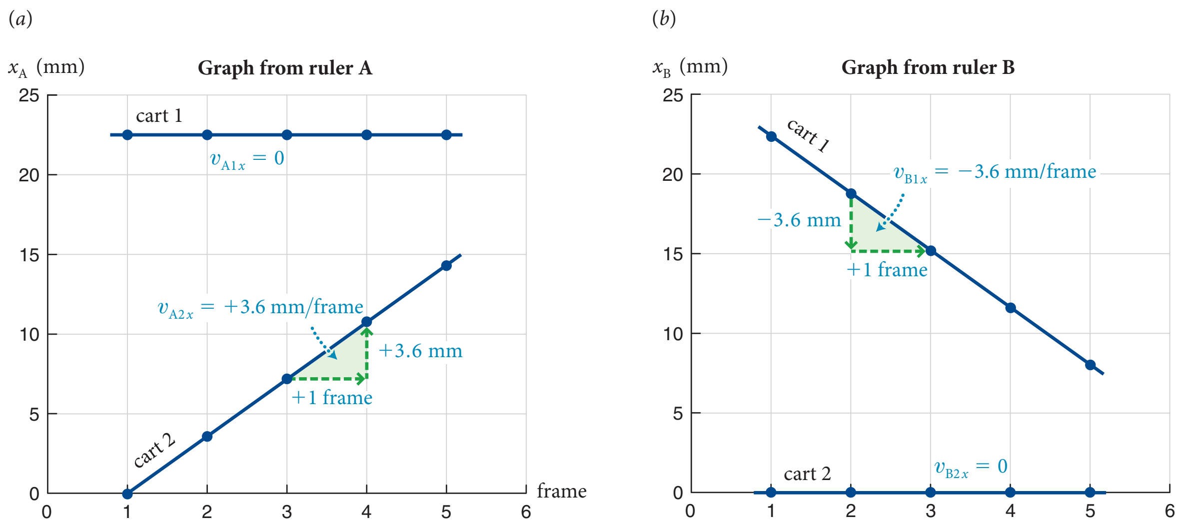
Example 6.1 Moving at my own speed

Suppose that in the situation shown in Figure 6.2, a third ruler is affixed to some device (not shown) that moves to the right along the track at a speed of 2.0 mm/frame. If in the Earth reference frame cart 2 again moves at +3.6 mm/frame and cart 1 is again at rest, what are the cart velocities according to an observer moving along with ruler C?

❶ **GETTING STARTED** If I arbitrarily let ruler C start in the position shown in frame 1 of Figure 6.2*b*—the zero mark on ruler C aligned with the front of cart 2—this ruler is shifted 2.0 mm to the right in the second frame and another 2.0 mm to the right in each successive frame. I have to use the readings from this ruler to determine the cart velocities.

❷ **DEVISE PLAN** I begin by making a table showing the position of each cart in each frame according to an observer moving along with ruler C. Because the ruler moves 2.0 mm to the right in each frame, I can obtain the readings on this ruler by subtracting 2.0 mm from each cart position in frame 2 of Figure 6.2*a*, subtracting 4.0 mm from each cart position in frame 3, subtracting 6.0 mm from each cart position in frame 4, and so on. I can then use these positions to make a position-versus-frame graph like the ones in Figure 6.3 and determine the velocities of the carts from the slopes of the curves.

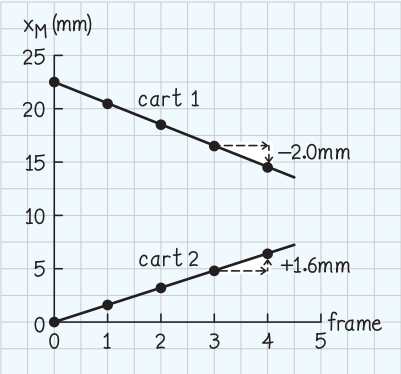
Figure 6.3 Position-versus-frame graphs for the carts of Figure 6.2 as determined from readings on (*a*) ruler A and (*b*) ruler B.



3 **EXECUTE PLAN** My table of the cart positions measured with ruler C is shown in **Figure 6.4**. By plotting these data, I see that the x component of the velocity of cart 1 is -2.0 mm/frame and that of cart 2 is $+1.6$ mm/frame. ✔

Figure 6.4

x_{M2} (mm)	x_{M1} (mm)
0	22.5
1.6	20.5
3.2	18.5
4.8	16.5
6.4	14.5



4 **EVALUATE RESULT** The observer moving with ruler C sees cart 1 moving to the left, meaning that the x component of its velocity should be negative, as I found. This observer and cart 2 both move in the positive x direction in the Earth reference frame, but the speed of cart 2 is higher than that of ruler C, and so cart 2 moves to the right relative to the observer. Therefore the x component of its velocity should be positive, in agreement with what I found. Because the velocity of ruler C is different from the velocity of either cart, neither cart is at rest relative to ruler C.

This example and the text preceding it show that motion is a relative concept. In the Earth reference frame, cart 1 is at rest and cart 2 is moving. Relative to the moving ruler in Figure 6.2*b*, cart 1 is moving and cart 2 is at rest. And relative to the ruler of Example 6.1, both carts are moving. In all three reference frames, however, we discover that the carts move at constant velocity. We could just as well have used a ruler moving at $+1.0$ mm/frame, $+4.0$ mm/frame, -0.1 mm/frame, or any other constant velocity. In each case, we would reach the same conclusion: The carts move at constant velocity. So, if an object moves at constant velocity in the Earth reference frame, its motion observed from *any reference frame moving at constant velocity relative to Earth* is also at constant velocity.

Comparing Figures 6.3*a* and 6.3*b*, we see that the x component of each cart's velocity according to ruler A (Figure 6.3*a*) is 3.6 mm/frame higher than the velocity according to ruler B (Figure 6.3*b*). So, an object's velocity $\vec{v}_{Ao}$ determined from the readings on ruler A is equal to the sum of the velocity $\vec{v}_{AB}$ of ruler B relative to ruler A and the velocity $\vec{v}_{Bo}$ of the object relative to ruler B: $\vec{v}_{Ao} = \vec{v}_{AB} + \vec{v}_{Bo}$. For cart 1, we have $0 = +3.6$ mm/frame $+ (-3.6$ mm/frame); for cart 2, $+3.6$ mm/frame $= +3.6$ mm/frame $+ 0$. You can readily verify that the same is true for the velocities determined with ruler C in Example 6.1.

6.1 What is the velocity of each cart in Figure 6.2 measured by an observer moving at -3.0 mm/frame in the Earth reference frame?

6.2 Inertial reference frames

How does the choice of reference frame affect our accounting methods for momentum and energy? To answer this question, let's consider the two carts of Figure 6.2 from the point of view of two observers. To an observer E, who is at rest relative to Earth, cart 1 is at rest and cart 2 is moving at constant velocity (**Figure 6.5*a***). To an observer M, who is moving along with cart 2, cart 2 is at rest and cart 1 is moving at constant velocity (Figure 6.5*b*). Even though the observers obtain different values for the carts' velocities, these values are constant, and therefore both observers conclude that the momentum of each cart is constant. Both observers also agree that the two carts are isolated, which means that conservation of momentum yields $\Delta\vec{p} = \vec{0}$ (Eq. 4.17) for each cart, in agreement with their observations.

6.2 From the point of view of each observer in Figure 6.5, (*a*) is the energy of each cart constant? (*b*) Is the isolated system containing cart 1 closed? (*c*) Is the isolated system containing cart 2 closed?

Although we checked for only the simple case of carts moving at constant velocity on a low-friction track, Figure 6.5 and Checkpoint 6.2 suggest that the accounting procedures and principles we introduced in the preceding two chapters can be applied in any reference frame moving at constant velocity relative to Earth. Indeed, when we are moving at constant velocity relative to Earth, things around us behave the same way they behave when we are at rest on the ground. For example, imagine being in a plane flying at a constant speed of 260 m/s in a straight line. The surface of the coffee in the cup in front of you looks no different

Figure 6.5 The carts of Figure 6.2 seen by (*a*) an observer in the Earth reference frame and (*b*) an observer moving along with cart 2.

(*a*) **Earth reference frame (observer E)** (*b*) **Reference frame of cart 2 (observer M)**

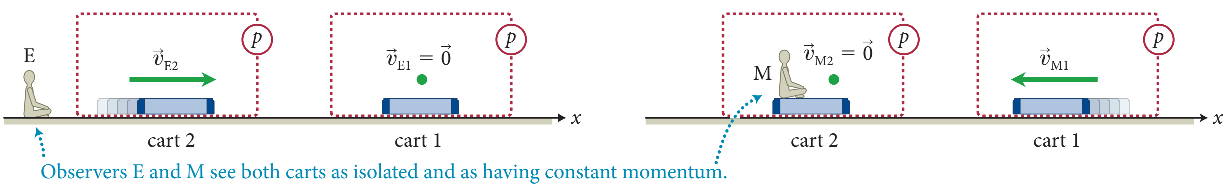
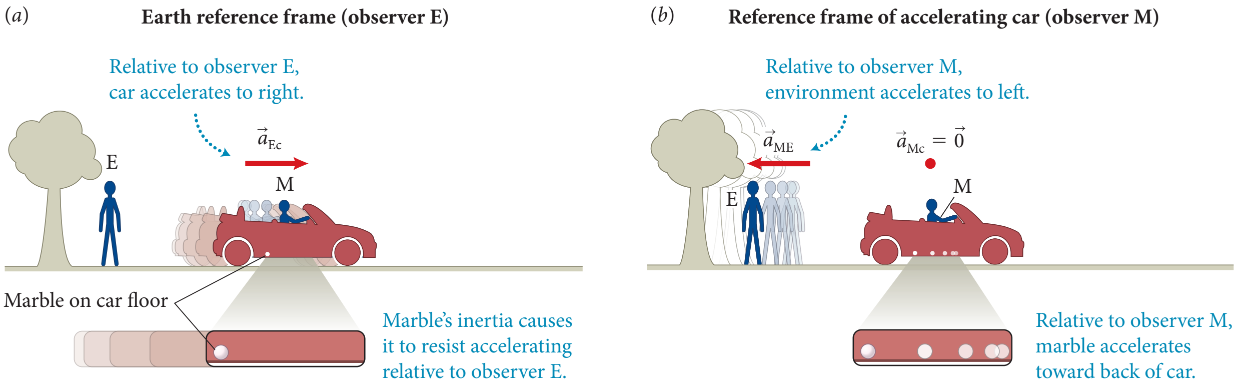


Figure 6.6 Motion of a marble in an accelerating car viewed from (*a*) the Earth reference frame and (*b*) a reference frame affixed to the car. $\vec{a}_{Ec}$ is the Earth observer's measurement of the car's acceleration, $\vec{a}_{ME}$ is the moving observer's measurement of Earth's acceleration, and $\vec{a}_{Mc}$ is the moving observer's measurement of the car's acceleration.



from the way it looks at home. If you drop your keys, they fall straight down just as they would if the plane were at rest on the ground. And when the flight attendant pours coffee into your cup while you all move at 260 m/s, you don't expect him to spill it on you any more than you do when a waiter pours your coffee in a restaurant.

In fact, if the engines are very quiet, the ride is smooth, and the window shades are down, it is impossible for you to determine whether or not the plane is moving. Here is the simplest possible experiment: Put a marble on the floor of the plane. Provided the floor is level relative to the ground, the marble remains at rest where you placed it. This is true whether the plane is at rest or cruising at a constant 260 m/s. When the plane is sitting at the gate, inertia keeps the marble at rest relative to the floor (which is at rest relative to the ground), and so $\vec{a} = \vec{0}$. In the air, inertia keeps the marble moving along with the plane at a constant velocity relative to the ground (or, put differently, inertia keeps the marble at rest relative to the floor), and so again $\vec{a} = \vec{0}$.

The Earth reference frame or any reference frame moving at constant velocity relative to Earth is called an **inertial reference frame.** We can tell whether or not a reference frame is inertial by testing whether or not the **law of inertia** holds:

> **In an inertial reference frame, any isolated object that is at rest remains at rest, and any isolated object in motion keeps moving at a constant velocity.**

The law of inertia does not hold in a reference frame that is accelerating relative to Earth. For example, imagine sitting in the passenger seat of a car moving at constant velocity with a marble on the floor near your feet. As long as the car is on a horizontal surface and keeps going straight at a constant speed, the marble remains at rest. When the driver suddenly accelerates forward, however, everything in the car lurches backward, with the marble probably ending up somewhere in the back of the car. According to an observer at rest in the Earth reference frame (**Figure 6.6*a***), the marble resists being accelerated forward because of its inertia, and unless you have glued it to the floor or restrained it in some other way, it fails to keep up with the accelerating car. Seen from inside the car, the isolated marble suddenly accelerates backward, and so the law of inertia does not hold in the reference frame of the accelerating car (Figure 6.6*b*). Reference frames in which the law of inertia does not hold are called *noninertial reference frames*.

We have singled out the Earth reference frame as the basic inertial reference frame. There is nothing fundamentally special about the Earth reference frame, however, other than that we perform most experiments in this frame. Strictly speaking, Earth is not an inertial reference frame because it revolves around a north-south axis and orbits the Sun in a nearly circular orbit. Because motion on a curved path means that the direction of the velocity is changing and so the velocity is not constant, Earth is accelerating and therefore is a noninertial reference frame. For most cases, the acceleration is too small to be noticeable, and so we may, for most practical purposes, consider the Earth reference frame to be inertial.

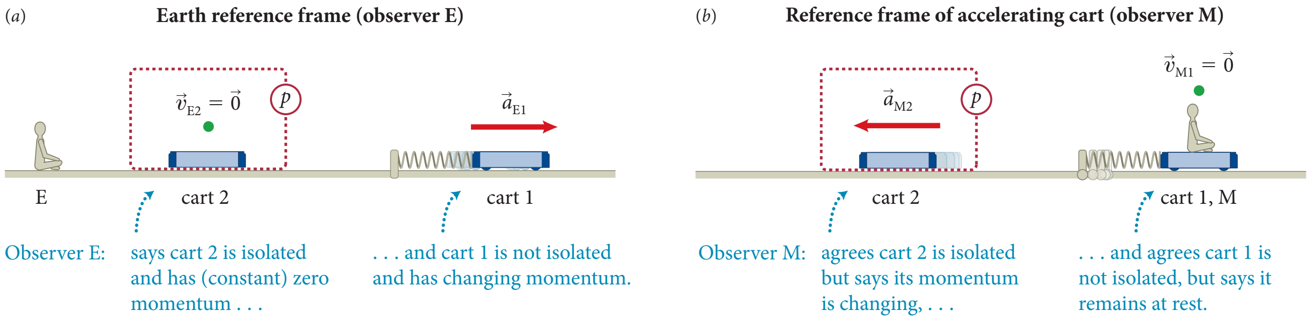
Exercise 6.2 Inertial or not?

Which of these reference frames are inertial: one affixed to (*a*) a merry-go-round, (*b*) the space shuttle orbiting Earth, (*c*) an airplane taking off, (*d*) a train moving at constant speed along a straight track?

SOLUTION (*b*) and (*d*). If I imagine placing a marble at rest in each reference frame and then observing the marble's motion, I can tell whether the law of inertia holds in that reference frame. From experience, I know that a marble remains at rest on the floor of a train moving at constant speed along a straight track, and so the train's reference frame is inertial. I also know that a marble moves away from where I place it on a merry-go-round and in an airplane taking off, so those reference frames are noninertial. Having never been on the space shuttle, I have no direct experience with such a reference frame. However, I have seen astronauts and objects floating in the space shuttle, and this motion suggests that objects stay at rest or keep moving as the law of inertia states, suggesting that the reference frame of the shuttle is inertial.* ✔

*Just like the Earth reference frame, the shuttle reference frame is only approximately inertial.

Figure 6.7 A stationary cart and an accelerating cart viewed from (*a*) the Earth reference frame and (*b*) a reference frame affixed to the accelerating cart.



Our accounting procedures for momentum and energy cannot be used in noninertial reference frames. Consider, for example, the two carts shown in **Figure 6.7**. Cart 1 is being accelerated by a spring, and cart 2 is at rest in the Earth reference frame. Cart 2 constitutes an isolated system, but cart 1 is not isolated because it interacts with the spring. To observer E in the Earth reference frame (Figure 6.7*a*), the behavior of both carts is in agreement with the momentum law: The momentum of the nonisolated cart 1 changes, while the momentum of the isolated cart 2 is constant. For observer M, however, who is accelerating along with cart 1 (Figure 6.7*b*), things don't quite add up. From this observer's perspective, cart 1 remains at rest even though it interacts with the spring, and the momentum of cart 2 changes even though that cart is isolated. Equations 4.17 ($\Delta\vec{p} = \vec{0}$) and 5.23 ($\Delta E = 0$), which embody the laws of conservation of momentum and energy, do not hold in the noninertial reference frame of observer M in Figure 6.7*b*.

Are we going to run into problems because the laws of the universe are different in noninertial reference frames? No, because nothing prescribes the reference frame; we get to choose it. So for now we just avoid using noninertial reference frames. For the accelerating car, for instance, we would choose not a reference frame affixed to the car but the Earth reference frame.

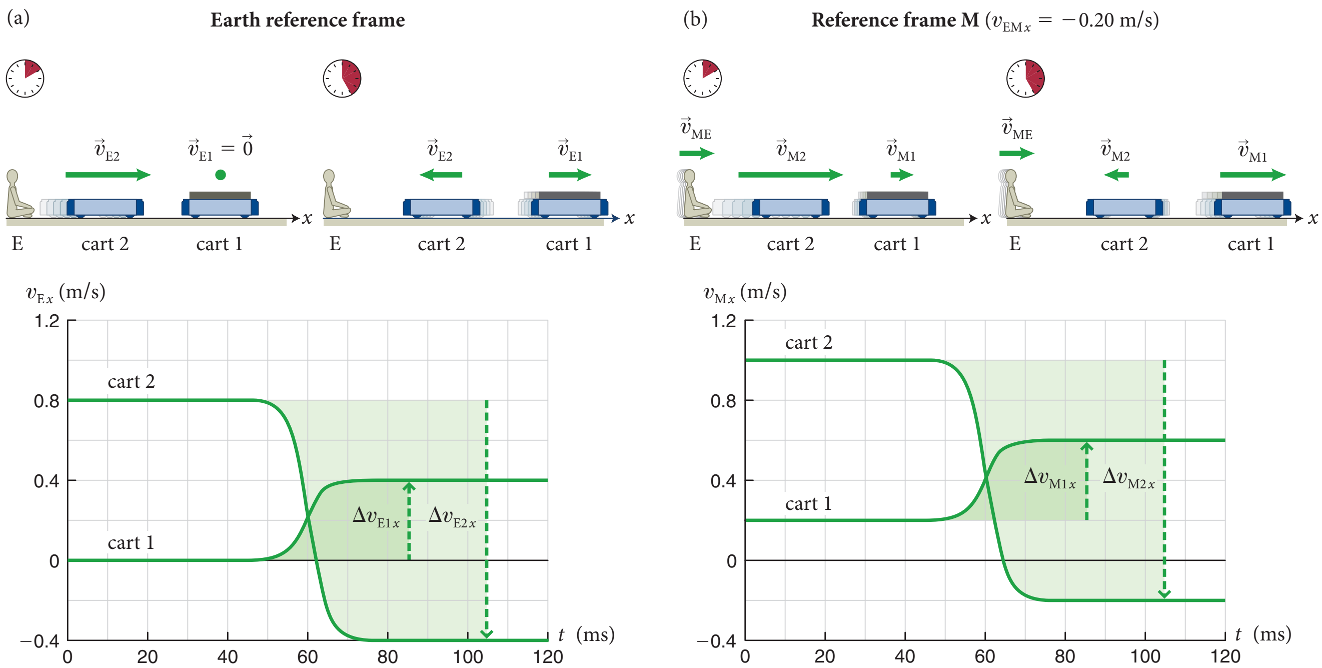
6.3 From the point of view of each observer in Figure 6.7, (*a*) is the energy of each cart constant? (*b*) Is the isolated system containing cart 1 closed? (*c*) Is the isolated system containing cart 2 closed? (*d*) Do the observations made by each observer agree with the conservation of energy law?

6.3 Principle of relativity

In the preceding section, we saw that the conservation laws apply for single objects in inertial reference frames. Let us now test the conservation laws for interacting objects.

Figure 6.8 shows velocity-versus-time graphs for two-cart collisions. The values in Figure 6.8*a* were measured by an observer in the Earth reference frame. As you saw in

Figure 6.8 Velocity-versus-time graphs for two carts colliding on a low-friction track as seen (*a*) from the Earth reference frame and (*b*) from a reference frame moving along the track at $v_{EMx} = -0.20$ m/s relative to Earth. The inertias are 0.36 kg for cart 1 and 0.12 kg for cart 2.



Checkpoint 6.1, we can obtain the velocities of the carts in reference frame M of Figure 6.8*b* by subtracting the velocities in reference frame M from the velocities in the Earth reference frame. Because reference frame M in Figure 6.8*b* moves at a constant -0.20 m/s relative to Earth, we must subtract -0.20 m/s (that is, add 0.20 m/s) from the x component of each velocity in the Earth reference frame to obtain the corresponding velocity in reference frame M.

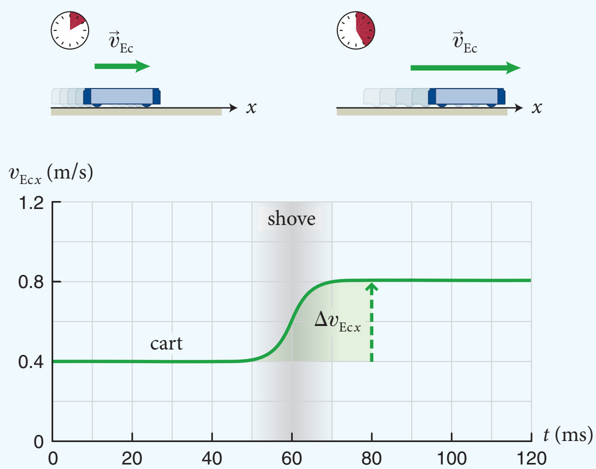
The only difference between Figures 6.8*a* and 6.8*b* is that the curves in Figure 6.8*b* are shifted upward by 0.20 m/s, the relative speed of the two reference frames. However, the *changes* in the x components of the cart's velocities, Δv_{1x} and Δv_{2x}, are not affected by the velocity of reference frame M. Because the inertia of any object is not affected by the motion of an observer studying the object, we observe that the changes in the x component of the carts' momenta, $\Delta p_{1x} = m_1 \Delta v_{1x}$ and $\Delta p_{2x} = m_2 \Delta v_{2x}$, are the same in both reference frames. So, because conservation of momentum requires that $\Delta \vec{p} = \vec{0}$ in the Earth reference frame, we also obtain $\Delta \vec{p} = \vec{0}$ in reference frame M.

If reference frame M in Figure 6.8*b* had moved at any other constant velocity relative to Earth, the vertical shift of the curves would be different but the shape of the curves would be the same. Again we would obtain the same momentum changes for the two carts, and again we would conclude that the collision does not change the momentum of the system. So, if the change in the system momentum is zero in the Earth reference frame, it must be zero in all reference frames moving at a constant velocity relative to Earth. Experiments show that this statement can be extended to any inertial reference frame, not just the Earth reference frame.

Example 6.3 Momentum shove

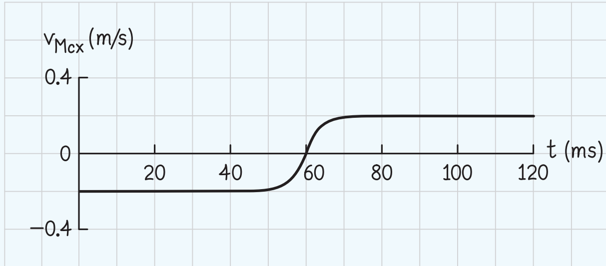
A 0.12-kg cart moving on a straight, low-friction track gets a shove so that its speed changes. Its initial and final speeds measured in the Earth reference frame are 0.40 m/s and 0.80 m/s (**Figure 6.9**). Determine the change in the cart's momentum as seen from (*a*) the Earth reference frame and (*b*) a reference frame M moving in the same direction as the cart at a speed of 0.60 m/s relative to the track.

Figure 6.9 Example 6.3.



❶ **GETTING STARTED** To determine the momentum change in either case, I need to know the cart's initial and final velocities in the two reference frames. In the Earth reference frame, $v_{Ecx,i} = +0.40$ m/s and $v_{Ecx,f} = +0.80$ m/s. In reference frame M, which moves at $v_{MEx} = +0.60$ m/s relative to Earth, these values are 0.60 m/s lower: $v_{Mcx,i} = -0.20$ m/s and $v_{Mcx,f} = +0.20$ m/s. I then make a velocity-versus-time graph for the cart as viewed from reference frame M (**Figure 6.10**).

Figure 6.10



❷ **DEVISE PLAN** Now that I have the x components of the initial and final velocities in both reference frames, I can calculate the x components of the initial and final momenta and determine the change in momentum.

❸ **EXECUTE PLAN** (*a*) In the Earth reference frame, the x components of the initial and final momenta are $p_{Ecx,i} = mv_{Ecx,i} = (0.12 \text{ kg})(+0.40 \text{ m/s}) = +0.048 \text{ kg} \cdot \text{m/s}$ and $p_{Ecx,f} = mv_{Ecx,f} = (0.12 \text{ kg})(+0.80 \text{ m/s}) = +0.096 \text{ kg} \cdot \text{m/s}$. So the x component of the change in momentum is $\Delta p_{Ecx} = (+0.096 \text{ kg} \cdot \text{m/s}) - (+0.048 \text{ kg} \cdot \text{m/s}) = +0.048 \text{ kg} \cdot \text{m/s}$. ✔

(*b*) In reference frame M, $p_{Mcx,i} = mv_{Mcx,i} = (0.12 \text{ kg}) \times (-0.20 \text{ m/s}) = -0.024 \text{ kg} \cdot \text{m/s}$ and $p_{Mcx,f} = mv_{Mcx,f} = (0.12 \text{ kg})(+0.20 \text{ m/s}) = +0.024 \text{ kg} \cdot \text{m/s}$. So $\Delta p_{Mcx} = (+0.024 \text{ kg} \cdot \text{m/s}) - (-0.024 \text{ kg} \cdot \text{m/s}) = +0.048 \text{ kg} \cdot \text{m/s}$. ✔

❹ **EVALUATE RESULT** The momentum change is the same in the two reference frames, as I expect. In reference frame M, the $v_x(t)$ curve shifts up or down, but its shape does not change, and therefore the values of $\Delta \vec{v}$ and $\Delta \vec{p}$ must also remain the same.

Example 6.3 reaffirms what we have already seen:

The change in a system's momentum is the same in any inertial reference frame.

6.4 In Example 6.3, what is the change in the cart's kinetic energy due to the shove (*a*) in the Earth reference frame, (b) in a reference frame moving in the same direction as the cart at 0.60 m/s relative to Earth, and (*c*) in a reference frame moving in the same direction as the cart at 0.80 m/s relative to Earth?

Unlike changes in momentum, changes in kinetic energy are not the same in two reference frames moving relative to each other. As Checkpoint 6.4 shows, the cart's kinetic energy increases, decreases, or stays the same depending on the reference frame.

The observation that changes in kinetic energy depend on the reference frame is disturbing. What does this result imply about conservation of energy? To begin addressing this question, let's look first at an elastic collision, such as

Table 6.1 An elastic collision seen from two reference frames

Cart	Inertia (kg)	v_x (m/s) before	v_x (m/s) after	Δv_x		Kinetic energy (10^{-3} J) before	Kinetic energy (10^{-3} J) after	ΔK
Earth reference frame								
1	0.36	0	+0.40	+0.40		0	29	+29
2	0.12	+0.80	−0.40	−1.2		38 +	9 +	−29 +
					K	**38**	**38**	**0**
Reference frame moving at −0.20 m/s relative to Earth								
1	0.36	+0.20	+0.60	+0.40		7	65	+58
2	0.12	+1.0	−0.20	−1.2		60 +	2 +	−58 +
					K	**67**	**67**	**0**

the one in Figure 6.8. In **Table 6.1** I have listed the velocities and kinetic energy of both carts. Note that even though the change in velocity Δv_x for each cart is the same in the two reference frames, the change in kinetic energy ΔK for each cart is not. In a given reference frame, however, the absolute value of ΔK is the same for both carts, and so the change in the kinetic energy K of the two-cart system is zero in each frame.

A simple way of arguing that the kinetic energy of the two-cart system *has* to remain unchanged in elastic collisions in any inertial reference frame is to look at how we formulated kinetic energy in Chapter 5 in terms of the relative velocities of two objects in a collision. As we saw earlier, differences in velocity do not depend on the velocity of the reference frame (**Figure 6.11**). In particular, our experimental observation that the relative speed of two colliding objects is unchanged in an elastic collision holds when we measure that relative speed in a reference frame that is moving at constant velocity. More generally, we observe:

The kinetic energy of a system of two elastically colliding objects does not change in any inertial reference frame.

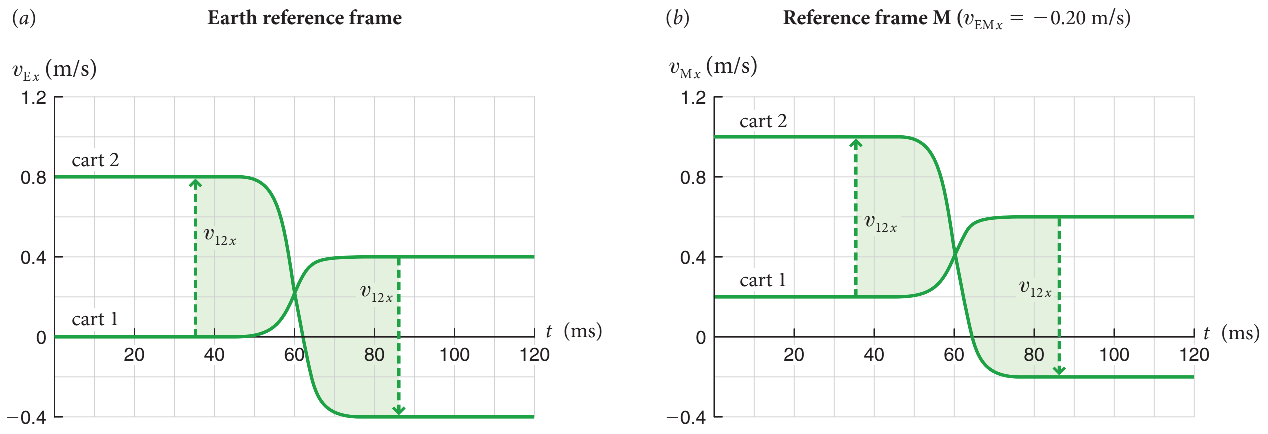
Even though K does not change in elastic collisions, the fact that the individual ΔK values depend on the reference frame could be a big problem for conservation of energy in inelastic collisions. This reference-frame dependence for the individual ΔK values could mean that ΔK for the system also depends on the reference frame. Recall from Section 5.3 that in inelastic collisions some kinetic energy is converted to internal energy, which is a measure of the state of the system. Because changes in kinetic energy are reference-frame-dependent, the amount that gets converted could be reference-frame-dependent as well and that would mean that the final state of the system would depend on the reference frame! To determine whether or not this is the case, we examine an inelastic collision in the next example.

Example 6.4 Energy conversion

Consider a collision between the two carts of Table 6.1, starting from the same initial velocities, but with $v_{\mathrm{E}1x,\mathrm{f}} = +0.30$ m/s. Make a table like Table 6.1 for this situation, and compare the amount of kinetic energy converted to internal energy in the Earth reference frame and in a reference frame M moving at -0.20 m/s relative to the track.

❶ GETTING STARTED Because all the initial values are the same, I can copy all the "before" values from Table 6.1. I am given the final velocity of cart 1, and so I need to determine the final velocity of cart 2 in the Earth reference frame, then determine the final velocities in reference frame M, calculate the corresponding kinetic energies, and determine the change in kinetic energy in each reference frame.

Figure 6.11 The relative velocity of the two colliding carts of Figure 6.8 is the same in (*a*) the Earth reference frame and (*b*) a reference frame moving along the track at $v_{\mathrm{EM}x} = -0.20$ m/s relative to Earth.



❷ **DEVISE PLAN** To determine $v_{E2x,f}$, I apply conservation of momentum to the isolated system of the two carts, which tells me that $\Delta\vec{p} = \vec{0}$. To calculate the two final velocities in reference frame M, I subtract the velocity of this reference frame from the carts' velocities in the Earth reference frame. Then I determine the kinetic energies from $K = \frac{1}{2}mv^2$ and complete the table by determining the changes in kinetic energy.

❸ **EXECUTE PLAN** Because $\vec{p}_i = \vec{p}_f$, I have

$$m_1 v_{E1x,i} + m_2 v_{E2x,i} = m_1 v_{E1x,f} + m_2 v_{E2x,f}.$$

Substituting $v_{E1x,i} = 0$ and solving for $v_{E2x,f}$, I get

$$v_{E2x,f} = \frac{m_2 v_{E2x,i} - m_1 v_{E1x,f}}{m_2}.$$

Substituting the given values into this expression, I obtain $v_{E2x,f} = -0.10$ m/s.

Because $v_{EMx} = -0.20$ m/s, I must add 0.20 m/s to the *x* components of the carts' velocities in the Earth reference frame to determine the corresponding values in reference frame M:

$$v_{M1x,i} = 0 + (+0.20 \text{ m/s}) = +0.20 \text{ m/s}$$

$$v_{M2x,i} = (+0.80 \text{ m/s}) + (+0.20 \text{ m/s}) = +1.00 \text{ m/s}$$

$$v_{M1x,f} = (+0.30 \text{ m/s}) + (+0.20 \text{ m/s}) = +0.50 \text{ m/s}$$

$$v_{M2x,f} = (-0.10 \text{ m/s}) + (+0.20 \text{ m/s}) = +0.10 \text{ m/s}.$$

Now I can calculate all the kinetic energies and complete the table (**Table 6.2**). ✔

❹ **EVALUATE RESULT** Each cart's kinetic energy change is different in the two reference frames because, as just noted in the text, changes in kinetic energy are reference-frame-dependent. However, the change in the *system*'s kinetic energy is the same in the two reference frames: −21 mJ in both cases. This tells me that the amount of kinetic energy converted to internal energy—and therefore the final state of the two carts—does not depend on the reference frame, as I expect.

6.5 Repeat Example 6.4 but let the collision be totally inelastic.

The results of Example 6.4 and Checkpoint 6.5 are reassuring. Imagine that the converted kinetic energy becomes thermal energy, with the result that the temperature of each cart increases. You wouldn't expect these temperature changes to be different when you view the collision from reference frames that are moving at constant speed relative to each other. Likewise, in a collision between two cars, where the converted kinetic energy goes into crushing the fenders, you wouldn't expect the bending of the metal to depend on the reference frame. The amount of kinetic energy converted to internal energy during an inelastic collision does not depend on the velocity of the reference frame relative to the colliding objects.

These observations, together with the results we obtained in the examples in this section, tell us that the same laws and principles we discussed in Chapters 4 and 5 hold in any inertial reference frame. In particular, the momentum and energy laws can be used in any inertial reference frame. Earlier we saw that it is not possible to tell the difference between various inertial reference frames—that is, reference frames in which the law of inertia holds. It is an experimental fact that things behave the same way in all inertial reference frames. We should therefore expect the laws that describe that behavior to be the same too. This leads us to formulate the **principle of relativity:**

> **The laws of the universe are the same in all inertial reference frames moving at constant velocity relative to each other.**

The principle of relativity provides a criterion for judging theories: For a theory to be valid, it must prescribe the same behavior in all inertial reference frames. Although observers in different inertial reference frames record different values for the velocities of carts involved in a collision and therefore different momenta and kinetic energies, they agree that the momentum and energy of the two-cart system do not change.

It follows from the principle of relativity that no experiment carried out entirely in one inertial reference frame can tell what the motion of that reference frame is relative to another reference frame. Returning to the example of Section 6.2: Sitting in a very quiet airplane during a smooth, turbulence-free ride, it is impossible to tell if you are flying

CONCEPTS

Table 6.2 **An inelastic collision seen from two reference frames**

		v_x (m/s)				Kinetic energy (10^{-3} J)		
Cart	Inertia (kg)	before	after	Δv_x		before	after	ΔK
Earth reference frame								
1	0.36	0	+0.30	+0.30		0	16	+16
2	0.12	+0.80	−0.10	−0.90		38	1	−37
					K	38	17	−21
Reference frame moving at −0.20 m/s relative to Earth								
1	0.36	+0.20	+0.50	+0.30		7	45	+38
2	0.12	+1.0	+0.10	−0.90		60	1	−59
					K	67	46	−21

or if the airplane is sitting on the runway with its engines running. Of course, you can tell you are moving relative to Earth's surface by peeking out of the window and observing the landscape move by. By timing the motion of some reference points on Earth, you could even determine the velocity of the airplane relative to Earth. By peeking out of the window, however, your observations are no longer confined to a single reference frame. To determine your velocity relative to Earth's surface, you need to take measurements in two reference frames.

A consequence of the principle of relativity is that it is not possible to deduce from measurements taken entirely in one reference frame the motion of that reference frame relative to other reference frames.

This statement also means that no inertial reference frame is preferred over any other; it is not possible to determine the *absolute* velocity of any reference frame or object. The laws of the universe are the same in all inertial reference frames, and there is no reference frame that is "at rest" in some absolute sense.

When you are standing on Earth's surface, are you "at rest" or moving at 30 km/s (the speed of Earth in its orbit around the Sun), or is your speed higher still—say, the speed of the Sun in its galactic orbit? None of these statements is meaningful because no experiments can be done to determine your speed in empty space. You can only speak of your velocity or speed *relative to something else*.

When specifying the velocity of an object, we must therefore always state relative to what we measure that velocity. For simplicity, when the Earth reference frame is the only reference frame of interest, we will omit this relative statement and the subscript E. The phrase "his velocity was $\vec{v}$" will be understood to mean the velocity is measured relative to Earth.

6.6 Is the coefficient of restitution e different in two inertial reference frames that are moving at constant velocity relative to each other? (See Eq. 5.18 if you have forgotten the definition of e.)

CONCEPTS

6.4 Zero-momentum reference frame

Let me end this first part of the chapter by addressing a practical question. If you can choose whichever inertial reference frame you wish, is any choice better than another? The reference frame you are in (usually one at rest relative to Earth) is a logical choice. By choosing some other reference frame, however, you could adjust the value of the momentum of a system up or down by some fixed amount without violating the conservation laws. You could, for instance, adjust the velocity of your reference frame in such a way that the momentum of the system you are observing becomes zero. Such a reference frame is called the system's **zero-momentum reference frame.**

To distinguish the zero-momentum reference frame from other reference frames, we'll refer to quantities measured in that reference frame with the capital letter Z. As we shall see, examining collisions from this reference frame often simplifies things.

The prescription for calculating the velocity of the zero-momentum reference frame is as follows:

The velocity $\vec{v}_{EZ}$ of a system's zero-momentum reference frame relative to Earth is equal to the momentum of the system in the Earth reference frame divided by the inertia of the system.

Example 6.5 Zeroing out the momentum

For the colliding carts in Figure 6.8, (*a*) determine the velocity of the zero-momentum reference frame relative to Earth and (*b*) show that the system momentum measured in the zero-momentum reference frame is zero both before and after the collision.

❶ GETTING STARTED This example requires me to apply the prescription for calculating the velocity of the zero-momentum reference frame and then verify that the system momentum is indeed zero in that reference frame.

❷ DEVISE PLAN To determine the velocity of the zero-momentum reference frame relative to Earth, I calculate the momentum of the two-cart system in the Earth reference frame and then divide this result by the sum of the inertias of the two carts.

Once I have determined the velocity of the zero-momentum reference frame, I subtract it from the carts' velocities in the Earth reference frame to obtain their velocities in the zero-momentum reference frame. I then use these velocities to calculate the carts' initial and final momenta.

❸ EXECUTE PLAN (*a*) In the Earth reference frame, cart 1 (inertia 0.36 kg) is initially at rest and cart 2 (inertia 0.12 kg) is moving at +0.80 m/s. The x component of the combined momentum of the two carts is therefore $(0.12\ \text{kg})(+0.80\ \text{m/s}) = +0.096\ \text{kg}\cdot\text{m/s}$. The x component of the velocity of the zero-momentum reference frame is thus

$$\frac{0.096\ \text{kg}\cdot\text{m/s}}{0.36\ \text{kg} + 0.12\ \text{kg}} = +0.20\ \text{m/s}. \checkmark$$

(*b*) To obtain the x component of the velocities of the carts in the zero-momentum reference frame, I must subtract 0.20 m/s from their values in the Earth reference frame: $v_{Z1x,i} = -0.20\ \text{m/s}$ and $v_{Z2x,i} = +0.60\ \text{m/s}$. Then $p_{Z1x,i} = -0.072\ \text{kg}\cdot\text{m/s}$ and $p_{Z2x,i} = +0.072\ \text{kg}\cdot\text{m/s}$, making the initial momentum of the system measured from this reference frame zero. ✔

For the x component of the final velocities in the zero-momentum reference frame, I get $v_{Z1x,f} = +0.20\ \text{m/s}$ and $v_{Z2x,f} = -0.60\ \text{m/s}$. So $p_{Z1x,f} = +0.072\ \text{kg}\cdot\text{m/s}$ and $p_{Z2x,f} = -0.072\ \text{kg}\cdot\text{m/s}$, making the system's final momentum measured from this reference frame zero. ✔

❹ EVALUATE RESULT In the zero-momentum reference frame, the momentum of the two-cart system before the collision is zero, as it must be from the definition of the zero-momentum frame. I also observe that it is zero after the collision, as I expect given that the system is isolated.

Figure 6.12 Collision between two carts as seen (*a*) from the Earth reference frame and (*b*) from the zero-momentum reference frame. The inertias of the carts are $m_1 = 0.36$ kg and $m_2 = 0.12$ kg and the x axis points to the right in the diagrams.

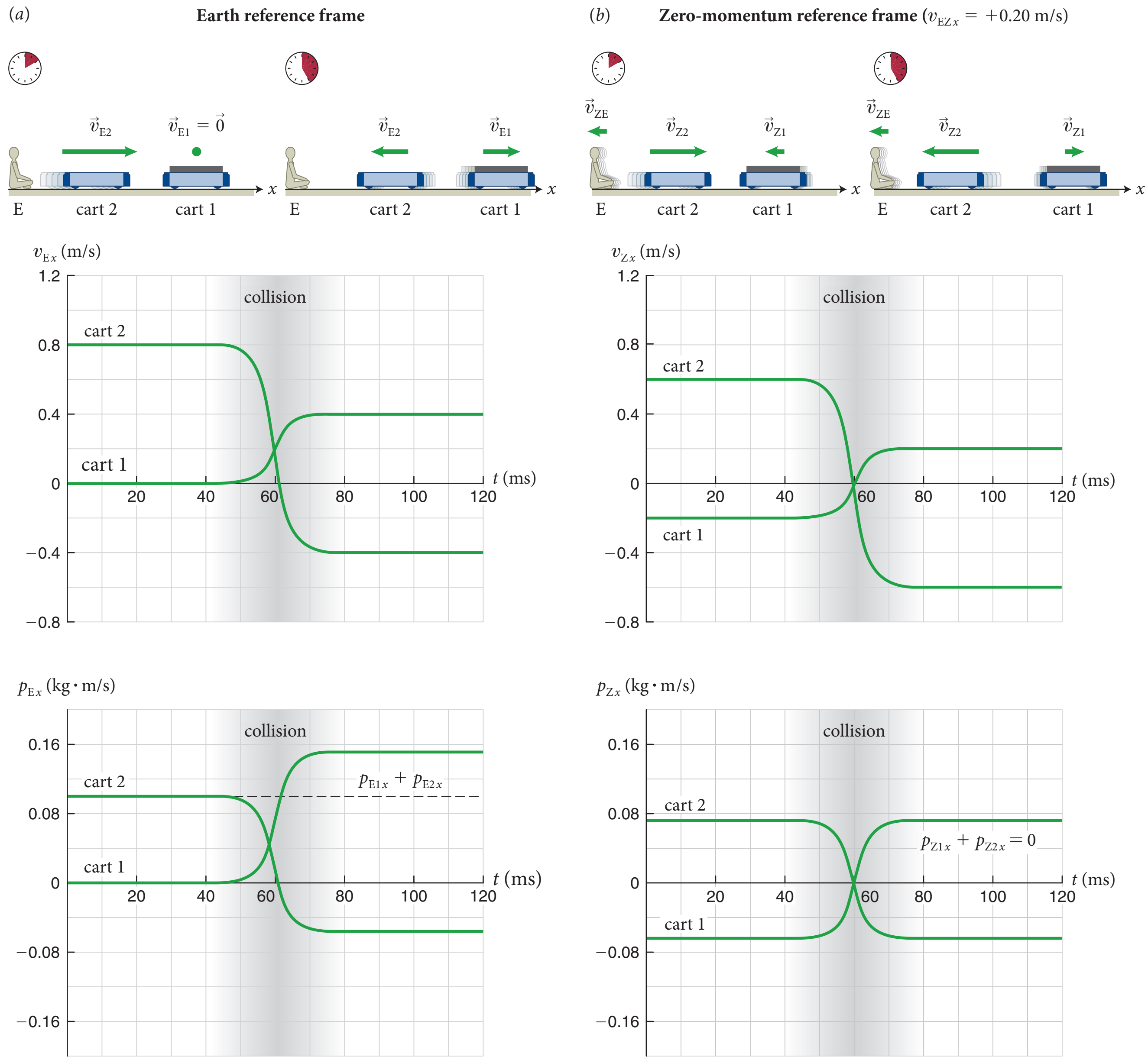


Figure 6.12 and **Table 6.3** summarize the results of Example 6.5. Note how the shift from the Earth reference frame to the zero-momentum reference frame increases symmetry in the momentum-versus-time graph. In the zero-momentum reference frame, the velocity and momentum of each cart simply change sign in the collision. In this frame, the system as a whole is at rest, allowing us to concentrate on the motion of the objects within the system.

6.7 Is the kinetic energy of the two-cart system in Figure 6.12 in the zero-momentum reference frame less than, equal to, or greater than the system's kinetic energy in the Earth reference frame?

Table 6.3 Symmetry in a zero-momentum reference frame

	Velocity change (m/s)		Momentum change (kg · m/s)	
Cart	Earth reference frame	Zero-momentum reference frame	Earth reference frame	Zero-momentum reference frame
1	$0 \rightarrow +0.40$	$-0.20 \rightarrow +0.20$	$0 \rightarrow +0.14$	$-0.072 \rightarrow +0.072$
2	$+0.80 \rightarrow -0.40$	$+0.60 \rightarrow -0.60$	$+0.096 \rightarrow -0.048$	$+0.072 \rightarrow -0.072$

Self-quiz

1. A space traveler discovers an object that accelerates in her reference frame. Which conclusion is correct? (*a*) Her reference frame is noninertial. (*b*) The object is not isolated. (*c*) You cannot tell.
2. A jogger starts from rest along a straight track. Consider the jogger-Earth system to be isolated. As the jogger's speed increases, does the speed of Earth change or remain constant?
3. A driver slams on the brakes of a car in which a marble is glued to the dashboard. To an observer standing by the side of the road, the marble slows down and has a nonzero acceleration. This happens because (*a*) the observer is in a noninertial reference frame, (*b*) the marble is not isolated, (*c*) some other reason.
4. Is it always possible to choose a zero-momentum reference frame for an isolated system that contains more than two objects?
5. On a low-friction track, two carts with unequal inertias move toward each other. After the collision, the two stick together and are at rest relative to the track. Before the collision, a zero-momentum reference frame would (*a*) move in the direction of the cart that has the greater inertia, (*b*) move in the direction of the faster cart, (*c*) not move relative to the track. (*d*) Can't tell—you need to know the carts' inertias and velocities.

Answers

1. (*c*). To claim that her reference frame is inertial, you need to know whether or not an *isolated* object remains at rest in that reference frame. From the information given, you don't know whether the object is accelerating because it is not isolated or because the reference frame is noninertial.
2. It changes. Strange as it sounds, Earth's velocity does change every time you start moving! Otherwise, conservation of momentum would not hold. Can you verify this statement experimentally? Not for the jogger-Earth case, because the ratio $\Delta v_{\text{jogger}}/\Delta v_{\text{Earth}}$ is equal to the inverse ratio of the inertias. Because Earth's inertia is enormous, its velocity change is infinitesimally small. This being so, how can you be sure you have the correct answer? Certainty comes from knowing that the law of conservation of momentum has been found to hold everywhere it has ever been tested—including on a cosmic scale.
3. (*b*). From the point of view of the observer, if the marble were not glued to the dashboard, the inertia of the marble would keep it moving at the velocity it had before the driver hit the brakes. The marble slows down only because the car's dashboard holds it in place. In other words, the dashboard and the marble interact with each other, and therefore the marble is not isolated.
4. Yes. The momentum of any system of objects is the sum of the individual momenta. To obtain the momentum of an object of inertia m_o in a reference frame moving at velocity $\vec{v}_{\text{EM}}$ relative to Earth, we must subtract $m_o\vec{v}_{\text{EM}}$ from the momentum of that object in the Earth reference frame. The system's momentum in that reference frame is then equal to the momentum of the system in the Earth reference frame minus the product $(m_1 + m_2 + m_3 + \cdots)\vec{v}_{\text{EM}}$. We can always choose a velocity $\vec{v}_{\text{EM}}$ such that the magnitude of this product is equal to the magnitude of the momentum of the system in the Earth reference frame and the algebraic signs are opposites.
5. (*c*). After the collision, both carts come to rest in the reference frame of the track, which means that the momentum of the two-cart system after the collision is zero. This means that the system's momentum before the collision was also zero. Therefore the reference frame of the track is a zero-momentum reference frame.

6.5 Galilean relativity

Consider two observers, A and B, moving at constant velocity relative to each other. Suppose they observe the same event and describe it relative to their respective reference frames and clocks (**Figure 6.13**). Let the origins of the two observers' reference frames coincide at $t = 0$ (Figure 6.13*a*). Observer A sees the event as happening at position $\vec{r}_{\mathrm{Ae}}$ at clock reading t_{Ae} (Figure 6.13*b*).* Observer B sees the event at position $\vec{r}_{\mathrm{Be}}$ at clock reading t_{Be}. What is the relationship between these clock readings and positions?

If, as we discussed in Chapter 1, we assume time is absolute—the same everywhere—and if the two observers have synchronized their (identical) clocks, they both observe the event at the same clock readings, which means

$$t_{\mathrm{Ae}} = t_{\mathrm{Be}}. \tag{6.1}$$

Because the clock readings of the two observers always agree, we can omit the subscripts referring to the reference frames:

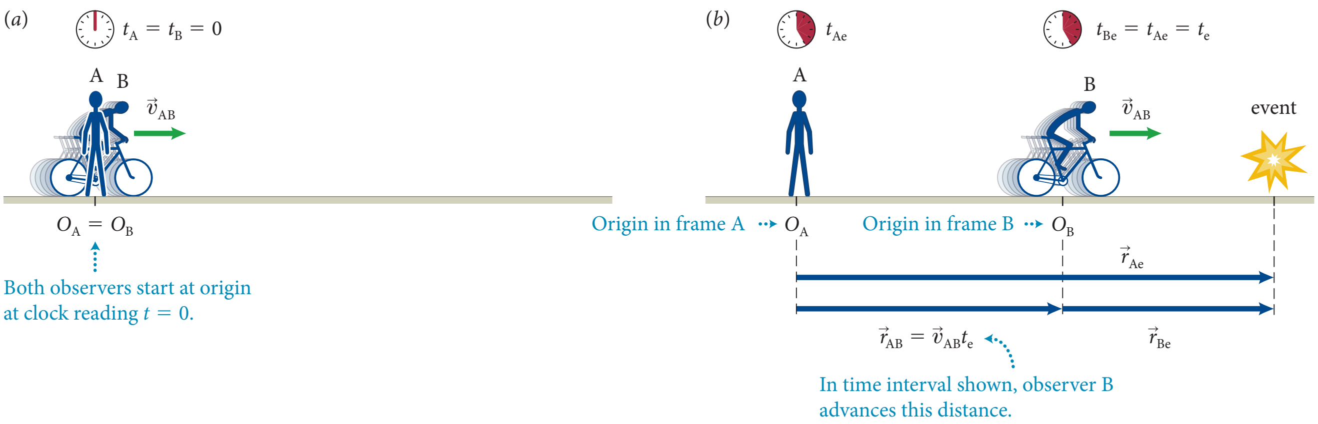
$$t_{\mathrm{A}} = t_{\mathrm{B}} = t. \tag{6.2}$$

From Figure 6.13 we see that the position $\vec{r}_{\mathrm{AB}}$ of observer B in reference frame A at instant t_{e} is equal to B's displacement over the time interval $\Delta t = t_{\mathrm{e}} - 0 = t_{\mathrm{e}}$, and so $\vec{r}_{\mathrm{AB}} = \vec{v}_{\mathrm{AB}}\, t_{\mathrm{e}}$ because B moves at constant velocity $\vec{v}_{\mathrm{AB}}$. Therefore

$$\vec{r}_{\mathrm{Ae}} = \vec{r}_{\mathrm{AB}} + \vec{r}_{\mathrm{Be}} = \vec{v}_{\mathrm{AB}}\, t_{\mathrm{e}} + \vec{r}_{\mathrm{Be}}. \tag{6.3}$$

Equations 6.2 and 6.3 allow us to relate event data collected in one reference frame to data on the same event e collected in a reference frame that moves at constant velocity relative to the first one (neither of these has to be at rest relative to Earth, but their origins must coincide at $t = 0$). To this end we rewrite these equations so that they give the values of time and position in reference frame B

Figure 6.13 Two observers moving relative to each other observe the same event. Observer B moves at constant velocity $\vec{v}_{\mathrm{AB}}$ relative to observer A. (*a*) The origins *O* of the two reference frames overlap at instant $t = 0$. (*b*) At instant t_{e}, when the event occurs, the origin of observer B's reference frame has a displacement $\vec{v}_{\mathrm{AB}} t_{\mathrm{e}}$ relative to reference frame A.



*Remember our subscript form: The capital letter refers to the reference frame; the lowercase e is for "event." Thus the vector $\vec{r}_{\mathrm{Ae}}$ represents observer **A**'s measurement of the position at which the **e**vent occurs.

QUANTITATIVE TOOLS

in terms of quantities measured in reference frame A:

$$t_B = t_A = t \tag{6.4}$$

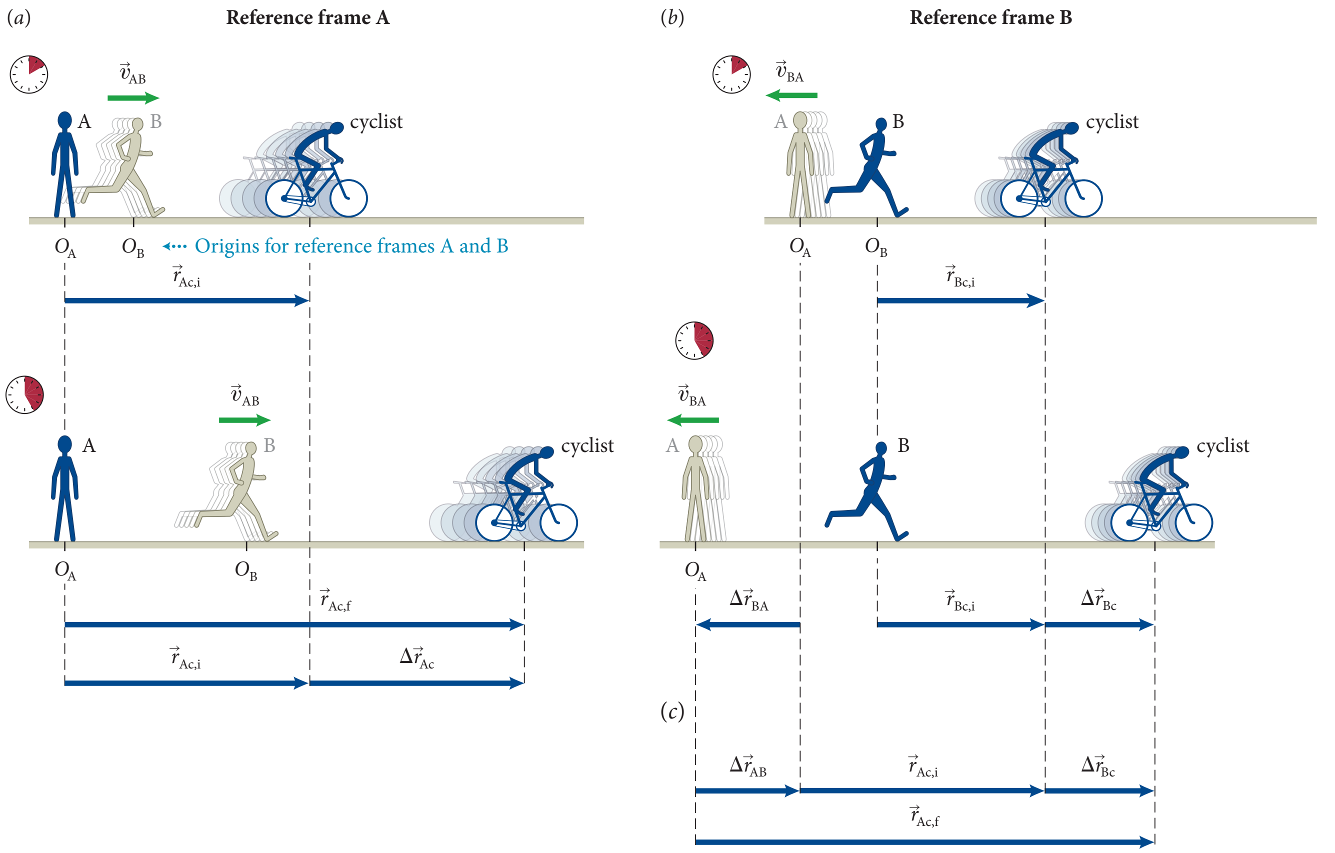
$$\vec{r}_{Be} = \vec{r}_{Ae} - \vec{v}_{AB}\, t_e. \tag{6.5}$$

These two equations, called the **Galilean transformation equations,** form the basis for *Galilean relativity*; they allow us to relate observations made in different reference frames. The Galilean transformation equations are predicated on the assumption that measurements of time intervals and lengths are not affected by motion. Even though this assumption sounds reasonable, it has been invalidated by experimental confirmation of the theory of special relativity (see Chapter 14). For most everyday phenomena, however, when speeds are much lower than the speed of light (about 3.0×10^8 m/s, see Section 1.3), Eqs. 6.4 and 6.5 remain excellent approximations.

6.8 A jogger runs in place on a treadmill whose belt moves at $v_{EBx} = +2.0$ m/s relative to Earth. Let the origins of the Earth reference frame and the reference frame B moving along with the top surface of the belt coincide at $t = 0$. (*a*) What is the jogger's position in the reference frame of the belt at $t = 10$ s? (*b*) Use Eq. 6.3 to show that an Earth observer's measurement of the jogger's position is $r_{Ejx} = 0$ at all instants.

Now that we have determined the relationship between positions in two reference frames, let us examine the relationship between the velocities of a cyclist relative to two observers, A and B, who are moving at constant velocity $\vec{v}_{AB}$ relative to each other (**Figure 6.14**). Each observer determines the velocity of the cyclist by measuring the cyclist's displacement during a certain time interval Δt.

Figure 6.14 The cyclist's displacement $\Delta\vec{r}_{Ac}$ measured by observer A is different from the displacement $\Delta\vec{r}_{Bc}$ measured by observer B, who is moving relative to observer A.



According to observer A, the bicycle undergoes a displacement $\Delta\vec{r}_{Ac}$ in the time interval Δt (Figure 6.14*a*). The bicycle's displacement $\Delta\vec{r}_{Bc}$ measured by observer B, however, is different because B undergoes a displacement $\Delta\vec{r}_{AB}$ relative to A (Figure 6.14*b*). Comparing the displacements, we obtain (Figure 6.14*c*)

$$\vec{r}_{Ac,f} - \vec{r}_{Ac,i} = \Delta\vec{r}_{Ac} = \Delta\vec{r}_{AB} + \Delta\vec{r}_{Bc}. \tag{6.6}$$

If we divide the two sides of this equation by the time interval Δt, we obtain

$$\frac{\Delta\vec{r}_{Ac}}{\Delta t} = \frac{\Delta\vec{r}_{AB}}{\Delta t} + \frac{\Delta\vec{r}_{Bc}}{\Delta t}, \tag{6.7}$$

and if we let Δt approach zero, we get

$$\vec{v}_{Ac} = \vec{v}_{AB} + \vec{v}_{Bc}. \tag{6.8}$$

Rewriting this equation so that it gives the velocity of an object o in reference frame B in terms of quantities measured in reference frame A, we get

$$\vec{v}_{Bo} = \vec{v}_{Ao} - \vec{v}_{AB}. \tag{6.9}$$

Equation 6.8 tells us that velocities obtained in different reference frames are additive. If an object moves at a velocity $\vec{v}_{Bo}$ in a reference frame that is moving at a velocity $\vec{v}_{AB}$ relative to some observer A, then the velocity $\vec{v}_{Ao}$ of that object in A's reference frame is the sum $\vec{v}_{AB} + \vec{v}_{Bo}$.

As an example, imagine walking in a train that moves along a straight track at 25 m/s relative to Earth. If you are walking forward at 1.0 m/s relative to the train, what is your velocity relative to Earth? In 1.0 s, the train moves 25 m relative to Earth, and in that same time interval you move that same 25 m plus an additional 1.0 m. Because you move (25 m) + (1.0 m) = 26 m each second, your velocity relative to Earth is 26 m/s.

To determine the relationship between an object's acceleration in the two reference frames, we begin by writing what a change in the object's velocity in reference frame B corresponds to in reference frame A. Using Eq. 6.8 and keeping in mind that the relative velocity $\vec{v}_{AB}$ of the two reference frames is constant, we get

$$\begin{aligned}\Delta\vec{v}_{Ao} &= \vec{v}_{Ao,f} - \vec{v}_{Ao,i} = (\vec{v}_{AB} + \vec{v}_{Bo,f}) - (\vec{v}_{AB} + \vec{v}_{Bo,i}) \\ &= \vec{v}_{Bo,f} - \vec{v}_{Bo,i} = \Delta\vec{v}_{Bo}.\end{aligned} \tag{6.10}$$

In words, *changes in velocity are the same in any two reference frames moving at constant relative velocity*—a conclusion we reached in Section 6.3.

Dividing both sides of Eq. 6.10 by Δt and taking the limit as Δt approaches zero, we determine that the accelerations in the two reference frames are identical, too:

$$\vec{a}_{Ao} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}_{Ao}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}_{Bo}}{\Delta t} \equiv \vec{a}_{Bo}. \tag{6.11}$$

In particular, if an object moves at constant velocity in the Earth reference frame ($\vec{a}_{Eo} = \vec{0}$), then it moves at constant velocity ($\vec{a}_{Mo} = \vec{0}$) in any reference frame M that moves at constant velocity relative to Earth, confirming what we concluded in Section 6.1.

Two bullet trains passing each other. If each train has a speed of 50 m/s relative to the track, at what speed does the oncoming train approach the one from which the photo is taken?

Answer: 100 m/s.

To simplify working with quantities in various reference frames, note from Figure 6.13*b* that

$$\vec{r}_{\rm Ae} = \vec{r}_{\rm AB} + \vec{r}_{\rm Be} \tag{6.12}$$

The first and last subscripts on either side of this equation are the same, as indicated by the arrows. Moreover, if you imagine canceling the two identical subscripts B that follow each other on the right side and contracting the two vectors $\vec{r}$ into one, you get

$$\vec{r}_{\rm A\cancel{B}} + \vec{r}_{\rm \cancel{B}e} = \vec{r}_{\rm Ae}, \tag{6.13}$$

which is the same as Eq. 6.12. This "subscript cancellation" works with any relative quantity. For example, observer A's measurement of the velocity of an object is equal to observer A's measurement of the velocity of observer B plus observer B's measurement of the velocity of the object:

$$\vec{v}_{\rm Ao} = \vec{v}_{\rm A\cancel{B}} + \vec{v}_{\rm \cancel{B}o}, \tag{6.14}$$

which is the same as Eq. 6.8.

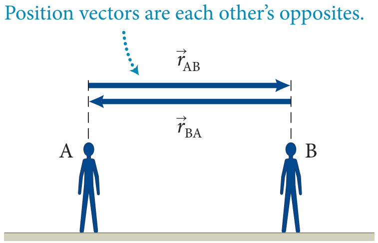
Figure 6.15 shows another useful subscript operation: Reversing the order of the subscripts reverses the direction of a relative vector $\vec{r}_{\rm AB}$. The vector describing the position of observer B relative to A has the same magnitude as the vector $\vec{r}_{\rm BA}$ describing the position of observer A relative to B, but it points in the opposite direction:

$$\vec{r}_{\rm AB} = -\vec{r}_{\rm BA}, \tag{6.15}$$

and by taking the derivative with respect to time of this equation, we obtain a similar expression for the velocity:

$$\vec{v}_{\rm AB} = -\vec{v}_{\rm BA}. \tag{6.16}$$

Figure 6.15 The position vectors of two observers relative to each other are of equal magnitude and point in opposite directions.



Procedure: Applying Galilean relativity

In problems dealing with more than one reference frame, you need to keep track not only of objects, but also of reference frames. For this reason, each quantity is labeled with two subscripts. The first subscript denotes the observer; the second denotes the object of interest. For example, if we have an observer on a train and also a car somewhere on the ground but in sight of the train, then $\vec{a}_{\rm Tc}$ is the train observer's measurement of the acceleration of the car. Once you understand this notation and a few basic operations, working with relative quantities is easy and straightforward.

observer A's measurement of velocity of car:

$\vec{v}_{\rm Ac}$

observer .... object of interest

1. Begin by listing the quantities given in the problem, using this double-subscript notation.
2. Write the quantities you need to determine in the same notation.
3. Use subscript cancellation (Eq. 6.13) to write an equation for each quantity you need to determine, keeping the first and the last subscripts on each side the same. For example, in a problem where you need to determine $\vec{v}_{\rm Tc}$ involving a moving observer B, write

$$\vec{v}_{\rm Tc} = \vec{v}_{\rm TB} + \vec{v}_{\rm Bc}\,.$$

4. If needed, use subscript reversal (Eq. 6.15) to eliminate any unknowns.
5. Use the kinematics relationships from Chapters 2 and 3 to solve for any remaining unknowns, making sure you stay in one reference frame.

You can use this procedure and the subscript operations for any of the three basic kinematic quantities (position, velocity, and acceleration).

Example 6.6 Who is moving?

You are driving at 25 m/s on a straight, horizontal road when a truck going 30 m/s in the same direction overtakes you. Let the positive x direction point in the direction of travel, and let the origins of the reference frames affixed to your car and the truck coincide at the instant the truck overtakes you. (*a*) What is your car's velocity as measured by someone in the truck? (*b*) What is the velocity of the truck relative to your car? (*c*) What is your car's position as measured by someone in the truck 60 s after overtaking you?

❶ GETTING STARTED This example involves two reference frames: the Earth reference frame (E) and the reference frame moving along with the truck (T). The velocities of the car and the truck are given in the Earth reference frame: $v_{\mathrm{Ec}x} = +25$ m/s and $v_{\mathrm{ET}x} = +30$ m/s. I need to determine the velocity of my car $\vec{v}_{\mathrm{Tc}}$ relative to the truck, the velocity of the truck $\vec{v}_{\mathrm{cT}}$ relative to my car, and my car's position $\vec{r}_{\mathrm{Tc}}$ relative to the truck 60 s after the overtaking.

❷ DEVISE PLAN I begin by using subscript cancellation to write an equation that gives the velocity $\vec{v}_{\mathrm{Tc}}$ of my car relative to the truck in terms of velocities involving the Earth reference frame: $\vec{v}_{\mathrm{Tc}} = \vec{v}_{\mathrm{TE}} + \vec{v}_{\mathrm{Ec}}$. The value of $\vec{v}_{\mathrm{Ec}}$ is given $(+25\ \mathrm{m/s})$, but I don't have a value for $\vec{v}_{\mathrm{TE}}$. However, I can use subscript reversal to write $v_{\mathrm{TE}x} = -v_{\mathrm{ET}x} = -30\ \mathrm{m/s}$. Once I have $\vec{v}_{\mathrm{Tc}}$, I can use subscript reversal once more to determine $\vec{v}_{\mathrm{cT}}$. The product of $\vec{v}_{\mathrm{Tc}}$ and the 60-s time interval gives me $\vec{r}_{\mathrm{Tc}}$ (from Eq. 2.16 $\Delta x = v_x \Delta t$).

❸ EXECUTE PLAN (*a*) Using subscript cancellation and reversal, I get $\vec{v}_{\mathrm{Tc}} = \vec{v}_{\mathrm{TE}} + \vec{v}_{\mathrm{Ec}} = -\vec{v}_{\mathrm{ET}} + \vec{v}_{\mathrm{Ec}}$ or, in terms of x components,

$$v_{\mathrm{Tc}x} = -v_{\mathrm{ET}x} + v_{\mathrm{Ec}x} = -(+30\ \mathrm{m/s}) + (+25\ \mathrm{m/s})$$

$$= -5.0\ \mathrm{m/s}. ✔$$

(*b*) $v_{\mathrm{cT}x} = -v_{\mathrm{Tc}x} = +5.0\ \mathrm{m/s}$ ✔

(*c*) My displacement in the truck's reference frame is $\Delta x_{\mathrm{Tc}} = (-5.0\ \mathrm{m/s})(60\ \mathrm{s}) = -300\ \mathrm{m}$. Thus 60 s after the truck passes you, a person in the truck measures your position as $r_{\mathrm{Tc}x} =$ 300 m behind the truck. ✔

❹ EVALUATE RESULT The truck is moving faster than my car, and so relative to the truck my car moves in the negative x direction, confirming the minus sign on my answers to parts *a* and *c*. Relative to me, the truck moves in the positive x direction, in agreement with the plus sign of my answer to part *b*.

6.9 In a train moving due north at 3.1 m/s relative to Earth, a passenger carrying a suitcase walks forward down the aisle at 1.2 m/s relative to the train. A spider crawls along the bottom of the suitcase at 0.5 m/s southward relative to the suitcase. What is the velocity of the spider relative to Earth?

6.6 Center of mass

To relate momentum in two reference frames moving relative to each other, we must examine inertia. Equation 4.2 told us how to obtain the inertia m_{o} of an object by letting the object collide with the standard inertia:

$$m_{\mathrm{o}} \equiv -\frac{\Delta v_{\mathrm{s}x}}{\Delta v_{\mathrm{o}x}} m_{\mathrm{s}}. \tag{6.17}$$

Suppose this inertia m_{o} is measured by two observers, A and B, who are moving at constant velocity $\vec{v}_{\mathrm{AB}}$ relative to each other. Even though the velocity values measured by the two observers differ, Eq. 6.10 tells us that the moving observer obtains the same velocity changes $\Delta v_{\mathrm{s}x}$ and $\Delta v_{\mathrm{o}x}$ as those measured in the Earth reference frame. This means that we can omit the subscript referring to the reference frame and write

$$m_{\mathrm{Ao}} = m_{\mathrm{Bo}} = m_{\mathrm{o}}. \tag{6.18}$$

Therefore the momenta of an object measured in two reference frames A and B are related by

$$\vec{p}_{\mathrm{Ao}} \equiv m_{\mathrm{o}}\vec{v}_{\mathrm{Ao}} = m_{\mathrm{o}}(\vec{v}_{\mathrm{AB}} + \vec{v}_{\mathrm{Bo}}) \equiv m_{\mathrm{o}}\vec{v}_{\mathrm{AB}} + \vec{p}_{\mathrm{Bo}}, \tag{6.19}$$

where we have used Eq. 6.8 to relate the velocities in the two reference frames. Equation 6.19 shows that the momentum measured in reference frame B is

QUANTITATIVE TOOLS

different from the momentum measured in reference frame A. In particular, if the velocity of reference frame B relative to reference frame A is $\vec{v}_{AB} = \vec{p}_{Ao}/m_o$, the momentum measured in reference frame B is zero: $\vec{p}_{Bo} = \vec{0}$ (and so reference frame B is a zero-momentum reference frame for that object).

We can make the momentum of a system of objects zero in the same way. Suppose we have a system made up of objects that have inertias $m_1, m_2, \ldots,$ and we let the momentum of the system in reference frame A be $\vec{p}_{A\,sys}$. Using Eq. 6.19, we can write for the momentum of the system

$$\begin{aligned}\vec{p}_{A\,sys} &= \vec{p}_{A1} + \vec{p}_{A2} + \cdots \\ &= (m_1\vec{v}_{AB} + \vec{p}_{B1}) + (m_2\vec{v}_{AB} + \vec{p}_{B2}) + \cdots \\ &= (m_1 + m_2 + \cdots)\vec{v}_{AB} + (\vec{p}_{B1} + \vec{p}_{B2} + \cdots) = m\vec{v}_{AB} + \vec{p}_{B\,sys}, \quad (6.20)\end{aligned}$$

where $m = m_1 + m_2 + \cdots$ is the inertia of the system. If we adjust the velocity $\vec{v}_{AB}$ of reference frame B relative to reference frame A such that

$$\vec{v}_{AB} = \frac{\vec{p}_{A\,sys}}{m}, \quad (6.21)$$

then, according to Eq. 6.20, reference frame B is a zero-momentum reference frame:

$$\vec{p}_{B\,sys} = \vec{p}_{A\,sys} - \mathrm{m}\vec{v}_{AB} = \vec{p}_{A\,sys} - m\frac{\vec{p}_{A\,sys}}{m} = \vec{0}. \quad (6.22)$$

Equations 6.21 and 6.22 show that relative to Earth the velocity of the zero-momentum reference frame Z for a system of objects is equal to the system's momentum measured in the Earth reference frame divided by the inertia of the system (as we stated in Section 6.4):

$$\vec{v}_{EZ} = \frac{\vec{p}_{E\,sys}}{m} = \frac{m_1\vec{v}_{E1} + m_2\vec{v}_{E2} + \cdots}{m_1 + m_2 + \cdots}. \quad \text{(zero-momentum reference frame)} \quad (6.23)$$

The velocity of the zero-momentum reference frame is related to the position of the **center of mass** of a system. This position is defined as

$$\vec{r}_{cm} \equiv \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}, \quad (6.24)$$

where $\vec{r}_1, \vec{r}_2, \ldots$ represent the positions of the objects of inertia $m_1, m_2, \ldots$ in the system.* Equation 6.24 applies to the values of the positions measured in *any* reference frame (and for this reason we can omit the reference-frame subscripts). To show the relationship between the velocity of the zero-momentum reference frame and the position of the center of mass, we take the derivative of the left and right sides of Eq. 6.24 with respect to time:

$$\frac{d\vec{r}_{cm}}{dt} = \frac{m_1(d\vec{r}_1/dt) + m_2(d\vec{r}_2/dt) + \cdots}{m_1 + m_2 + \cdots}. \quad (6.25)$$

*Even though I pointed out in Chapter 5 that *mass* and *inertia* are synonyms, we should—to be consistent with our use of the word *inertia* for the quantity m—call $\vec{r}_{cm}$ the position of the *center of inertia*. Historically, however, $\vec{r}_{cm}$ has always been called the position of the *center of mass*, a tradition we shall follow.

Because $d\vec{r}/dt = \vec{v}$ (Eq. 2.25), we can rewrite Eq. 6.25 in the form

$$\vec{v}_{\rm cm} \equiv \frac{d\vec{r}_{\rm cm}}{dt} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots}{m_1 + m_2 + \cdots}, \tag{6.26}$$

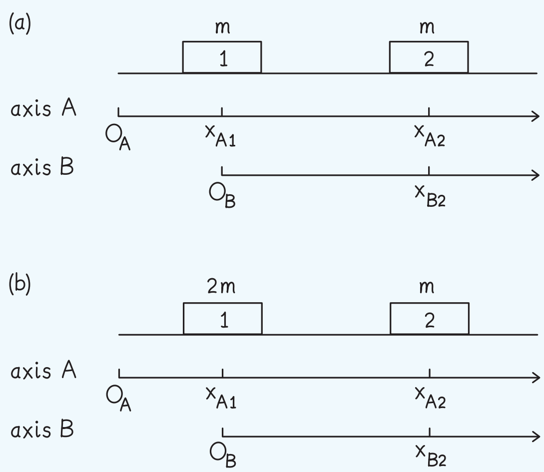
where $\vec{v}_{\rm cm}$ is the system's **center-of-mass velocity.** Note that this velocity is precisely the velocity of the zero-momentum reference frame in Eq. 6.23. Thus we conclude that, when measured from a reference frame moving at the same velocity as the center of mass of a system, the system's momentum is automatically zero. (In other words, such a reference frame is a zero-momentum reference frame.) In figures we shall denote the center of mass by a circle with a cross in it: ⊗.

Example 6.7 Two-cart center of mass

The positions of two identical carts at rest on a low-friction track are measured relative to two axes oriented in the same direction along the track. On axis A, cart 1 is at $x_{\rm A1}$ and cart 2 is at $x_{\rm A2}$. On axis B, cart 1 is at the origin and cart 2 is at $x_{\rm B2}$. (*a*) Determine the position of the center of mass of the two-cart system relative to each axis. (*b*) If instead of identical carts you have $m_1 = 2m_2$, what is the position of the center of mass on each axis?

1 GETTING STARTED I begin by making a sketch of the situation (**Figure 6.16**). Each axis corresponds to a reference frame that is at rest relative to Earth. Because the origins of the two axes don't coincide, the x components of the cart positions on axis A are different from those on axis B. I have to use these x components to determine the position of the system's center of mass on each axis for two cases: when the cart inertias are identical and when they are not.

Figure 6.16



2 DEVISE PLAN The position of the center of mass of a system of objects is given by Eq. 6.24. To use this equation in this problem, I must write it in terms of components. For axis A, I have

$$x_{\rm A\,cm} = \frac{m_1x_{\rm A1} + m_2x_{\rm A2}}{m_1 + m_2}, \tag{1}$$

and for axis B I replace the subscripts A with B. To solve this problem, I substitute the given values into this equation.

3 EXECUTE PLAN (*a*) Substituting $m_1 = m_2 = m$ into Eq. 1, I get for axis A

$$x_{\rm A\,cm} = \frac{mx_{\rm A1} + mx_{\rm A2}}{m + m} = \frac{m(x_{\rm A1} + x_{\rm A2})}{2m} = \frac{x_{\rm A1} + x_{\rm A2}}{2}. \checkmark$$

This position is halfway between the the two carts (see Figure 6.16). For axis B, I get

$$x_{\rm B\,cm} = \frac{mx_{\rm B1} + mx_{\rm B2}}{m + m} = \frac{m(0 + x_{\rm B2})}{2m} = \frac{x_{\rm B2}}{2}, \checkmark$$

which is again the position midway between the two carts.

(*b*) Now I have $m_1 = 2m_2 = 2m$, and so for axis A,

$$x_{\rm A\,cm} = \frac{2mx_{\rm A1} + mx_{\rm A2}}{2m + m} = \frac{2x_{\rm A1} + x_{\rm A2}}{3}. \checkmark$$

This position is one-third of the way from $x_{\rm A1}$ to $x_{\rm A2}$ on axis A because $(2x_{\rm A1} + x_{\rm A2})/3 = x_{\rm A1} + (x_{\rm A2} - x_{\rm A1})/3$. For axis B,

$$x_{\rm B\,cm} = \frac{2m\cdot(0) + mx_{\rm B2}}{2m + m} = \frac{x_{\rm B2}}{3}, \checkmark$$

which is one-third of the way between the origin of axis B and cart 2. If you go back to the carts in Figure 6.16*b* and mark the center-of-mass position on both axes, you'll see that the position $x_{\rm B\,cm} = x_{\rm B2}/3$ is aligned with the position $x_{\rm A\,cm} = (2x_{\rm A1} + x_{\rm A2})/3$, as it must be. The center of mass of any given system can be at one position only, regardless of which reference frame is used to locate that position.

4 EVALUATE RESULT When $m_1 = m_2$, the center of mass lies at the center of the system (midway between the carts) regardless of the choice of axis, which makes sense. When $m_1 = 2m_2$, the center of mass lies twice as close to cart 1 as to cart 2. This makes sense because I expect the center of mass to shift toward the object with the larger inertia. If, for example, the inertia of cart 1 were so large that the inertia of cart 2 was negligible, I could drop the second term in the numerator and denominator of Eq. 1, which would make the position of the center of mass be that of cart 1.

6.10 In Example 6.7, let $m_1 = 3m_2$. (*a*) Where on axis A is the center of mass of the two-cart system? (*b*) Where on axis A would you need to place a third cart of inertia $m_3 = m_1$ so that the center of mass of the three-cart system is at the position of cart 2?

Example 6.7 demonstrates an important point: The position of the center of mass of a system is a property of the system that is independent of the choice of reference frame. Equation 6.24 suggests that the center of mass represents a sort of "average" position of a system, but the center of mass is more important than that. Back in Chapter 2 we began our study of motion by specifying the position of an object at each instant by the position vector $\vec{r}$. Real objects, however, are *extended* rather than *pointlike,* and so to say "the position" of a system or of a real object is not a precise statement unless we specify a fixed reference point in the system or on the object. For example, the statement "I parked my car 3 m from the fire hydrant" could mean that the front end of the car is 3 m away from the hydrant, or the midpoint of the car is 3 m from the hydrant, or any other point on the car is 3 m from the hydrant. The center of mass allows us to specify a fixed position in a system according to an exact prescription given by Eq. 6.24.

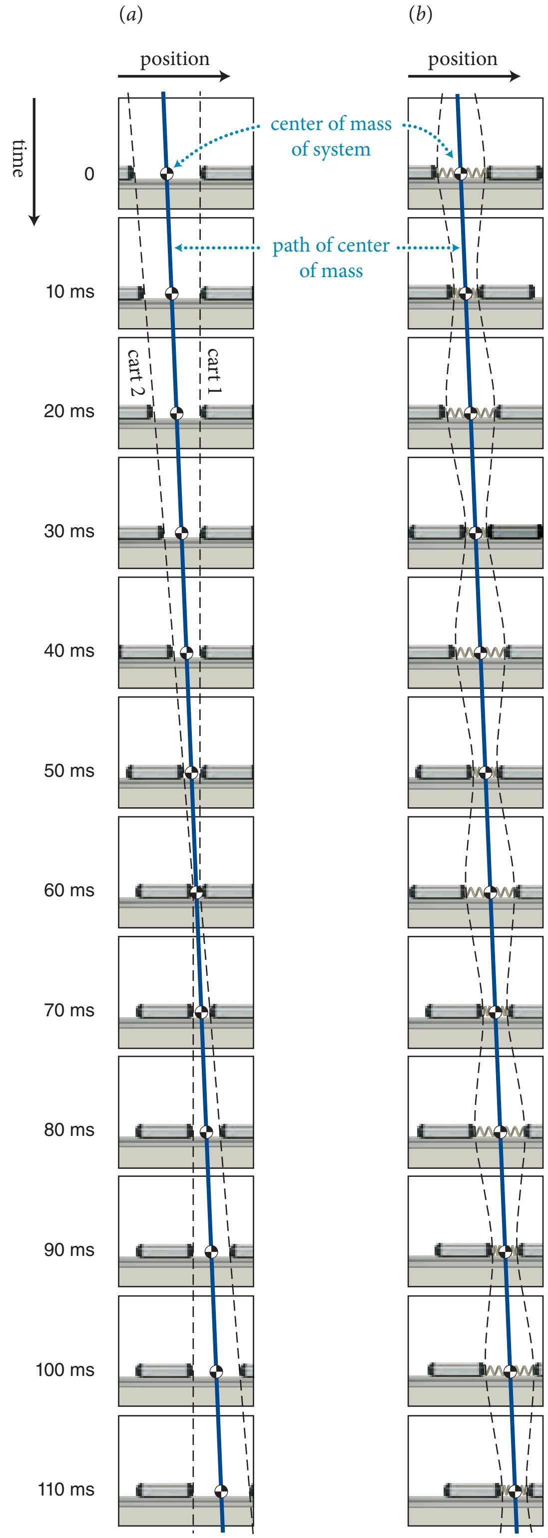
For extended objects, we can apply Eq. 6.24 by dividing the object into small pieces and substituting the position and inertia of each piece into the equation. This method is described in Appendix D. For symmetrical objects that have a uniformly distributed inertia, however, we don't need to carry out the calculation because the center of mass lies on an axis of symmetry and on any plane of symmetry. For example, the center of mass of a uniform rod lies at the center of the rod, midway between its ends, and the center of mass of a uniform sphere or cube lies at the geometric center of the sphere or cube.

Center of mass is also an important tool in our quest for simplification: It allows us to be precise even for systems where the motion is complex. To take a specific example, consider the motions shown in **Figure 6.17**. Because the two carts are identical, the center of mass of the two-cart system is always halfway between them. Notice that the center-of-mass motion is unaffected by the collision in Figure 6.17*a*. This is so because the momentum of the system doesn't change, and so the motion of the system must be constant. The center of mass of the system moves at velocity $\vec{v}_{cm}$ both before and after the collision.

The importance of the center of mass becomes even more evident in Figure 6.17*b*. Now the carts bounce back and forth because of the action of the spring while the two-cart system as a whole moves to the right. The motion of the center of mass of the system is entirely independent of the motion of the carts relative to each other and independent of the interactions between the carts. Even though the system continuously changes shape (the distance between the carts keeps changing), the center of mass provides a fixed reference point that moves at constant velocity.

6.11 (*a*) Determine the center-of-mass velocity of the two carts in Figure 6.8*a* before and after the collision, and verify that it is equal to the velocity of the carts at the point where the two $v_x(t)$ curves intersect. (*b*) Is the velocity of the carts at the point where the two $v_x(t)$ curves intersect always equal to the center-of-mass velocity? (*c*) Determine the velocities of the two carts in the zero-momentum reference frame, before and after the collision, and show that in this reference frame the momentum of the system is always zero.

Figure 6.17 Two identical carts moving on a low-friction track. (*a*) The carts collide elastically. (*b*) The carts are connected by a spring and bounce back and forth about the center of mass. In both cases, the center of mass moves at constant velocity, so that the $x(t)$ curve for the center of mass is always a straight line.



6.7 Convertible kinetic energy

We can derive some additional useful expressions involving the center-of-mass velocity. Keeping in mind that $\vec{v}_{\text{Eo}} = \vec{v}_{\text{EZ}} + \vec{v}_{\text{Zo}}$ (Eq. 6.8) and that for the zero-momentum reference frame $\vec{v}_{\text{EZ}} = \vec{v}_{\text{cm}}$ (Eqs. 6.23 and 6.26), we can derive an expression that gives, for a system of objects, the kinetic energy $K_{\text{E sys}}$ measured in the Earth reference frame in terms of the corresponding kinetic energy $K_{\text{Z sys}}$ measured in the zero-momentum reference frame:

$$K_{\text{E sys}} = \tfrac{1}{2}m_1 v_{\text{E1}x}^2 + \tfrac{1}{2}m_2 v_{\text{E2}x}^2 + \cdots$$

$$= \tfrac{1}{2}m_1(v_{\text{cm}\,x} + v_{\text{Z1}x})^2 + \tfrac{1}{2}m_2(v_{\text{cm}\,x} + v_{\text{Z2}x})^2 + \cdots. \tag{6.27}$$

Working out the first term on the right gives

$$\tfrac{1}{2}m_1(v_{\text{cm}\,x} + v_{\text{Z1}x})^2 = \tfrac{1}{2}m_1 v_{\text{cm}}^2 + m_1 v_{\text{cm}\,x} v_{\text{Z1}x} + \tfrac{1}{2}m_1 v_{\text{Z1}}^2, \tag{6.28}$$

where I dropped the subscripts x on the terms that contain squares of the x component of the velocity because $v_x^2 = v^2$. Working out the other terms in Eq. 6.27 and gathering terms that contain equal powers of $v_{\text{cm}\,x}$, we obtain

$$K_{\text{E sys}} = \tfrac{1}{2}(m_1 + m_2 + \cdots)v_{\text{cm}}^2 + (m_1 v_{\text{Z1}x} + m_2 v_{\text{Z2}x} + \cdots)v_{\text{cm}\,x}$$

$$+ \left(\tfrac{1}{2}m_1 v_{\text{Z1}}^2 + \tfrac{1}{2}m_2 v_{\text{Z2}}^2 + \cdots\right). \tag{6.29}$$

From the definition of the zero-momentum reference frame, we have

$$m_1 v_{\text{Z1}x} + m_2 v_{\text{Z2}x} + \cdots = p_{\text{Z1}x} + p_{\text{Z2}x} + \cdots = (p_{\text{Z sys}})_x = 0, \tag{6.30}$$

so that the middle term on the right in Eq. 6.29 disappears. The last term in Eq. 6.29 is the kinetic energy $K_{\text{Z sys}}$ of the system measured in the zero-momentum reference frame, and so substituting $m = m_1 + m_2 + \cdots$, we have

$$K_{\text{E sys}} = \tfrac{1}{2}mv_{\text{cm}}^2 + K_{\text{Z sys}}. \tag{6.31}$$

The first term on the right in this equation, called the system's **translational kinetic energy,** is the kinetic energy associated with the motion of the center of mass of the system:

$$K_{\text{cm}} \equiv \tfrac{1}{2}mv_{\text{cm}}^2. \tag{6.32}$$

This term has the form of the kinetic energy of an object of inertia m moving at speed v_{cm}.

For an isolated system, the translational kinetic energy K_{cm} is *nonconvertible*, which means that it cannot be converted to internal energy. To see why this is true, note that this kinetic energy is a function of the system's center-of-mass velocity, which cannot change for an isolated system (see Section 6.8).

The remainder of the kinetic energy—the $K_{\text{Z sys}}$ term in Eq. 6.31—is the system's **convertible kinetic energy,** the amount that can be converted to internal energy without changing the momentum of the system. It is equal to the system's kinetic energy minus the (nonconvertible) translational kinetic energy:

$$K_{\text{conv}} \equiv K_{\text{Z sys}} = K_{\text{E sys}} - \tfrac{1}{2}mv_{\text{cm}}^2$$

$$= \left(\tfrac{1}{2}m_1 v_{\text{E1}}^2 + \tfrac{1}{2}m_2 v_{\text{E2}}^2 + \cdots\right) - \tfrac{1}{2}(m_1 + m_2 + \cdots)v_{\text{cm}}^2. \tag{6.33}$$

With this last equation, we can eliminate any reference to the zero-momentum reference frame in Eq. 6.31, leaving only quantities measured in the Earth reference frame:

$$K_{\text{E sys}} = K_{\text{cm}} + K_{\text{conv}}. \tag{6.34}$$

If we had started not from an expression for the kinetic energy in the Earth reference frame (Eq. 6.27) but from the corresponding expression in any other inertial reference frame A, we would have obtained the same results, with A substituted for all the subscripts E. For this reason, we can omit the subscript E and rewrite Eqs. 6.31–6.34 in a form that holds in any inertial reference frame:

$$K = K_{\text{cm}} + K_{\text{conv}}, \tag{6.35}$$

where $$K_{\text{conv}} = \left(\tfrac{1}{2}m_1 v_1^2 + \tfrac{1}{2}m_2 v_2^2 + \cdots\right) - \tfrac{1}{2}mv_{\text{cm}}^2. \tag{6.36}$$

The kinetic energy of a system can be split into a convertible part and a nonconvertible part. The nonconvertible part is the system's translational kinetic energy $K_{\text{cm}} = \frac{1}{2}mv_{\text{cm}}^2$. The remainder of the kinetic energy is convertible.

For a system of two colliding objects, the convertible kinetic energy is

$$K_{\text{conv}} = K - \tfrac{1}{2}mv_{\text{cm}}^2 = \left(\tfrac{1}{2}m_1 v_1^2 + \tfrac{1}{2}m_2 v_2^2\right) - \tfrac{1}{2}(m_1 + m_2)v_{\text{cm}}^2. \tag{6.37}$$

With some algebra (see Checkpoint 6.12), substituting the Eq. 6.26 form of v_{cm} into this expression yields

$$K_{\text{conv}} = \tfrac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)v_{12}^2 \quad \text{(two-object system)}. \tag{6.38}$$

If I now write $$\mu \equiv \frac{m_1 m_2}{m_1 + m_2}, \tag{6.39}$$

then the convertible kinetic energy for two objects takes the simple form

$$K_{\text{conv}} = \tfrac{1}{2}\mu v_{12}^2 \quad \text{(two-object system)}, \tag{6.40}$$

where $\vec{v}_{12} = \vec{v}_2 - \vec{v}_1$ is the relative velocity of the two objects (see Eq. 5.1). The right side of this expression has the form of the kinetic energy of an object of inertia μ moving at a speed v_{12}. The quantity represented by the Greek letter μ (mu), which we introduced to simplify our notation, thus has the same units as inertia. It is called the **reduced inertia,** or *reduced mass*, because it is less than the inertia of either of the two colliding objects.

6.12 Verify that Eq. 6.38 is valid by substituting Eq. 6.26 for v_{cm} into Eq. 6.37 and working through the algebra.

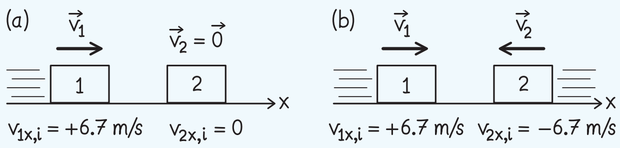
Example 6.8 Crash test

Two collisions are carried out to crash-test a 1000-kg car. (*a*) While moving at 15 mi/h, the car strikes an identical car initially at rest. (*b*) While moving at 15 mi/h, the car strikes an identical car moving toward it and also traveling at 15 mi/h. For each collision, how much kinetic energy can be converted to internal energy, and what fraction of the initial kinetic energy of the two-car system does this represent?

❶ **GETTING STARTED** I am given the inertias of two cars and their initial velocities in two collisions. I must calculate two values for each collision: the amount of kinetic energy converted during the collision and what fraction of the system's energy this converted amount is. I begin by making a sketch of the situation before each collision (**Figure 6.18**). I assume that the car that is initially at rest is free to roll (its parking brake is not on), and I ignore any friction so that I can consider the two-car system to be isolated. I also convert the speed from miles per hour to meters per second:

$$\frac{15 \text{ mile}}{1 \text{ hour}} \times \frac{1 \text{ hour}}{3600 \text{ s}} \times \frac{1609 \text{ m}}{1 \text{ mile}} = 6.7 \text{ m/s}.$$

Figure 6.18



❷ **DEVISE PLAN** The amount of energy that can be converted is given by Eq. 6.40, which requires me to calculate the reduced inertia of the two-car system (Eq. 6.39) and the relative speed of the two cars (Eq. 5.1). To determine the fraction of the initial kinetic energy that this converted amount of energy represents, I divide the converted amount by the sum of the initial kinetic energies $\frac{1}{2}mv^2$ of the two cars.

❸ **EXECUTE PLAN** (*a*) Substituting $m_1 = m_2 = 1000$ kg into Eq. 6.39, I obtain the reduced inertia of the two-car system: $\mu = 500$ kg. The initial relative speed of the two cars is

$$v_{12} = |v_{2x} - v_{1x}| = |0 - (+6.7 \text{ m/s})| = 6.7 \text{ m/s}$$

and so

$$K_{\text{conv}} = \tfrac{1}{2}(500 \text{ kg})(6.7 \text{ m/s})^2 = 11 \text{ kJ}. ✔$$

The initial kinetic energy of the system is $K_i = K_{1,i} = \frac{1}{2}(1000 \text{ kg})(6.7 \text{ m/s})^2 = 22 \text{ kJ}$, and so the fraction of energy converted is $K_{\text{conv}}/K_i = (11 \text{ kJ})/(22 \text{ kJ}) = 0.50$. ✔

(*b*) The initial relative speed of the two cars is

$$v_{12} = |v_{2x} - v_{1x}| = |6.7 \text{ m/s} - (-6.7 \text{ m/s})| = 13.4 \text{ m/s}$$

and so $K_{\text{conv}} = \frac{1}{2}(500 \text{ kg})(13.4 \text{ m/s})^2 = 45 \text{ kJ}.$ ✔

The initial kinetic energy of the system is $K_i = K_{1,i} + K_{2,i} = \frac{1}{2}(1000 \text{ kg})(6.7 \text{ m/s})^2 + \frac{1}{2}(1000 \text{ kg})(6.7 \text{ m/s})^2 = 45 \text{ kJ}$, and so all of the initial kinetic energy is convertible: $K_{\text{conv}}/K_i = (45 \text{ kJ})/(45 \text{ kJ}) = 1.0$. ✔

❹ **EVALUATE RESULT** When both cars move at 15 mi/h, the system has twice as much kinetic energy as when only one car moves at that speed, which makes sense. When only one car moves, however, the momentum of the system is nonzero, which means that not all the kinetic energy can be converted, as I found. In the second collision, the momentum of the system is zero, and so all of the kinetic energy can be converted, in agreement with my answer.

For an inelastic collision, v_{12} changes and so K changes, too. Using Eq. 6.35, we can write for the change in the kinetic energy of the system:

$$\Delta K = \Delta K_{\text{cm}} + \Delta K_{\text{conv}}. \tag{6.41}$$

The first term on the right is zero because the system is isolated, and so its translational kinetic energy cannot change. Substituting Eq. 6.40 into the last term, we have

$$\Delta K = \tfrac{1}{2}\mu v_{12\text{f}}^2 - \tfrac{1}{2}\mu v_{12\text{i}}^2 = \tfrac{1}{2}\mu(v_{12\text{f}}^2 - v_{12\text{i}}^2). \tag{6.42}$$

If we pull $v_{12\text{i}}^2$ out of the parentheses, we can write this expression in terms of the coefficient of restitution e (see Eq. 5.18):

$$\Delta K = \tfrac{1}{2}\mu v_{12\text{i}}^2\left(\frac{v_{12\text{f}}^2}{v_{12\text{i}}^2} - 1\right) = \tfrac{1}{2}\mu v_{12\text{i}}^2(e^2 - 1). \tag{6.43}$$

This value represents the amount of kinetic energy converted to internal energy during the inelastic collision. The maximum change in the system's kinetic energy occurs in a totally inelastic collision ($e = 0$):

$$\Delta K = -\tfrac{1}{2}\mu v_{12\mathrm{i}}^2 \quad \text{(totally inelastic collision)}. \tag{6.44}$$

So, when two colliding objects stick together, their relative velocity becomes zero and all of the convertible kinetic energy is converted to internal energy (compare Eqs. 6.40 and 6.44). Even though the collision is totally inelastic, however, the two-object system still has some kinetic energy because the nonconvertible part of the system's kinetic energy ($\tfrac{1}{2}mv_{\mathrm{cm}}^2$) is unaffected by the collision.

For an elastic collision, $e = 1$, and so Eq. 6.43 becomes $\Delta K = 0$: The kinetic energy is the same before and after the collision, as expected.

6.13 A moving object that has inertia m strikes a stationary object that has inertia $0.5m$. (*a*) What fraction of the initial kinetic energy of the system is convertible? (*b*) Why can't the rest be converted?

6.8 Conservation laws and relativity

In Section 6.3 we introduced the principle of relativity, which states that the laws of the universe are the same in any inertial reference frame. In this section we prove this statement for the momentum and energy laws, which embody the laws of conservation of momentum and energy.

Let us consider a system from two reference frames, A and B, that are moving at velocity $\vec{v}_{\mathrm{AB}}$ relative to each other. In Section 6.6 we saw that the momentum of an object is different in two reference frames that are moving relative to each other (Eq. 6.19), but now we are interested in *changes* in momentum. For an object of inertia m_{o}, we can write

$$\Delta\vec{p}_{\mathrm{Ao}} = m_{\mathrm{o}}\Delta\vec{v}_{\mathrm{Ao}}. \tag{6.45}$$

Because a change in velocity in reference frame A is the same as a change in reference frame B (Eq. 6.10), we have

$$\Delta\vec{p}_{\mathrm{Ao}} = m_{\mathrm{o}}\Delta\vec{v}_{\mathrm{Ao}} = m_{\mathrm{o}}\Delta\vec{v}_{\mathrm{Bo}} = \Delta\vec{p}_{\mathrm{Bo}}, \tag{6.46}$$

which tells us that the change in momentum of an object is the same in reference frames A and B. So, for the momentum of a system of objects, we have

$$\Delta\vec{p}_{\mathrm{A\,sys}} = \Delta\vec{p}_{\mathrm{B\,sys}}. \tag{6.47}$$

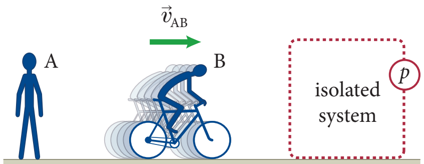
In words, *changes in the momentum of a system are the same in any two reference frames moving at constant velocity relative to each other*.

Equation 6.47 tells us that if the change in momentum of a system is zero in reference frame A, then it is also zero in reference frame B. So, if a system is isolated in reference frame A, it is also isolated in reference frame B, as required by the principle of relativity (**Figure 6.19**).

Next we examine changes in the energy of a system. Let us first consider a change in the system's internal energy. In Section 5.3 we introduced internal energy as a quantitative measure of a change in state. Given that the state of any object or system cannot depend on the motion of the observer, we must conclude that any change in internal energy must be independent of the reference frame:

$$\Delta E_{\mathrm{A\,int}} = \Delta E_{\mathrm{B\,int}}. \tag{6.48}$$

Figure 6.19 Whether or not a system is isolated does not depend on the motion of the observer, as long as the reference frames are inertial relative to each other.



Unfortunately, the situation is not so simple for a change in kinetic energy, as we saw in Section 6.3. If an object's velocity in reference frame A increases to $v_{\text{Ao}x,\text{i}} + \Delta v_{\text{Ao}x}$ from an initial value $v_{\text{Ao}x,\text{i}}$, its kinetic energy increases by an amount

$$\Delta K_{\text{Ao}} = \tfrac{1}{2}m_{\text{o}}\left[(v_{\text{Ao}x,\text{i}} + \Delta v_{\text{Ao}x})^2 - (v_{\text{Ao}x,\text{i}})^2\right]$$
$$= m_{\text{o}}v_{\text{Ao}x,\text{i}}\Delta v_{\text{Ao}x} + \tfrac{1}{2}m_{\text{o}}(\Delta v_{\text{Ao}x})^2. \quad (6.49)$$

Substituting Eqs. 6.8 and 6.10 into the right side, we get

$$\Delta K_{\text{Ao}} = m_{\text{o}}(v_{\text{AB}x} + v_{\text{Bo}x,\text{i}})\Delta v_{\text{Bo}x} + \tfrac{1}{2}m_{\text{o}}(\Delta v_{\text{Bo}x})^2. \quad (6.50)$$

Applying Eq. 6.49 to reference frame B, we have

$$\Delta K_{\text{Bo}} = m_{\text{o}}v_{\text{Bo}x,\text{i}}\,\Delta v_{\text{Bo}x} + \tfrac{1}{2}m_{\text{o}}(\Delta v_{\text{Bo}x})^2. \quad (6.51)$$

When we expand Eq. 6.50, we get

$$\Delta K_{\text{Ao}} = m_{\text{o}}v_{\text{AB}x}\Delta v_{\text{Bo}x} + m_{\text{o}}v_{\text{Bo}x,\text{i}}\Delta v_{\text{Bo}x} + \tfrac{1}{2}m_{\text{o}}(\Delta v_{\text{Bo}x})^2. \quad (6.52)$$

Because the second and third terms on the right are exactly what we have on the right in Eq. 6.51, we see that

$$\Delta K_{\text{Ao}} = m_{\text{o}}v_{\text{AB}x}\Delta v_{\text{Bo}x} + \Delta K_{\text{Bo}} \neq \Delta K_{\text{Bo}} \quad \text{(single object)}, \quad (6.53)$$

which means that the change in an object's kinetic energy depends on the reference frame in which that change is measured.

To investigate whether or not ΔK, the change in the kinetic energy of a system made up of two colliding objects, is independent of the reference frame in which the energy is measured, let us examine the kinetic energy converted to internal energy, given by Eq. 6.42. In reference frame A we have

$$\Delta K_{\text{A}} = \tfrac{1}{2}\mu(v_{\text{A12,f}}^2 - v_{\text{A12,i}}^2) \quad \text{(two-object system)}. \quad (6.54)$$

Because the relative velocities are *differences* in velocities, Eq. 6.10 tells us that $v_{\text{B12}} = v_{\text{A12}}$, and so the change in the system's kinetic energy must also be the same in any two inertial reference frames:

$$\Delta K_{\text{B}} = \Delta K_{\text{A}} \quad (6.55)$$

(in spite of the fact that the changes in kinetic energy are different for the individual objects!). Combining Eqs. 6.55 and 6.48 and generalizing the result to a system of more than two objects, we obtain

$$\Delta K_{\text{B}} + \Delta E_{\text{B int}} = \Delta K_{\text{A}} + \Delta E_{\text{A int}} \quad (6.56)$$

or

$$\Delta E_{\text{A sys}} = \Delta E_{\text{B sys}}. \quad (6.57)$$

In words, *changes in the energy of a system are the same in any two reference frames moving at constant velocity relative to each other*.

Equation 6.57 tells us that if the change in energy of a system is zero in reference frame A, then it is also zero in reference frame B. So, if a system is closed in reference frame A, it is also closed in reference frame B, as required by the principle of relativity.

6.14 Objects 1 ($m_1 = 1.0$ kg) and 2 ($m_2 = 3.0$ kg) collide inelastically. The velocities are $v_{1x,\mathrm{i}} = +4.0$ m/s, $v_{2x,\mathrm{i}} = 0$, $v_{1x,\mathrm{f}} = -0.50$ m/s, and $v_{2x,\mathrm{f}} = +1.5$ m/s. (*a*) What is the coefficient of restitution *e?* (*b*) Make a table like Table 6.1 showing the kinetic energy converted to internal energy both in the Earth reference frame and in a reference frame moving at $v_{\mathrm{EM}x} = -1.0$ m/s relative to Earth.

Chapter Glossary

SI units of physical quantities are given in parentheses.

Center of mass A reference-frame-independent reference point on a system of objects, whose position is given by

$$\vec{r}_{\mathrm{cm}} \equiv \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}. \tag{6.24}$$

Center-of-mass velocity $\vec{v}_{\mathrm{cm}}$ (m/s) Velocity of the center of mass of a system of objects:

$$\vec{v}_{\mathrm{cm}} \equiv \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots}{m_1 + m_2 + \cdots}. \tag{6.26}$$

Convertible kinetic energy K_{conv} (J) The portion of the kinetic energy of a system that can be converted to internal energy:

$$K_{\mathrm{conv}} \equiv K - \tfrac{1}{2}mv_{\mathrm{cm}}^2. \tag{6.36}$$

The convertible kinetic energy measured for a system is the same in any inertial reference frame.

Earth reference frame A reference frame at rest relative to Earth.

Galilean transformation equations The relationships between the coordinates in time and space of an event in two reference frames A and B moving at constant velocity $\vec{v}_{\mathrm{AB}}$ relative to each other:

$$t_{\mathrm{B}} = t_{\mathrm{A}} = t \tag{6.4}$$

$$\vec{r}_{\mathrm{Be}} = \vec{r}_{\mathrm{Ae}} - \vec{v}_{\mathrm{AB}}t_{\mathrm{e}}. \tag{6.5}$$

See also the Procedure box on page 136.

Inertial reference frame A reference frame in which the law of inertia holds.

Law of inertia In an inertial reference frame, any isolated object that is at rest remains at rest, and any isolated object that is in motion keeps moving at a constant velocity.

Observer A person (real or imaginary) carrying out measurements or observations in a reference frame.

Principle of relativity The laws of the universe are the same in any inertial reference frame.

Reduced inertia μ (kg) A scalar defined as

$$\mu \equiv \frac{m_1 m_2}{m_1 + m_2}. \tag{6.39}$$

Reference frame An axis and an origin that define a direction in space and a unit of length relative to which one can observe and measure motion.

Translational (nonconvertible) kinetic energy K_{cm} (J) The kinetic energy associated with the motion of the center of mass of a system:

$$K_{\mathrm{cm}} \equiv \tfrac{1}{2}mv_{\mathrm{cm}}^2. \tag{6.32}$$

The translational kinetic energy of an isolated system cannot be converted to internal energy because if it were, the system's momentum would not be constant.

Zero-momentum reference frame A reference frame in which the momentum of the system is zero. The velocity of this reference frame relative to another reference frame is equal to the system's center-of-mass velocity relative to that reference frame.