25 Work and Energy in Electrostatics

CONCEPTS

QUANTITATIVE TOOLS

As we saw in our study of mechanics, it is often easier to solve problems by using the concepts of energy and work than by using forces. In this chapter we study how to apply energy considerations to electric interactions. Because there are two types of charge—positive and negative—the energy changes associated with changes in charge configurations are a bit more complicated than those associated with changes in gravitational configurations. We first analyze the potential energy associated with a stationary charge distribution and then introduce a new quantity, *potential difference,* that is related to potential energy and that plays an important role in electronics because, unlike potential energy, it can be measured directly.

25.1 Electric potential energy

Figure 25.1 shows the energy changes that occur in closed systems of two charged objects. In Figure 25.1*a*, a positively charged particle is released from rest in the constant electric field of a large stationary object carrying a negative charge. The attractive electric interaction between the particle and the object accelerates the particle toward the object, and so the kinetic energy of the system increases. This increase in kinetic energy must be due to a decrease in **electric potential energy,** the potential energy associated with the relative positions of charged objects. As we can see, the electric potential energy of two oppositely charged objects decreases with decreasing separation between the two.

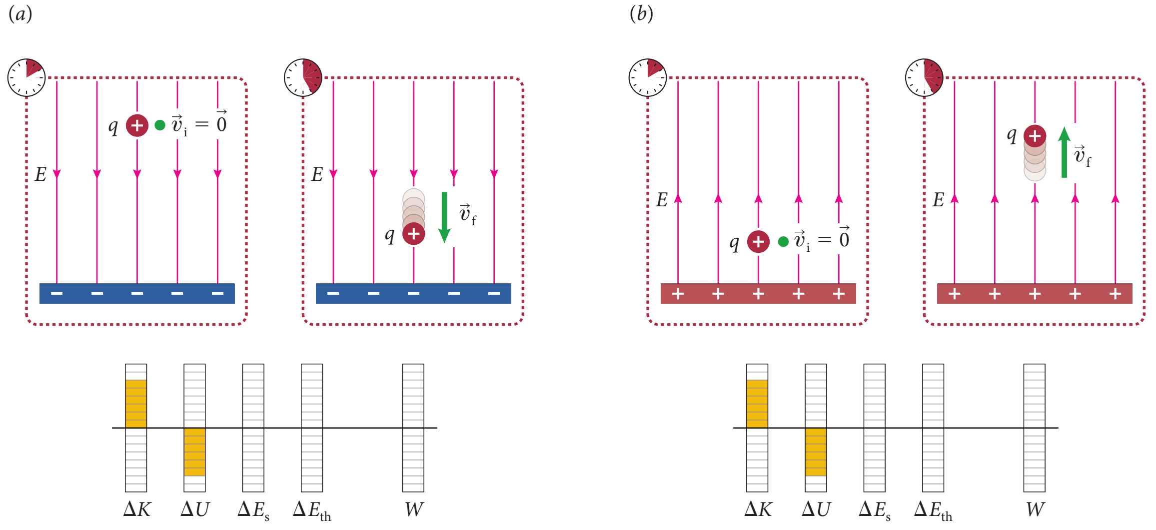
Figure 25.1*b* shows what happens in a system of two objects carrying like charges. In this case the particle is accelerated away from the object, and so the kinetic energy increases and the electric potential energy decreases with *increasing* separation between the two.

The situation depicted in Figure 25.1*a* is the electric equivalent of free fall. While all objects in free fall near Earth's surface experience the same acceleration, objects in electric fields experience *different* accelerations. Consider, for example, two particles with different masses in free fall near Earth's surface (**Figure 25.2*a***). Even though the particle with the greater mass is subject to a greater gravitational force, its acceleration is the same as that of the particle with the smaller mass. The reason is that the particle's inertia (its resistance to acceleration) is equal to its mass (see Chapter 13). Now consider the situation illustrated in Figure 25.2*b*. Two particles carrying the same charge $+q$, but of different mass, are released near the surface of a large negatively charged object. The electric forces exerted by the negatively charged object on the two particles are equal in magnitude, but because the masses of the two particles are different, the particles' accelerations are different as well.

25.1 Suppose both particles in Figure 25.2*b* are released from rest. Let $m_2 > m_1$ and consider only electric interactions. (*a*) How do their kinetic energies compare after they have both undergone the same displacement? (*b*) How do their momenta compare? (*c*) How do their kinetic energies and momenta compare at some fixed instant after they have been released? (*d*) How would you need to adjust the charges on the particles in order for the two particles to have the same acceleration upon release?

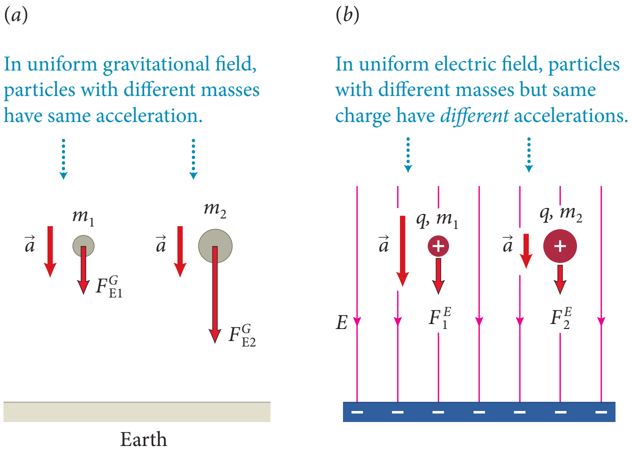
Changes in electric potential energy can also be associated with changes in the orientation of charged objects. Consider, for example, the situation illustrated in **Figure 25.3**. An electric dipole is held near the surface of a large, positively charged object. If the electric field of the

Figure 25.1 Energy diagrams for closed systems in which a positively charged particle is released from rest near a large stationary object that carries (*a*) a negative or (*b*) positive charge.



CONCEPTS

Figure 25.2 The free motion of charged particles in an electric field is different from the free fall of objects in a gravitational field.



large charged object is uniform, then the dipole begins rotating as shown (see Sections 23.4 and 23.8). As it begins rotating, the dipole gains rotational kinetic energy. This means that the electric potential energy of the system must be changing as the orientation of the dipole changes. This change occurs because the positive side of the dipole gets farther away from the positively charged object, while the negative side gets closer to it; as we've seen, both of these motions correspond to a decrease of electric potential energy.

25.2 As the dipole in Figure 25.3 continues to rotate, it reaches the point where its axis is aligned with the electric field of the large object. (*a*) What happens to the electric potential energy as the dipole moves beyond that point? (*b*) Describe the motion of the dipole beyond that point. (*c*) How would the motion of the dipole change if it were released with a different orientation from the one shown in Figure 25.3?

Figure 25.3 Energy diagram for a system in which a dipole is released from rest near a positively charged stationary object.

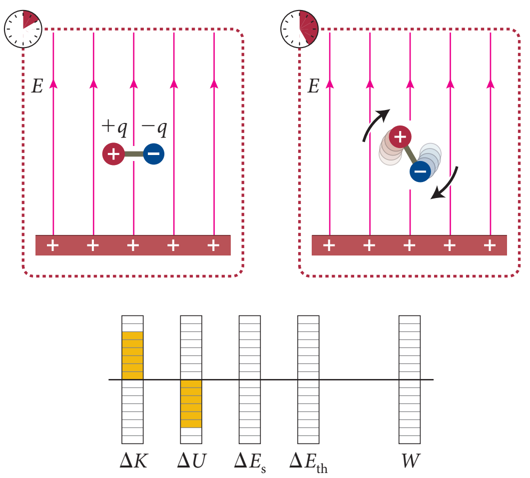
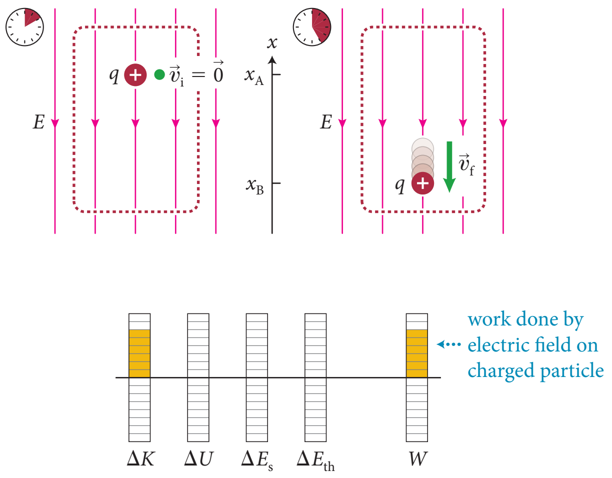


Figure 25.4 Energy diagram for a positively charged particle in the uniform electric field of a stationary, negatively charged object, that is not part of the system.

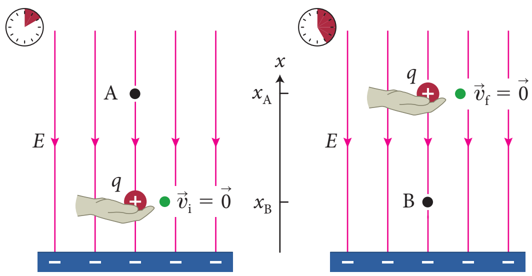


25.2 Electrostatic work

In general, we shall be considering the motion of a charged object through the constant electric field created by other stationary charged objects (such a field is sometimes called an *electrostatic field*). Therefore, our system—the charged object whose motion we are considering—is not closed. The energy of the system is not constant, and we must take into account the work done by the electric field on the system. For example, considering just the particle in Figure 25.1*a* as our system, we obtain the energy diagram shown in **Figure 25.4**. The particle can have only kinetic energy, so its increase in kinetic energy is now due to work done by the electric field on it.

25.3 (*a*) Suppose the particle in Figure 25.4 moves along the x axis from point A at $x = x_A$ to point B at $x = x_B$. How much work is done by the electric field $\vec{E}$ on it? (*b*) Suppose now that an external agent moves the particle back from B to A, starting and ending at rest as shown in **Figure 25.5**. How much work does the electric field do on the particle as it is moved? (*c*) How much work does the agent do on the particle while it is moved? (*d*) What is the combined work done by the agent and by the electric field on the particle as it is moved? (*e*) Draw an energy diagram to illustrate the energy changes of the particle as the agent moves it from B to A.

Figure 25.5 Checkpoint 25.3.



Checkpoint 25.3 illustrates the importance of distinguishing between the work done by the electric field on a charged particle and the work done by the agent moving it. We shall refer to the work done by an electrostatic field as **electrostatic work.** If the particle begins and ends at rest, the electrostatic work is equal in magnitude and opposite in sign to the work done by the agent doing the moving. Then the total work done on the particle—the sum of the electrostatic and mechanical work—is zero. Indeed, the kinetic energy of the particle does not change, and because the particle possesses no other form of energy, its energy remains unchanged: $\Delta E = W = 0$.

Note that the electric force between charged particles, just like the gravitational force, is a central force: Its line of action always lies along the line connecting the two interacting particles (see Section 13.2). A direct consequence of this fact is:

> **The electrostatic work done on a charged particle as it moves from one point to another is independent of the path taken by the particle and depends on only the positions of the endpoints of the path.**

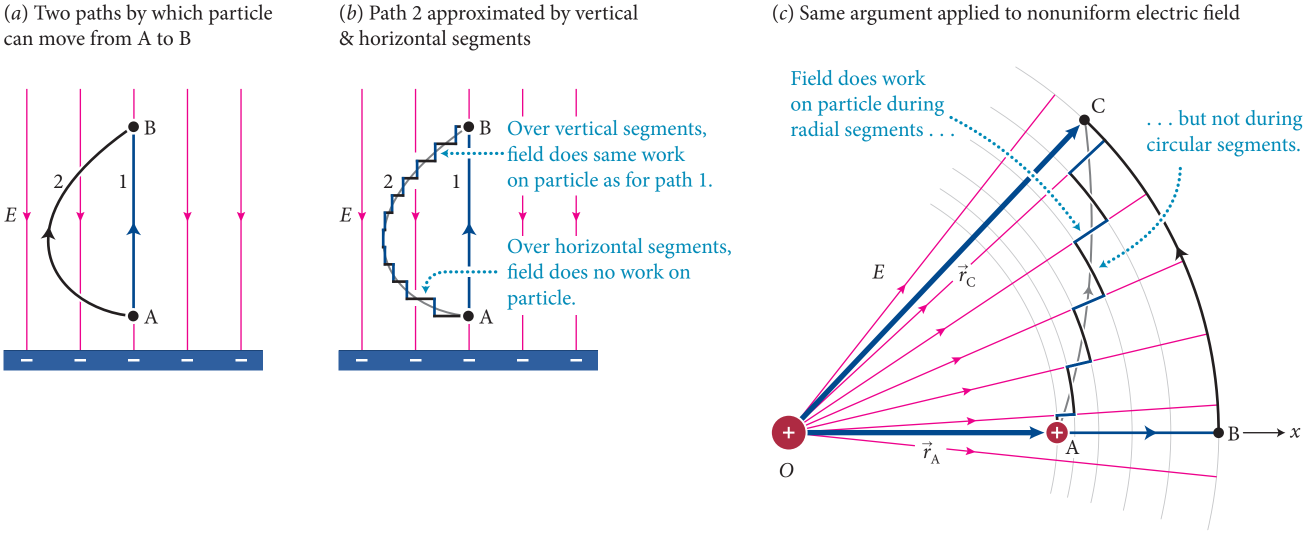
The proof of this statement parallels the one for gravitational forces in Section 13.6. Imagine, for example, lifting a particle from A to B along curved path 2 in **Figure 25.6*a*** instead of along straight path 1. As shown in Figure 25.6*b*, the path can be approximated by small straight horizontal and vertical segments. Along the horizontal segments, the electric force is perpendicular to the force displacement, so the electrostatic work on the particle along these segments is zero. Along the vertical segments, the electrostatic work on the particle is nonzero, but note that each vertical segment corresponds to an equivalent vertical segment on path 1. Thus, the displacements along all the vertical segments of path 2 add up to precisely the displacement along path 1. In other words, the electrostatic work on the particle along path 2 (or any other path from A to B) is equal to that along path 1.

Figure 25.6*c* shows how this argument can be generalized to a nonuniform electric field. Imagine a particle being moved from point A to point C along the gray trajectory in the electric field caused by an object carrying a charge at the origin. We can approximate the trajectory by a succession of small circular arcs centered about the origin and small straight radial segments. The electrostatic work done on the particle along the circular arcs is zero because the force exerted on the particle is perpendicular to the force displacement. The radial segments, on the other hand, contribute to the electrostatic work done on the particle. The sum of all the radial segments, however, is equal to the radial displacement from A to B. Because no electrostatic work is done on the particle along the circular path from B to C, we thus see that the electrostatic work done on the particle along the gray trajectory from A to C is equal to the electrostatic work done along the path from A to B. The electrostatic work done on the particle along *any* path from A to C is thus the same as the electrostatic work done from A to B.

25.4 Suppose the electrostatic work done on a charged particle as it moves along the gray path from A to C in Figure 25.6*c* is W. What is the electrostatic work done on the particle (*a*) along the path from C to B to A and (*b*) along the closed path from A to C to B and back to A?

Checkpoint 25.4 shows that the electrostatic work done on a charged particle that moves around a closed path—*any* closed path—in an electrostatic field is zero. We obtained a similar result for the work done by the gravitational force (Eq. 13.17). The physical reason for this result is that the

Figure 25.6 The electrostatic work done on a charged particle as the particle moves from point A to point B is independent of the path taken; it depends only on the positions of the endpoints of the path.



electric force between two charged particles, like the gravitational force between two objects that have mass, is nondissipative: When charged particles are moved around in electrostatic fields, no energy is irreversibly converted to other forms of energy. This is so because the electric interaction is nondissipative.

25.5 (*a*) If the electrostatic work done on a charged particle as it is moves around a closed path is zero, does this also mean that the electrostatic work done on the particle is zero as it moves along a piece of the closed path? (*b*) In Checkpoint 25.3*a* we found that the electrostatic work done on the particle is the product of the *x* components of the electric force and the force displacement. Is the same true for the electrostatic work done on a particle that is moved along path AB in Figure 25.6*c*? (*c*) What happens to this electrostatic work if (*i*) the charge on the particle and (*ii*) the mass of the particle is doubled?

The third part of Checkpoint 25.5 demonstrates a very important point: The electrostatic work done on a charged particle is proportional to the charge carried by that particle. This means that once we have calculated the electrostatic work done on a particle carrying a charge q along some path in an electrostatic field, we don't need to carry out the whole calculation again if we are interested in the electrostatic work done on another particle carrying a charge $2q$. We know that the electrostatic work done on the second particle is twice the electrostatic work done on the first. Thus, if we know the electrostatic work done on a particle carrying a unit positive charge along some path, then we know the electrostatic work done on a particle carrying *any* charge between the same two points! We therefore introduce a new quantity, called **electrostatic potential difference** (or simply **potential difference),** defined as:

The potential difference between point A and point B in an electrostatic field is equal to the negative of the electrostatic work per unit charge done on a charged particle as it moves from A to B.

The potential difference is a scalar, and because the electrostatic work done on a charged particle as it moves from one position to another can be positive or negative, the potential difference can also be positive or negative; the potential difference between any two points B and A is the negative of that between points A and B. It is important to keep in mind that potential difference *is not a form of energy*—it is electrostatic work done per unit charge and therefore has SI units of J/C.

You may be wondering why the potential difference is defined in terms of the *negative* of the electrostatic work done on a particle and not in terms of the energy required to move the particle, which has the opposite sign. The reason is that the energy required to move the particle depends on the change in the particle's kinetic energy. If the particle starts at rest and ends at a nonzero speed, the particle's kinetic energy increases and so the energy required is greater than when it starts and ends at rest. The electrostatic work done on a particle, on the other hand, is independent of any change in the particle's kinetic energy.

25.6 (*a*) Is the potential difference along any path from A to C in Figure 25.6*c* positive, negative, or zero? (*b*) Along any path from C to B? (*c*) Along the straight path from B to A? (*d*) In Figure 25.4, is the potential difference between the particle's initial and final positions positive, negative, or zero? (*e*) Express this potential difference in terms of the change in the particle's kinetic energy ΔK and its charge q.

In principle, only the potential *difference* between the endpoints of a path is meaningful. We can, however, assign a value to the **potential** at each of these endpoints by choosing a reference point. Specifically, if there is a positive potential difference between points A and B, then A is at a lower potential than B; by assigning a value to the potential at one of the two points, the value of the potential at the other point is fixed. Potential and potential difference, which are immensely useful in solving problems in electrostatics, are discussed in more detail in Section 25.5.

25.3 Equipotentials

As we have seen, the electrostatic work done on a charged particle along the horizontal segments in Figure 25.6*b* and the circular arcs in Figure 25.6*c* is zero. Consequently, the potential difference between any two points on such an arc or horizontal segment is zero. In other words, the potential has the same value at all points along these arcs or segments. Such paths are said to be **equipotential lines:**

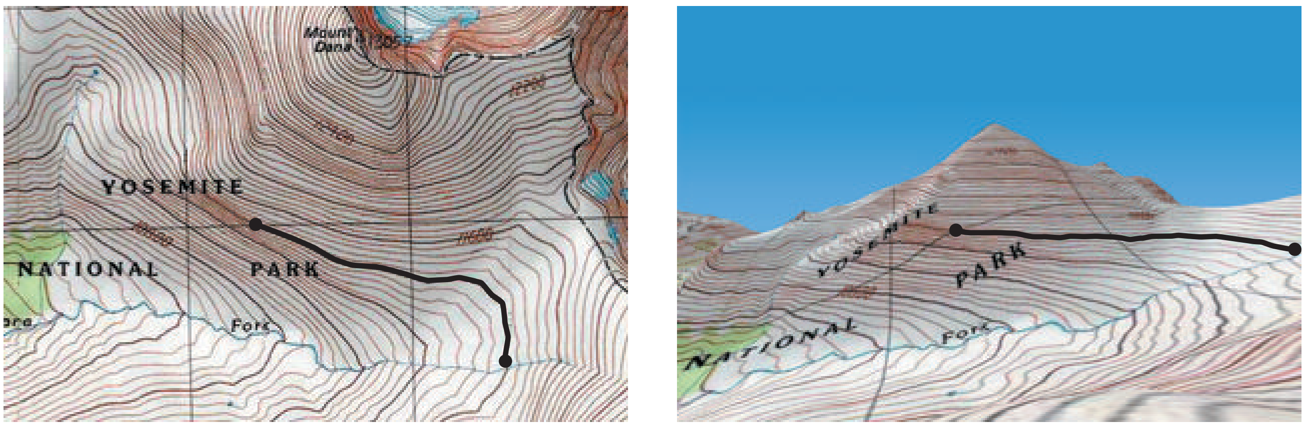
An equipotential line is a line along which the value of the electrostatic potential does not change. The electrostatic work done on a charged particle as it moves along an equipotential line is zero.

The equipotential lines in Figure 25.6 are much like contour lines on a topographical map. For example, the contour lines in **Figure 25.7** on the next page connect points of equal elevation. If you follow a contour line, your elevation remains the same, and so the gravitational potential energy associated with the separation between Earth and your body remains constant. Consequently, no gravitational work is done by Earth on you while you follow contour lines—these lines represent "gravitational equipotential lines."

Returning to the electrostatic case, note that Figure 25.6 is a two-dimensional representation of a three-dimensional situation. Thus, the equipotential line segments shown are really parts of **equipotential surfaces.** In Figure 25.6*a*, for example, the electrostatic work done on a charged particle is zero along *any* displacement parallel to the surface of the negatively charged object, into or out of the plane of the drawing. Consequently, as shown in **Figure 25.8** on the next page, any surface parallel to the surface of the charged sheet causing the electric field is an equipotential surface. Often we'll use the term *equipotential* to denote an equipotential line or surface.

Figure 25.7 Contour lines on a map are analogous to equipotential lines. If you hike along a contour line, you neither gain nor lose gravitational potential energy.

(*a*) (*b*)



Just as with vector fields and field line diagrams, it is impossible to draw equipotential surfaces at all locations. In general, equipotentials are drawn with some fixed potential difference between them, just as contour lines represent a fixed difference in altitude. Keep in mind, however, that at any point between the equipotential surfaces shown in a figure we can, in principle, draw another equipotential surface.

25.7 Consider a single charged particle. Are there any equipotential lines or equipotential surfaces surrounding this particle?

The equipotential surfaces in Figure 25.8 and those in Checkpoint 25.7 are perpendicular to the electric field lines. This is true for *any* stationary charge distribution:

The equipotential surfaces of a stationary charge distribution are everywhere perpendicular to the corresponding electric field lines.

The proof of this statement is straightforward: If the electric field line were not perpendicular to the equipotential surface, then the electric field would have a nonzero component along the surface. This means there would be a nonzero component of electric force along the surface. By definition, however, the electrostatic work done on a charged particle is zero along an equipotential surface, and so there cannot be such a component.

Figure 25.9 shows a two-dimensional view of the equipotential surfaces of a more complicated stationary charge distribution. Note how, at every point in the diagram, the equipotentials are, indeed, perpendicular to the field lines.

Recall from Section 24.5 that in electrostatic equilibrium, the electric field inside the bulk of a conducting object is zero, regardless of the shape of the object or any charge carried by it. This means that no electrostatic work is done on a charged particle inside a charged or uncharged conducting object. Thus, the entire volume of the conducting object

Figure 25.8 Equipotential surfaces in a uniform electric field in (*a*) two dimensions; and (*b*) three dimensions.

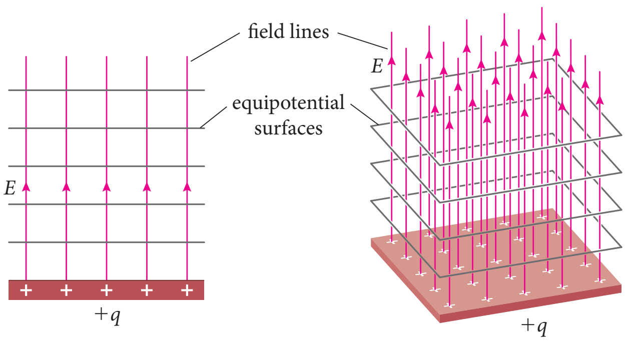
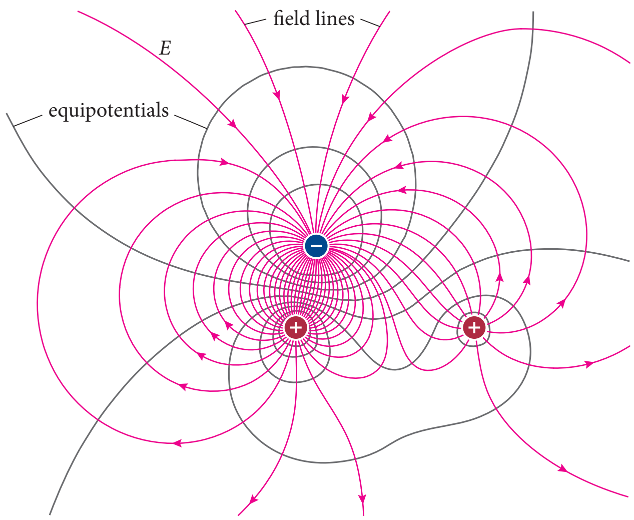


Figure 25.9 Field lines and equipotentials for three stationary charged particles.



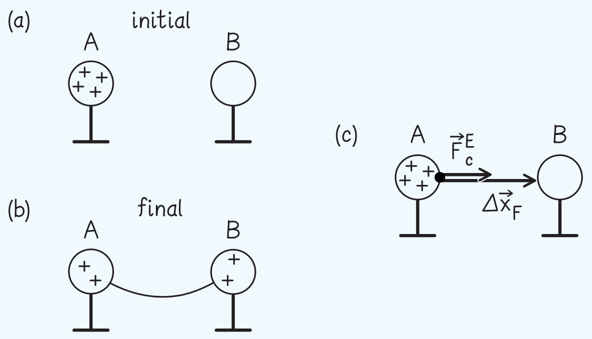
is an *equipotential volume*. In electrostatic equilibrium, all points within a conducting object are at the same electrostatic potential.

Example 25.1 Potential differences

Two metallic spheres A and B are placed on nonconducting stands. Sphere A carries a positive charge, and sphere B is electrically neutral. The two spheres are connected to each other via a wire, and the charge carriers reach a new electrostatic equilibrium. (*a*) Is the electric potential energy of the charge configuration after the spheres are connected greater than, smaller than, or equal to that of the original configuration? (*b*) Before the spheres are connected, is the potential difference between A and B positive, negative, or zero? Is it positive, negative, or zero after the spheres are connected?

❶ **GETTING STARTED** I need to evaluate the electric potential energy—the energy associated with the configuration of charge carriers—and the potential difference—the negative of the electrostatic work per unit charge done on a charge carrier. I begin with two sketches: one showing the charge distribution before the spheres are connected, and one showing the distribution after they are connected (**Figure 25.10*a*** and *b*). Once the spheres are connected, they form one conducting object, and the positive charge initially on A spreads out over both spheres.

Figure 25.10.



❷ **DEVISE PLAN** To evaluate the potential difference between A and B, I need to determine the negative of the electrostatic work done on a charged particle as it is moved from A to B and then divide the result by the charge on the particle.

❸ **EXECUTE PLAN** (*a*) After the two spheres are connected, the charge carriers spread out over both spheres, so they are farther apart than they were before the connection was made. Because I know that energy is required to push positively charged particles together, I conclude that the electric potential energy associated with the charge configuration in Figure 25.10*b* is smaller than that associated with the configuration in Figure 25.10*a*. ✔

(*b*) When a positive charge carrier is moved from A to B, the electrostatic work done on the carrier is *positive* because the electric force exerted on the carrier (directed away from sphere A) and the force displacement (from A to B) point in the same direction (Figure 25.10*c*). Consequently, the potential difference between A and B must be negative. ✔

Once the spheres are connected, they form one conducting object. Because the electric field is zero inside the entire object, no energy is required to move charge carriers around inside A and B, or from A to B, or vice versa. Thus, the potential difference between A and B after they are connected is zero. ✔

❹ **EVALUATE RESULT** I know that a closed system always tends to arrange itself so as to lower the system's potential energy (see Section 7.8). It therefore makes sense that the electric potential energy of the system is smaller after the spheres are connected. That the potential difference between A and B is negative before they are connected is a direct consequence of the definition of potential difference. After A and B are connected, they form one large equipotential volume and so it makes sense that the potential difference is zero.

Example 25.1 allows us to make another very useful observation:

> **An electrostatic field is directed from points of higher potential to points of lower potential.**

Because the electric field gives the direction of the force exerted on a positively charged particle, a direct consequence of this fact is:

> **In an electrostatic field, positively charged particles tend to move toward regions of lower potential, whereas negatively charged particles tend to move toward regions of higher potential.**

25.8 When you hold a positively charged rod above a metallic sphere without touching it, a surplus of negative charge carriers accumulates at the top of the sphere, leaving a surplus of positive charge carriers at the bottom. Is the potential difference between the top and the bottom of the sphere positive, negative, or zero?

Self-quiz

1. Consider the situation illustrated in **Figure 25.11**. A positively charged particle is lifted against the uniform electric field of a negatively charged plate. Ignoring any gravitational interactions, draw energy diagrams for the following choices of systems: (*a*) particle and plate, $v_f = 0$; (*b*) particle only, $v_f = 0$; (*c*) particle, plate, and person lifting, $v_f = 0$; (*d*) particle and plate, $v_f \neq 0$.
2. A positively charged particle is moved from point A to point B in the electric field of the large, stationary, positively charged object in **Figure 25.12**. (*a*) Is the electrostatic work done on the particle positive, negative or zero? (*b*) How is the electrostatic work done on the particle along the straight path from A to B different from the electrostatic work done on the particle along a path from A to B via C?
3. **Figure 25.13** shows both the electric field lines and the equipotentials associated with the given charge distribution. (*a*) Is the potential at point A higher than, lower than, or the same as the potential at point B? (*b*) Is the potential at point C higher than, lower than, or the same as the potential at point B? (*c*) Is the potential at point C higher than, lower than, or the same as the potential at point A?

Figure 25.11

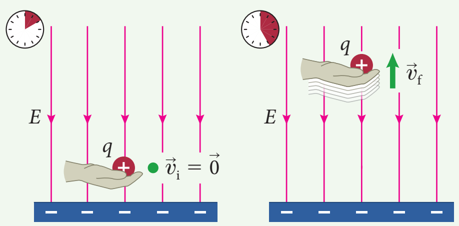


Figure 25.12

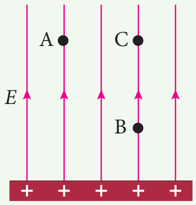


Figure 25.13

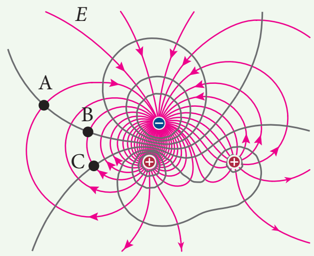
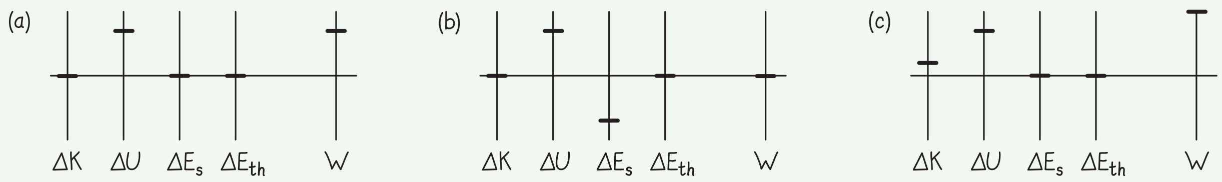


Figure 25.14



Answers

1. Only the person involves the conversion of source energy, so $\Delta E_s = 0$ in cases *a*, *b*, and *d*. When $v_f = 0$, $\Delta K = 0$; in *d*, $v_f \neq 0$ and so $\Delta K > 0$. For case *a*, the electric potential energy of the system increases as a result of positive work done by the agent (hand) on the particle, see **Figure 25.14*a***. For case *b*, the (kinetic) energy of the particle alone does not change because the positive work done by the agent on the particle and the negative electrostatic work done on it are equal in magnitude, so all bars are zero. For case *c*, a decrease in the source energy (provided by the agent) is responsible for the increase in electric potential energy of the system (Figure 25.14*b*). For case *d*, the positive work done by the agent increases both the kinetic energy of the particle and the electric potential energy of the system (Figure 25.14*c*).
2. (*a*) The electrostatic work done on the particle while it moves from point A to point B is negative because the angle between the force exerted on the particle and the force displacement is between 90° and 180° (see Eqs. 10.35 and 10.33). (*b*) The electrostatic work done along the two paths is the same because the electrostatic work done on a particle between two points is independent of the path taken.
3. (*a*) The same, because these points lie along an equipotential surface. (*b*) Higher. The potential increases in a direction opposite to the direction of the electric field, so point C is at a higher potential than point B. (*c*) Higher. Points A and B are at the same potential, and point C is at a higher potential than point B.

25.4 Calculating work and energy in electrostatics

To quantify the electrostatic work done on a charged particle, consider charged particles 1 and 2 in **Figure 25.15**. Particle 2 is moved from point A to point B through the nonuniform electric field of particle 1, which is held stationary. The electrostatic work done by particle 1 on particle 2 as it is moved along the solid path from A to B is given by (Eq. 10.44)

$$W_{12}(\mathrm{A} \rightarrow \mathrm{B}) = \int_{\mathrm{A}}^{\mathrm{B}} \vec{F}^{E}_{12} \cdot d\vec{\ell}, \tag{25.1}$$

where $\vec{F}^{E}_{12}$ is the electric force exerted by particle 1 on particle 2 and $d\vec{\ell}$ is an infinitesimal segment of the path. This line integral (see Appendix B) is generally not easy to calculate because the magnitude of the electric force and the angle between the force and the path segment $d\vec{\ell}$ vary along the path. However, as we saw in Section 25.2, the electrostatic work done on particle 2 along the solid path from A to B in Figure 25.15 is the same as that done along the dashed path from A to C to B. The circular path from A to C is along an equipotential (see Checkpoint 25.7), so the electrostatic work done on the particle along that path is zero. The electric force is given by Coulomb's law (Eq. 22.7). If we take particle 1 to be at the origin, we can write $r_{12} = r$ for the distance between the two particles and $\hat{r}_{12} = \hat{r}$ for the unit vector pointing from particle 1 to particle 2. Along the radial path from C to B, an infinitesimal segment of the path can be written as $d\vec{\ell} = dr\,\hat{r}$, and so with $k = 1/(4\pi\epsilon_0)$ (Eq. 24.7), the integrand in Eq. 25.1 becomes

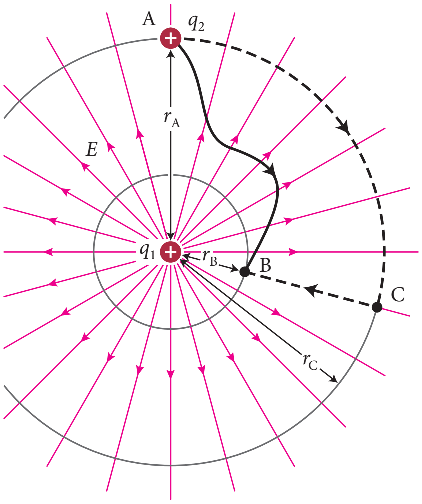


Figure 25.15 The electrostatic work done by particle 1 on particle 2 as the latter is moved from A to B is the same for the meandering solid path and for the dashed path ACB.

$$\vec{F}^{E}_{12} \cdot d\vec{\ell} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\,\hat{r} \cdot (dr\,\hat{r}) = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\,dr. \tag{25.2}$$

The line integral in Eq. 25.1 thus becomes

$$W_{12}(\mathrm{A} \rightarrow \mathrm{B}) = W_{12}(\mathrm{C} \rightarrow \mathrm{B}) = \frac{q_1 q_2}{4\pi\epsilon_0}\int_{r_{\mathrm{C}}}^{r_{\mathrm{B}}} \frac{1}{r^2}\,dr$$

$$= -\frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_{r_{\mathrm{C}}}^{r_{\mathrm{B}}} = \frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r_{\mathrm{C}}} - \frac{1}{r_{\mathrm{B}}}\right]. \tag{25.3}$$

The distance from particle 1 to point A is the same as the distance from particle 1 to point C, so we have $r_{\mathrm{A}} = r_{\mathrm{C}}$ and

$$W_{12}(\mathrm{A} \rightarrow \mathrm{B}) = \frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r_{\mathrm{A}}} - \frac{1}{r_{\mathrm{B}}}\right]. \tag{25.4}$$

Generalizing this result to arbitrary initial and final points, we get

$$W_{12} = \frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r_{12,\mathrm{i}}} - \frac{1}{r_{12,\mathrm{f}}}\right], \tag{25.5}$$

where $r_{12,\mathrm{i}}$ and $r_{12,\mathrm{f}}$ are the initial and final values of the distance separating particles 1 and 2. Note that this expression is independent of the path taken: The electrostatic work done on particle 2 depends on only the distance between the two particles at the endpoints. Equation 25.5 also does not require particle 1 to be at the origin.

25.9 (*a*) Using Eq. 25.5, determine whether the electrostatic work done on particle 2 along path CB in Figure 25.15 is positive, negative, or zero. (*b*) Does moving the particle carrying charge q_2 along path CB involve positive, negative, or zero mechanical work done on the particle? Assume the particle begins and ends at rest; verify the consistency of your answer with part *a*. (*c*) By how much does the electric potential energy of the two charged particles change as particle 2 is moved from C to B? Is this change positive, negative, or zero?

As Checkpoint 25.9 demonstrates, the change in electric potential energy of the system that comprises the two particles in Figure 25.15 is the negative of the electrostatic work done by particle 1 on the system that comprises particle 2 only. (This relationship between electrostatic work and change in electric potential energy can also be seen in Figures 25.1 and Figure 25.4.) Taking the negative of Eq. 25.5, we thus obtain

$$\Delta U^E = \frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r_{12,\mathrm{f}}} - \frac{1}{r_{12,\mathrm{i}}}\right]. \qquad (25.6)$$

Equation 25.6 gives only the *change* in electric potential energy, not *the* potential energy for a given configuration of charge. To obtain such an expression, we must first choose a zero of potential energy. It is customary to choose this zero to correspond to the configuration for which the force between the interacting particles is zero—that is to say, for infinite separation: $U^E = 0$ when $r_{12} = \infty$. Substituting this choice of reference point for the initial point in Eq. 25.6, we obtain

$$\Delta U^E \equiv U_\mathrm{f}^E - U_\mathrm{i}^E = U_\mathrm{f}^E - 0 = \frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r_{12,\mathrm{f}}} - 0\right]. \qquad (25.7)$$

Thus, the **electric potential energy** for two particles carrying charges q_1 and q_2 and separated by distance r_{12} is

$$U^E = \frac{q_1 q_2}{4\pi\epsilon_0}\frac{1}{r_{12}} \quad (U^E \text{ zero at infinite separation}). \qquad (25.8)$$

As this expression shows, the electric potential energy associated with two positively charged particles is positive: If they are brought together starting from infinite separation, the electric potential energy of the two particles increases. This is as it should be, because the two particles repel each other and so energy must be added to the system to bring it together ($W > 0$). The same idea applies to two negatively charged particles because the product $q_1 q_2$ is still positive. For a system of oppositely charged particles, on the other hand, $q_1 q_2 < 0$ and so $U^E < 0$. Indeed, the interaction is attractive and the potential energy decreases as the two are brought together.

25.10 Suppose we keep particle 2 stationary and move particle 1 in so that their final separation is the same as that in Figure 25.15. Is the electrostatic work done on particle 1 as it is moved in also given by the right-hand side of Eq. 25.4?

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Example 25.2 Putting two charged particles together

Two small pith balls, initially separated by a large distance, are each given a positive charge of 5.0 nC. By how much does the electric potential energy of the two-ball system change if the balls are brought together to a separation distance of 2.0 mm?

❶ GETTING STARTED I am asked to calculate a change in the electric potential energy of a system. I take *large* to mean a separation distance great enough that the balls don't interact. So the initial state is one in which the two pith balls have infinite separation. In the final state they are 2.0 mm apart.

❷ **DEVISE PLAN** To calculate the change in electric potential energy I can use Eq. 25.6.

❸ **EXECUTE PLAN** Substituting $q_1 = q_2 = 5.0 \times 10^{-9}$ C, $r_{12,i} \approx \infty$, $r_{12,f} = 2.0 \times 10^{-3}$ m, and $k = 1/(4\pi\epsilon_0) = 9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}$ into Eq. 25.6, I get

$$\Delta U^E = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})(5.0 \times 10^{-9}\ \mathrm{C})^2\left[\frac{1}{2.0 \times 10^{-3}\ \mathrm{m}} - 0\right]$$
$$= 1.1 \times 10^{-4}\ \mathrm{J}.$$

❹ **EVALUATE RESULT** My answer is positive, which makes sense because the balls repel each other and so work must be done on the system as they are brought together. This work increases the potential energy of the system. The magnitude of the potential energy change is small, but so is the magnitude of the force between the balls: Substituting q_1, q_2, and $r_{12,f}$ into Coulomb's law (Eq. 22.1), I obtain a force magnitude of 0.056 N. This is the maximum force between the two balls, so it makes sense that the energy associated with this interaction is small.

We can readily generalize the expressions for electrostatic work and electric potential energy for situations involving more than two charged particles. To determine the electric potential energy of a system of three charged particles in a certain configuration, for example, we calculate the electrostatic work done while assembling the system in its final configuration, starting from a situation where all three particles are far apart. Placing the first particle, carrying charge q_1, in its final position involves no electrostatic work because the other two particles are far away and therefore not interacting with particle 1. Next we bring in particle 2, carrying charge q_2, as illustrated in **Figure 25.16*a***. The electrostatic work done while moving particle 2 can be found by substituting infinity for $r_{12,i}$ and r_{12} for $r_{12,f}$ in Eq. 25.5:

$$W_{12} = -\frac{q_1 q_2}{4\pi\epsilon_0}\frac{1}{r_{12}}. \quad (25.9)$$

Finally we bring in particle 3, carrying charge q_3, as shown in Figure 25.16*b*. The electrostatic work done while moving particle 3 is given by Eq. 25.1:

$$W_3 = \int_i^f (\Sigma\vec{F}_3^E) \cdot d\vec{\ell}, \quad (25.10)$$

where $\Sigma\vec{F}_3^E$ is the vector sum of the forces exerted on particle 3. Particle 3 is subject to two forces, one exerted by particle 1 and one by particle 2, so

$$\int_i^f (\Sigma\vec{F}_3^E) \cdot d\vec{\ell} = \int_i^f (\vec{F}_{13}^E + \vec{F}_{23}^E) \cdot d\vec{\ell}$$

$$= \int_i^f \vec{F}_{13}^E \cdot d\vec{\ell} + \int_i^f \vec{F}_{23}^E \cdot d\vec{\ell} = W_{13} + W_{23}. \quad (25.11)$$

In other words, the electrostatic work done as 3 is moved to its final position is the sum of the electrostatic work done when only 1 is present plus that done when only 2 is present. Now we can apply Eq. 25.5 to each term in Eq. 25.11:

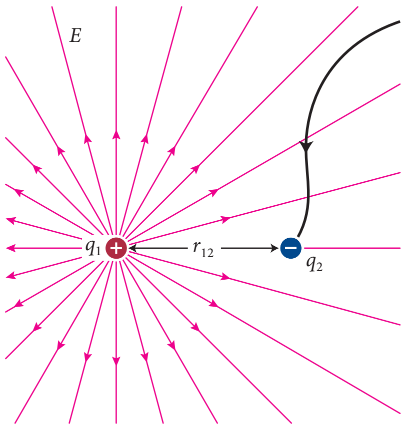
$$W_{13} + W_{23} = -\frac{q_1 q_3}{4\pi\epsilon_0}\frac{1}{r_{13}} - \frac{q_2 q_3}{4\pi\epsilon_0}\frac{1}{r_{23}}. \quad (25.12)$$

The total electrostatic work done on the three-particle system while assembling the charge configuration is the sum of Eqs. 25.9 and Eq. 25.12:

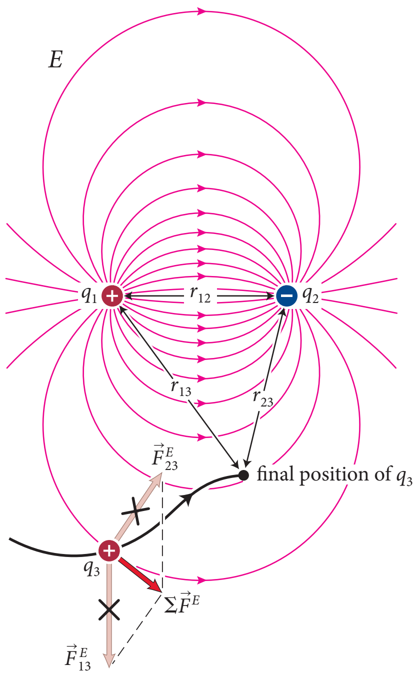
$$W = -\frac{q_1 q_2}{4\pi\epsilon_0}\frac{1}{r_{12}} - \frac{q_1 q_3}{4\pi\epsilon_0}\frac{1}{r_{13}} - \frac{q_2 q_3}{4\pi\epsilon_0}\frac{1}{r_{23}}. \quad (25.13)$$

Figure 25.16 To obtain the electrostatic potential energy of a system of three charged particles, we assemble the system one particle at a time.

(*a*) We bring in second charged particle



(*b*) We bring in third charged particle



Alternatively, with our choice of zero at infinity, the electric potential energy of the system is

$$U^E = \frac{q_1 q_2}{4\pi\epsilon_0}\frac{1}{r_{12}} + \frac{q_1 q_3}{4\pi\epsilon_0}\frac{1}{r_{13}} + \frac{q_2 q_3}{4\pi\epsilon_0}\frac{1}{r_{23}}$$

$$(U^E \text{ zero at infinite separation}). \quad (25.14)$$

In words, to determine the electric potential energy of a system of charged particles, we need to add the electric potential energies for each pair of particles.

25.11 Suppose the charged particles in Figure 25.16 are assembled in a different order—say, 3 first, then 1, and finally 2. Do you obtain the same result as in Eq. 25.13 and Eq. 25.14?

25.5 Potential difference

The negative of the electrostatic work per unit charge done on a particle that carries a positive charge q from one point to another is defined as the **potential difference** between those points:

$$V_{AB} \equiv V_B - V_A \equiv \frac{-W_q(A \rightarrow B)}{q}. \quad (25.15)$$

Potential difference is a scalar, and the SI units of potential difference are joules per coulomb (J/C). In honor of Alessandro Volta (1745–1827), who developed the first battery, this derived unit is given the name **volt** (V):

$$1\ \text{V} \equiv 1\ \text{J/C}. \quad (25.16)$$

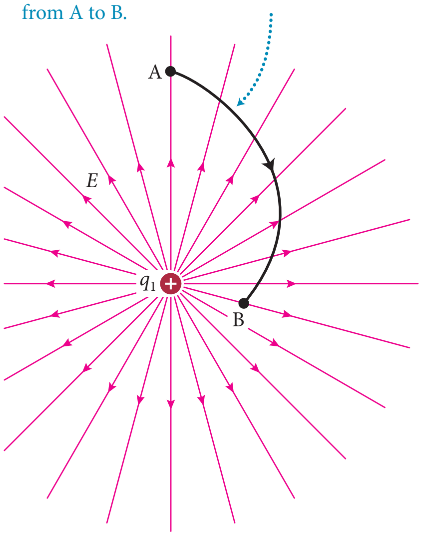


Figure 25.17 Once we know the electrostatic work done by a particle carrying a charge q_1 on a particle carrying a charge q_2 as the latter is moved from A to B in Figure 25.15, we can determine the potential difference between A and B in the electric field of particle 1.

For example, if the electric field in **Figure 25.17** does -12 J of electrostatic work on a particle carrying charge $q_2 = +2.0$ C as it is moved from point A to point B (in other words, it requires $+12$ J of work by an external agent without the particle gaining any kinetic energy), then the potential difference V_{AB} between A and B is $-(-12\ \text{J})/(2.0\ \text{C}) = +6.0$ V. That is, the potential at B is 6.0 V higher than that at A.

Once we know the potential difference V_{AB} between A and B, we can obtain the electrostatic work done on *any* object carrying a charge q as it is moved along any path from A to B:

$$W_q(A \rightarrow B) = -qV_{AB}. \quad (25.17)$$

Keep in mind that the subscripts AB mean "from A to B." Because B is the final position, we write $V_{AB} \equiv V_B - V_A$. So when we refer to the "potential difference between A and B," we always mean the potential at B minus the potential at A.

Potential is important in practical applications because the potential difference between two points can be measured readily with a device called a *voltmeter* (we'll encounter these devices when we discuss electrical circuits in Chapters 31 and 32.) In the next chapter we shall discuss the operation of another familiar device, the *battery*, which allows one to maintain a constant potential difference between two points. A 9-V battery, for example, maintains a +9-V-potential difference between its negative and positive terminals. (The positive terminal is at the higher potential, and thus the potential difference is *positive* when going from − to +.) For example, when a particle carrying charge +1 C is moved from the negative terminal of a 9-V battery to the positive terminal, the particle undergoes a potential difference $V_{-+} = V_+ - V_- = +9$ V. Equation 25.17 tells us that the electrostatic work done on the particle is equal to

$$W_q(- \rightarrow +) = -q\, V_{-+} = -(+1\text{ C})(+9\text{ V}) = -9\text{ J}. \qquad (25.18)$$

That this quantity is negative indicates that the agent moving the particle must do a positive amount of work on the particle. Likewise, when a particle carrying a charge of -2 C is moved from the $-$ terminal to the $+$ terminal of a 9-V battery, the electrostatic work done on the particle is $-(-2\text{ C})(+9\text{ V}) = +18$ J.

For simplicity, we shall denote the magnitude of the potential difference between the terminals of a battery by V_{batt}. Therefore

$$V_{\text{batt}} = V_+ - V_-. \qquad (25.19)$$

Just as with potential energy, only potential *differences* are physically relevant. If we choose a reference point, however, we can determine the value for the potential at any other point. In the preceding section we chose infinity as the reference point for the electric potential energy for charged particles because they do not interact at infinite distance $U^E(\infty) = 0$. The same choice can be made for the potential, but when we deal with electrical circuits it is customary to assign zero potential to Earth (ground) because Earth is a good and very large conducting object through which the motion of charge carriers requires negligible energy.

Exercise 25.3 Potential and potential difference

The negative terminal of a 9-V battery is connected to ground via a wire. (*a*) What is the potential of the negative terminal? (*b*) What is the potential of the positive terminal? (*c*) What is the potential of the negative terminal if the positive terminal is connected to ground?

SOLUTION (*a*) I know from Section 25.3 that any conducting objects that are in electrical contact with each other form an equipotential. Once they are in contact, therefore, the ground (which is conducting), the wire, and the negative terminal are all at the same potential. If the potential of the ground is zero (an arbitrary but customary choice), then the potential of the negative terminal is also zero. ✔

(*b*) The potential difference between the negative and positive terminals is $+9$ V, meaning that the potential of the positive terminal is 9 V higher than that of the negative terminal: $V_{\text{batt}} = V_+ - V_- = +9$ V. If V_- is zero, then the potential of the positive terminal is $V_+ = +9\text{ V} + V_- = (+9\text{ V}) + (0\text{ V}) = +9$ V. ✔

(*c*) With the positive terminal connected to ground, that terminal's potential becomes zero. Because the battery maintains a potential difference of $+9$ V between the negative and positive terminals, I now have $V_{\text{batt}} = V_+ - V_- = 0 - V_- = +9$ V and so the negative terminal is at a potential of -9 V. ✔

To obtain an explicit expression for the potential difference between two points A and B in the electric field of particle 1 carrying charge q_1, we start with the expression for the electrostatic work done by particle 1 on a particle 2 carrying charge q_2 as it is moved from A to B (Eq. 25.4). All we need to do is add a minus sign and divide by q_2:

$$V_{\text{AB}} \equiv \frac{-W_{12}(\text{A} \rightarrow \text{B})}{q_2} = \frac{q_1}{4\pi\epsilon_0}\left[\frac{1}{r_{\text{B}}} - \frac{1}{r_{\text{A}}}\right]. \qquad (25.20)$$

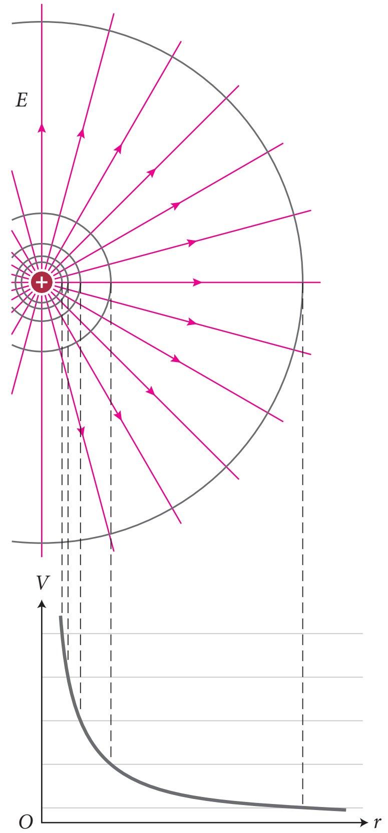


Figure 25.18 Equipotentials, field lines, and graph of potential for a charged particle.

For charged particles, the choice of ground as a zero for the potential is not very meaningful. However, if we set the zero for potential at infinity and let r_A be at infinity, we obtain for the potential at a distance $r = r_B$ from a single charged particle

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r} \quad \text{(potential zero at infinity).} \qquad (25.21)$$

Equation 25.21 confirms that the potential is constant on any spherical surface centered on the charged particle (that is, any surface for which r is constant), as we concluded in Checkpoint 25.7. **Figure 25.18** shows the potential for a charged particle in graphical form. The bottom of the figure shows the $1/r$ dependence of the potential; the top shows a two-dimensional view of the equipotentials around the charged particle. The $1/r$ dependence of the potential is reflected by the increasing spacing of the equipotentials (with a fixed potential difference between them) as r increases.

25.12 (*a*) Using Eq. 25.20, determine whether the potential difference between A and B in Figure 25.17 is positive, negative, or zero. (*b*) From the directions of the electric force and the force displacement, is the electrostatic work done on a positively charged particle as it is moved along a straight path from A to B positive, negative, or zero? Verify that your answer is consistent with the answer in part *a*.

Example 25.4 Atomic potential difference

A (simplistic) model of the hydrogen atom treats the electron as a particle carrying a charge $-e$ orbiting a proton (a particle carrying a charge $+e$) in a circle of radius $r_H = 0.53 \times 10^{-10}$ m. (*a*) How much energy is required to completely separate the electron from the proton? For simplicity, ignore the electron's kinetic energy. (*b*) Across what potential difference does the electron travel as it is separated from the proton?

❶ **GETTING STARTED** To completely separate the electron from the proton, I must increase the distance between them from r_H to infinity (at which point their interaction is reduced to zero). By increasing the separation, I increase the potential energy of the electron-proton system. The energy I must add to the system in order to separate the particles is equal to the increase in potential energy of the system.

❷ **DEVISE PLAN** I can calculate the potential energy increase from Eq. 25.6, setting $r_i = r_H$ and $r_f = \infty$. I can obtain the potential difference between the initial and final positions of the electron by taking the negative of the electrostatic work done by the electric field of the proton on the electron during the separation and dividing this value by the charge on the electron.

❸ **EXECUTE PLAN** (*a*) The change in the electric potential energy of the two-particle system is

$$\Delta U^E = \frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r_f} - \frac{1}{r_i}\right] = \frac{-e^2}{4\pi\epsilon_0}\left[0 - \frac{1}{r_H}\right]$$

$$= (9.0 \times 10^9\ \mathrm{N\cdot m^2/C^2})\frac{(1.6 \times 10^{-19}\ \mathrm{C})^2}{0.53 \times 10^{-10}\ \mathrm{m}} = 4.3 \times 10^{-18}\ \mathrm{J}.$$

Because there are no changes in any other forms of energy in the system, the energy required to separate the electron and the proton in a hydrogen atom is 4.3×10^{-18} J. ✔

(*b*) The answer I obtained in part *a* is the mechanical work an external agent must do on the electron to separate it from the proton. I know that the energy of the electron does not change because the electron gains no kinetic energy. Considering just the electron as my system, I thus have $\Delta E = 0$ and so the work done on the electron is zero. This work has two parts: mechanical work done by the external agent and electrostatic work done by the electric field of the proton. Because the sum of these two parts is zero, I know that the electrostatic work done by the electric field of the proton on the electron must be the negative of the mechanical work done by the external agent, so $W_{pe}(r_H \to \infty) = -4.3 \times 10^{-18}$ J. The potential difference is thus (Eq. 25.15)

$$V_{H\infty} = \frac{-W_{pe}(r_H \to \infty)}{q} = \frac{-(-4.3 \times 10^{-18}\ \mathrm{J})}{-1.6 \times 10^{-19}\ \mathrm{C}} = -27\ \mathrm{V}. ✔$$

❹ **EVALUATE RESULT** The positive sign on ΔU^E in part *a* means that the potential energy of the proton-electron system increases when the two are moved apart. Therefore mechanical work must be done to pull them apart, as I expect because the electron and proton attract each other. The value I obtain is extremely small, but I know that 1 m^3 of matter contains about 10^{29} atoms (see Exercise 1.6), so the electric potential energy in a cubic meter of matter is on the order of $(10^{28})(4.3 \times 10^{-18}\ \text{J}) \approx 10^{11}$ J. In the box "Coherent versus incoherent energy" on page 158 I learned that the amount of chemical energy in a pencil is on the order of 10^5 J. Given that chemical energy is derived from electric potential energy stored in chemical bonds and that the volume of a pencil is about 10^{-5} m^3, my answer for part *a* is not unreasonable.

For part *b*, I could have used Eq. 25.20 directly. Substituting the values given into that equation, I obtain the same answer, which gives me confidence in the answer I obtained.

To obtain a more general result for the potential difference between one point and another in an arbitrary electric field $\vec{E}$, consider the situation illustrated in **Figure 25.19**. A particle carrying a charge q is moved from point A to point B in an electric field due to some charge distribution (not visible in the illustration). The electrostatic work done on the particle is

$$W_q(\text{A} \rightarrow \text{B}) = \int_{\text{A}}^{\text{B}} \vec{F}_q^E \cdot d\vec{\ell}. \tag{25.22}$$

The vector sum of the forces exerted on the particle is equal to the product of the electric field and the charge q (Eq. 23.6):

$$\vec{F}_q^E = q\vec{E}, \tag{25.23}$$

so

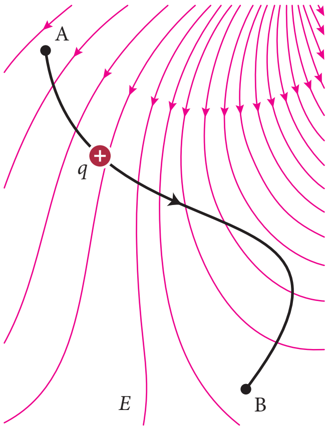
$$W_q(\text{A} \rightarrow \text{B}) = q\int_{\text{A}}^{\text{B}} \vec{E} \cdot d\vec{\ell}. \tag{25.24}$$

The potential difference between point A and point B is therefore

$$V_{\text{AB}} \equiv \frac{-W_q(\text{A} \rightarrow \text{B})}{q} = -\int_{\text{A}}^{\text{B}} \vec{E} \cdot d\vec{\ell}. \tag{25.25}$$

In evaluating this line integral, we must keep in mind that the integral does not depend on the path taken but only on the endpoints. It therefore pays to choose a path that facilitates evaluating the integral. The Procedure box below and the next example will help you gain practice calculating potential differences between two points.

Figure 25.19 To determine the potential difference between two points in an electric field, we must evaluate the electrostatic work done on a charged particle as the particle is moved along a path between those points.



Procedure: Calculating the potential difference between two points in an electric field

The potential difference between two points in an electric field is given by Eq. 25.25. The following steps will help you evaluate the integral:

1. Begin by making a sketch of the electric field, indicating the points corresponding to the two points between which you wish to determine the potential difference.
2. To facilitate evaluating the scalar product $\vec{E} \cdot d\vec{\ell}$, choose a path between the two points so that $\vec{E}$ is either parallel or perpendicular to the path. If necessary, break the path into segments. If $\vec{E}$ has a constant value along the path (or a segment of the path), you can pull it out of the integral; the remaining integral is then equal to the length of the corresponding path (or the segment of the path).
3. Remember that to determine V_{AB} ("the potential difference between points A and B"), your path begins at *A* and ends at *B*. The vector $d\vec{\ell}$ therefore is tangent to the path, in the direction that leads from A to B (see also Appendix B).

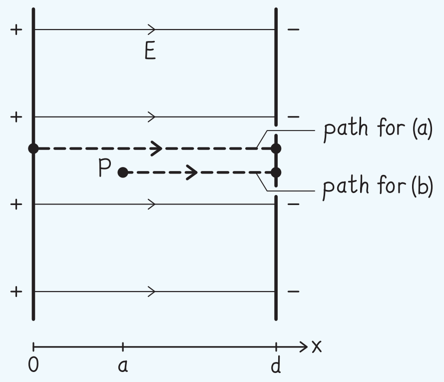
At this point you can substitute the expression for the electric field and carry out the integral. Once you are done, you may want to verify the algebraic sign of the result you obtained: *negative* when a positively charged particle moves along the path in the direction of the electric field, and *positive* when it moves in the opposite direction.

Example 25.5 Electrostatic potential in a uniform field

Consider a uniform electric field of magnitude E between two parallel charged plates separated by a distance d. (a) What is the potential difference between the positive plate and the negative plate? (b) What is the value of the potential at a point P that lies between the plates and is a distance $a < d$ from the positive plate, if the potential of the negative plate is zero?

❶ **GETTING STARTED** I begin by making a sketch of the two parallel plates and the electric field (**Figure 25.20**). For part a I must calculate the potential difference between the positive and negative plates, so I choose a path that begins at the positive plate and ends at the negative plate. For part b I am asked to determine, relative to the potential at the negative plate, the potential at a point P located between the plates a distance a from the positive plate, so I choose a path that runs from P to the negative plate. I indicate the endpoints for both paths in my drawing and choose my x axis to be parallel with the electric field with the positive plate at $x = 0$.

Figure 25.20



❷ **DEVISE PLAN** To obtain the potential difference between the endpoints of each path, I apply Eq. 25.25. With my choice of x axis the $d\vec{\ell}$ factor in Eq. 25.25 becomes $d\vec{\ell} = dx\,\hat{\imath}$. Because $\vec{E} = E\,\hat{\imath}$ I thus have $\vec{E} \cdot d\vec{\ell} = E\,dx$, and because the field is uniform, E is constant. For part b, once I have the potential difference between the endpoints of the path from P to the negative plate, I can determine the potential at P because I am told that the potential at the negative plate is zero.

❸ **EXECUTE PLAN** (a) Applying Eq. 25.25 to the path from $x_i = 0$ (positive plate) to $x_f = d$ (negative plate), I get

$$V_{0d} = -\int_0^d E\,dx = -E\int_0^d dx = -Ed, \checkmark \qquad (1)$$

where I have pulled E out of the integral because it is constant.

(b) If I start the line integral in Eq. 1 at point P, I obtain

$$V(d) - V(a) = -E\int_a^d dx = -E(d-a)$$

or, because $V(d) = 0$,

$$V(a) = E(d-a). \checkmark$$

❹ **EVALUATE RESULT** The negative sign in Eq. 1 means that the potential at the end of the path (that is, at the negative plate) is *lower* than the potential at the beginning (the positive plate). This negative potential difference is in agreement with the sign of the electrostatic work done on a positively charged particle: Positive electrostatic work is done on the particle as it is moved from the positive plate to the negative plate, and so, according to Eq. 25.15, the potential difference should indeed be negative.

My result for part b indicates that the potential is positive at $x = a$ ($a < d$), decreases linearly with the distance a to the positive plate, and goes to zero at $a = d$. That makes sense, because I know that the potential of the positive plate must be higher than that of the negative plate and if $a = d$, I get $V(d) = 0$, as expected.

With an appropriate choice of reference point for the potential, Eq. 25.25 allows us to assign values to the potential at every point surrounding a charge distribution. This "potential field" is related to the electric field: Each can be determined from the other. A drawing that shows a set of equipotentials for a charge distribution is equivalent to a drawing that shows a set of field lines for that charge distribution. In Section 25.7 we shall show how the electric field can be derived from the potential field. The potential field, however, has advantages over the electric field. First, it is a scalar field whereas the electric field is a vector field, and so calculations involving the potential are generally simpler. Second, while no devices exist to measure electric field strength directly, we can measure the potential difference between two points with a voltmeter.

25.13 Verify that Eq. 25.25 is consistent with Eq. 25.20 by substituting the expression for the electric field of a charged particle.

By following the same procedure as in Checkpoint 25.13, we can now obtain the potential for a group of charged particles. Recall that the electric field due to

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a group of charged particles is equal to the sum of the electric fields of the individual charged particles (Eq. 23.5),

$$\vec{E} = \sum_n \vec{E}_n. \tag{25.26}$$

This gives us

$$V_{\text{AB}} = \int_{\text{A}}^{\text{B}} \vec{E} \cdot d\vec{\ell} = -\int_{\text{A}}^{\text{B}} \left(\sum_n \vec{E}_n\right) \cdot d\vec{\ell}. \tag{25.27}$$

Because the integral of a sum is equal to a sum of integrals, we have

$$-\int_{\text{A}}^{\text{B}} \left(\sum_n \vec{E}_n\right) \cdot d\vec{\ell} = -\sum_n \int_{\text{A}}^{\text{B}} \vec{E}_n \cdot d\vec{\ell}. \tag{25.28}$$

The line integral after the summation sign is the negative of the potential difference due to the field $\vec{E}_n$, so we see that the total potential difference is the sum of the potential differences caused by the individual particles:

$$V_{\text{AB}} = \sum_n V_{\text{AB},n}, \tag{25.29}$$

where $V_{\text{AB},n}$ is the potential difference caused by particle n. Substituting the potential of a single particle, Eq. 25.21, and again letting the potential at infinity be zero, we get

$$V_{\text{P}} = \frac{1}{4\pi\epsilon_0} \sum_n \frac{q_n}{r_{n\text{P}}} \quad \text{(potential zero at infinity)}, \tag{25.30}$$

where q_n is the charge carried by particle n and $r_{n\text{P}}$ is the distance of particle n from the point P at which we are evaluating the potential (**Figure 25.21**).

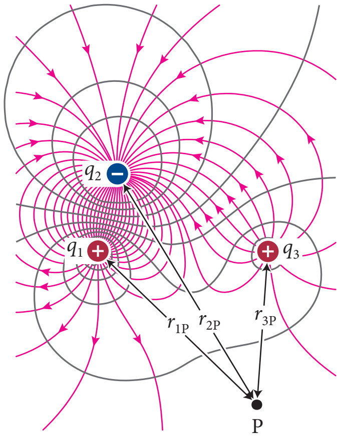


Figure 25.21 The potential at point P due to a group of charged particles is the algebraic sum of the potentials at P due to the individual particles.

The line integral on the right in Eq. 25.25 has an important significance. As we argued in Section 25.2, the electrostatic work done on a charged particle moving around a closed path (as in **Figure 25.22**) is zero. Therefore, Eq. 25.24 must yield zero for a closed path:

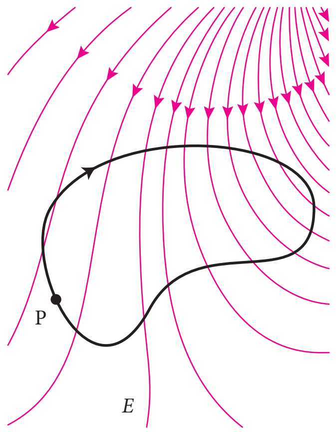
$$W_q(\text{closed path}) = q\oint \vec{E} \cdot d\vec{\ell} = 0, \tag{25.31}$$

where the circle through the integral sign indicates that the integration is to be taken around a closed path. Because Eq. 25.31 holds for any value of q, we have

$$\oint \vec{E} \cdot d\vec{\ell} = 0 \quad \text{(electrostatic field)}. \tag{25.32}$$

In other words, for any electrostatic field, the line integral of the electric field around a closed path is zero. This is equivalent to saying that you cannot extract energy from an electrostatic field by moving a charged particle around a closed path. In terms of potential, this means that if we start at some point P on the closed path and the potential has a value V_{P} at that point, then the potential can take on other values as we go around the closed path, but as we return to P, the value of the potential must once again be V_{P}. As we shall see later in Chapter 30, it *is* possible to get energy out of electric fields, however. Here we have shown only that it is *not* possible to extract energy by moving a charged particle around a closed path in *electrostatic* fields—that is, those due to stationary charge distributions.

Figure 25.22 The electrostatic work done on a charged particle as the particle is moved around a closed path starting and ending at some point P is zero.



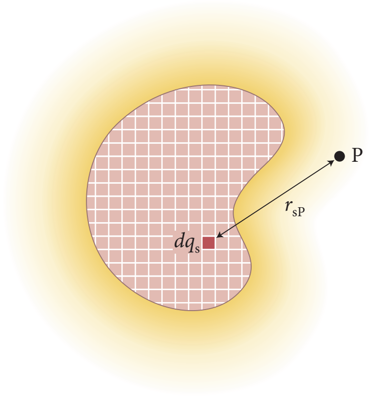
25.14 Describe how the potential varies as you go around the closed path in Figure 25.22 in the direction shown, starting from a potential V_P at P.

25.6 Electrostatic potentials of continuous charge distributions

For extended objects with continuous charge distributions, we cannot use Eq. 25.30 directly to calculate the potential. Instead we must divide the object into infinitesimally small segments, each carrying charge dq_s (which we can treat as a charged particle), and then integrate over the entire object.

Consider, for example the object shown in **Figure 25.23**. Let the zero of potential again be at infinity. Treating each segment as a charged particle, we calculate its contribution to the potential at P (Eq. 25.21):

Figure 25.23 The potential due to an extended object is the algebraic sum of the potentials of all the infinitesimally small segments that make up the object.



$$dV_s = \frac{1}{4\pi\epsilon_0}\frac{dq_s}{r_{sP}}, \tag{25.33}$$

where r_{sP} is the distance between P and dq_s. The potential due to the entire object is then given by the sum over all the segments that make up the object. For infinitesimally small segments, this yields the integral

$$V_P = \int dV_s = \frac{1}{4\pi\epsilon_0}\int\frac{dq_s}{r_{sP}}, \quad \text{(potential zero at infinity)}, \tag{25.34}$$

where the integral is taken over the entire object. Note the parallel between this integral and the integral in Eq. 23.15 for calculating the electric field of a continuous charge distribution. Indeed, the procedure for calculating the potential of a continuous charge distribution is very similar to that for calculating the electric field of a continuous charge distribution (see Section 23.7). However, the potential in Eq. 25.34 is much easier to evaluate because it is a scalar and does not involve any unit vectors. The Procedure box below provides some helpful steps in evaluating Eq. 25.34, and the next two examples show how to put the procedure into practice.

PROCEDURE: Calculating the electrostatic potentials of continuous charge distributions

To calculate the electrostatic potential of a continuous charge distribution (relative to zero potential at infinity), you need to evaluate the integral in Eq. 25.34. The following steps will help you work out the integral:

1. Begin by making a sketch of the charge distribution. Mentally divide the distribution into infinitesimally small segments, each carrying a charge dq_s. Indicate one such segment in your drawing.
2. Choose a coordinate system that allows you to express the position of dq_s in the charge distribution in terms of a minimum number of coordinates (x, y, z, r, or θ). These coordinates are the integration variables. For example, use a radial coordinate system for a charge distribution with radial symmetry. Never place the representative segment dq_s at the origin.
3. Indicate the point at which you wish to determine the potential. Express the factor $1/r_{sP}$, where r_{sP} is the distance between dq_s and the point of interest, in terms of the integration variable(s).
4. Determine whether the charge distribution is one dimensional (a straight or curved wire), two dimensional (a flat or curved surface), or three dimensional (any bulk object). Express dq_s in terms of the corresponding charge density of the object and the integration variable(s).

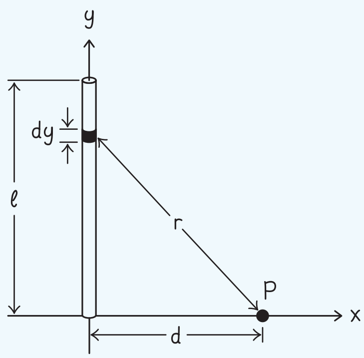
At this point you can substitute your expressions for dq_s and $1/r_{sP}$ into Eq. 25.34 and work out the integral.

Example 25.6 Electrostatic potential of a uniformly charged thin rod

A thin rod of length ℓ carries a uniformly distributed charge q. What is the potential V_P at point P a distance d from the rod along a line that runs perpendicular to the long axis of the rod and passes through one end of the rod?

❶ GETTING STARTED I begin by making a sketch. I let the y axis be along the rod, with the origin at the bottom of the rod. Point P then lies on the x axis (**Figure 25.24**). What I must calculate is the electrostatic potential V_P at P.

Figure 25.24



❷ DEVISE PLAN Because the rod is thin and uniformly charged, I can treat it as a one-dimensional object that has a linear charge density $\lambda = q/\ell$. To determine the electrostatic potential at point P, I divide the rod lengthwise into a large number of infinitesimally small segments, each of length dy. Each segment contributes to the potential at P an amount given by Eq. 25.33. To calculate the potential at P due to the entire rod, I can then use Eq. 25.34 to integrate my result over the entire length of the rod.

❸ EXECUTE PLAN Each length segment dy carries a charge $dq_s = \lambda dy = (q/\ell)dy$. To calculate the potential at P due to the whole rod, I substitute the distance between each segment dy and P, $r_{sP} = \sqrt{y^2+d^2}$, and my expression for dq_s into Eq. 25.34 and integrate from the bottom of the rod at $y = 0$ to the top at $y = \ell$:

$$V_P = \frac{1}{4\pi\epsilon_0}\frac{q}{\ell}\int_0^\ell \frac{dy}{\sqrt{y^2 + d^2}}.$$

Looking up the solution of the integral, I obtain

$$V_P = \frac{1}{4\pi\epsilon_0}\frac{q}{\ell}\left[\ln\left(y + \sqrt{y^2 + d^2}\right)\right]_0^\ell$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{\ell}\left[\ln\left(\ell + \sqrt{\ell^2 + d^2}\right) - \ln d\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{\ell}\ln\left(\frac{\ell + \sqrt{\ell^2 + d^2}}{d}\right). ✔$$

❹ EVALUATE RESULT: If I let ℓ go to zero, my answer should become the same as that for a particle (Eq. 25.21). When $\ell \ll d$, I can ignore the term ℓ^2 in my answer and so the argument of the logarithm becomes $(\ell + d)/d = 1 + \ell/d$. Because $\ln(1 + \epsilon) \approx \epsilon$ when $\epsilon \ll 1$ (see Appendix B), my answer becomes

$$V_P \approx \frac{1}{4\pi\epsilon_0}\frac{q}{\ell}\ln\left(\frac{\ell + d}{d}\right) \approx \frac{1}{4\pi\epsilon_0}\frac{q}{\ell}\frac{\ell}{d} = \frac{1}{4\pi\epsilon_0}\frac{q}{d},$$

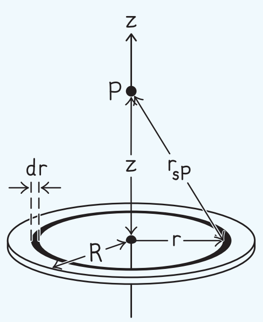
which is indeed equal to the electrostatic potential at a distance $r = d$ from a particle.

Example 25.7 Electrostatic potential of a uniformly charged disk

A thin disk of radius R carries a uniformly distributed charge. The surface charge density on the disk is σ. What is the electrostatic potential due to the disk at point P that lies a distance z from the plane of the disk along an axis that runs through the disk center and is perpendicular to the plane of the disk?

❶ GETTING STARTED I begin by making a sketch of the disk (**Figure 25.25**). I let the disk be in the xy plane, with the origin at the center of the disk and point P on the z axis.

Figure 25.25



❷ DEVISE PLAN Because of the circular symmetry of the disk, I divide it into a large number of thin circular ring segments, each of radius r and thickness dr. All parts of a given ring are the same distance r_{sP} from point P, so each part makes the same contribution to the potential at P. I can therefore calculate the contribution of an entire ring segment to the potential at P using Eq. 25.33, substituting the charge dq_s on the segment and the distance from a point on the segment to P. The charge dq_s on the ring segment is given by the product of its area (circumference times thickness) and the surface charge density: $dq_s = (2\pi r)\,dr\,\sigma$. To calculate the potential at point P due to the entire disk, I can use Eq. 25.34 to integrate my result over the radius of the disk, using r as my integration variable.

❸ EXECUTE PLAN: The distance from a point on any ring segment to P is $r_{sP} = \sqrt{z^2 + r^2}$, and so each segment's contribution to the potential is given by

$$dV_P = \frac{1}{4\pi\epsilon_0}\frac{(2\pi r)\sigma\,dr}{\sqrt{z^2 + r^2}}.$$

(Continued)

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To calculate the potential at P due to the whole disk, I integrate this expression from $r = 0$ to $r = R$:

$$V_P = \int dV_P = \frac{1}{4\pi\epsilon_0}\int_0^R \frac{2\pi r\sigma}{\sqrt{z^2 + r^2}}\,dr.$$

Looking up the solution of the integral, I get

$$V_P = \frac{2\pi\sigma}{4\pi\epsilon_0}\int_0^R \frac{r\,dr}{\sqrt{z^2 + r^2}} = \frac{\sigma}{2\epsilon_0}\left(\sqrt{z^2 + R^2} - |z|\right). \checkmark$$

❹ **EVALUATE RESULT** When z is very large relative to R, the disk should resemble a particle and my result should reduce to that for a particle (Eq. 25.21). For large $z > 0$ I can use the binomial expansion (see Appendix B) to write the factor in parentheses as

$$\sqrt{z^2(1 + R^2/z^2)} - z \approx z\left(1 + \tfrac{1}{2}\frac{R^2}{z^2}\right) - z = \tfrac{1}{2}\frac{R^2}{z}.$$

Because $\sigma\pi R^2$ is equal to the charge q on the disk, my expression for V becomes

$$V_P \approx \frac{\sigma}{2\epsilon_0}\left(\tfrac{1}{2}\frac{R^2}{z}\right) = \frac{\sigma\pi R^2}{4\pi\epsilon_0 z} = \frac{1}{4\pi\epsilon_0}\frac{q}{z},$$

which is equal to the result for the potential along a z axis due to a particle located at the origin.

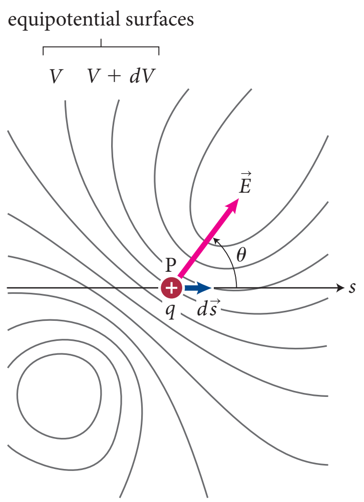
You may have noticed that the two examples above are parallel to corresponding examples in Section 23.7 in which we calculated the electric fields due to a thin charged rod or disk. If you compare the calculations, however, the advantage of working with the potential becomes obvious: Because the potential is a scalar you don't need to take vector sums. Haven't we thrown away some information, though? After all, the answer we get is also just a scalar, not a vector like the electric field. Figure 25.9 gives us some idea of the answer to this question: Because field lines and equipotentials are always perpendicular to each other, you can draw equipotentials if you know the field line pattern or, conversely, draw field lines if you know the equipotentials. It turns out that even though the potential is a scalar and the electric field is a vector, it is possible to determine one from the other.

25.15 Verify that the potentials obtained in Examples 25.6 and 25.7 have the correct sign.

25.7 Obtaining the electric field from the potential

For the potential to be a useful quantity, we must be able to determine the electric field (and therefore the forces exerted by this field) from the potential. For example, let the equipotentials in **Figure 25.26** represent the potential of some charge distribution. How can we use the known potential of the charge distribution to determine the value of the electric field at any point P?

Figure 25.26 To determine the component of electric field along an axis, we calculate the electrostatic work done on a charged particle as the particle is moved over a short segment along that axis.



We know that the electric field is perpendicular to the equipotentials, and so the electric field at P must be along the direction indicated in the figure. To determine the magnitude of the electric field, imagine moving a particle carrying a charge q over an infinitesimally small displacement $d\vec{s}$ along some arbitrary axis s. Let the particle be displaced from P, where the potential is V, to a point P′ where the potential is $V + dV$. According to Eq. 25.17, the electrostatic work done on the particle is

$$W_q(\mathrm{P} \to \mathrm{P'}) = -qV_{\mathrm{PP'}} = -q(V_{\mathrm{P'}} - V_{\mathrm{P}}) = -q\,dV \qquad (25.35)$$

because the potential difference between P and P′ is $(V + dV) - V = dV$. On the other hand, we also know that the electrostatic work done on the particle is equal to the scalar product of the electric force exerted on the particle and the force displacement $d\vec{r}_F = d\vec{s}$:

$$\begin{aligned} W_q(\mathrm{P} \to \mathrm{P'}) &= \vec{F}^E_q \cdot d\vec{s} = (q\vec{E}) \cdot d\vec{s} \\ &= q(\vec{E} \cdot d\vec{s}) = qE\cos\theta\,ds, \end{aligned} \qquad (25.36)$$

where we have assumed that the force displacement is small enough that $\vec{E}$ can be considered constant between P and P′. Note that θ is the angle between $\vec{E}$ and the s axis, so $E \cos \theta$ is the component of the electric field along the s axis. We can write $E \cos \theta = E_s$, and equating the two expressions for the electrostatic work done on the particle, Eqs. 25.35 and 25. 36, we get

$$-q\,dV = qE_s\,ds \tag{25.37}$$

or

$$E_s = -\frac{dV}{ds}. \tag{25.38}$$

In other words, the component of the electric field along the s axis is given by the negative of the derivative of the potential with respect to s. The faster V varies (the more closely spaced the equipotentials), the greater the magnitude of the electric field.

Equation 25.38 gives only the component of the electric field along the (arbitrary) axis s. To determine all the components of the electric field, we must repeat this procedure for each of the three Cartesian coordinates:

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}. \tag{25.39}$$

Note the partial derivatives; these are necessary because the function $V(x, y, z)$ depends on all three Cartesian coordinates. When you take the partial derivative with respect to one coordinate, just remember to keep the other coordinates constant. (For example, if $V = x^2y$, then $\partial V/\partial x = 2xy$, $\partial V/\partial y = x^2$, and $\partial V/\partial z = 0$.)

Once the components of the electric field are determined, we can write the electric field in vectorial form:

$$\vec{E} = -\frac{\partial V}{\partial x}\,\hat{\imath} - \frac{\partial V}{\partial y}\,\hat{\jmath} - \frac{\partial V}{\partial z}\,\hat{k}. \tag{25.40}$$

Equation 25.40 then tells us how to obtain the electric field from the potential.

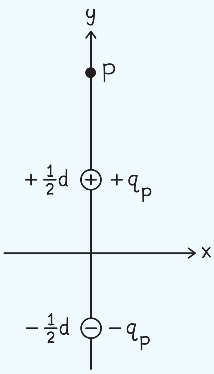
25.16 Apply Eq. 25.40 to the potential you obtained for the uniform field between two parallel charged plates in Example 25.5, and verify that you get the correct expression for the electric field between the plates.

Example 25.8 The electrostatic potential and electric field due to a dipole

A permanent dipole consists of a particle carrying a charge $+q_p$ at $x = 0$, $y = +\frac{1}{2}d$ and another particle carrying a charge $-q_p$ at $x = 0$, $y = -\frac{1}{2}d$. Use the electrostatic potential at a point P on the axis of the dipole to determine the electric field at that point.

1 GETTING STARTED I begin by making a sketch of the dipole (**Figure 25.27**). I place the dipole along my y axis, with the midpoint of the dipole length at the origin, so that the particles are at the coordinates given in the problem. I choose a point P on the positive y axis, and I let the y coordinate of P be y. Because the two charged particles lie on the y axis, the electric field they create at P must be directed along the y axis.

Figure 25.27



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QUANTITATIVE TOOLS

❷ **DEVISE PLAN** The potential at P is the sum of the potentials due to the individual charged particles. To calculate these potentials, I can use Eq. 25.30. Once I have calculated the potential at P, I can use Eq. 25.40 to determine the electric field.

❸ **EXECUTE PLAN** The potential at *P* is

$$V_P = \frac{1}{4\pi\epsilon_0}\sum\frac{q_n}{r_{nP}} = \frac{1}{4\pi\epsilon_0}\left[\frac{+q_P}{y-\frac{1}{2}d} + \frac{-q_P}{y+\frac{1}{2}d}\right]. \quad (1)$$

Now I use this result to determine the electric field. Because I am working with the y axis, I work with the y component of Eq. 25.40:

$$E_y = -\frac{\partial V}{\partial y} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_P}{(y-\frac{1}{2}d)^2} - \frac{q_P}{(y+\frac{1}{2}d)^2}\right]. ✔$$

Because the electric field at P is directed along the y axis, the other components of the electric field are zero: $E_x = E_z = 0$. ✔

❹ **EVALUATE RESULT:** This is the same result as in Eq. 23.11. (Remember $k = 1/(4\pi\epsilon_0)$.)

In comparing the derivation in Example 25.8 with the derivation of the electric field in Section 23.6, note that the calculation of the electric field via the potential does not involve any vector addition. The scalar nature of the potential therefore greatly simplifies calculations.

25.17 Calculate the electric field at any point on the axis of a thin charged disk from the potential we obtained in Example 25.7. Compare your answer to the result we obtained by direct integration in Section 23.7.

Chapter Glossary

SI units of physical quantities are given in parentheses.

Electric potential energy U^E (J) The form of potential energy associated with the configuration of stationary objects that carry electrical charge. When the reference point for the electric potential energy is set at infinity, the potential energy for two particles carrying charges q_1 and q_2 and separated by a distance r_{12} is

$$U^E = \frac{q_1q_2}{4\pi\epsilon_0}\frac{1}{r_{12}} \quad (U^E \text{ zero at infinite separation}). \quad (25.8)$$

Electrostatic work W_q (J) Work done by an electrostatic field on a charged particle or object moving through that field. The electrostatic work depends on only the endpoints of the path. For a particle of charge q that is moved from point A to point B in an electric field, the electrostatic work is

$$W_q(\mathrm{A}\to\mathrm{B}) = q\int_{\mathrm{A}}^{\mathrm{B}}\vec{E}\cdot d\vec{\ell}. \quad (25.24)$$

Equipotentials Lines or surfaces along which the value of the potential is constant. The equipotential surfaces of a charge distribution are always perpendicular to the corresponding electric field lines. The electrostatic work done on a charged particle or object is zero as it is moved along an equipotential.

Potential V_P (V) Potential differences can be turned into values of the potential at every point in space by choosing a reference point where the potential is taken to be zero. Common choices of reference point are Earth (or *ground*) and infinity. The potential of a collection of charged particles (measured with respect to zero at infinity) at some point P can be found by taking the algebraic sum of the potentials due to the individual particles at P:

$$V_P = \frac{1}{4\pi\epsilon_0}\sum\frac{q_n}{r_{nP}} \quad \text{(potential zero at infinity)}, \quad (25.30)$$

where q_n is the charge carried by particle n and r_{nP} is the distance from P to that particle. For continuous charge distributions, the sum can be replaced by an integral:

$$V_P = \frac{1}{4\pi\epsilon_0}\int\frac{dq_s}{r_{sP}} \quad \text{(potential zero at infinity)}. \quad (25.34)$$

The electric field can be obtained from the potential by taking the partial derivatives:

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k}. \quad (25.40)$$

Potential difference V_{AB} (V) The potential difference between points A and B is equal to the negative of the electrostatic work per unit charge done on a charged particle as it is moved along a path from A to B:

$$V_{AB} \equiv \frac{-W_q(\mathrm{A}\to\mathrm{B})}{q} = -\int_{\mathrm{A}}^{\mathrm{B}}\vec{E}\cdot d\vec{\ell}. \quad (25.25)$$

For electrostatic fields, the potential difference around a closed path is zero:

$$\oint\vec{E}\cdot d\vec{\ell} = 0 \quad \text{(electrostatic field)}. \quad (25.32)$$

Volt (V) The derived SI unit of potential defined as $1\ \mathrm{V} \equiv 1\ \mathrm{J/C}$.

26 Charge Separation and Storage

This chapter deals with generating and storing electric potential energy. To produce charged objects, positive and negative charge carriers must be pulled apart and then kept separate. Work is required to pull apart charge carriers, just as work is required to stretch a spring. In each case, this work results in energy storage in the system. We now look at what kind of changes in energy are involved in the separation of positive and negative charge carriers and how charge carriers that have been separated can be stored in simple arrangements of conductors.

26.1 Charge separation

Whenever objects are "charged" (by separating strips of Scotch tape, rubbing objects against each other, using batteries, etc.), the basic phenomenon is the same: Some process (pulling, rubbing, chemical reactions) separates positive and negative charge carriers from one another. As a concrete example, consider a rubber rod and a piece of fur. If you rub the two together and then separate them, they become oppositely charged because the rod pulls electrons away from the fur: The rod ends up with a surplus of electrons and the fur with a deficit. Provided none of the electrons on the rod leak away (to the air, your hand, etc.), the magnitude of the negative charge on the rod is equal to that of the positive charge on the fur.

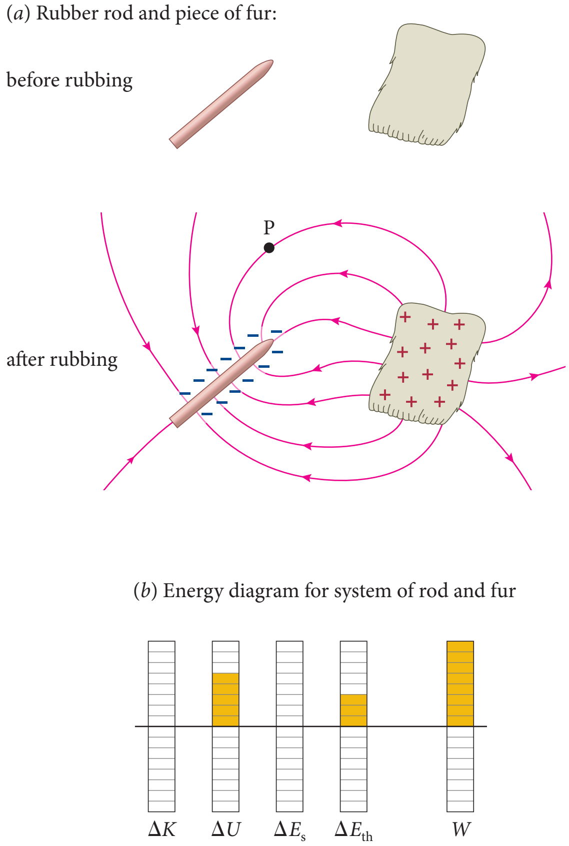
What is the change in energy associated with this charge separation? Consider the rubber-fur system in its initial and final states (Figure 26.1*a*). To separate the positive and negative charge carriers, they must be pulled apart against an attractive electric force, just as the ends of a stretched spring are pulled apart against an elastic force. Because work must be done on the rod-fur system, the electric potential energy of the system is greater in the final state. This energy is supplied by you while you rub the two objects together and then increase their separation. Not all of the energy you put into the system goes into electric potential energy; the friction involved in the rubbing not only produces charge separation but also heats up the rod and the fur, so part of the work you do on the system increases the thermal energy. An energy diagram for the rod-fur system is shown in Figure 26.1*b*.

26.1 Suppose you repeat the charging (starting again with uncharged rod and fur), but this time you rub longer and twice as much charge accumulates at each point on the two objects. How do the following quantities compare to what they were after the first charging: (*i*) the direction and magnitude of the electric field at point P in Figure 26.1*a*; (*ii*) the potential difference between two fixed points on the rod and the fur; and (*iii*) the electric potential energy in the rod-fur system?

Checkpoint 26.1 highlights the essence of this chapter. Be sure not to confuse *potential difference* and *electric potential energy*:

- The system's electric potential energy depends on the configuration of the positive and negative charge carriers in the system.

Figure 26.1 When we charge a rubber rod and a piece of fur by rubbing them together, we do work to separate charge and hence increase the electric potential energy of the system comprising the rod and fur.



- The potential difference between points on the rod and the fur is a measure of the electrostatic work done on a particle carrying a unit of charge (not part of the system) while moving between those points.

In the next section we examine the proportionality between potential difference and charge separation in detail. In this section we concentrate on the relationship between charge separation and electric potential energy.

The crucial point to take away from Checkpoint 26.1 is:

Positive work must be done on a system to cause a charge separation of the positive and negative charge carriers in the system. This work increases the system's electric potential energy.

26.2 If you double the separation between the charged rod and fur in Figure 26.1*a*, does the electric potential energy of the rod-fur system increase, decrease, or stay the same?

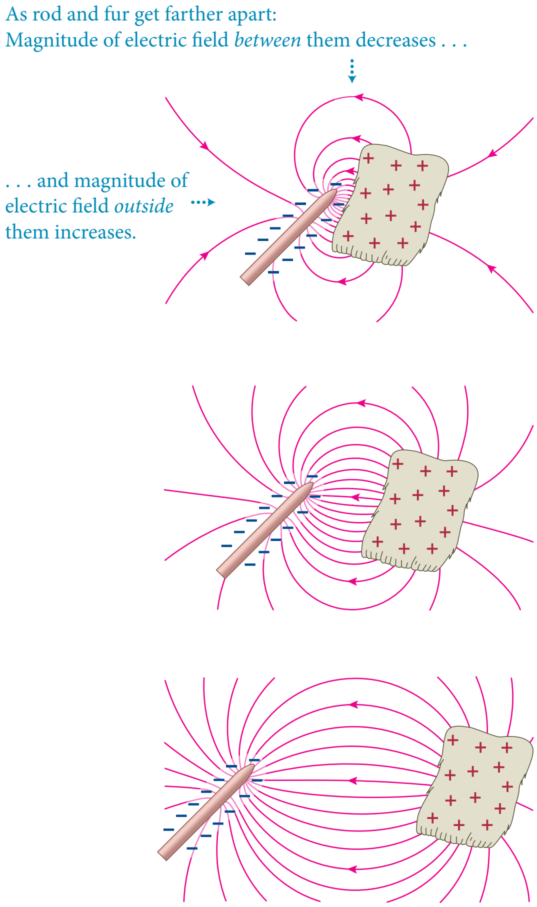
The amount of stored electric potential energy depends on the amount of charge that is separated and the distance that separates the charge carriers. More charge or a greater separation means more electric potential energy is stored. These arguments apply to *all* devices that separate charge, such as Van de Graaff generators (see the box below) and batteries (see Section 26.4). Once electric potential energy has been generated by separating charge carriers, this energy can be used for other processes, such as lighting a lamp, operating a radio, and so on.

Every **charge-separating device** (or **charging device**) has some mechanism that moves charge carriers *against* an electric field—a process that requires work to be done on the system of charge carriers. For the rod-fur system, this work is mechanical and is supplied by the person doing the rubbing and separating the objects. In a battery, chemical reactions drive charge carriers through a region where the electric field opposes their motion.

26.3 If you include the person doing the rubbing in the system considered in Figure 26.1, what is the resulting energy diagram?

Where is the electric potential energy of the rod-fur system stored? As you may recall from Section 7.2, potential energy is stored in reversible changes in the configuration of interacting components of a system. Electric potential energy, therefore, is associated with the configuration of the charge carriers in a system. A look at the electric field pattern suggests an alternative view, however. **Figure 26.2** shows how the electric field line pattern changes as the distance between the charged rod and the fur increases. Note how more of the space around the system becomes filled with field lines (indicating that the magnitude of the electric field there increases), while the density of the field lines between

Figure 26.2 Change in the electric field pattern as the distance between the rod and fur increases.



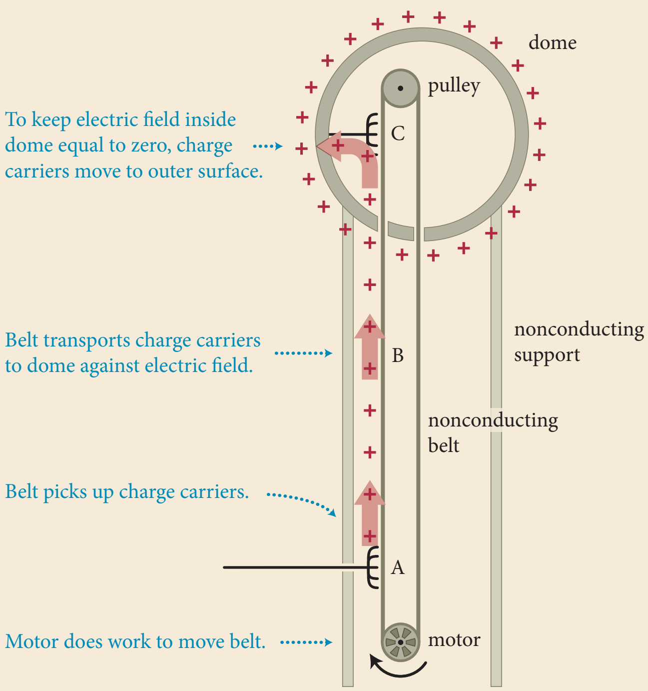
Van de Graaff generator

Figure 26.3 shows a schematic diagram of a Van de Graaff generator—a mechanical device invented in the 1930s by Robert J. Van de Graaff to separate large amounts of electrical charge. The basic principle is extremely simple: A nonconducting belt delivers charge carriers to a hollow conducting dome that rests on a nonconducting support. Machines of this type are used to generate the very large potential differences required in particle accelerators and for the generation of x rays.

Operation of the generator involves three important steps. The first step is a transfer of charge carriers to the belt at A. This transfer can be done by literally "spraying" charged particles onto the belt or simply by rubbing the rubber belt against some appropriate material.

The second step transports the charge carriers to the dome. This step is possible because the belt is nonconducting, so the charge carriers are not mobile—they are stuck to the belt, which is driven by a motor around a pulley inside the dome. The motor must do work on the charge carriers to move them against the electric field of the dome. (In the example shown in the diagram, positive charge carriers at B must be transported upward against the downward electric field of the positively charged dome.)

Figure 26.3 Schematic diagram of a Van de Graaff generator.



The third step transfers the charge carriers from the belt onto the dome. As we saw in Section 24.5, the electric field inside a hollow conductor is always zero and any charge inside a conductor moves toward the outer surface. Therefore, once the charge carriers are inside the dome, they tend to move to the outer surface of the dome. For this purpose, a comb of conducting needles is placed close to the belt at C. If the charge carriers on the belt are electrons, the electrons hop onto the comb and move via the connecting wire toward the outside of the metal dome, causing the dome to acquire a negative charge. Alternatively, the charge carriers on the belt can be positively ionized air molecules, in which case electrons in the comb are attracted toward the ions. These electrons then jump from the comb onto the belt, neutralizing the ions on the belt while leaving a positive charge behind on the outside of the dome.

Construction of a huge double Van de Graaff generator for the MIT Physics Department in South Dartmouth, Massachusetts, in 1933. These generators, currently at the Boston Museum of Science, generated opposite charge and were able to produce potential differences of 10,000,000 V between the two 4.5-m domes.

the rod and the fur decreases. Thus, as the distance between the rod and the fur changes, the electric potential energy changes and the electric field changes.

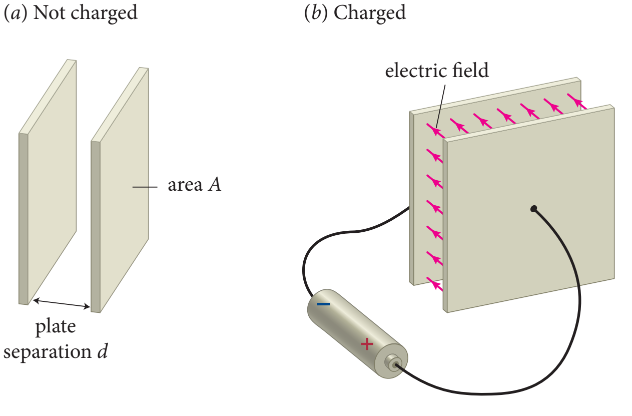
As we shall see later in this chapter, we can relate a change in the electric potential energy of a system to a change in the system's electric field (integrated over all of space), which suggests that the electric potential energy of a system is stored in its electric field. In other words, the electric potential energy of the rod-fur system is spread throughout the space around it. As long as the two objects are held stationary, the field is stationary, so the exact "location" of the energy is not very important because there is no way we can determine it experimentally. If we shake the charged rod or fur, however, the shaking causes a wavelike disturbance in the electric field. This disturbance propagates away from the rod or fur and carries with it energy that we can detect. For now we don't need to concern ourselves with such waves. It suffices to know that electric fields store electric potential energy.

26.2 Capacitors

Any system of two charged objects, such as the rod-fur system in the preceding section, stores electric potential energy. To study how much electric potential energy can be stored in a system of two objects, let's begin by considering the simple arrangement of two parallel conducting plates shown in **Figure 26.4a**. A system for storing electric potential energy that consists of two conductors is called a **capacitor**; the arrangement in Figure 26.4*a* is called a *parallel-plate capacitor*

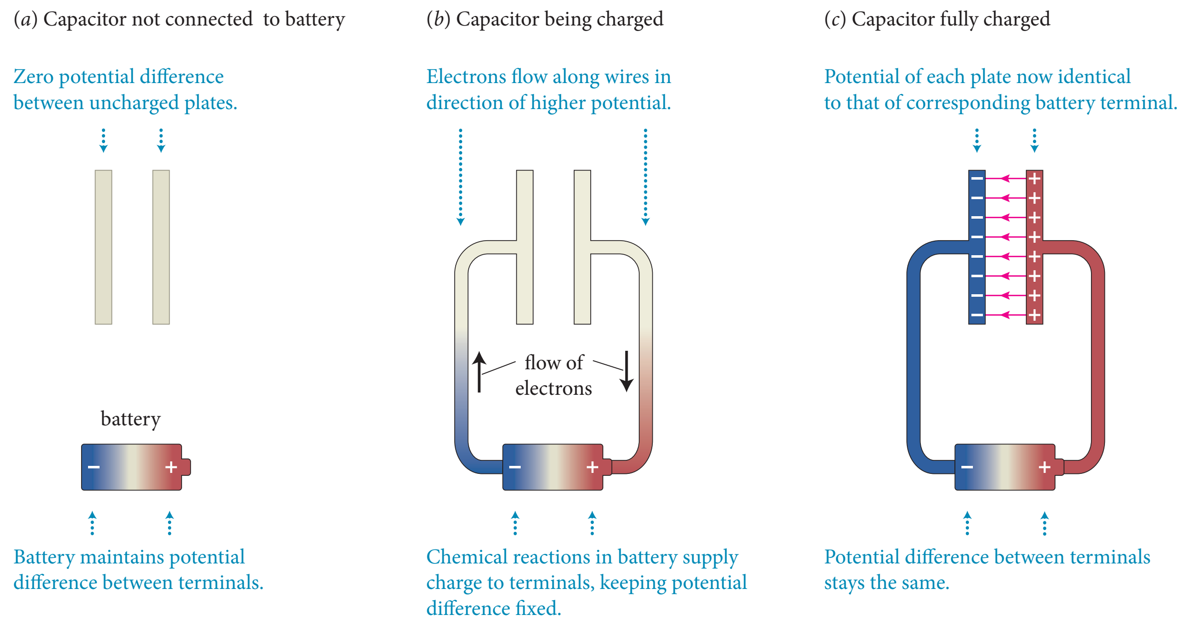
Figure 26.4*b* illustrates a simple method for charging such a capacitor. Each plate is connected by a wire to a terminal of a battery, which maintains a fixed potential difference between its terminals. **Figure 26.5** shows what happens when the connection is made between the battery and the capacitor. If the capacitor plates are far enough away from

Figure 26.4 A parallel-plate capacitor.



CONCEPTS

Figure 26.5 Charging a capacitor.



the battery, the potential difference between the plates initially is zero (Figure 26.5*a*). Immediately after the wires are connected, there is a potential difference between the ends of each wire. This difference in potential causes electrons (which are mobile in metal) in the wires to flow as indicated by the arrows in Figure 26.5*b*. A positive charge builds up on the plate connected to the positive terminal, and a negative charge of equal magnitude builds up on the other plate. As electrons leave one plate and accumulate on the other, the potential difference between the plates changes. This process continues until the potential is the same at both ends of each wire—that is, when the potential difference between the plates is equal to that between the terminals of the battery (Figure 26.5*c*). Because there is no longer any potential difference from one end of the wire to the other, the flow of electrons stops and the capacitor is said to be *fully charged*. In the process of achieving this state, the battery has done work on the electrons; this work has now become electric potential energy stored in the capacitor.

The time interval it takes to fully charge a capacitor depends on the properties of the capacitor, the battery, and the way the capacitor is connected to the battery. Typically, only a fraction of a second is needed for charging, although the time interval it takes to charge very large capacitors can be minutes (more on this in Chapter 32).

26.4 (*a*) Suppose that we disconnect the wires from the plates after the capacitor is charged as shown in Figure 26.5*c*. How does the potential difference between the plates after the wires are disconnected compare to that just before they are disconnected? (*b*) If we replace the battery in Figure 26.5 by a battery that maintains a greater potential difference between its terminals, is the magnitude of the charge on the plates greater than, smaller than, or the same as when the first battery is connected?

When a capacitor is not connected to anything, as in Checkpoint 26.4*a*, it is said to be *isolated*. For an isolated capacitor, the *quantity of charge on each plate* is fixed because the charge carriers have nowhere to go. In contrast, for a capacitor that is connected to a battery, the *potential difference across the capacitor* is fixed—the charge carriers on the plates always adjust themselves in such a way as to ensure that the potential difference across the capacitor is equal to that across the battery.

Figure 26.6 shows the electric fields of two isolated charged parallel-plate capacitors. The field is nearly uniform in the region between the plates, but it is nonuniform at the edges. When the spacing between the plates is small compared to the

Figure 26.6 Effect of plate separation in relation to plate area on the field of a parallel-plate capacitor.

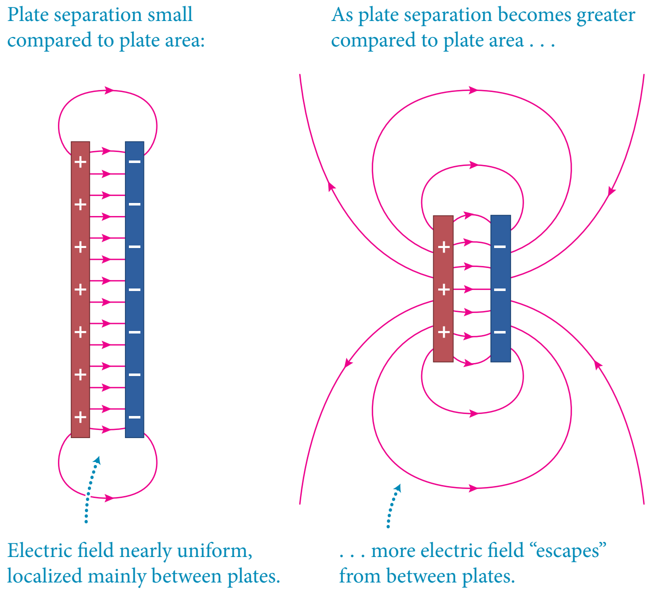
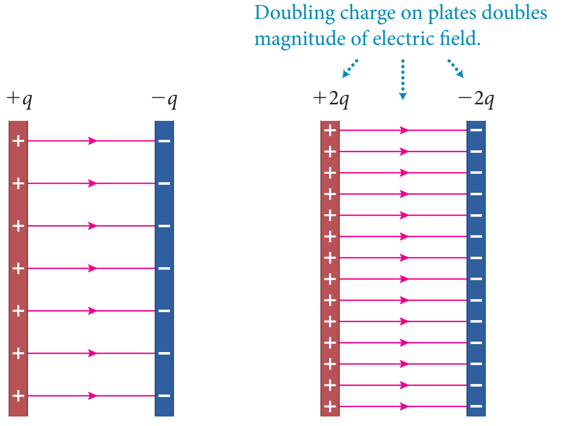


Figure 26.7 Doubling the charge on a parallel-plate capacitor.

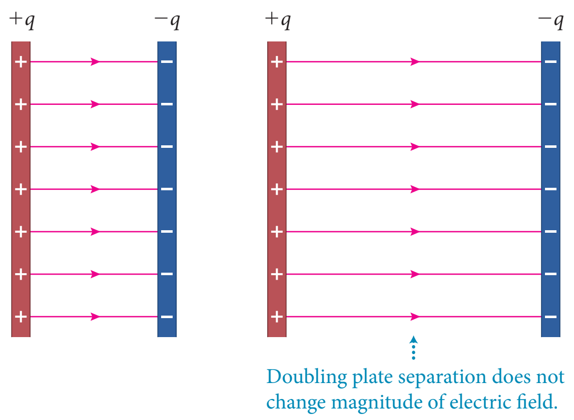


area of the plates, the effect of the nonuniform field is negligible: The electric field is confined almost entirely to the region between the plates, and for all practical purposes we can consider this field to be uniform. When discussing parallel-plate capacitors, we shall ignore the nonuniform fields at the edges and assume the electric field is entirely uniform between the plates. This simplification is justified by the geometry of most capacitors used in electronic applications.

Let us now examine the relationship between the magnitude of the charge on the plates of a parallel-plate capacitor and the potential difference between them. **Figure 26.7** shows an isolated parallel-plate capacitor carrying a positive charge $+q$ on one plate and a negative charge $-q$ on the other. If we double the magnitude of the charge on each plate, then the electric field between the plates doubles, too (see Checkpoint 23.14). Consequently, the electric force exerted on a charged particle between the plates doubles, so the electrostatic work done in moving a charged particle from one plate to the other also doubles. According to Eq. 25.15, the potential difference between the plates doubles as well. In other words, the potential difference between the plates is proportional to the magnitude of the charge on the plates.

What happens if we increase the plate separation of an isolated parallel-plate capacitor, as illustrated in **Figure 26.8**? The electric field remains the same because it is determined by the

Figure 26.8 Doubling the plate separation of a parallel-plate capacitor.



surface charge density on the plates, which doesn't change (see Checkpoint 23.14). Because the distance between the plates increases, however, the electrostatic work done in moving a charged particle from one plate to the other increases—more work is required to move the particle over a greater distance—so the potential difference between the plates increases too, confirming the result we obtained in Example 25.5.

26.5 Suppose the two capacitors in Figure 26.8 are each connected to a 9-V battery. (*a*) Which of the two capacitors stores the greater amount of charge? (*b*) If, instead of the separation increasing, the area of the plates of the capacitor is halved and then the capacitor is connected to a 9-V battery, does the capacitor store more charge, less charge, or the same amount of charge as before the area of the plates was halved?

As Checkpoint 26.5 illustrates, the geometry of the capacitor determines its capacity to store charge. In general:

For a given potential difference between the plates of a parallel-plate capacitor, the amount of charge stored on its plates increases with increasing plate area and decreases with increasing plate separation.

Does this mean that we can increase the amount of charge stored on a parallel-plate capacitor indefinitely simply by making the plate separation infinitesimally small? The answer is *no*, because if the plate spacing is decreased while the potential difference between the capacitor plates is fixed, the charge on each plate increases and thus the magnitude of the electric field in the capacitor increases. When the electric field is about 3×10^6 V/m, the air molecules between the plates become *ionized* and the air becomes conducting, allowing a direct transfer of charge carriers between the plates. Once such a so-called **electrical breakdown** occurs, the capacitor loses all its stored energy in the form of a spark.

The opening page of this chapter shows an electrical breakdown of air between the charged dome of a very large Van de Graaff generator and a nearby metal object. The breakdown limits the maximum potential difference across a capacitor and thus the maximum amount of charge that can be stored on it. The electric field at which electrical breakdown occurs is called the *breakdown threshold*.

The breakdown threshold can be raised by inserting a nonconducting material between the capacitor plates. As we shall see in the next section, such a nonconducting material also greatly increases the amount of charge that can be stored by a capacitor. To understand why this is so, we begin by considering a simpler situation: the insertion of a conductor between the plates of a parallel-plate capacitor.

Figure 26.9*a* shows an isolated charged capacitor. Suppose we now insert a conducting slab between the plates of this capacitor (Figure 26.9*b*). As we saw in Section 24.5, the charge carriers in the conductor rearrange themselves in such a fashion as to eliminate the field inside the bulk of the conductor.

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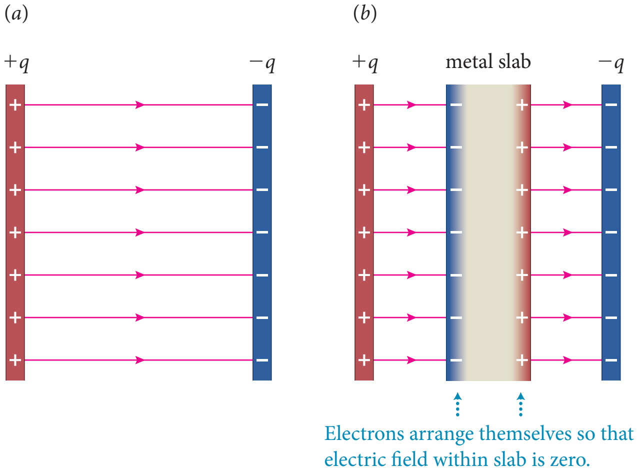
Figure 26.9 Inserting a conducting slab between the plates of a parallel-plate capacitor.



26.6 Suppose the capacitor in Figure 26.9*a* is charged and then disconnected from the battery. (*a*) As the conducting slab is inserted in the capacitor, as in Figure 26.9*b*, does the amount of charge on the capacitor plates increase, decrease, or stay the same? (*b*) How much charge accumulates on each side of the slab, once it is inserted? (*c*) What is the potential difference across the metal slab? (*d*) As the slab is inserted, does the magnitude of the potential difference between the capacitor plates increase, decrease, or stay the same?

As Checkpoint 26.6 demonstrates, the effect of the slab is to make the electric field in part of the space between the capacitor plates zero, thus reducing the magnitude of the potential difference between the plates for a given amount of charge on them. As the next example shows, the converse of this fact is that, for a given potential difference between its plates, the capacitor can store more energy with the slab inserted than it can without the slab. In other words, the slab increases the capacitor's capacity to store charge.

Example 26.1 Metal-slab capacitor

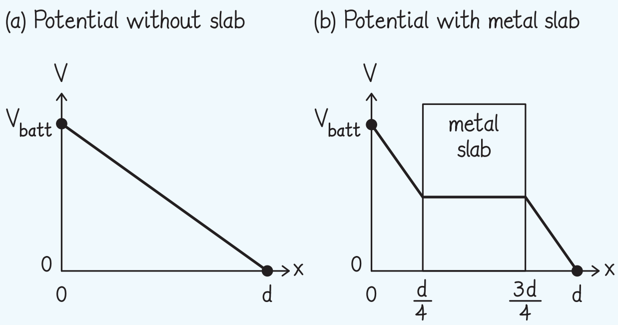
Suppose the capacitor in Figure 26.9 has a plate separation distance d and the plates carry charges $+q$ and $-q$ when the capacitor is connected to a battery that maintains a potential difference V_{batt} between its terminals. If a metal slab of thickness $d/2$ is inserted midway between the plates while the battery remains connected, what happens to (*a*) the magnitude of the electric field between the plates and (*b*) the quantity of charge on the plates?

❶ GETTING STARTED I am given the plate separation distance and plate charges for a capacitor connected to a battery, and I must determine how the electric field magnitude between the plates and the quantity of charge on each plate change when a metal slab is inserted. Because the capacitor remains connected to the battery, the potential difference across the capacitor is fixed. Because the slab is conducting, the electric field inside the slab is always zero.

❷ DEVISE PLAN The electric field magnitude determines the electrostatic work, which in turn determines the potential difference between the plates (which I know). To determine how the electric field magnitude changes when the metal slab is inserted, therefore, I must first determine how the potential between the plates changes when the slab is inserted. Once I know the electric field, I can determine how the slab affects the charge on the plates because I know that the magnitude of the electric field between the plates is proportional to the surface charge density σ (see Checkpoint 23.14, $E = 4k\pi\sigma$).

❸ EXECUTE PLAN To compare the potentials before and after the slab is inserted, I must plot V as a function of position between the plates (**Figure 26.10**). I choose my x axis to be parallel to the electric field, with the positive plate at $x = 0$ and the negative plate at $x = d$ as the zero of potential. In the absence of the metal slab, the field is uniform between the plates, and so the potential decreases linearly from $x = 0$ to $x = d$ (Figure 26.10*a*). Because the electric field inside the slab is zero, the potential does not vary across the slab (it is an equipotential; see Section 25.3). Because the battery keeps the potential at each plate constant, the potential-versus-distance curve must take on the zigzag form shown in Figure 26.10*b*.

Figure 26.10



(*a*) Insertion of the metal slab affects the electric field between the plates because the slab is an electrical conductor and so the electric field inside it must always be zero. However, because V_{batt} is constant, I know that the electrostatic work, $W_q = F^E d = qV_{batt}$, done on a particle carrying a charge q to move the particle from one plate to the other must be the same whether or not the slab is in place there. Because no electrostatic work is done to move the particle through the slab where the electric field is zero, the field outside the slab must be greater to make up for the smaller distance over which the particle is moved. More precisely, because the distance over which the electric field is nonzero is reduced to $d/2$, the magnitude of the field must be twice what it was before the slab was inserted. ✔

(*b*) If the field doubles, then the charge per unit area must also double. Given that the area of the plates does not change, this means that the charge on the plates must double. ✔

❹ EVALUATE RESULT Inserting the metal slab with the battery connected is equivalent to halving the separation distance between the plates while keeping the potential difference constant. I know from Checkpoint 26.5 that, for a constant potential difference, the quantity of charge stored on a capacitor increases with decreasing plate separation, as I found.

26.7 (*a*) Does the position of the slab in Figure 26.9 affect the potential difference across the capacitor? Consider, in particular, the case in which the slab is moved all the way to one side and makes electrical contact with one of the plates. (*b*) Sketch the potential $V(x)$ as a function of x, with the slab off-center.

26.3 Dielectrics

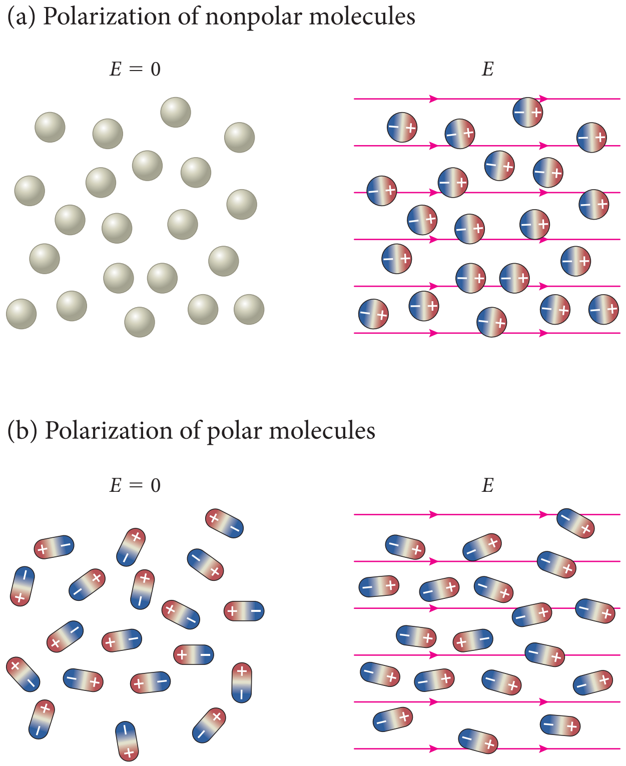
As we just saw, decreasing the space inside an isolated charged capacitor where the electric field is nonzero increases its capacity to store electrical charge for a given potential difference across its plates. With a conducting slab inserted, however, the gap between either plate and the slab face nearest it is smaller than the plate-to-plate gap before the slab was inserted. Because a decreased gap with the potential difference held constant means E increases, we still have the problem of electrical breakdown.

Suppose, however, that we insert a nonconducting material—a **dielectric**—between the plates of a capacitor. As we discussed in Section 22.4, the electric field between the plates of the capacitor polarizes the dielectric. What effect does this polarization have? To answer this question we must first look in more detail at what happens in a polarized dielectric material.

We should distinguish between two general types of dielectric materials. A *polar* dielectric consists of molecules that have a permanent electric dipole moment; each molecule is electrically neutral, but the centers of its positive and negative charge distributions do not coincide (see Section 23.4). The atoms or molecules in a *nonpolar* dielectric have no dipole moment in the absence of an electric field.

Figure 26.11*a* shows the polarization of a nonpolar dielectric. In the presence of an electric field, the electrons in a nonpolar dielectric are displaced in the direction opposite to $\vec{E}$, inducing a dipole moment on each molecule.

Figure 26.11 Polarization of nonpolar and polar molecules in an electric field.



26.8 Why are the electrons displaced in a direction *opposite* the electric field?

The polarization of a polar dielectric is shown in Figure 26.11*b*. In the absence of an electric field, the individual molecules' dipole moments are randomly aligned, so the material as a whole is not polarized. In the presence of an electric field, however, the molecular dipoles are subject to a torque (see Section 23.8) that tends to align the dipoles with the electric field, giving rise to a macroscopic polarization. In general, the polarization of polar dielectrics is much greater than that of nonpolar ones because the permanent dipole moments of the molecules in a polar dielectric are much greater than the induced dipole moments in a nonpolar dielectric.

Figure 26.12 illustrates the effect of the uniform polarization of the atoms or molecules in a polar or nonpolar dielectric. The charge enclosed by any volume that lies entirely inside the polarized dielectric is zero: The positive and negative charge carriers may not coincide exactly, but on average they occur in equal numbers. However, this is not true for a small volume at the surface of the dielectric. In the volume on the right in Figure 26.12, for example, a surplus of positive charge appears at the surface of the material. Thus, a polarized dielectric has a very thin sliver of surplus positive charge on one side and a sliver containing an equal amount of surplus negative charge on the other side (see also Figure 22.21). The surface charge on either side of the polarized dielectric is said to be **bound** because the charge carriers that cause it are not free to roam around in the material. In contrast, the charge on the capacitor plates is **free**

Figure 26.12 The reason a polarized dielectric exhibits a macroscopic polarization.

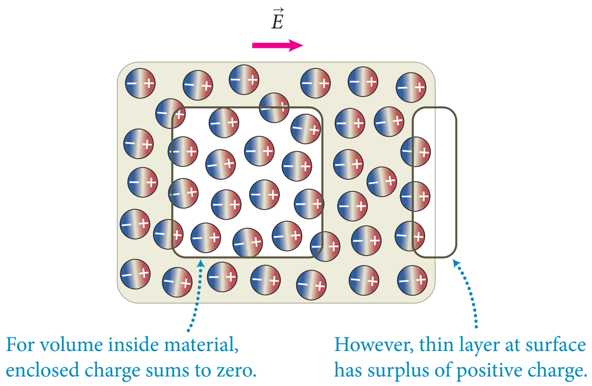


Figure 26.13 The polarization induced on a dielectric in a parallel-plate capacitor is equivalent to two thin sheets carrying opposite charge.

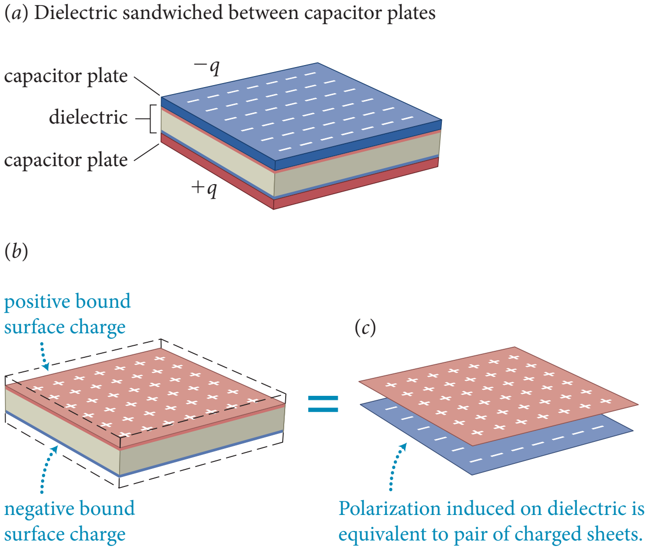
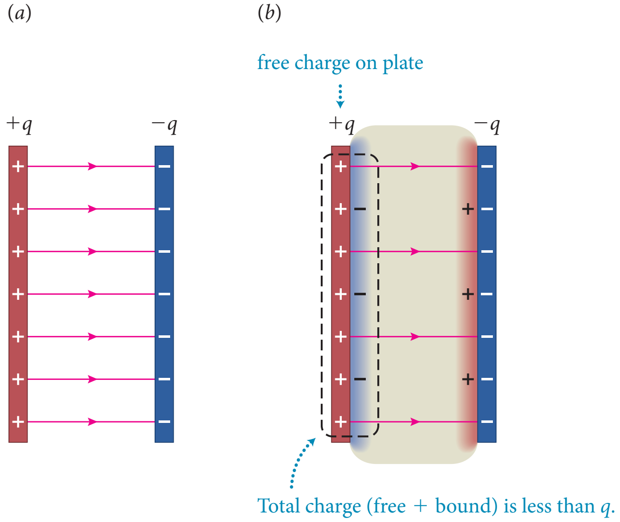


Figure 26.14 The presence of a polarized dielectric reduces the strength of the electric field between the plates of a capacitor.



because the charge carriers that cause it can move around freely. From a macroscopic point of view, a uniformly polarized dielectric differs from an unpolarized dielectric only by the presence of this bound surface charge.

What is the effect of the bound surface charge on the electric field inside the capacitor? Consider a dielectric slab inside an isolated charged capacitor (**Figure 26.13*a***). The dielectric is polarized by the electric field of the capacitor; that is, a positive bound surface charge appears on the surface near the negatively charged capacitor plate, and a negative bound surface charge appears on the other side (Figure 26.13*b*). Imagine now that we could "freeze in" the polarization and consider just the slab by itself. Except for two sheets of charge at the top and bottom, the bulk of the dielectric is neutral. Thus, for all practical purposes, the polarized dielectric is equivalent to two very thin sheets carrying opposite charge (Figure 26.13*c*).

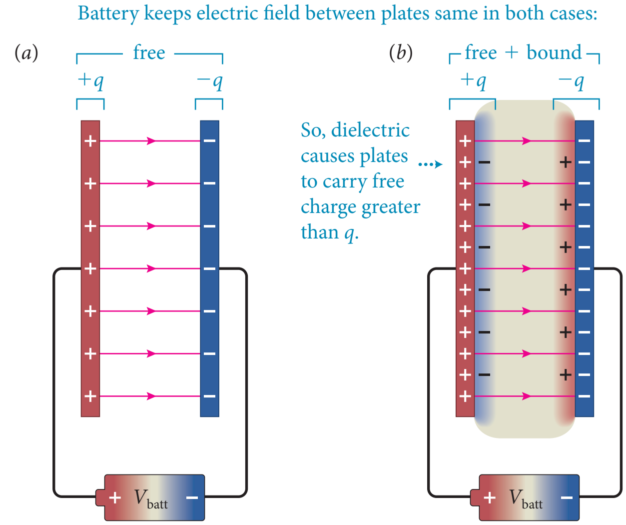
26.9 (*a*) In which direction does the electric field due to the bound surface charge point at a location above the top surface in Figure 26.13*c*? (*b*) In which direction does it point at a location between the top and bottom surfaces?

We can now obtain the electric field of the capacitor with the dielectric by superposition: It is equal to the electric field of the capacitor without the dielectric, plus the electric field of the polarized dielectric by itself. As you found in Checkpoint 26.9, the direction of the electric field due to the polarized dielectric is opposite that of the capacitor, so the presence of the dielectric decreases the electric field strength in the capacitor. Alternatively, we can say that each of the bound surface charges compensates for part of the free charge on the adjoining capacitor plate, so, in effect, the total charge (free and bound) on each side of the capacitor is reduced (**Figure 26.14**). This reduction in charge, in turn, gives rise to a smaller electric field inside the capacitor. For some materials, the field inside can be reduced by a factor of several thousand.

26.10 (*a*) If the magnitude of the bound surface charge on the dielectric slab in Figure 26.14*b* were equal to the magnitude of the free charge on the capacitor plates, what would be the electric field inside the capacitor? (*b*) Could the magnitude of the bound surface charge ever be *greater* than the magnitude of the free charge on the plates?

Figure 26.15 shows what happens when a dielectric-filled capacitor is connected to a battery. The battery keeps the potential difference between the capacitor plates the same regardless of the presence of the dielectric. Because

Figure 26.15 The presence of a polarized dielectric increases the charge on the plates of a capacitor connected to a battery.



the electric field in the capacitor is equal to the potential difference divided by the distance between the plates (see Example 25.5), it follows that, as long as the capacitor is connected to the battery, the electric field must be the same regardless of the dielectric. The electric field can be the same only if the distribution of charge causing the electric field is the same. In other words, regardless of the presence of the dielectric, we must have the same amount of total charge (free and bound) on each side of the capacitor. Let the magnitude of the free charge on the capacitor plates without the dielectric be q. As shown in Figure 26.15*b*, the polarization of the dielectric causes a negative bound surface charge next to the positive capacitor plate; the sum of the free charge on the conductor and the adjoining bound surface charge must still be equal to $+q$. Similarly, the sum of the negative free charge on the opposite plate and the positive bound surface charge on the adjoining dielectric still must be $-q$. Consequently, the magnitude of the free charge on each plate by itself can be much greater than without the dielectric. This extra charge is supplied by the battery.

26.11 Given that the electric field is the same in both capacitors in Figure 26.15, which stores the greater amount of electric potential energy?

If the answer to Checkpoint 26.11 surprises you—after all, the electric fields are the same in the two capacitors—remember that the amount of *charge separation* is not the same. The charge on the capacitor plates polarizes the dielectric, and this polarization is the result of charge separation in the molecules of the dielectric. Thus, instead of empty space without charge separation between the capacitor plates, we now have (in addition to a greater charge on the plates) a lot of additional charge separation on the microscopic scale. Most of the energy stored in the capacitor is not due to the separation of charge on the plates, but to the separation of charge in the dielectric between the plates. The pulling apart of the positive and negative charge distributions in the dielectric increases the electric potential energy stored in the dielectric, much like stretching a spring by pulling its ends apart stores elastic potential energy. This tells us that an electric field of a given magnitude in a dielectric stores more energy than an equal field in vacuum.

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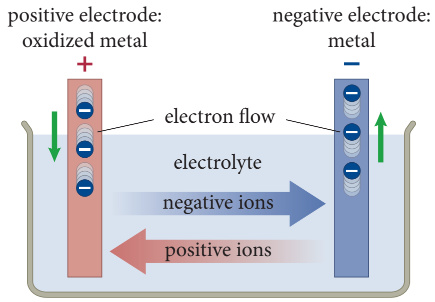
26.4 Voltaic cells and batteries

Electric potential energy is generated by separating charged particles. Earlier in this chapter we discussed two means of accomplishing such charge separation: charging by rubbing and the Van de Graaff generator. Another common way to generate electric potential energy is by means of a *voltaic cell*, the first of which was constructed by Alessandro Volta in around 1800. Assemblies of voltaic cells are called *batteries*. A standard 9-V alkaline battery, for example, consists of six 1.5-V cells connected together. While there are many types of voltaic cells and batteries, all have a common operating principle: Chemical reactions turn chemical energy into electric potential energy by accumulating electrons on one side of the cell (the negative terminal) and removing electrons from the other side (the positive terminal).

Figure 26.16 General operating principle of a voltaic cell. Electrons flow when the cell is connected to an electronic device.

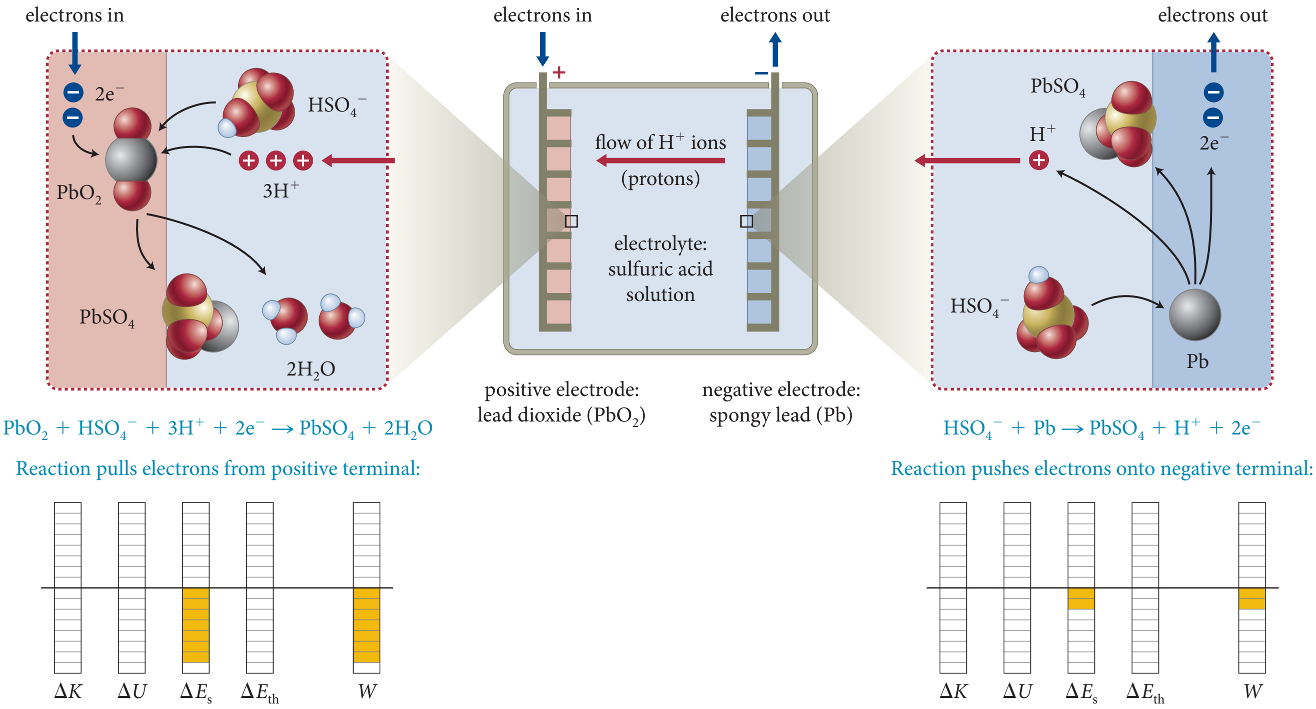


The general principle of a voltaic cell is illustrated in Figure 26.16. Two conducting terminals, or *electrodes*, are submerged in an *electrolyte*—a solvent that contains mobile ions. One electrode is usually made from an oxidized metal; the oxidized metal reacts by accepting positive ions from the electrolyte and electrons from the electrode. The other electrode is generally metallic; it oxidizes by taking in negative ions and giving up electrons. Because of these reactions, a surplus of electrons builds up on the metallic terminal and a deficit of electrons builds up on the oxidized-metal terminal, causing a potential difference between the two. The reactions stop when the potential difference between the electrodes reaches a certain value called the *cell potential difference*. This value is determined by the type of chemicals in the cell and is typically on the order of a few volts. As we saw in Section 25.5, this potential difference can be used to do electrostatic work on charge carriers when a battery is connected to some device—such as a light bulb, a motor, or a capacitor.

As long as the cell is not connected to anything and its chemicals do not deteriorate, the cell remains in the same state indefinitely. When the cell is connected to a capacitor or to some other device, however, the surplus of electrons is removed from the negative electrode and electrons are supplied to the positive electrode. Then the chemical reactions resume in order to maintain the cell potential difference between the terminals. As the reactions proceed, the electrolyte becomes more dilute and the compositions of the electrodes change. The cell is exhausted when all the ions in the electrolyte have been depleted. For some types of cells, the chemical reactions can be reversed by supplying a potential difference to the terminals of the cell; electric potential energy is then converted back to chemical energy. Such cells are used in rechargeable batteries, which can be reused repeatedly to store and recover electric potential energy.

As we noted in Section 26.1, any charge-separating device involves the motion of charge carriers against the direction of an electric field. This is where work is done on the charge

Figure 26.17 Schematic diagram of a lead-acid cell and of the reactions taking place at the positive and negative electrodes.



carriers and where some form of energy is converted to electric potential energy. Inside a voltaic cell, electrons must be pulled away from the positively charged terminal and deposited onto the negatively charged terminal. This process occurs at the surface of the electrodes where the chemical reactions take place. The chemical reactions move charge carriers against strong opposing electric forces. The chemical energy released in the reactions provides the energy necessary to move the charge carriers against the electric field. The work done per unit charge is called the **emf** (pronounced e-m-f)* of the device:

The emf of a charge-separating device is the work per unit charge done by nonelectrostatic interactions in separating positive and negative charge carriers inside the device.

All charge-separating devices—batteries, voltaic cells, generators, solar cells—have some *nonelectrostatic* means to separate charge carriers and thereby create a potential difference across the terminals of the device.

26.12 As electrons leave one terminal and are added to the other, ions in the electrolyte must flow in the direction indicated in Figure 26.16 to maintain an even distribution of charge. What must be the direction of the electric field in the bulk of the electrolyte to cause this flow?

Figure 26.17 illustrates the operation of a lead-acid cell used in automobile batteries. A 12-V automobile battery consists of six such cells, each producing a potential difference of 2.1 V. The negative electrode of a lead-acid cell is composed of spongy lead (Pb) packed on a metal grid; the positive electrode contains lead dioxide (PbO_2) packed on a metal grid. The electrodes are immersed in sulfuric acid and chemical reactions convert the lead, the lead dioxide, and the sulfuric acid into lead sulfate ($PbSO_4$) and water. For every molecule of lead sulfate that is produced in these reactions, one electron is removed from the positive terminal and one is added to the negative terminal. The left and right sides of Figure 26.17 show energy diagrams for the species undergoing chemical reactions at each of the electrodes. For each reaction, the chemical energy of the species involved in the reaction decreases; this energy is used to do work on the electrons in the electrodes.

26.13 Given that the cell does *positive* work on the electrons, why is it that the work in both energy diagrams in Figure 26.17 is *negative*?

**emf* stands for *electromotive force*, a misnomer because this quantity bears no relation to the concept of force. For this reason we shall always refer to this quantity by its abbreviation, rather than its original meaning.

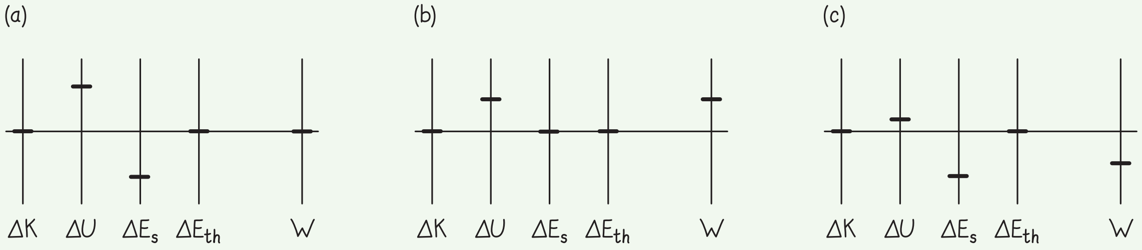
Self-quiz

1. Consider again Figure 26.2 and imagine moving one more electron from the fur to the rod. (*a*) Is the work that must be done on the rod-fur system to accomplish this transfer positive, negative, or zero? (*b*) Is the electrostatic work positive, negative, or zero? (*c*) Does the electric potential energy of the rod-fur system increase, decrease, or remain the same?
2. You have probably seen pictures in which a person's hair stands out from his or her head because of "electrostatic charge." Look back at the discussion of Van de Graaff generators and discuss how this can happen when a person makes contact with the globe of the generator but is insulated from the ground.
3. A parallel-plate capacitor is connected to a battery. If the distance between the plates is decreased, do the magnitudes of the following quantities increase, decrease, or stay the same: (*i*) the potential difference between the negative plate and the positive plate, (*ii*) the electric field between the plates, and (*iii*) the charge on the plates?
4. When a dielectric is inserted between the plates of an isolated charged capacitor, do the magnitudes of the following quantities increase, decrease, or stay the same: (*i*) the charge on the plates, (*ii*) the electric field between the plates, and (*iii*) the potential difference between the negative plate and the positive plate?
5. Draw an energy diagram for the process of charging a capacitor with a dielectric as shown in Figure 26.15*b* for the following systems: (*a*) battery, capacitor, and dielectric; (*b*) dielectric only; (*c*) battery and capacitor. Ignore any dissipation of energy.

ANSWERS:

1. (*a*) To displace the electron toward the rod, you must apply a force directed toward the rod. Because the force and force displacement are in the same direction, you (an external agent) must do positive work. (*b*) The electric force exerted on the electron is directed toward the fur, opposite the direction of the force displacement, so the electrostatic work is negative. (*c*) The electric potential energy of the system increases because separating charge carriers increases a system's electric potential energy.
2. If a person is in contact with the globe of the generator but insulated from the ground, then the person acts as an extension of the globe. Electrical charge spreads out over the surface of the person, including the surface of each hair as well. Because each hair has a surplus of the same type of charge, the hairs repel each other and stand out, getting as far away from each other as possible.
3. (*i*) Stays the same. The battery keeps the potential difference across the capacitor constant. (*ii*) To keep a constant potential difference when the distance between the plates decreases, the magnitude of the electric field between the plates must increase because $Ed = V_{batt}$ (see Example 25.5). (*iii*) For the magnitude of the electric field to increase, the charge on the plates must increase.
4. (*i*) Because the capacitor is isolated, the charge on the plates must remain the same—there is no path for the charge to travel elsewhere. (*ii*) When the dielectric is inserted, the electric field due to the bound surface charge is in the opposite direction of the electric field due to the free charge on the plates and decreases the magnitude of the electric field between the plates. (*iii*) Because the magnitude of the electric field decreases and the separation between the plates is constant, the magnitude of the potential difference between the negative plate and the positive plate must also decrease.
5. See **Figure 26.18**. (*a*) During charging, a decrease in source energy (from the battery) increases the electric potential energy (more charge separation in the dielectric and on the capacitor plates). (*b*) The electric potential energy of the dielectric increases due to work done on it by the battery and the capacitor. The electric potential energy stored in the dielectric is smaller than that stored in part *a* because some electric potential energy is stored on the capacitor plates. (*c*) The decrease in source energy is the same as in part *a*. The electric potential energy stored on the capacitor is smaller than in part *a* because most of the converted source energy ends up in the dielectric, which is not part of the system considered. This energy leaves the system as negative work.

Figure 26.18



SELF-QUIZ

26.5 Capacitance

Figure 26.19 shows three capacitors, each one consisting of a pair of conducting objects carrying opposite charges of magnitude q. For each arrangement, the potential difference between the objects is proportional to q; that is, doubling q doubles the potential difference across the capacitor. The ratio of the magnitude of the charge on one of the objects to the magnitude of the potential difference across them is defined as the **capacitance** of the arrangement:

$$C \equiv \frac{q}{V_{\text{cap}}}. \tag{26.1}$$

In Eq. 26.1, q represents *the magnitude of the charge on each conducting object* and V_{cap} is *the magnitude of the potential difference between the conducting objects*. Because both these quantities are positive, C is always positive.

The value of C depends on the size, shape, and separation of the conductors. In Figure 26.19, for example, the values of V would typically be different for the three capacitors, even though q is the same for each. Below we'll examine how to determine C for a given set of conductors.

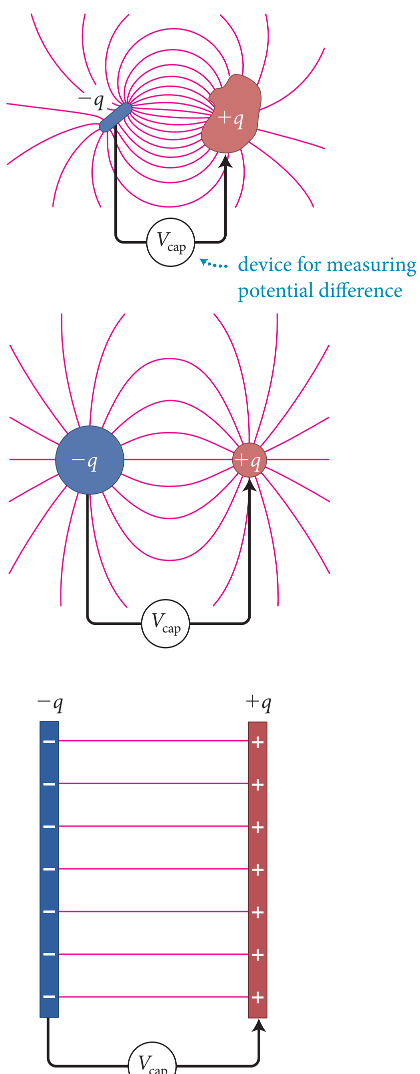


Figure 26.19 The electric fields and potential differences of three different capacitors.

26.14 Two capacitors, A and B, are each connected to a 9-V battery. If $C_A > C_B$, which capacitor stores the greater amount of charge?

The answer to Checkpoint 26.14 suggests a simple interpretation of C. As its name suggests, C represents the capacitor's *capacity to store charge*: The greater C, the greater the amount of charge stored for a given value of V_{cap}.

As you can see from Eq. 26.1, capacitance has SI units of coulomb per volt. This derived unit is given the name **farad** (F), in honor of the English physicist Michael Faraday:

$$1\ \text{F} \equiv 1\ \text{C/V}.$$

As you will see in Checkpoint 26.15, a capacitance of 1 F is enormous. The capacitance of capacitors commonly found in electronic devices is expressed in microfarads ($1\ \mu\text{F} = 1 \times 10^{-6}$ F) and picofarads ($1\ \text{pF} = 1 \times 10^{-12}$ F).

Figure 26.19 suggests a simple procedure for determining the capacitance of a given set of conductors: Determine the potential difference V_{cap} between the two conductors when they carry some given charge q, and use Eq. 26.1 to calculate C. Note that because conductors are equipotentials, V_{cap} represents the potential difference between *any* two points on the conductors measured along *any* path. The Procedure box below gives one procedure for determining the capacitance of a given set of conductors. In the next examples we apply this procedure to some simple configurations of conductors.

Procedure: Calculating the capacitance of a pair of conductors

To calculate the capacitance of a pair of conductors:

1. Let the conductors carry opposite charges of magnitude q.
2. Use Gauss's law, Coulomb's law, or direct integration to determine the electric field along a path leading from the negatively charged conductor to the positively charged conductor.
3. Calculate the electrostatic work W done on a test particle carrying a charge q_t along this path (Eq. 25.24) and determine the potential difference across the capacitor from Eq. 25.15:

$$V_{\text{cap}} = -W_{q_t}(- \to +)/q_t.$$

4. Use Eq. 26.1, $C \equiv q/V_{\text{cap}}$, to determine C.

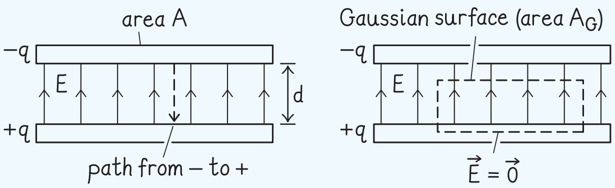
Example 26.2 Parallel-plate capacitor

What is the capacitance of a parallel-plate capacitor that has a plate area A and a plate separation distance d?

❶ **GETTING STARTED** I begin by making a sketch of the capacitor, showing the electric field between the plates (**Figure 26.20*a***). The problem doesn't specify the plate shape, so I simply show the capacitor from the side, representing each plate by a horizontal line. If I assume that the separation distance d is small, then the electric field is uniform and confined between the plates.

Figure 26.20

(a) (b)



❷ **DEVISE PLAN** I can use the steps of the Procedure box on page 697 to determine the capacitance. The first step is to determine the electric field between the capacitor plates when they carry opposite charges of magnitude q. The second step is to obtain the electrostatic work done on a test particle moved from one plate to the other; I can use Eq. 25.24 to obtain this work. Because the field is uniform, it is most convenient to choose a path along a field line for the path over which the electrostatic work is done. As specified in the Procedure box, the path runs from the negatively charged plate to the positively charged plate. Once I know the electrostatic work, I know the potential difference across the capacitor and so can calculate the capacitance.

❸ **EXECUTE PLAN** Because of the planar symmetry, I can use Gauss's law to determine the electric field. I choose a cylindrical Gaussian surface straddling the surface of the positively charged plate. The cylinder height is less than d, and the area of the end surfaces is A_G (Figure 26.20*b*). The electric flux through this Gaussian surface is zero everywhere except through the top surface, where $\Phi = EA_G$. (The bottom surface is inside the conducting metal plate, where the electric field is zero.)

To apply Gauss's law, I also need to know the charge enclosed by the Gaussian surface. The positive plate carries a charge $+q$ distributed over a surface of area A, so the surface charge density is $\sigma = +q/A$ and the charge enclosed by the Gaussian surface is $q_{enc} = \sigma A_G = (q/A)A_G$. Applying Gauss's law, $\Phi = q_{enc}/\epsilon_0$ (Eq. 24.8), I get

$$EA_G = \frac{q}{\epsilon_0 A}A_G \quad \text{or} \quad E = \frac{q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0} \tag{1}$$

in agreement with Eq. 24.17.

Now that I know E, I can calculate the electrostatic work required to move a test particle carrying a charge $+q_t$ from the negatively charged to the positively charged plate. The electric force exerted on the test particle is upward in Figure 26.20, and the force displacement is downward because the particle moves from negative plate to positive plate. Because these two vectors point in opposite directions, the electrostatic work done on the test particle is negative, $W_{q_t} = -q_t Ed$, and the potential difference across the capacitor is, from Eq. 25.15,

$$V_{cap} \equiv \frac{-W_{q_t}(- \to +)}{q_t} = \frac{q_t Ed}{q_t} = Ed$$

or, substituting E from Eq. 1,

$$V_{cap} = \frac{qd}{\epsilon_0 A}.$$

Note that the potential difference is proportional to the magnitude of the charge on each plate, q. Using the definition of capacitance, I obtain

$$C \equiv \frac{q}{V_{cap}} = \frac{q}{qd/(\epsilon_0 A)} = \frac{\epsilon_0 A}{d}. \checkmark$$

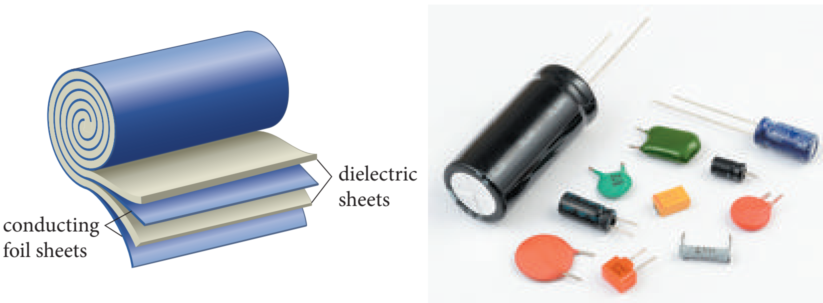
❹ **EVALUATE RESULT** My result agrees with the conclusions we drew in Section 26.2: The capacitance (or quantity of charge stored for a given potential difference) increases with increasing plate area A and decreasing plate separation distance d. Also, I note that the electric field—and therefore the capacitance—do not depend on the plate: Circular or square plates give the same result.

26.15 The plate spacing in a typical parallel-plate capacitor is about 50 μm. (*a*) What is the plate area in a 1-μF capacitor? (*b*) Given that the electric field at which electrical breakdown occurs in air is about 3×10^6 V/m, what is the maximum charge that this capacitor can hold? (*c*) How many electrons does this charge correspond to? (The electron's charge is $e = 1.6 \times 10^{-19}$ C.)

Figure 26.21

(*a*) One way to design a compact capacitor with a large surface area

(*b*) Some capacitors used in electronic circuits

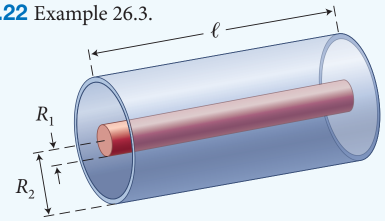


As Checkpoint 26.15 shows, even modest capacitances require very large plate areas. Various techniques are used to keep the overall size of capacitors small, one of which involves rolling up two thin conducting sheets that are separated by thin sheets of a dielectric material (**Figure 26.21*a***). Figure 26.21*b* shows a number of different capacitors used in electronic circuits.

Example 26.3 Coaxial cylindrical capacitor

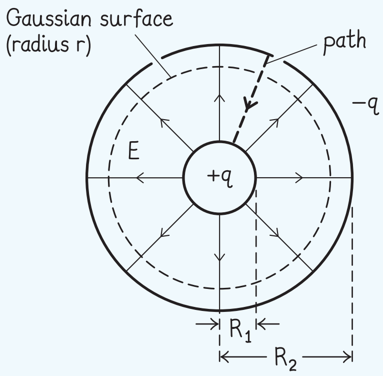
Figure 26.22 shows a *coaxial capacitor* consisting of two concentric metal cylinders 1 and 2, of radii R_1 and $R_2 > R_1$, and both of length $\ell \gg R_2$. Both cylinders are made of metal. What is the capacitance of this arrangement?

Figure 26.22 Example 26.3.



❶ **GETTING STARTED** To determine the capacitance, I must let the two cylinders carry opposite charges of magnitude q, which I assume to be uniformly distributed over each cylinder. If I let cylinder 1 carry a charge $+q$ and cylinder 2 carry a charge $-q$, the electric field points radially outward from cylinder 1 to cylinder 2 (**Figure 26.23**). Because the cylinders are very long relative to their separation distance $R_2 - R_1$, I assume that the electric field is confined to the volume between the cylinders.

Figure 26.23



❷ **DEVISE PLAN** Again I refer to the Procedure box on page 697 to calculate the capacitance. For the path over which electrostatic work is done, I choose a straight path that goes radially from cylinder 2 to cylinder 1.

❸ **EXECUTE PLAN** Because of the cylindrical symmetry, I choose a cylindrical Gaussian surface (Figure 26.23). The length of the Gaussian surface is ℓ_G, and its radius is r ($R_2 > r > R_1$). The electric flux Φ through the curved portion of the Gaussian surface is equal to the product of the electric field strength E_r at a distance r from the common axis of cylinders 1 and 2 and the surface area of the Gaussian surface $A_G = (2\pi r)\ell_G$. Therefore $\Phi = 2\pi r \ell_G E_r$. Because the linear charge density on cylinder 1 is $+q/\ell$, the quantity of charge enclosed by the Gaussian surface is given by the product of the linear charge density and the length of the Gaussian surface: $q_{enc} = +(q/\ell)\ell_G$. Applying Gauss's law, I get

$$2\pi r \ell_G E_r = \frac{q\ell_G}{\epsilon_0 \ell},$$

or $E_r = q/(2\pi\epsilon_0 \ell r)$, in agreement with the result we obtained in Exercise 24.7, $E = 2k\lambda/r$, because $k = 1/(4\pi\epsilon_0)$ and $q/\ell = \lambda$.

Now that I know E_r, I can calculate the electrostatic work required to move a test particle carrying a charge q_t from cylinder 2 to cylinder 1. Integrating the electric force exerted on the test particle over the force displacement from cylinder 2 (negatively charged) to cylinder 1 (positively charged), I get

$$W_{q_t} = \int_{R_2}^{R_1} \frac{qq_t}{2\pi\epsilon_0 \ell r}\,dr.$$

Working out the integral, I obtain

$$W_{q_t} = \frac{qq_t}{2\pi\epsilon_0 \ell}\Big[\ln r\Big]_{R_2}^{R_1} = \frac{qq_t}{2\pi\epsilon_0 \ell}\ln\left(\frac{R_1}{R_2}\right).$$

The potential difference between the negative cylinder 2 and the positive cylinder 1 is thus

$$V_{cap} \equiv \frac{-W_{q_t}}{q_t} = -\frac{q}{2\pi\epsilon_0 \ell}\ln\left(\frac{R_1}{R_2}\right) = \frac{q}{2\pi\epsilon_0 \ell}\ln\left(\frac{R_2}{R_1}\right).$$

Because $R_2 > R_1$, the logarithm is positive and therefore V_{cap} is positive, as it should be because I am bringing a quantity q_t of positive charge from a location of low potential on negatively charged cylinder 2 to a location of high potential on positively charged cylinder 1.

According to Eq. 26.1, the capacitance of the coaxial capacitor is thus

$$C \equiv \frac{q}{V_{cap}} = \frac{2\pi\epsilon_0 \ell}{\ln(R_2/R_1)}. ✔$$

❹ **EVALUATE RESULT** My result shows that the capacitance is proportional to ℓ, which makes sense: The longer the coaxial cylinders, the greater the quantity of charge that can be stored on them. Decreasing R_1 or increasing R_2 is equivalent to increasing the plate separation distance d in a parallel-plate capacitor, which decreases the capacitance. Indeed, my result shows a decreasing capacitance for decreasing R_1 or increasing R_2. (The dependence on R_1 and R_2 is a bit more complicated than the dependence on d in a parallel-plate capacitor because the electric field in the coaxial capacitor is nonuniform and because changing the radii of the cylinders affects their surface areas.)

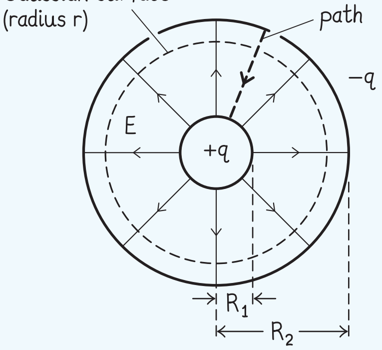
26.16 Coaxial cables used for cable television typically have a central metallic core of 0.20-mm radius, surrounded by a cylindrical metallic sheath of 2.0-mm radius. The two are separated by a plastic spacer. If the effect of the spacer can be ignored (that is, assuming the two conductors are separated by air), what is the capacitance of a 100-m-long cable?

Example 26.4 Spherical capacitor

What is the capacitance of a spherical capacitor consisting of two concentric conducting spherical shells of radii R_1 and $R_2 > R_1$?

❶ **GETTING STARTED** If I let the inner sphere carry a positive charge $+q$ and the outer one a negative charge $-q$, my sketch of the capacitor looks identical to the sketch of the coaxial capacitor of Example 26.3 (**Figure 26.24**). The calculation, however, will not be the same because now the electric field has spherical, not cylindrical, symmetry.

Figure 26.24



❷ **DEVISE PLAN** For the path over which electrostatic work is done, I choose a straight path that goes radially from the outer sphere to the inner sphere. The outer sphere does not contribute to the electric field between the spheres because the field inside a hollow conductor is always zero (see Section 24.5). The electric field created by the inner sphere is the same as that created by a charged particle (Eq. 24.15, $E = kq/r^2$), so I can use this expression to follow steps 3 and 4 in the Procedure box on page 697.

❸ **EXECUTE PLAN** The electrostatic work done in moving a test particle carrying a charge q_t from the outer sphere to the inner sphere is

$$W_{q_t}(- \to +) = \int_{R_2}^{R_1} \frac{qq_t}{4\pi\epsilon_0 r^2}\, dr.$$

Working out the integral, I get

$$W_{q_t}(- \to +) = -\frac{qq_t}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_{R_2}^{R_1} = -\frac{qq_t}{4\pi\epsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right].$$

The potential difference between the outer and inner spheres is thus

$$V_{cap} \equiv \frac{-W_{q_t}(- \to +)}{q_t} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right],$$

so the capacitance is

$$C \equiv \frac{q}{V_{cap}} = 4\pi\epsilon_0\left[\frac{1}{R_1} - \frac{1}{R_2}\right]^{-1} = 4\pi\epsilon_0\left[\frac{R_2 - R_1}{R_1 R_2}\right]^{-1}$$

$$= 4\pi\epsilon_0\left[\frac{R_1 R_2}{R_2 - R_1}\right]. ✔$$

❹ **EVALUATE RESULT** I expect the capacitance to go up as the separation distance $R_2 - R_1$ between the spheres decreases, and this is just what my result shows. If I increase the spheres' radii while keeping their separation distance $R_2 - R_1$ fixed, the surface area $A = 4\pi R^2$ of each sphere increases and the capacitance should increase, in agreement with my result.

26.17 (*a*) To calculate the "capacitance" of an isolated sphere, evaluate the result we obtained in Example 26.4 in the limit that R_2 goes to infinity. (*b*) What is the capacitance of the spherical metal dome of a Van de Graaff generator like the one shown in the chapter-opening photo, which has a radius of about 2.5 m. (*c*) Given that air breaks down in an electric field with a magnitude of about 3.0×10^6 V/m, what is the maximum amount of charge that can be stored on the dome before the air breaks down?

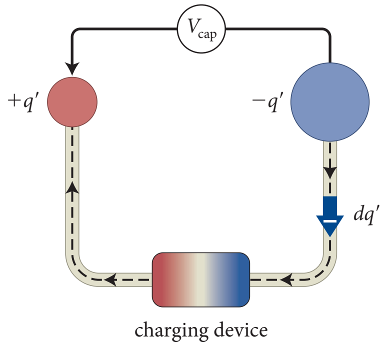


Figure 26.25 To determine the electric potential energy stored in a capacitor, we calculate the energy required to transfer charge from one conductor to the other.

26.6 Electric field energy and emf

How much electric potential energy is stored in a charged capacitor? To answer this question, consider a simple capacitor consisting of two conducting objects. In order to charge the capacitor, some charge-separating device must transfer charge from one conductor to the other (**Figure 26.25**). During this transfer, the charge-separating device does work on the capacitor and this energy ends up as electric potential energy "stored in the capacitor."* One complication in the calculation of the work done by the charge-separating device is that as the magnitude of the charge on each conductor increases, the potential difference increases too, so the work required to transfer a unit of charge increases.

*We are assuming that there is no dissipation of energy, so that all the work done on the system ends up as electric potential energy. In practice this is a reasonable assumption.

Let us therefore break down the transfer of charge from one conductor to the other into small increments of charge dq', so that the potential difference is essentially constant during the transfer of a single increment. Consider some instant during the charging when the magnitude of the charge on each conductor is q'. The potential difference between the negative and positive conductors is then given by Eq. 26.1: $V_{cap} = q'/C$. As an additional increment of charge dq' is moved from the negative to the positive conductor, the electrostatic work done on it is $dW = -dq' V_{cap}$ (Eq. 25.17). Because the charge-separating device must do work on charge carriers against the electric force, the work done by the charge-separating device on the charge carriers is the negative of the electrostatic work, so the change in electric potential energy of the capacitor during the transfer is

$$dU^E = -dW = V_{cap}\,dq' = \frac{q'}{C}\,dq'. \tag{26.2}$$

When the magnitude of the charge on each conductor has increased from zero to its final value q, the electric potential energy stored in the capacitor is

$$U^E = \int dU^E = \int_0^q \frac{q'}{C}\,dq' = \frac{1}{C}\int_0^q q'\,dq' = \frac{1}{2}\frac{q^2}{C}. \tag{26.3}$$

Often it is more convenient to express the electric potential energy not in terms of the magnitude of the charge q on the capacitor, but in terms of the potential difference across it:

$$U^E = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV_{cap}^2 = \frac{1}{2}qV_{cap}. \tag{26.4}$$

Note that Eqs. 26.3 and 26.4 hold for any type of capacitor, regardless of the configuration of the conductors. All that enters into these expressions besides the charge or the potential difference is the capacitance, which depends on the size, shape, and the separation of the conductors.

26.18 A 1.0-μF parallel-plate capacitor with a plate spacing of 50 μm is charged up to the breakdown threshold. (*a*) If the electric field in the air between the capacitor plates is 3.0×10^6 V/m, how much energy is stored in the capacitor? Express your answer in joules. (*b*) How high must you raise this book ($m \approx 2$ kg) to increase the gravitational potential energy of the Earth-book system by the same amount?

As we discussed in Section 26.1, we can imagine electric potential energy to be stored either in the configuration of charge in the capacitor or in the electric field. We can use Eq. 26.4 and our knowledge about the electric field in a capacitor to relate electric potential energy to the electric field. From Example 25.5 we know that the magnitude of the potential difference between the plates of a parallel-plate capacitor is given by Ed, so, using the expression for C in Example 26.2 and Eq. 26.4, we can write for the electric potential energy stored in a parallel-plate capacitor

$$U^E = \frac{1}{2}CV_{cap}^2 = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\epsilon_0 E^2 (Ad). \tag{26.5}$$

The term in parentheses on the right side, Ad, is equal to the volume of the space between the capacitor plates—that is, the region to which the electric field is confined. Therefore, the energy per unit volume stored in the electric field—the **energy density** of the electric field—is

$$u_E \equiv \frac{U^E}{\text{volume}} = \tfrac{1}{2}\epsilon_0 E^2. \tag{26.6}$$

Although we derived this expression for the special case of a parallel-plate capacitor, it holds true for any electric field in vacuum. Any given region of space where a uniform electric field is present can be viewed as containing an amount of electric potential energy equal to $\frac{1}{2}\epsilon_0$ times the square of the magnitude of the electric field in that region times the volume. If the electric field is nonuniform, we must subdivide the volume of interest into small enough segments that E can be considered uniform within each segment, then apply Eq. 26.6 to each segment and take the sum of all the contributions. (This corresponds to integrating the energy density over the volume of the region that contains the electric field.)

26.19 A parallel-plate capacitor has plates of area A separated by a distance d. The magnitude of the charge on each plate is q. (*a*) Determine the magnitude of the force exerted by the positively charged plate on the negatively charged one. (*b*) Suppose you increase the separation between the plates by an amount Δx. How much work do you need to do on the capacitor to achieve this increase? (*c*) What is the change in the electric potential energy of the capacitor? (*d*) Moving the plate adds additional space with electric field between the plates. Show that the energy stored in the electric field in this additional space is equal to the work done on the capacitor.

The energy stored in a capacitor is supplied to it by the charging device—such as a generator, a battery, or a solar cell. Inside this device, nonelectrostatic interactions cause a separation of charge by doing work on charged particles. The work per unit charge done by the nonelectrostatic interactions on the charge carriers inside the device is called the emf and is denoted by $\mathscr{E}$:

$$\mathscr{E} \equiv \frac{W_{\text{nonelectrostatic}}}{q}. \tag{26.7}$$

The SI unit of emf is the same as that of potential: the volt. The rating of a battery—1.5 V or 9 V—gives its emf.*

If no energy is dissipated inside the charging device, *all* of the energy can be transferred to charge carriers outside the device. This transfer takes place through electric interactions. In Figure 26.25, for example, electric forces remove electrons from one object and push them onto the other, charging the capacitor. In the absence of any energy dissipation, the nonelectrostatic work done on charge carriers inside the device is equal to the electrostatic work done on charge carriers outside it. Because the electrostatic work per unit charge is the potential difference between the negative and positive terminals of the charging device, we have, for an ideal charging device,

$$V_{\text{device}} = \mathscr{E} \quad \text{(ideal device)}. \tag{26.8}$$

In practice, some energy is always dissipated inside the device, so not all of the nonelectrostatic work done on charge carriers inside the device can be turned into electrostatic work. Consequently, for most devices, $V_{\text{device}} < \mathscr{E}$.

*The term *voltage* is sometimes used to refer to a potential difference or to an emf (such as the rating of a battery). Potential difference, however, is related to *electrostatic* work done on charge carriers, whereas emf deals with *nonelectrostatic* work done on them. Thus, electrostatic work brings opposite charge carriers together, while nonelectrostatic work causes charge separation. To maintain this important distinction, we shall avoid the term *voltage*.

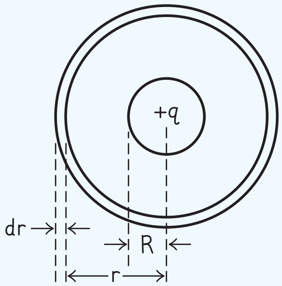
Example 26.5 Van de Graaff energy

The radius of the dome on the Van de Graaff generator shown on the opening page of this chapter is about 2.5 m, and air breaks down when the field magnitude is about 3.0×10^6 V/m. How much electric potential energy is stored in the electric field surrounding the dome just before the air there breaks down?

❶ **GETTING STARTED** I am given the radius of a Van de Graaff dome and asked to calculate how much potential energy is in the electric field surrounding the dome just before the field causes the air to break down. If I approximate the dome as a uniformly charged spherical shell, then the electric field surrounding it is the same as that surrounding a particle carrying the same charge [Eq. 24.15, $E = q/(4\pi\epsilon_0 r^2)$]. The magnitude of the field around the dome is greatest at the dome surface, so I take this E_{surf} value as the maximum value just before the air breaks down.

❷ **DEVISE PLAN** Equation 26.6 gives me the energy density of the electric field around the dome. I can substitute the Eq. 24.15 expression for the electric field of a sphere carrying a charge q into Eq. 26.6 to obtain an expression for the energy density of the electric field at an arbitrary distance r from the dome center. Because the electric field has the same magnitude at any location a distance r from the dome center, I can divide the space outside the dome into a series of thin spherical shells, each of thickness dr and all concentric with the dome (**Figure 26.26**), and then integrate over all shells from $r = R$ to $r = \infty$ to obtain the energy stored in the electric field surrounding the dome in terms of the charge q. I can then use Eq. 24.15 to eliminate q from my result and express the energy stored in terms of E_{surf}, which is given.

Figure 26.26



❸ **EXECUTE PLAN** Substituting Eq. 24.15 into Eq. 26.6, I get

$$u_E = \tfrac{1}{2}\epsilon_0 E^2 = \tfrac{1}{2}\epsilon_0\left(\frac{q}{4\pi\epsilon_0 r^2}\right)^2.$$

The volume of a thin spherical shell of radius r and thickness dr centered on the dome is equal to the surface area of a sphere of radius r times the shell thickness: $(4\pi r^2)dr$. The energy in that volume is thus $dU^E = u_E(4\pi r^2)dr$, and so the electric potential energy in the space around the dome is

$$U^E = \int_R^\infty dU^E = \int_R^\infty u_E(4\pi r^2)\,dr = \tfrac{1}{2}\epsilon_0\int_R^\infty\left(\frac{q}{4\pi\epsilon_0 r^2}\right)^2(4\pi r^2)dr$$

$$= \frac{q^2}{8\pi\epsilon_0}\int_R^\infty \frac{1}{r^2}\,dr = \frac{q^2}{8\pi\epsilon_0}\left[\frac{-1}{r}\right]_R^\infty = \frac{q^2}{8\pi\epsilon_0 R}. \qquad (1)$$

Now I have an expression for U^E, the quantity I must determine, but I have no value for q. Given that the electric field at the dome surface is given by Eq. 24.15, $E_{\text{surf}} = q/(4\pi\epsilon_0 R^2)$, I can rearrange this expression to $q = E_{\text{surf}}(4\pi\epsilon_0 R^2)$ and rewrite Eq. 1 as $U^E = 2\pi\epsilon_0 E_{\text{surf}}^2 R^3$. Because E_{surf} is the maximum electric field magnitude around the dome, I know that this magnitude must be the breakdown value for air. Substituting the values given, I get

$$U^E = 2\pi(8.85 \times 10^{-12}\,\text{C}^2/(\text{N}\cdot\text{m}^2))(3.0 \times 10^6\,\text{V/m})^2(2.5\,\text{m})^3$$

$$= 7.8\,\text{kJ}. ✔$$

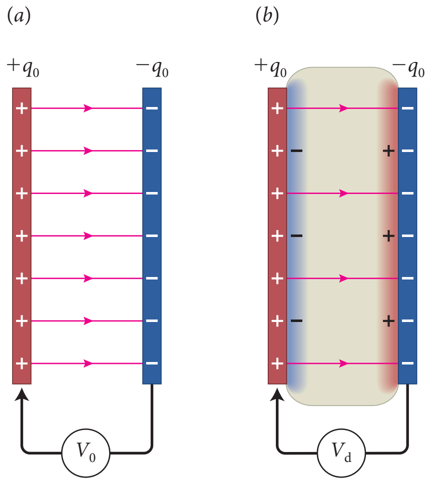
❹ **EVALUATE RESULT** As a check on my work, I can calculate the electric potential energy of the charge stored on the dome using Eq. 26.4. In Checkpoint 26.17, you found that the capacitance of an isolated sphere is $C_{\text{sphere}} = 4\pi\epsilon_0 R$ and that the potential of a sphere is related to the electric field at its surface by $V_{\text{cap}} = ER$. Therefore $U^E = \tfrac{1}{2}CV_{\text{cap}}^2 = 2\pi\epsilon_0 R^3 E_{\text{surf}}^2$, which is the same result I obtained.

26.20 The flash unit on a typical camera uses a 100-μF capacitor to store electric potential energy. The capacitor is charged to a potential of 300 V. When the flash is fired, the energy in the capacitor is released to a bulb in a burst of about 1.0-ms duration. (*a*) How much energy is stored in the fully charged capacitor before it is fired? (*b*) What is the average power of the flash firing?

26.7 Dielectric constant

As we saw in Section 26.3, the capacitance of a capacitor can be increased by inserting a dielectric between the two conductors. For example, inserting a slab of mica between the plates of an isolated charged capacitor (**Figure 26.27**) decreases the potential difference across the capacitor by a factor of 5. This tells us that the mica reduces the electric field inside the isolated capacitor by a factor of 5. By definition, the magnitude of the potential difference V_0 across the isolated

Figure 26.27 The potential difference across an isolated parallel-plate capacitor is greater (*a*) without a dielectric between the plates than it is (*b*) with a dielectric.



capacitor without a dielectric divided by the magnitude of the potential difference V_d with the dielectric is called the **dielectric constant** κ:

$$\kappa \equiv \frac{V_0}{V_d}. \tag{26.9}$$

Given that the magnitude q_0 of the charge on each plate of the isolated capacitor is not affected by the dielectric, we see from Eqs. 26.9 and 26.1 that

$$\kappa \equiv \frac{V_0}{V_d} = \frac{V_0/q_0}{V_d/q_0} = \frac{1/C_0}{1/C_d} = \frac{C_d}{C_0}, \tag{26.10}$$

where C_d is the capacitance of the capacitor with the dielectric and C_0 that without a dielectric. Therefore, the capacitance changes by the factor κ when a dielectric is inserted:

$$C_d = \kappa C_0. \tag{26.11}$$

The dielectric constant is always greater than 1 ($\kappa > 1$) because the presence of a dielectric decreases the electric field inside the capacitor. The greater the polarization of the dielectric material, the more reduced the electric field inside the dielectric and the greater the dielectric constant κ. Table 26.1 gives the dielectric constants for several commonly used dielectric materials. The dielectric constant for vacuum—that is, no material between the plates—is unity by definition. Because air is very dilute, the dielectric constant of air is nearly unity as well. If the dielectric is composed of polar molecules that can align themselves (such as the water molecules in liquid water), then the overall polarization is much greater than in nonpolar dielectrics, so the dielectric constant is large. For some polar materials, the dielectric constant can be in the thousands.

The electric field $\vec{E}$ inside the dielectric is the superposition of the electric field due to the free charge on the plates, $\vec{E}_{free}$, and the electric field due to the bound surface charge on the dielectric, $\vec{E}_{bound}$: $\vec{E} = \vec{E}_{free} + \vec{E}_{bound}$ (Figure 26.28*a*). We designate the magnitude of the free charge on the capacitor plates by q_{free} and the magnitude of the bound charge on the surfaces of the dielectric by q_{bound} (Figure 26.28*b* and *c*). With this notation, both q_{free} and q_{bound} are always positive. Using the expression for the electric field of a sheet of charge, we can thus write for the magnitude of the electric field inside the capacitor in the absence of a dielectric

$$E_{free} = \frac{\sigma_{free}}{\epsilon_0} = \frac{q_{free}}{\epsilon_0 A}, \tag{26.12}$$

Figure 26.28 (*a*) The electric field inside a dielectric-filled capacitor is the vector sum of the electric field due to the charged plates and that due to the polarized dielectric. (*b*) and (*c*) Bound and free charge on a vacuum-filled and a dielectric-filled isolated parallel-plate capacitor.

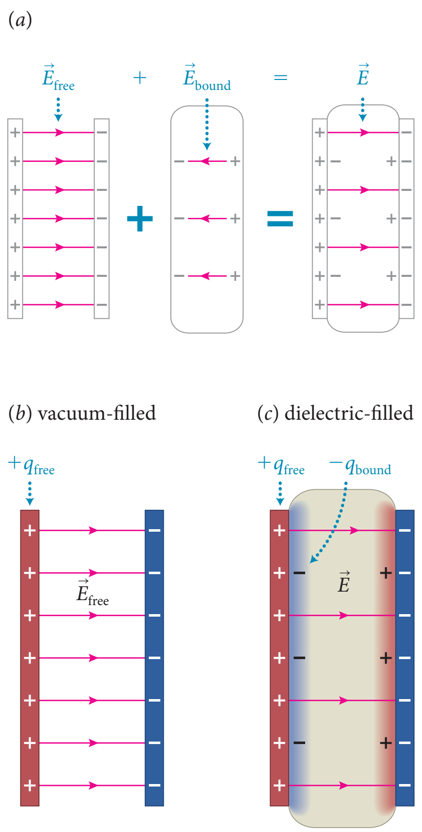


Table 26.1 Dielectric properties

Material	Dielectric constant κ	Breakdown threshold E_{max} (V/m)
Air (1 atm)	1.00059	3.0×10^6
Paper	1.5–3	4.0×10^7
Mylar (polyester)	3.3	4.3×10^8
Quartz	4.3	8×10^6
Mica	5	2×10^8
Oil	2.2–2.7	
Porcelain	6–8	
Water (distilled, 20 °C)	80.2	$6.5–7 \times 10^7$
Titania ceramic	126	8×10^6
Strontium titanate	322	
Barium titanate	1200	8×10^7

where $\sigma_{\text{free}} = q_{\text{free}}/A$ is the magnitude of the free surface charge density and A is the area of either capacitor plate. Likewise, the magnitude of the electric field due to the bound surface charge is

$$E_{\text{bound}} = \frac{\sigma_{\text{bound}}}{\epsilon_0} = \frac{q_{\text{bound}}}{\epsilon_0 A}, \tag{26.13}$$

where σ_{bound} is the magnitude of the bound surface charge density. Because $\vec{E}_{\text{free}}$ and $\vec{E}_{\text{bound}}$ point in opposite directions, the magnitude of the electric field $\vec{E}$ inside the dielectric is then

$$E = E_{\text{free}} - E_{\text{bound}} = \frac{\sigma_{\text{free}} - \sigma_{\text{bound}}}{\epsilon_0} = \frac{q_{\text{free}} - q_{\text{bound}}}{\epsilon_0 A}. \tag{26.14}$$

Let us determine the magnitude of the bound charge q_{bound}. If the plate separation is d, we can write Eq. 26.10 in the form

$$\kappa \equiv \frac{V_0}{V_{\text{d}}} = \frac{E_{\text{free}}\, d}{E\, d} = \frac{E_{\text{free}}}{E}, \tag{26.15}$$

where E_{free} is the magnitude of the electric field due to the free charge only, and E is the magnitude of the electric field inside the dielectric. In other words, the dielectric reduces the electric field by the factor κ: $E = E_{\text{free}}/\kappa$. Substituting Eqs. 26.12 and 26.14 into this expression, we get

$$\frac{q_{\text{free}}}{\kappa\epsilon_0 A} = \frac{q_{\text{free}} - q_{\text{bound}}}{\epsilon_0 A} \tag{26.16}$$

or

$$\frac{q_{\text{free}}}{\kappa} = q_{\text{free}} - q_{\text{bound}}. \tag{26.17}$$

Solving this expression for q_{bound}:

$$q_{\text{bound}} = \frac{\kappa - 1}{\kappa} q_{\text{free}}. \tag{26.18}$$

Because κ is always greater than 1, we see that the magnitude of the bound surface charge is always smaller than the magnitude of the free charge that causes it.

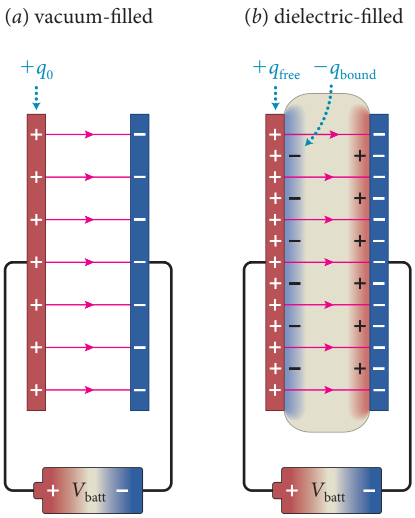
Next we consider the situation of a capacitor connected to a battery (**Figure 26.29**). In this situation, the potential difference across the capacitor is constant, but the charge on the plates changes when the dielectric is inserted. As we have seen in Section 26.3, the electric field inside the dielectric must be the same as before the dielectric was inserted. In other words, the sum of the free and bound charges must still be equal to q_0; that is, $q_0 = q_{\text{free}} - q_{\text{bound}}$ (Figure 26.29). Because the definition of capacitance involves only the charge on the capacitor plates, we can write

$$C_{\text{d}} \equiv \frac{q_{\text{free}}}{V_{\text{d}}} \tag{26.19}$$

and likewise

$$C_0 \equiv \frac{q_0}{V_0}. \tag{26.20}$$

Figure 26.29 Bound and free charge on a vacuum-filled and a dielectric-filled parallel-plate capacitor connected to a battery.



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Note the difference between q_{free} and q_0. Even though both represent free charge, they are not equal because when the dielectric is inserted, the battery increases the charge on the plate in order to maintain a constant potential difference across the capacitor, so $q_{free} > q_0$. Because $C_d = \kappa C_0$ (Eq. 26.11), we see from Eqs. 26.19 and 26.20 that the dielectric increases the charge on the capacitor plates by the factor κ:

$$q_{free} = \kappa q_0. \tag{26.21}$$

26.21 (*a*) In Figure 26.29, what is the magnitude of q_{bound}? Express your answer in terms of q_0 and the properties of the dielectric. (*b*) What is the bound surface charge density on the dielectric? Express your answer in terms of the electric field E.

Example 26.6 Capacitor with dielectric

A parallel-plate capacitor consists of two conducting plates with a surface area of 1.0 m^2 and a plate separation distance of 50 μm. (*a*) Determine the capacitance and the energy stored in the capacitor when it is charged by connecting it to a 9.0-V battery. (*b*) With the capacitor fully charged and disconnected from the battery, a 50-μm-thick sheet of Mylar is inserted between the plates. Determine the potential difference across the capacitor and the energy stored in it. (*c*) If the Mylar-filled capacitor is connected to the battery, how much work does the battery do to fully charge the capacitor?

❶ **GETTING STARTED** I am given information about a capacitor and a battery used to charge it. From this information, I must determine the capacitance and the energy stored in the capacitor with and without a sheet of Mylar between the plates connected to the battery, and determine what happens to the potential with and without a sheet of Mylar between the plates and with and without the battery connected to the capacitor. When connected to the capacitor, the battery maintains a constant potential across the capacitor. When the battery is not connected to the capacitor, the charge on the plates remains constant.

❷ **DEVISE PLAN** To calculate the energy stored in the capacitor I can use Eq. 26.4; I calculated the capacitance of a parallel-plate capacitor in Example 26.2. When the dielectric is added, the potential and the capacitance are given by Eqs. 26.9 and 26.11. From Table 26.1, I see that the dielectric constant of Mylar is $\kappa = 3.3$.

❸ **EXECUTE PLAN** (*a*) Using the result of Example 26.2, I get

$$C = \frac{[8.85 \times 10^{-12}\,\mathrm{C^2/(N \cdot m^2)}](1.0\,\mathrm{m^2})}{50 \times 10^{-6}\,\mathrm{m}}$$

$$= 0.18 \times 10^{-6}\,\frac{\mathrm{C^2}}{\mathrm{N \cdot m}} = 0.18\,\mu\mathrm{F}, ✔$$

and Eq. 26.4 gives

$$U^E = \tfrac{1}{2}C(V_0)^2 = \tfrac{1}{2}(0.18\,\mu\mathrm{F})(9.0\,\mathrm{V})^2 = 7.2\,\mu\mathrm{J}. ✔$$

(*b*) Because of the bound surface charge on the dielectric, the electric field between the capacitor plates decreases, and so the potential difference across the capacitor decreases, too. From the definition of the dielectric constant (Eq. 26.9), I have

$$V_d = \frac{V_0}{\kappa} = \frac{9.0\,\mathrm{V}}{3.3} = 2.7\,\mathrm{V}, ✔$$

where I obtained my value for κ from Table 26.1. To calculate the energy in the presence of the dielectric, I must first obtain an expression for the capacitance of the dielectric-filled capacitor. Substituting the expression for the capacitance of a parallel-plate capacitor (see Example 26.2) into Eq. 26.11 yields

$$C_d = \kappa C_0 = \frac{\kappa \epsilon_0 A}{d}$$

$$= \frac{(3.3)[8.85 \times 10^{-12}\,\mathrm{C^2/(N \cdot m^2)}](1.0\,\mathrm{m^2})}{50 \times 10^{-6}\,\mathrm{m}}$$

$$= 0.58\,\mu\mathrm{F}.$$

The stored energy is thus

$$U^E = \tfrac{1}{2}CV_d^2 = \tfrac{1}{2}(0.58\,\mu\mathrm{F})(2.9\,\mathrm{V})^2 = 2.2\,\mu\mathrm{J}. ✔$$

(*c*) The energy stored in the fully charged dielectric-filled capacitor is

$$U^E = \tfrac{1}{2}CV_{batt}^2 = \tfrac{1}{2}(0.58\,\mu\mathrm{F})(9.0\,\mathrm{V})^2 = 24\,\mu\mathrm{J}.$$

From part *b* I know that before it was connected to the battery, the capacitor stored 2.2 μJ, and so the work done by the battery in charging the capacitor must be 24 μJ $-$ 2.2 μJ $=$ 22 μJ. ✔

❹ **EVALUATE RESULT** My answers to parts *a* and *b* show that the amount of energy stored decreases when the dielectric is inserted. That makes sense because, as the dielectric is brought

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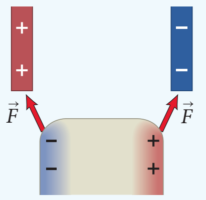
near the plates, the charged plates induce a polarization on the dielectric and consequently it is pulled into the space between the plates (**Figure 26.30**). Therefore, the capacitor does *positive* work on whoever is holding the dielectric, and the energy in the capacitor decreases as the dielectric enters the space between the plates. This work is equal to the difference in energy between parts *a* and *b*: $W = 7.2\ \mu J - 2.2\ \mu J = 5.0\ \mu J$. My answer to part *c* is about three times greater than the value I calculated for U^E in part *a*, which is what I expect given that the capacitance is increased by the factor $\kappa = 3.3$ once the dielectric is inserted.

Figure 26.30



26.22 Verify that in the solution to part *a* of Example 26.6, (*a*) the ratio of units $C^2/(N \cdot m)$ is equivalent to the unit F and (*b*) the product of units $F \cdot V^2$ is equivalent to the unit J.

26.8 Gauss's law in dielectrics

Can we apply Gauss's law to calculate the electric fields inside dielectric materials? The answer is *yes,* because Gauss's law is a fundamental law that follows directly from the $1/r^2$ dependence of Coulomb's law. Thus, the presence of a dielectric cannot affect its validity.

Consider the situation illustrated in **Figure 26.31**. To determine the magnitude of the electric field E inside the dielectric, we consider the cylindrical Gaussian surface with cross-sectional area A shown in the figure. The electric flux is zero except through the right flat surface of the cylinder, so

$$\oint \vec{E} \cdot d\vec{A} = EA. \tag{26.22}$$

The charge enclosed by the Gaussian surface is not just the enclosed charge on the plate—we must also take into account the enclosed bound charge on the dielectric. The enclosed charge is thus $q_{\text{enc}} = q_{\text{free, enc}} - q_{\text{bound, enc}}$, and Gauss's law then gives

$$\oint \vec{E} \cdot d\vec{A} = EA = \frac{q_{\text{free, enc}} - q_{\text{bound, enc}}}{\epsilon_0}. \tag{26.23}$$

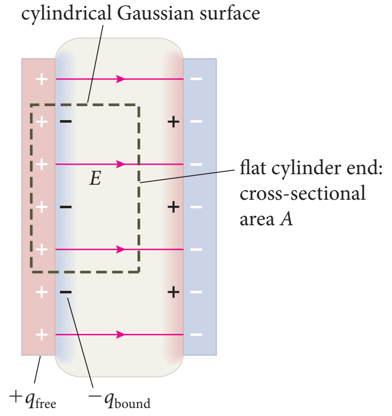
In this form, Gauss's law is not very useful, because in order to extract E from Eq. 26.23, we need to know the magnitude of the bound surface charge. Generally, we don't know the contribution from the bound charge in a given situation.

Substituting the relationship between the free and bound charges (Eq. 26.18), however, we can rewrite Eq. 26.23 in the form

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{free, enc}}}{\epsilon_0 \kappa}. \tag{26.24}$$

This result—Gauss's law in dielectrics—is remarkable. The left side contains the electric flux of the electric field *inside the dielectric*. We can obtain this field, however, just by accounting for the enclosed *free* charge (and we already know how to deal with that charge). This relationship is valid because the effect of the bound charge is completely accounted for by the dielectric constant in the denominator. As Eq. 26.17 shows, dividing q_{free} by κ gives the difference of the free and bound charges.

Figure 26.31 A cylindrical Gaussian surface used to calculate the electric field inside a dielectric-filled parallel-plate capacitor.



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Because the dielectric constant affects the value of the electric field, **Gauss's law in matter** is usually written in the form

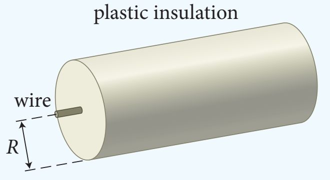
$$\oint \kappa \vec{E} \cdot d\vec{A} = \frac{q_{\text{free, enc}}}{\epsilon_0}. \qquad (26.25)$$

This form of Gauss's law is very general: Even though we derived it for the special case of a parallel-plate capacitor, it holds in any situation, even one without a dielectric. In the absence of matter (that is, in vacuum), $\kappa = 1$, and because there is no bound charge we have $q_{\text{free, enc}} = q_{\text{enc}}$. Then Eq. 26.25 becomes identical to the familiar form of Gauss's law (Eq. 24.8).

Example 26.7 Electric field surrounding a charged insulated wire

A thin, long, straight wire is surrounded by plastic insulation of radius R and dielectric constant κ (**Figure 26.32**). The wire carries a uniform distribution of charge with a positive linear charge density λ. If the wire has a diameter d, what is the potential difference between the outer surface of the wire and the outer surface of the insulation?

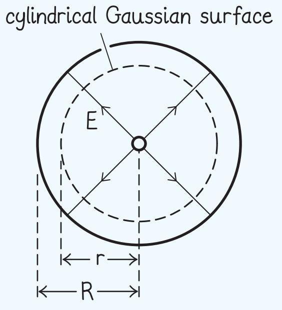
Figure 26.32 Example 26.7.



❶ **GETTING STARTED:** The insulation reduces the electric field created by the charge in the wire, so the potential difference between the wire surface and any location a distance R from the wire (in other words, any location on the outer surface of the insulation) is smaller than when there is no insulation around the wire.

❷ **DEVISE PLAN:** The potential difference between two locations A and B can be obtained from Eq. 25.25, $V_{AB} = -\int_A^B \vec{E} \cdot d\vec{\ell}$, but using this expression requires me to know the electric field. To calculate the electric field inside the insulation, I can apply Eq. 26.25 to a cylindrical Gaussian surface that has radius r and length L and is concentric with the wire, as shown in **Figure 26.33**.

Figure 26.33



❸ **EXECUTE PLAN:** Because of the cylindrical symmetry of the wire, the electric field has the same magnitude E everywhere on the curved region of the Gaussian surface. The electric flux through that region of the Gaussian surface is equal to the product of the electric field at a distance r from the wire and the surface area: $\Phi = EA = E(2\pi rL)$ (Eq. 3 in Exercise 24.7). The free charge enclosed by the cylinder is λL, and with these substitutions Eq. 26.25 becomes

$$\kappa E(2\pi rL) = \frac{\lambda L}{\epsilon_0}$$

and

$$E = \frac{\lambda}{2\pi\kappa\epsilon_0 r}. \qquad (1)$$

Substituting this expression for E into Eq. 25.25, I obtain for the potential difference between the outer surface of the wire and the outer surface of the insulation

$$V_{dR} = -\int_{d/2}^{R} \vec{E} \cdot d\vec{r} = -\frac{\lambda}{2\pi\kappa\epsilon_0}\int_{d/2}^{R} \frac{1}{r}\,dr$$
$$= -\frac{\lambda}{2\pi\kappa\epsilon_0} \ln \frac{2R}{d}. \checkmark$$

❹ **EVALUATE RESULT:** Because $\ln(2R/d)$ is positive, the potential difference is negative, as it should be because I am moving away from the positively charged wire. As an additional check, if I set $\kappa = 1$ in Eq. 1, my result for the electric field becomes identical to the result I obtained for a thin wire without insulation in Exercise 24.7.

26.23 Show that, if you account for the free and bound charges, Gauss's law in vacuum (Eq. 24.8) yields the same result for the electric field outside the insulation as Gauss's law in matter (Eq. 26.25) does.

Chapter Glossary

SI units of physical quantities are given in parentheses.

Bound charge A surplus of charge in polarized matter due to charge carriers that are bound to atoms and cannot move freely within the bulk of the material.

Capacitance C (F) The ratio of the magnitude of the charge q on a pair of oppositely charged conductors and the magnitude V_{cap} of the potential difference between them:

$$C \equiv \frac{q}{V_{\text{cap}}}. \tag{26.1}$$

The capacitance is a measure of a capacitor's capacity to store charge (or, equivalently, electric potential energy).

Capacitor A pair of conducting objects separated by a nonconducting material or vacuum. Any such pair of objects stores electric potential energy when charge has been transferred from one object to the other.

Charge-separating device A device that transfers charge from one object to another. To achieve this charge transfer, the device must move charge carriers against an electric field, requiring the device to do work on the charge carriers. This work can be supplied from a variety of sources, such as mechanical or chemical energy. Examples of charge-separating devices are voltaic cells, batteries, and Van de Graaff generators.

Dielectric A nonconducting material inserted between the plates of a capacitor. Often used more broadly to describe any nonconducting material. *Polar* dielectrics are made up of molecules that have a nonzero dipole moment, whereas *nonpolar* dielectrics consist of nonpolar molecules.

Dielectric constant κ (unitless) The factor by which the potential across an isolated capacitor is reduced by the insertion of a dielectric:

$$\kappa \equiv \frac{V_0}{V_{\text{d}}}. \tag{26.9}$$

Electrical breakdown When a dielectric material is subject to a very large electric field, the molecules in the material may ionize, temporarily turning the dielectric into a conductor. The electric field magnitude at which breakdown occurs is called the *breakdown threshold*.

Emf $\mathscr{E}$ (V) The emf of a charge-separating device is the work per unit charge done by nonelectrostatic interactions in separating positive and negative charge carriers inside the device:

$$\mathscr{E} \equiv \frac{W_{\text{nonelectrostatic}}}{q}. \tag{26.7}$$

Energy density of the electric field u_E (J/m^3) The energy per unit volume contained in an electric field. In vacuum:

$$u_E = \tfrac{1}{2}\epsilon_0 E^2. \tag{26.6}$$

Farad (F) The derived SI unit of capacitance:

$$1\ \text{F} \equiv 1\ \text{C/V}.$$

Free charge A surplus of charge due to charge carriers that can move freely within the bulk of a material.

Gauss's law in matter For electric fields inside matter, Gauss's law can be written in the form

$$\oint \kappa \vec{E} \cdot d\vec{A} = \frac{q_{\text{free, enc}}}{\epsilon_0}. \tag{26.25}$$