A MODERN ALMAGEST:
An Updated Version of Ptolemy's Model of the Solar System

Richard Fitzpatrick

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Preface

This book is devoted to a re-examination of Claudius Ptolemy's Almagest, one of the foundational scientific works of antiquity. Although often contrasted unfavorably with Euclid's Elements, and frequently dismissed as overly complex, and misguided in its geocentric approach, the Almagest remains a remarkable achievement: namely, a mathematically sophisticated and observationally grounded attempt to describe the motions of the heavens. The aim of my work is to reassess the scientific merits of Ptolemy's model by reconstructing it in a modern framework. Using contemporary mathematical methods, as well as standard astronomical conventions and terminology, I seek to render the structure and logic of the Ptolemaic system more accessible to the modern reader. In the process, I demonstrate that many common criticisms of the Almagest are either overstated or based on misunderstandings, and that, when properly interpreted, Ptolemy's model can be viewed as a surprisingly accurate approximation to the later Keplerian description of planetary motion. I do not attempt to reproduce every aspect of the original Almagest. Rather than revisiting the construction of trigonometric tables or ancient observational techniques, I focus on the essential geometric and kinematic ideas underlying the model. Certain known deficiencies in Ptolemy's system are corrected, and, where appropriate, his methods are reformulated in terms of equivalent but more transparent schemes. The result is a streamlined yet faithful reconstruction of the Ptolemaic system—one that preserves its historical character while demonstrating its continued mathematical and scientific interest. It is my hope that this approach will allow readers to better appreciate both the ingenuity of Ptolemy's achievement and its place in the development of astronomical science.

Richard Fitzpatrick

The University of Texas at Austin

1. Introduction

1.1 Euclid's Elements and Ptolemy's Almagest

The modern world inherited two major scientific treaties from the civilization of ancient Greece. The first of these, the *Elements* (Στοιχεῖα) of Euclid (Εὐκλείδης), is a large compendium of mathematical theorems concerning geometry, proportion, and number theory. These theorems were not necessarily discovered by Euclid himself. In fact, the theorems were largely the work of earlier mathematicians, such as Pythagoras (Πυθαγόρας) and his school, Eudoxos of Cnidus (Εὔδοξος ὁ Κνίδιος), and Theaetetus of Athens (Θεαίτητος ὁ Ἀθηναῖος). However, Euclid is credited with arranging the theorems in a logical manner, such that a given theorem only depends on ones appearing before it in the treatise. Euclid also demonstrated that all of the theorems follow from a set of definitions combined with five simple, but unproved, axioms. The Elements is rightly regarded as the first, largely successful, attempt to construct an axiomatic system in mathematics, and is still held in high esteem within the scientific community.

The second treatise, the *Almagest*[1] of Claudius Ptolemy (Κλαύδιος Πτολεμαῖος), is an attempt to find a simple geometric explanation for the apparent motions of the Sun, the Moon, and the five visible (to the naked eye) planets in the Earth's sky. On the basis of his own naked-eye observations, combined with those of earlier astronomers such as Hipparchus of Nicaea (Ἵππαρχος ὁ Νικαεύς), Ptolemy proposed a model of the Solar System in which the Earth is stationary. According to this model, the Sun moves in a circular orbit, (nearly) centered on the Earth, that maintains a fixed inclination of about 23° to the terrestrial equator. Furthermore, each planet moves on the rim of a small circle called an *epicycle* (ἐπίκυκλος in Greek), whose center revolves around the Earth on a large eccentric circle called a *deferent* (ἀποφορά in Greek). (See Figure 8.2.) The planetary deferents and epicycles also maintain fixed inclinations,[2] which are all fairly close to 23°, to the terrestrial equator.

The scientific reputation of the Almagest has not fared as well as that of Euclid's Elements. Nowadays, it is a commonly held belief, even amongst scientists, that Ptolemy's mistaken adherence to the tenets of Aristotelian philosophy—in particular, the immovability of the Earth, and the necessity for heavenly bodies to move uniformly in circles—led him to construct an over-complicated, unwieldy, and faintly ridiculous model of planetary motion. As is well known, Ptolemy's model was superseded in 1543 AD by the heliocentric model of Nicolaus Copernicus, in which the planets revolve about the Sun in circular orbits.[3] The Copernican model was,

[1] The true title of this work is Μαθηματικὴ Σύνταξις, which means "Mathematical Treatise". The treatise was later called Ἡ Μεγάλη Σύνταξις, meaning "The Great Treatise", and the superlative form of the adjective, μεγίστη, with the Arabic article "al" prepended, lies behind the Arabic name from which the English name Almagest derives.

[2] Actually, Ptolemy erroneously allowed the inclinations of the deferents and epicycles to vary slightly.

[3] In fact, the planets revolve on small circular epicycles, whose centers revolve around the Sun on eccentric circular deferents.

in turn, superseded in the early 1600's AD by the, ultimately correct, model of Johannes Kepler, in which the planets revolve about the Sun in eccentric elliptical orbits.

The aim of this treatise is to re-examine the scientific merits of Ptolemy's Almagest.

1.2 Ptolemy's model of the Solar System

Claudius Ptolemy lived and worked in the city of Alexandria, capital of the Roman province of Egypt, during the reigns of the later Flavian and the Antonine emperors. Ptolemy was heir—via the writings of Euclid, and later mathematicians such as Apollonius of Perga (Ἀπολλώνιος ὁ Περγαῖος), and Archimedes of Syracuse (Ἀρχιμήδης ὁ Συρακούσιος)—to the considerable mathematical knowledge of geometry and arithmetic acquired by the civilization of ancient Greece. Ptolemy also inherited an extensive Babylonian and ancient Greek body of knowledge concerning observational and theoretical astronomy. The most important astronomer prior to Ptolemy was undoubtedly Hipparchus of Nicaea (second century BC), who produced the first circular chord table (which is key to Ptolemy's trigonometry), compiled the first star catalog, developed the theory of solar motion used by Ptolemy, discovered the precession of the equinoxes, and collected an extensive set of astronomical observations—some of which he made himself, and some of which he acquired from Babylonian records—which were available to Ptolemy (possibly via the famous Library of Alexandria). Other astronomers who made significant contributions prior to Ptolemy include Meton of Athens (Μέτων ὁ Ἀθηναῖος, 5th century BC), Eudoxos of Cnidus (5th/4th century BC), Callipus of Cyzicus (Καλλίππος ὁ Κυζίκιος, 4th century BC), Aristarchus of Samos (Ἀρίσταρχος ὁ Σάμιος, 4th/3rd century BC), Eratosthenes of Cyrene (Ἐρατοσθένης ὁ Κυρηναῖος, 3rd/2nd century BC), and Menelaus of Alexandria (Μενέλαος ὁ Ἀλεξανδρεύς, 1st century AD).

Ptolemy's aim in the Almagest is to construct a kinematic model of the Solar System, as seen from the Earth. In other words, the Almagest outlines a relatively simple geometric model that describes the apparent motions of the Sun, Moon, and planets, relative to the Earth, but does not attempt to explain why these motions occur. (The models of Copernicus and Kepler are similar to that of Ptolemy in this respect.) As such, the fact that the model described in the Almagest is geocentric in nature is a non-issue, because the Earth is stationary in its own frame of reference. This is not to say that the heliocentric hypothesis is without advantages. As we shall see, the assumption of heliocentricity allowed Copernicus to determine, for the first time, the ratios of the mean radii of the various planets in the Solar System.

We now know, from the work of Kepler, that planetary orbits are actually ellipses that are confocal with the Sun. Such orbits possess two main properties. First, they are eccentric; that is, the Sun is displaced from the geometric center of the orbit. Second, they are elliptical; that is, the orbit is elongated along a particular axis. Now, Keplerian orbits are characterized by a quantity, e, known as the *eccentricity*, that measures their deviation from circularity. It is easily demonstrated that the eccentricity of a Keplerian orbit scales as e, whereas the corresponding degree of elongation scales as e^2. Because the orbits of the visible planets in the Solar System all possess relatively small values of e (that is, $e \leq 0.21$), it follows that, to an excellent approximation, these orbits can be represented as eccentric circles; that is, circles that are not quite concentric with the Sun. In other words, we can neglect the ellipticities of planetary orbits compared to their eccentricities. This is exactly what Ptolemy does in the Almagest. It follows that Ptolemy's assumption that heavenly bodies move in circles is actually one of the main strengths of his model, rather than being the main weakness, as is commonly supposed.

Kepler's second law of planetary motion states that the radius vector connecting a planet to the Sun sweeps out equal areas in equal time intervals. In the approximation in which planetary orbits are represented as eccentric circles, this law implies that a typical planet revolves around the Sun at a non-uniform rate. However, it is easily demonstrated that the non-uniform rotation of the radius vector connecting the planet to the Sun implies a uniform rotation of the radius vector connecting the planet to the so-called *equant*; that is, the point diagrammatically opposite to the Sun with respect to the geometric center of the orbit. (See Figure 1.1.) Ptolemy discovered the equant scheme empirically, and used it to control the non-uniform rotation of the planets in his model. In fact, this discovery is one of Ptolemy's main claims to fame.

It follows, from the previous discussion, that the geocentric model of Ptolemy is equivalent to a heliocentric model in which the various planetary orbits are represented as eccentric circles, and in which the radius vector connecting a given planet to its corresponding equant revolves at a uniform rate. In fact, Ptolemy's model of planetary motion can be thought of as a version of Kepler's model that is accurate to first order in the planetary eccentricities. (See Chapter 4.) According to the Ptolemaic scheme, from the point of view of the Earth, the orbit of the Sun is described by a single circular motion, whereas that of a planet is described by a combination of two circular motions. In reality, the single circular motion of the Sun represents the (approximately) circular motion of the Earth around the Sun, whereas the two circular motions of a typical planet represent a combination of the planet's (approximately) circular motion around the Sun, and the Earth's motion around the Sun. Incidentally, the popular story that Ptolemy's scheme requires an absurdly large number of circles in order to fit the observational data to any degree of accuracy has no basis in fact. Actually, Ptolemy's model of the Sun and the planets, which fits the data very well, only contains 12 circles (that is, 6 deferents and 6 epicycles). (There are 6 epicycles because Mercury possesses an additional spurious epicycle.) Ptolemy's lunar model contains an additional 3 circles. The additional common tale that medieval astronomers added more and more epicycles to Ptolemy's model in a vain attempt to make it fit the observations better also has no basis in fact.

Ptolemy is often accused of slavish adherence to the tenants of Aristotelian philosophy, to the overall detriment of his model. However, despite Ptolemy's conventional geocentrism, his model of the Solar System deviates from orthodox Aristotelian philosophy in a number of crucially important respects. In his treatise "On the Heavens" (Περὶ Οὐρανοῦ) Aristotle (Ἀριστοτέλης) argues, from a purely philosophical standpoint, that heavenly bodies should move in single uniform circles. However, in the Ptolemaic system, the motion of the planets is a combination of two circular motions. Moreover, at least one of these motions is non-uniform. Aristotle also argues, again from purely philosophical grounds, that the Earth is located at the exact center of the universe, about which all heavenly bodies orbit in concentric circles. However, in the Ptolemaic system, the Earth is slightly displaced from the center of the universe. Indeed, there is no

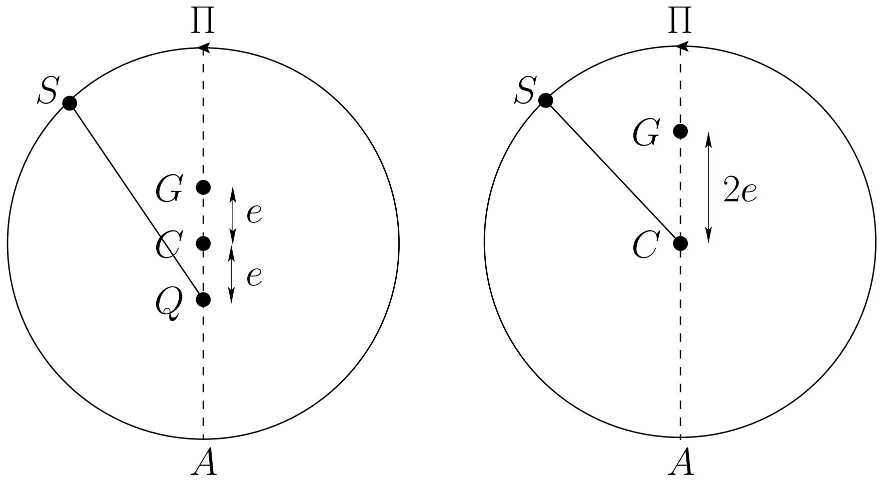

Figure 1.1: Hipparchus' (and Ptolemy's) model of the Sun's apparent orbit about the Earth (right) compared to the optimal model (left). The radius vectors in both models rotate uniformly. Here, S is the Sun, G the Earth, C the geometric center of the orbit, Q the equant, Π the perigee, and A the apogee. The radius of the orbit is normalized to unity.

unique center of the universe, because the circular orbit of the Sun and the circular planetary deferents all have slightly different geometric centers, none of which coincide with the Earth. As described in the Almagest, the non-orthodox (from the point of view of Aristotelian philosophy) aspects of Ptolemy's model were ultimately dictated by observations. This suggests that, although Ptolemy's world-view was based on Aristotelian philosophy, he did not hesitate to deviate from this standpoint when required to by observational data.

From our heliocentric point of view, it is easily appreciated that the epicycles of the *superior planets* (that is, the planets farther from the Sun than the Earth) in Ptolemy's model actually represent the Earth's orbit around the Sun, whereas the deferents represent the planets' orbits around the Sun. (See Figure 8.1.) It follows that the epicycles of the superior planets should all be the same size (that is, the size of the Earth's orbit), and that the radius vectors connecting the centers of the epicycles to the planets should always all point in the same direction as the vector connecting the Earth to the Sun.

We can also appreciate that the deferents of the *inferior planets* (that is, the planets closer to the Sun than the Earth) in Ptolemy's model actually represent the Earth's orbit around the Sun, whereas the epicycles represent the planets' orbits around the Sun. (See Figure 9.1.) It follows that the deferents of the inferior planets should all be the same size (that is, the size of the Earth's orbit), and that the centers of the epicycles (relative to the Earth) should all correspond to the position of the Sun (relative to the Earth).

The geocentric model of the Solar System outlined previously represents a perfected version of Ptolemy's model, constructed with a knowledge of the true motions of the planets around the Sun. Not surprisingly, the model actually described in the Almagest deviates somewhat from this ideal form. In the following, we shall refer to these deviations as "errors", but this should not be understood in a pejorative sense.

Ptolemy's first error lies in his model of the Sun's apparent motion around the Earth, which he inherited from Hipparchus. Figure 1.1 compares what Ptolemy actually did, in this respect, compared to what he should have done in order to be completely consistent with the rest of his model. Let us normalize the mean radius of the Sun's apparent orbit to unity, for the sake of clarity. Ptolemy should have adopted the model shown on the left in Figure 1.1, in which the Earth is displaced from the center of the Sun's orbit a distance $e = 0.0167$ (the eccentricity of the Earth's orbit around the Sun) towards the perigee (the point of the Sun's closest approach to the Earth), and the equant is displaced the same distance in the opposite direction. The instantaneous angular position of the Sun is then obtained by allowing the radius vector connecting the equant to the Sun to rotate uniformly at the Sun's mean orbital angular velocity. Of course, this implies that the Sun rotates non-uniformly about the Earth. Ptolemy actually adopted the Hipparchian model shown on the right in Figure 1.1. In this model, the Earth is displaced a distance $2e$ from the center of the Sun's orbit in the direction of the perigee, and the Sun rotates at a uniform rate (that is, the radius vector CS rotates uniformly). It turns out that, to first order in e, these two models are equivalent in terms of their ability to predict the angular position of the Sun relative to the Earth. (See Chapter 4.) Nevertheless, the Hipparchian model is incorrect, because it predicts too large (by a factor of two) a variation in the radial distance of the Sun from the Earth (and, hence, the angular size of the Sun) during the course of a year. (See Chapter 4.) Ptolemy probably adopted the Hipparchian model because his Aristotelian leanings prejudiced him in favor of uniform circular motion whenever this was consistent with observations. (It should be noted that Ptolemy was not especially interested in explaining the relatively small variations in the angular size of the Sun during the year; presumably, because this effect would have been almost impossible for him to accurately measure.)

Ptolemy's next error was to neglect the non-uniform rotation of the superior planets on their epicycles. This is equivalent to neglecting the orbital eccentricity of the Earth (recall that the epicycles of the superior planets actually represent the Earth's orbit) compared to those of the superior planets. It turns out that this is a fairly good approximation, because the superior planets all have significantly greater orbital eccentricities than the Earth. Nevertheless, neglecting the non-uniform rotation of the superior planets on their epicycles has the unfortunate effect of obscuring the tight coupling between the apparent motions of these planets, and that of the Sun. The radius vectors connecting the epicycle centers of the superior planets to the planets themselves should always all point exactly in the same direction as that of the Sun relative to the Earth. When the aforementioned non-uniform rotation is neglected, the radius vectors instead point in the direction of the mean Sun relative to the Earth. The *mean Sun* is a fictitious body that has the same apparent orbit around the Earth as the real Sun, but that circles the Earth at a uniform rate. The mean Sun only coincides with the real Sun twice a year.

Ptolemy's third error is associated with his treatment of the inferior planets. As we have seen, in going from the superior to the inferior planets, deferents and epicycles effectively swap roles.

For instance, it is the deferents of the inferior planets, rather than the epicycles, that represent the Earth's orbit. Hence, for the sake of consistency with his treatment of the superior planets, Ptolemy should have neglected the non-uniform rotation of the epicycle centers around the deferents of the inferior planets, and retained the non-uniform rotation of the planets themselves around the epicycle centers. Instead, he did exactly the opposite. This is equivalent to neglecting the inferior planets' orbital eccentricities relative to that of the Earth. It follows that this approximation only works when an inferior planet has a significantly smaller orbital eccentricity than that of the Earth. It turns out that this is indeed the case for Venus, which has the smallest eccentricity of any planet in the Solar System. Thus, Ptolemy was able to successfully account for the apparent motion of Venus. Mercury, on the other hand, has a much larger orbital eccentricity than the Earth. Moreover, it is particularly difficult to obtain good naked-eye positional data for Mercury, because this planet always appears very close to the Sun in the sky. Consequently, Ptolemy's Mercury data was highly inaccurate. Not surprisingly, Ptolemy was not able to account for the apparent motion of Mercury using his standard deferent-epicycle approach. Instead, in order to fit the data, he was forced to introduce an additional, and entirely spurious, epicycle into his model of Mercury's orbit.

Ptolemy's fourth, and possibly largest, error is associated with his treatment of the Moon. It should be noted that the Moon's motion around the Earth is extremely complicated in nature, because it is strongly perturbed by the Sun, and was not fully understood until the early 20th century AD. Ptolemy constructed an ingenious geometric model of the Moon's orbit that was capable of predicting the lunar ecliptic longitude to reasonable accuracy. Unfortunately, this model necessitates a monthly variation in the Earth-Moon distance by a factor of about two, which implies a similarly large variation in the Moon's angular diameter. However, the observed variation in the Moon's diameter is very much smaller than this. Hence, Ptolemy's model of lunar motion is not even approximately correct.

Ptolemy's fifth error is associated with his treatment of planetary ecliptic latitudes. Given that the deferents and epicycles of the superior planets represent the orbits of the planets themselves around the Sun, and the Sun's apparent orbit around the Earth, respectively, it follows that one should take the slight inclination of planetary orbits to the ecliptic plane (that is, the plane of the Sun's apparent orbit) into account by tilting the deferents of superior planets, while keeping their epicycles parallel to the ecliptic. Similarly, given that the epicycles and deferents of inferior planets represent the orbits of the planets themselves around the Sun, and the Sun's apparent orbit around the Earth, respectively, one should tilt the epicycles of inferior planets, while keeping their deferents parallel to the ecliptic. Finally, because the inclination of planetary orbits are all essentially constant in time, the inclinations of the epicycles and deferents should also be constant. Unfortunately, when Ptolemy constructed his theory of planetary latitudes, he tilted the both deferents and epicycles of all of the planets. Even worse, he allowed the inclinations of the epicycles to the ecliptic plane to vary in time. The net result is a theory that is far more complicated than is necessary.

The final failing in Ptolemy's model of the Solar System lies in its scale invariance. Using angular position data alone, Ptolemy was able to determine the ratio of the epicycle radius to that of the deferent for each planet, but was not able to determine the relative sizes of the deferents of different planets. In order to break this scale invariance it is necessary to make an additional assumption; namely, that the Earth orbits the Sun. This brings us to Copernicus.

1.3 Copernicus's model of the Solar System

The Polish astronomer Nicolaus Copernicus (Mikołaj Kopernik, 1473–1543 AD) studied the Almagest assiduously, but eventually became dissatisfied with Ptolomy's approach. The main reason for this dissatisfaction was not the geocentric nature of Ptolomy's model, but rather the fact that it mandates that heavenly bodies execute non-uniform circular motion. Copernicus, like Aristotle, was convinced that the supposed perfection of the heavens requires such bodies to execute uniform circular motion only. Copernicus was thus spurred to construct his own model of the Solar System, which was described in his book "On the Revolutions of the Heavenly Spheres" (De Revolutionibus Orbium Coelestium), published in the year of his death.

The most well-known aspect of Copernicus's model is the fact that it is heliocentric. As has already been mentioned, when describing the motion of the Sun, Moon, and planets relative to the Earth, it makes little practical difference whether one adopts a geocentric or a heliocentric model of the Solar System. Having said this, the heliocentric approach does have one large advantage. If we accept that the Sun, and not the Earth, is stationary, then it immediately follows that the epicycles of the superior planets, and the deferents of the inferior planets, represent the Earth's orbit around the Sun. Hence, all of these circles must be the same size. This realization allows us to break the scale invariance which is one of the main failings of Ptolemy's model. Thus, the ratio of the deferent radius to that of the epicycle for a superior planet, which is easily inferred from observations, actually corresponds to the ratio of planet's orbital radius to that of the Earth. Likewise, the ratio of the epicycle radius to that of the deferent for an inferior planet, which is again easily determined observationally, also corresponds to the ratio of the planet's orbital radius to that of the Earth. Using this type of reasoning, Copernicus was able to construct the first accurate scale model of the Solar System, and to firmly establish the order in which the planets orbit the Sun. In some sense, this was his main achievement.

Copernicus's insistence that heavenly bodies should only move in uniform circles lead him to reject Ptolemy's equant scheme, and to replace it with the scheme illustrated in Figure 1.2. According to Copernicus, a heliocentric planetary orbit is a combination of two circular motions. The first is the motion of the planet around a small circular epicycle, and the second is the motion of the center of the epicycle around the Sun on a circular deferent. Both motions are uniform, and in the same direction. However, the former motion is twice as fast as the latter. In addition, the Sun is displaced from the center of the deferent in the direction of the perihelion, the displacement being proportional to the orbital eccentricity. Furthermore, the Sun's displacement is three times greater than the radius of the epicycle. Finally, the radius of the deferent is equal to the major radius of the planetary orbit. It turns out that Copernicus's scheme is a marginally less accurate approximation than Ptolemy's to a low eccentricity Keplerian orbit. (See Chapter 4.)

Copernicus modeled the orbit of the Earth around the Sun using an Hipparchian scheme (see Figure 1.1) in which the Earth moves uniformly around an eccentric circle. Unfortunately,

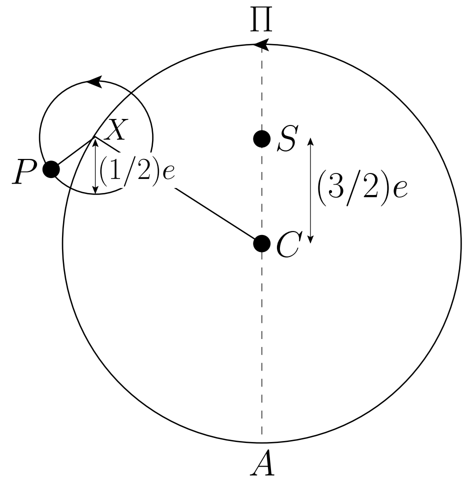

Figure 1.2: Copernicus's model of a heliocentric planetary orbit. Here, S is the Sun, P the planet, C the geometric center of the deferent, X the center of the epicycle, Π the perihelion, and A the aphelion. The radius vectors CX and XP both rotate uniformly in the same direction, but XP rotates twice as fast as CX. The major radius of the orbit is normalized to unity.

such a scheme exaggerates the variation in the radial distance between the Earth and the Sun during the course of a year by a factor of two, and so introduces significant errors into the calculation of the parallax of the planets due to the motion of the Earth. On the other hand, Copernicus's model of the Moon's orbit around the Earth is a considerable improvement on Ptolemy's, because it does not grossly exaggerate the monthly variation in the Earth-Moon distance. Like Ptolemy, Copernicus introduced an additional spurious epicycle into his model of Mercury's orbit, and erroneously allowed the inclination of his planetary orbits to vary slightly in time.

In summary, Copernicus's model of the Solar System contains approximately the same number of circles as Ptolemy's, the only difference being that Copernicus's epicycles are much smaller than Ptolemy's. Indeed, the model of Copernicus is about as complicated, and not appreciably more accurate (except in the case of the Moon), than that described in the Almagest. In this respect, Copernicus cannot be said to have demonstrated the correctness of his heliocentric approach on the basis of observational data.

1.4 Kepler's model of the Solar System

Johannes Kepler (1571–1630 AD) was fortunate to inherit an extensive set of naked-eye solar, lunar, and planetary angular position data from the Danish astronomer Tycho Brahe (1546–1601 AD). This data extended over many decades, and was of unprecedented accuracy.

Although Kepler adopted the heliocentric approach of Copernicus, what he effectively first did was to perfect Ptolemy's model of the Solar System (or, rather, its heliocentric equivalent). Thus, Kepler replaced Ptolemy's erroneous equantless model of the Sun's apparent orbit around the Earth with a corrected version containing an equant; in the process, halving the eccentricity of the orbit. (See Figure 1.1.) Kepler also introduced equants into the epicycles of the superior and inferior planets. Once he had perfected Ptolemy's model, the heliocentric nature of the Solar System became manifestly apparent to Kepler. For instance, he found that the epicycles of the superior planets, the Sun's apparent orbit around the Earth, and the deferents of the inferior planets, all had exactly the same eccentricity. The obvious implication is that these circles all correspond to some common motion within the Solar System; in fact, the motion of the Earth around the Sun.

Once Kepler had corrected the Almagest model, he compared its predictions with his observational data. In particular, Kepler investigated the apparent motion of Mars in the night sky. Kepler found that his model performed extremely well, but that there remained small differences between its predictions and the observational data. The maximum discrepancy was about $8'$; that is, about one quarter of the apparent size of the Sun. By the standards of naked-eye astronomy, this was a very small discrepancy. Nevertheless, given the incredible accuracy of Tycho Brahe's observations, the discrepancy was still significant. Thus, Kepler embarked on an epic new series of calculations which eventually lead him to the conclusion that the planetary orbits are actually eccentric ellipses, rather than eccentric circles. Kepler published the results of his research in his treatise "New Astronomy" (Astronomia Nova) in 1609 AD. It is interesting to note that had Tycho's data been a little less accurate, or had the orbit of Mars been a little less eccentric, Kepler might well have settled for a model which was kinematically equivalent to a perfected version of the model described in the Almagest. We can also appreciate that, given the far less accurate observational data available to Ptolemy, there was no way in which he could have discerned the very small difference between elliptical planetary orbits and the eccentric circular orbits employed in the Almagest.

1.5 Purpose of book

As we have seen, misconceptions abound regarding the details of Ptolemy's model of the Solar System, as well as its scientific merit. Part of the reason for this is that the Almagest is an extremely difficult book for a modern reader to comprehend. For instance, virtually all of its theoretical results are justified via lengthy and opaque geometric proofs. Moreover, the plane and spherical trigonometry employed by Ptolemy is of a rather primitive nature, and, consequently, somewhat unwieldy. Dates are also a major stumbling block, because three different systems are used in the Almagest, all of which are archaic, and essentially meaningless to the modern reader. Another difficulty is the unfamiliar, and far from optimal, ancient Greek method of representing numbers and fractions. Finally, the terminology employed in the Almagest is, in many instances, significantly different to that used in modern astronomy textbooks.

The aim of this book is to reconstruct Ptolemy's model of the Solar System employing modern mathematical methods, standard dates, and conventional astronomical terminology. It is hoped that the resulting model will enable the reader to comprehend the full extent of Ptolemy's scientific achievement. In fact, the model described in this work is a somewhat improved version of Ptolemy's, in that all of the previously mentioned deficiencies have been corrected. Furthermore, Ptolemy's equant scheme has been replaced by a Keplerian scheme, expanded to second order in the planetary eccentricities. It should be noted, however, that these two schemes are essentially indistinguishable for small eccentricity orbits. Certain aspects of the Almagest have not been reproduced. For instance, it was not thought necessary to instruct the reader on how to construct trigonometric tables, or primitive astronomical instruments. Furthermore, no attempted has been made to derive any of the model parameters directly from observational data, because the orbital elements and physical properties of the Sun, Moon, and planets are, by now, extremely well established. Any detailed discussion of the fixed stars has also been omitted, because stellar positions are also very well established, and the apparent motion of the stars in the sky is comparatively straightforward compared to those of the Sun, the Moon, and the planets. What remains is a mathematical model of the Solar System that is surprisingly accurate (the maximum errors in the ecliptic longitudes of the Sun, Moon, Mercury, Venus, Mars, Jupiter, and Saturn during the years 1995–2006 AD are $0.7'$, $14'$, $28'$, $10'$, $14'$, $4'$, and $1'$, respectively), yet sufficiently simple that all of the necessary calculations can be performed by hand, with the aid of tables. The form of the calculations, as well as the layout of the tables, is, for the most part, fairly similar to those found in the Almagest. Many examples of the use of the tables are provided.

2. Spherical astronomy

2.1 The celestial sphere

It is often helpful to imagine that celestial objects are attached to a vast sphere centered on the Earth. This fictitious construction is known as the *celestial sphere*. The Earth's dimensions are assumed to be infinitesimally small compared to those of the sphere. (Because the distance of a typical celestial object from the Earth is very much larger than the Earth's radius.) It follows that only half of the sphere is visible from any particular observation site on the Earth's surface. Furthermore, the angular position of a given celestial object (relative to some fixed celestial reference) is the same at all such sites. In other words, there is negligible parallax associated with viewing the same celestial object from different observation sites on the surface of the Earth.[1]

2.2 Celestial motions

Celestial objects exhibit two distinct types of motion. The first motion is such that the whole celestial sphere, and all of the celestial objects attached to it, rotates uniformly from east to west once every 24 (sidereal) hours, about a fixed axis passing through the Earth's north and south poles. This type of motion is called *diurnal motion*, and is a consequence of the Earth's daily rotation. Diurnal motion preserves the relative angular positions of all celestial objects. However, certain celestial objects, such as the Sun, the Moon, and the planets, possess a second motion, superimposed on the first, that causes their angular positions to slowly change relative to one another, and to the fixed stars. This *intrinsic motion* of objects in the Solar System is due to a combination of the Earth's orbital motion about the Sun, and the orbital motions of the Moon and the planets about the Earth and the Sun, respectively.

Incidentally, the terms for the Earth, the Sun, and the Moon in Greek are ἡ γῆ, ὁ ἥλιος, and ἡ σελήνη, respectively. Likewise, the terms for the planets Mercury, Venus, Mars, Jupiter, and Saturn are ὁ τοῦ Ἑρμοῦ [the (star) of Hermes], ὁ τῆς Ἀφροδίτης [the (star) of Aphrodite], ὁ τοῦ Ἄρεως [the (star) of Ares], ὁ τοῦ Διός [the (star) of Zeus], and ὁ τοῦ Κρόνου [the (star) of Cronus]. Note that the word star was usually taken as read in Greek.

2.3 Celestial coordinates

Consider Figure 2.1. The celestial sphere rotates about the celestial axis, PP', which is the imagined extension of the Earth's axis of rotation. This axis intersects the celestial sphere at the *north*

[1] The one exception to this rule is the Moon, which is sufficiently close to the Earth that its parallax is significant. See Section 6.9.

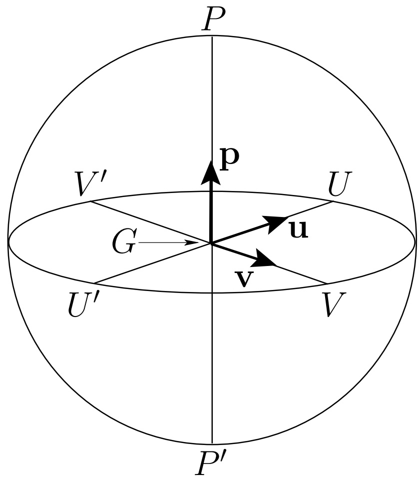

Figure 2.1: The celestial sphere. Here, G, P, P', V, and V' represent the Earth, the north celestial pole, the south celestial pole, the vernal equinox, and the autumnal equinox, respectively. Moreover, $VUV'U'$ is the celestial equator, and PP' the celestial axis.

celestial pole, P, and the *south celestial pole*, P'. It follows that the two celestial poles are unaffected by diurnal motion, and remain fixed in the sky.

The *celestial equator*, $VUV'U'$, is the intersection of the Earth's equatorial plane with the celestial sphere, and is therefore perpendicular to the celestial axis. The so-called *vernal equinox*, V, is a particular point on the celestial equator that is used as the origin of celestial longitude. Furthermore, the *autumnal equinox*, V', is a point that lies directly opposite the vernal equinox on the celestial equator. Let the line UU' lie in the plane of the celestial equator such that it is perpendicular to VV', as shown in the figure.

It is helpful to define three, right-handed, mutually perpendicular, unit vectors; $\mathbf{v}$, $\mathbf{u}$, and $\mathbf{p}$. Here, $\mathbf{v}$ is directed from the Earth to the vernal equinox, $\mathbf{u}$ from the Earth to point U, and $\mathbf{p}$ from the Earth to the north celestial pole. See Figure 2.1.

Let G be the Earth. Consider a general celestial object, R. See Figure 2.2. The location of R on the celestial sphere is conveniently specified by two angular coordinates, δ and α. Let GR' be the projection of GR onto the equatorial plane. The coordinate δ, which is known as *declination*, is the angle subtended between GR' and GR. Objects north of the celestial equator have positive declinations, and vice versa. It follows that objects on the celestial equator have declinations of 0°, whereas the north and south celestial poles have declinations of $+90^\circ$ and -90°, respectively. The coordinate α, which is known as *right ascension*, is the angle subtended between GV and GR'. Right ascension increases from west to east (that is, in the opposite direction to the celestial sphere's diurnal rotation). Thus, the vernal and autumnal equinoxes have right ascensions of 0° and 180°, respectively. Note that α lies in the range 0° to 360°. Right

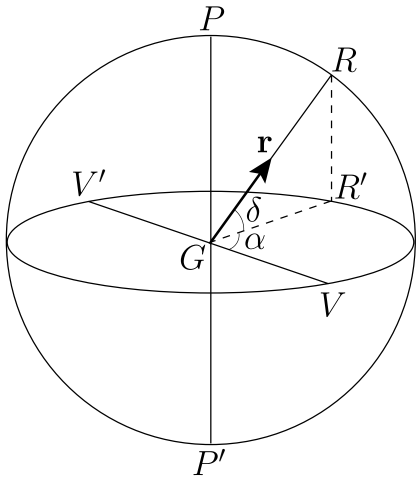

Figure 2.2: Celestial coordinates. Here, G is the Earth, R a celestial object, and R' the object's projection onto the plane of the celestial equator, $VR'V'$.

ascension is sometimes measured in hours, instead of degrees, with one hour corresponding to 15° (because it takes 24 (sidereal) hours for the celestial sphere to complete one diurnal rotation). In this scheme, the vernal and autumnal equinoxes have right ascensions of 0 hours and 12 hours, respectively. Moreover, α lies in the range 0 to 24 hours. (Incidentally, in this book, α is measured relative to the mean equinox at date, unless otherwise specified.) Finally, let **r** be a unit vector that is directed from the Earth to R. See Figure 2.2. It is easily demonstrated that

$$\mathbf{r} = \cos\delta\,\cos\alpha\,\mathbf{v} + \cos\delta\,\sin\alpha\,\mathbf{u} + \sin\delta\,\mathbf{p}, \tag{2.1}$$

and

$$\sin\delta = \mathbf{r}\cdot\mathbf{p}, \tag{2.2}$$

$$\tan\alpha = \frac{\mathbf{r}\cdot\mathbf{u}}{\mathbf{r}\cdot\mathbf{v}}. \tag{2.3}$$

2.4 The ecliptic circle

During the course of a year, the Sun's intrinsic motion causes it to trace out a fixed circle that bisects the celestial sphere. This circle is known as the *ecliptic*. The Sun travels around the ecliptic from west to east (that is, in the opposite direction to the celestial sphere's diurnal rotation). Moreover, the ecliptic circle is inclined at a fixed angle of $\epsilon = 23^\circ 26'$ to the celestial equator. This angle actually represents the fixed inclination of the Earth's axis of rotation to the normal

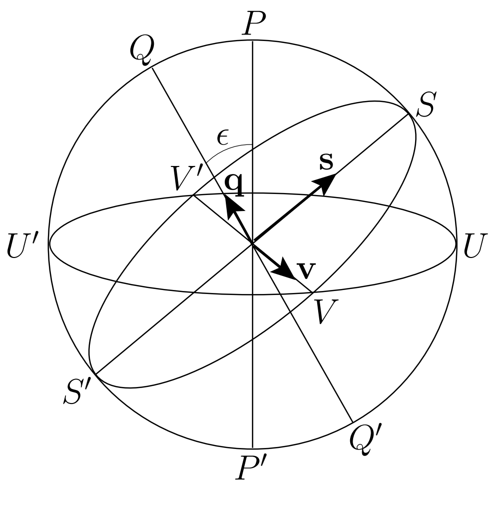

Figure 2.3: The ecliptic circle. Here, P, P', Q, Q', V, V', S, and S' denote the north celestial pole, the south celestial pole, the north ecliptic pole, the south ecliptic pole, the vernal equinox, the autumnal equinox, the summer solstice, and the winter solstice, respectively. Moreover, $VUV'U'$ is the celestial equator, $VSV'S'$ the ecliptic, and PP' the celestial axis.

to its orbital plane.[2]

The vernal equinox, V, is defined as the point at which the ecliptic crosses the celestial equator from south to north (in the direction of the Sun's ecliptic motion). See Figure 2.3. Likewise, the autumnal equinox, V', is the point at which the ecliptic crosses the celestial equator from north to south. In addition, the *summer solstice*, S, is the point on the ecliptic that is furthest north of the celestial equator, whereas the *winter solstice*, S', is the point that is furthest south. It follows that the lines VV' and SS' are perpendicular. Let QQ' be the normal to the plane of the ecliptic that passes through the Earth, as shown in Figure 2.3. Here, Q is termed the *northern ecliptic pole*, and Q' the *southern ecliptic pole*. It is easily demonstrated that

$$\mathbf{s} = \cos\epsilon\,\mathbf{u} + \sin\epsilon\,\mathbf{p}, \tag{2.4}$$

$$\mathbf{q} = -\sin\epsilon\,\mathbf{u} + \cos\epsilon\,\mathbf{p}, \tag{2.5}$$

where $\mathbf{s}$ is a unit vector that is directed from the Earth to the summer solstice, and $\mathbf{q}$ a unit vector that is directed from the Earth to the north ecliptic pole. See Figure 2.3. We can also write

$$\mathbf{u} = \cos\epsilon\,\mathbf{s} - \sin\epsilon\,\mathbf{q}, \tag{2.6}$$

$$\mathbf{p} = \sin\epsilon\,\mathbf{s} + \cos\epsilon\,\mathbf{q}. \tag{2.7}$$

[2] In fact, ϵ is very slowly decreasing in time. The value of ϵ used in the Almagest is $23^\circ 51'$. However, the true value of ϵ in Ptolemy's day was $23^\circ 41'$.

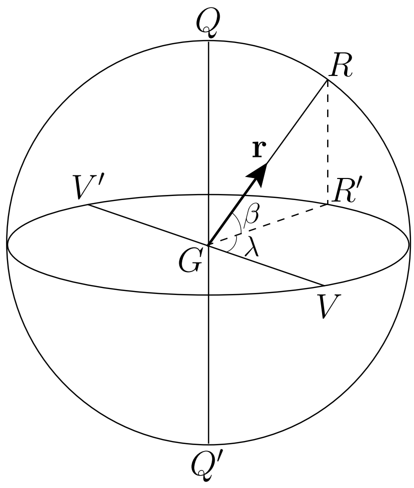

Figure 2.4: Ecliptic coordinates. Here, G is the Earth, R a celestial object, and R' the object's projection onto the ecliptic plane, $VR'V'$.

Thus, **v**, **s**, and **q** constitute another right-handed, mutually perpendicular, set of unit vectors.

2.5 Ecliptic coordinates

It is convenient to specify the positions of the Sun, Moon, and planets in the sky using a pair of angular coordinates, β and λ, that are measured with respect to the ecliptic, rather than the celestial equator. Let G and R denote the Earth and a celestial object, respectively, and let GR' denote the projection of the line GR onto the plane of the ecliptic, $VR'V'$. See Figure 2.4. The coordinate β, which is known as *ecliptic latitude*, is the angle subtended between GR' and GR. Objects north of the ecliptic plane have positive ecliptic latitudes, and vice versa. The coordinate λ, which is known as *ecliptic longitude*, is the angle subtended between GV and GR'. Ecliptic longitude increases from west to east (that is, in the same direction that the Sun travels around the ecliptic). (Again, in this book, λ is measured relative to the mean equinox at date, unless specified otherwise.) Note that the basis vectors in the ecliptic coordinate system are **v**, **s**, and **q**, whereas the corresponding basis vectors in the celestial coordinate system are **v**, **u**, and **p**. See Figures 2.1 and 2.3. By analogy with Equations (2.1)–(2.3), we can write

$$\mathbf{r} = \cos\beta\,\cos\lambda\,\mathbf{v} + \cos\beta\,\sin\lambda\,\mathbf{s} + \sin\beta\,\mathbf{q}, \tag{2.8}$$

$$\sin\beta = \mathbf{r}\cdot\mathbf{q}, \tag{2.9}$$

$$\tan\lambda = \frac{\mathbf{r}\cdot\mathbf{s}}{\mathbf{r}\cdot\mathbf{v}}, \tag{2.10}$$

where **r** is a unit vector that is directed from G to R. Hence, it follows from Equations (2.1), (2.4), and (2.5) that

$$\sin\beta = \cos\epsilon\,\sin\delta - \sin\epsilon\,\cos\delta\,\sin\alpha, \tag{2.11}$$

$$\tan\lambda = \frac{\cos\epsilon\,\cos\delta\,\sin\alpha + \sin\epsilon\,\sin\delta}{\cos\delta\,\cos\alpha}. \tag{2.12}$$

These expressions specify the transformation from celestial to ecliptic coordinates. The inverse transformation follows from Equations (2.2), (2.3), and (2.6)–(2.8):

$$\sin\delta = \cos\epsilon\,\sin\beta + \sin\epsilon\,\cos\beta\,\sin\lambda, \tag{2.13}$$

$$\tan\alpha = \frac{\cos\epsilon\,\cos\beta\,\sin\lambda - \sin\epsilon\,\sin\beta}{\cos\beta\,\cos\lambda}. \tag{2.14}$$

Figure 2.13 shows all stars of visible magnitude less than $+6$ lying within 15° of the ecliptic. Table 2.1 gives the ecliptic longitudes (relative to the start of the appropriate zodiacal sign; see next section), ecliptic latitudes, and visible magnitudes of a selection of these stars that lie within 10° of the ecliptic. The figure and the table can be used to convert ecliptic longitude and latitude into approximate position in the sky against the backdrop of the fixed stars.

Figure 2.13 and Table 2.1 contain more or less the same information as that contained in Section 5 of Book VII of the Almagest (although we have restricted our attention to stars that lie relatively close to the ecliptic). Incidentally, the ancient Greeks did not individually name stars, describing them instead in figurative terms, such as "the bright, reddish star on the right shoulder of Orion" (Betelgeuse).

2.6 The signs of the zodiac

The *signs of the zodiac* are a well-known set of names given to 30° long segments of the ecliptic circle. Thus, the sign of Aries extends over the range of ecliptic longitudes 0°–30°, the sign of Taurus over the range 30°–60°, and so on. Note that, as a consequence of the precession of the equinoxes, the signs of the zodiac no longer coincide with the constellations of the same name. See Figure 2.13. (Incidentally, Ptolemy was well aware of this fact because, even in his day, the signs of the zodiac did not coincide with the constellations. The signs coincided with the constellations in the time of the ancient Mesopotamians; that is, about 1500 BC.) The 12 zodiacal signs are listed in the following table. It can be seen from the table that ecliptic longitude 72° corresponds to the twelfth degree of Gemini, and ecliptic longitude 242° to the second degree of Sagittarius, et cetera.

Incidentally, the term zodiac comes from the Greek ζῳδιακός κύκλος, meaning "circle of little animals". The Greek names for the zodiacal signs were Κριός (Ram), Ταῦρος (Bull), Δίδυμοι (Twins), Κάρκινος (Crab), Λέων (Lion), Παρθένος (Virgin), Ζυγός (Balance/scale), Σκορπιός (Scorpion), Τοξότης (Archer), Αἰγόκερως (Goat-horned), Ὑδροχόος (Water-pourer), and Ἰχθύες (Fishes). The familiar names listed in the previous table are merely latinized versions of the Greek names.

Sign	Abbr.	Longitude	Sign	Abbr.	Longitude	Sign	Abbr.	Longitude
Aries	AR	000°–030°	Leo	LE	120°–150°	Sagittarius	SG	240°–270°
Taurus	TA	030°–060°	Virgo	VI	150°–180°	Capricorn	CP	270°–300°
Gemini	GE	060°–090°	Libra	LI	180°–210°	Aquarius	AQ	300°–330°
Cancer	CN	090°–120°	Scorpio	SC	210°–240°	Pisces	PI	330°–360°

2.7 Ecliptic declinations and right ascensions.

According to Equations (2.13) and (2.14), the celestial coordinates of a point on the ecliptic circle (which corresponds to $\beta = 0$) that has ecliptic longitude λ are specified by

$$\sin\delta = \sin\epsilon\,\sin\lambda, \tag{2.15}$$

$$\tan\alpha = \cos\epsilon\,\tan\lambda. \tag{2.16}$$

The previous formulae have been used to construct Table 2.2, which lists the declinations and right ascensions of a set of equally spaced points on the ecliptic circle. Note that λ is measured relative to the start of the appropriate zodiacal sign.

Table 2.2 contains essentially the same information as is contained in the "Table of inclinations" (Κανόνιον λοξώσεως) that appears in Section 15 of Book I of the Almagest. The slight difference between the entries appearing in our tables and those appearing in Ptolemy's table is due to the fact that Ptolemy adopted the value $23^\circ 51' 20''$ for the obliquity of the ecliptic, whereas we are using the modern (and correct) value $23^\circ 26'$.

2.8 Local horizon and meridian

Consider a general observation site X located on the surface of the Earth. (Note that, in the following, it is tacitly assumed that the site lies the Earth's northern hemisphere. However, the analysis also applies to sites situated in the the southern hemisphere.) The local *zenith*, Z, is the point on the celestial sphere that is directly overhead at X, whereas the *nadir*, Z', is the point that is directly underfoot. See Figure 2.5. The *horizon* is the tangent plane to the Earth at X, and divides the celestial sphere into two halves. The upper half, containing the zenith, is visible from site X, whereas the lower half is invisible.

Figure 2.6 shows the visible half of the celestial sphere at observation site X. Here, $NESW$ is the local horizon, and N, E, S, and W are the north, east, south, and west compass points, respectively. The plane $NPZS$, which passes through the north and south compass points, as well as the zenith, is known as the local *meridian*. The meridian is perpendicular to the horizon. The north celestial pole lies in the meridian plane, and is elevated an angular distance L above the north compass point. See Figures 2.5 and 2.6. Here, L is the terrestrial *latitude* of observation site X. It is helpful to define three, right-handed, mutually perpendicular, local unit vectors; **e**, **n**, and **z**. Here, **e** is directed toward the east compass point, **n** toward the north compass point, and **z** toward the zenith. See Figures 2.6.

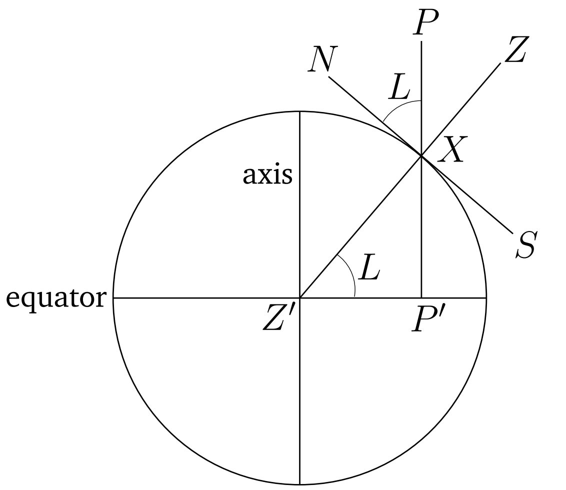

Figure 2.5: A general observation site X, of latitude L, on the surface of the Earth. Here, P, P', Z, and Z' denote the directions to the north celestial pole, south celestial pole, zenith, and nadir, respectively. The line NS represents the local horizon.

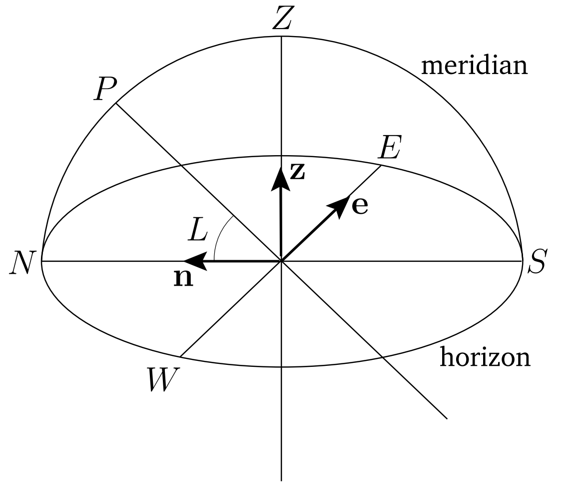

Figure 2.6: The local horizon and meridian. Here, N, S, E, W denote the north. south, east, and west compass points, Z the zenith, and P the north celestial pole. Moreover, $NESW$ is the horizon, and $NPZS$ the meridian.

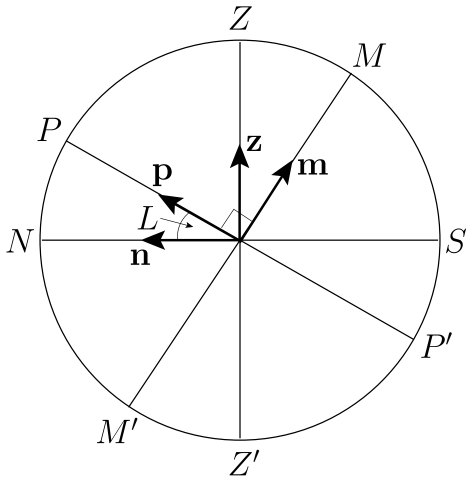

Figure 2.7: The local meridian.

Figure 2.7 shows the meridian plane at X. Let the line MM' lie in this plane such that it is perpendicular to the celestial axis, PP'. Moreover, let M lie in the visible hemisphere. It is helpful to define the unit vector **m** which is directed toward M, as shown in the diagram. It is easily seen that

$$\mathbf{n} = \cos L\,\mathbf{p} - \sin L\,\mathbf{m}, \tag{2.17}$$

$$\mathbf{z} = \sin L\,\mathbf{p} + \cos L\,\mathbf{m}. \tag{2.18}$$

Figure 2.8 shows the celestial equator viewed from observation site X. Here, α_0 is the right ascension of the celestial objects culminating (that is, attaining their highest altitude in the sky) on the meridian at the time of observation. Incidentally, it is easily demonstrated that all objects culminating on the meridian at any instant in time have the same right ascension. Note that the angle α_0 increases uniformly in time, at the rate of 15° a (sidereal) hour, due to the diurnal motion of the celestial sphere. It can be seen from the diagram that

$$\mathbf{m} = \sin\alpha_0\,\mathbf{u} + \cos\alpha_0\,\mathbf{v}, \tag{2.19}$$

$$\mathbf{e} = \cos\alpha_0\,\mathbf{u} - \sin\alpha_0\,\mathbf{v}. \tag{2.20}$$

Thus, from Equations (2.17) and (2.18),

$$\mathbf{e} = -\sin\alpha_0\,\mathbf{v} + \cos\alpha_0\,\mathbf{u}, \tag{2.21}$$

$$\mathbf{n} = -\sin L\,\cos\alpha_0\,\mathbf{v} - \sin L\,\sin\alpha_0\,\mathbf{u} + \cos L\,\mathbf{p}, \tag{2.22}$$

$$\mathbf{z} = \cos L\,\cos\alpha_0\,\mathbf{v} + \cos L\,\sin\alpha_0\,\mathbf{u} + \sin L\,\mathbf{p}. \tag{2.23}$$

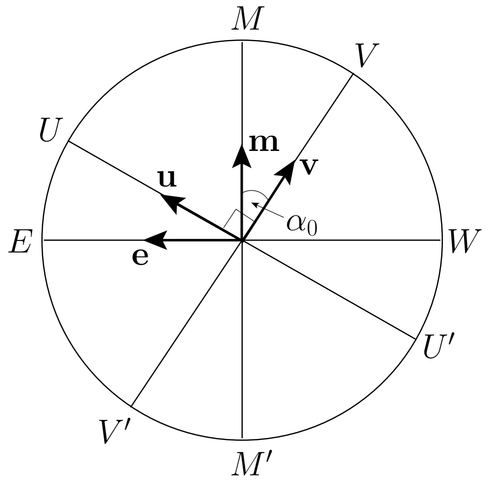

Figure 2.8: The local celestial equator.

Similarly, from Equations (2.6) and (2.7),

$$\begin{aligned}
\mathbf{e} &= -\sin\alpha_0\,\mathbf{v} + \cos\epsilon\,\cos\alpha_0\,\mathbf{s} - \sin\epsilon\,\cos\alpha_0\,\mathbf{q}, &(2.24)\\
\mathbf{n} &= -\sin L\,\cos\alpha_0\,\mathbf{v} + (\cos L\,\sin\epsilon - \sin L\,\cos\epsilon\,\sin\alpha_0)\,\mathbf{s}\\
&\quad + (\cos L\,\cos\epsilon + \sin L\,\sin\epsilon\,\sin\alpha_0)\,\mathbf{q}, &(2.25)\\
\mathbf{z} &= \cos L\,\cos\alpha_0\,\mathbf{v} + (\sin L\,\sin\epsilon + \cos L\,\cos\epsilon\,\sin\alpha_0)\,\mathbf{s}\\
&\quad + (\sin L\,\cos\epsilon - \cos L\,\sin\epsilon\,\sin\alpha_0)\,\mathbf{q}. &(2.26)
\end{aligned}$$

2.9 Horizontal coordinates

It is convenient to specify the positions of celestial objects in the sky, when viewed from a particular observation site, X, on the Earth's surface, using a pair of angular coordinates, a and A, that are measured with respect to the local horizon. Let R denote a celestial object, and XR' the projection of the line XR onto the horizontal plane, $NESW$. See Figure 2.9. The coordinate a, which is known as *altitude*, is the angle subtended between XR' and XR. Objects above the horizon have positive altitudes, whereas objects below the horizon have negative altitudes. The zenith has altitude 90°, and the horizon altitude 0°. The coordinate A, which is known as *azimuth*, is the angle subtended between XN and XR'. Azimuth increases from the north towards the east. Thus, the north, east, south, and west compass points have azimuths of 0°, 90°, 180°, and 270°, respectively. Note that the basis vectors in the horizontal coordinate system are **e**, **n**, and **z**, whereas the corresponding basis vectors in the celestial coordinate system are **v**, **u**, and **p**. See Figures 2.1 and 2.6. By analogy with Equations (2.1)–(2.3), we can write

$$\mathbf{r} = \cos a\,\sin A\,\mathbf{e} + \cos a\,\cos A\,\mathbf{n} + \sin a\,\mathbf{z}, \tag{2.27}$$

$$\sin a = \mathbf{r}\cdot\mathbf{z}, \tag{2.28}$$

$$\tan A = \frac{\mathbf{r}\cdot\mathbf{e}}{\mathbf{r}\cdot\mathbf{n}}, \tag{2.29}$$

where **r** is a unit vector directed from X to R. Hence, it follows from Equations (2.1), and (2.22)–(2.23), that

$$\sin a = \sin L\,\sin\delta + \cos L\,\cos\delta\,\cos(\alpha-\alpha_0), \tag{2.30}$$

$$\tan A = \frac{\cos\delta\,\sin(\alpha-\alpha_0)}{\cos L\,\sin\delta - \sin L\,\cos\delta\,\cos(\alpha-\alpha_0)}. \tag{2.31}$$

These expressions allow us to calculate the altitude and azimuth of a celestial object of declination δ and right ascension α that is viewed from an observation site on the Earth's surface of terrestrial latitude L at an instance in time when celestial objects of right ascension α_0 are culminating at the meridian.. According to Equations (2.8), and (2.25)–(2.26), the altitude and azimuth of a similarly viewed point on the ecliptic (which corresponds to $\beta = 0$) of ecliptic longitude λ are given by

$$\sin a = \cos L\,\cos\lambda\,\cos\alpha_0 + \sin L\,\sin\epsilon\,\sin\lambda + \cos L\,\cos\epsilon\,\sin\lambda\,\sin\alpha_0, \tag{2.32}$$

$$\tan A = \frac{\cos\epsilon\,\sin\lambda\,\cos\alpha_0 - \cos\lambda\,\sin\alpha_0}{\cos L\,\sin\epsilon\,\sin\lambda - \sin L\,\cos\lambda\,\cos\alpha_0 - \sin L\,\cos\epsilon\,\sin\lambda\,\sin\alpha_0}. \tag{2.33}$$

2.10 Meridian transits

Consider a celestial object, of declination δ and right ascension α, that is viewed from an observation site on the Earth's surface of terrestrial latitude L. According to Equation (2.30), the object culminates, or attains its highest altitude in the sky, when $\alpha_0 = \alpha$. This event is known as an *upper transit*. Furthermore, the object attains its lowest altitude in the sky when $\alpha_0 = 180^\circ + \alpha$. This event is known as a *lower transit*. Both upper and lower transits take place as the object in question passes through the meridian plane.

According to Equation (2.30), the altitude of a celestial object at its upper transit satisfies $\sin a_+ = \cos(L-\delta)$, implying that

$$a_+ = 90^\circ - |L-\delta|. \tag{2.34}$$

Likewise, the altitude at its lower transit satisfies $\sin a_- = -\cos(L+\delta)$, giving

$$a_- = |L+\delta| - 90^\circ. \tag{2.35}$$

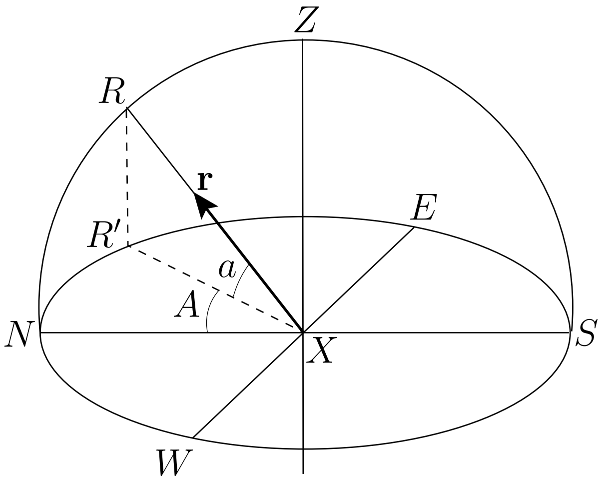

Figure 2.9: Horizontal coordinates. Here, R is a celestial object, and R' its projection onto the horizontal plane, $NESW$.

The previous two expressions allow us to group celestial objects into three classes. Objects with declinations satisfying $|L+\delta| > 90^\circ$ never set; that is, their lower transits lie above the horizon. Objects with declinations satisfying $|L-\delta| > 90^\circ$ never rise; that is, their upper transits lie below the horizon. Finally, objects with declinations that satisfy neither of the two previous inequalities both rise and set during the course of a day. It follows that all celestial objects appear to rise and set when viewed from an observation site on the terrestrial equator (which corresponds to $L = 0^\circ$) . On the other hand, when viewed from an observation site at the north pole (which corresponds to $L = 90^\circ$), objects north of the celestial equator never set, while objects south of the celestial equator never rise, and vice versa for objects viewed from the south pole. All three classes of celestial object are present when the sky is viewed from an observation site on the Earth's surface of intermediate latitude.

2.11 Principal terrestrial latitude circles

According to Equation (2.15), the Sun's declination varies between $-\epsilon$ and $+\epsilon$ during the course of a year. It follows from Equation (2.34) that it is only possible for the Sun to have an upper transit at the zenith in a region of the Earth whose latitude lies between $-\epsilon$ and ϵ. The circles of latitude bounding this region are known as the *tropics*. Thus, the *tropic of Capricorn*—so-called because the Sun is at the winter solstice, and, therefore, at the first point of Capricorn (that is, the zeroth degree of Capricorn), when it culminates at the zenith at this latitude—lies at $L = -23^\circ 26'$. Moreover, the *tropic of Cancer*—so-called because the Sun is at the summer solstice, and, therefore, at the first point of Cancer, when it culminates at the zenith at this latitude—lies

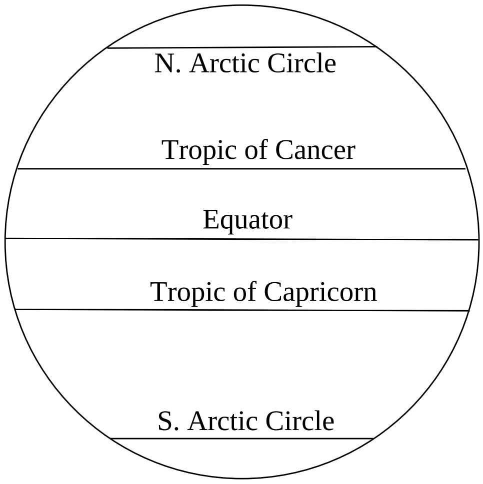

Figure 2.10: The principal latitude circles of the Earth.

at $L = +23^\circ 26'$. Incidentally, the word "tropic" derives from the Greek τροπή, meaning a turn or change in direction.

Equations (2.34) and (2.35) imply that the Sun does not rise for part of the year, and does not set for part of the year, in two regions of the Earth whose terrestrial latitudes satisfy $|L| > 90^\circ - \epsilon$. These two regions are bounded by the poles and two circles of latitude known as the *arctic circles*. The *south arctic circle* lies at $L = -66^\circ 34'$. Likewise, the *north arctic circle* lies at $L = +66^\circ 34'$. Incidentally, the word "arctic" comes from the Greek ἀρκτικός, meaning "of the bear", and is so called because the most prominent northern constellations are the "great bear" and the "little bear".

The equator, the two tropics, and the two arctic circles constitute the five *principal latitude circles* of the Earth, and are shown in Figure 2.10.

2.12 Equinoxes and solstices

The ecliptic longitude of the Sun when it reaches the vernal equinox is $\lambda = 0^\circ$. It follows, from Equation (2.32), that the altitude of the Sun on the day of the equinox is given by $\sin a = \cos L \, \cos\alpha_0$. Thus, the Sun rises when $\alpha_0 = -90^\circ$, culminates at an altitude of $90^\circ - |L|$ when $\alpha_0 = 0^\circ$, and sets when $\alpha_0 = 90^\circ$. We conclude that the length of the equinoctial day is 180 time-degrees, which is equivalent to 12 hours (because 15° of right ascension cross the meridian in one hour). Thus, day and night are equally long on the day of the vernal equinox. (The word equinox is derived from the Latin word *aequinoctium*, which means "equal night".) It is easily demonstrated that the same is true on the day of the autumnal equinox.

The ecliptic longitude of the Sun when it reaches the summer solstice is $\lambda = 90^\circ$. It follows that the altitude of the Sun on the day of the solstice is given by $\sin a = \sin L \sin\epsilon + \cos L \cos\epsilon \sin\alpha_0$. Thus, the Sun rises when $\alpha_0 = -\sin^{-1}(\tan L \tan\epsilon)$, culminates at an altitude of $90^\circ - |L-\epsilon|$ when $\alpha_0 = 90^\circ$, and sets when $\alpha_0 = 180^\circ + \sin^{-1}(\tan L \tan\epsilon)$. We conclude that the length of the longest day of the year in the Earth's northern hemisphere (which, of course, occurs when the Sun reaches the summer solstice) is $180^\circ + 2\sin^{-1}(\tan L \tan\epsilon)$ time-degrees. Likewise, the length of the shortest night (which also occurs at the summer solstice) is $180 - 2\sin^{-1}(\tan L \tan\epsilon)$ time-degrees. These formulae are only valid for northern latitudes below the arctic circle. At higher latitudes, the Sun never sets for part of the year, and the longest "day" is consequently longer than 24 hours. It is easily demonstrated that the shortest day in the Earth's northern hemisphere, which takes place when the Sun reaches the winter solstice, is equal to the shortest night, and the longest night (which also occurs at the winter solstice) to the longest day. Moreover, the Sun culminates at an altitude of $90^\circ - |L+\epsilon|$ on day of the winter solstice. Again, at latitudes above the arctic circle, the Sun never rises for part of the year, and the longest "night" is consequently longer than 24 hours.

Consider an observation site on the Earth's surface of latitude L that lies above the northern arctic circle. The declination of the Sun on the first day after the spring equinox on which it fails to set is $\delta = 90^\circ - L$. According to Equation (2.15), the Sun's ecliptic longitude on this day is $\sin^{-1}(\cos L/\sin\epsilon)$. Likewise, the declination of the Sun on the day when it starts to set again is $\delta = 90^\circ - L$, and its ecliptic longitude is $180^\circ - \sin^{-1}(\cos L/\sin\epsilon)$. Assuming that the Sun travels around the ecliptic circle at a uniform rate (which is approximately true), the fraction of a year that the Sun stays above the horizon in summer is $0.5 - \sin^{-1}(\cos L/\sin\epsilon)/180^\circ$. It is easily demonstrated that the fraction of a year that the Sun stays below the horizon in winter is also $0.5 - \sin^{-1}(\cos L/\sin\epsilon)/180^\circ$.

2.13 Terrestrial climes

Table 2.3 specifies the length of the longest day, as well as the altitude of the Sun when it culminates at the meridian on the days of the equinoxes and solstices, calculated for a set of observation sites in the northern hemisphere with equally spaced terrestrial latitudes. This table was constructed using the formulae in the previous section. The table can be adapted to observation sites in the Earth's southern hemisphere via the following simple transformation: $L \to -L$, Summer $\leftrightarrow$ Winter, N $\leftrightarrow$ S. For instance, at a latitude of -10°, the longest day, which corresponds to the winter solstice, is of length 12^h35^m. Moreover, on this day, the Sun's upper transit is south of the zenith, at an altitude of $+76^\circ 34'$.

Table 2.3 contains essentially the same information as is contained in the discussion that appears in Section 6 of Book II of the Almagest. Incidentally, the word "clime", meaning a region defined by its weather, comes from the Greek κλίμα, meaning slope of inclination. Of course, the inclination in question is the inclination of the Sun's rays to the vertical at a given location on the Earth.

2.14 Ecliptic ascensions

Consider the rising, or *ascension*, of celestial objects at the eastern horizon, as viewed from a particular observation site on the Earth's surface. If the observation site lies on the terrestrial equator then all celestial objects appear to ascend at right angles to the horizon. This process is known as *right ascension*. On the other hand, if the observation site does not lie at the equator then celestial objects appear to ascend at an oblique angle to the horizon. This process is known as *oblique ascension*. For the case of right ascension, it is easily demonstrated that all celestial objects with the same celestial longitude ascend simultaneously. Indeed, celestial longitude is generally known as "right ascension" because, in the case of right ascension, the celestial longitude of an object (in hours) is simply the time elapsed between the ascension of the vernal equinox, and the ascension of the object in question.

Let us now consider the ascension of points on the ecliptic. Applying Equation (2.30) to a point on the celestial equator (which corresponds to $\delta = 0$) of right ascension α, we obtain

$$\sin a = \cos L\, \cos(\alpha - \alpha_0) = \cos L\, \sin(\alpha_0 - \alpha + 90^\circ). \tag{2.36}$$

It follows that we can write

$$\alpha_0 = \alpha - 90^\circ, \tag{2.37}$$

where α is the right ascension of the point on the celestial equator that ascends at the eastern horizon (that is, $a = 0$ and $da/dt > 0$) at the same time that celestial objects of right ascension α_0 are culminating at the meridian. Substituting this result into Equation (2.32), we get

$$\sin a = \cos L\, \cos\lambda\, \sin\alpha + \sin L\, \sin\epsilon\, \sin\lambda - \cos L\, \cos\epsilon\, \sin\lambda\, \cos\alpha, \tag{2.38}$$

which implies that if $a = 0$ and $da/dt > 0$ then

$$\tan\lambda = \frac{\cos L\, \sin\alpha}{\cos L\, \cos\epsilon\, \cos\alpha - \sin\epsilon\, \sin L}. \tag{2.39}$$

This expression specifies the ecliptic longitude, λ, of the point on the ecliptic circle that ascends simultaneously with a point on the celestial equator of right ascension α. Note, incidentally, that points on the celestial equator ascend at a uniform rate of 15° an hour at all viewing sites on the Earth's surface (except the poles, where the celestial equator does not ascend at all). The same is not true of points on the ecliptic. Expression (2.39) can be inverted to give

$$\alpha = \tan^{-1}(\tan\lambda\, \cos\epsilon) - \sin^{-1}\left[\frac{\sin\lambda\, \sin\epsilon\, \tan L}{(1 - \sin^2\lambda\, \sin^2\epsilon)^{1/2}}\right]. \tag{2.40}$$

The solution of Equation (2.40) for observation sites lying above the arctic circle is complicated by the fact that, at such sites, a section of the ecliptic never sets, or descends, and a section never ascends. It is easily demonstrated that the section that never descends lies between ecliptic longitudes λ_c and $180^\circ - \lambda_c$, whereas the section that never ascends lies between longitudes

$180^\circ + \lambda_c$ and $360^\circ - \lambda_c$. Here, $\lambda_c = \sin^{-1}(\cos L/\sin\epsilon)$. Points on the ecliptic of longitude λ_c, $180^\circ - \lambda_c$, $180^\circ + \lambda_c$, and $360^\circ - \lambda_c$ ascend simultaneously with points on the celestial equator of right ascension $360^\circ - \alpha_c$, α_c, $360^\circ - \alpha_c$, and α_c, respectively. Here, $\alpha_c = \cos^{-1}(1/\tan L\, \tan\epsilon)$.

Tables 2.4–2.16 list the ascensions of a series of equally spaced points on the ecliptic circle, as viewed from a set of observation sites in the Earth's northern hemisphere with different terrestrial latitudes. The tables were calculated with the aid of formula (2.40). Let us now illustrate the use of these tables.

Consider a day on which the Sun is at ecliptic longitude 14LE00 (that is, $14^\circ 00'$ into the sign of Leo). What is the length of the day (that is, the period between sunrise and sunset) at an observation site on the Earth's surface of latitude $+30^\circ$. Consulting Table 2.7, we find that the Sun ascends simultaneously with a point on the celestial equator of right ascension $126^\circ 32'$. Now, the ecliptic is a great circle on the celestial sphere. Hence, exactly half of the ecliptic is visible from any observation site on the Earth's surface. This implies that when a given point on the ecliptic circle is ascending, the point directly opposite it on the circle is descending, and vice versa. Let us term the directly opposite point the *complementary point*. By definition, the difference in ecliptic longitude between a given point on the ecliptic circle and its complementary point is 180°. Thus, the complementary point to 14LE00 is 14AQ00. It follows that 14AQ00 ascends at the same time that 14LE00 descends. In other words, the Sun sets when 14AQ00 ascends. Consulting Table 2.7, we find that the Sun sets at the same time that a point on the celestial equator of right ascension $326^\circ 23'$ rises. Thus, in the time interval between the rising and setting of the Sun, a $326^\circ 23' - 126^\circ 32' = 199^\circ 51'$ section of the celestial equator ascends at the eastern horizon. However, points on the celestial equator ascend at the uniform rate of 15° an hour. Thus, the length, in hours, of the period between the rising and setting of the Sun is $199^\circ 51'/15^\circ = 13^h 19^m$. In other words, the length of the day in question is $13^h 19^m$.

The previous calculation is slightly inaccurate for a number of reasons. Firstly, it neglects the fact that the Sun is continuously moving on the ecliptic circle at the rate of about 1° a day. Secondly, it neglects the fact that the celestial equator ascends at the rate of 15° per sidereal, rather than solar, hour. A sidereal hour is $1/24$th of a *sidereal day*, which is the time between successive upper transits of a fixed celestial object, such as a star. On the other hand, a solar hour is $1/24$th of a *solar day*, which is the mean time between successive upper transits of the Sun. A sidereal day is shorter than a solar day by 4 minutes. Fortunately, it turns out that these first two inaccuracies largely cancel one another out. Another source of inaccuracy is the fact that, due to refraction of light by the atmosphere, the Sun is actually 1° below the horizon when it appears to rise or set. The final source of inaccuracy is the fact that the Sun has a finite angular extent (of about half a degree), and that, strictly speaking, dawn and dusk commence when the Sun's upper limb rises and sets, respectively. Of course, our calculation only deals with the rising and setting of the center of the Sun. Taken together, the previously mentioned inaccuracies can cause the true length of a day differ from that calculated from the ascension tables by up to 15 minutes.

Tables 2.4–2.16 also effectively list the descents of a series of equally spaced points on the ecliptic circle, as viewed from a set of observation sites in the Earth's southern hemisphere with different terrestrial latitudes (which are minus those specified in the various tables). For instance, Table 2.5 gives the right ascensions of points on the celestial equator that set simultaneously with points on the ecliptic, as seen from an observation site at latitude -10°.

Consider a day on which the Sun is at ecliptic longitude 08SC00. Let us calculate the length of the day at an observation site on the Earth's surface of latitude -50°. Consulting Table 2.9, we find that the Sun sets simultaneously with a point on the celestial equator of right ascension $233^\circ 09'$. Now, the complementary point on the ecliptic to 08SC00 is 08TA00. Consulting Table 2.9 again, we find that this point sets simultaneously with a point on the celestial equator of right ascension $18^\circ 07'$. It follows that the Sun rises simultaneously with the latter point. Thus, the time interval between the rising and setting of the Sun is $233^\circ 09' - 18^\circ 07' = 215^\circ 02'$ time-degrees, or $14^h 20^m$.

The *ascendent*, or *horoscope*, is defined as the point on the ecliptic that is ascending at the eastern horizon. Suppose that we wish to find the ascendent 2.6 hours after sunrise, as seen from an observation site of latitude $+55^\circ$, on a day on which the Sun has ecliptic longitude 16SC00. Of course, knowledge of the ascendent at the time of birth is key to drawing up a natal chart in astrology. Hence, this type of calculation was of great importance to the ancient Greeks. (It was of particular importance to Ptolemy, because he wrote the standard treatise on Greek natal astrology; the so-called *Tetrabiblos*.) Consulting Table 2.10, we find that, on the day in question, the Sun rises simultaneously with a point on the celestial equator of right ascension $248^\circ 46'$. Now, 2.6 hours corresponds to $39^\circ 00'$. Thus, the ascendent rises simultaneously with a point on the celestial equator of right ascension $248^\circ 46' + 39^\circ 00' = 287^\circ 46'$. Consulting Table 2.10 again, we find that, to the nearest degree, the ascendent at the time in question has ecliptic longitude 13SG00.

Suppose, next, that we wish to find the right ascension, α, of the point on the celestial equator that culminates simultaneously with a given point on the ecliptic of ecliptic longitude λ. From Equation (2.33), we can see that if $A = 180^\circ$ then $\tan A = 0$, and $\tan\lambda\,\cos\epsilon = \tan\alpha$, or

$$\alpha = \tan^{-1}(\tan\lambda\,\cos\epsilon). \tag{2.41}$$

However, this expression is identical to expression (2.40), when the latter is evaluated for the special case $L = 0^\circ$. It follows that our problem can be solved by consulting Table 2.4, which is the ascension table for the case of right ascension. For instance, on a day on which the ecliptic longitude of the Sun is 08TA00, we find from Table 2.4 that the right ascension of the point on the celestial equator that culminates simultaneously with the Sun (in other words, that culminates at local noon) is $35^\circ 38'$. Moreover, this is the case for observation sites at all terrestrial latitudes. Note that we have effectively calculated the right ascension of the Sun on the day in question.

Suppose, finally, that we wish to find the point on the ecliptic that culminates 7 hours after local noon on the aforementioned day. Because 7 hours corresponds to 105°, the right ascension of the point on the celestial equator that culminates simultaneously with the point is question is $35^\circ 38' + 105^\circ 00' = 140^\circ 38'$. Consulting Table 2.4 again, we find that, to the nearest degree, the ecliptic longitude of the point in question is 18LE00.

Note that Tables 2.4–2.16 contain essentially the same information as is contained in the "Table of ascensions by tenth parts" (Κανόνιον τῶν κατὰ δεκαμοιρίαν ἀναφορῶν) that appears in

Section 8 of Book II of the Almagest.

2.15 Azimuth of ecliptic ascension point

Consider the azimuth of the point on the ecliptic circle that is ascending at the eastern horizon. According to Equation (2.27), the azimuth of any point on the horizon (which corresponds to $a = 0^\circ$) satisfies $\cos A = \mathbf{r}\cdot\mathbf{n}$. It follows from Equations (2.8) and (2.25) that

$$\cos A = -\cos\lambda\,\sin L\,\sin\alpha + \sin\lambda\,\cos L\,\sin\epsilon + \sin\lambda\,\sin L\,\cos\epsilon\,\cos\alpha. \tag{2.42}$$

Here, we have made use of the fact that the point in question also lies on the ecliptic (which corresponds to $\beta = 0$), as well as the fact that $\alpha_0 = \alpha - 90^\circ$, where α is the right ascension of the simultaneously rising point on the celestial equator. Moreover, λ is the ecliptic longitude of the point in question, and L the terrestrial latitude of the observation site. Now, λ and α satisfy Equation (2.39), as well as the previous equation. Thus, eliminating α between these two equations, we obtain

$$\cos A = \frac{\sin\lambda\,\sin\epsilon}{\cos L}. \tag{2.43}$$

This expression gives the azimuth, A, of the ascending point of the ecliptic as a function of its ecliptic longitude, λ, and the latitude, L, of the observation site.

For instance, suppose that we wish to find the azimuth of the point at which the Sun rises on the eastern horizon at an observation site of terrestrial latitude $+60^\circ$, on a day on which the Sun's ecliptic longitude is 08PI00. It follows from Equation (2.43) that

$$A = \cos^{-1}[\sin(338^\circ)\,\sin(23^\circ 26')/\cos(60^\circ)] = 107^\circ 20'.$$

We conclude that the Sun rises $17^\circ 20'$ to the south of the east compass point on the day in question. It is easily demonstrated that the Sun sets $17^\circ 20'$ south of the west compass point on the same day (neglecting the slight change in the Sun's ecliptic latitude during the course of the day.) Likewise, it can easily be shown that, at an observation site of terrestrial latitude -60°, the Sun also rises $17^\circ 20'$ to the south of the east compass point on the day in question, and sets $17^\circ 20'$ to the south of the west compass point.

2.16 Ecliptic altitude and orientation

Consider a point on the ecliptic circle of ecliptic longitude λ. We wish to determine the altitude of this point, as well as the angle subtended there between the ecliptic and the vertical, t hours before or after it culminates at the meridian, as seen from an observation site on the Earth's surface of latitude L.

The situation is as shown in Figure 2.11. Here, Y is the point in question, and ZYC an altitude circle (that is, a great circle passing through the zenith) drawn through it. We wish to determine the altitude $a \equiv CY$ of point Y, as well as the angle $\mu \equiv ZYB$. Note that μ is

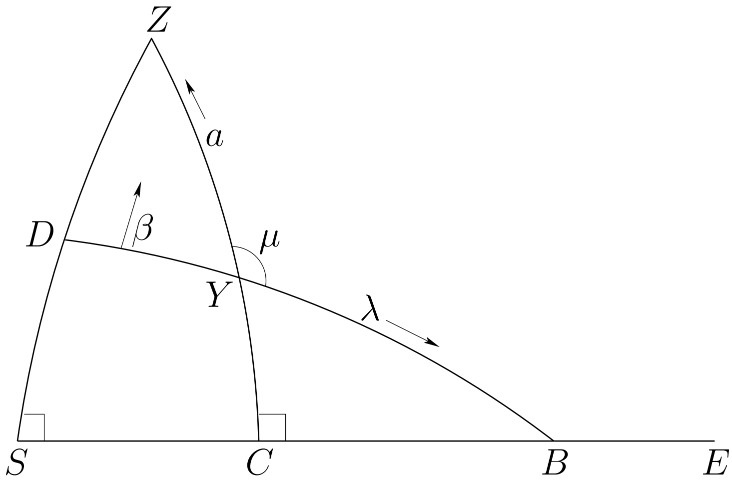

Figure 2.11: Parallactic angle in the case where increasing altitude corresponds to increasing ecliptic latitude. Here, $SCBE$ is the southern horizon, with S and E the south and east compass points, respectively. Moreover, DYB is the ecliptic, ZDS the meridian, Z the zenith, and ZYC an altitude circle.

defined such that it lies between the ecliptic in the direction of increasing ecliptic longitude and the altitude circle in the direction of increasing altitude. Moreover, μ is acute when increasing altitude, a, corresponds to increasing ecliptic latitude, β, and obtuse when increasing a corresponds to decreasing β. See Figures 2.11 and 2.12. Incidentally, this definition is adopted in order to simplify the calculation of lunar parallax. See Section 6.9. In the following, we shall refer to μ as the *parallactic angle*. However, it should be noted that, according to the modern definition, the parallactic angle is $90^\circ - \mu$.

From Equations (2.15) and (2.16), the declination and right ascension of point Y are given by

$$\sin\delta = \sin\epsilon\,\sin\lambda, \tag{2.44}$$

$$\tan\,\alpha = \cos\epsilon\,\tan\lambda, \tag{2.45}$$

respectively. We can also write $\alpha_0 = \alpha - t$, where α_0 is the right ascension of the point on the ecliptic that is culminating (that is, point D in the diagram), and t is measured in time-degrees. Note that if t is positive then it measures time before culmination, whereas if it is negative then its magnitude measures time after culmination. It follows from Equations (2.30) and (2.31) that

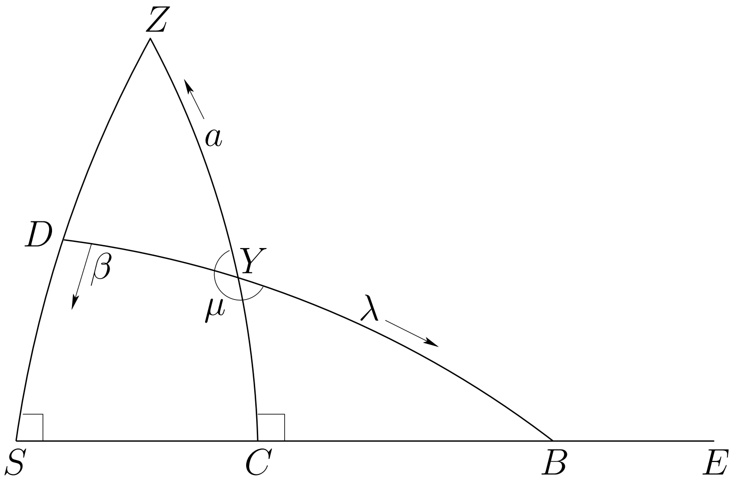

Figure 2.12: Parallactic angle in the case where increasing altitude corresponds to decreasing ecliptic latitude. Here, $SCBE$ is the southern horizon, with S and E the south and east compass points, respectively. Moreover, DYB is the ecliptic, ZDS the meridian, Z the zenith, and ZYC an altitude circle.

the altitude and azimuth of point Y satisfy

$$\sin a = \sin L\, \sin\delta + \cos L\, \cos\delta\, \cos t, \tag{2.46}$$

$$\tan A = \frac{\cos\delta\, \sin t}{\cos L\, \sin\delta - \sin L\, \cos\delta\, \cos t}, \tag{2.47}$$

respectively.

From Equation (2.8), the unit vector

$$\mathbf{r} = \cos\lambda\,\mathbf{v} + \sin\lambda\,\mathbf{s} \tag{2.48}$$

is directed from the observation site to point Y. Furthermore, the unit vector

$$\frac{\partial\mathbf{r}}{\partial\lambda} = -\sin\lambda\,\mathbf{v} + \cos\lambda\,\mathbf{s} \tag{2.49}$$

is tangent to the ecliptic circle, at point Y, in the direction of increasing ecliptic longitude. From Equation (2.27), the unit vector

$$\mathbf{r} = \cos a\, \sin A\,\mathbf{e} + \cos a\, \cos A\,\mathbf{n} + \sin a\,\mathbf{z} \tag{2.50}$$

is directed from the observation site to point Y. Here, a and A are the altitude and azimuth, respectively, of this point in the sky. Moreover, the unit vector

$$\frac{\partial\mathbf{r}}{\partial a} = -\sin a\, \sin A\,\mathbf{e} - \sin a\, \cos A\,\mathbf{n} + \cos a\,\mathbf{z} \equiv (\cos A\,\mathbf{e} - \sin A\,\mathbf{n})\times\mathbf{r} \tag{2.51}$$

is a tangent to the altitude circle passing through point Y in the direction of increasing altitude. It follows from the definition of parallactic angle, and elementary vector algebra, that

$$\cos\mu = \frac{\partial\mathbf{r}}{\partial\lambda}\cdot\frac{\partial\mathbf{r}}{\partial a} = -\sin\lambda\,\cos A\,\mathbf{v}\times\mathbf{e}\cdot\mathbf{r} + \sin\lambda\,\sin A\,\mathbf{v}\times\mathbf{n}\cdot\mathbf{r}$$
$$+\cos\lambda\,\cos A\,\mathbf{s}\times\mathbf{e}\cdot\mathbf{r} - \cos\lambda\,\sin A\,\mathbf{s}\times\mathbf{n}\cdot\mathbf{r}. \tag{2.52}$$

However, according to Equations (2.24), (2.25), and (2.48),

$$\mathbf{v}\times\mathbf{e}\cdot\mathbf{r} = \sin\lambda\,\sin\epsilon\,\cos\alpha_0, \tag{2.53}$$

$$\mathbf{v}\times\mathbf{n}\cdot\mathbf{r} = -\sin\lambda\,(\cos L\,\cos\epsilon + \sin L\,\sin\epsilon\,\sin\alpha_0), \tag{2.54}$$

$$\mathbf{s}\times\mathbf{e}\cdot\mathbf{r} = -\cos\lambda\,\sin\epsilon\,\cos\alpha_0, \tag{2.55}$$

$$\mathbf{s}\times\mathbf{n}\cdot\mathbf{r} = \cos\lambda\,(\cos L\,\cos\epsilon + \sin L\,\sin\epsilon\,\sin\alpha_0). \tag{2.56}$$

The previous five equations can be combined to give

$$\cos\mu = -\cos A\,\sin\epsilon\,\cos(\alpha - t) - \sin A\,[\cos L\,\cos\epsilon + \sin L\,\sin\epsilon\,\sin(\alpha - t)]\,. \tag{2.57}$$

Now, it follows from Equation (2.26) that

$$\mathbf{z}\cdot\mathbf{q} = \sin L\,\cos\epsilon - \cos L\,\sin\epsilon\,\sin(\alpha - t). \tag{2.58}$$

This quantity is significant because if $\mathbf{z}\cdot\mathbf{q} > 0$ then increasing altitude corresponds to increasing ecliptic latitude, whereas if $\mathbf{z}\cdot\mathbf{q} < 0$ then increasing altitude corresponds to decreasing ecliptic latitude. Thus, in the former case, μ is the solution of Equation (2.57) that lies in the range $0^\circ \leq \mu \leq 180^\circ$, whereas in the latter case it is the solution that lies in the range $180^\circ \leq \mu \leq 360^\circ$.

According to Equation (2.46), the critical value of t at which point Y reaches the horizon is given by

$$\cos t_h = -\tan L\,\tan\delta. \tag{2.59}$$

Of course, the previous equation is only soluble if $|\tan L\,\tan\delta| < 1$. However, it is easily demonstrated that if $\tan L\,\tan\delta < -1$ then point Y never sets, whereas if $\tan L\,\tan\delta > 1$ then point Y never rises.

Note that the value of μ at $t = 0$ represents the inclination of the ecliptic to the vertical as point Y culminates. Furthermore, the values of μ at $t = t_h$ (corresponding to $a = 0^\circ$) represent the inclination of the ecliptic to the vertical as point Y rises and sets.

Tables 2.17–2.25 show the altitudes of twelve equally spaced points on the ecliptic, as well as the parallactic angle at these points, as functions of time, calculated for a series of observation sites in the Earth's northern hemisphere with equally spaced terrestrial latitudes. The twelve points correspond to the start of the twelve zodiacal signs, and are named accordingly. Thus, "Aries" corresponds to ecliptic longitude 0°, "Taurus" to ecliptic longitude 30°, et cetera. For each point, four columns of data are provided. The first column corresponds to the time (in hours and minutes) either before or after the culmination of the point, the second column gives the altitude of the point (which is the same in both cases), the third column gives the parallactic angle, μ, for the case in which the first column indicates time prior to the culmination of the point, and the fourth column gives the parallactic angle for the opposite case. Data is only provided for cases in which the various points on the ecliptic lie on or above the horizon.

Now, it can be seen, from the previous analysis, that if $L \rightarrow -L$, $t \rightarrow t$, $\lambda \rightarrow \lambda + 180^\circ$ then $\delta \rightarrow -\delta$, $\alpha \rightarrow \alpha + 180^\circ$, $A \rightarrow 180^\circ - A$, $\cos\mu \rightarrow \cos\mu$, $\mathbf{z}\cdot\mathbf{q} \rightarrow -\mathbf{z}\cdot\mathbf{q}$, and so $a \rightarrow a$, $\mu \rightarrow 360^\circ - \mu$. It follows that Tables 2.17–2.25 can also be used to calculate altitudes and parallactic angles of points on the ecliptic, as functions of time, for observation sites in the Earth's southern hemisphere. For example, suppose that we wish to determine the altitude and parallactic angle of the first point of Gemini (that is, $\lambda = 60^\circ$), as seen from an observation site of terrestrial latitude -10°, 3 hours before and after it culminates at the meridian. In order to do this, we must examine the Sagittarius (that is, $\lambda = 240^\circ$) entry in the $L = +10^\circ$ ecliptic altitude table; that is, Table 2.18 (because $\lambda \rightarrow \lambda + 180^\circ$ when $L \rightarrow -L$). The fourth row of this entry tells us that, $t = 03{:}00$ hours before culmination, the altitude and parallactic angle of the first point of Gemini are $a = 36^\circ 26'$ and $\mu = 360^\circ - 162^\circ 11' = 197^\circ 49'$, respectively (because $a \rightarrow a$ and $\mu \rightarrow 360^\circ - \mu$ as $L \rightarrow -L$). This row also tells us that, $t = 03{:}00$ hours after culmination, the altitude and parallactic angle of the first point of Gemini are $a = 36^\circ 26'$ and $\mu = 360^\circ - 042^\circ 17' = 317^\circ 43'$, respectively.

Note that Tables 2.17–2.25 contain essentially the same information as is contained in the "Tabulation of angles and arcs according to parallels" (Ἔκθεσις τῶν κατὰ παράλληλον γωνιῶν καὶ περιφερειῶν) that appears in Section 13 of Book II of the Almagest.

2.17 Maps and tables

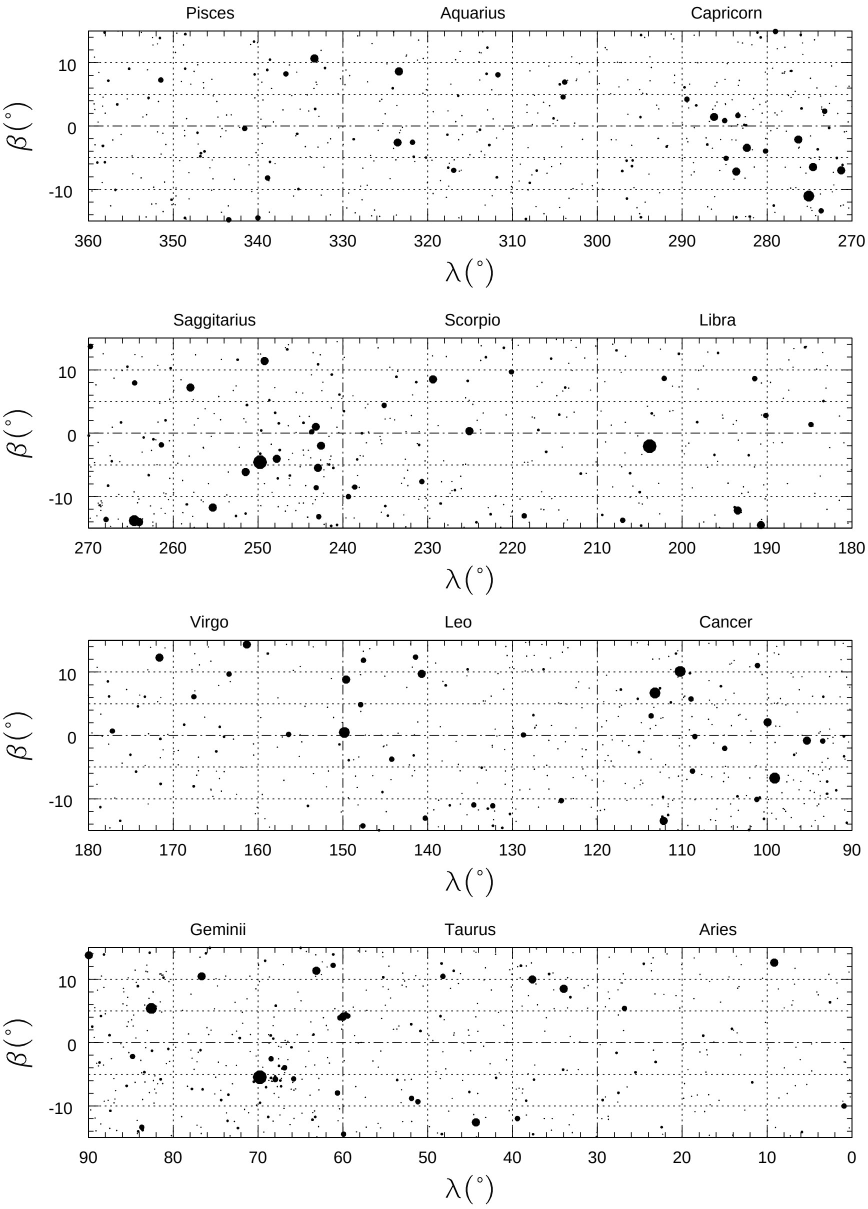

Figure 2.13: Map showing all stars of visual magnitude less than $+6$ lying within 15° of the ecliptic plane.

Aries

λ	β	Mag.	Name
$09^\circ09'$	$+12^\circ36'$	$+2.8$	γ PEG
$26^\circ49'$	$+05^\circ23'$	$+3.6$	η PSC

Libra

λ	β	Mag.	Name
$04^\circ50'$	$+01^\circ22'$	$+3.9$	η VIR
$10^\circ08'$	$+02^\circ46'$	$+3.5$	γ VIR
$11^\circ28'$	$+08^\circ37'$	$+3.4$	δ VIR
$22^\circ08'$	$+08^\circ38'$	$+3.4$	ζ VIR
$23^\circ51'$	$-02^\circ03'$	$+1.0$	α VIR

Taurus

λ	β	Mag.	Name
$03^\circ58'$	$+08^\circ29'$	$+2.6$	β ARI
$07^\circ39'$	$+09^\circ58'$	$+2.0$	α ARI

Scorpio

λ	β	Mag.	Name
$15^\circ5'$	$+00^\circ20'$	$+2.8$	α LIB
$19^\circ22'$	$+08^\circ30'$	$+2.6$	β LIB

Gemini

λ	β	Mag.	Name
$00^\circ00'$	$+04^\circ03'$	$+2.9$	η TAU
$09^\circ47'$	$-05^\circ28'$	$+0.9$	α TAU
$22^\circ34'$	$+05^\circ23'$	$+1.7$	β TAU

Sagittarius

λ	β	Mag.	Name
$02^\circ34'$	$-01^\circ59'$	$+2.3$	δ SCO
$02^\circ56'$	$-05^\circ29'$	$+2.9$	π SCO
$03^\circ11'$	$+01^\circ00'$	$+2.6$	β SCO
$07^\circ48'$	$-04^\circ02'$	$+2.9$	σ SCO
$09^\circ46'$	$-04^\circ34'$	$+1.0$	α SCO
$11^\circ27'$	$-06^\circ08'$	$+2.8$	τ SCO
$17^\circ58'$	$+07^\circ12'$	$+2.4$	η OPH

Cancer

λ	β	Mag.	Name
$05^\circ18'$	$-00^\circ49'$	$+2.9$	μ GEM
$09^\circ06'$	$-06^\circ44'$	$+1.9$	γ GEM
$09^\circ56'$	$+02^\circ04'$	$+3.0$	ϵ GEM
$20^\circ14'$	$+10^\circ06'$	$+2.0$	α GEM
$23^\circ13'$	$+06^\circ41'$	$+1.1$	β GEM

Capricorn

λ	β	Mag.	Name
$01^\circ16'$	$-07^\circ00'$	$+3.0$	γ SGR
$04^\circ34'$	$-06^\circ28'$	$+2.7$	δ SGR
$06^\circ19'$	$-02^\circ08'$	$+2.8$	λ SGR
$12^\circ23'$	$-03^\circ27'$	$+2.0$	σ SGR
$13^\circ38'$	$-07^\circ11'$	$+2.6$	ζ SGR
$16^\circ15'$	$+01^\circ26'$	$+2.9$	π SGR

Leo

λ	β	Mag.	Name
$20^\circ47'$	$+09^\circ43'$	$+3.0$	ϵ LEO
$29^\circ37'$	$+08^\circ49'$	$+2.6$	γ LEO
$29^\circ50'$	$+00^\circ28'$	$+1.4$	α LEO

Aquarius

λ	β	Mag.	Name
$23^\circ24'$	$+08^\circ37'$	$+2.9$	β AQR
$23^\circ33'$	$-02^\circ36'$	$+2.9$	δ CAP

Virgo

λ	β	Mag.	Name
$06^\circ23'$	$+00^\circ09'$	$+3.9$	ρ LEO
$13^\circ25'$	$+09^\circ40'$	$+3.3$	θ LEO
$17^\circ34'$	$+06^\circ06'$	$+3.9$	ι LEO
$27^\circ10'$	$+00^\circ42'$	$+3.6$	β VIR

Pisces

λ	β	Mag.	Name
$06^\circ43'$	$+08^\circ14'$	$+3.8$	γ AQR
$08^\circ52'$	$-08^\circ12'$	$+3.3$	δ AQR
$11^\circ34'$	$-00^\circ23'$	$+3.7$	λ AQR
$21^\circ27'$	$+07^\circ15'$	$+3.7$	γ PSC

Table 2.1: Ecliptic longitudes (relative to the mean equinox at the J2000 epoch), ecliptic latitudes, and visual magnitudes of selected bright stars lying within 10° of the ecliptic plane.

Aries			Taurus			Gemini		
λ	δ	α	λ	δ	α	λ	δ	α
00°	+00°00′	000°00′	00°	+11°28′	027°55′	00°	+20°09′	057°49′
02°	+00°48′	001°50′	02°	+12°10′	029°50′	02°	+20°33′	059°54′
04°	+01°35′	003°40′	04°	+12°51′	031°45′	04°	+20°57′	062°00′
06°	+02°23′	005°30′	06°	+13°31′	033°41′	06°	+21°18′	064°07′
08°	+03°10′	007°21′	08°	+14°10′	035°38′	08°	+21°38′	066°14′
10°	+03°58′	009°11′	10°	+14°49′	037°36′	10°	+21°57′	068°22′
12°	+04°45′	011°02′	12°	+15°26′	039°34′	12°	+22°13′	070°30′
14°	+05°31′	012°53′	14°	+16°02′	041°33′	14°	+22°28′	072°39′
16°	+06°18′	014°44′	16°	+16°37′	043°32′	16°	+22°42′	074°48′
18°	+07°04′	016°36′	18°	+17°11′	045°32′	18°	+22°54′	076°57′
20°	+07°49′	018°28′	20°	+17°44′	047°33′	20°	+23°03′	079°07′
22°	+08°34′	020°20′	22°	+18°16′	049°35′	22°	+23°12′	081°17′
24°	+09°19′	022°13′	24°	+18°46′	051°38′	24°	+23°18′	083°28′
26°	+10°02′	024°07′	26°	+19°15′	053°41′	26°	+23°22′	085°39′
28°	+10°46′	026°00′	28°	+19°43′	055°45′	28°	+23°25′	087°49′
30°	+11°28′	027°55′	30°	+20°09′	057°49′	30°	+23°26′	090°00′

Cancer			Leo			Virgo		
λ	δ	α	λ	δ	α	λ	δ	α
00°	+23°26′	090°00′	00°	+20°09′	122°11′	00°	+11°28′	152°05′
02°	+23°25′	092°11′	02°	+19°43′	124°15′	02°	+10°46′	154°00′
04°	+23°22′	094°21′	04°	+19°15′	126°19′	04°	+10°02′	155°53′
06°	+23°18′	096°32′	06°	+18°46′	128°22′	06°	+09°19′	157°47′
08°	+23°12′	098°43′	08°	+18°16′	130°25′	08°	+08°34′	159°40′
10°	+23°03′	100°53′	10°	+17°44′	132°27′	10°	+07°49′	161°32′
12°	+22°54′	103°03′	12°	+17°11′	134°28′	12°	+07°04′	163°24′
14°	+22°42′	105°12′	14°	+16°37′	136°28′	14°	+06°18′	165°16′
16°	+22°28′	107°21′	16°	+16°02′	138°27′	16°	+05°31′	167°07′
18°	+22°13′	109°30′	18°	+15°26′	140°26′	18°	+04°45′	168°58′
20°	+21°57′	111°38′	20°	+14°49′	142°24′	20°	+03°58′	170°49′
22°	+21°38′	113°46′	22°	+14°10′	144°22′	22°	+03°10′	172°39′
24°	+21°18′	115°53′	24°	+13°31′	146°19′	24°	+02°23′	174°30′
26°	+20°57′	118°00′	26°	+12°51′	148°15′	26°	+01°35′	176°20′
28°	+20°33′	120°06′	28°	+12°10′	150°10′	28°	+00°48′	178°10′
30°	+20°09′	122°11′	30°	+11°28′	152°05′	30°	+00°00′	180°00′

Libra			Scorpio			Sagittarius		
λ	δ	α	λ	δ	α	λ	δ	α
00°	−00°00′	180°00′	00°	−11°28′	207°55′	00°	−20°09′	237°49′
02°	−00°48′	181°50′	02°	−12°10′	209°50′	02°	−20°33′	239°54′
04°	−01°35′	183°40′	04°	−12°51′	211°45′	04°	−20°57′	242°00′
06°	−02°23′	185°30′	06°	−13°31′	213°41′	06°	−21°18′	244°07′
08°	−03°10′	187°21′	08°	−14°10′	215°38′	08°	−21°38′	246°14′
10°	−03°58′	189°11′	10°	−14°49′	217°36′	10°	−21°57′	248°22′
12°	−04°45′	191°02′	12°	−15°26′	219°34′	12°	−22°13′	250°30′
14°	−05°31′	192°53′	14°	−16°02′	221°33′	14°	−22°28′	252°39′
16°	−06°18′	194°44′	16°	−16°37′	223°32′	16°	−22°42′	254°48′
18°	−07°04′	196°36′	18°	−17°11′	225°32′	18°	−22°54′	256°57′
20°	−07°49′	198°28′	20°	−17°44′	227°33′	20°	−23°03′	259°07′
22°	−08°34′	200°20′	22°	−18°16′	229°35′	22°	−23°12′	261°17′
24°	−09°19′	202°13′	24°	−18°46′	231°38′	24°	−23°18′	263°28′
26°	−10°02′	204°07′	26°	−19°15′	233°41′	26°	−23°22′	265°39′
28°	−10°46′	206°00′	28°	−19°43′	235°45′	28°	−23°25′	267°49′
30°	−11°28′	207°55′	30°	−20°09′	237°49′	30°	−23°26′	270°00′

Capricorn			Aquarius			Pisces		
λ	δ	α	λ	δ	α	λ	δ	α
00°	−23°26′	270°00′	00°	−20°09′	302°11′	00°	−11°28′	332°05′
02°	−23°25′	272°11′	02°	−19°43′	304°15′	02°	−10°46′	333°60′
04°	−23°22′	274°21′	04°	−19°15′	306°19′	04°	−10°02′	335°53′
06°	−23°18′	276°32′	06°	−18°46′	308°22′	06°	−09°19′	337°47′
08°	−23°12′	278°43′	08°	−18°16′	310°25′	08°	−08°34′	339°40′
10°	−23°03′	280°53′	10°	−17°44′	312°27′	10°	−07°49′	341°32′
12°	−22°54′	283°03′	12°	−17°11′	314°28′	12°	−07°04′	343°24′
14°	−22°42′	285°12′	14°	−16°37′	316°28′	14°	−06°18′	345°16′
16°	−22°28′	287°21′	16°	−16°02′	318°27′	16°	−05°31′	347°07′
18°	−22°13′	289°30′	18°	−15°26′	320°26′	18°	−04°45′	348°58′
20°	−21°57′	291°38′	20°	−14°49′	322°24′	20°	−03°58′	350°49′
22°	−21°38′	293°46′	22°	−14°10′	324°22′	22°	−03°10′	352°39′
24°	−21°18′	295°53′	24°	−13°31′	326°19′	24°	−02°23′	354°30′
26°	−20°57′	297°60′	26°	−12°51′	328°15′	26°	−01°35′	356°20′
28°	−20°33′	300°06′	28°	−12°10′	330°10′	28°	−00°48′	358°10′
30°	−20°09′	302°11′	30°	−11°28′	332°05′	30°	−00°00′	360°00′

Table 2.2: Declinations and right ascensions of points on the ecliptic circle.

L	Longest Day	Summer Solstice Noon Altitude of Sun	Equinoctial Noon Altitude of Sun	Winter Solstice Noon Altitude of Sun
$+00^\circ$	12^h00^m	$+66^\circ34'$ N	$+90^\circ00'$ S	$+66^\circ34'$ S
$+05^\circ$	12^h17^m	$+71^\circ34'$ N	$+85^\circ00'$ S	$+61^\circ34'$ S
$+10^\circ$	12^h35^m	$+76^\circ34'$ N	$+80^\circ00'$ S	$+56^\circ34'$ S
$+15^\circ$	12^h53^m	$+81^\circ34'$ N	$+75^\circ00'$ S	$+51^\circ34'$ S
$+20^\circ$	13^h13^m	$+86^\circ34'$ N	$+70^\circ00'$ S	$+46^\circ34'$ S
$+25^\circ$	13^h33^m	$+88^\circ26'$ S	$+65^\circ00'$ S	$+41^\circ34'$ S
$+30^\circ$	13^h56^m	$+83^\circ26'$ S	$+60^\circ00'$ S	$+36^\circ34'$ S
$+35^\circ$	14^h21^m	$+78^\circ26'$ S	$+55^\circ00'$ S	$+31^\circ34'$ S
$+40^\circ$	14^h51^m	$+73^\circ26'$ S	$+50^\circ00'$ S	$+26^\circ34'$ S
$+45^\circ$	15^h25^m	$+68^\circ26'$ S	$+45^\circ00'$ S	$+21^\circ34'$ S
$+50^\circ$	16^h09^m	$+63^\circ26'$ S	$+40^\circ00'$ S	$+16^\circ34'$ S
$+55^\circ$	17^h06^m	$+58^\circ26'$ S	$+35^\circ00'$ S	$+11^\circ34'$ S
$+60^\circ$	18^h29^m	$+53^\circ26'$ S	$+30^\circ00'$ S	$+06^\circ34'$ S
$+65^\circ$	21^h07^m	$+48^\circ26'$ S	$+25^\circ00'$ S	$+01^\circ34'$ S
$+70^\circ$	62^d06^h	$+43^\circ26'$ S	$+20^\circ00'$ S	$-03^\circ26'$ S
$+75^\circ$	100^d06^h	$+38^\circ26'$ S	$+15^\circ00'$ S	$-08^\circ26'$ S
$+80^\circ$	130^d02^h	$+33^\circ26'$ S	$+10^\circ00'$ S	$-13^\circ26'$ S
$+85^\circ$	156^d22^h	$+28^\circ26'$ S	$+05^\circ00'$ S	$-18^\circ26'$ S
$+90^\circ$	182^d15^h	$+23^\circ26'$ S	$+00^\circ00'$ S	$-23^\circ26'$ S

Table 2.3: Terrestrial climes in the Earth's northern hemisphere. The superscripts h, m, and d stand for hours, minutes, and days, respectively. The symbols S and N indicated that the upper transit of the Sun occurs to the south and north of the zenith, respectively.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	027°55′	00°	057°49′	00°	090°00′	00°	122°11′	00°	152°05′
02°	001°50′	02°	029°50′	02°	059°54′	02°	092°11′	02°	124°15′	02°	154°00′
04°	003°40′	04°	031°45′	04°	062°00′	04°	094°21′	04°	126°19′	04°	155°53′
06°	005°30′	06°	033°41′	06°	064°07′	06°	096°32′	06°	128°22′	06°	157°47′
08°	007°21′	08°	035°38′	08°	066°14′	08°	098°43′	08°	130°25′	08°	159°40′
10°	009°11′	10°	037°36′	10°	068°22′	10°	100°53′	10°	132°27′	10°	161°32′
12°	011°02′	12°	039°34′	12°	070°30′	12°	103°03′	12°	134°28′	12°	163°24′
14°	012°53′	14°	041°33′	14°	072°39′	14°	105°12′	14°	136°28′	14°	165°16′
16°	014°44′	16°	043°32′	16°	074°48′	16°	107°21′	16°	138°27′	16°	167°07′
18°	016°36′	18°	045°32′	18°	076°57′	18°	109°30′	18°	140°26′	18°	168°58′
20°	018°28′	20°	047°33′	20°	079°07′	20°	111°38′	20°	142°24′	20°	170°49′
22°	020°20′	22°	049°35′	22°	081°17′	22°	113°46′	22°	144°22′	22°	172°39′
24°	022°13′	24°	051°38′	24°	083°28′	24°	115°53′	24°	146°19′	24°	174°30′
26°	024°07′	26°	053°41′	26°	085°39′	26°	118°00′	26°	148°15′	26°	176°20′
28°	026°00′	28°	055°45′	28°	087°49′	28°	120°06′	28°	150°10′	28°	178°10′
30°	027°55′	30°	057°49′	30°	090°00′	30°	122°11′	30°	152°05′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	207°55′	00°	237°49′	00°	270°00′	00°	302°11′	00°	332°05′
02°	181°50′	02°	209°50′	02°	239°54′	02°	272°11′	02°	304°15′	02°	333°60′
04°	183°40′	04°	211°45′	04°	242°00′	04°	274°21′	04°	306°19′	04°	335°53′
06°	185°30′	06°	213°41′	06°	244°07′	06°	276°32′	06°	308°22′	06°	337°47′
08°	187°21′	08°	215°38′	08°	246°14′	08°	278°43′	08°	310°25′	08°	339°40′
10°	189°11′	10°	217°36′	10°	248°22′	10°	280°53′	10°	312°27′	10°	341°32′
12°	191°02′	12°	219°34′	12°	250°30′	12°	283°03′	12°	314°28′	12°	343°24′
14°	192°53′	14°	221°33′	14°	252°39′	14°	285°12′	14°	316°28′	14°	345°16′
16°	194°44′	16°	223°32′	16°	254°48′	16°	287°21′	16°	318°27′	16°	347°07′
18°	196°36′	18°	225°32′	18°	256°57′	18°	289°30′	18°	320°26′	18°	348°58′
20°	198°28′	20°	227°33′	20°	259°07′	20°	291°38′	20°	322°24′	20°	350°49′
22°	200°20′	22°	229°35′	22°	261°17′	22°	293°46′	22°	324°22′	22°	352°39′
24°	202°13′	24°	231°38′	24°	263°28′	24°	295°53′	24°	326°19′	24°	354°30′
26°	204°07′	26°	233°41′	26°	265°39′	26°	298°00′	26°	328°15′	26°	356°20′
28°	206°00′	28°	235°45′	28°	267°49′	28°	300°06′	28°	330°10′	28°	358°10′
30°	207°55′	30°	237°49′	30°	270°00′	30°	302°11′	30°	332°05′	30°	360°00′

Table 2.4: Right ascensions of the ecliptic for latitude 0°.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	025°52′	00°	054°07′	00°	085°37′	00°	118°28′	00°	150°02′
02°	001°42′	02°	027°39′	02°	056°07′	02°	087°48′	02°	120°38′	02°	152°04′
04°	003°23′	04°	029°27′	04°	058°08′	04°	089°59′	04°	122°47′	04°	154°06′
06°	005°05′	06°	031°16′	06°	060°10′	06°	092°11′	06°	124°56′	06°	156°07′
08°	006°47′	08°	033°05′	08°	062°13′	08°	094°23′	08°	127°05′	08°	158°08′
10°	008°29′	10°	034°55′	10°	064°17′	10°	096°34′	10°	129°13′	10°	160°09′
12°	010°12′	12°	036°46′	12°	066°22′	12°	098°46′	12°	131°20′	12°	162°09′
14°	011°55′	14°	038°38′	14°	068°28′	14°	100°58′	14°	133°27′	14°	164°09′
16°	013°38′	16°	040°31′	16°	070°34′	16°	103°10′	16°	135°33′	16°	166°08′
18°	015°21′	18°	042°25′	18°	072°41′	18°	105°22′	18°	137°39′	18°	168°08′
20°	017°05′	20°	044°19′	20°	074°49′	20°	107°34′	20°	139°44′	20°	170°07′
22°	018°49′	22°	046°15′	22°	076°58′	22°	109°45′	22°	141°49′	22°	172°06′
24°	020°34′	24°	048°11′	24°	079°07′	24°	111°57′	24°	143°53′	24°	174°04′
26°	022°19′	26°	050°09′	26°	081°16′	26°	114°07′	26°	145°57′	26°	176°03′
28°	024°05′	28°	052°07′	28°	083°26′	28°	116°18′	28°	148°00′	28°	178°01′
30°	025°52′	30°	054°07′	30°	085°37′	30°	118°28′	30°	150°02′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	209°58′	00°	241°32′	00°	274°23′	00°	305°53′	00°	334°08′
02°	181°59′	02°	212°00′	02°	243°42′	02°	276°34′	02°	307°53′	02°	335°55′
04°	183°57′	04°	214°03′	04°	245°53′	04°	278°44′	04°	309°51′	04°	337°41′
06°	185°56′	06°	216°07′	06°	248°03′	06°	280°53′	06°	311°49′	06°	339°26′
08°	187°54′	08°	218°11′	08°	250°15′	08°	283°02′	08°	313°45′	08°	341°11′
10°	189°53′	10°	220°16′	10°	252°26′	10°	285°11′	10°	315°41′	10°	342°55′
12°	191°52′	12°	222°21′	12°	254°38′	12°	287°19′	12°	317°35′	12°	344°39′
14°	193°52′	14°	224°27′	14°	256°50′	14°	289°26′	14°	319°29′	14°	346°22′
16°	195°51′	16°	226°33′	16°	259°02′	16°	291°32′	16°	321°22′	16°	348°05′
18°	197°51′	18°	228°40′	18°	261°14′	18°	293°38′	18°	323°14′	18°	349°48′
20°	199°51′	20°	230°47′	20°	263°26′	20°	295°43′	20°	325°05′	20°	351°31′
22°	201°52′	22°	232°55′	22°	265°37′	22°	297°47′	22°	326°55′	22°	353°13′
24°	203°53′	24°	235°04′	24°	267°49′	24°	299°50′	24°	328°44′	24°	354°55′
26°	205°54′	26°	237°13′	26°	270°01′	26°	301°52′	26°	330°33′	26°	356°37′
28°	207°56′	28°	239°22′	28°	272°12′	28°	303°53′	28°	332°21′	28°	358°18′
30°	209°58′	30°	241°32′	30°	274°23′	30°	305°53′	30°	334°08′	30°	360°00′

Table 2.5: Oblique ascensions of the ecliptic for latitude $+10^\circ$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	023°41′	00°	050°09′	00°	080°55′	00°	114°30′	00°	147°51′
02°	001°33′	02°	025°20′	02°	052°04′	02°	083°07′	02°	116°46′	02°	150°02′
04°	003°06′	04°	026°59′	04°	054°00′	04°	085°18′	04°	119°01′	04°	152°12′
06°	004°38′	06°	028°40′	06°	055°57′	06°	087°31′	06°	121°16′	06°	154°22′
08°	006°12′	08°	030°22′	08°	057°56′	08°	089°44′	08°	123°31′	08°	156°31′
10°	007°45′	10°	032°04′	10°	059°56′	10°	091°58′	10°	125°46′	10°	158°40′
12°	009°18′	12°	033°48′	12°	061°57′	12°	094°12′	12°	128°00′	12°	160°49′
14°	010°52′	14°	035°32′	14°	063°59′	14°	096°27′	14°	130°14′	14°	162°58′
16°	012°26′	16°	037°18′	16°	066°02′	16°	098°42′	16°	132°27′	16°	165°06′
18°	014°01′	18°	039°04′	18°	068°07′	18°	100°57′	18°	134°40′	18°	167°14′
20°	015°36′	20°	040°52′	20°	070°13′	20°	103°12′	20°	136°53′	20°	169°22′
22°	017°12′	22°	042°41′	22°	072°19′	22°	105°28′	22°	139°06′	22°	171°30′
24°	018°48′	24°	044°31′	24°	074°27′	24°	107°44′	24°	141°18′	24°	173°37′
26°	020°25′	26°	046°23′	26°	076°35′	26°	109°59′	26°	143°29′	26°	175°45′
28°	022°02′	28°	048°15′	28°	078°45′	28°	112°15′	28°	145°40′	28°	177°53′
30°	023°41′	30°	050°09′	30°	080°55′	30°	114°30′	30°	147°51′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	212°09′	00°	245°30′	00°	279°05′	00°	309°51′	00°	336°19′
02°	182°07′	02°	214°20′	02°	247°45′	02°	281°15′	02°	311°45′	02°	337°58′
04°	184°15′	04°	216°31′	04°	250°01′	04°	283°25′	04°	313°37′	04°	339°35′
06°	186°23′	06°	218°42′	06°	252°16′	06°	285°33′	06°	315°29′	06°	341°12′
08°	188°30′	08°	220°54′	08°	254°32′	08°	287°41′	08°	317°19′	08°	342°48′
10°	190°38′	10°	223°07′	10°	256°48′	10°	289°47′	10°	319°08′	10°	344°24′
12°	192°46′	12°	225°20′	12°	259°03′	12°	291°53′	12°	320°56′	12°	345°59′
14°	194°54′	14°	227°33′	14°	261°18′	14°	293°58′	14°	322°42′	14°	347°34′
16°	197°02′	16°	229°46′	16°	263°33′	16°	296°01′	16°	324°28′	16°	349°08′
18°	199°11′	18°	232°00′	18°	265°48′	18°	298°03′	18°	326°12′	18°	350°42′
20°	201°20′	20°	234°14′	20°	268°02′	20°	300°04′	20°	327°56′	20°	352°15′
22°	203°29′	22°	236°29′	22°	270°16′	22°	302°04′	22°	329°38′	22°	353°48′
24°	205°38′	24°	238°44′	24°	272°29′	24°	304°03′	24°	331°20′	24°	355°22′
26°	207°48′	26°	240°59′	26°	274°42′	26°	306°00′	26°	333°01′	26°	356°54′
28°	209°58′	28°	243°14′	28°	276°53′	28°	307°56′	28°	334°40′	28°	358°27′
30°	212°09′	30°	245°30′	30°	279°05′	30°	309°51′	30°	336°19′	30°	360°00′

Table 2.6: Oblique ascensions of the ecliptic for latitude $+20°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	021°11′	00°	045°36′	00°	075°30′	00°	109°57′	00°	145°22′
02°	001°23′	02°	022°41′	02°	047°24′	02°	077°42′	02°	112°19′	02°	147°42′
04°	002°45′	04°	024°11′	04°	049°14′	04°	079°55′	04°	114°41′	08°	154°40′
06°	004°08′	06°	025°43′	06°	051°06′	06°	082°08′	06°	117°04′	06°	152°21′
08°	005°31′	08°	027°15′	08°	053°00′	08°	084°23′	08°	119°26′	08°	154°40′
10°	006°54′	10°	028°49′	10°	054°55′	10°	086°39′	10°	121°48′	10°	156°59′
12°	008°17′	12°	030°23′	12°	056°51′	12°	088°56′	12°	124°10′	12°	159°18′
14°	009°41′	14°	031°59′	14°	058°50′	14°	091°14′	14°	126°32′	14°	161°37′
16°	011°05′	16°	033°37′	16°	060°49′	16°	093°32′	16°	128°54′	16°	163°55′
18°	012°30′	18°	035°15′	18°	062°51′	18°	095°51′	18°	131°16′	18°	166°13′
20°	013°55′	20°	036°55′	20°	064°54′	20°	098°11′	20°	133°38′	20°	168°31′
22°	015°21′	22°	038°36′	22°	066°58′	22°	100°32′	22°	135°59′	22°	170°49′
24°	016°47′	24°	040°19′	24°	069°04′	24°	102°52′	24°	138°20′	24°	173°07′
26°	018°15′	26°	042°03′	26°	071°12′	26°	105°14′	26°	140°41′	26°	175°25′
28°	019°42′	28°	043°48′	28°	073°20′	28°	107°35′	28°	143°01′	28°	177°42′
30°	021°11′	30°	045°36′	30°	075°30′	30°	109°57′	30°	145°22′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	214°38′	00°	250°03′	00°	284°30′	00°	314°24′	00°	338°49′
02°	182°18′	02°	216°59′	02°	252°25′	02°	286°40′	02°	316°12′	02°	340°18′
04°	184°35′	04°	219°19′	04°	254°46′	04°	288°48′	04°	317°57′	04°	341°45′
06°	186°53′	06°	221°40′	06°	257°08′	06°	290°56′	06°	319°41′	06°	343°13′
08°	189°11′	08°	224°01′	08°	259°28′	08°	293°02′	08°	321°24′	08°	344°39′
10°	191°29′	10°	226°22′	10°	261°49′	10°	295°06′	10°	323°05′	10°	346°05′
12°	193°47′	12°	228°44′	12°	264°09′	12°	297°09′	12°	324°45′	12°	347°30′
14°	196°05′	14°	231°06′	14°	266°28′	14°	299°11′	14°	326°23′	14°	348°55′
16°	198°23′	16°	233°28′	16°	268°46′	16°	301°10′	16°	328°01′	16°	350°19′
18°	200°42′	18°	235°50′	18°	271°04′	18°	303°09′	18°	329°37′	18°	351°43′
20°	203°01′	20°	238°12′	20°	273°21′	20°	305°05′	20°	331°11′	20°	353°06′
22°	205°20′	22°	240°34′	22°	275°37′	22°	307°00′	22°	332°45′	22°	354°29′
24°	207°39′	24°	242°56′	24°	277°52′	24°	308°54′	24°	334°17′	24°	355°52′
26°	209°59′	26°	245°19′	26°	280°05′	26°	310°46′	26°	335°49′	26°	357°15′
28°	212°18′	28°	247°41′	28°	282°18′	28°	312°36′	28°	337°19′	28°	358°37′
30°	214°38′	30°	250°03′	30°	284°30′	30°	314°24′	30°	338°49′	30°	360°00′

Table 2.7: Oblique ascensions of the ecliptic for latitude $+30^\circ$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	018°07′	00°	039°54′	00°	068°40′	00°	104°15′	00°	142°17′
02°	001°10′	02°	019°24′	02°	041°34′	02°	070°52′	02°	106°46′	02°	144°49
04°	002°20′	04°	020°43′	04°	043°16′	04°	073°06′	04°	109°17′	04°	147°21′
06°	003°30′	06°	022°03′	06°	045°01′	06°	075°21′	06°	111°48′	06°	149°52′
08°	004°41′	08°	023°24′	08°	046°48′	08°	077°38′	08°	114°20′	08°	152°24′
10°	005°52′	10°	024°46′	10°	048°36′	10°	079°57′	10°	116°53′	10°	154°55′
12°	007°03′	12°	026°10′	12°	050°27′	12°	082°18′	12°	119°25′	12°	157°26′
14°	008°14′	14°	027°35′	14°	052°20′	14°	084°39′	14°	121°57′	14°	159°57′
16°	009°26′	16°	029°02′	16°	054°15′	16°	087°03′	16°	124°30′	16°	162°28′
18°	010°38′	18°	030°30′	18°	056°12′	18°	089°27′	18°	127°03′	18°	164°58′
20°	011°51′	20°	031°59′	20°	058°12′	20°	091°53′	20°	129°35′	20°	167°29′
22°	013°05′	22°	033°31′	22°	060°13′	22°	094°19′	22°	132°08′	22°	169°59′
24°	014°19′	24°	035°04′	24°	062°17′	24°	096°47′	24°	134°40′	24°	172°29′
26°	015°34′	26°	036°38′	26°	064°23′	26°	099°16′	26°	137°13′	26°	175°00′
28°	016°50′	28°	038°15′	28°	066°31′	28°	101°45′	28°	139°45′	28°	177°30′
30°	018°07′	30°	039°54′	30°	068°40′	30°	104°15′	30°	142°17′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	217°43′	00°	255°45′	00°	291°20′	00°	320°06′	00°	341°53′
02°	182°30′	02°	220°15′	02°	258°15′	02°	293°29′	02°	321°45′	02°	343°10′
04°	185°00′	04°	222°47′	04°	260°44′	04°	295°37′	04°	323°22′	04°	344°26′
06°	187°31′	06°	225°20′	06°	263°13′	06°	297°43′	06°	324°56′	08°	346°55′
08°	190°01′	08°	227°52′	08°	265°41′	08°	299°17′	08°	326°29′	08°	346°55′
10°	192°31′	10°	230°25′	10°	268°07′	10°	301°48′	10°	328°01′	10°	348°09′
12°	195°02′	12°	232°57′	12°	270°33′	12°	303°48′	12°	329°30′	12°	349°22′
14°	197°32′	14°	235°30′	14°	272°57′	14°	305°45′	14°	330°58′	14°	350°34′
16°	200°03′	16°	238°03′	16°	275°21′	16°	307°40′	16°	332°25′	16°	351°46′
18°	202°34′	18°	240°35′	18°	277°42′	18°	309°33′	18°	333°50′	18°	352°57′
20°	205°05′	20°	243°07′	20°	280°03′	20°	311°24′	20°	335°14′	20°	354°08′
22°	207°36′	22°	245°40′	22°	282°22′	22°	313°12′	22°	336°36′	22°	355°19′
24°	210°08′	24°	248°12′	24°	284°39′	24°	314°59′	24°	337°57′	24°	356°30′
26°	212°39′	26°	250°43′	26°	286°54′	26°	316°44′	26°	339°17′	26°	357°40′
28°	215°11′	28°	253°14′	28°	289°08′	28°	318°26′	28°	340°36′	28°	358°50′
30°	217°43′	30°	255°45′	30°	291°20′	30°	320°06′	30°	341°53′	30°	360°00′

Table 2.8: Oblique ascensions of the ecliptic for latitude $+40°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	013°55′	00°	031°54′	00°	058°54′	00°	096°15′	00°	138°06′
02°	000°53′	02°	014°56′	02°	033°22′	02°	061°06′	02°	098°59′	02°	140°54′
04°	001°47′	04°	015°59′	04°	034°52′	04°	063°21′	04°	101°44′	04°	143°43′
06°	002°40′	06°	017°02′	06°	036°25′	06°	065°40′	06°	104°29′	06°	146°31′
08°	003°34′	08°	018°07′	08°	038°01′	08°	068°00′	08°	107°15′	08°	149°19′
10°	004°27′	10°	019°13′	10°	039°40′	10°	070°24′	10°	110°02′	10°	152°07′
12°	005°22′	12°	020°21′	12°	041°22′	12°	072°50′	12°	112°50′	12°	154°55′
14°	006°16′	14°	021°31′	14°	043°06′	14°	075°18′	14°	115°37′	14°	157°42′
16°	007°11′	16°	022°42′	16°	044°54′	16°	077°49′	16°	118°26′	16°	160°30′
18°	008°07′	18°	023°54′	18°	046°45′	18°	080°22′	18°	121°14′	18°	163°17′
20°	009°03′	20°	025°09′	20°	048°38′	20°	082°57′	20°	124°02′	20°	166°05′
22°	010°00′	22°	026°26′	22°	050°35′	22°	085°33′	22°	126°51′	22°	168°52′
24°	010°57′	24°	027°44′	24°	052°35′	24°	088°12′	24°	129°40′	24°	171°39′
26°	011°56′	26°	029°05′	26°	054°38′	26°	090°51′	26°	132°28′	26°	174°26′
28°	012°55′	28°	030°28′	28°	056°45′	28°	093°33′	28°	135°17′	28°	177°13′
30°	013°55′	30°	031°54′	30°	058°54′	30°	096°15′	30°	138°06′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	221°54′	00°	263°45′	00°	301°06′	00°	328°06′	00°	346°05′
02°	182°47′	02°	224°43′	02°	266°27′	02°	303°15′	02°	329°32′	02°	347°05′
04°	185°34′	04°	227°32′	04°	269°09′	04°	305°22′	04°	330°55′	04°	348°04′
06°	188°21′	06°	230°20′	06°	271°48′	06°	307°25′	06°	332°16′	06°	349°03′
08°	191°08′	08°	233°09′	08°	274°27′	08°	309°25′	08°	333°34′	08°	350°00′
10°	193°55′	10°	235°58′	10°	277°03′	10°	311°22′	10°	334°51′	10°	350°57′
12°	196°43′	12°	238°46′	12°	279°38′	12°	313°15′	12°	336°06′	12°	351°53′
14°	199°30′	14°	241°34′	14°	282°11′	14°	315°06′	14°	337°18′	14°	352°49′
16°	202°18′	16°	244°23′	16°	284°42′	16°	316°54′	16°	338°29′	16°	353°44′
18°	205°05′	18°	247°10′	18°	287°10′	18°	318°38′	18°	339°39′	18°	354°38′
20°	207°53′	20°	249°58′	20°	289°36′	20°	320°20′	20°	340°47′	20°	355°33′
22°	210°41′	22°	252°45′	22°	292°00′	22°	321°59′	22°	341°53′	22°	356°26′
24°	213°29′	24°	255°31′	24°	294°20′	24°	323°35′	24°	342°58′	24°	357°20′
26°	216°17′	26°	258°16′	26°	296°39′	26°	325°08′	26°	344°01′	26°	358°13′
28°	219°06′	28°	261°01′	28°	298°54′	28°	326°38′	28°	345°04′	28°	359°07′
30°	221°54′	30°	263°45′	30°	301°06′	30°	328°06′	30°	346°05′	30°	360°00′

Table 2.9: Oblique ascensions of the ecliptic for latitude $+50^\circ$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	011°04′	00°	026°14′	00°	051°45′	00°	090°35′	00°	135°15′
02°	000°42′	02°	011°54′	02°	027°31′	02°	053°58′	02°	093°29′	02°	138°15′
04°	001°24′	04°	012°44′	04°	028°52′	04°	056°15′	04°	096°24′	04°	141°15′
06°	002°06′	06°	013°36′	06°	030°16′	06°	058°35′	06°	099°21′	06°	144°15′
08°	002°48′	08°	014°30′	08°	031°44′	08°	060°59′	08°	102°18′	08°	147°14′
10°	003°31′	10°	015°24′	10°	033°14′	10°	063°27′	10°	105°16′	10°	150°14′
12°	004°14′	12°	016°21′	12°	034°48′	12°	065°57′	12°	108°14′	12°	153°13′
14°	004°57′	14°	017°18′	14°	036°26′	14°	068°31′	14°	111°14′	14°	156°12′
16°	005°41′	16°	018°18′	16°	038°07′	16°	071°08′	16°	114°13′	16°	159°11′
18°	006°25′	18°	019°19′	18°	039°52′	18°	073°48′	18°	117°13′	18°	162°10′
20°	007°10′	20°	020°23′	20°	041°41′	20°	076°31′	20°	120°13′	20°	165°08′
22°	007°55′	22°	021°28′	22°	043°34′	22°	079°16′	22°	123°14′	22°	168°07′
24°	008°41′	24°	022°36′	24°	045°31′	24°	082°03′	24°	126°14′	24°	171°05′
26°	009°28′	26°	023°46′	26°	047°32′	26°	084°52′	26°	129°14′	26°	174°03′
28°	010°15′	28°	024°58′	28°	049°37′	28°	087°43′	28°	132°14′	28°	177°02′
30°	011°04′	30°	026°14′	30°	051°45′	30°	090°35′	30°	135°15′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	224°45′	00°	269°25′	00°	308°15′	00°	333°46′	00°	348°56′
02°	182°58′	02°	227°46′	02°	272°17′	02°	310°23′	02°	335°02′	02°	349°45′
04°	185°57′	04°	230°46′	04°	275°08′	04°	312°28′	04°	336°14′	04°	350°32′
06°	188°55′	06°	233°46′	06°	277°57′	06°	314°29′	06°	337°24′	06°	351°19′
08°	191°53′	08°	236°46′	08°	280°44′	08°	316°26′	08°	338°32′	08°	352°05′
10°	194°52′	10°	239°47′	10°	283°29′	10°	318°19′	10°	339°37′	10°	352°50′
12°	197°50′	12°	242°47′	12°	286°12′	12°	320°08′	12°	340°41′	12°	353°35′
14°	200°49′	14°	245°47′	14°	288°52′	14°	321°53′	14°	341°42′	14°	354°19′
16°	203°48′	16°	248°46′	16°	291°29′	16°	323°34′	16°	342°42′	16°	355°03′
18°	206°47′	18°	251°46′	18°	294°03′	18°	325°12′	18°	343°39′	18°	355°46′
20°	209°46′	20°	254°44′	20°	296°33′	20°	326°46′	20°	344°36′	20°	356°29′
22°	212°46′	22°	257°42′	22°	299°01′	22°	328°16′	22°	345°30′	22°	357°12′
24°	215°45′	24°	260°39′	24°	301°25′	24°	329°44′	24°	346°24′	24°	357°54′
26°	218°45′	26°	263°36′	26°	303°45′	26°	331°08′	26°	347°16′	26°	358°36′
28°	221°45′	28°	266°31′	28°	306°02′	28°	332°29′	28°	348°06′	28°	359°18′
30°	224°45′	30°	269°25′	30°	308°15′	30°	333°46′	30°	348°56′	30°	360°00′

Table 2.10: Oblique ascensions of the ecliptic for latitude $+55^\circ$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	007°20′	00°	018°22′	00°	041°21′	00°	082°44′	00°	131°31′
02°	000°27′	02°	007°54′	02°	019°24′	02°	043°34′	02°	085°54′	02°	134°47′
04°	000°55′	04°	008°29′	04°	020°29′	04°	045°54′	04°	089°06′	04°	138°02′
06°	001°23′	06°	009°05′	06°	021°38′	06°	048°18′	06°	092°19′	06°	141°17′
08°	001°50′	08°	009°42′	08°	022°50′	08°	050°48′	08°	095°33′	08°	144°32′
10°	002°18′	10°	010°20′	10°	024°07′	10°	053°23′	10°	098°48′	10°	147°47′
12°	002°46′	12°	011°00′	12°	025°27′	12°	056°03′	12°	102°04′	12°	151°01′
14°	003°15′	14°	011°41′	14°	026°52′	14°	058°47′	14°	105°20′	14°	154°15′
16°	003°44′	16°	012°24′	16°	028°22′	16°	061°35′	16°	108°36′	16°	157°29′
18°	004°13′	18°	013°08′	18°	029°57′	18°	064°27′	18°	111°52′	18°	160°42′
20°	004°43′	20°	013°55′	20°	031°38′	20°	067°23′	20°	115°09′	20°	163°55′
22°	005°13′	22°	014°43′	22°	033°23′	22°	070°22′	22°	118°26′	22°	167°09′
24°	005°44′	24°	015°34′	24°	035°14′	24°	073°24′	24°	121°42′	24°	170°22′
26°	006°15′	26°	016°28′	26°	037°11′	26°	076°28′	26°	124°59′	26°	173°34′
28°	006°47′	28°	017°23′	28°	039°13′	28°	079°35′	28°	128°15′	28°	176°47′
30°	007°20′	30°	018°22′	30°	041°21′	30°	082°44′	30°	131°31′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	228°29′	00°	277°16′	00°	318°39′	00°	341°38′	00°	352°40′
02°	183°13′	02°	231°45′	02°	280°25′	02°	320°47′	02°	342°37′	02°	353°13′
04°	186°26′	04°	235°01′	04°	283°32′	04°	322°49′	04°	343°32′	04°	353°45′
06°	189°38′	06°	238°18′	06°	286°36′	06°	324°46′	06°	344°26′	06°	354°16′
08°	192°51′	08°	241°34′	08°	289°38′	08°	326°37′	08°	345°17′	08°	354°47′
10°	196°05′	10°	244°51′	10°	292°37′	10°	328°22′	10°	346°05′	10°	355°17′
12°	199°18′	12°	248°08′	12°	295°33′	12°	330°03′	12°	346°52′	12°	355°47′
14°	202°31′	14°	251°24′	14°	298°25′	14°	331°38′	14°	347°36′	14°	356°16′
16°	205°45′	16°	254°40′	16°	301°13′	16°	333°08′	16°	348°19′	16°	356°45′
18°	208°59′	18°	257°56′	18°	303°57′	18°	334°33′	18°	349°00′	18°	357°14′
20°	212°13′	20°	261°12′	20°	306°37′	20°	335°53′	20°	349°40′	20°	357°42′
22°	215°28′	22°	264°27′	22°	309°12′	22°	337°10′	22°	350°18′	22°	358°10′
24°	218°43′	24°	267°41′	24°	311°42′	24°	338°22′	24°	350°55′	24°	358°37′
26°	221°58′	26°	270°54′	26°	314°06′	26°	339°31′	26°	351°31′	26°	359°05′
28°	225°13′	28°	274°06′	28°	316°26′	28°	340°36′	28°	352°06′	28°	359°33′
30°	228°29′	30°	277°16′	30°	318°39′	30°	341°38′	30°	352°40′	30°	360°00′

Table 2.11: Oblique ascensions of the ecliptic for latitude $+60°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	002°07′	00°	005°56′	00°	021°39′	00°	070°18′	00°	126°18′
02°	000°08′	02°	002°17′	02°	006°22′	02°	023°56′	02°	074°04′	02°	129°57′
04°	000°16′	04°	002°28′	04°	006°51′	04°	026°25′	04°	077°50′	04°	133°35′
06°	000°23′	06°	002°39′	06°	007°22′	06°	029°06′	06°	081°36′	06°	137°12′
08°	000°31′	08°	002°51′	08°	007°57′	08°	031°58′	08°	085°22′	08°	140°49′
10°	000°39′	10°	003°03′	10°	008°36′	10°	034°59′	10°	089°08′	10°	144°25′
12°	000°47′	12°	003°16′	12°	009°19′	12°	038°09′	12°	092°54′	12°	148°00′
14°	000°55′	14°	003°29′	14°	010°07′	14°	041°26′	14°	096°39′	14°	151°35′
16°	001°04′	16°	003°44′	16°	011°02′	16°	044°50′	16°	100°24′	16°	155°09′
18°	001°12′	18°	003°59′	18°	012°04′	18°	048°19′	18°	104°08′	18°	158°43′
20°	001°21′	20°	004°15′	20°	013°14′	20°	051°52′	20°	107°52′	20°	162°16′
22°	001°29′	22°	004°32′	22°	014°33′	22°	055°29′	22°	111°35′	22°	165°50′
24°	001°38′	24°	004°51′	24°	016°02′	24°	059°08′	24°	115°17′	24°	169°22′
26°	001°48′	26°	005°11′	26°	017°42′	26°	062°50′	26°	118°58′	26°	172°55′
28°	001°57′	28°	005°33′	28°	019°34′	28°	066°33′	28°	122°38′	28°	176°28′
30°	002°07′	30°	005°56′	30°	021°39′	30°	070°18′	30°	126°18′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	233°42′	00°	289°42′	00°	338°21′	00°	354°04′	00°	357°53′
02°	183°32′	02°	237°22′	02°	293°27′	02°	340°26′	02°	354°27′	02°	358°03′
04°	187°05′	04°	241°02′	04°	297°10′	04°	342°18′	04°	354°49′	04°	358°12′
06°	190°38′	06°	244°43′	06°	300°52′	06°	343°58′	06°	355°09′	06°	358°22′
08°	194°10′	08°	248°25′	08°	304°31′	08°	345°27′	08°	355°28′	08°	358°31′
10°	197°44′	10°	252°08′	10°	308°08′	10°	346°46′	10°	355°45′	10°	358°39′
12°	201°17′	12°	255°52′	12°	311°41′	12°	347°56′	12°	356°01′	12°	358°48′
14°	204°51′	14°	259°36′	14°	315°10′	14°	348°58′	14°	356°16′	14°	358°56′
16°	208°25′	16°	263°21′	16°	318°34′	16°	349°53′	16°	356°31′	16°	359°05′
18°	212°00′	18°	267°06′	18°	321°51′	18°	350°41′	18°	356°44′	18°	359°13′
20°	215°35′	20°	270°52′	20°	325°01′	20°	351°24′	20°	356°57′	20°	359°21′
22°	219°11′	22°	274°38′	22°	328°02′	22°	352°03′	22°	357°09′	22°	359°29′
24°	222°48′	24°	278°24′	24°	330°54′	24°	352°38′	24°	357°21′	24°	359°37′
26°	226°25′	26°	282°10′	26°	333°35′	26°	353°09′	26°	357°32′	26°	359°44′
28°	230°03′	28°	285°56′	28°	336°04′	28°	353°38′	28°	357°43′	28°	359°52′
30°	233°42′	30°	289°42′	30°	338°21′	30°	354°04′	30°	357°53′	30°	360°00′

Table 2.12: Oblique ascensions of the ecliptic for latitude $+65°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	360°00′	00°	354°02′	00°	—	00°	—	00°41′	032°53′	00°	118°13′
02°	359°39′	02°	353°30′	02°	—	02°	—	02°	044°26′	02°	122°31′
04°	359°18′	04°	352°57′	04°	—	04°	—	04°	052°41′	04°	126°47′
06°	358°57′	06°	352°21′	06°	—	06°	—	06°	059°22′	06°	131°01′
08°	358°35′	08°	351°42′	08°	—	08°	—	08°	065°22′	08°	135°13′
10°	358°14′	10°	351°00′	10°	—	10°	—	10°	070°57′	10°	139°22′
12°	357°52′	12°	350°14′	12°	—	12°	—	12°	076°15′	12°	143°31′
14°	357°29′	14°	349°23′	14°	—	14°	—	14°	081°21′	14°	147°37′
16°	357°06′	16°	348°26′	16°	—	16°	—	16°	086°18′	16°	151°43′
18°	356°43′	18°	347°20′	18°	—	18°	—	18°	091°07′	18°	155°47′
20°	356°18′	20°	346°04′	20°	—	20°	—	20°	095°49′	20°	159°51′
22°	355°53′	22°	344°32′	22°	—	22°	—	22°	100°26′	22°	163°54′
24°	355°27′	24°	342°37′	24°	—	24°	—	24°	104°58′	24°	167°56′
26°	355°00′	26°	340°03′	26°	—	26°	—	26°	109°27′	26°	171°57′
28°	354°32′	28°	335°55′	28°	—	28°	—	28°	113°51′	28°	175°59′
30°	354°02′	29°19′	327°07′	30°	—	30°	—	30°	118°13′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	241°47′	00°	—	00°	—	00°41′	032°53′	00°	005°58′
02°	184°01′	02°	246°09′	02°	—	02°	—	02°	024°05′	02°	005°28′
04°	188°03′	04°	250°33′	04°	—	04°	—	04°	019°57′	04°	005°00′
06°	192°04′	06°	255°02′	06°	—	06°	—	06°	017°23′	06°	004°33′
08°	196°06′	08°	259°34′	08°	—	08°	—	08°	015°28′	08°	004°07′
10°	200°09′	10°	264°11′	10°	—	10°	—	10°	013°56′	10°	003°42′
12°	204°13′	12°	268°53′	12°	—	12°	—	12°	012°40′	12°	003°17′
14°	208°17′	14°	273°42′	14°	—	14°	—	14°	011°34′	14°	002°54′
16°	212°23′	16°	278°39′	16°	—	16°	—	16°	010°37′	16°	002°31′
18°	216°29′	18°	283°45′	18°	—	18°	—	18°	009°46′	18°	002°08′
20°	220°38′	20°	289°03′	20°	—	20°	—	20°	009°00′	20°	001°46′
22°	224°47′	22°	294°38′	22°	—	22°	—	22°	008°18′	22°	001°25′
24°	228°59′	24°	300°38′	24°	—	24°	—	24°	007°39′	24°	001°03′
26°	233°13′	26°	307°19′	26°	—	26°	—	26°	007°03′	26°	000°42′
28°	237°29′	28°	315°34′	28°	—	28°	—	28°	006°30′	28°	000°21′
30°	241°47′	29°19′	327°07′	30°	—	30°	—	30°	005°58′	30°	000°00′

Table 2.13: Oblique ascensions of the ecliptic for latitude $+70^\circ$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	360°00′	00°	338°42′	00°	—	00°	—	00°	—	00°	102°52′
02°	358°52′	02°	336°16′	02°	—	02°	—	02°	—	02°	108°49′
04°	357°44′	04°	333°24′	04°	—	04°	—	04°	—	04°	114°32′
06°	356°35′	06°	329°54′	06°	—	06°	—	06°	—	06°	120°04′
08°	355°25′	08°	325°10′	08°	—	08°	—	08°	—	08°	125°27′
10°	354°13′	10°	316°55′	10°	—	10°	—	10°	—	10°	130°43′
12°	353°00′	10°36′	308°11′	12°	—	12°	—	12°	—	12°	135°52′
14°	351°44′	14°	—	14°	—	14°	—	14°	—	14°	140°57′
16°	350°26′	16°	—	16°	—	16°	—	16°	—	16°	145°58′
18°	349°05′	18°	—	18°	—	18°	—	19°24′	051°49′	18°	150°56′
20°	347°39′	20°	—	20°	—	20°	—	20°	061°44′	20°	155°50′
22°	346°08′	22°	—	22°	—	22°	—	22°	073°54′	22°	160°43′
24°	344°30′	24°	—	24°	—	24°	—	24°	082°31′	24°	165°34′
26°	342°45′	26°	—	26°	—	26°	—	26°	089°54′	26°	170°23′
28°	340°50′	28°	—	28°	—	28°	—	28°	096°36′	28°	175°12′
30°	338°42′	30°	—	30°	—	30°	—	30°	102°52′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	257°08′	00°	—	00°	—	00°	—	00°	021°18′
02°	184°48′	02°	263°24′	02°	—	02°	—	02°	—	02°	019°10′
04°	189°37′	04°	270°06′	04°	—	04°	—	04°	—	04°	017°15′
06°	194°26′	06°	277°29′	06°	—	06°	—	06°	—	06°	015°30′
08°	199°17′	08°	286°06′	08°	—	08°	—	08°	—	08°	013°52′
10°	204°10′	10°	298°16′	10°	—	10°	—	10°	—	10°	012°21′
12°	209°04′	10°36′	308°11′	12°	—	12°	—	12°	—	12°	010°55′
14°	214°02′	14°	—	14°	—	14°	—	14°	—	14°	009°34′
16°	219°03′	16°	—	16°	—	16°	—	16°	—	16°	008°16′
18°	224°08′	18°	—	18°	—	18°	—	19°24′	051°49′	18°	007°00′
20°	229°17′	20°	—	20°	—	20°	—	20°	043°05′	20°	005°47′
22°	234°33′	22°	—	22°	—	22°	—	22°	034°50′	22°	004°35′
24°	239°56′	24°	—	24°	—	24°	—	24°	030°06′	24°	003°25′
26°	245°28′	26°	—	26°	—	26°	—	26°	026°36′	26°	002°16′
28°	251°11′	28°	—	28°	—	28°	—	28°	023°44′	28°	001°08′
30°	257°08′	30°	—	30°	—	30°	—	30°	021°18′	30°	000°00′

Table 2.14: Oblique ascensions of the ecliptic for latitude $+75°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	360°00′	00°	—	00°	—	00°	—	00°	—	00°	—
02°	357°19′	02°	—	02°	—	02°	—	02°	—	02°	—
04°	354°37′	04°	—	04°	—	04°	—	04°	—	04°07′	066°00′
06°	351°52′	06°	—	06°	—	06°	—	06°	—	06°	089°25′
08°	349°02′	08°	—	08°	—	08°	—	08°	—	08°	100°58′
10°	346°04′	10°	—	10°	—	10°	—	10°	—	10°	110°24′
12°	342°58′	12°	—	12°	—	12°	—	12°	—	12°	118°47′
14°	339°39′	14°	—	14°	—	14°	—	14°	—	14°	126°33′
16°	336°02′	16°	—	16°	—	16°	—	16°	—	16°	133°52′
18°	331°59′	18°	—	18°	—	18°	—	18°	—	18°	140°54′
20°	327°20′	20°	—	20°	—	20°	—	20°	—	20°	147°42′
22°	321°39′	22°	—	22°	—	22°	—	22°	—	22°	154°20′
24°	313°51′	24°	—	24°	—	24°	—	24°	—	24°	160°51′
25°53′	294°00′	26°	—	26°	—	26°	—	26°	—	26°	167°16′
28°	—	28°	—	28°	—	28°	—	28°	—	28°	173°39′
30°	—	30°	—	30°	—	30°	—	30°	—	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	—	00°	—	00°	—	00°	—	00°	—
02°	186°21′	02°	—	02°	—	02°	—	02°	—	02°	—
04°	192°44′	04°	—	04°	—	04°	—	04°	—	04°07′	066°00′
06°	199°09′	06°	—	06°	—	06°	—	06°	—	06°	046°09′
08°	205°40′	08°	—	08°	—	08°	—	08°	—	08°	038°21′
10°	212°18′	10°	—	10°	—	10°	—	10°	—	10°	032°40′
12°	219°06′	12°	—	12°	—	12°	—	12°	—	12°	028°01′
14°	226°08′	14°	—	14°	—	14°	—	14°	—	14°	023°58′
16°	233°27′	16°	—	16°	—	16°	—	16°	—	16°	020°21′
18°	241°13′	18°	—	18°	—	18°	—	18°	—	18°	017°02′
20°	249°36′	20°	—	20°	—	20°	—	20°	—	20°	013°56′
22°	259°02′	22°	—	22°	—	22°	—	22°	—	22°	010°58′
24°	270°35′	24°	—	24°	—	24°	—	24°	—	24°	008°08′
25°53′	294°00′	26°	—	26°	—	26°	—	26°	—	26°	005°23′
28°	—	28°	—	28°	—	28°	—	28°	—	28°	002°41′
30°	—	30°	—	30°	—	30°	—	30°	—	30°	000°00′

Table 2.15: Oblique ascensions of the ecliptic for latitude $+80°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	360°00′	00°	—	00°	—	00°	—	00°	—	00°	—
02°	352°42′	02°	—	02°	—	02°	—	02°	—	02°	—
04°	345°11′	04°	—	04°	—	04°	—	04°	—	04°	—
06°	337°07′	06°	—	06°	—	06°	—	06°	—	06°	—
08°	328°02′	08°	—	08°	—	08°	—	08°	—	08°	—
10°	316°53′	10°	—	10°	—	10°	—	10°	—	10°	—
12°	299°32′	12°	—	12°	—	12°	—	12°	—	12°	—
12°40′	281°39′	14°	—	14°	—	14°	—	14°	—	14°	—
16°	—	16°	—	16°	—	16°	—	16°	—	17°20′	078°21′
18°	—	18°	—	18°	—	18°	—	18°	—	18°	097°28′
20°	—	20°	—	20°	—	20°	—	20°	—	20°	118°31′
22°	—	22°	—	22°	—	22°	—	22°	—	22°	133°20′
24°	—	24°	—	24°	—	24°	—	24°	—	24°	146°06′
26°	—	26°	—	26°	—	26°	—	26°	—	26°	157°50′
28°	—	28°	—	28°	—	28°	—	28°	—	28°	169°02′
30°	—	30°	—	30°	—	30°	—	30°	—	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	—	00°	—	00°	—	00°	—	00°	—
02°	190°58′	02°	—	02°	—	02°	—	02°	—	02°	—
04°	202°10′	04°	—	04°	—	04°	—	04°	—	04°	—
06°	213°54′	06°	—	06°	—	06°	—	06°	—	06°	—
08°	226°40′	08°	—	08°	—	08°	—	08°	—	08°	—
10°	241°29′	10°	—	10°	—	10°	—	10°	—	10°	—
12°	262°32′	12°	—	12°	—	12°	—	12°	—	12°	—
12°39′	281°39′	14°	—	14°	—	14°	—	14°	—	14°	—
16°	—	16°	—	16°	—	16°	—	16°	—	17°20′	078°21′
18°	—	18°	—	18°	—	18°	—	18°	—	18°	060°28′
20°	—	20°	—	20°	—	20°	—	20°	—	20°	043°07′
22°	—	22°	—	22°	—	22°	—	22°	—	22°	031°58′
24°	—	24°	—	24°	—	24°	—	24°	—	24°	022°53′
26°	—	26°	—	26°	—	26°	—	26°	—	26°	014°49′
28°	—	28°	—	28°	—	28°	—	28°	—	28°	007°18′
30°	—	30°	—	30°	—	30°	—	30°	—	30°	000°00′

Table 2.16: Oblique ascensions of the ecliptic for latitude $+85°$.

Aries

00:00	90°00′	156°34′	336°34′
01:00	75°00′	156°34′	336°34′
02:00	60°00′	156°34′	336°34′
03:00	45°00′	156°34′	336°34′
04:00	30°00′	156°34′	336°34′
05:00	15°00′	156°34′	336°34′
06:00	00°00′	156°34′	336°34′

Taurus

00:00	78°32′	249°26′	249°26′
01:00	71°12′	196°00′	302°51′
02:00	58°04′	178°26′	320°25′
03:00	43°52′	170°40′	328°11′
04:00	29°20′	165°58′	332°53′
05:00	14°42′	162°29′	336°23′
06:00	00°00′	159°26′	339°26′

Gemini

00:00	69°51′	257°46′	257°46′
01:00	65°04′	219°53′	295°39′
02:00	54°24′	198°35′	316°57′
03:00	41°36′	186°47′	328°46′
04:00	28°00′	179°01′	336°32′
05:00	14°04′	173°03′	342°30′
06:00	00°00′	167°46′	347°46′

Cancer

00:00	66°34′	270°00′	270°00′
01:00	62°24′	236°02′	303°58′
02:00	52°37′	214°34′	325°26′
03:00	40°27′	201°41′	338°19′
04:00	27°18′	192°56′	347°04′
05:00	13°44′	186°05′	353°55′
06:00	00°00′	180°00′	000°00′

Leo

00:00	69°51′	282°14′	282°14′
01:00	65°04′	244°21′	320°07′
02:00	54°24′	223°03′	341°25′
03:00	41°36′	211°14′	353°13′
04:00	28°00′	203°28′	000°59′
05:00	14°04′	197°30′	006°57′
06:00	00°00′	192°14′	012°14′

Virgo

00:00	78°32′	290°34′	290°34′
01:00	71°12′	237°09′	344°00′
02:00	58°04′	219°35′	001°34′
03:00	43°52′	211°49′	009°20′
04:00	29°20′	207°07′	014°02′
05:00	14°42′	203°37′	017°31′
06:00	00°00′	200°34′	020°34′

Libra

00:00	90°00′	203°26′	023°26′
01:00	75°00′	203°26′	023°26′
02:00	60°00′	203°26′	023°26′
03:00	45°00′	203°26′	023°26′
04:00	30°00′	203°26′	023°26′
05:00	15°00′	203°26′	023°26′
06:00	00°00′	203°26′	023°26′

Scorpio

00:00	78°32′	110°34′	110°34′
01:00	71°12′	164°00′	057°09′
02:00	58°04′	181°34′	039°35′
03:00	43°52′	189°20′	031°49′
04:00	29°20′	194°02′	027°07′
05:00	14°42′	197°31′	023°37′
06:00	00°00′	200°34′	020°34′

Sagittarius

00:00	69°51′	102°14′	102°14′
01:00	65°04′	140°07′	064°21′
02:00	54°24′	161°25′	043°03′
03:00	41°36′	173°13′	031°14′
04:00	28°00′	180°59′	023°28′
05:00	14°04′	186°57′	017°30′
06:00	00°00′	192°14′	012°14′

Capricorn

00:00	66°34′	090°00′	090°00′
01:00	62°24′	123°58′	056°02′
02:00	52°37′	145°26′	034°34′
03:00	40°27′	158°19′	021°41′
04:00	27°18′	167°04′	012°56′
05:00	13°44′	173°55′	006°05′
06:00	00°00′	180°00′	000°00′

Aquarius

00:00	69°51′	077°46′	077°46′
01:00	65°04′	115°39′	039°53′
02:00	54°24′	136°57′	018°35′
03:00	41°36′	148°46′	006°47′
04:00	28°00′	156°32′	359°01′
05:00	14°04′	162°30′	353°03′
06:00	00°00′	167°46′	347°46′

Pisces

00:00	78°32′	069°26′	069°26′
01:00	71°12′	122°51′	016°00′
02:00	58°04′	140°25′	358°26′
03:00	43°52′	148°11′	350°40′
04:00	29°20′	152°53′	345°58′
05:00	14°42′	156°23′	342°29′
06:00	00°00′	159°26′	339°26′

Table 2.17: Ecliptic altitude and parallactic angle for latitude 0°.

	Aries		
00:00	80°00′	066°34′	066°34′
01:00	72°02′	122°18′	010°50′
02:00	58°32′	137°08′	356°00′
03:00	44°08′	142°34′	350°34′
04:00	29°30′	145°03′	348°05′
05:00	14°46′	146°13′	346°55′
06:00	00°00′	146°34′	346°34′
	Taurus		
00:00	88°32′	249°26′	249°26′
01:00	75°11′	163°41′	335°10′
02:00	60°30′	159°21′	339°30′
03:00	45°48′	156°49′	342°02′
04:00	31°08′	154°35′	344°16′
05:00	16°31′	152°16′	346°35′
06:00	01°59′	149°37′	349°14′
06:08	00°00′	149°13′	349°38′
	Gemini		
00:00	79°51′	257°46′	257°46′
01:00	72°20′	200°37′	314°55′
02:00	59°22′	182°38′	332°54′
03:00	45°32′	174°04′	341°29′
04:00	31°28′	168°13′	347°20′
05:00	17°24′	163°15′	352°18′
06:00	03°26′	158°22′	357°10′
06:15	00°00′	157°07′	358°26′
	Cancer		
00:00	76°34′	270°00′	270°00′
01:00	70°22′	220°40′	319°20′
02:00	58°23′	200°04′	339°56′
03:00	45°04′	189°35′	350°25′
04:00	31°23′	182°27′	357°33′
05:00	17°38′	176°31′	003°29′
06:00	03°58′	170°49′	009°11′
06:18	00°00′	169°05′	010°55′
	Leo		
00:00	79°51′	282°14′	282°14′
01:00	72°20′	225°05′	339°23′
02:00	59°22′	207°06′	357°22′
03:00	45°32′	198°31′	005°56′
04:00	31°28′	192°40′	011°47′
05:00	17°24′	187°42′	016°45′
06:00	03°26′	182°50′	021°38′
06:15	00°00′	181°34′	022°53′
	Virgo		
00:00	88°32′	290°34′	290°34′
01:00	75°11′	204°50′	016°19′
02:00	60°30′	200°30′	020°39′
03:00	45°48′	197°58′	023°11′
04:00	31°08′	195°44′	025°25′
05:00	16°31′	193°25′	027°44′
06:00	01°59′	190°46′	030°23′
06:08	00°00′	190°22′	030°47′
	Libra		
00:00	80°00′	113°26′	113°26′
01:00	72°02′	169°10′	057°42′
02:00	58°32′	184°00′	042°52′
03:00	44°08′	189°26′	037°26′
04:00	29°30′	191°55′	034°57′
05:00	14°46′	193°05′	033°47′
06:00	00°00′	193°26′	033°26′
	Scorpio		
00:00	68°32′	110°34′	110°34′
01:00	63°52′	145°55′	075°13′
02:00	53°15′	165°58′	055°11′
03:00	40°23′	176°40′	044°29′
04:00	26°37′	183°07′	038°01′
05:00	12°26′	187°30′	033°39′
05:52	00°00′	190°22′	030°47′
	Sagittarius		
00:00	59°51′	102°14′	102°14′
01:00	56°26′	129°41′	074°47′
02:00	47°48′	149°23′	055°05′
03:00	36°26′	162°11′	042°17′
04:00	23°44′	170°55′	033°32′
05:00	10°20′	177°27′	027°00′
05:45	00°00′	181°34′	022°53′
	Capricorn		
00:00	56°34′	090°00′	090°00′
01:00	53°29′	115°22′	064°38′
02:00	45°31′	134°39′	045°21′
03:00	34°44′	147°56′	032°04′
04:00	22°30′	157°24′	022°36′
05:00	09°29′	164°40′	015°20′
05:42	00°00′	169°05′	010°55′
	Aquarius		
00:00	59°51′	077°46′	077°46′
01:00	56°26′	105°13′	050°19′
02:00	47°48′	124°55′	030°37′
03:00	36°26′	137°43′	017°49′
04:00	23°44′	146°28′	009°05′
05:00	10°20′	153°00′	002°33′
05:45	00°00′	157°07′	358°26′
	Pisces		
00:00	68°32′	069°26′	069°26′
01:00	63°52′	104°47′	034°05′
02:00	53°15′	124°49′	014°02′
03:00	40°23′	135°31′	003°20′
04:00	26°37′	141°59′	356°53′
05:00	12°26′	146°21′	352°30′
05:52	00°00′	149°13′	349°38′

Table 2.18: Ecliptic altitude and parallactic angle for latitude $+10°$.

Aries

00:00	70°00′	066°34′	066°34′
01:00	65°11′	101°59′	031°09′
02:00	54°28′	120°31′	012°37′
03:00	41°38′	129°20′	003°48′
04:00	28°01′	133°46′	359°22′
05:00	14°05′	135°55′	357°13′
06:00	00°00′	136°34′	356°34′

Taurus

00:00	81°28′	069°26′	069°26′
01:00	73°15′	126°58′	011°53′
02:00	59°57′	139°10′	359°41′
03:00	45°59′	142°26′	356°25′
04:00	31°54′	142°53′	355°58′
05:00	17°50′	141°53′	356°58′
06:00	03°54′	139°48′	359°03′
06:17	00°00′	139°00′	359°51′

Gemini

00:00	89°51′	257°46′	257°46′
01:00	75°55′	165°46′	349°46′
02:00	61°52′	162°48′	352°44′
03:00	47°52′	159°52′	355°41′
04:00	33°59′	156°42′	358°51′
05:00	20°15′	153°07′	002°26′
06:00	06°46′	148°54′	006°38′
06:31	00°00′	146°24′	009°08′

Cancer

00:00	86°34′	270°00′	270°00′
01:00	75°39′	190°58′	202°37′
02:00	61°58′	181°12′	204°00′
03:00	48°13′	175°44′	150°21′
04:00	34°33′	171°08′	143°47′
05:00	21°03′	166°33′	137°02′
06:00	07°49′	161°32′	130°26′
06:36	00°00′	158°07′	126°33′

Leo

00:00	89°51′	282°14′	282°14′
01:00	75°55′	190°14′	014°14′
02:00	61°52′	187°16′	017°12′
03:00	47°52′	184°19′	020°08′
04:00	33°59′	181°09′	023°18′
05:00	20°15′	177°34′	026°53′
06:00	06°46′	173°22′	031°06′
06:31	00°00′	170°52′	033°36′

Virgo

00:00	81°28′	110°34′	110°34′
01:00	73°15′	168°07′	053°02′
02:00	59°57′	180°19′	040°50′
03:00	45°59′	183°35′	037°34′
04:00	31°54′	184°02′	037°07′
05:00	17°50′	183°02′	038°07′
06:00	03°54′	180°57′	040°12′
06:17	00°00′	180°09′	041°00′

Libra

00:00	70°00′	113°26′	113°26′
01:00	65°11′	148°51′	078°01′
02:00	54°28′	167°23′	059°29′
03:00	41°38′	176°12′	050°40′
04:00	28°01′	180°38′	046°14′
05:00	14°05′	182°47′	044°05′
06:00	00°00′	183°26′	043°26′

Scorpio

00:00	58°32′	110°34′	110°34′
01:00	55°14′	135°49′	085°19′
02:00	46°51′	153°58′	067°11′
03:00	35°41′	165°27′	055°42′
04:00	23°06′	172°48′	048°21′
05:00	09°48′	177°40′	043°29′
05:43	00°00′	180°09′	041°00′

Sagittarius

00:00	49°51′	102°14′	102°14′
01:00	47°15′	123°13′	081°14′
02:00	40°15′	140°14′	064°14′
03:00	30°24′	152°37′	051°50′
04:00	18°52′	161°33′	042°55′
05:00	06°21′	168°11′	036°16′
05:29	00°00′	170°52′	033°36′

Capricorn

00:00	46°34′	090°00′	090°00′
01:00	44°10′	109°49′	070°11′
02:00	37°38′	126°24′	053°36′
03:00	28°16′	138°59′	041°01′
04:00	17°10′	148°24′	031°36′
05:00	05°00′	155°40′	024°20′
05:24	00°00′	158°07′	021°53′

Aquarius

00:00	49°51′	077°46′	077°46′
01:00	47°15′	098°46′	056°47′
02:00	40°15′	115°46′	039°46′
03:00	30°24′	128°10′	027°23′
04:00	18°52′	137°05′	018°27′
05:00	06°21′	143°44′	011°49′
05:29	00°00′	146°24′	009°08′

Pisces

00:00	58°32′	069°26′	069°26′
01:00	55°14′	094°41′	044°11′
02:00	46°51′	112°49′	026°02′
03:00	35°41′	124°18′	014°33′
04:00	23°06′	131°39′	007°12′
05:00	09°48′	136°31′	002°20′
05:43	00°00′	139°00′	359°51′

Table 2.19: Ecliptic altitude and parallactic angle for latitude $+20°$.

	Aries		
00:00	60°00′	066°34′	066°34′
01:00	56°46′	090°43′	042°25′
02:00	48°35′	107°28′	025°40′
03:00	37°46′	117°20′	015°48′
04:00	25°40′	122°53′	010°15′
05:00	12°57′	125°42′	007°26′
06:00	00°00′	126°34′	006°34′
	Taurus		
00:00	71°28′	069°26′	069°26′
01:00	66°49′	104°08′	034°43′
02:00	56°33′	121°13′	017°38′
03:00	44°24′	128°24′	010°27′
04:00	31°35′	131°07′	007°44′
05:00	18°36′	131°23′	007°28′
06:00	05°42′	129°55′	008°56′
06:27	00°00′	128°45′	010°06′
	Gemini		
00:00	80°09′	077°46′	077°46′
01:00	73°15′	128°48′	026°45′
02:00	61°12′	141°47′	013°46′
03:00	48°20′	144°53′	010°40′
04:00	35°22′	144°39′	010°54′
05:00	22°30′	142°39′	012°54′
06:00	09°55′	139°19′	016°14′
06:49	00°00′	135°36′	019°57′
	Cancer		
00:00	83°26′	090°00′	090°00′
01:00	75°06′	150°38′	029°22′
02:00	62°30′	159°40′	020°20′
03:00	49°32′	160°38′	019°22′
04:00	36°36′	159°05′	020°55′
05:00	23°52′	156°10′	023°50′
06:00	11°28′	152°05′	027°55′
06:58	00°00′	146°59′	033°01′
	Leo		
00:00	80°09′	102°14′	102°14′
01:00	73°15′	153°15′	051°12′
02:00	61°12′	166°14′	038°13′
03:00	48°20′	169°20′	035°07′
04:00	35°22′	169°06′	035°21′
05:00	22°30′	167°06′	037°21′
06:00	09°55′	163°46′	040°41′
06:49	00°00′	160°03′	044°24′
	Virgo		
00:00	71°28′	110°34′	110°34′
01:00	66°49′	145°17′	075°52′
02:00	56°33′	162°22′	058°47′
03:00	44°24′	169°33′	051°36′
04:00	31°35′	172°16′	048°53′
05:00	18°36′	172°32′	048°37′
06:00	05°42′	171°04′	050°05′
06:27	00°00′	169°54′	051°15′
	Libra		
00:00	60°00′	113°26′	113°26′
01:00	56°46′	137°35′	089°17′
02:00	48°35′	154°20′	072°32′
03:00	37°46′	164°12′	062°40′
04:00	25°40′	169°45′	057°07′
05:00	12°57′	172°34′	054°18′
06:00	00°00′	173°26′	053°26′
	Scorpio		
00:00	48°32′	110°34′	110°34′
01:00	46°05′	129°26′	091°43′
02:00	39°28′	144°41′	076°27′
03:00	30°03′	155°36′	065°33′
04:00	18°58′	163°03′	058°06′
05:00	06°54′	168°00′	053°09′
05:33	00°00′	169°54′	051°15′
	Sagittarius		
00:00	39°51′	102°14′	102°14′
01:00	37°49′	118°43′	085°45′
02:00	32°08′	132°59′	071°28′
03:00	23°45′	144°13′	060°14′
04:00	13°33′	152°43′	051°44′
05:00	02°11′	159°04′	045°23′
05:11	00°00′	160°03′	044°24′
	Capricorn		
00:00	36°34′	090°00′	090°00′
01:00	34°40′	105°49′	074°11′
02:00	29°18′	119°46′	060°14′
03:00	21°17′	131°05′	048°55′
04:00	11°27′	139°56′	040°04′
05:00	00°23′	146°47′	033°13′
05:02	00°00′	146°59′	033°01′
	Aquarius		
00:00	39°51′	077°46′	077°46′
01:00	37°49′	094°15′	061°17′
02:00	32°08′	108°32′	047°01′
03:00	23°45′	119°46′	035°47′
04:00	13°33′	128°16′	027°17′
05:00	02°11′	134°37′	020°56′
05:11	00°00′	135°36′	019°57′
	Pisces		
00:00	48°32′	069°26′	069°26′
01:00	46°05′	088°17′	050°34′
02:00	39°28′	103°33′	035°19′
03:00	30°03′	114°27′	024°24′
04:00	18°58′	121°54′	016°57′
05:00	06°54′	126°51′	012°00′
05:33	00°00′	128°45′	010°06′

Table 2.20: Ecliptic altitude and parallactic angle for latitude $+30°$.

	Aries		
00:00	50°00′	066°34′	066°34′
01:00	47°44′	083°43′	049°25′
02:00	41°34′	097°21′	035°47′
03:00	32°48′	106°41′	026°27′
04:00	22°31′	112°28′	020°40′
05:00	11°26′	115°35′	017°33′
06:00	00°00′	116°34′	016°34′
	Taurus		
00:00	61°28′	069°26′	069°26′
01:00	58°32′	091°45′	047°06′
02:00	51°05′	106°59′	031°52′
03:00	41°12′	115°28′	023°23′
04:00	30°13′	119°34′	019°17′
05:00	18°47′	120°50′	018°01′
06:00	07°21′	120°00′	018°51′
06:39	00°00′	118°26′	020°25′
	Gemini		
00:00	70°09′	077°46′	077°46′
01:00	66°21′	107°24′	048°09′
02:00	57°35′	123°23′	032°10′
03:00	46°53′	130°11′	025°21′
04:00	35°31′	132°22′	023°11′
05:00	24°03′	131°54′	023°39′
06:00	12°47′	129°33′	026°00′
07:00	02°01′	125°32′	030°00′
07:12	00°00′	124°34′	030°59′
	Cancer		
00:00	73°26′	090°00′	090°00′
01:00	69°09′	123°52′	056°08′
02:00	59°48′	139°36′	040°24′
03:00	48°49′	145°21′	034°39′
04:00	37°23′	146°36′	033°24′
05:00	25°57′	145°23′	034°37′
06:00	14°49′	142°24′	037°36′
07:00	04°14′	137°54′	042°06′
07:25	00°00′	135°32′	044°28′
	Leo		
00:00	70°09′	102°14′	102°14′
01:00	66°21′	131°51′	072°36′
02:00	57°35′	147°50′	056°37′
03:00	46°53′	154°39′	049°49′
04:00	35°31′	156°49′	047°38′
05:00	24°03′	156°21′	048°06′
06:00	12°47′	154°00′	050°27′
07:00	02°01′	150°00′	054°28′
07:12	00°00′	149°01′	055°26′
	Virgo		
00:00	61°28′	110°34′	110°34′
01:00	58°32′	132°54′	088°15′
02:00	51°05′	148°08′	073°01′
03:00	41°12′	156°37′	064°32′
04:00	30°13′	160°43′	060°26′
05:00	18°47′	161°59′	059°10′
06:00	07°21′	161°09′	060°00′
06:39	00°00′	159°35′	061°34′
	Libra		
00:00	50°00′	113°26′	113°26′
01:00	47°44′	130°35′	096°17′
02:00	41°34′	144°13′	082°39′
03:00	32°48′	153°33′	073°19′
04:00	22°31′	159°20′	067°32′
05:00	11°26′	162°27′	064°25′
06:00	00°00′	163°26′	063°26′
	Scorpio		
00:00	38°32′	110°34′	110°34′
01:00	36°41′	124°53′	096°16′
02:00	31°29′	137°16′	083°53′
03:00	23°46′	146°52′	074°17′
04:00	14°20′	153°47′	067°22′
05:00	03°49′	158°26′	062°42′
05:21	00°00′	159°35′	061°34′
	Sagittarius		
00:00	29°51′	102°14′	102°14′
01:00	28°15′	115°14′	089°13′
02:00	23°40′	126°57′	077°30′
03:00	16°41′	136°40′	067°47′
04:00	07°57′	144°17′	060°10′
04:48	00°00′	149°01′	055°26′
	Capricorn		
00:00	26°34′	090°00′	090°00′
01:00	25°03′	102°38′	077°22′
02:00	20°41′	114°10′	065°50′
03:00	13°58′	123°56′	056°04′
04:00	05°30′	131°48′	048°12′
04:35	00°00′	135°32′	044°28′
	Aquarius		
00:00	29°51′	077°46′	077°46′
01:00	28°15′	090°47′	064°46′
02:00	23°40′	102°30′	053°03′
03:00	16°41′	112°13′	043°20′
04:00	07°57′	119°50′	035°43′
04:48	00°00′	124°34′	030°59′
	Pisces		
00:00	38°32′	069°26′	069°26′
01:00	36°41′	083°44′	055°07′
02:00	31°29′	096°07′	042°44′
03:00	23°46′	105°43′	033°08′
04:00	14°20′	112°38′	026°13′
05:00	03°49′	117°18′	021°34′
05:21	00°00′	118°26′	020°25′

Table 2.21: Ecliptic altitude and parallactic angle for latitude $+40°$.

Aries			
00:00	40°00′	066°34′	066°34′
01:00	38°23′	078°49′	054°19′
02:00	33°50′	089°20′	043°48′
03:00	27°02′	097°15′	035°53′
04:00	18°45′	102°34′	030°34′
05:00	09°35′	105°36′	027°32′
06:00	00°00′	106°34′	026°34′
Taurus			
00:00	51°28′	069°26′	069°26′
01:00	49°32′	084°17′	054°34′
02:00	44°15′	096°05′	042°46′
03:00	36°43′	103°58′	034°53′
04:00	27°52′	108°27′	030°24′
05:00	18°23′	110°17′	028°34′
06:00	08°46′	110°00′	028°51′
06:56	00°00′	108°01′	030°50′
Gemini			
00:00	60°09′	077°46′	077°46′
01:00	57°51′	096°00′	059°33′
02:00	51°51′	109°08′	046°25′
03:00	43°40′	116°42′	038°50′
04:00	34°26′	120°14′	035°19′
05:00	24°50′	120°57′	034°36′
06:00	15°18′	119°34′	035°59′
07:00	06°11′	116°25′	039°08′
07:44	00°00′	113°05′	042°27′
Cancer			
00:00	63°26′	090°00′	090°00′
01:00	60°58′	110°03′	069°57′
02:00	54°38′	123°43′	056°17′
03:00	46°12′	131°02′	048°58′
04:00	36°50′	134°04′	045°56′
05:00	27°13′	134°17′	045°43′
06:00	17°44′	132°27′	047°33′
07:00	08°45′	128°55′	051°05′
08:00	00°34′	123°50′	056°10′
08:04	00°00′	123°24′	056°36′
Leo			
00:00	60°09′	102°14′	102°14′
01:00	57°51′	120°27′	084°00′
02:00	51°51′	133°35′	070°52′
03:00	43°40′	141°10′	063°18′
04:00	34°26′	144°41′	059°46′
05:00	24°50′	145°24′	059°03′
06:00	15°18′	144°01′	060°26′
07:00	06°11′	140°52′	063°35′
07:44	00°00′	137°33′	066°55′
Virgo			
00:00	51°28′	110°34′	110°34′
01:00	49°32′	125°26′	095°43′
02:00	44°15′	137°14′	083°55′
03:00	36°43′	145°07′	076°02′
04:00	27°52′	149°36′	071°33′
05:00	18°23′	151°26′	069°43′
06:00	08°46′	151°09′	070°00′
06:56	00°00′	149°10′	071°59′
Libra			
00:00	40°00′	113°26′	113°26′
01:00	38°23′	125°41′	101°11′
02:00	33°50′	136°12′	090°40′
03:00	27°02′	144°07′	082°45′
04:00	18°45′	149°26′	077°26′
05:00	09°35′	152°28′	074°24′
06:00	00°00′	153°26′	073°26′
06:00	00°00′	153°26′	073°26′
Scorpio			
00:00	28°32′	110°34′	110°34′
01:00	27°08′	121°21′	099°48′
02:00	23°09′	131°02′	090°07′
03:00	17°03′	138°58′	082°11′
04:00	09°22′	144°55′	076°14′
05:00	00°37′	148°57′	072°11′
05:04	00°00′	149°10′	071°59′
Sagittarius			
00:00	19°51′	102°14′	102°14′
01:00	18°36′	112°20′	092°07′
02:00	15°00′	121°40′	082°48′
03:00	09°22′	129°39′	074°48′
04:00	02°10′	136°05′	068°22′
04:16	00°00′	137°33′	066°55′
Capricorn			
00:00	16°34′	090°00′	090°00′
01:00	15°22′	099°56′	080°04′
02:00	11°54′	109°10′	070°50′
03:00	06°27′	117°13′	062°47′
03:56	00°00′	123°24′	056°36′
Aquarius			
00:00	19°51′	077°46′	077°46′
01:00	18°36′	087°53′	067°40′
02:00	15°00′	097°12′	058°20′
03:00	09°22′	105°12′	050°21′
04:00	02°10′	111°38′	043°55′
04:16	00°00′	113°05′	042°27′
Pisces			
00:00	28°32′	069°26′	069°26′
01:00	27°08′	080°12′	058°39′
02:00	23°09′	089°53′	048°58′
03:00	17°03′	097°49′	041°02′
04:00	09°22′	103°46′	035°05′
05:00	00°37′	107°49′	031°03′
05:04	00°00′	108°01′	030°50′

Table 2.22: Ecliptic altitude and parallactic angle for latitude $+50°$.

	Aries		
00:00	30°00′	066°34′	066°34′
01:00	28°53′	075°04′	058°04′
02:00	25°40′	082°40′	050°28′
03:00	20°42′	088°46′	044°22′
04:00	14°29′	093°08′	040°00′
05:00	07°26′	095°43′	037°25′
06:00	00°00′	096°34′	036°34′
	Taurus		
00:00	41°28′	069°26′	069°26′
01:00	40°12′	079°11′	059°40′
02:00	36°37′	087°35′	051°17′
03:00	31°15′	093°51′	045°00′
04:00	24°40′	097°53′	040°58′
05:00	17°24′	099°50′	039°01′
06:00	09°55′	099°56′	038°55′
07:00	02°36′	098°20′	040°31′
07:22	00°00′	097°20′	041°31′
	Gemini		
00:00	50°09′	077°46′	077°46′
01:00	48°44′	089°05′	066°27′
02:00	44°49′	098°24′	057°08′
03:00	39°04′	104°52′	050°41′
04:00	32°12′	108°33′	046°59′
05:00	24°49′	109°55′	045°37′
06:00	17°21′	109°22′	046°11′
07:00	10°11′	107°09′	048°23′
08:00	03°39′	103°29′	052°03′
08:38	00°00′	100°29′	055°04′
	Cancer		
00:00	53°26′	090°00′	090°00′
01:00	51°57′	102°07′	077°53′
02:00	47°53′	111°53′	068°07′
03:00	41°58′	118°24′	061°36′
04:00	35°01′	121°55′	058°05′
05:00	27°35′	123°01′	056°59′
06:00	20°09′	122°11′	057°49′
07:00	13°03′	119°43′	060°17′
08:00	06°36′	115°51′	064°09′
09:00	01°09′	110°43′	069°17′
09:15	00°00′	109°17′	070°43′
	Leo		
00:00	50°09′	102°14′	102°14′
01:00	48°44′	113°33′	090°55′
02:00	44°49′	122°52′	081°36′
03:00	39°04′	129°19′	075°08′
04:00	32°12′	133°01′	071°27′
05:00	24°49′	134°23′	070°05′
06:00	17°21′	133°49′	070°38′
07:00	10°11′	131°37′	072°51′
08:00	03°39′	127°57′	076°31′
08:38	00°00′	124°56′	079°31′
	Virgo		
00:00	41°28′	110°34′	110°34′
01:00	40°12′	120°20′	100°49′
02:00	36°37′	128°43′	092°25′
03:00	31°15′	135°00′	086°09′
04:00	24°40′	139°02′	082°07′
05:00	17°24′	140°59′	080°10′
06:00	09°55′	141°05′	080°04′
07:00	02°36′	139°29′	081°40′
07:22	00°00′	138°29′	082°40′
	Libra		
00:00	30°00′	113°26′	113°26′
01:00	28°53′	121°56′	104°56′
02:00	25°40′	129°32′	097°20′
03:00	20°42′	135°38′	091°14′
04:00	14°29′	140°00′	086°52′
05:00	07°26′	142°35′	084°17′
06:00	00°00′	143°26′	083°26′
	Scorpio		
00:00	18°32′	110°34′	110°34′
01:00	17°31′	118°22′	102°46′
02:00	14°36′	125°33′	095°36′
03:00	10°02′	131°37′	089°32′
04:00	04°11′	136°18′	084°51′
04:38	00°00′	138°29′	082°40′
	Sagittarius		
00:00	09°51′	102°14′	102°14′
01:00	08°56′	109°45′	094°42′
02:00	06°13′	116°48′	087°40′
03:00	01°56′	122°57′	081°31′
03:22	00°00′	124°56′	079°31′
	Capricorn		
00:00	06°34′	090°00′	090°00′
01:00	05°40′	097°28′	082°32′
02:00	03°02′	104°30′	075°30′
02:45	00°00′	109°17′	070°43′
	Aquarius		
00:00	09°51′	077°46′	077°46′
01:00	08°56′	085°18′	070°15′
02:00	06°13′	092°20′	063°12′
03:00	01°56′	098°29′	057°03′
03:22	00°00′	100°29′	055°04′
	Pisces		
00:00	18°32′	069°26′	069°26′
01:00	17°31′	077°14′	061°38′
02:00	14°36′	084°24′	054°27′
03:00	10°02′	090°28′	048°23′
04:00	04°11′	095°09′	043°42′
04:38	00°00′	097°20′	041°31′

Table 2.23: Ecliptic altitude and parallactic angle for latitude $+60°$.

	Aries		
00:00	20°00′	066°34′	066°34′
01:00	19°17′	071°57′	061°11′
02:00	17°14′	076°53′	056°15′
03:00	14°00′	081°00′	052°08′
04:00	09°51′	084°04′	049°04′
05:00	05°05′	085°56′	047°12′
06:00	00°00′	086°34′	046°34′
	Taurus		
00:00	31°28′	069°26′	069°26′
01:00	30°42′	075°20′	063°31′
02:00	28°30′	080°39′	058°12′
03:00	25°05′	084°55′	053°56′
04:00	20°46′	087°54′	050°58′
05:00	15°53′	089°31′	049°20′
06:00	10°46′	089°48′	049°03′
07:00	05°45′	088°49′	050°02′
08:00	01°06′	086°40′	052°12′
08:16	00°00′	085°55′	052°56′
	Gemini		
00:00	40°09′	077°46′	077°46′
01:00	39°20′	084°21′	071°12′
02:00	37°00′	090°08′	065°25′
03:00	33°25′	094°37′	060°56′
04:00	28°58′	097°34′	057°59′
05:00	24°00′	098°58′	056°34′
06:00	18°53′	098°58′	056°35′
07:00	13°55′	097°40′	057°52′
08:00	09°23′	095°15′	060°18′
09:00	05°33′	091°50′	063°43′
10:00	02°37′	087°38′	067°55′
11:00	00°46′	082°51′	072°42′
12:00	00°09′	077°46′	077°46′
	Cancer		
00:00	43°26′	090°00′	090°00′
01:00	42°36′	096°54′	083°06′
02:00	40°12′	102°56′	077°04′
03:00	36°33′	107°31′	072°29′
04:00	32°03′	110°27′	069°33′
05:00	27°04′	111°47′	068°13′
06:00	21°57′	111°38′	068°22′
07:00	17°00′	110°13′	069°47′
08:00	12°31′	107°40′	072°20′
09:00	08°44′	104°10′	075°50′
10:00	05°51′	099°54′	080°06′
11:00	04°03′	095°05′	084°55′
12:00	03°26′	090°00′	090°00′
	Leo		
00:00	40°09′	102°14′	102°14′
01:00	39°20′	108°48′	095°39′
02:00	37°00′	114°35′	089°52′
03:00	33°25′	119°04′	085°23′
04:00	28°58′	122°01′	082°26′
05:00	24°00′	123°26′	081°02′
06:00	18°53′	123°25′	081°02′
07:00	13°55′	122°08′	082°20′
08:00	09°23′	119°42′	084°45′
09:00	05°33′	116°17′	088°10′
10:00	02°37′	112°05′	092°22′
11:00	00°46′	107°18′	097°09′
12:00	00°09′	102°14′	102°14′
	Virgo		
00:00	31°28′	110°34′	110°34′
01:00	30°42′	116°29′	104°40′
02:00	28°30′	121°48′	099°21′
03:00	25°05′	126°04′	095°05′
04:00	20°46′	129°02′	092°06′
05:00	15°53′	130°40′	090°29′
06:00	10°46′	130°57′	090°12′
07:00	05°45′	129°58′	091°11′
08:00	01°06′	127°48′	093°20′
08:16	00°00′	127°04′	094°05′
	Libra		
00:00	20°00′	113°26′	113°26′
01:00	19°17′	118°49′	108°03′
02:00	17°14′	123°45′	103°07′
03:00	14°00′	127°52′	099°00′
04:00	09°51′	130°56′	095°56′
05:00	05°05′	132°48′	094°04′
06:00	00°00′	133°26′	093°26′
	Scorpio		
00:00	08°32′	110°34′	110°34′
01:00	07°52′	115°42′	105°27′
02:00	05°56′	120°28′	100°40′
03:00	02°53′	124°35′	096°34′
03:44	00°00′	127°04′	094°05′
	Pisces		
00:00	08°32′	069°26′	069°26′
01:00	07°52′	074°33′	064°18′
02:00	05°56′	079°20′	059°32′
03:00	02°53′	083°26′	055°25′
03:44	00°00′	085°55′	052°56′

Table 2.24: Ecliptic altitude and parallactic angle for latitude $+70°$.

	Aries		
00:00	10°00′	066°34′	066°34′
01:00	09°39′	069°11′	063°57′
02:00	08°39′	071°36′	061°32′
03:00	07°03′	073°40′	059°28′
04:00	04°59′	075°15′	057°53′
05:00	02°35′	076°14′	056°54′
06:00	00°00′	076°34′	056°34′
	Taurus		
00:00	21°28′	069°26′	069°26′
01:00	21°07′	072°11′	066°40′
02:00	20°04′	074°44′	064°07′
03:00	18°26′	076°52′	061°59′
04:00	16°19′	078°26′	060°25′
05:00	13°53′	079°23′	059°29′
06:00	11°18′	079°38′	059°14′
07:00	08°44′	079°12′	059°39′
08:00	06°21′	078°08′	060°43′
09:00	04°20′	076°30′	062°21′
10:00	02°47′	074°25′	064°26′
11:00	01°48′	072°00′	066°51′
12:00	01°28′	069°26′	069°26′
	Gemini		
00:00	30°09′	077°46′	077°46′
01:00	29°47′	080°44′	074°48′
02:00	28°43′	083°27′	072°05′
03:00	27°02′	085°42′	069°51′
04:00	24°53′	087°19′	068°14′
05:00	22°25′	088°14′	067°19′
06:00	19°50′	088°25′	067°08′
07:00	17°17′	087°53′	067°39′
08:00	14°56′	086°44′	068°49′
09:00	12°56′	085°01′	070°32′
10:00	11°25′	082°51′	072°41′
11:00	10°28′	080°24′	075°09′
12:00	10°09′	077°46′	077°46′
	Cancer		
00:00	33°26′	090°00′	090°00′
01:00	33°04′	093°04′	086°56′
02:00	31°59′	095°53′	084°07′
03:00	30°17′	098°10′	081°50′
04:00	28°07′	099°49′	080°11′
05:00	25°39′	100°43′	079°17′
06:00	23°03′	100°53′	079°07′
07:00	20°31′	100°19′	079°41′
08:00	18°11′	099°06′	080°54′
09:00	16°12′	097°21′	082°39′
10:00	14°42′	095°09′	084°51′
11:00	13°45′	092°39′	087°21′
12:00	13°26′	090°00′	090°00′
	Leo		
00:00	30°09′	102°14′	102°14′
01:00	29°47′	105°12′	099°16′
02:00	28°43′	107°55′	096°33′
03:00	27°02′	110°09′	094°18′
04:00	24°53′	111°46′	092°41′
05:00	22°25′	112°41′	091°46′
06:00	19°50′	112°52′	091°35′
07:00	17°17′	112°21′	092°07′
08:00	14°56′	111°11′	093°16′
09:00	12°56′	109°28′	094°59′
10:00	11°25′	107°19′	097°09′
11:00	10°28′	104°51′	099°36′
12:00	10°09′	102°14′	102°14′
	Virgo		
00:00	21°28′	110°34′	110°34′
01:00	21°07′	113°20′	107°49′
02:00	20°04′	115°53′	105°16′
03:00	18°26′	118°01′	103°08′
04:00	16°19′	119°35′	101°34′
05:00	13°53′	120°31′	100°37′
06:00	11°18′	120°46′	100°22′
07:00	08°44′	120°21′	100°48′
08:00	06°21′	119°17′	101°52′
09:00	04°20′	117°39′	103°30′
10:00	02°47′	115°34′	105°35′
11:00	01°48′	113°09′	108°00′
12:00	01°28′	110°34′	110°34′
	Libra		
00:00	10°00′	113°26′	113°26′
01:00	09°39′	116°03′	110°49′
02:00	08°39′	118°28′	108°24′
03:00	07°03′	120°32′	106°20′
04:00	04°59′	122°07′	104°45′
05:00	02°35′	123°06′	103°46′
06:00	00°00′	123°26′	103°26′

Table 2.25: Ecliptic altitude and parallactic angle for latitude $+80°$.

3. Dates and times

3.1 *Julian and Gregorian calendars*

The Julian calendar was instituted by Julius Caesar in 45 BC. However, it was not correctly implemented until 8 AD. The Julian calendar assumes that the length of a tropical year (that is, the time between successive passages of the Sun through the vernal equinox) is 365.25 days. Unfortunately, the true length of the tropical year is 365.2422 days. This slight error caused the date of the vernal equinox to regress through the calendar over the course of many centuries. In order to correct this problem, the Gregorian calendar was promulgated by Pope Gregory XIII in 1582. The new calendar was immediately adopted by all of the Catholic counties in Europe, and was eventually (in some cases, after a delay of hundreds of years) also adopted by all of the Protestant countries. According to the Gregorian calendar, the length of a tropical year is 365.2425 days, which is very close to the correct length.

3.2 *Julian day number*

Following modern astronomical practice, we shall specify dates in this treatise by means of *Julian day numbers*. According to this scheme, days are numbered consecutively from January 1, 4713 BC (in the Julian calendar), which is designated day zero. For instance, October 14, 1066 AD (the Julian date of the battle of Hastings) is day 2 110 701. Each Julian day commences at 12:00 universal time (UT). (Universal time is virtually identical to Greenwich mean time.)

3.3 *Determination of Julian day numbers*

The Julian day number of a given day can be determined from Tables 3.1–3.3. The date must be expressed in terms of the Gregorian calendar.

The procedure is as follows:

1. Enter the table of century years (Table 3.1) with the century year immediately preceding the date in question, and take out the tabular value. If the century year is marked with a †, note this fact for use in step 2.

2. Enter the table of years of the century (Table 3.2) with the last two digits of the year in question, and take out the tabular value. If the century year used in step 1 was marked with a †, diminish the tabular value by one day, unless the tabular value is zero. If the year in question is a leap year, marked with a $*$, note this fact for use in step 3.

3. Enter the table of the days of the year (Table 3.3) with the day in question, and take out the tabular value. If the year in question is a leap year and the table entry falls after February 28, add one day to the tabular value. The sum of the values obtained in steps 1, 2, and 3 then gives the Julian day number of the date in question.

Example 1: June 10, 1992 AD:

1. Century year	$^{\dagger}1900$		$2\,415\,020$
2. Year of the century	$^{*}92$	$33\,603 - 1 =$	$33\,602$
3. Day of the year	June 10	$161 + 1 =$	162
Julian day number			$2\,448\,784$

Observe that in step 2 the tabular value has been diminished by 1 because 1900 is a common year (marked with a † in Table 3.1). In step 3, the tabular value has been increased by 1 because 1992 is a leap year (marked with a $*$ in Table 3.2), and the date falls after February 28.

Example 2: January 18, 1824 AD:

1. Century year	$^{\dagger}1800$		$2\,378\,496$
2. Year of the century	$^{*}24$	$8\,766 - 1 =$	$8\,765$
3. Day of the year	January 18	$18 =$	18
Julian day number			$2\,387\,279$

Observe that in step 2 the tabular value has been diminished by 1 because 1800 is a common year (marked with a † in Table 3.1). In step 3, the tabular value has not been increased by 1, despite the fact that 1824 is a leap year (marked with an $*$ in Table 3.2), because the date falls before February 28.

We can specify the time of day (in universal time), as well as the date, by means of fractional Julian day numbers. For instance, $t = 2\,448\,784.0$ JD corresponds to 12:00 UT on June 10, 1992 AD, whereas $t = 2\,448\,784.5$ JD corresponds to 24:00 UT later the same day.

3.4 Tables

†1700	2 341 972
†1800	2 378 496
†1900	2 415 020
2000	2 451 544
†2100	2 488 069
†2200	2 524 593
†2300	2 561 117

Table 3.1: Julian Day Number: Century Years. † Common years. All years are AD. Table reproduced, with permission, from Evans (1998).

§0	0	*20	7 305	*40	14 610	*60	21 915	*80	29 220
1	336	21	7 671	41	14 976	61	22 281	81	29 586
2	731	22	8 036	42	15 341	62	22 646	82	29 951
3	1 096	23	8 401	43	15 706	63	22 011	83	30 316
*4	1 461	*24	8 766	*44	16 071	*64	23 376	*84	30 681
5	1 827	25	9 132	45	16 437	65	23 742	85	31 047
6	2 192	26	9 497	46	16 802	66	24 107	86	31 412
7	2 557	27	9 862	47	17 167	67	24 472	87	31 777
*8	2 922	*28	10 227	*48	17 532	*68	24 837	*88	32 142
9	3 288	29	10 593	49	17 898	69	25 203	89	32 508
10	3 653	30	10 958	50	18 263	70	25 568	90	32 873
11	4 018	31	11 323	51	18 628	71	25 933	91	33 238
*12	4 383	*32	11 688	*52	18 993	*72	26 298	*92	33 603
13	4 749	33	12 054	53	19 359	73	26 664	93	33 969
14	5 114	34	12 419	54	19 724	74	27 029	94	34 334
15	5 479	35	12 784	55	20 089	75	27 394	95	34 699
*16	5 844	*36	13 149	*56	20 454	*76	27 759	*96	35 064
17	6 210	37	13 515	57	20 820	77	28 125	97	35 430
18	6 575	38	13 880	58	21 185	78	28 490	98	35 795
19	6 940	39	14 245	59	21 550	79	28 855	99	36 160

Table 3.2: Julian Day Number: Years of the Century. * Leap year. § Leap year unless century is marked †. In centuries marked †, subtract one day from the tabulated values for the years 1 through 99. Reproduced, with permission, from Evans (1998).

Day	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	285	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29	$*$	88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

Table 3.3: Julian Day Number: Days of the Year. $*$ In leap year, after February 28, add 1 to the tabulated value. Reproduced, with permission, from Evans (1998).

4. Geometric planetary orbit models

4.1 *The model of Kepler*

In this chapter, Kepler's geometric model of a geocentric planetary orbit is examined in detail, and then compared to the less accurate geometric models of Hipparchus, Ptolemy, and Copernicus. In the following, all orbits are viewed from the northern ecliptic pole.

Kepler's geometric model of a heliocentric planetary orbit is summed up in his three well-known laws of planetary motion. According to Kepler's first law, all planetary orbits are ellipses that are confocal with the Sun and lie in a fixed plane. Moreover, according to Kepler's second law, the radius vector which connects the Sun to a given planet sweeps out equal areas in equal time intervals.

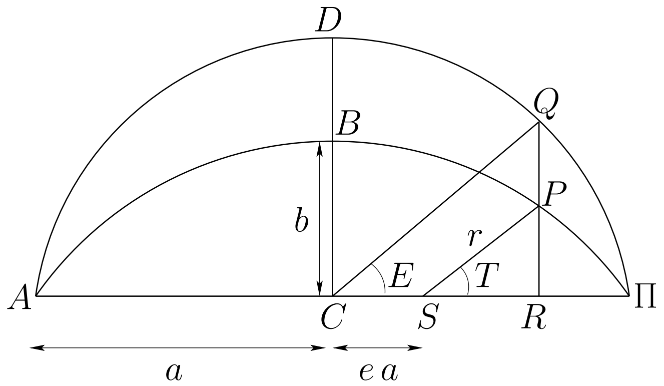

Figure 4.1: A Keplerian orbit.

Consider Figure 4.1. The curve ΠPBA is half of an elliptical planetary orbit. Furthermore, C is the geometric center of the orbit, S the focus at which the Sun is located, P the instantaneous position of the planet, Π the perihelion point (that is, the planet's point of closest approach to the Sun), and A the aphelion point (that is, the point of furthest distance from the Sun). The ellipse is symmetric about ΠA, which is termed the *major axis*, and about CB, which is termed the *minor axis*. The length $CA \equiv a$ is called the orbital *major radius*. The length CS represents the displacement of the Sun from the geometric center of the orbit, and is generally written $e\,a$, where e is termed the orbital *eccentricity*, and $0 \leq e \leq 1$. The length $CB \equiv b = a\,(1-e^2)^{1/2}$ is called the orbital *minor radius*. The length $SP \equiv r$ represents the radial distance of the planet from the Sun. Finally, the angle $RSP \equiv T$ is the angular bearing of the planet from the Sun, relative to the major axis of the orbit, and is termed the *true anomaly*.

The curve ΠQDA is half of a circle whose geometric center is C, and whose radius is a. Hence, the circle passes through the perihelion and aphelion points. R is the point at which the perpendicular from P meets the major axis ΠA. The point where RP produced meets circle ΠQDA is denoted Q. Finally, the angle $SCQ \equiv E$ is called the *elliptic anomaly*.

Now, the equation of the ellipse ΠPBA is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \tag{4.1}$$

where x and y are the perpendicular distances from the minor and major axes, respectively. Likewise, the equation of the circle ΠQDA is

$$\frac{x'^{\,2}}{a^2} + \frac{y'^{\,2}}{a^2} = 1. \tag{4.2}$$

Hence, if $x = x'$ then

$$\frac{y}{y'} = \frac{b}{a}, \tag{4.3}$$

and it follows that

$$\frac{RP}{RQ} = \frac{b}{a}. \tag{4.4}$$

Now, $CS = e\,a$. Furthermore, it is easily demonstrated that $SR = r\,\cos T$, $RP = r\,\sin T$, $CR = a\,\cos E$, and $RQ = a\,\sin E$. Consequently, Equation (4.4) yields

$$r\,\sin T = b\,\sin E = a\,(1-e^2)^{1/2}\,\sin E. \tag{4.5}$$

Also, because $SR = CR - CS$, we have

$$r\,\cos T = a\,(\cos E - e). \tag{4.6}$$

Taking the square root of the sum of the squares of the previous two equations, we obtain

$$r = a\,(1 - e\,\cos E), \tag{4.7}$$

which can be combined with Equation (4.6) to give

$$\cos T = \frac{\cos E - e}{1 - e\,\cos E}. \tag{4.8}$$

Now, according to Kepler's second law,

$$\frac{\text{Area}\,\Pi PS}{\pi\,a\,b} = \frac{t - t_\ast}{\tau}, \tag{4.9}$$

where t is the time at which the planet passes point P, t_* the time at which it passes the perihelion point, and τ the *orbital period*. However,

$$\text{Area}\,\Pi PS = \text{Area}\,SRP + \text{Area}\,\Pi RP = \frac{1}{2}\,r^2\,\cos T\,\sin T + \text{Area}\,\Pi RP. \tag{4.10}$$

But,

$$\text{Area}\,\Pi RP = \frac{b}{a}\,\text{Area}\,\Pi RQ, \tag{4.11}$$

because $RP/RQ = b/a$ for all values of T. In addition,

$$\text{Area}\,\Pi RQ = \text{Area}\,\Pi QC - \text{Area}\,RQC = \frac{1}{2}\,E\,a^2 - \frac{1}{2}\,a^2\,\cos E\,\sin E. \tag{4.12}$$

Hence, we can write

$$\left(\frac{t-t_*}{\tau}\right)\pi\,a\,b = \frac{1}{2}\,r^2\,\cos T\,\sin T + \frac{b}{a}\,\frac{a^2}{2}\,(E - \cos E\,\sin E). \tag{4.13}$$

According to Equations (4.5) and (4.6), $r\,\sin T = b\,\sin E$, and $r\,\cos T = a\,(\cos E - e)$, so the previous expression reduces to

$$M = E - e\,\sin E, \tag{4.14}$$

where

$$M = \left(\frac{2\pi}{\tau}\right)(t - t_*) \tag{4.15}$$

is an angle that is zero at the perihelion point, increases uniformly in time, and has a repetition period that matches the period of the planetary orbit. This angle is termed the *mean anomaly*.

In summary, the radial and angular polar coordinates, r and T, respectively, of a planet in a Keplerian orbit about the Sun are specified as implicit functions of the mean anomaly, which is a linear function of time, by the following three equations:

$$M = E - e\,\sin E, \tag{4.16}$$

$$r = a\,(1 - e\,\cos E), \tag{4.17}$$

$$\cos T = \frac{\cos E - e}{1 - e\,\cos E}. \tag{4.18}$$

It turns out that the Earth and the five visible (to the naked eye) planets all possess low eccentricity orbits characterized by $0 < e \ll 1$. Hence, it is a good approximation to expand the previous three equations using e as a small parameter. To second order, we get

$$E = M + e\,\sin M + (1/2)\,e^2\,\sin 2M, \tag{4.19}$$

$$r = a\,(1 - e\,\cos T - e^2\,\sin^2 T), \tag{4.20}$$

$$T = E + e\,\sin E + (1/4)\,e^2\,\sin 2E. \tag{4.21}$$

Finally, these equations can be combined to give r and T as explicit functions of the mean anomaly:

$$\frac{r}{a} = 1 - e\,\cos M + e^2\,\sin^2 M, \tag{4.22}$$

$$T = M + 2\,e\,\sin M + (5/4)\,e^2\,\sin 2M. \tag{4.23}$$

4.2 *The model of Hipparchus*

Hipparchus's geometric model of the apparent orbit of the Sun around the Earth can also be used to describe a heliocentric planetary orbit. The model is illustrated in Figure 4.2. The orbit of the planet corresponds to the circle ΠPDA, where Π is the perihelion point, P the planet's instantaneous position, and A the aphelion point. The diameter ΠSCA is the effective major axis of the orbit (to be more exact, it is the line of apsides), where C is the geometric center of circle ΠPDA, and S the fixed position of the Sun. The radius CP of circle ΠPDA is the effective major radius, a, of the orbit. The distance SC is equal to $2\,e\,a$, where e is the orbit's effective eccentricity. The angle $PC\Pi$ is identified with the mean anomaly, M, and increases linearly in time. In other words, as seen from C, the planet P moves uniformly around circle ΠPDA in a counterclockwise direction. Finally, SP is the radial distance, r, of the planet from the Sun, and angle $PS\Pi$ is the planet's true anomaly, T.

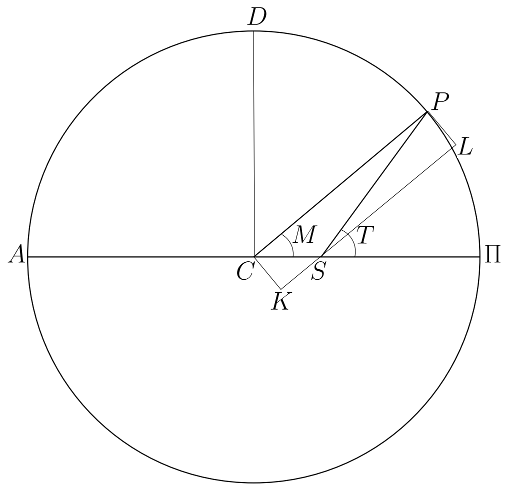

Figure 4.2: A Hipparchian orbit.

Let us draw the straight-line KSL parallel to CP, and passing through point S, and then complete the rectangle $PCKL$. Simple geometry reveals that $CK = PL = 2\,e\,a\,\sin M$, $KS = 2\,e\,a\,\cos M$, and $SL = a - 2\,e\,a\,\cos M$. Moreover, according to the theorem of

Pythagoras, $SP^2 = SL^2 + PL^2$, which implies that

$$\frac{r}{a} = (1 - 4\,e\,\cos M + 4\,e^2)^{1/2}. \tag{4.24}$$

Now, $T = M + q$, where q is angle PSL. However,

$$\sin q = \frac{PL}{SP} = \frac{2\,e\,\sin M}{(1 - 4\,e\,\cos M + 4\,e^2)^{1/2}}. \tag{4.25}$$

Finally, expanding the previous two equations to second order in the small parameter e, we obtain

$$\frac{r}{a} = 1 - 2\,e\,\cos M + 2\,e^2\,\sin^2 M, \tag{4.26}$$

$$T = M + 2\,e\,\sin M + 2\,e^2\,\sin 2M. \tag{4.27}$$

It can be seen, by comparison with Equations (4.22) and (4.23), that the relative radial distance, r/a, in the Hipparchian model deviates from that in the (correct) Keplerian model to first order in e (in fact, the variation of r/a is greater by a factor of two in the former model), whereas the true anomaly, T, only deviates to second order in e. We conclude that Hipparchus's geometric model of a heliocentric planetary orbit does a reasonably good job at predicting the angular position of the planet, relative to the Sun, but significantly exaggerates (by a factor of two) the variation in the radial distance between the two during the course of a complete orbital rotation.

4.3 The model of Ptolemy

Ptolemy's geometric model of the motion of the center of an epicycle around a deferent can also be used to describe a heliocentric planetary orbit. The model is illustrated in Figure 4.3. The orbit of the planet corresponds to the circle ΠPDA (only half of which is shown), where Π is the perihelion point, P the planet's instantaneous position, and A the aphelion point. The diameter $\Pi SCQA$ is the effective major axis of the orbit, where C is the geometric center of circle ΠPDA, S the fixed position of the Sun, and Q the location of the so-called *equant*. The radius CP of circle ΠPDA is the effective major radius, a, of the orbit. The distances SC and CQ are both equal to $e\,a$, where e is the orbit's effective eccentricity. The angle $PQ\Pi$ is identified with the mean anomaly, M, and increases linearly in time. In other words, as seen from Q, the planet P moves uniformly around circle ΠPDA in a counterclockwise direction. Finally, SP is the radial distance, r, of the planet from the Sun, and angle $PS\Pi$ is the planet's true anomaly, T.

Let us draw the straight-line KSL parallel to QP, and passing through point S, and then complete the rectangle $PQKL$. Simple geometry reveals that $QK = PL = 2\,e\,a\,\sin M$, $KS = 2\,e\,a\,\cos M$, and $SL = \rho - 2\,e\,a\,\cos M$, where $\rho = QP$. The cosine rule applied to triangle CQP yields $CP^2 = CQ^2 + QP^2 - 2\,CQ\,QP\,\cos M$, or $\rho^2 - 2\,e\,a\,\cos M\,\rho -$

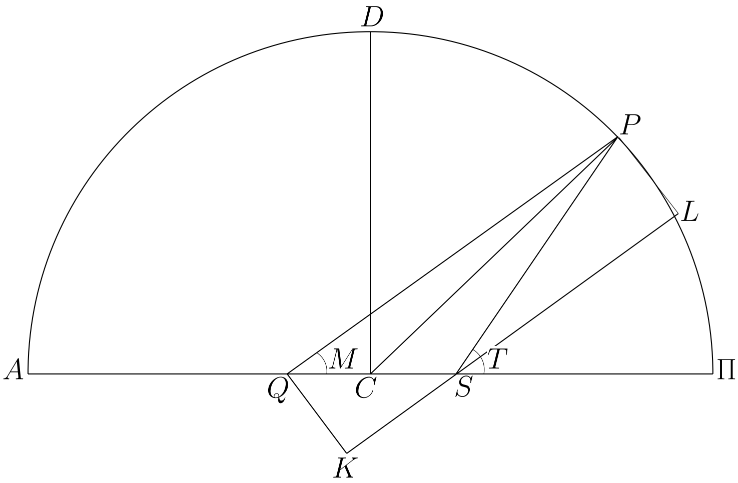

Figure 4.3: A Ptolemaic orbit.

$a^2\,(1-e^2) = 0$, which can be solved to give $\rho/a = e\,\cos M+(1-e^2\,\sin^2 M)^{1/2}$. Moreover, according to the theorem of Pythagoras, $SP^2 = SL^2 + PL^2$, which implies that

$$\frac{r}{a} = [1-2\,e\,\cos M\,(1-e^2\,\sin^2 M)^{1/2} + e^2 + 2\,e^2\,\sin^2 M]^{1/2}. \tag{4.28}$$

Now, $T = M + q$, where q is angle PSL. However,

$$\sin q = \frac{PL}{SP} = \frac{2\,e\,\sin M}{[1-2\,e\,\cos M\,(1-e^2\,\sin^2 M)^{1/2} + e^2 + 2\,e^2\,\sin^2 M]^{1/2}}. \tag{4.29}$$

Finally, expanding the previous two equations to second order in the small parameter e, we obtain

$$\frac{r}{a} = 1 - e\,\cos M + (3/2)\,e^2\,\sin^2 M, \tag{4.30}$$

$$T = M + 2\,e\,\sin M + e^2\,\sin 2M. \tag{4.31}$$

It can be seen, by comparison with Equations (4.22)–(4.23) and (4.26)–(4.27), that Ptolemy's geometric model of a heliocentric planetary orbit is significantly more accurate than Hipparchus's model, because the relative radial distance, r/a, and the true anomaly, T, in the former model both only deviate from those in the (correct) Keplerian model to second order in e.

4.4 The model of Copernicus

Copernicus's geometric model of a heliocentric planetary orbit is illustrated in Figure 4.4. The planet P rotates on a circular epicycle YP whose center X moves around the Sun on the eccentric circle ΠXDA (only half of which is shown). The diameter ΠSCA is the effective major axis of the orbit, where C is the geometric center of circle ΠXDA, and S the fixed position of the Sun. When X is at Π or A the planet is at its perihelion or aphelion points, respectively. The radius CX of circle ΠXDA is the effective major radius, a, of the orbit. The distance SC is equal to $(3/2)\,e\,a$, where e is the orbit's effective eccentricity. Moreover, the radius XP of the epicycle is equal to $(1/2)\,e\,a$. The angle $XC\Pi$ is identified with the mean anomaly, M, and increases linearly in time. In other words, as seen from C, the center of the epicycle X moves uniformly around circle ΠXDA in a counterclockwise direction. The angle PXY, where Y is point at which CX produced meets the epicycle, is equal to the mean anomaly M. In other words, the planet P moves uniformly around the epicycle YP, in an counterclockwise direction, at twice the speed that point X moves around circle ΠXDA. Finally, SP is the radial distance, r, of the planet from the Sun, and angle $PS\Pi$ is the planet's true anomaly, T.

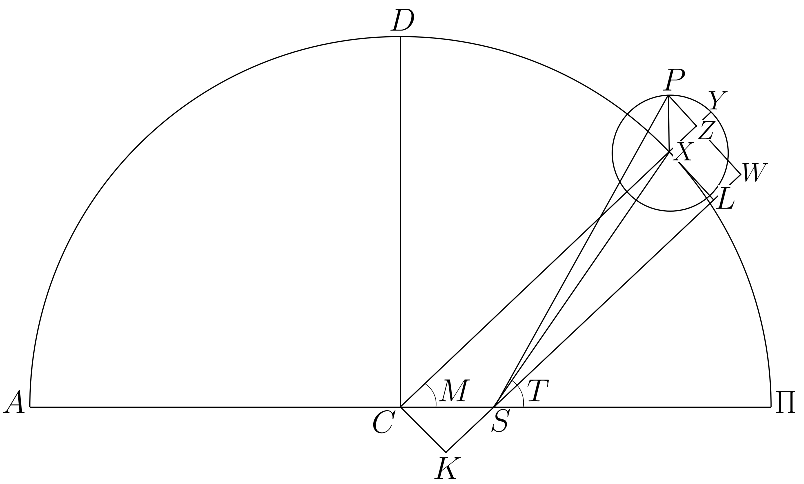

Figure 4.4: A Copernican orbit.

Let us draw the straight-line KSL parallel to CX, and passing through point S, and then complete the rectangle $XCKL$. Simple geometry reveals that $CK = XL = (3/2)\,e\,a\,\sin M$, $KS = (3/2)\,e\,a\,\cos M$, and $SL = a - (3/2)\,e\,a\,\cos M$. Let PZ be drawn normal to XY, and let it meet KSL produced at point W. Simple geometry reveals that $ZW = XL$, $ZP = (1/2)\,e\,a\,\sin M$, and $XZ = LW = (1/2)\,e\,a\,\cos M$. It follows that $WP = ZW + ZP = XL + ZP = 2\,e\,a\,\sin M$, and $SW = SL + LW = SL + XZ = a - e\,a\,\cos\,M$. Moreover, according to the theorem of Pythagoras, $SP^2 = SW^2 + WP^2$, which implies that

$$\frac{r}{a} = (1 - 2\,e\,\cos M + e^2 + 3\,e^2\,\sin^2 M)^{1/2}. \tag{4.32}$$

Now, $T = M + q$, where q is angle PSW. However,

$$\sin q = \frac{WP}{SP} = \frac{2\,e\,\sin M}{(1 - 2\,e\,\cos M + e^2 + 3\,e^2\,\sin^2 M)^{1/2}}. \tag{4.33}$$

Finally, expanding the previous two equations to second order in the small parameter e, we obtain

$$\frac{r}{a} = 1 - e\,\cos M + 2\,e^2\,\sin^2 M, \tag{4.34}$$

$$T = M + 2\,e\,\sin M + e^2\,\sin 2M. \tag{4.35}$$

It can be seen, by comparison with Equations (4.22)–(4.23) and (4.30)–(4.31), that, as is the case for Ptolemy's model, both the relative radial distance, r/a, and the true anomaly, T, in Copernicus's geometric model of a heliocentric planetary orbit only deviate from those in the (correct) Keplerian model to second order in e. However, the deviation in the Ptolemaic model is slightly smaller than that in the Copernican model. To be more exact, the maximum deviation in r/a is $(1/2)\,e^2$ in the former model, and e^2 in the latter. On the other hand, the maximum deviation in T is $(1/4)\,e^2$ in both models.

5. The Sun

5.1 Solar ecliptic longitude model

Our solar longitude model is sketched in Figure 5.1. From a geocentric point of view, the Sun, S, appears to execute a (counterclockwise) Keplerian orbit of major radius a, and eccentricity e, about the Earth, G. As has already been mentioned, the circle traced out by the Sun on the celestial sphere is known as the ecliptic circle. This circle is inclined at $23^\circ 26'$ to the celestial equator, which is the projection of the Earth's equator onto the celestial sphere. Suppose that the angle subtended at the Earth between the vernal equinox (that is, the point at which the ecliptic crosses the celestial equator from south to north) and the Sun's perigee (that is, the point of closest approach to the Earth) is ϖ. This angle is termed the *longitude of the perigee*, and is assumed to vary linearly with time: that is,

$$\varpi = \varpi_0 + \varpi_1\,(t - t_0). \tag{5.1}$$

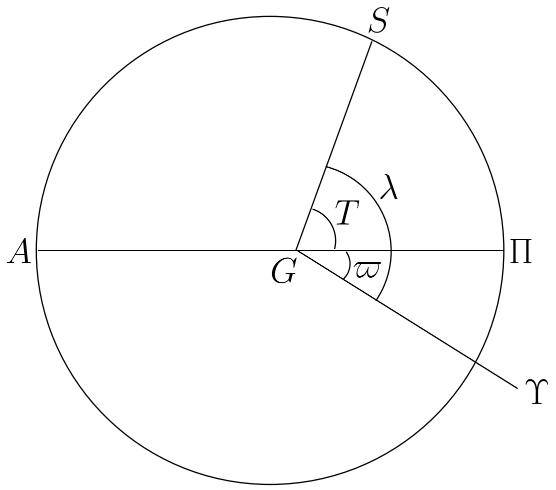

Figure 5.1: The apparent orbit of the Sun about the Earth. Here, S, G, Π, A, ϖ, T, λ, and Υ represent the Sun, the Earth, the perigee, the apogee, the longitude of the perigee, the true anomaly, the ecliptic longitude, and The vernal equinox, respectively. the view is from northern ecliptic pole. The Sun orbits counterclockwise.

The Sun's *ecliptic longitude* is defined as the angle subtended at the Earth between the vernal equinox and the Sun. Hence, from Figure 5.1,

$$\lambda = \varpi + T, \tag{5.2}$$

where T is the true anomaly. (See Chapter 4.) By analogy, the *mean longitude* is written

$$\bar{\lambda} = \varpi + M, \tag{5.3}$$

where M is the mean anomaly. (See Chapter 4.) It follows from Equation (4.23) that

$$\lambda = \bar{\lambda} + q, \tag{5.4}$$

where

$$q = 2\,e\,\sin M + (5/4)\,e^2\,\sin 2M, \tag{5.5}$$

is called the *equation of center*. Note that λ, $\bar{\lambda}$, T, and M are usually written as angles in the range 0° to 360°, whereas q is generally written as an angle in the range -180° to $+180^\circ$.

The mean longitude increases uniformly with time (because both ϖ and M increase uniformly with time) as

$$\bar{\lambda} = \bar{\lambda}_0 + n\,(t - t_0), \tag{5.6}$$

where $\bar{\lambda}_0$ is termed the *mean longitude at epoch*, n the *rate of motion in mean longitude*, and t_0 the *epoch*. We can also write

$$M = M_0 + \tilde{n}\,(t - t_0), \tag{5.7}$$

where

$$M_0 = \bar{\lambda}_0 - \varpi_0 \tag{5.8}$$

is called the *mean anomaly at epoch*, and

$$\tilde{n} = n - \varpi_1 \tag{5.9}$$

the *rate of motion in mean anomaly*.

Our procedure for determining the ecliptic longitude of the Sun is as follows. The requisite orbital elements (that is, e, n, $\tilde{n}$, $\bar{\lambda}_0$, and M_0) for the J2000 epoch (that is, 12:00 UT on January 1, 2000 AD, which corresponds to $t_0 = 2\,451\,545.0$ JD) are listed in Table 5.1. These elements are calculated on the assumption that the vernal equinox precesses at the uniform rate of -3.8246×10^{-5} °/day. The ecliptic longitude of the Sun is specified by the following formulae:

$$\bar{\lambda} = \bar{\lambda}_0 + n\,(t - t_0), \tag{5.10}$$

$$M = M_0 + \tilde{n}\,(t - t_0), \tag{5.11}$$

$$q = 2\,e\,\sin M + (5/4)\,e^2\,\sin 2M, \tag{5.12}$$

$$\lambda = \bar{\lambda} + q. \tag{5.13}$$

These formulae are capable of matching NASA ephemeris data [1] during the years 1995–2006 AD with a mean error of $0.2'$ and a maximum error of $0.7'$.

The ecliptic longitude of the Sun can be calculated with the aid of Tables 5.3 and 5.4. Table 5.3 allows the mean longitude, $\bar{\lambda}$, and mean anomaly, M, of the Sun to be determined as functions of time. Table 5.4 specifies the equation of center, q, as a function of the mean anomaly.

Note that Table 5.3 contains essentially the same information as that contained in the "Table of the Sun's mean motion" (Κανόνιον τῆς ὁμαλῆς τοῦ ἡλίου κινήσεως) that appears in Section 2 of Book III of the Almagest. Likewise, Table 5.4 contains essentially the same information as that contained in the "Table of the solar anomaly" (Κανόνιον τῆς ἡλιακῆς ἀνωμαλίας) that appears in Section 6 of Book III of the Almagest.

5.2 Determination of solar ecliptic longitude

The procedure for using Tables 5.3 and 5.4 to determine solar longitude is as follows:

1. Determine the fractional Julian day number, t, corresponding to the date and time at which the Sun's ecliptic longitude is to be calculated with the aid of Tables 3.1–3.3. Form $\Delta t = t - t_0$, where $t_0 = 2\,451\,545.0$ is the epoch.

2. Enter Table 5.3 with the digit for each power of 10 in Δt and take out the corresponding values of $\Delta\bar{\lambda}$ and ΔM. If Δt is negative then the corresponding values are also negative. The value of the mean longitude, $\bar{\lambda}$, is the sum of all the $\Delta\bar{\lambda}$ values plus the value of $\bar{\lambda}$ at the epoch. Likewise, the value of the mean anomaly, M, is the sum of all the ΔM values plus the value of M at the epoch. Add as many multiples of 360° to $\bar{\lambda}$ and M as is required to make them both fall in the range 0° to 360°. Round M to the nearest degree.

3. Enter Table 5.4 with the value of M and take out the corresponding value of the equation of center, q, and the radial anomaly, ζ. (The latter step is only necessary if the ecliptic longitude of the Sun is to be used to determine that of a planet.) It is necessary to interpolate if M is odd.

4. The ecliptic longitude, λ, is the sum of the mean longitude, $\bar{\lambda}$, and the equation of center, q. If necessary, convert λ into an angle in the range 0° to 360°. The decimal fraction can be converted into arc minutes using Table 5.2. Round to the nearest arc minute.

Two examples of the use of this procedure are given in the next section.

5.3 Example solar longitude calculations

Example 1: May 5, 2005 AD, 00:00 UT:

[1] See `http://ssd.jpl.nasa.gov/`

According to Tables 3.1–3.3, $t = 2\,453\,495.5$ JD. Hence, $t - t_0 = 2\,453\,495.5 - 2\,451\,545.0 = 1\,950.5$ JD. Making use of Table 5.3, we find:

t(JD)	$\bar{\lambda}(^\circ)$	$M(^\circ)$
$+1\,000$	265.647	265.600
$+900$	167.083	167.040
$+50$	49.282	49.280
$+.5$	0.493	0.493
Epoch	280.458	357.588
	762.963	840.001
Modulus	42.963	120.001

Rounding the mean anomaly to the nearest degree, we obtain $M \simeq 120^\circ$. It follows from Table 5.4 that

$$q(120^\circ) = 1.641^\circ,$$

so

$$\lambda = \bar{\lambda} + q = 42.961 + 1.641 = 44.602 \simeq 44^\circ 36'.$$

Here, we have converted the decimal fraction into arc minutes using Table 5.2, and then rounded the final result to the nearest arc minute.

Following the practice of the Ancient Greeks (and modern-day astrologers), we shall express ecliptic longitudes in terms of the signs of the zodiac, which are listed in Section 2.6. The ecliptic longitude $44^\circ 36'$ is conventionally written 14TA36; that is, $14^\circ 36'$ into the sign of Taurus. Thus, we conclude that the position of the Sun at 00:00 UT on May 5, 2005 AD was 14TA36.

Example 2: December 25, 1800 AD, 00:00 UT:

According to Tables 3.1–3.3, $t = 2\,378\,854.5$ JD. Hence, $t - t_0 = 2\,378\,854.5 - 2\,451\,545.0 = -72\,690.5$ JD. Making use of Table 5.3, we find:

t(JD)	$\bar{\lambda}(^\circ)$	$M(^\circ)$
$-70\,000$	-235.315	-232.017
$-2\,000$	-171.295	-171.200
-600	-231.388	-231.360
-90	-88.708	-88.704
$-.5$	-0.493	-0.493
Epoch	280.458	357.588
	-446.741	-366.186
Modulus	273.259	353.814

We conclude that $M \simeq 354^\circ$. From Table 5.4,

$$q(354^\circ) = -0.204^\circ,$$

so

$$\lambda = \bar{\lambda} + q = 273.259 - 0.204 = 273.055 \simeq 273^\circ 03'.$$

Thus, the position of the Sun at 00:00 UT on December 25, 1800 AD was 3CP03.

5.4 Determination of equinox and solstice dates

We can also use Tables 5.3 and 5.4 to calculate the dates of the equinoxes and solstices, and, hence, the lengths of the seasons, in a given year. The *vernal equinox* (that is, the point on the Sun's apparent orbit at which it passes through the celestial equator from south to north) corresponds to $\lambda = 0^\circ$, the *summer solstice* (that is, the point at which the Sun is farthest north of the celestial equator) to $\lambda = 90^\circ$, the *autumnal equinox* (that is, the point at which the Sun passes through the celestial equator from north to south) to $\lambda = 180^\circ$, and the *winter solstice* (that is, the point at which the Sun is farthest south of the celestial equator) to $\lambda = 270^\circ$. See Figure 5.2. Furthermore, *spring* is defined as the period between the spring equinox and the summer solstice, *summer* as the period between the summer solstice and the autumnal equinox, *autumn* as the period between the autumnal equinox and the winter solstice, and *winter* as the period between the winter solstice and the following vernal equinox. Consider the year 2000 AD. For the case of the vernal equinox, we can first estimate the time at which this event takes place by approximating the solar longitude as the mean solar longitude; that is,

$$\lambda \simeq \bar{\lambda} = \bar{\lambda}_0 + n\,(t - t_0) = 280.458 + 0.98564735\,(t - t_0),$$

We obtain

$$t \simeq t_0 + (360 - 280.458)/0.98564735 \simeq t_0 + 81\,\text{JD}.$$

Calculating the true solar longitude at this time, using Tables 5.3 and 5.4, we get $\lambda = 2.177^\circ$. Now, the actual vernal equinox occurs when $\lambda = 0^\circ$. Thus, a much better estimate for the date of the vernal equinox is

$$t = t_0 + 81 - 2.177/0.98564735 \simeq t_0 + 78.8\,\text{JD},$$

which corresponds to 7:00 UT on March 20. Similar calculations show that the summer solstice takes place at

$$t = t_0 + 171.6\,\text{JD},$$

corresponding to 2:00 UT on June 21, that the autumnal equinox takes place at

$$t = t_0 + 265.2\,\text{JD},$$

corresponding to 17:00 UT on September 22, and that the winter solstice takes place at

$$t = t_0 + 355.1\,\text{JD},$$

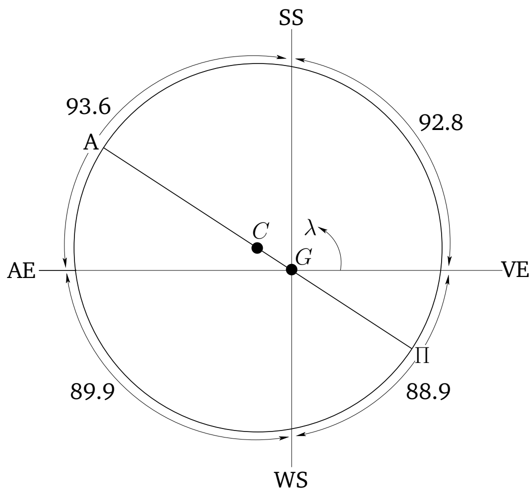

Figure 5.2: The Sun's apparent orbit around the Earth, G, showing the vernal equinox (VE), summer solstice (SS), autumnal equinox (AE), and winter solstice (WS). Here, λ, Π, A, and C are the ecliptic longitude, the perigee, the apogee, and the geometric center of the orbit, respectively. The lengths of the seasons (in days) are indicated.

corresponding to 14:00 UT on December 21. Thus, the length of spring is 92.8 days, the length of summer 93.6 days, and the length of autumn 89.9 days. Finally, the length of winter is the length of the tropical year (that is, the time period between successive vernal equinoxes), which is $360/0.98564735 = 325.24$ days, minus the sum of the lengths of the other three seasons. This gives 88.9 days.

Figure 5.2 illustrates the relationship between the equinox and solstice points, and the lengths of the seasons. The Earth is displaced from the geometric center of the Sun's apparent orbit in the direction of the solar perigee, which presently lies between the winter solstice and the vernal equinox. This displacement (which is greatly exaggerated in the figure) has two effects. Firstly, it causes the arc of the Sun's apparent orbit between the summer solstice and autumnal equinox to be longer than that between the winter solstice and the vernal equinox. Secondly, it causes the Sun to appear to move faster in winter than in summer, in accordance with Kepler's second law, because the Sun is closer to the Earth in the former season. Both of these effects tend to lengthen summer, and shorten winter. Hence, summer is presently the longest season, and winter the shortest.

In Section 4 of Book III of the Almagest, Ptolemy effectively performs the calculation described in this section in reverse, in order to determine the eccentricity of the Sun's apparent orbit about the Earth, as well as the location of the solar perigee, from the observed differences in the lengths of the seasons. However, Ptolemy's figures for the lengths of spring, summer, autumn, and winter, which he inherited from Hipparchus, were 94.5, 92.5, 88.125, and 90.125 days, respectively. The lengths of the seasons have changed since the time of Hipparchus because the solar perigee has rotated by about 38° (in the direction of the Sun's apparent motion) over the last 2 200 years.

5.5 The equation of time

At any particular observation site on the Earth's surface, *local noon* is defined as the instant in time when the Sun culminates at the meridian. However, as a consequence of the inclination of the ecliptic to the celestial equator, as well as the uneven motion of the Sun around the ecliptic, the time interval between successive local noons, which is known as a *solar day*, is not constant, but varies throughout the year. Hence, if we were to define a second as $1/86\,400$ of a solar day then the length of a second would also vary throughout the year, which is clearly undesirable. In order to avoid this problem, astronomers have invented a fictitious body called the *mean Sun*. The mean Sun travels around the celestial equator (from west to east) at a constant rate that is such that it completes one orbit every tropical year. Moreover, the mean Sun and the true Sun coincide at the spring equinox. *Local mean noon* at a particular observation site is defined as the instance in time when the mean Sun culminates at the meridian. Because the orbit of the mean Sun is not inclined to the celestial equator, and the mean Sun travels around the celestial equator at a uniform rate, the time interval between successive mean noons, which is known as a *mean solar day*, takes the constant value of 24 hours, or 86 400 seconds, throughout the year. *Universal time* (UT) is defined such that 12:00 UT coincides with mean noon every day at an observation site of terrestrial longitude 0°. If we define *local time* (LT) as $LT = UT-\phi(^\circ)/15^\circ$ hrs., where ϕ is the terrestrial longitude of the observation site, then 12:00 LT coincides with mean noon every day at a general observation site on the Earth's surface.

According to the previous definition, the right ascension, $\bar{\alpha}$, of the mean Sun satisfies

$$\bar{\alpha} = \bar{\lambda}, \tag{5.14}$$

where $\bar{\lambda}$ is the Sun's mean ecliptic longitude. Moreover, it follows from Equations (2.16) and (5.4) that the right ascension of the true Sun is given by

$$\tan\alpha = \cos\epsilon\,\tan(\bar{\lambda}+q), \tag{5.15}$$

where ϵ is the inclination of the ecliptic to the celestial equator, $q(M)$ the Sun's equation of center, and M its mean anomaly. Now, neglecting the small time variation of the longitude of the Sun's perigee [that is, setting $\varpi_1 = 0$ in Equation (5.1)], we can write [see Equations (5.6), (5.7), and (5.9), as well as Table 5.1]

$$M = \bar{\lambda} + M_0 - \bar{\lambda}_0 = \bar{\lambda} + 77.213^\circ. \tag{5.16}$$

It follows that, to first order in the solar eccentricity, e, we have

$$\Delta\alpha = \bar{\alpha} - \alpha = \lambda - \tan^{-1}(\cos\epsilon\,\tan\lambda) - 2\,e\,\sin\,M, \tag{5.17}$$

where

$$M = \lambda + 77.213^\circ. \tag{5.18}$$

Now,

$$\Delta t = \Delta\alpha(^\circ)/15^\circ \tag{5.19}$$

represents the time difference (in hours) between local noon and mean local noon (because right ascension crosses the meridian at the uniform rate of 15° an hour), and is known as the *equation of time*. If Δt is positive then local noon occurs before mean local noon, and vice versa.

The equation of time specifies the difference between time calculated using a sundial or sextant—which is known as *solar time*—and time obtained from an accurate clock—which is known as *mean solar time*. Table 5.5 shows the equation of time as a function of the Sun's ecliptic longitude. It can be seen that the difference between solar time and mean solar time can be as much as 16 minutes, and attains its maximum value between the autumnal equinox and the winter solstice, and its minimum value between the winter solstice and vernal equinox.

The equation of time is discussed in Section 9 of Book III of the Almagest.

5.6 Solar distance model

The distance, r, of the Sun from the Earth varies slightly over the course of a year because the Earth is not quite at the geometric center of the Sun's apparent orbit. It follows that the apparent angular size of the solar disk in the Earth's sky also exhibits an annual variation. We need to understand this variation in order to accurately predict lunar and solar eclipses. According to Equation (4.22),

$$\frac{r}{a} = 1 - \zeta \tag{5.20}$$

where

$$\zeta = e\,\cos\,M - e^2\,\sin^2\,M. \tag{5.21}$$

Here, $a = 1.498\times 10^8$ km is the Sun's (apparent) orbital major radius, and ζ is known as a *radial anomaly*. The solar radial anomaly is tabulated as a function of its argument (M) in Table 5.4.

The apparent radius of the solar disk is

$$\rho_S = \frac{R}{r}, \tag{5.22}$$

where $R = 6.957\times 10^5$ km is the mean physical radius of the Sun. Here, use has been made of the small angle approximation. It follows from Equations (5.20) and (5.22) that

$$\rho_S = \frac{15.987'}{1-\zeta}. \tag{5.23}$$

Clearly, the mean diameter of the solar disk is about half a degree.

As an example of a solar apparent radius calculation, we have already seen that at 00:00 UT on May 5, 2005 AD the mean anomaly of the Sun was $M \simeq 120^\circ$. It follows from Table 5.4 that the solar radial anomaly was $\zeta = -0.856 \times 10^{-2}$. Hence, the apparent radius of the solar disk was

$$\rho_S = \frac{15.987}{1 + 0.856 \times 10^{-2}} \simeq 15.85'. \tag{5.24}$$

As a second example of a solar radius calculation, we have also seen that at 00:00 UT on December 25, 1800 AD the mean anomaly of the Sun was $M \simeq 354^\circ$. It follows from Table 5.4 that the solar radial anomaly was $\zeta = 1.662 \times 10^{-2}$. Hence, the apparent radius of the solar disk was

$$\rho_S = \frac{15.987}{1 - 1.662 \times 10^{-2}} \simeq 16.26'. \tag{5.25}$$

5.7 Tables

Object	a (AU)	e	n (°/day)	$\tilde{n}$ (°/day)	$\bar{\lambda}_0$ (°)	M_0 (°)
Mercury	0.387098	0.205636	4.09237703	4.09233439	252.087	174.693
Venus	0.723334	0.006777	1.60216872	1.60213040	181.973	49.237
Sun	1.000000	0.016711	0.98564735	0.98560025	280.458	357.588
Mars	1.523706	0.093394	0.52407118	0.52402076	355.460	19.388
Jupiter	5.202873	0.048386	0.08312507	0.08308100	34.365	19.348
Saturn	9.536651	0.053862	0.03350830	0.03348152	50.059	317.857

Table 5.1: Keplerian orbital elements for the Sun and the five visible planets at the J2000 epoch (that is, 12:00 UT, January 1, 2000 AD, which corresponds to $t_0 = 2\,451\,545.0$ JD). The elements are optimized for use in the time period 1800 AD to 2050 AD. Source: Jet Propulsion Laboratory (NASA), `http://ssd.jpl.nasa.gov/`. The motion rates have been converted into tropical motion rates assuming a uniform precession of the equinoxes of 3.8246×10^{-5} ° per day. An astronomical unit (AU) is 1.496×10^8 km.

00.0′	.000	10.0′	.167	20.0′	.333	30.0′	.500	40.0′	.667	50.0′	.833
00.2′	.003	10.2′	.170	20.2′	.337	30.2′	.503	40.2′	.670	50.2′	.837
00.4′	.007	10.4′	.173	20.4′	.340	30.4′	.507	40.4′	.673	50.4′	.840
00.6′	.010	10.6′	.177	20.6′	.343	30.6′	.510	40.6′	.677	50.6′	.843
00.8′	.013	10.8′	.180	20.8′	.347	30.8′	.513	40.8′	.680	50.8′	.847
01.0′	.017	11.0′	.183	21.0′	.350	31.0′	.517	41.0′	.683	51.0′	.850
01.2′	.020	11.2′	.187	21.2′	.353	31.2′	.520	41.2′	.687	51.2′	.853
01.4′	.023	11.4′	.190	21.4′	.357	31.4′	.523	41.4′	.690	51.4′	.857
01.6′	.027	11.6′	.193	21.6′	.360	31.6′	.527	41.6′	.693	51.6′	.860
01.8′	.030	11.8′	.197	21.8′	.363	31.8′	.530	41.8′	.697	51.8′	.863
02.0′	.033	12.0′	.200	22.0′	.367	32.0′	.533	42.0′	.700	52.0′	.867
02.2′	.037	12.2′	.203	22.2′	.370	32.2′	.537	42.2′	.703	52.2′	.870
02.4′	.040	12.4′	.207	22.4′	.373	32.4′	.540	42.4′	.707	52.4′	.873
02.6′	.043	12.6′	.210	22.6′	.377	32.6′	.543	42.6′	.710	52.6′	.877
02.8′	.047	12.8′	.213	22.8′	.380	32.8′	.547	42.8′	.713	52.8′	.880
03.0′	.050	13.0′	.217	23.0′	.383	33.0′	.550	43.0′	.717	53.0′	.883
03.2′	.053	13.2′	.220	23.2′	.387	33.2′	.553	43.2′	.720	53.2′	.887
03.4′	.057	13.4′	.223	23.4′	.390	33.4′	.557	43.4′	.723	53.4′	.890
03.6′	.060	13.6′	.227	23.6′	.393	33.6′	.560	43.6′	.727	53.6′	.893
03.8′	.063	13.8′	.230	23.8′	.397	33.8′	.563	43.8′	.730	53.8′	.897
04.0′	.067	14.0′	.233	24.0′	.400	34.0′	.567	44.0′	.733	54.0′	.900
04.2′	.070	14.2′	.237	24.2′	.403	34.2′	.570	44.2′	.737	54.2′	.903
04.4′	.073	14.4′	.240	24.4′	.407	34.4′	.573	44.4′	.740	54.4′	.907
04.6′	.077	14.6′	.243	24.6′	.410	34.6′	.577	44.6′	.743	54.6′	.910
04.8′	.080	14.8′	.247	24.8′	.413	34.8′	.580	44.8′	.747	54.8′	.913

05.0′	.083	15.0′	.250	25.0′	.417	35.0′	.583	45.0′	.750	55.0′	.917
05.2′	.087	15.2′	.253	25.2′	.420	35.2′	.587	45.2′	.753	55.2′	.920
05.4′	.090	15.4′	.257	25.4′	.423	35.4′	.590	45.4′	.757	55.4′	.923
05.6′	.093	15.6′	.260	25.6′	.427	35.6′	.593	45.6′	.760	55.6′	.927
05.8′	.097	15.8′	.263	25.8′	.430	35.8′	.597	45.8′	.763	55.8′	.930
06.0′	.100	16.0′	.267	26.0′	.433	36.0′	.600	46.0′	.767	56.0′	.933
06.2′	.103	16.2′	.270	26.2′	.437	36.2′	.603	46.2′	.770	56.2′	.937
06.4′	.107	16.4′	.273	26.4′	.440	36.4′	.607	46.4′	.773	56.4′	.940
06.6′	.110	16.6′	.277	26.6′	.443	36.6′	.610	46.6′	.777	56.6′	.943
06.8′	.113	16.8′	.280	26.8′	.447	36.8′	.613	46.8′	.780	56.8′	.947
07.0′	.117	17.0′	.283	27.0′	.450	37.0′	.617	47.0′	.783	57.0′	.950
07.2′	.120	17.2′	.287	27.2′	.453	37.2′	.620	47.2′	.787	57.2′	.953
07.4′	.123	17.4′	.290	27.4′	.457	37.4′	.623	47.4′	.790	57.4′	.957
07.6′	.127	17.6′	.293	27.6′	.460	37.6′	.627	47.6′	.793	57.6′	.960
07.8′	.130	17.8′	.297	27.8′	.463	37.8′	.630	47.8′	.797	57.8′	.963
08.0′	.133	18.0′	.300	28.0′	.467	38.0′	.633	48.0′	.800	58.0′	.967
08.2′	.137	18.2′	.303	28.2′	.470	38.2′	.637	48.2′	.803	58.2′	.970
08.4′	.140	18.4′	.307	28.4′	.473	38.4′	.640	48.4′	.807	58.4′	.973
08.6′	.143	18.6′	.310	28.6′	.477	38.6′	.643	48.6′	.810	58.6′	.977
08.8′	.147	18.8′	.313	28.8′	.480	38.8′	.647	48.8′	.813	58.8′	.980
09.0′	.150	19.0′	.317	29.0′	.483	39.0′	.650	49.0′	.817	59.0′	.983
09.2′	.153	19.2′	.320	29.2′	.487	39.2′	.653	49.2′	.820	59.2′	.987
09.4′	.157	19.4′	.323	29.4′	.490	39.4′	.657	49.4′	.823	59.4′	.990
09.6′	.160	19.6′	.327	29.6′	.493	39.6′	.660	49.6′	.827	59.6′	.993
09.8′	.163	19.8′	.330	29.8′	.497	39.8′	.663	49.8′	.830	59.8′	.997

Table 5.2: Arc minute to decimal fraction conversion table.

Δt(JD)	$\Delta\bar{\lambda}(^\circ)$	$\Delta M(^\circ)$	Δt(JD)	$\Delta\bar{\lambda}(^\circ)$	$\Delta M(^\circ)$	Δt(JD)	$\Delta\bar{\lambda}(^\circ)$	$\Delta M(^\circ)$
10 000	136.474	136.002	1 000	265.647	265.600	100	98.565	98.560
20 000	272.947	272.005	2 000	171.295	171.200	200	197.129	197.120
30 000	49.421	48.007	3 000	76.942	76.801	300	295.694	295.680
40 000	185.894	184.010	4 000	342.589	342.401	400	34.259	34.240
50 000	322.367	320.012	5 000	248.237	248.001	500	132.824	132.800
60 000	98.841	96.015	6 000	153.884	153.601	600	231.388	231.360
70 000	235.315	232.017	7 000	59.531	59.202	700	329.953	329.920
80 000	11.788	8.020	8 000	325.179	324.802	800	68.518	68.480
90 000	148.262	144.022	9 000	230.826	230.402	900	167.083	167.040
10	9.856	9.856	1	0.986	0.986	0.1	0.099	0.099
20	19.713	19.712	2	1.971	1.971	0.2	0.197	0.197
30	29.569	29.568	3	2.957	2.957	0.3	0.296	0.296
40	39.426	39.424	4	3.943	3.942	0.4	0.394	0.394
50	49.282	49.280	5	4.928	4.928	0.5	0.493	0.493
60	59.139	59.136	6	5.914	5.914	0.6	0.591	0.591
70	68.995	68.992	7	6.900	6.899	0.7	0.690	0.690
80	78.852	78.848	8	7.885	7.885	0.8	0.789	0.788
90	88.708	88.704	9	8.871	8.870	0.9	0.887	0.887

Table 5.3: Mean motion of the Sun. Here, $\Delta t = t - t_0$, $\Delta\bar{\lambda} = \bar{\lambda} - \bar{\lambda}_0$, and $\Delta M = M - M_0$. At epoch ($t_0 = 2\,451\,545.0$ JD), $\bar{\lambda}_0 = 280.458^\circ$, and $M_0 = 357.588^\circ$.

$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$
0	0.000	1.671	90	1.915	−0.028	180	0.000	−1.671	270	−1.915	−0.028
2	0.068	1.670	92	1.912	−0.086	182	−0.065	−1.670	272	−1.915	0.030
4	0.136	1.667	94	1.907	−0.144	184	−0.131	−1.667	274	−1.913	0.089
6	0.204	1.662	96	1.900	−0.202	186	−0.196	−1.662	276	−1.909	0.147
8	0.272	1.654	98	1.891	−0.260	188	−0.261	−1.655	278	−1.902	0.205
10	0.339	1.645	100	1.879	−0.317	190	−0.326	−1.647	280	−1.893	0.263
12	0.406	1.633	102	1.865	−0.374	192	−0.390	−1.636	282	−1.881	0.321
14	0.473	1.620	104	1.849	−0.431	194	−0.454	−1.623	284	−1.867	0.378
16	0.538	1.604	106	1.830	−0.486	196	−0.517	−1.608	286	−1.851	0.435
18	0.604	1.587	108	1.809	−0.542	198	−0.580	−1.592	288	−1.833	0.491
20	0.668	1.567	110	1.787	−0.596	200	−0.642	−1.574	290	−1.812	0.547
22	0.731	1.545	112	1.762	−0.650	202	−0.703	−1.553	292	−1.789	0.602
24	0.794	1.522	114	1.735	−0.703	204	−0.764	−1.531	294	−1.764	0.656
26	0.855	1.497	116	1.705	−0.755	206	−0.824	−1.507	296	−1.737	0.710
28	0.916	1.469	118	1.674	−0.806	208	−0.882	−1.482	298	−1.707	0.763
30	0.975	1.440	120	1.641	−0.856	210	−0.940	−1.454	300	−1.676	0.815
32	1.033	1.409	122	1.606	−0.906	212	−0.997	−1.425	302	−1.642	0.865
34	1.089	1.377	124	1.569	−0.954	214	−1.052	−1.394	304	−1.606	0.915
36	1.145	1.342	126	1.530	−1.001	216	−1.107	−1.362	306	−1.568	0.964
38	1.198	1.306	128	1.490	−1.046	218	−1.160	−1.327	308	−1.528	1.011
40	1.251	1.269	130	1.447	−1.091	220	−1.211	−1.292	310	−1.487	1.058
42	1.301	1.229	132	1.403	−1.134	222	−1.261	−1.254	312	−1.443	1.103
44	1.350	1.189	134	1.358	−1.175	224	−1.310	−1.216	314	−1.397	1.146
46	1.397	1.146	136	1.310	−1.216	226	−1.358	−1.175	316	−1.350	1.189
48	1.443	1.103	138	1.261	−1.254	228	−1.403	−1.134	318	−1.301	1.229
50	1.487	1.058	140	1.211	−1.292	230	−1.447	−1.091	320	−1.251	1.269
52	1.528	1.011	142	1.160	−1.327	232	−1.490	−1.046	322	−1.198	1.306
54	1.568	0.964	144	1.107	−1.362	234	−1.530	−1.001	324	−1.145	1.342
56	1.606	0.915	146	1.052	−1.394	236	−1.569	−0.954	326	−1.089	1.377
58	1.642	0.865	148	0.997	−1.425	238	−1.606	−0.906	328	−1.033	1.409
60	1.676	0.815	150	0.940	−1.454	240	−1.641	−0.856	330	−0.975	1.440
62	1.707	0.763	152	0.882	−1.482	242	−1.674	−0.806	332	−0.916	1.469
64	1.737	0.710	154	0.824	−1.507	244	−1.705	−0.755	334	−0.855	1.497
66	1.764	0.656	156	0.764	−1.531	246	−1.735	−0.703	336	−0.794	1.522
68	1.789	0.602	158	0.703	−1.553	248	−1.762	−0.650	338	−0.731	1.545
70	1.812	0.547	160	0.642	−1.574	250	−1.787	−0.596	340	−0.668	1.567
72	1.833	0.491	162	0.580	−1.592	252	−1.809	−0.542	342	−0.604	1.587
74	1.851	0.435	164	0.517	−1.608	254	−1.830	−0.486	344	−0.538	1.604
76	1.867	0.378	166	0.454	−1.623	256	−1.849	−0.431	346	−0.473	1.620
78	1.881	0.321	168	0.390	−1.636	258	−1.865	−0.374	348	−0.406	1.633
80	1.893	0.263	170	0.326	−1.647	260	−1.879	−0.317	350	−0.339	1.645
82	1.902	0.205	172	0.261	−1.655	262	−1.891	−0.260	352	−0.272	1.654
84	1.909	0.147	174	0.196	−1.662	264	−1.900	−0.202	354	−0.204	1.662
86	1.913	0.089	176	0.131	−1.667	266	−1.907	−0.144	356	−0.136	1.667
88	1.915	0.030	178	0.065	−1.670	268	−1.912	−0.086	358	−0.068	1.670
90	1.915	−0.028	180	0.000	−1.671	270	−1.915	−0.028	360	−0.000	1.671

Table 5.4: Anomalies of the Sun.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	Δt	λ	Δt	λ	Δt	λ	Δt	λ	Δt	λ	Δt
00°	-07^m28^s	00°	$+01^m02^s$	00°	$+03^m31^s$	00°	-01^m42^s	00°	-06^m27^s	00°	-02^m44^s
02°	-06^m52^s	02°	$+01^m28^s$	02°	$+03^m22^s$	02°	-02^m09^s	02°	-06^m30^s	02°	-02^m11^s
04°	-06^m15^s	04°	$+01^m51^s$	04°	$+03^m11^s$	04°	-02^m36^s	04°	-06^m31^s	04°	-01^m36^s
06°	-05^m38^s	06°	$+02^m12^s$	06°	$+02^m57^s$	06°	-03^m03^s	06°	-06^m29^s	06°	$+00^m59^s$
08°	-05^m01^s	08°	$+02^m32^s$	08°	$+02^m42^s$	08°	-03^m28^s	08°	-06^m24^s	08°	$+00^m21^s$
10°	-04^m25^s	10°	$+02^m49^s$	10°	$+02^m24^s$	10°	-03^m53^s	10°	-06^m16^s	10°	$+00^m18^s$
12°	-03^m48^s	12°	$+03^m04^s$	12°	$+02^m05^s$	12°	-04^m17^s	12°	-06^m06^s	12°	$+00^m59^s$
14°	-03^m12^s	14°	$+03^m17^s$	14°	$+01^m44^s$	14°	-04^m39^s	14°	-05^m53^s	14°	$+01^m40^s$
16°	-02^m37^s	16°	$+03^m27^s$	16°	$+01^m21^s$	16°	-04^m59^s	16°	-05^m38^s	16°	$+02^m23^s$
18°	-02^m02^s	18°	$+03^m35^s$	18°	$+00^m58^s$	18°	-05^m18^s	18°	-05^m20^s	18°	$+03^m06^s$
20°	-01^m28^s	20°	$+03^m40^s$	20°	$+00^m33^s$	20°	-05^m35^s	20°	-05^m00^s	20°	$+03^m49^s$
22°	$+00^m55^s$	22°	$+03^m43^s$	22°	$+00^m07^s$	22°	-05^m50^s	22°	-04^m37^s	22°	$+04^m33^s$
24°	$+00^m24^s$	24°	$+03^m44^s$	24°	$+00^m20^s$	24°	-06^m03^s	24°	-04^m12^s	24°	$+05^m17^s$
26°	$+00^m06^s$	26°	$+03^m42^s$	26°	$+00^m47^s$	26°	-06^m14^s	26°	-03^m45^s	26°	$+06^m01^s$
28°	$+00^m35^s$	28°	$+03^m38^s$	28°	-01^m14^s	28°	-06^m22^s	28°	-03^m15^s	28°	$+06^m45^s$
30°	$+01^m02^s$	30°	$+03^m31^s$	30°	-01^m42^s	30°	-06^m27^s	30°	-02^m44^s	30°	$+07^m28^s$

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	Δt	λ	Δt	λ	Δt	λ	Δt	λ	Δt	λ	Δt
00°	$+07^m28^s$	00°	$+15^m40^s$	00°	$+13^m55^s$	00°	$+01^m42^s$	00°	-10^m58^s	00°	-13^m58^s
02°	$+08^m11^s$	02°	$+15^m55^s$	02°	$+13^m22^s$	02°	$+00^m43^s$	02°	-11^m33^s	02°	-13^m47^s
04°	$+08^m53^s$	04°	$+16^m08^s$	04°	$+12^m46^s$	04°	$+00^m16^s$	04°	-12^m03^s	04°	-13^m32^s
06°	$+09^m34^s$	06°	$+16^m17^s$	06°	$+12^m08^s$	06°	-01^m14^s	06°	-12^m31^s	06°	-13^m15^s
08°	$+10^m15^s$	08°	$+16^m23^s$	08°	$+11^m26^s$	08°	-02^m12^s	08°	-12^m55^s	08°	-12^m56^s
10°	$+10^m53^s$	10°	$+16^m27^s$	10°	$+10^m42^s$	10°	-03^m08^s	10°	-13^m17^s	10°	-12^m34^s
12°	$+11^m31^s$	12°	$+16^m26^s$	12°	$+09^m55^s$	12°	-04^m04^s	12°	-13^m35^s	12°	-12^m11^s
14°	$+12^m07^s$	14°	$+16^m23^s$	14°	$+09^m07^s$	14°	-04^m58^s	14°	-13^m50^s	14°	-11^m45^s
16°	$+12^m41^s$	16°	$+16^m16^s$	16°	$+08^m16^s$	16°	-05^m51^s	16°	-14^m02^s	16°	-11^m18^s
18°	$+13^m14^s$	18°	$+16^m06^s$	18°	$+07^m23^s$	18°	-06^m42^s	18°	-14^m10^s	18°	-10^m49^s
20°	$+13^m44^s$	20°	$+15^m53^s$	20°	$+06^m29^s$	20°	-07^m31^s	20°	-14^m16^s	20°	-10^m18^s
22°	$+14^m12^s$	22°	$+15^m36^s$	22°	$+05^m33^s$	22°	-08^m17^s	22°	-14^m18^s	22°	-09^m46^s
24°	$+14^m38^s$	24°	$+15^m15^s$	24°	$+04^m36^s$	24°	-09^m02^s	24°	-14^m18^s	24°	-09^m13^s
26°	$+15^m01^s$	26°	$+14^m52^s$	26°	$+03^m39^s$	26°	-09^m44^s	26°	-14^m14^s	26°	-08^m39^s
28°	$+15^m22^s$	28°	$+14^m25^s$	28°	$+02^m40^s$	28°	-10^m23^s	28°	-14^m08^s	28°	-08^m04^s
30°	$+15^m40^s$	30°	$+13^m55^s$	30°	$+01^m42^s$	30°	-10^m58^s	30°	-13^m58^s	30°	-07^m28^s

Table 5.5: The equation of time. The superscripts m and s denote minutes and seconds.

6. The Moon

6.1 Lunar ecliptic longitude model

The orbit of the Moon around the Earth is strongly perturbed by the gravitational influence of the Sun. It follows that we cannot derive an accurate lunar longitude model from Keplerian orbit theory alone. Instead, we shall employ a greatly simplified version of modern lunar theory. According to such theory, the time variation of the ecliptic longitude of the Moon is fairly well represented by the following formulae:[1]

$$\bar{\lambda} = \bar{\lambda}_0 + n\,(t - t_0), \tag{6.1}$$

$$M = M_0 + \tilde{n}\,(t - t_0), \tag{6.2}$$

$$\bar{F} = \bar{F}_0 + \breve{n}\,(t - t_0), \tag{6.3}$$

$$\tilde{D} = \bar{\lambda} - \lambda_S, \tag{6.4}$$

$$q_1 = 2\,e\,\sin M + 1.2379\,e^2\,\sin 2M, \tag{6.5}$$

$$q_2 = 0.4052\,e\,\sin(2\tilde{D} - M), \tag{6.6}$$

$$q_3 = 0.2094\,e\,(\sin 2\tilde{D} - 0.0527\,\sin\tilde{D}), \tag{6.7}$$

$$q_4 = -0.0589\,e\,\sin M_S, \tag{6.8}$$

$$q_5 = -0.0364\,e\,\sin 2\bar{F}, \tag{6.9}$$

$$\lambda = \bar{\lambda} + q_1 + q_2 + q_3 + q_4 + q_5. \tag{6.10}$$

Here, λ_S and M_S are the longitude and mean anomaly of the Sun, respectively. Moreover, e, λ, $\bar{\lambda}$, $\bar{F}$, and q_i are the eccentricity, longitude, mean longitude, mean argument of latitude, and ith anomaly of the Moon, respectively. The Moon's first anomaly is due to the eccentricity of its orbit, and is very similar in form to that obtained from Keplerian orbit theory. (See Chapter 4.) The Moon's second, third, and fourth anomalies are knows as *evection*, *variation*, and the *annual inequality*, respectively, and originate from the perturbing influence of the Sun. Finally, the Moon's fifth anomaly is called the *reduction to the ecliptic*, and is a consequence of the fact that the Moon's orbit is slightly tilted with respect to the plane of the ecliptic. Note that Ptolemy's lunar theory only takes the first two lunar anomalies into account. (Evection was discovered by Ptolemy. However, variation was only discovered in the 1580s by Tycho Brahe, who also discovered the annual inequality about a decade later.)

The Moon's orbital elements—e, n, $\tilde{n}$, $\breve{n}$, $\bar{\lambda}_0$, M_0, and F_0—for the J2000 epoch are listed in Table 6.1. Note that the lunar perigee precesses in the direction of the Moon's orbital motion

[1] See Meeus (1998) and Fitzpatrick (2012).

at the rate of $n - \tilde{n} = 0.11140^\circ$ per day, or 360° in 8.85 years. This very large precession rate (more than 2000 times the corresponding precession rate for the Sun's apparent orbit) is another consequence of the strong perturbing influence of the Sun on the Moon's orbit. The previous formulae are capable of matching NASA ephemeris data during the years 1995–2006 AD with a mean error of $5'$ and a maximum error of $14'$.

The ecliptic longitude of the Moon can be calculated with the aid of Tables 6.2 and 6.3. Table 6.2 allows the lunar mean longitude, $\bar{\lambda}$, the mean anomaly, M, and the mean argument of latitude, $\bar{F}$, to be determined as functions of time. Table 6.3 specifies the lunar anomalies, q_1–q_5, as functions of their various arguments.

Incidentally, Table 6.2 contains essentially the same information as is contained in the "Table of the Moon's mean motion" (*Κανόνες τῶν τῆς σελήνης μέσων κινήσεων*) that appears in Section 4 of Book IV of the Almagest. Moreover, Table 6.3 contains equivalent information to that contained in the "Table of the whole lunar anomaly" (Κανόνιον τῆς καθόλου σεληνακῆς ἀνωμαλίας) that appears in Section 8 of Book V of the Almagest.

6.2 Determination of lunar ecliptic longitude

The procedure for using Tables 6.1 and 6.3 to determine lunar longitude is as follows:

1. Determine the fractional Julian day number, t, corresponding to the date and time at which the Moon's ecliptic longitude is to be calculated with the aid of Tables 3.1–3.3. Form $\Delta t = t - t_0$, where $t_0 = 2\,451\,545.0$ is the epoch.

2. Calculate the ecliptic longitude, λ_S, and the mean anomaly, M_S, of the Sun using the procedure set out in Section 5.2.

3. Enter Table 6.2 with the digit for each power of 10 in Δt and take out the corresponding values of $\Delta\bar{\lambda}$, ΔM, and $\Delta\bar{F}$. If Δt is negative then the values are minus those shown in the table. The value of the mean longitude, $\bar{\lambda}$, is the sum of all the $\Delta\bar{\lambda}$ values plus the value of $\bar{\lambda}$ at the epoch. Likewise, the value of the mean anomaly, M, is the sum of all the ΔM values plus the value of M at the epoch. Finally, the value of the mean argument of latitude, $\bar{F}$, is the sum of all the $\Delta\bar{F}$ values plus the value of $\bar{F}$ at the epoch. Add as many multiples of 360° to $\bar{\lambda}$, M, and $\bar{F}$ as is required to make them all fall in the range 0° to 360°.

4. Form $\tilde{D} = \bar{\lambda} - \lambda_S$.

5. Form the five arguments $a_1 = M$, $a_2 = 2\tilde{D} - M$, $a_3 = \tilde{D}$, $a_4 = M_S$, $a_5 = 2\bar{F}$. Add as many multiples of 360° to the arguments as is required to make them all fall in the range 0° to 360°. Round each argument to the nearest degree.

6. Enter Table 6.3 with the value of each of the five arguments a_1–a_5 and take out the value of each of the five corresponding anomalies q_1–q_5. It is necessary to interpolate if the arguments are odd.

7. The Moon's ecliptic longitude is given by $\lambda = \bar{\lambda} + q_1 + q_2 + q_3 + q_4 + q_5$. If necessary, convert λ into an angle in the range 0° to 360°. The decimal fraction can be converted into arc minutes using Table 5.2. Round to the nearest arc minute.

Two examples of the use of the procedure that has just been described are given in the following section.

6.3 *Example lunar longitude calculations*

Example 1: May 5, 2005 AD, 00:00 UT:

From Section 5.2, $t - t_0 = 1950.5$ JD, $\lambda_S = 44.602^\circ$, and $M_S = 120.001^\circ$. Making use of Table 6.2, we find:

t(JD)	$\bar{\lambda}(^\circ)$	$M(^\circ)$	$\bar{F}(^\circ)$
+1 000	216.396	104.993	269.350
+900	338.757	238.494	26.415
+50	298.820	293.250	301.468
+.5	6.588	6.532	6.615
Epoch	218.322	134.916	93.284
	1078.883	778.185	697.132
Modulus	358.883	58.185	337.132

It follows that

$$\tilde{D} = \bar{\lambda} - \lambda_S = 358.883 - 44.602 = 314.281^\circ.$$

Thus,

$$a_1 = M \simeq 58^\circ, \quad a_2 = 2\tilde{D} - M = 2 \times 314.281 - 58.185 = 570.377 \simeq 210^\circ,$$

$$a_3 = \tilde{D} \simeq 314^\circ, \quad a_4 = M_S \simeq 120^\circ,$$

$$a_5 = 2\bar{F} = 2 \times 337.132 = 674.264 \simeq 314^\circ.$$

Table 6.3 yields

$$q_1(a_1) = 5.525^\circ, \quad q_2(a_2) = -0.637^\circ, \quad q_3(a_3) = -0.633^\circ,$$

$$q_4(a_4) = -0.160^\circ, \quad q_5(a_5) = 0.082^\circ.$$

Hence,

$$\begin{aligned}\lambda &= \bar{\lambda} + q_1 + q_2 + q_3 + q_4 + q_5\\ &= 358.883 + 5.525 - 0.637 - 0.633 - 0.160 + 0.082\\ &= 363.060^\circ,\end{aligned}$$

or

$$\lambda = 3.060 \simeq 3^\circ 04'.$$

Thus, the ecliptic longitude of the Moon at 00:00 UT on May 5, 2005 AD was 3AR04.

Example 2: December 25, 1800 AD, 00:00 UT:

From Section 5.2, $t - t_0 = -72\,690.5$ JD, $\lambda_S = 273.055^\circ$, and $M_S = 353.814^\circ$. Making use of Table 6.2, we find:

t(JD)	$\bar{\lambda}(^\circ)$	$M(^\circ)$	$\bar{F}(^\circ)$
$-70\,000$	-27.752	-149.506	-134.519
$-2\,000$	-72.793	-209.986	-178.701
-600	-345.838	-278.996	-17.610
-90	-105.876	-95.849	-110.642
$-.5$	-6.588	-6.532	-6.615
Epoch	218.322	134.916	93.284
	-340.525	-605.953	-354.803
Modulus	19.475	114.047	5.197

It follows that

$$\tilde{D} = \bar{\lambda} - \lambda_S = 19.475 - 273.055 = -253.580^\circ.$$

Thus,

$$a_1 = M \simeq 114^\circ, \quad a_2 = 2\tilde{D} - M = -2 \times 253.580 - 114.047 = -621.207 \simeq 99^\circ,$$

$$a_3 = \tilde{D} \simeq 106^\circ, \quad a_4 = M_S \simeq 354^\circ,$$

$$a_5 = 2\bar{F} = 2 \times 5.197 = 10.394 \simeq 10^\circ.$$

Table 6.3 yields

$$q_1(a_1) = 5.586^\circ, \;\; q_2(a_2) = 1.259^\circ, \;\; q_3(a_3) = -0.382^\circ,$$

$$q_4(a_4) = 0.019^\circ, \;\; q_5(a_5) = -0.020^\circ.$$

Hence,

$$\begin{aligned}\lambda &= \bar{\lambda} + q_1 + q_2 + q_3 + q_4 + q_5 \\ &= 19.475 + 5.586 + 1.259 - 0.382 + 0.019 - 0.020 \\ &= 25.937^\circ,\end{aligned}$$

or

$$\lambda = 25.937 \simeq 25^\circ 56'.$$

Thus, the ecliptic longitude of the Moon at 00:00 UT on December 25, 1800 AD was 25AR56.

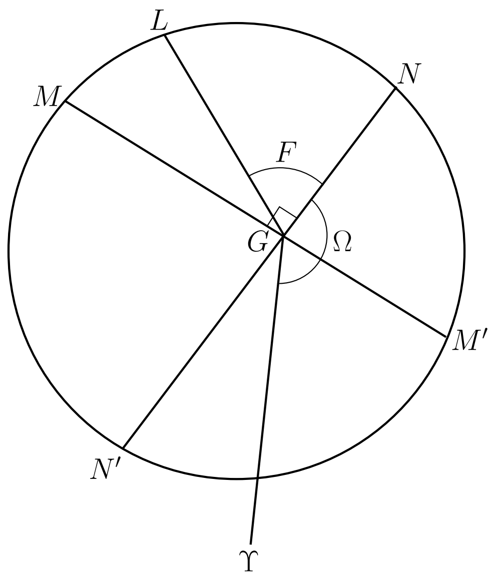

Figure 6.1: The orbit of the Moon about the Earth. Here, G, L, N, N', Ω, F, and Υ represent the Earth, the Moon, the ascending node, the descending node, the longitude of the ascending node, the argument of latitude, and the vernal equinox, respectively. The view is from northern ecliptic pole. The Moon orbits counterclockwise.

6.4 Lunar ecliptic latitude model

A model of the Moon's ecliptic latitude is needed in order to predict the occurrence of solar and lunar eclipses. Figure 6.1 shows the Moon's orbit about the Earth. The plane of this orbit is fixed, but slightly tilted with respect to the plane of the ecliptic (that is, the plane of the Sun's apparent orbit about the Earth). Let the two planes intersect along the line of nodes, NGN'. Here, N is the point at which the orbit crosses the ecliptic plane from south to north (in the direction of the Moon's orbital motion), and is termed the *ascending node*. Likewise, N' is the point at which the orbit crosses the ecliptic plane from north to south, and is called the *descending node*. Incidentally, the line of nodes must pass through point G, because the Earth is common to the ecliptic plane and the plane of the lunar orbit. The angle, Ω, subtended between the radius vector $G\Upsilon$, connecting the Earth to the vernal equinox, and the line GN, is known as the *longitude of the ascending node*. Note, incidentally, that the ascending node precesses in the opposite direction to the Moon's orbital motion at the rate $\breve{n} - n = 5.2954 \times 10^{-2}\,^\circ$ per day, or 360° in 18.6 years. This unusually large precession rate is another consequence of the Sun's strong perturbing influence on the Moon's orbit. Let the line MGM' lie in the plane of the Moon's orbit such that it is perpendicular to NGN'. The inclination, i, of the Moon's orbital plane is the angle that GM subtends with its projection onto the ecliptic plane. Likewise, the Moon's ecliptic longitude, β, is the angle that GL subtends with its projection onto the ecliptic plane. Simple geometry yields $\sin\beta = \sin i\ \sin F$, where F is the angle between GN and GL. This angle is termed the *argument of latitude.* Now, it is easily seen that $F \simeq \lambda - \Omega$, where λ is the Moon's ecliptic longitude (that is, the angle subtended between $G\Upsilon$ and GL). Here, we are assuming that the orbital inclination i is relatively small. The *mean argument of latitude* is defined $\bar{F} = \bar{\lambda} - \Omega$. Hence, our model for the Moon's ecliptic latitude becomes

$$F = \bar{F} + q_1 + q_2 + q_3 + q_4 + q_5, \tag{6.11}$$

$$\sin\beta = \sin i\ \sin F. \tag{6.12}$$

The value of the lunar orbital inclination, i, for the J2000 epoch is specified in Table 6.1. The previous model is capable of matching NASA ephemeris data during the years 1995-2006 AD with a mean error of $6'$, and a maximum error of $11'$.

6.5 Determination of lunar ecliptic latitude

The ecliptic latitude of the Moon can be calculated with the aid of Table 6.4. The procedure for using this table is as follows:

1. Determine the fractional Julian day number, t, corresponding to the date and time at which the Moon's ecliptic latitude is to be calculated with the aid of Tables 3.1–3.3. Form $\Delta t = t - t_0$, where $t_0 = 2\,451\,545.0$ is the epoch.

2. Calculate the lunar mean argument of latitude, $\bar{F}$, and the five lunar anomalies, q_1–q_5, using the procedure outlined earlier in this section.

3. Form the argument $F = \bar{F} + q_1 + q_2 + q_3 + q_4 + q_5$. Add as many multiples of 360° to F as is required to make it fall in the range 0° to 360°. Round F to the nearest degree.

4. Enter Table 6.4 with the value of F and take out the lunar ecliptic latitude, β. It is necessary to interpolate if F is odd.

Two examples of the use of the procedure that has just been described are given in the following section.

6.6 Example lunar latitude calculations

Example 1: May 5, 2005 AD, 00:00 UT:

We have already seen in Section 6.3 that at 00:00 UT on May 5, 2005 AD the lunar mean argument of latitude, and the lunar anomalies, were $\bar{F} = 337.132^\circ$, and $q_1 = 5.525^\circ$, $q_2 =$

-0.637°, $q_3 = -0.633^\circ$, $q_4 = -0.160^\circ$, and $q_5 = 0.082^\circ$, respectively. Hence,

$$\begin{aligned} F &= \bar{F} + q_1 + q_2 + q_3 + q_4 + q_5 \\ &= 337.132 + 5.525 - 0.637 - 0.633 - 0.160 + 0.082 \\ &\simeq 341^\circ. \end{aligned}$$

Thus, according to Table 6.4, the ecliptic latitude of the Moon at 00:00 UT on May 5, 2005 AD was $-1.668^\circ \simeq -1^\circ 40'$.

Example 2: December 25, 1800 AD, 00:00 UT:

We have already seen in Section 6.3 that at 00:00 UT on December 25, 1800 AD the lunar mean argument of latitude, and the lunar anomalies, were $\bar{F} = 5.197^\circ$, and $q_1 = 5.586^\circ$, $q_2 = 1.259^\circ$, $q_3 = -0.382^\circ$, $q_4 = -0.019^\circ$, and $q_5 = -0.020^\circ$, respectively. Hence,

$$\begin{aligned} F &= \bar{F} + q_1 + q_2 + q_3 + q_4 + q_5 \\ &= 5.197 + 5.586 + 1.259 - 0.382 + 0.019 - 0.020 \\ &\simeq 12^\circ. \end{aligned}$$

Thus, according to Table 6.4, the ecliptic latitude of the Moon at 00:00 UT on December 25, 1800 AD was $+1.065^\circ \simeq +1^\circ 04'$.

6.7 The length of a month

A *sidereal month* is the mean period of time between successive passages of the Moon through the vernal equinox, and is $360/n = 360/13.17639646 = 27.322$ days in length, where n comes from Table 6.1. A *synodic month* is the mean period of time between successive new moons, or successive full moons, and is $360/(n - n_S) = 360/(13.17639646 - 0.98564735) = 29.531$ days in length, where n_S (which is the mean motion of the Sun) comes from Table 5.1. An *anomalistic month* is the mean period of time between successive passages of the Moon through its perigee, and is $360/\tilde{n} = 360/13.06499295 = 27.555$ days in length, where $\tilde{n}$ comes from Table 6.1. Finally, a *draconic month* is the mean period of time between successive passages of the Moon through its ascending node, and is $360/\breve{n} = 360/13.22935027 = 27.212$ days in length, where $\breve{n}$ comes from Table 6.1. Incidentally, the lengths of the various months are discussed in Section 3 of Book IV of the Almagest. Impressively, Ptolemy's estimates of the various lengths match those just given to three decimal places.

6.8 Lunar distance model

According to a simplified version of modern lunar theory, the variation of the distance, r, of the Moon from the Earth is fairly well represented by the following formulae: [2]

$$\frac{r}{a_M} = 1 - \zeta_1 - \zeta_2 - \zeta_3 - \zeta_4 - \zeta_5, \tag{6.13}$$

$$\zeta_1 = 0.9894\, e\, \cos M + 0.4915\, e^2\, \cos 2M, \tag{6.14}$$

$$\zeta_2 = 0.1751\, e\, \cos(2\tilde{D} - M), \tag{6.15}$$

$$\zeta_3 = 0.1399\, e\, (\cos 2\tilde{D} - 0.0368\, \cos \tilde{D}), \tag{6.16}$$

$$\zeta_4 = -0.0023\, e\, \cos M_S, \tag{6.17}$$

$$\zeta_5 = 0.0001\, e\, \cos 2\bar{F}, \tag{6.18}$$

where $a_M = 3.850 \times 10^5$ km is the mean Earth-Moon distance, and ζ_i is termed the ith radial anomaly of the Moon. As before, the first anomaly is due to the eccentricity of the lunar orbit, and is very similar to that obtained from Keplerian orbit theory. The second, third, and fourth anomalies are due to the perturbing action of the Sun. Finally, the fifth anomaly is due to the slight inclination of the lunar orbit to the ecliptic plane. The radial anomalies of the Moon are specified as functions of their arguments in Table 6.5.

Our lunar distance model can be used to calculate the apparent radius of the Moon's disk (were it fully illuminated) in the Earth's sky. In fact, the apparent radius is

$$\rho_M = \frac{R_M}{r}, \tag{6.19}$$

where $R_M = 1737$ km is the mean physical radius of the Moon, and use has been made of the small angle approximation. It follows that

$$\rho_M = \frac{15.510'}{1 - \zeta_1 - \zeta_2 - \zeta_3 - \zeta_4 - \zeta_5}. \tag{6.20}$$

Clearly, the mean diameter of the (illuminated) lunar disk is about half a degree.

As an example of an apparent lunar radius calculation, we have seen that at 00:00 UT on May 5, 2005 AD the arguments of the lunar radial anomalies took the values

$$a_1 = M \simeq 58^\circ, \quad a_2 = 2\tilde{D} - M \simeq 210^\circ,$$

$$a_3 = \tilde{D} \simeq 314^\circ, \quad a_4 = M_S \simeq 120^\circ \quad a_5 = 2\bar{F} \simeq 314^\circ.$$

Table 6.5 yields

$$100\,\zeta_1(a_1) = 2.813, \;\; 100\,\zeta_2(a_2) = -0.832, \;\; 100\,\zeta_3(a_3) = 0.046,$$

[2] See Meeus (1998) and Fitzpatrick (2012).

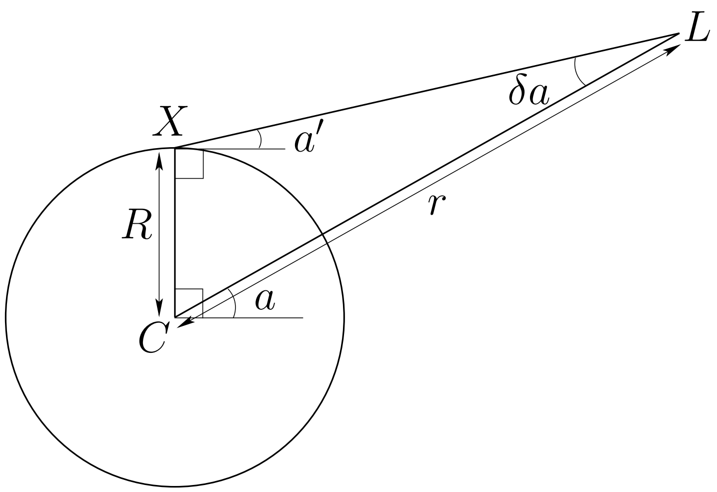

Figure 6.2: The Moon, L, as viewed by a hypothetical observer, C, at the center of the Earth, and a real observer, X, on the surface of the Earth.

$$100\,\zeta_4(a_4) = 0.006,\;\; 100\,\zeta_5(a_5) = -0.001.$$

Hence, we deduce that the apparent lunar radius was

$$\rho_M = \frac{15.510'}{1-(2.813-0.832+0.046+0.006-0.001)/100} = 15.83'.$$

As a second example of a lunar radius calculation, we have seen that at 00:00 UT on December 25, 1800 AD the arguments of the lunar radial anomalies took the values

$$a_1 = M \simeq 114^\circ,\;\;\; a_2 = 2\tilde{D} - M \simeq 99^\circ,$$

$$a_3 = \tilde{D} \simeq 106^\circ,\;\;\; a_4 = M_S \simeq 354^\circ\;\;\; a_5 = 2\bar{F} \simeq 10^\circ.$$

Table 6.5 yields

$$100\,\zeta_1(a_1) = -2.308,\;\; 100\,\zeta_2(a_2) = 0.151,\;\; 100\,\zeta_3(a_3) = -0.643,$$

$$100\,\zeta_4(a_4) = 0.013,\;\; 100\,\zeta_5(a_5) = 0.001.$$

Hence, we deduce that the apparent lunar radius was

$$\rho_M = \frac{15.510'}{1-(-2.308+0.151-0.643+0.013+0.001)/100} = 15.09'.$$

6.9 Lunar parallax

Now, it turns out that the Moon is sufficiently close to the Earth that its position in the sky is significantly modified by *parallax*. All of our previous analysis applies to a hypothetical observer situated at the center of the Earth. Consider a real observer situated on the Earth's surface. It can be seen from Figure 6.2 that the altitude of the Moon is a' for the real observer, and a for the hypothetical observer. Simple trigonometry reveals that $a' = a - \delta a$, which implies that the real observer sees the Moon at a lower altitude than the hypothetical observer. Let $R_E = 6371$ km be the mean radius of the Earth, and r the distance from the center of the Earth to the Moon. More simple trigonometry yields

$$\sin\delta a = \frac{R_E}{r}\,\cos a'. \tag{6.21}$$

Making use of Equation (6.13), as well as the small angle approximation, we deduce that

$$\delta a \simeq \delta_M\,\cos a, \tag{6.22}$$

where

$$\delta_M = \frac{56.888'}{1-\zeta_1-\zeta_2-\zeta_3-\zeta_4-\zeta_5}. \tag{6.23}$$

Note that lunar parallax increases with decreasing lunar altitude, reaching a maximum value of about $57'$ when the Moon is close to the horizon.

We have seen that at 00:00 UT on May 5, 2005 AD the lunar radial anomalies were $100\,\zeta_1 = 2.813$, $100\,\zeta_2 = -0.832$, $100\,\zeta_3 = 0.046$, $100\,\zeta_4 = 0.006$, and $100\,\zeta_5 = -0.001$. Thus, the maximum lunar parallax was

$$\delta_M = \frac{56.888'}{1-(2.813-0.832+0.046+0.006-0.001)/100} = 58.08'.$$

Likewise, we have also seen that at 00:00 UT on December 15, 1800 AD the lunar radial anomalies were $100\,\zeta_1 = -2.308$, $100\,\zeta_2 = 0.151$, $100\,\zeta_3 = -0.0643$, $100\,\zeta_4 = 0.013$, and $100\,\zeta_5 = 0.001$. Thus, the maximum lunar parallax was

$$\delta_M = \frac{56.888'}{1-(-2.308+0.151-0.643+0.013+0.001)/100} = 55.35'.$$

Incidentally, the information contained in this section is equivalent to that contained in the "Parallax table" (Κανὼν παραλλακτικός) that appears in Section 18 of Book V of the Almagest. Moreover, in Section 15 of the same book, Ptolemy uses the measured lunar parallax to determine the distance of the Moon from the Earth. Ptolemy finds that the Moon is 59 Earth radii from the center of the Earth. (The correct Earth-Moon distance is 60.33 Earth radii.) Furthermore, in Section 16 of the same book, Ptolemy uses his Earth-Moon distance, in combination with the apparent size of the Moon in the sky, to estimate that the Earth's radius is 3.4 times that of the Moon. (The correct answer is that the Earth's radius is 3.67 times that of the Moon.)

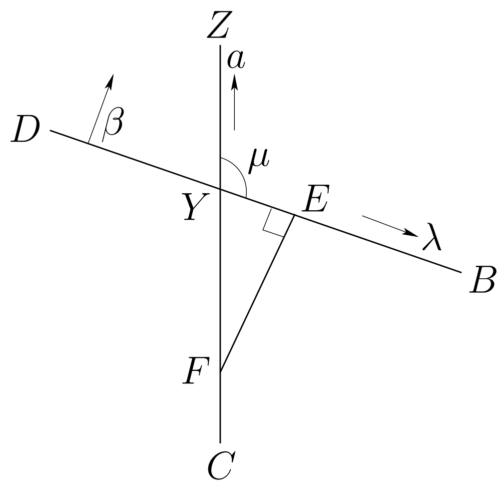

Figure 6.3: Parallactic shifts in the Moon's ecliptic longitude and latitude.

It now remains to investigate how parallax affects the Moon's ecliptic longitude and latitude. Figure 6.3 shows a detail of Figure 2.11. Point Y is the Moon's geocentric position on the celestial sphere. DB is a line passing through this point that is parallel to the local ecliptic circle, whereas ZC is a small section of an altitude circle passing through Y. The angle subtended between the ecliptic and the altitude circle is the parallactic angle, μ. Let F be the true position of the Moon. It follows that $\delta a = YF$. The changes in the Moon's ecliptic longitude and latitude are $\delta\lambda = YE$ and $-\delta\beta = EF$, respectively. Here, we are considering the case where increasing altitude corresponds to increasing ecliptic latitude. Assuming that the arcs δa, $\delta\lambda$, and $\delta\beta$ are all fairly small, the triangle YEF can be treated as a plane triangle. Hence, we obtain

$$\delta\lambda = -\delta a\,\cos\mu, \tag{6.24}$$

$$\delta\beta = -\delta a\,\sin\mu. \tag{6.25}$$

As is easily demonstrated, the previous formulae also apply to the case in which increasing altitude corresponds to decreasing ecliptic latitude.

For example, consider a day on which the geocentric ecliptic longitude of the Moon is $\lambda = 210^\circ$ (that is, 00SC00). Let the maximum lunar parallax be $\delta_M = 56.89'$. Suppose that the Moon is viewed from an observation site located at terrestrial latitude $+10^\circ$. The "Scorpio" entry in Table 2.18 gives the Moon's geocentric altitude, a, as a function of time, as well as the value of the parallactic angle μ. Making use of this data, in combination with Equations (6.22), (6.24), and (6.25), we can calculate the parallax-induced changes in the Moon's ecliptic longitude and latitude as it transits the sky. Data from such a calculation is given in the following table. The first column specifies time since the Moon's upper transit (thus, $t = +1$ hrs. means one hour after the upper transit), the second column gives the Moon's geocentric altitude, the third column the parallactic angle, the fourth column the decrease in the Moon's real altitude due to parallax, and the fifth and sixth columns the parallax-induced changes in its ecliptic longitude and latitude, respectively. It can be seen that parallax causes the Moon's apparent location to shift by almost 2° relative to the fixed stars as it transits the sky. Note that the previous calculation is somewhat inaccurate because it does not take into account the Moon's motion along the ecliptic (which can easily amount to 6° during the course of a night). However, the calculation does illustrate how the data contained in Tables 2.17–2.25, in combination with the data in Table 6.5, permits the parallax-induced shift in the Moon's ecliptic position to be calculated for a wide range of different lunar phases, observation sites, and observation times.

t (hrs.)	a	μ	δa	$\delta\lambda$	$\delta\beta$
-5.52	$00^\circ 00'$	$190^\circ 22'$	$57'$	$+56'$	$+10'$
-5.00	$12^\circ 26'$	$187^\circ 30'$	$56'$	$+55'$	$+07'$
-4.00	$26^\circ 37'$	$183^\circ 07'$	$51'$	$+51'$	$+03'$
-3.00	$40^\circ 23'$	$176^\circ 40'$	$43'$	$+43'$	$-03'$
-2.00	$53^\circ 15'$	$165^\circ 58'$	$34'$	$+33'$	$-08'$
-1.00	$63^\circ 52'$	$145^\circ 55'$	$25'$	$+21'$	$-14'$
$+0.00$	$68^\circ 32'$	$110^\circ 34'$	$21'$	$+07'$	$-20'$
$+1.00$	$63^\circ 52'$	$075^\circ 13'$	$25'$	$-06'$	$-24'$
$+2.00$	$53^\circ 15'$	$055^\circ 11'$	$34'$	$-19'$	$-28'$
$+3.00$	$40^\circ 23'$	$044^\circ 29'$	$43'$	$-31'$	$-30'$
$+4.00$	$26^\circ 37'$	$038^\circ 01'$	$51'$	$-40'$	$-31'$
$+5.00$	$12^\circ 26'$	$033^\circ 39'$	$56'$	$-46'$	$-31'$
$+5.52$	$00^\circ 00'$	$030^\circ 47'$	$57'$	$-49'$	$-29'$

6.10 Tables

e	$n(^\circ/\text{day})$	$\tilde{n}(^\circ/\text{day})$	$\breve{n}(^\circ/\text{day})$	$\bar{\lambda}_0(^\circ)$	$M_0(^\circ)$	$F_0(^\circ)$	$i(^\circ)$
0.054881	13.17639646	13.06499295	13.22935027	218.322	134.916	93.284	5.128

Table 6.1: Orbital elements of the Moon for the J2000 epoch (that is, 12:00 UT, January 1, 2000 AD, which corresponds to $t_0 = 2\,451\,545.0$ JD).

Δt(JD)	$\Delta\bar{\lambda}(^\circ)$	$\Delta M(^\circ)$	$\Delta\bar{F}(^\circ)$	Δt(JD)	$\Delta\bar{\lambda}(^\circ)$	$\Delta M(^\circ)$	$\Delta\bar{F}(^\circ)$
10 000	3.965	329.930	173.503	1 000	216.396	104.993	269.350
20 000	7.929	299.859	347.005	2 000	72.793	209.986	178.701
30 000	11.894	269.788	160.508	3 000	289.189	314.979	88.051
40 000	15.858	239.718	334.011	4 000	145.586	59.972	357.401
50 000	19.823	209.648	147.513	5 000	1.982	164.965	266.751
60 000	23.788	179.577	321.016	6 000	218.379	269.958	176.102
70 000	27.752	149.506	134.519	7 000	74.775	14.951	85.452
80 000	31.717	119.436	308.022	8 000	291.172	119.944	354.802
90 000	35.681	89.366	121.524	9 000	147.568	224.937	264.152
100	237.640	226.499	242.935	10	131.764	130.650	132.294
200	115.279	92.999	125.870	20	263.528	261.300	264.587
300	352.919	319.498	8.805	30	35.292	31.950	36.881
400	230.559	185.997	251.740	40	167.056	162.600	169.174
500	108.198	52.496	134.675	50	298.820	293.250	301.468
600	345.838	278.996	17.610	60	70.584	63.900	73.761
700	223.478	145.495	260.545	70	202.348	194.550	206.055
800	101.117	11.994	143.480	80	334.112	325.199	338.348
900	338.757	238.494	26.415	90	105.876	95.849	110.642
1	13.176	13.065	13.229	0.1	1.318	1.306	1.323
2	26.353	26.130	26.459	0.2	2.635	2.613	2.646
3	39.529	39.195	39.688	0.3	3.953	3.919	3.969
4	52.706	52.260	52.917	0.4	5.271	5.226	5.292
5	65.882	65.325	66.147	0.5	6.588	6.532	6.615
6	79.058	78.390	79.376	0.6	7.906	7.839	7.938
7	92.235	91.455	92.605	0.7	9.223	9.145	9.261
8	105.411	104.520	105.835	0.8	10.541	10.452	10.583
9	118.588	117.585	119.064	0.9	11.859	11.758	11.906

Table 6.2: Mean motion of the Moon. Here, $\Delta t = t - t_0$, $\Delta\bar{\lambda} = \bar{\lambda} - \bar{\lambda}_0$, $\Delta M = M - M_0$, and $\Delta\bar{F} = \bar{F} - \bar{F}_0$. At epoch ($t_0 = 2\,451\,545.0$ JD), $\bar{\lambda}_0 = 218.322^\circ$, $M_0 = 134.916^\circ$, and $\bar{F}_0 = 93.284^\circ$.

Arg. (°)	$q_1(^\circ)$	$q_2(^\circ)$	$q_3(^\circ)$	$q_4(^\circ)$	$q_5(^\circ)$
000(360)	$+0.000$	$+0.000$	$+0.000$	-0.000	-0.000
002(358)	$+0.234$	$+0.044$	$+0.045$	-0.006	-0.004
004(356)	$+0.468$	$+0.089$	$+0.089$	-0.013	-0.008
006(354)	$+0.702$	$+0.133$	$+0.133$	-0.019	-0.012
008(352)	$+0.934$	$+0.177$	$+0.177$	-0.026	-0.016
010(350)	$+1.165$	$+0.221$	$+0.219$	-0.032	-0.020
012(348)	$+1.394$	$+0.265$	$+0.261$	-0.039	-0.024
014(346)	$+1.622$	$+0.308$	$+0.301$	-0.045	-0.028
016(344)	$+1.847$	$+0.351$	$+0.339$	-0.051	-0.032
018(342)	$+2.069$	$+0.394$	$+0.376$	-0.057	-0.035
020(340)	$+2.288$	$+0.436$	$+0.411$	-0.063	-0.039
022(338)	$+2.504$	$+0.477$	$+0.444$	-0.069	-0.043
024(336)	$+2.717$	$+0.518$	$+0.475$	-0.075	-0.047
026(334)	$+2.925$	$+0.559$	$+0.504$	-0.081	-0.050
028(332)	$+3.130$	$+0.598$	$+0.530$	-0.087	-0.054
030(330)	$+3.329$	$+0.637$	$+0.553$	-0.093	-0.057
032(328)	$+3.525$	$+0.675$	$+0.573$	-0.098	-0.061
034(326)	$+3.715$	$+0.712$	$+0.591$	-0.104	-0.064
036(324)	$+3.900$	$+0.749$	$+0.606$	-0.109	-0.067
038(322)	$+4.079$	$+0.784$	$+0.618$	-0.114	-0.070
040(320)	$+4.253$	$+0.819$	$+0.626$	-0.119	-0.074
042(318)	$+4.421$	$+0.853$	$+0.632$	-0.124	-0.077
044(316)	$+4.582$	$+0.885$	$+0.634$	-0.129	-0.080
046(314)	$+4.737$	$+0.917$	$+0.633$	-0.133	-0.082
048(312)	$+4.886$	$+0.947$	$+0.629$	-0.138	-0.085
050(310)	$+5.028$	$+0.976$	$+0.622$	-0.142	-0.088
052(308)	$+5.163$	$+1.004$	$+0.612$	-0.146	-0.090
054(306)	$+5.291$	$+1.031$	$+0.598$	-0.150	-0.093
056(304)	$+5.412$	$+1.056$	$+0.582$	-0.154	-0.095
058(302)	$+5.525$	$+1.081$	$+0.562$	-0.157	-0.097
060(300)	$+5.631$	$+1.103$	$+0.540$	-0.160	-0.099

Arg. (°)	$q_1(^\circ)$	$q_2(^\circ)$	$q_3(^\circ)$	$q_4(^\circ)$	$q_5(^\circ)$
060(300)	$+5.631$	$+1.103$	$+0.540$	-0.160	-0.099
062(298)	$+5.730$	$+1.125$	$+0.515$	-0.164	-0.101
064(296)	$+5.821$	$+1.145$	$+0.488$	-0.166	-0.103
066(294)	$+5.904$	$+1.164$	$+0.458$	-0.169	-0.105
068(292)	$+5.979$	$+1.181$	$+0.425$	-0.172	-0.106
070(290)	$+6.047$	$+1.197$	$+0.391$	-0.174	-0.108
072(288)	$+6.107$	$+1.212$	$+0.354$	-0.176	-0.109
074(286)	$+6.158$	$+1.225$	$+0.316$	-0.178	-0.110
076(284)	$+6.202$	$+1.236$	$+0.275$	-0.180	-0.111
078(282)	$+6.238$	$+1.246$	$+0.234$	-0.181	-0.112
080(280)	$+6.266$	$+1.255$	$+0.191$	-0.182	-0.113
082(278)	$+6.287$	$+1.262$	$+0.147$	-0.183	-0.113
084(276)	$+6.299$	$+1.267$	$+0.102$	-0.184	-0.114
086(274)	$+6.303$	$+1.271$	$+0.057$	-0.185	-0.114
088(272)	$+6.300$	$+1.273$	$+0.011$	-0.185	-0.114
090(270)	$+6.289$	$+1.274$	-0.035	-0.185	-0.114
092(268)	$+6.270$	$+1.273$	-0.081	-0.185	-0.114
094(266)	$+6.244$	$+1.271$	-0.126	-0.185	-0.114
096(264)	$+6.210$	$+1.267$	-0.171	-0.184	-0.114
098(262)	$+6.169$	$+1.262$	-0.216	-0.183	-0.113
100(260)	$+6.120$	$+1.255$	-0.259	-0.182	-0.113
102(258)	$+6.065$	$+1.246$	-0.302	-0.181	-0.112
104(256)	$+6.002$	$+1.236$	-0.343	-0.180	-0.111
106(254)	$+5.932$	$+1.225$	-0.382	-0.178	-0.110
108(252)	$+5.856$	$+1.212$	-0.420	-0.176	-0.109
110(250)	$+5.772$	$+1.197$	-0.456	-0.174	-0.108
112(248)	$+5.683$	$+1.181$	-0.490	-0.172	-0.106
114(246)	$+5.586$	$+1.164$	-0.521	-0.169	-0.105
116(244)	$+5.484$	$+1.145$	-0.550	-0.166	-0.103
118(242)	$+5.376$	$+1.125$	-0.577	-0.164	-0.101
120(240)	$+5.261$	$+1.103$	-0.600	-0.160	-0.099

Arg. (°)	$q_1(^\circ)$	$q_2(^\circ)$	$q_3(^\circ)$	$q_4(^\circ)$	$q_5(^\circ)$
120(240)	$+5.261$	$+1.103$	-0.600	-0.160	-0.099
122(238)	$+5.141$	$+1.081$	-0.621	-0.157	-0.097
124(236)	$+5.016$	$+1.056$	-0.639	-0.154	-0.095
126(234)	$+4.885$	$+1.031$	-0.654	-0.150	-0.093
128(232)	$+4.748$	$+1.004$	-0.666	-0.146	-0.090
130(230)	$+4.607$	$+0.976$	-0.675	-0.142	-0.088
132(228)	$+4.461$	$+0.947$	-0.681	-0.138	-0.085
134(226)	$+4.310$	$+0.917$	-0.683	-0.133	-0.082
136(224)	$+4.155$	$+0.885$	-0.682	-0.129	-0.080
138(222)	$+3.996$	$+0.853$	-0.678	-0.124	-0.077
140(220)	$+3.832$	$+0.819$	-0.671	-0.119	-0.074
142(218)	$+3.665$	$+0.784$	-0.660	-0.114	-0.070
144(216)	$+3.493$	$+0.749$	-0.647	-0.109	-0.067
146(214)	$+3.319$	$+0.712$	-0.630	-0.104	-0.064
148(212)	$+3.141$	$+0.675$	-0.610	-0.098	-0.061
150(210)	$+2.959$	$+0.637$	-0.588	-0.093	-0.057
152(208)	$+2.775$	$+0.598$	-0.562	-0.087	-0.054
154(206)	$+2.589$	$+0.559$	-0.534	-0.081	-0.050
156(204)	$+2.399$	$+0.518$	-0.503	-0.075	-0.047
158(202)	$+2.207$	$+0.477$	-0.470	-0.069	-0.043
160(200)	$+2.014$	$+0.436$	-0.435	-0.063	-0.039
162(198)	$+1.818$	$+0.394$	-0.398	-0.057	-0.035
164(196)	$+1.620$	$+0.351$	-0.358	-0.051	-0.032
166(194)	$+1.421$	$+0.308$	-0.318	-0.045	-0.028
168(192)	$+1.221$	$+0.265$	-0.275	-0.039	-0.024
170(190)	$+1.019$	$+0.221$	-0.231	-0.032	-0.020
172(188)	$+0.816$	$+0.177$	-0.186	-0.026	-0.016
174(186)	$+0.613$	$+0.133$	-0.141	-0.019	-0.012
176(184)	$+0.409$	$+0.089$	-0.094	-0.013	-0.008
178(182)	$+0.205$	$+0.044$	-0.047	-0.006	-0.004
180(180)	$+0.000$	$+0.000$	-0.000	-0.000	-0.000

Table 6.3: Longitudinal anomalies of the Moon. The common argument corresponds to M, $2\tilde{D}-M$, $\tilde{D}$, M_S, and $2\bar{F}$ for the case of q_1, q_2, q_3, q_4, and q_5, respectively. If the argument is in parentheses then the anomalies are minus the values shown in the table.

$F(^\circ)$	$\beta(^\circ)$	$F(^\circ)$	$F(^\circ)$	$\beta(^\circ)$	$F(^\circ)$
000/180	0.000	(180)/(360)	046/134	3.686	(226)/(314)
002/178	0.179	(182)/(358)	048/132	3.809	(228)/(312)
004/176	0.357	(184)/(356)	050/130	3.926	(230)/(310)
006/174	0.535	(186)/(354)	052/128	4.039	(232)/(308)
008/172	0.713	(188)/(352)	054/126	4.147	(234)/(306)
010/170	0.889	(190)/(350)	056/124	4.250	(236)/(304)
012/168	1.065	(192)/(348)	058/122	4.347	(238)/(302)
014/166	1.239	(194)/(346)	060/120	4.440	(240)/(300)
016/164	1.412	(196)/(344)	062/118	4.527	(242)/(298)
018/162	1.583	(198)/(342)	064/116	4.608	(244)/(296)
020/160	1.752	(200)/(340)	066/114	4.684	(246)/(294)
022/158	1.919	(202)/(338)	068/112	4.754	(248)/(292)
024/156	2.083	(204)/(336)	070/110	4.818	(250)/(290)
026/154	2.246	(206)/(334)	072/108	4.877	(252)/(288)
028/152	2.405	(208)/(332)	074/106	4.929	(254)/(286)
030/150	2.561	(210)/(330)	076/104	4.975	(256)/(284)
032/148	2.715	(212)/(328)	078/102	5.016	(258)/(282)
034/146	2.865	(214)/(326)	080/100	5.050	(260)/(280)
036/144	3.012	(216)/(324)	082/098	5.078	(262)/(278)
038/142	3.155	(218)/(322)	084/096	5.100	(264)/(276)
040/140	3.294	(220)/(320)	086/094	5.116	(266)/(274)
042/138	3.429	(222)/(318)	088/092	5.125	(268)/(272)
044/136	3.560	(224)/(316)	090/090	5.128	(270)/(270)

Table 6.4: Ecliptic latitude of the Moon. The latitude is minus the value shown in the table if the argument is in parentheses.

Arg. (°)	$100\,\zeta_1$	$100\,\zeta_2$	$100\,\zeta_3$	$100\,\zeta_4$	$100\,\zeta_5$
000(360)	$+5.578$	-0.961	$+0.740$	-0.013	$+0.001$
002(358)	$+5.574$	-0.960	$+0.738$	-0.013	$+0.001$
004(356)	$+5.563$	-0.959	$+0.732$	-0.013	$+0.001$
006(354)	$+5.545$	-0.956	$+0.723$	-0.013	$+0.001$
008(352)	$+5.519$	-0.952	$+0.710$	-0.012	$+0.001$
010(350)	$+5.487$	-0.946	$+0.694$	-0.012	$+0.001$
012(348)	$+5.447$	-0.940	$+0.674$	-0.012	$+0.001$
014(346)	$+5.399$	-0.932	$+0.650$	-0.012	$+0.001$
016(344)	$+5.345$	-0.924	$+0.624$	-0.012	$+0.001$
018(342)	$+5.284$	-0.914	$+0.594$	-0.012	$+0.001$
020(340)	$+5.216$	-0.903	$+0.562$	-0.012	$+0.001$
022(338)	$+5.141$	-0.891	$+0.526$	-0.012	$+0.001$
024(336)	$+5.060$	-0.878	$+0.488$	-0.012	$+0.001$
026(334)	$+4.972$	-0.864	$+0.447$	-0.011	$+0.001$
028(332)	$+4.877$	-0.848	$+0.404$	-0.011	$+0.001$
030(330)	$+4.776$	-0.832	$+0.359$	-0.011	$+0.001$
032(328)	$+4.670$	-0.815	$+0.313$	-0.011	$+0.001$
034(326)	$+4.557$	-0.797	$+0.264$	-0.010	$+0.001$
036(324)	$+4.439$	-0.777	$+0.214$	-0.010	$+0.001$
038(322)	$+4.315$	-0.757	$+0.163$	-0.010	$+0.001$
040(320)	$+4.185$	-0.736	$+0.112$	-0.010	$+0.001$
042(318)	$+4.051$	-0.714	$+0.059$	-0.009	$+0.001$
044(316)	$+3.911$	-0.691	$+0.006$	-0.009	$+0.001$
046(314)	$+3.767$	-0.668	-0.046	-0.009	$+0.001$
048(312)	$+3.618$	-0.643	-0.099	-0.008	$+0.001$
050(310)	$+3.465$	-0.618	-0.151	-0.008	$+0.001$
052(308)	$+3.307$	-0.592	-0.203	-0.008	$+0.001$
054(306)	$+3.146$	-0.565	-0.254	-0.007	$+0.000$
056(304)	$+2.981$	-0.537	-0.303	-0.007	$+0.000$
058(302)	$+2.813$	-0.509	-0.352	-0.007	$+0.000$
060(300)	$+2.641$	-0.480	-0.398	-0.006	$+0.000$

Arg. (°)	$100\,\zeta_1$	$100\,\zeta_2$	$100\,\zeta_3$	$100\,\zeta_4$	$100\,\zeta_5$
060(300)	$+2.641$	-0.480	-0.398	-0.006	$+0.000$
062(298)	$+2.466$	-0.451	-0.443	-0.006	$+0.000$
064(296)	$+2.289$	-0.421	-0.485	-0.006	$+0.000$
066(294)	$+2.109$	-0.391	-0.525	-0.005	$+0.000$
068(292)	$+1.928$	-0.360	-0.563	-0.005	$+0.000$
070(290)	$+1.744$	-0.329	-0.598	-0.004	$+0.000$
072(288)	$+1.558$	-0.297	-0.630	-0.004	$+0.000$
074(286)	$+1.371$	-0.265	-0.659	-0.003	$+0.000$
076(284)	$+1.183$	-0.232	-0.685	-0.003	$+0.000$
078(282)	$+0.994$	-0.200	-0.707	-0.003	$+0.000$
080(280)	$+0.804$	-0.167	-0.726	-0.002	$+0.000$
082(278)	$+0.613$	-0.134	-0.742	-0.002	$+0.000$
084(276)	$+0.423$	-0.100	-0.754	-0.001	$+0.000$
086(274)	$+0.232$	-0.067	-0.762	-0.001	$+0.000$
088(272)	$+0.042$	-0.034	-0.767	-0.000	$+0.000$
090(270)	-0.148	-0.000	-0.768	-0.000	$+0.000$
092(268)	-0.337	$+0.034$	-0.765	$+0.000$	-0.000
094(266)	-0.525	$+0.067$	-0.758	$+0.001$	-0.000
096(264)	-0.712	$+0.100$	-0.748	$+0.001$	-0.000
098(262)	-0.898	$+0.134$	-0.734	$+0.002$	-0.000
100(260)	-1.082	$+0.167$	-0.717	$+0.002$	-0.000
102(258)	-1.264	$+0.200$	-0.696	$+0.003$	-0.000
104(256)	-1.444	$+0.232$	-0.671	$+0.003$	-0.000
106(254)	-1.622	$+0.265$	-0.643	$+0.003$	-0.000
108(252)	-1.798	$+0.297$	-0.612	$+0.004$	-0.000
110(250)	-1.971	$+0.329$	-0.578	$+0.004$	-0.000
112(248)	-2.141	$+0.360$	-0.542	$+0.005$	-0.000
114(246)	-2.308	$+0.391$	-0.502	$+0.005$	-0.000
116(244)	-2.471	$+0.421$	-0.460	$+0.006$	-0.000
118(242)	-2.632	$+0.451$	-0.416	$+0.006$	-0.000
120(240)	-2.789	$+0.480$	-0.370	$+0.006$	-0.000

Arg. (°)	$100\,\zeta_1$	$100\,\zeta_2$	$100\,\zeta_3$	$100\,\zeta_4$	$100\,\zeta_5$
120(240)	-2.789	$+0.480$	-0.370	$+0.006$	-0.000
122(238)	-2.942	$+0.509$	-0.322	$+0.007$	-0.000
124(236)	-3.092	$+0.537$	-0.272	$+0.007$	-0.000
126(234)	-3.237	$+0.565$	-0.221	$+0.007$	-0.000
128(232)	-3.379	$+0.592$	-0.168	$+0.008$	-0.001
130(230)	-3.516	$+0.618$	-0.115	$+0.008$	-0.001
132(228)	-3.649	$+0.643$	-0.061	$+0.008$	-0.001
134(226)	-3.777	$+0.668$	-0.007	$+0.009$	-0.001
136(224)	-3.901	$+0.691$	$+0.047$	$+0.009$	-0.001
138(222)	-4.020	$+0.714$	$+0.101$	$+0.009$	-0.001
140(220)	-4.134	$+0.736$	$+0.155$	$+0.010$	-0.001
142(218)	-4.243	$+0.757$	$+0.208$	$+0.010$	-0.001
144(216)	-4.347	$+0.777$	$+0.260$	$+0.010$	-0.001
146(214)	-4.446	$+0.797$	$+0.311$	$+0.010$	-0.001
148(212)	-4.540	$+0.815$	$+0.361$	$+0.011$	-0.001
150(210)	-4.628	$+0.832$	$+0.408$	$+0.011$	-0.001
152(208)	-4.712	$+0.848$	$+0.454$	$+0.011$	-0.001
154(206)	-4.789	$+0.864$	$+0.498$	$+0.011$	-0.001
156(204)	-4.861	$+0.878$	$+0.540$	$+0.012$	-0.001
158(202)	-4.928	$+0.891$	$+0.578$	$+0.012$	-0.001
160(200)	-4.989	$+0.903$	$+0.615$	$+0.012$	-0.001
162(198)	-5.044	$+0.914$	$+0.648$	$+0.012$	-0.001
164(196)	-5.094	$+0.924$	$+0.678$	$+0.012$	-0.001
166(194)	-5.138	$+0.932$	$+0.705$	$+0.012$	-0.001
168(192)	-5.176	$+0.940$	$+0.729$	$+0.012$	-0.001
170(190)	-5.208	$+0.946$	$+0.749$	$+0.012$	-0.001
172(188)	-5.235	$+0.952$	$+0.766$	$+0.012$	-0.001
174(186)	-5.255	$+0.956$	$+0.779$	$+0.013$	-0.001
176(184)	-5.270	$+0.959$	$+0.788$	$+0.013$	-0.001
178(182)	-5.279	$+0.960$	$+0.794$	$+0.013$	-0.001
180(180)	-5.282	$+0.961$	$+0.796$	$+0.013$	-0.001

Table 6.5: Radial anomalies of the Moon. The common argument corresponds to M, $2\tilde{D}-M$, $\tilde{D}$, M_S, and $2\bar{F}$ for the case of ζ_1, ζ_2, ζ_3, ζ_4, and ζ_5, respectively. If the argument is in parentheses then the anomalies are minus the values shown in the table.

7. Lunar-Solar Syzygies and Eclipses

7.1 Syzygies

Let λ_S and λ_M represent the ecliptic longitudes of the Sun and the Moon, respectively. The lunar-solar *elongation* is defined

$$D = \lambda_M - \lambda_S. \tag{7.1}$$

Because the Moon is only visible because of light reflected from the Sun,[1] there is a fairly obvious relationship between lunar-solar elongation and lunar phase. See Figure 7.1. For instance, a *new moon* corresponds to $D = 0^\circ$, a *quarter moon* to $D = 90^\circ$ or 270°, and a *full moon* to $D = 180^\circ$. New moons and full moons are collectively known as lunar-solar *syzygies*. (From the Greek συζυγία, meaning "yoking together" or conjuction.)

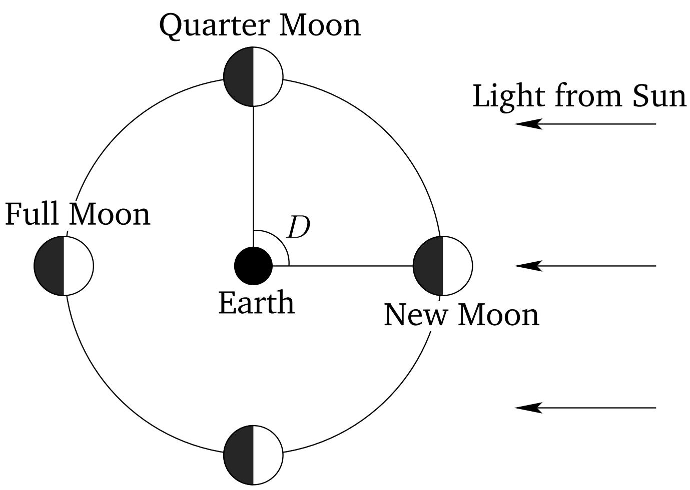

Figure 7.1: The phases of the Moon.

[1] Anaxagoras of Clazomenae (Ἀναξαγόρας ὁ Κλαζομενίας), who lived in the 5th century BC, is generally credited with this discovery.

7.2 Lunar-solar elongation model

We can determine the lunar-solar elongation by combining the solar and lunar models described in the previous two chapters. Our elongation model is as follows:

$$\bar{D} = \bar{\lambda}_M - \bar{\lambda}_S, \tag{7.2}$$

$$q_1 = 2\,e_M\,\sin M_M + 1.2379\,e_M^2\,\sin 2M_M, \tag{7.3}$$

$$q_2 = 0.4052\,e_M\,\sin(2\bar{D} - M_M), \tag{7.4}$$

$$q_3 = 0.2094\,e_M\,(\sin 2\bar{D} - 0.0527\,\sin \bar{D}), \tag{7.5}$$

$$q_4 = -(0.0589\,e_M + 2\,e_S)\,\sin M_S - (5/4)\,e_S^2\,\sin 2M_S, \tag{7.6}$$

$$q_5 = -0.0364\,e_M\,\sin 2\bar{F}_M, \tag{7.7}$$

$$D = \bar{D} + q_1 + q_2 + q_3 + q_4 + q_5. \tag{7.8}$$

Here, e_S, M_S, and $\bar{\lambda}_S$ are the eccentricity, mean anomaly, and mean longitude of the Sun's apparent orbit about the Earth, respectively. Moreover, e_M, M_M, $\bar{\lambda}_M$, and $\bar{F}_M$ are the eccentricity, mean anomaly, mean longitude, and mean argument of latitude of the Moon's orbit, respectively.

The lunar-solar elongation can be calculated with the aid of Tables 7.1 and 7.2. Table 7.1 allows the mean lunar-solar elongation, $\bar{D}$, the mean lunar argument of latitude, $\bar{F}_M$, the mean anomaly of the Sun, M_S, and the mean anomaly of the Moon, M_M, to be determined as functions of time. Table 7.2 specifies the anomalies q_1–q_5 as functions of their various arguments.

Tables 7.1 and 7.2 contain equivalent information to that contained in the "Table of conjunctions" (Συνόδων κανόνιον) and the "Table of full moons" (Πανσελήνων κανόνιον) that appear in Section 3 of Book VI of the Almagest.

7.3 Determination of lunar-solar elongation

The procedure for using Tables 7.1 and 7.2 to determine the lunar-solar elongation is as follows:

1. Determine the fractional Julian day number, t, corresponding to the date and time at which the lunar-solar elongation is to be calculated with the aid of Tables 3.1–3.3. Form $\Delta t = t - t_0$, where $t_0 = 2\,451\,545.0$ is the epoch.

2. Enter Table 7.1 with the digit for each power of 10 in Δt and take out the corresponding values of $\Delta\bar{D}$, $\Delta\bar{F}_M$, ΔM_S, and ΔM_M. If Δt is negative then the values are minus those shown in the table. The value of the mean lunar-solar elongation, $\bar{D}$, is the sum of all the $\Delta\bar{D}$ values plus the value of $\bar{D}$ at the epoch. Likewise, the value of the mean lunar argument of latitude, $\bar{F}_M$, is the sum of all the $\Delta\bar{F}_M$ values plus the value of $\bar{F}_M$ at the epoch. Moreover, the value of the solar mean anomaly, M_S, is the sum of all the ΔM_S values plus the value of M_S at the epoch. Finally, the value of the lunar mean anomaly,

M_M, is the sum of all the ΔM_M values plus the value of M_M at the epoch. Add as many multiples of 360° to $\bar{D}$, $\bar{F}_M$, M_S, and M_M as is required to make them all fall in the range 0° to 360°.

3. Form the five arguments $a_1 = M_M$, $a_2 = 2\bar{D} - M_M$, $a_3 = \bar{D}$, $a_4 = M_S$, $a_5 = 2\bar{F}_M$. Add as many multiples of 360° to the arguments as is required to make them all fall in the range 0° to 360°. Round each argument to the nearest degree.

4. Enter Table 7.2 with the value of each of the five arguments a_1–a_5 and take out the value of each of the five corresponding anomalies q_1–q_5. It is necessary to interpolate if the arguments are odd.

5. The lunar-solar elongation is given by $D = \bar{D} + q_1 + q_2 + q_3 + q_4 + q_5$. If necessary, convert D into an angle in the range 0° to 360°. The decimal fraction can be converted into arc minutes using Table 5.2.

In order to facilitate the calculation of syzygies, the previous model has been used to contruct Table 7.3, which lists the dates and fractional Julian day numbers of the first new moons of the years 1900–2099 CE. Two examples of syzygy calculations are given in the following section. Incidentally, our model is capable of predicting the times of new moons and full moons to within 10 minutes.

7.4 Example syzygy calculations

Example 1: Sixth new moon in 2004 AD:

From Table 7.3, the date of first new moon in 2004 AD is 2453026.4 JD. Now, the lunar-solar elongation increases at the mean rate $n_M - n_S = 13.17639646 - 0.98564735 = 12.1907491^\circ$ per day, or 360° in 29.53 days; the latter time period is known as a synodic month. Hence, a rough estimate for the date of the sixth new moon in 2004 AD is five synodic months after that of the first: that is, $2453026.4 + 5 \times 29.53 \simeq 2453174.1$ JD. It follows that $\Delta t = 2453174.1 - 2451545.0 = 1629.1$ JD. Let us calculate the lunar-solar elongation at this date. From Table 7.1:

t(JD)	$\bar{D}(^\circ)$	$\bar{F}_M(^\circ)$	$M_S(^\circ)$	$M_M(^\circ)$
+1 000	310.749	269.350	265.600	104.993
+600	114.449	17.610	231.360	278.996
+20	243.815	264.587	19.712	261.300
+9	109.717	119.064	8.870	117.585
+.1	1.219	1.323	0.099	1.306
Epoch	297.864	93.284	357.588	134.916
	1077.813	765.218	883.229	899.096
Modulus	357.813	45.218	163.229	179.096

Thus,

$$a_1 = M_M \simeq 179^\circ, \quad a_2 = 2\bar{D} - M_M = 2 \times 357.813 - 179.082 \simeq 177^\circ,$$

$$a_3 = \bar{D} \simeq 358^\circ, \quad a_4 = M_S \simeq 163^\circ,$$

$$a_5 = 2\bar{F}_M = 2 \times 45.218 \simeq 90^\circ.$$

Table 7.2 yields

$$q_1(a_1) = 0.103^\circ, \;\; q_2(a_2) = 0.067^\circ, \;\; q_3(a_3) = -0.045^\circ,$$

$$q_4(a_4) = -0.603^\circ, \;\; q_5(a_5) = -0.114^\circ.$$

Hence,

$$\begin{aligned} D &= \bar{D} + q_1 + q_2 + q_3 + q_4 + q_5 \\ &= 357.813 + 0.103 + 0.067 - 0.045 - 0.603 - 0.114 \\ &\simeq 357.22^\circ. \end{aligned}$$

Now, the actual new moon takes place when $D = 360.00^\circ$. Thus, a far better estimate for the date of the sixth new moon in 2004 AD is $2453174.10 + (360.00 - 357.22)/12.1907491 = 2453174.33$ JD. This corresponds to 19:55 UT on June 17th.

Example 2: Third full moon in 1982 AD

From Table 7.3, the fractional Julian day number of first new moon in 1982 AD is 2444994.7 JD, which corresponds to January 25th. Because there is more than half a synodic month between this event and the start of year, we conclude that the first full moon in 1982 AD took place before January 25th. Hence, a rough estimate for the date of the third full moon in 1982 AD is one and a half synodic months after that of the first new moon; that is, $2444994.7 + 1.5 \times 29.53 \simeq 2445039.0$ JD. It follows that $\Delta t = 2445039.0 - 2451545.0 = -6506.0$ JD. Let us calculate the lunar-solar elongation at this date. From Table 7.1:

t(JD)	$\bar{D}(^\circ)$	$\bar{F}_M(^\circ)$	$M_S(^\circ)$	$M_M(^\circ)$
$-6\,000$	-64.495	-176.102	-153.601	-269.958
-500	-335.375	-134.675	-132.800	-52.496
-6	-73.144	-79.376	-5.914	-78.390
Epoch	297.864	93.284	357.588	134.916
	-175.150	-296.869	65.273	-265.928
Modulus	184.131	63.062	65.273	94.072

Thus,

$$a_1 = M_M \simeq 94^\circ,\ \ a_2 = 2\bar{D} - M_M = 2 \times 184.850 - 94.072 \simeq 276^\circ,$$
$$a_3 = \bar{D} \simeq 185^\circ,\ \ a_4 = M_S \simeq 65^\circ,$$
$$a_5 = 2\bar{F}_M = 2 \times 63.062 \simeq 126^\circ.$$

Table 7.2 yields

$$q_1(a_1) = 6.244^\circ,\ q_2(a_2) = -1.267^\circ,\ q_3(a_3) = 0.118^\circ,$$
$$q_4(a_4) = -1.918^\circ,\ q_5(a_5) = -0.093^\circ.$$

Hence,

$$\begin{aligned} D &= \bar{D} + q_1 + q_2 + q_3 + q_4 + q_5 \\ &= 184.850 + 6.244 - 1.267 + 0.118 - 1.918 - 0.093 \\ &\simeq 187.93^\circ. \end{aligned}$$

Now, the actual full moon takes place when $D = 180.00^\circ$. Thus, a far better estimate for the date of the third full moon in 1982 AD is $2445039.0 + (180.00 - 187.93)/12.1907491 = 2445038.35$ JD. This corresponds to 20:04 UT on March 9th.

7.5 Solar and lunar eclipses

Eclipses are discussed in Book VI of the Almagest. A *solar eclipse*—or, more accurately, a lunar-solar occultation—occurs when the Moon blocks the light of the Sun. Clearly, this is only possible at a new moon. See Figure 7.1. On the other hand, an umbral *lunar eclipse* occurs when the Moon falls into the umbra of the Earth. Of course, this is only possible at a full moon. It follows that eclipses can only take place at lunar-solar syzygies.

In order to determine whether a particular lunar-solar syzygy coincides with an eclipse, we first need to calculate the angular radii of the Sun, the Moon, and the Earth's umbra in the sky. We have already seen that the angular radius of the Sun, ρ_M, is given by formula (5.23). Likewise, the angular radius of the Moon, ρ_M, is given by formula (6.20). Let R_S and R_E be the physical radii of the Sun and the Earth, respectively. Moreover, let r_S and r_M be the Earth-Sun and the Earth-Moon distances, respectively. The ratio $\delta_M = R_E/r_M$ is the maximum parallax of the Moon, and is specified in formula (6.23). Simple trigonometry reveals that the angular size of the Earth's umbra at the radius of the Moon's orbit is

$$\rho_U = \delta_M - \rho_S. \tag{7.9}$$

This can be seen from Figure 7.2. The radius of the umbra at the position of the Moon is $R_U = R_E - x = R_E - r_M\,\rho_S$. Hence, the angular radius of the umbra is $\rho_U = R_U/r_M =$

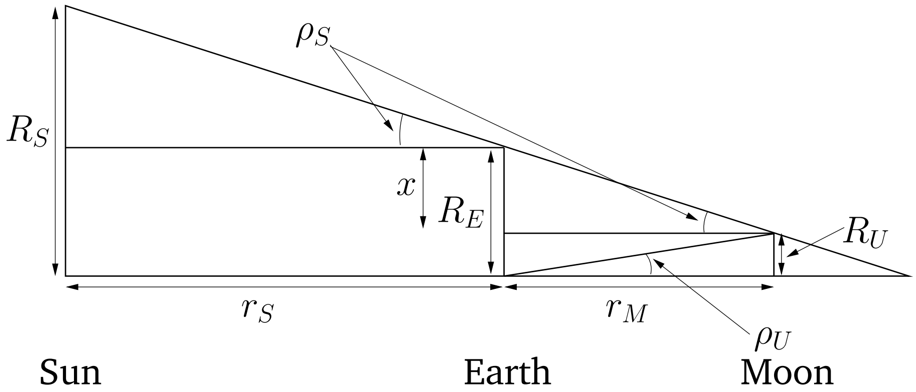

Figure 7.2: The Earth's umbra. Here, R_S, R_E, R_U, r_S, and r_M are the physical radius of the Sun, the physical radius of the Earth, the physical radius of the Earth's umbra at the Moon, the Earth-Sun distance, and the Earth-Moon distances, respectively.

$\delta_M - \rho_S$. Incidentally, the identification of two of the angles in the figure with $\rho_S = R_S/r_S$ follows because $R_S \gg R_E$.

A solar eclipse does not take place every new moon, nor a lunar eclipse every full moon, because of the inclination of the Moon's orbit to the ecliptic plane, which causes the Moon to pass either above or below the Sun, or the Earth's umbra, respectively, in the majority of cases. It follows that the critical parameter that determines the occurrence of eclipses is the ecliptic latitude of the Moon at syzygy, β_{syz}. Of course, once the date and time of a syzygy has been established, β_{syz} can be calculated from Table 6.4. However, the lunar argument of latitude, F, must first be determined using

$$F = \bar{F}_M + q_1 + q_2 + q_3 + q_{4'} + q_5, \tag{7.10}$$

where $\bar{F}_M$ comes from Table 7.1, q_1, q_2, q_3, and q_5 are obtained from Table 7.2, and $q_{4'}$ is the q_4 from Table 6.3. For instance, we have seen that for the third full moon of 1982 AD, $\bar{F}_M = 63.062, M_S \simeq 65^\circ, q_1 = 6.244^\circ, q_2 = -1.267^\circ, q_3 = 0.118^\circ$, and $q_5 = -0.093^\circ$. According to Table 6.3, $q_{4'}(M_S) = -0.168^\circ$. Hence,

$$\begin{aligned} F &= \bar{F}_M + q_1 + q_2 + q_3 + q_{4'} + q_5 \\ &= 63.062 + 6.244 - 1.267 + 0.118 - 0.168 - 0.093 = 67.926 \\ &\simeq 68^\circ. \end{aligned}$$

It follows from Table 6.4 that $\beta_{syz} = 4.784^\circ \simeq 4^\circ 47'$.

The criterion for an umbral lunar eclipse is particularly simple, because it is not complicated by lunar parallax. A *total lunar eclipse*, in which the Moon is completely immersed in the Earth's

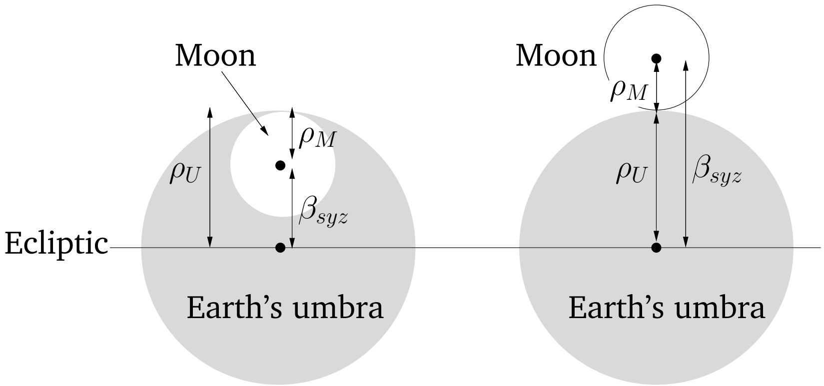

Figure 7.3: The limiting cases for a total lunar eclipse (left) and a partial lunar eclipse (right).

umbra, must take place at a full moon if $|\beta_{syz}| < \rho_U - \rho_M$ (see Figure 7.3), or equivalently

$$|\beta_{syz}| < \beta_{M\,t} \equiv \delta_M - \rho_M - \rho_S, \tag{7.11}$$

and either a total or a *partial lunar eclipse*, in which the Moon is only partially immersed in the Earth's umbra, must take place if $|\beta_{syz}| < \rho_U + \rho_M$ (see Figure 7.3), or equivalently

$$|\beta_{syz}| < \beta_M \equiv \delta_M + \rho_M - \rho_S. \tag{7.12}$$

Note that lunar eclipses are simultaneously visible at all observation sites on the Earth for which the Moon is above the horizon, because the Earth's umbra is larger than the Moon, and the relative position of the Moon and the Earth's umbra is not affected by parallax (because both the Moon and the umbra are the same distance from the Earth). The *magnitude* of a lunar eclipse is defined as the linear fraction of the Moon's diameter that lies in the Earth's umbra at maximum eclipse. It follows that

$$m = \frac{\rho_U + \rho_M - |\beta_{syz}|}{2\,\rho_M} = \frac{\beta_{M\,t} - |\beta_{syz}|}{\beta_M - \beta_{M\,t}}. \tag{7.13}$$

The criterion for a solar eclipse is modified by lunar parallax, which causes the angular position of the Moon relative to the Sun to shift by up to δ_M from its geocentric position. The amount of the shift depends on the observation site. However, a site can always be found at which the shift takes its maximum value in any particular direction. Note that the Sun has negligible parallax, because it is much farther away from the Earth than the Moon. Taking parallactic shifts into account, a *total solar eclipse*, in which the Sun is totally obscured by the Moon, must take place if $\rho_M > \rho_S$ and

$$|\beta_{syz}| < \beta_{S\,t} \equiv \delta_M + \rho_M - \rho_S, \tag{7.14}$$

an *annular solar eclipse*, in which all of the Sun apart from a thin outer ring is obscured by the Moon, must take place if $\rho_S > \rho_M$ and

$$|\beta_{syz}| < \beta_{S\,a} \equiv \delta_M - \rho_M + \rho_S, \tag{7.15}$$

and either a total, an annular, or a *partial solar eclipse*, in which the Sun is only partially obscured by the Moon, must take place if

$$|\beta_{syz}| < \beta_S \equiv \delta_M + \rho_M + \rho_S. \tag{7.16}$$

As a consequence of lunar parallax, and the fact that the angular sizes of the Sun and Moon in the sky are very similar, solar eclipses are only visible in very localized regions of the Earth. The magnitude of a solar eclipse is defined as the linear fraction of the Sun's diameter that is obscured by the Moon at maximum eclipse. It follows that

$$m = \frac{\rho_M}{\rho_S} = \frac{\beta_S - \beta_{S\,a}}{\beta_S - \beta_{S\,t}} \tag{7.17}$$

for a total or annular eclipse, and

$$m = \frac{\delta_M + \rho_S + \rho_M - |\beta_{syz}|}{2\,\rho_S} = \frac{\beta_S - |\beta_{syz}|}{\beta_S - \beta_{S\,t}}. \tag{7.18}$$

for a partial eclipse.

Note that the previous criteria represent necessary, but not sufficient, conditions for the occurrence of the various eclipses with which they are associated. This is the case because the point of closest approach of the Moon and the Earth's umbra, in the case of a lunar eclipse, and the Moon and Sun, in the case of a solar eclipse, does not necessarily occur exactly at the syzygy, due to the inclination of the Moon's orbit to the ecliptic. However, because the said inclination is fairly gentle, the previous criteria turn out to be very accurate predictors of eclipses.

7.6 Example eclipse calculations

Let us use our model to examine the lunar-solar syzygies of the year 1992 AD in order to see whether any of them were associated with solar or lunar eclipses. The following table shows the dates and times of the new moons of 1992 AD, calculated using the method described in Section 7.3. Also shown is the magnitude of the Moon's ecliptic latitude at each syzygy, $|\beta_{syz}|$, calculated from Equation (7.10) and Table 6.4, as well as the critical values of this parameter for a general, total, and annular solar eclipse. The latter are calculated from Equations (7.14)–(7.16), with the aid of Table 6.5. The eclipse magnitude is also shown. It can be seen that the criterion for a total solar eclipse (that is, $|\beta_{syz}| < \beta_{St}$ and $\beta_{St} > \beta_{Sa}$) is satisfied for the syzygy marked with a T, the criterion for an annular solar eclipse (that is, $|\beta_{syz}| < \beta_{Sa}$ and $\beta_{Sa} > \beta_{St}$) for the syzygy marked with an A, and the criterion for a partial solar eclipse (that is, $\beta_{St}, \beta_{Sa} < |\beta_{syz}| < \beta_S$) for the syzygy marked with a P. It is easily verified that a total solar eclipse, an annular solar eclipse, and a partial solar eclipse did indeed take place in 1992 AD at the dates and times indicated.

Date	Time (UT)	$\beta_S(')$	$\beta_{St}(')$	$\beta_{Sa}(')$	$\lvert\beta_{syz}\rvert(')$	Type	m
04/01/1992	23:15	85.0	52.5	55.5	022.9	A	0.91
03/02/1992	18:58	84.9	52.5	55.5	181.5		
04/03/1992	13:14	85.7	53.5	55.9	284.8		
03/04/1992	04:52	87.2	55.2	56.7	304.0		
02/05/1992	17:39	89.1	57.3	57.7	238.1		
01/06/1992	03:55	91.0	59.5	58.8	109.3		
30/06/1992	12:18	92.7	61.2	59.7	046.8	T	1.05
29/07/1992	19:35	93.7	62.2	60.3	190.4		
28/08/1992	02:43	93.8	62.2	60.4	285.2		
26/09/1992	10:44	93.1	61.2	60.0	304.6		
25/10/1992	20:38	91.5	59.3	59.2	240.0		
24/11/1992	09:14	89.5	57.1	58.1	105.4		
24/12/1992	00:49	87.4	54.9	56.9	061.9	P	0.78

The following table shows the dates and times of the full moons of 1992 AD. Also shown is the magnitude of the Moon's ecliptic latitude at each syzygy, as well as the critical values of this parameter for a general and a total lunar eclipse. The latter are calculated from Equations (7.11) and (7.12), with the aid of Table 6.5. The eclipse magnitude is also shown.. It can be seen that the criterion for a total lunar eclipse (that is, $|\beta_{syz}| < \beta_{Mt}$) is satisfied for the syzygy marked with a T, whereas the criterion for a partial lunar eclipse (that is, $\beta_{Mt} < |\beta_{syz}| < \beta_M$) is satisfied for the syzygy marked with a P. It is easily verified that a total lunar eclipse, and a partial lunar eclipse did indeed take place in 1992 AD at the dates and times indicated.

Date	Time (UT)	$\beta_M(')$	$\beta_{Mt}(')$	$\lvert\beta_{syz}\rvert(')$	Type	m
19/01/1992	21:25	61.9	28.4	106.4		
18/02/1992	07:51	61.4	28.2	241.2		
18/03/1992	18:07	60.1	27.5	305.2		
17/04/1992	04:43	58.2	26.5	282.5		
16/05/1992	16:14	56.3	25.4	183.0		
15/06/1992	05:06	54.6	24.5	035.6	P	0.63
14/07/1992	19:18	53.4	23.8	120.8		
13/08/1992	10:26	52.9	23.5	245.8		
12/09/1992	02:02	53.1	23.6	305.6		
11/10/1992	17:49	54.1	24.1	281.2		
10/11/1992	09:17	55.8	25.0	176.0		
09/12/1992	23:47	57.9	26.1	018.3	T	1.25
08/01/1993	12:43	59.9	27.3	145.4		

7.7 Eclipse statistics

Consider a very large collection of lunar-solar syzygies. For such a collection, we expect the lunar argument of latitude, F, the lunar mean anomaly, M_M, and the solar mean anomaly, M_S, to be statistically independent of one another, and randomly distributed in the range 0° to 360°. Using this insight, we can easily calculate the probability that a new moon is coincident with a solar eclipse, or a full moon with a lunar eclipse, using Equation (6.12) and the criteria (7.11), (7.12), and (7.14)–(7.16).

For a new moon we find:

Probability of total solar eclipse:	4.2%
Probability of annular solar eclipse:	7.7%
Probability of partial solar eclipse:	6.6%
Probability of any solar eclipse:	18.5%

For a full moon we get:

Probability of total lunar eclipse:	5.2%
Probability of partial lunar eclipse:	6.5%
Probability of any lunar eclipse:	11.7%

Thus, we can see that, over a long period of time, the ratio of the number of total/annular solar eclipses to the number of partial solar eclipses is about 9/5, whereas the ratio of the number of partial lunar eclipses to the number of total lunar eclipses is approximately 5/4. Furthermore, the ratio of the number of solar eclipses to the number of lunar eclipses is about 11/7. Because there are 12.37 synodic months in a year, the mean number of solar eclipses per year is approximately $12.37 \times 0.185 \simeq 2.3$, whereas the mean number of lunar eclipses per year is about $12.37 \times 0.117 \simeq 1.4$. Clearly, solar eclipses are more common that lunar eclipses. On the other hand, at a given observation site on the Earth, lunar eclipses are much more common than solar eclipses, because the former are visible over all regions of the Earth for which the Moon is above the horizon, whereas the latter are only visible in a very localized region.

7.8 Eclipse cycles

Now, 223 synodic months corresponds to 6585.413 days, which corresponds to 238.99 anomalistic months and 242.00 draconic months. In other words, 223 synodic months is almost exactly equal to 239 anomalistic months and 242 draconic months. This implies that if a solar eclipse takes place at a given new moon then, 223 new moons later, the Moon's perigee and ascending node will be found in almost the exact same positions, which suggests that the Moon's ecliptic latitude, as well as β_S, β_{St}, and β_{Sa}, will take almost exactly the same values. Hence, another solar eclipse is almost certain to happen 223 new moons after a given eclipse. The same reasoning leads to the conclusion that if a lunar eclipse takes place at a given full moon then another eclipse is almost certain to occur 223 full moons later. It can easily be demonstrated that there is no number of synodic months less than 233 that corresponds to near integer numbers of both anomalistic and draconic months. Hence, the time period of 223 synodic months, which is (mistakenly) called the *saros*, is the minimum (almost) guaranteed period on which solar and lunar eclipses repeat. Incidentally, Ptolemy, in Section 2 of Book IV of the Almagest, states that the near equality of 223 synodic months to 239 anomalistic months, as well as to 245 draconic months, was known to very ancient astronomers, which probably implies that the equality was discovered by Babylonian astronomers in the 7th or 6th century BC.

The following table gives details of a series of solar eclipses that take place at regular intervals of 223 new moons. This particular series, which is known as saros cycle 146, started in 1541 AD and will end in 2893 AD.

Date	Time (UT)	$\beta_S(')$	$\beta_{St}(')$	$\beta_{Sa}(')$	$\lvert\beta_{syz}\rvert(')$	Type	m
03/04/1848	22:53	93.9	61.9	60.5	76.3	P	0.55
15/04/1866	06:56	93.8	61.9	60.4	73.6	P	0.63
25/04/1884	14:51	93.6	61.9	60.3	70.6	P	0.73
07/05/1902	22:39	93.5	61.8	60.2	67.2	P	0.83
18/05/1920	06:20	93.3	61.7	60.1	63.5	P	0.94
29/05/1938	13:56	93.2	61.6	60.0	59.6	T	1.05
08/06/1956	21:27	93.0	61.5	59.9	55.5	T	1.05
20/06/1974	04:53	92.8	61.4	59.8	51.2	T	1.05
30/06/1992	12:18	92.7	61.2	59.7	46.8	T	1.05
11/07/2010	19:40	92.5	61.0	59.6	42.3	T	1.05
22/07/2028	03:03	92.3	60.8	59.5	37.9	T	1.04
02/08/2046	10:26	92.1	60.6	59.4	33.6	T	1.04
12/08/2064	17:52	91.9	60.4	59.3	29.5	T	1.03
24/08/2082	01:21	91.7	60.1	59.2	25.5	T	1.03
04/09/2100	08:54	91.5	59.8	59.1	21.8	T	1.02
15/09/2118	16:32	91.4	59.6	59.0	18.4	T	1.02
26/09/2136	00:17	91.2	59.3	58.9	15.3	T	1.01
07/10/2154	08:08	91.0	59.0	58.9	12.6	T	1.01

Likewise, the following table gives details of a series of lunar eclipses that take place at regular intervals of 223 full moons. This particular series, which is known as saros cycle 125, started in 1163 AD and will end in 2443 AD.

Date	Time (UT)	$\beta_M(')$	$\beta_{Mt}(')$	$\lvert\beta_{syz}\rvert(')$	Type	m
27/10/1920	14:17	58.9	26.7	13.9	T	1.40
07/11/1938	22:32	58.6	26.6	15.4	T	1.35
18/11/1956	06:52	58.4	26.4	16.6	T	1.31
29/11/1974	15:18	58.1	26.3	17.6	T	1.27
09/12/1992	23:47	57.9	26.1	18.3	T	1.25
21/12/2010	08:19	57.7	26.0	18.8	T	1.23
31/12/2028	16:52	57.5	25.9	19.3	T	1.21
12/01/2047	01:25	57.3	25.8	19.7	T	1.19
22/01/2065	09:55	57.1	25.7	20.2	T	1.17
02/02/2083	18:22	56.9	25.6	20.9	T	1.15
14/02/2101	02:43	56.7	25.5	21.7	T	1.12
25/02/2119	10:57	56.6	25.4	22.8	T	1.08
07/03/2137	19:04	56.4	25.3	24.2	T	1.04
19/03/2155	03:01	56.2	25.2	26.0	P	0.98
29/03/2173	10:49	56.1	25.2	28.1	P	0.91
09/04/2191	18:28	55.9	25.1	30.6	P	0.82

7.9 *Tables*

Δt(JD)	$\Delta\bar{D}(^\circ)$	$\Delta\bar{F}_M(^\circ)$	$\Delta M_S(^\circ)$	$\Delta M_M(^\circ)$	Δt(JD)	$\Delta\bar{D}(^\circ)$	$\Delta\bar{F}_M(^\circ)$	$\Delta M_S(^\circ)$	$\Delta M_M(^\circ)$
10 000	227.491	173.503	136.002	329.930	1 000	310.749	269.350	265.600	104.993
20 000	94.982	347.005	272.005	299.859	2 000	261.498	178.701	171.200	209.986
30 000	322.473	160.508	48.007	269.788	3 000	212.247	88.051	76.801	314.979
40 000	189.964	334.011	184.010	239.718	4 000	162.996	357.401	342.401	59.972
50 000	57.455	147.513	320.012	209.648	5 000	113.746	266.751	248.001	164.965
60 000	284.947	321.016	96.015	179.577	6 000	64.495	176.102	153.601	269.958
70 000	152.438	134.519	232.017	149.506	7 000	15.244	85.452	59.202	14.951
80 000	19.929	308.022	8.020	119.436	8 000	325.993	354.802	324.802	119.944
90 000	247.420	121.524	144.022	89.366	9 000	276.742	264.152	230.402	224.937
100	139.075	242.935	98.560	226.499	10	121.907	132.294	9.856	130.650
200	278.150	125.870	197.120	92.999	20	243.815	264.587	19.712	261.300
300	57.225	8.805	295.680	319.498	30	5.722	36.881	29.568	31.950
400	196.300	251.740	34.240	185.997	40	127.630	169.174	39.424	162.600
500	335.375	134.675	132.800	52.496	50	249.537	301.468	49.280	293.250
600	114.449	17.610	231.360	278.996	60	11.445	73.761	59.136	63.900
700	253.524	260.545	329.920	145.495	70	133.352	206.055	68.992	194.550
800	32.599	143.480	68.480	11.994	80	255.260	338.348	78.848	325.199
900	171.674	26.415	167.040	238.494	90	17.167	110.642	88.704	95.849
1	12.191	13.229	0.986	13.065	0.1	1.219	1.323	0.099	1.306
2	24.381	26.459	1.971	26.130	0.2	2.438	2.646	0.197	2.613
3	36.572	39.688	2.957	39.195	0.3	3.657	3.969	0.296	3.919
4	48.763	52.917	3.942	52.260	0.4	4.876	5.292	0.394	5.226
5	60.954	66.147	4.928	65.325	0.5	6.095	6.615	0.493	6.532
6	73.144	79.376	5.914	78.390	0.6	7.314	7.938	0.591	7.839
7	85.335	92.605	6.899	91.455	0.7	8.534	9.261	0.690	9.145
8	97.526	105.835	7.885	104.520	0.8	9.753	10.583	0.788	10.452
9	109.717	119.064	8.870	117.585	0.9	10.972	11.906	0.887	11.758

Table 7.1: Mean motion of the lunar-solar elongation. Here, $\Delta t = t - t_0$, $\Delta\bar{D} = \bar{D} - \bar{D}_0$, $\Delta\bar{F}_M = \bar{F}_M - \bar{F}_{M\,0}$, $\Delta M_S = M_S - M_{S\,0}$, and $\Delta M_M = M_M - M_{M\,0}$. At epoch ($t_0 = 2\,451\,545.0$ JD), $\bar{D}_0 = 297.864^\circ$, $\bar{F}_{M\,0} = 93.284^\circ$, $M_{S\,0} = 357.588^\circ$, and $M_{M\,0} = 134.916^\circ$.

Arg. (°)	$q_1(^\circ)$	$q_2(^\circ)$	$q_3(^\circ)$	$q_4(^\circ)$	$q_5(^\circ)$
000(360)	$+0.000$	$+0.000$	$+0.000$	-0.000	-0.000
002(358)	$+0.234$	$+0.044$	$+0.045$	-0.075	-0.004
004(356)	$+0.468$	$+0.089$	$+0.089$	-0.149	-0.008
006(354)	$+0.702$	$+0.133$	$+0.133$	-0.224	-0.012
008(352)	$+0.934$	$+0.177$	$+0.177$	-0.298	-0.016
010(350)	$+1.165$	$+0.221$	$+0.219$	-0.372	-0.020
012(348)	$+1.394$	$+0.265$	$+0.261$	-0.445	-0.024
014(346)	$+1.622$	$+0.308$	$+0.301$	-0.517	-0.028
016(344)	$+1.847$	$+0.351$	$+0.339$	-0.589	-0.032
018(342)	$+2.069$	$+0.394$	$+0.376$	-0.661	-0.035
020(340)	$+2.288$	$+0.436$	$+0.411$	-0.731	-0.039
022(338)	$+2.504$	$+0.477$	$+0.444$	-0.801	-0.043
024(336)	$+2.717$	$+0.518$	$+0.475$	-0.869	-0.047
026(334)	$+2.925$	$+0.559$	$+0.504$	-0.936	-0.050
028(332)	$+3.130$	$+0.598$	$+0.530$	-1.003	-0.054
030(330)	$+3.329$	$+0.637$	$+0.553$	-1.067	-0.057
032(328)	$+3.525$	$+0.675$	$+0.573$	-1.131	-0.061
034(326)	$+3.715$	$+0.712$	$+0.591$	-1.193	-0.064
036(324)	$+3.900$	$+0.749$	$+0.606$	-1.253	-0.067
038(322)	$+4.079$	$+0.784$	$+0.618$	-1.312	-0.070
040(320)	$+4.253$	$+0.819$	$+0.626$	-1.370	-0.074
042(318)	$+4.421$	$+0.853$	$+0.632$	-1.425	-0.077
044(316)	$+4.582$	$+0.885$	$+0.634$	-1.479	-0.080
046(314)	$+4.737$	$+0.917$	$+0.633$	-1.531	-0.082
048(312)	$+4.886$	$+0.947$	$+0.629$	-1.581	-0.085
050(310)	$+5.028$	$+0.976$	$+0.622$	-1.629	-0.088
052(308)	$+5.163$	$+1.004$	$+0.612$	-1.674	-0.090
054(306)	$+5.291$	$+1.031$	$+0.598$	-1.718	-0.093
056(304)	$+5.412$	$+1.056$	$+0.582$	-1.760	-0.095
058(302)	$+5.525$	$+1.081$	$+0.562$	-1.799	-0.097
060(300)	$+5.631$	$+1.103$	$+0.540$	-1.836	-0.099

Arg. (°)	$q_1(^\circ)$	$q_2(^\circ)$	$q_3(^\circ)$	$q_4(^\circ)$	$q_5(^\circ)$
060(300)	$+5.631$	$+1.103$	$+0.540$	-1.836	-0.099
062(298)	$+5.730$	$+1.125$	$+0.515$	-1.871	-0.101
064(296)	$+5.821$	$+1.145$	$+0.488$	-1.903	-0.103
066(294)	$+5.904$	$+1.164$	$+0.458$	-1.933	-0.105
068(292)	$+5.979$	$+1.181$	$+0.425$	-1.961	-0.106
070(290)	$+6.047$	$+1.197$	$+0.391$	-1.986	-0.108
072(288)	$+6.107$	$+1.212$	$+0.354$	-2.009	-0.109
074(286)	$+6.158$	$+1.225$	$+0.316$	-2.029	-0.110
076(284)	$+6.202$	$+1.236$	$+0.275$	-2.047	-0.111
078(282)	$+6.238$	$+1.246$	$+0.234$	-2.062	-0.112
080(280)	$+6.266$	$+1.255$	$+0.191$	-2.075	-0.113
082(278)	$+6.287$	$+1.262$	$+0.147$	-2.085	-0.113
084(276)	$+6.299$	$+1.267$	$+0.102$	-2.093	-0.114
086(274)	$+6.303$	$+1.271$	$+0.057$	-2.098	-0.114
088(272)	$+6.300$	$+1.273$	$+0.011$	-2.100	-0.114
090(270)	$+6.289$	$+1.274$	-0.035	-2.100	-0.114
092(268)	$+6.270$	$+1.273$	-0.081	-2.097	-0.114
094(266)	$+6.244$	$+1.271$	-0.126	-2.092	-0.114
096(264)	$+6.210$	$+1.267$	-0.171	-2.084	-0.114
098(262)	$+6.169$	$+1.262$	-0.216	-2.074	-0.113
100(260)	$+6.120$	$+1.255$	-0.259	-2.061	-0.113
102(258)	$+6.065$	$+1.246$	-0.302	-2.046	-0.112
104(256)	$+6.002$	$+1.236$	-0.343	-2.028	-0.111
106(254)	$+5.932$	$+1.225$	-0.382	-2.008	-0.110
108(252)	$+5.856$	$+1.212$	-0.420	-1.986	-0.109
110(250)	$+5.772$	$+1.197$	-0.456	-1.961	-0.108
112(248)	$+5.683$	$+1.181$	-0.490	-1.933	-0.106
114(246)	$+5.586$	$+1.164$	-0.521	-1.904	-0.105
116(244)	$+5.484$	$+1.145$	-0.550	-1.872	-0.103
118(242)	$+5.376$	$+1.125$	-0.577	-1.838	-0.101
120(240)	$+5.261$	$+1.103$	-0.600	-1.801	-0.099

Arg. (°)	$q_1(^\circ)$	$q_2(^\circ)$	$q_3(^\circ)$	$q_4(^\circ)$	$q_5(^\circ)$
120(240)	$+5.261$	$+1.103$	-0.600	-1.801	-0.099
122(238)	$+5.141$	$+1.081$	-0.621	-1.763	-0.097
124(236)	$+5.016$	$+1.056$	-0.639	-1.723	-0.095
126(234)	$+4.885$	$+1.031$	-0.654	-1.680	-0.093
128(232)	$+4.748$	$+1.004$	-0.666	-1.636	-0.090
130(230)	$+4.607$	$+0.976$	-0.675	-1.589	-0.088
132(228)	$+4.461$	$+0.947$	-0.681	-1.541	-0.085
134(226)	$+4.310$	$+0.917$	-0.683	-1.491	-0.082
136(224)	$+4.155$	$+0.885$	-0.682	-1.439	-0.080
138(222)	$+3.996$	$+0.853$	-0.678	-1.385	-0.077
140(220)	$+3.832$	$+0.819$	-0.671	-1.330	-0.074
142(218)	$+3.665$	$+0.784$	-0.660	-1.274	-0.070
144(216)	$+3.493$	$+0.749$	-0.647	-1.215	-0.067
146(214)	$+3.319$	$+0.712$	-0.630	-1.156	-0.064
148(212)	$+3.141$	$+0.675$	-0.610	-1.095	-0.061
150(210)	$+2.959$	$+0.637$	-0.588	-1.033	-0.057
152(208)	$+2.775$	$+0.598$	-0.562	-0.969	-0.054
154(206)	$+2.589$	$+0.559$	-0.534	-0.905	-0.050
156(204)	$+2.399$	$+0.518$	-0.503	-0.839	-0.047
158(202)	$+2.207$	$+0.477$	-0.470	-0.773	-0.043
160(200)	$+2.014$	$+0.436$	-0.435	-0.705	-0.039
162(198)	$+1.818$	$+0.394$	-0.398	-0.637	-0.035
164(196)	$+1.620$	$+0.351$	-0.358	-0.568	-0.032
166(194)	$+1.421$	$+0.308$	-0.318	-0.499	-0.028
168(192)	$+1.221$	$+0.265$	-0.275	-0.429	-0.024
170(190)	$+1.019$	$+0.221$	-0.231	-0.358	-0.020
172(188)	$+0.816$	$+0.177$	-0.186	-0.287	-0.016
174(186)	$+0.613$	$+0.133$	-0.141	-0.215	-0.012
176(184)	$+0.409$	$+0.089$	-0.094	-0.144	-0.008
178(182)	$+0.205$	$+0.044$	-0.047	-0.072	-0.004
180(180)	$+0.000$	$+0.000$	-0.000	-0.000	-0.000

Table 7.2: Anomalies of the lunar-solar elongation. The common argument corresponds to M_M, $2\bar{D} - M_M$, $\bar{D}$, M_S, and $2\bar{F}_M$ for the case of q_1, q_2, q_3, q_4, and q_5, respectively. If the argument is in parentheses then the anomalies are minus the values shown in the table.

Date	JD	Date	JD	Date	JD	Date	JD
01/01/1900	2415021.08	18/01/1950	2433299.83	06/01/2000	2451550.25	23/01/2050	2469829.71
20/01/1901	2415405.11	07/01/1951	2433654.34	24/01/2001	2451934.05	12/01/2051	2470184.29
09/01/1902	2415759.38	26/01/1952	2434038.43	13/01/2002	2452288.07	02/01/2052	2470538.62
28/01/1903	2416143.19	15/01/1953	2434393.09	02/01/2003	2452642.35	19/01/2053	2470922.46
17/01/1904	2416497.16	05/01/1954	2434747.60	21/01/2004	2453026.37	08/01/2054	2471276.44
05/01/1905	2416851.27	24/01/1955	2435131.54	10/01/2005	2453381.00	27/01/2055	2471660.24
24/01/1906	2417235.21	13/01/1956	2435485.62	29/01/2006	2453765.10	16/01/2056	2472014.43
14/01/1907	2417589.74	30/01/1957	2435869.40	19/01/2007	2454119.67	05/01/2057	2472368.91
03/01/1908	2417944.41	19/01/1958	2436223.43	08/01/2008	2454473.98	24/01/2058	2472753.00
22/01/1909	2418328.51	09/01/1959	2436577.73	26/01/2009	2454857.82	14/01/2059	2473107.67
11/01/1910	2418682.99	28/01/1960	2436961.76	15/01/2010	2455211.81	03/01/2060	2473462.20
30/01/1911	2419066.90	16/01/1961	2437316.40	04/01/2011	2455565.88	21/01/2061	2473846.13
19/01/1912	2419420.96	06/01/1962	2437671.03	23/01/2012	2455949.82	10/01/2062	2474200.24
07/01/1913	2419774.94	25/01/1963	2438055.07	11/01/2013	2456304.31	29/01/2063	2474584.02
26/01/1914	2420158.78	14/01/1964	2438409.36	30/01/2014	2456688.40	18/01/2064	2474938.03
15/01/1915	2420513.11	02/01/1965	2438763.38	20/01/2015	2457043.06	06/01/2065	2475292.30
05/01/1916	2420867.70	21/01/1966	2439147.16	10/01/2016	2457397.57	25/01/2066	2475676.34
23/01/1917	2421251.82	10/01/1967	2439501.26	28/01/2017	2457781.50	15/01/2067	2476030.97
12/01/1918	2421606.44	29/01/1968	2439885.19	17/01/2018	2458135.59	05/01/2068	2476385.61
02/01/1919	2421960.85	18/01/1969	2440239.70	06/01/2019	2458489.57	23/01/2069	2476769.65
21/01/1920	2422344.72	07/01/1970	2440594.36	24/01/2020	2458873.41	12/01/2070	2477123.97
09/01/1921	2422698.73	26/01/1971	2440978.46	13/01/2021	2459227.70	01/01/2071	2477478.01
27/01/1922	2423082.50	16/01/1972	2441332.95	02/01/2022	2459582.27	20/01/2072	2477861.78
17/01/1923	2423436.61	04/01/1973	2441687.15	21/01/2023	2459966.37	08/01/2073	2478215.85
06/01/1924	2423791.03	23/01/1974	2442070.96	11/01/2024	2460321.00	27/01/2074	2478599.78
24/01/1925	2424175.11	12/01/1975	2442424.94	29/01/2025	2460705.03	16/01/2075	2478954.27
14/01/1926	2424529.78	01/01/1976	2442779.11	18/01/2026	2461059.32	06/01/2076	2479308.93
03/01/1927	2424884.36	19/01/1977	2443163.09	07/01/2027	2461413.35	24/01/2077	2479693.03
22/01/1928	2425268.34	09/01/1978	2443517.66	26/01/2028	2461797.14	14/01/2078	2480047.55
11/01/1929	2425622.51	28/01/1979	2443901.77	14/01/2029	2462151.23	03/01/2079	2480401.78
29/01/1930	2426006.29	17/01/1980	2444256.40	04/01/2030	2462505.61	22/01/2080	2480785.58
18/01/1931	2426360.28	06/01/1981	2444610.81	23/01/2031	2462889.68	10/01/2081	2481139.55
07/01/1932	2426714.48	25/01/1982	2444994.70	12/01/2032	2463244.34	28/01/2082	2481523.37
25/01/1933	2427098.47	14/01/1983	2445348.72	30/01/2033	2463628.42	18/01/2083	2481877.66
15/01/1934	2427453.07	03/01/1984	2445702.73	20/01/2034	2463982.92	07/01/2084	2482232.22
05/01/1935	2427807.73	21/01/1985	2446086.61	09/01/2035	2464337.12	25/01/2085	2482616.34
24/01/1936	2428191.81	10/01/1986	2446441.01	28/01/2036	2464720.93	15/01/2086	2482970.98
12/01/1937	2428546.19	29/01/1987	2446825.07	16/01/2037	2465074.91	04/01/2087	2483325.42
01/01/1938	2428900.29	19/01/1988	2447179.73	05/01/2038	2465429.07	23/01/2088	2483709.31
20/01/1939	2429284.06	07/01/1989	2447534.31	24/01/2039	2465813.06	11/01/2089	2484063.34
09/01/1940	2429638.08	26/01/1990	2447918.31	14/01/2040	2466167.64	30/01/2090	2484447.11
27/01/1941	2430021.96	15/01/1991	2448272.49	02/01/2041	2466522.30	19/01/2091	2484801.19
16/01/1942	2430376.39	04/01/1992	2448626.47	21/01/2042	2466906.37	09/01/2092	2485155.57
06/01/1943	2430731.03	22/01/1993	2449010.28	11/01/2043	2467260.78	27/01/2093	2485539.64
25/01/1944	2431115.14	11/01/1994	2449364.46	30/01/2044	2467644.66	16/01/2094	2485894.30
14/01/1945	2431469.72	30/01/1995	2449748.45	18/01/2045	2467998.68	06/01/2095	2486248.90
03/01/1946	2431824.01	20/01/1996	2450103.03	07/01/2046	2468352.69	25/01/2096	2486632.91
22/01/1947	2432207.85	09/01/1997	2450457.69	26/01/2047	2468736.58	13/01/2097	2486987.12
11/01/1948	2432561.83	28/01/1998	2450841.76	15/01/2048	2469090.97	02/01/2098	2487341.11
29/01/1949	2432945.62	17/01/1999	2451196.16	04/01/2049	2469445.60	21/01/2099	2487724.89

Table 7.3: Dates and fractional Julian day numbers of the first new moons of the years 1900–2099 AD.

8. The superior planets

8.1 Planetary ecliptic longitude model

Figure 8.1 compares and contrasts heliocentric and geocentric models of the motion of a superior planet (that is, a planet that is farther from the Sun than the Earth), P, as seen from the Earth, G. The Sun is at S. In the heliocentric model, we can write the Earth-planet displacement vector, **P**, as the sum of the Earth-Sun displacement vector, **S**, and the Sun-planet displacement vector, $\mathbf{P}'$. The geocentric model, which is entirely equivalent to the heliocentric model as far as the relative motion, of the planet with respect to the Earth is concerned, and is much more convenient, relies on the simple vector identity

$$\mathbf{P} = \mathbf{S} + \mathbf{P}' \equiv \mathbf{P}' + \mathbf{S}. \tag{8.1}$$

In other words, we can get from the Earth to the planet by one of two different routes. The first route corresponds to the heliocentric model, and the second to the geocentric model. In the latter model, $\mathbf{P}'$ gives the displacement of the so-called *guide-point*, G', from the Earth. Because $\mathbf{P}'$ is also the displacement of the planet, P, from the Sun, S, it is clear that G' executes a Keplerian orbit about the Earth whose elements are the same as those of the orbit of the planet about the Sun. The ellipse traced out by G' is termed the *deferent*. The vector **S** gives the displacement of the planet from the guide-point. However, **S** is also the displacement of the Sun from the Earth. Hence, it is clear that the planet, P, executes a Keplerian orbit about the guide-point, G', whose elements are the same as the Sun's apparent orbit about the Earth. The ellipse traced out by P about G' is termed the *epicycle*.

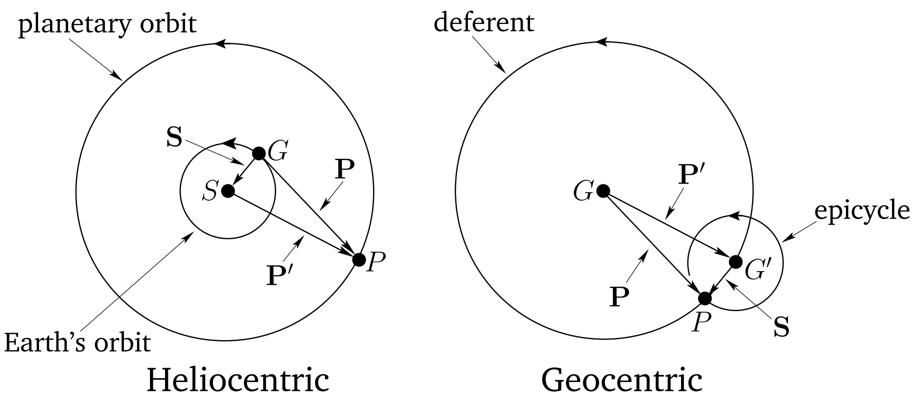

Figure 8.1: Heliocentric and geocentric models of the motion of a superior planet. Here, S is the Sun, G the Earth, and P the planet. The view is from the northern ecliptic pole.

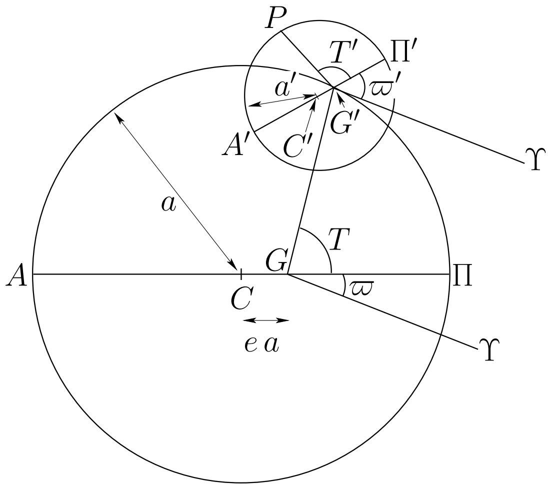

Figure 8.2: Planetary longitude model. View is from northern ecliptic pole.

Figure 8.2 illustrates in more detail how the deferent-epicycle model is used to determine the ecliptic longitude of a superior planet. The planet P orbits (counterclockwise) on a small Keplerian orbit $\Pi' P A'$ about guide-point G', which, in turn, orbits the Earth, G, (counterclockwise) on a large Keplerian orbit $\Pi G' A$. As has already been mentioned, the small orbit is termed the epicycle, and the large orbit the deferent. Both orbits are assumed to lie in the plane of the ecliptic. This approximation does not introduce a large error into our calculations because the orbital inclinations of the visible planets to the ecliptic plane are all fairly small. Let C, A, Π, a, e, ϖ, and T denote the geometric center, apocenter (that is, the point of farthest distance from the central object), pericenter (that is, the point of closest approach to the central object), major radius, eccentricity, longitude of the pericenter, and true anomaly of the deferent, respectively. Let C', A', Π', a', e', ϖ', and T' denote the corresponding quantities for the epicycle.

Let the line GG' be produced, and let the perpendicular PB be dropped to it from P, as shown in Figure 8.3. The angle $\mu \equiv PG'B$ is termed the *epicyclic anomaly* (see Figures 8.4), and takes the form

$$\mu = T' + \varpi' - T - \varpi = \bar{\lambda}' + q' - \bar{\lambda} - q, \tag{8.2}$$

where $\bar{\lambda}$ and q are the mean longitude and equation of center for the deferent, whereas $\bar{\lambda}'$ and q' are the corresponding quantities for the epicycle. See Chapter 5. The epicyclic anomaly is generally written in the range 0° to 360°. The angle $\theta \equiv PGG'$ is termed the *equation of the epicycle*, and is usually written in the range -180° to $+180^\circ$. It is clear from the figure that

$$\tan\theta = \frac{\sin\mu}{r/r' + \cos\mu}, \tag{8.3}$$

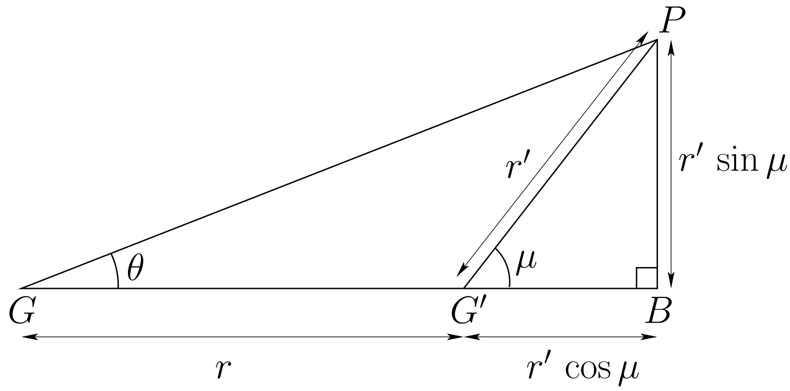

Figure 8.3: The triangle GBP.

where $r \equiv GG'$ and $r' \equiv G'P$ are the radial polar coordinates for the deferent and epicycle, respectively. Moreover, according to Equation (4.22), $r/r' = (a/a')\,z$, where

$$z = \frac{1-\zeta}{1-\zeta'}, \tag{8.4}$$

and

$$\zeta = e\,\cos\,M - e^{2}\,\sin^{2}\,M, \tag{8.5}$$

$$\zeta' = e'\cos\,M' - e'^{\,2}\sin^{2}\,M' \tag{8.6}$$

are termed *radial anomalies*. Finally, the ecliptic longitude of the planet is given by (see Figure 8.4)

$$\lambda = \bar{\lambda} + q + \theta. \tag{8.7}$$

Now,

$$\theta(\mu, z) \equiv \tan^{-1}\left[\frac{\sin\mu}{(a/a')\,z + \cos\mu}\right] \tag{8.8}$$

is a function of two variables, μ and z. It is impractical to tabulate such a function directly. Fortunately, while $\theta(\mu, z)$ has a strong dependence on μ, it only has a fairly weak dependence on z. In fact, it is easily seen that z varies between $z_{\rm min} = \bar{z} - \delta z$ and $z_{\rm max} = \bar{z} + \delta z$, where

$$\bar{z} = \frac{1 + e\,e'}{1 - e'^{\,2}}, \tag{8.9}$$

$$\delta z = \frac{e + e'}{1 - e'^{\,2}}. \tag{8.10}$$

Let us define

$$\xi = \frac{\bar{z} - z}{\delta z}. \tag{8.11}$$

This variable takes the value -1 when $z = z_{\rm max}$, the value 0 when $z = \bar{z}$, and the value $+1$ when $z = z_{\rm min}$. Thus, using quadratic interpolation, we can write

$$\theta(\mu, z) \simeq \Theta_-(\xi)\,\delta\theta_-(\mu) + \bar{\theta}(\mu) + \Theta_+(\xi)\,\delta\theta_+(\mu), \tag{8.12}$$

where

$$\bar{\theta}(\mu) = \theta(\mu, \bar{z}), \tag{8.13}$$

$$\delta\theta_-(\mu) = \theta(\mu, \bar{z}) - \theta(\mu, z_{\rm max}), \tag{8.14}$$

$$\delta\theta_+(\mu) = \theta(\mu, z_{\rm min}) - \theta(\mu, \bar{z}), \tag{8.15}$$

and

$$\Theta_-(\xi) = -(1/2)\,\xi\,(\xi - 1), \tag{8.16}$$

$$\Theta_+(\xi) = +(1/2)\,\xi\,(\xi + 1). \tag{8.17}$$

This scheme allows us to avoid having to tabulate a two-dimensional function, while ensuring that the exact value of $\theta(\mu, z)$ is obtained when $z = \bar{z}$, $z_{\rm min}$, or $z_{\rm max}$. The previous interpolation scheme is very similar to that adopted by Ptolemy in the Almagest.

Our procedure for determining the ecliptic longitude of a superior planet is described in the following. It is assumed that the ecliptic longitude, λ_S, and the radial anomaly, ζ_S, of the Sun have already been calculated. The latter quantity is tabulated as a function of the solar mean anomaly in Table 5.4. In the following, a, e, n, $\tilde{n}$, $\bar{\lambda}_0$, and M_0 represent elements of the orbit of the planet in question about the Sun, and e_S represents the eccentricity of the Sun's apparent orbit about the Earth. (In general, the subscript S denotes the Sun.) In particular, a is the major radius of the planetary orbit in units in which the major radius of the Sun's apparent orbit about the Earth is unity. The requisite elements for all of the superior planets at the J2000 epoch ($t_0 = 2\,451\,545.0$ JD) are listed in Table 5.1. The ecliptic longitude of a superior planet is specified by the following formulae:

$$\bar{\lambda} = \bar{\lambda}_0 + n\,(t - t_0), \tag{8.18}$$

$$M = M_0 + \tilde{n}\,(t - t_0), \tag{8.19}$$

$$q = 2\,e\,\sin\,M + (5/4)\,e^2\,\sin\,2M, \tag{8.20}$$

$$\zeta = e\,\cos M - e^2\,\sin^2 M, \tag{8.21}$$

$$\mu = \lambda_S - \bar{\lambda} - q, \tag{8.22}$$

$$\bar{\theta} = \theta(\mu, \bar{z}) \equiv \tan^{-1}\left(\frac{\sin\mu}{a\,\bar{z} + \cos\mu}\right), \tag{8.23}$$

$$\delta\theta_- = \theta(\mu, \bar{z}) - \theta(\mu, z_{\rm max}), \tag{8.24}$$

$$\delta\theta_+ = \theta(\mu, z_{\rm min}) - \theta(\mu, \bar{z}), \tag{8.25}$$

$$z = \frac{1 - \zeta}{1 - \zeta_S}, \tag{8.26}$$

$$\xi = \frac{\bar{z} - z}{\delta z}, \tag{8.27}$$

$$\theta = \Theta_-(\xi)\,\delta\theta_- + \bar{\theta} + \Theta_+(\xi)\,\delta\theta_+, \tag{8.28}$$

$$\lambda = \bar{\lambda} + q + \theta. \tag{8.29}$$

Here, $\bar{z} = (1 + e\,e_S)/(1 - e_S^2)$, $\delta z = (e + e_S)/(1 - e_S^2)$, $z_{\rm min} = \bar{z} - \delta z$, and $z_{\rm max} = \bar{z} + \delta z$. The constants $\bar{z}$, δz, $z_{\rm min}$, and $z_{\rm max}$ for each of the superior planets are listed in Table 8.1. Finally, the functions $\Theta_\pm$ are tabulated in Table 8.2.

For the case of Mars, the previous formulae are capable of matching NASA ephemeris data during the years 1995–2006 AD with a mean error of $3'$ and a maximum error of $14'$. For the case of Jupiter, the mean error is $1.6'$ and the maximum error $4'$. Finally, for the case of Saturn, the mean error is $0.5'$ and the maximum error $1'$.

8.2 *Determination of the ecliptic longitude of Mars*

The ecliptic longitude of Mars can be determined with the aid of Tables 8.3–8.5. Table 8.3 allows the mean longitude, $\bar{\lambda}$, and the mean anomaly, M, of Mars to be calculated as functions of time. Next, Table 8.4 permits the equation of center, q, and the radial anomaly, ζ, to be determined as functions of the mean anomaly. Finally, Table 8.5 allows the quantities $\delta\theta_-$, $\bar{\theta}$, and $\delta\theta_+$ to be calculated as functions of the epicyclic anomaly, μ.

The procedure for using the tables is as follows:

1. Determine the fractional Julian day number, t, corresponding to the date and time at which the ecliptic longitude is to be calculated with the aid of Tables 3.1–3.3. Form $\Delta t = t - t_0$, where $t_0 = 2\,451\,545.0$ is the epoch.

2. Calculate the ecliptic longitude, λ_S, and radial anomaly, ζ_S, of the Sun using the procedure set out in Section 5.2.

3. Enter Table 8.3 with the digit for each power of 10 in Δt and take out the corresponding values of $\Delta\bar{\lambda}$ and ΔM. If Δt is negative then the corresponding values are also negative. The value of the mean longitude, $\bar{\lambda}$, is the sum of all the $\Delta\bar{\lambda}$ values plus the value of $\bar{\lambda}$ at the epoch. Likewise, the value of the mean anomaly, M, is the sum of all the ΔM values plus the value of M at the epoch. Add as many multiples of 360° to $\bar{\lambda}$ and M as is required to make them both fall in the range 0° to 360°. Round M to the nearest degree.

4. Enter Table 8.4 with the value of M and take out the corresponding value of the equation of center, q, and the radial anomaly, ζ. It is necessary to interpolate if M is odd.

5. Form the epicyclic anomaly, $\mu = \lambda_S - \bar{\lambda} - q$. Add as many multiples of 360° to μ as is required to make it fall in the range 0° to 360°. Round μ to the nearest degree.

6. Enter Table 8.5 with the value of μ and take out the corresponding values of $\delta\theta_-$, $\bar{\theta}$, and $\delta\theta_+$. If $\mu > 180^\circ$ then it is necessary to make use of the identities $\delta\theta_\pm(360^\circ - \mu) = -\delta\theta_\pm(\mu)$ and $\bar{\theta}(360^\circ - \mu) = -\bar{\theta}(\mu)$.

7. Form $z = (1 - \zeta)/(1 - \zeta_S)$.

8. Obtain the values of $\bar{z}$ and δz from Table 8.1. Form $\xi = (\bar{z} - z)/\delta z$.

9. Enter Table 8.2 with the value of ξ and take out the corresponding values of Θ_- and Θ_+. If $\xi < 0$ then it is necessary to use the identities $\Theta_+(\xi) = -\Theta_-(-\xi)$ and $\Theta_-(\xi) = -\Theta_+(-\xi)$.

10. Form the equation of the epicycle, $\theta = \Theta_-\,\delta\theta_- + \bar{\theta} + \Theta_+\,\delta\theta_+$.

11. The ecliptic longitude, λ, is the sum of the mean longitude, $\bar{\lambda}$, the equation of center, q, and the equation of the epicycle, θ. If necessary convert λ into an angle in the range 0° to 360°. The decimal fraction can be converted into arc minutes using Table 5.2. Round to the nearest arc minute. The final result can be written in terms of the signs of the zodiac using the table in Section 2.6.

Two examples of this procedure are given in the following section

8.3 Example martian ecliptic longitude calculations

Example 1: May 5, 2005 AD, 00:00 UT:

From Section 5.2, $t - t_0 = 1\,950.5$ JD, $\lambda_S = 44.602^\circ$, $M_S \simeq 120^\circ$. Hence, it follows from Table 5.4 that $\zeta_S(M_S) = -8.56 \times 10^{-3}$. Making use of Table 8.3, we find:

t(JD)	$\bar{\lambda}(^\circ)$	$M(^\circ)$
$+1000$	164.071	164.021
$+900$	111.664	111.619
$+50$	26.204	26.201
$+.5$	0.262	0.262
Epoch	355.460	19.388
	657.661	321.491
Modulus	297.661	321.491

Given that $M \simeq 321^\circ$, Table 8.4 yields

$$q(321^\circ) = -7.345^\circ, \qquad \zeta(321^\circ) = 6.912 \times 10^{-2}.$$

Thus,

$$\mu = \lambda_S - \bar{\lambda} - q = 44.602 - 297.661 + 7.345 = 114.286 \simeq 114^\circ,$$

where we have rounded the epicyclic anomaly to the nearest degree. It follows from Table 8.5 that

$$\delta\theta_-(114^\circ) = 3.853^\circ, \qquad \bar{\theta}(114^\circ) = 39.209^\circ, \qquad \delta\theta_+(114^\circ) = 4.612^\circ.$$

Now,

$$z = (1 - \zeta)/(1 - \zeta_S) = (1 - 6.912 \times 10^{-2})/(1 + 8.56 \times 10^{-3}) = 0.9230.$$

However, from Table 8.1, $\bar{z} = 1.00184$ and $\delta z = 0.11014$, so

$$\xi = (\bar{z} - z)/\delta z = (1.00184 - 0.9230)/0.11014 \simeq 0.72.$$

According to Table 8.2,

$$\Theta_-(0.72) = 0.101, \qquad \Theta_+(0.72) = 0.619,$$

so

$$\begin{aligned}\theta &= \Theta_-\,\delta\theta_- + \bar{\theta} + \Theta_+\,\delta\theta_+ \\ &= 0.101 \times 3.853 + 39.209 + 0.619 \times 4.612 \\ &= 42.453^\circ.\end{aligned}$$

Finally,

$$\lambda = \bar{\lambda} + q + \theta = 297.661 - 7.345 + 42.453 = 332.769 \simeq 332^\circ 46'.$$

Thus, the ecliptic longitude of Mars at 00:00 UT on May 5, 2005 AD was 2PI46.

Example 2: December 25, 1800 AD, 00:00 UT:

From Section 5.2, $t - t_0 = -72,690.5$ JD, $\lambda_S = 273.055^\circ$, $M_S \simeq 354^\circ$. Hence, it follows from Table 5.4 that $\zeta_S(M_S) = 1.662 \times 10^{-2}$. Making use of Table 8.3, we find:

t(JD)	$\bar{\lambda}(^\circ)$	$M(^\circ)$
-70,000	-324.983	-321.453
-2,000	-328.142	-328.042
-600	-314.443	-314.412
-90	-47.166	-47.162
-.5	-0.262	-0.262
Epoch	355.460	19.388
	-659.536	-991.943
Modulus	60.464	88.057

Given that $M \simeq 88^\circ$, Table 8.4 yields

$$q(88^\circ) = 10.739^\circ, \qquad \zeta(88^\circ) = -5.45 \times 10^{-3},$$

so

$$\mu = \lambda_S - \bar{\lambda} - q = 273.055 - 60.464 - 10.739 = 201.852 \simeq 202^\circ.$$

It follows from Table 8.5 that

$$\delta\theta_-(202^\circ) = -5.980^\circ, \qquad \bar{\theta}(202^\circ) = -32.007^\circ, \qquad \delta\theta_+(202^\circ) = -8.955^\circ.$$

Now,

$$z = (1 - \zeta)/(1 - \zeta_S) = (1 + 5.45 \times 10^{-3})/(1 - 1.662 \times 10^{-2}) = 1.02244,$$

so

$$\xi = (\bar{z} - z)/\delta z = (1.00184 - 1.02244)/0.11014 \simeq -0.19.$$

According to Table 8.2,

$$\Theta_-(-0.19) = -0.113, \qquad \Theta_+(-0.19) = -0.077,$$

so

$$\begin{aligned}
\theta &= \Theta_-\,\delta\theta_- + \bar{\theta} + \Theta_+\,\delta\theta_+ \\
&= -0.113 \times 5.980 - 32.007 - 0.077 \times 8.955 \\
&= -30.642^\circ.
\end{aligned}$$

Finally,

$$\lambda = \bar{\lambda} + q + \theta = 60.464 + 10.739 - 30.642 = 40.561 \simeq 40°34'.$$

Thus, the ecliptic longitude of Mars at 00:00 UT on December 25, 1800 AD was 10TA34.

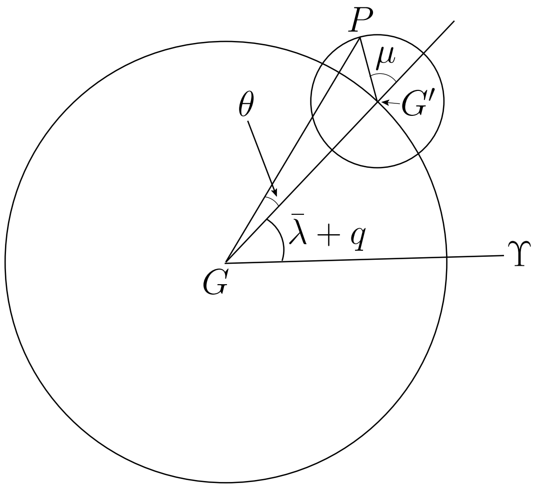

Figure 8.4: The geocentric orbit of a superior planet. Here, G, G', P, μ, θ, $\bar{\lambda}$, q, and Υ represent the Earth, guide-point, planet, epicyclic anomaly, equation of the epicycle, mean longitude, equation of center, and spring equinox, respectively. The view is from northern ecliptic pole. Both G' and P orbit counterclockwise.

8.4 Conjunction, opposition, and station dates

Figure 8.4 shows the geocentric orbit of a superior planet. Recall that the vector $G'P$ is always parallel to the vector connecting the Earth to the Sun. It follows that a so-called *conjunction*, at which the Sun lies directly between the planet and the Earth, occurs whenever the epicyclic anomaly, μ, takes the value $0°$. At a conjunction, the planet is farthest from the Earth, and has the same ecliptic longitude as the Sun, and is, therefore, invisible. Conversely, a so-called *opposition*, at which the Earth lies directly between the planet and the Sun, occurs whenever $\mu = 180°$. At an opposition, the planet is closest to the Earth, and also directly opposite the Sun in the sky, and, therefore, at its brightest. Now, a superior planet rotates around the epicycle at a faster angular velocity than its guide-point rotates around the deferent. Moreover, both the planet and guide-point rotate in the same direction. It follows that the planet is traveling backward in the sky (relative to the direction of its mean motion) at opposition. This phenomenon is called *retrograde motion*. The period of retrograde motion begins and ends at *stations*; so-called because when the planet reaches them it appears to stand still in the sky (relative to the fixed stars) for a few days while it reverses direction.

Tables 8.3–8.5 can be used to determine the dates of the conjunctions, oppositions, and stations of Mars. Consider the first conjunction after the epoch (January 1, 2000 AD). We can estimate the time at which this event occurs by approximating the epicyclic anomaly as the so-called *mean epicyclic anomaly*:

$$\mu \simeq \bar{\mu} = \bar{\lambda}_S - \bar{\lambda} = \bar{\lambda}_{0\,S} - \bar{\lambda}_0 + (n_S - n)\,(t - t_0)$$
$$= 284.998 + 0.46157617\,(t - t_0).$$

We obtain

$$t \simeq t_0 + (360 - 284.998)/0.46157617 \simeq t_0 + 162\,\text{JD}.$$

A calculation of the epicyclic anomaly at this time, using Tables 8.3–8.5, yields $\mu = -9.583^\circ$. Now, the actual conjunction occurs when $\mu = 0^\circ$. Hence, our next estimate is

$$t \simeq t_0 + 162 + 9.583/0.46157617 \simeq t_0 + 183\,\text{JD}.$$

A calculation of the epicyclic anomaly at this time gives 0.294°. Thus, our final estimate is

$$t = t_0 + 183 - 0.294/0.461557617 = t_0 + 182.4\,\text{JD},$$

which corresponds to July 1, 2000 AD.

Consider the first opposition of Mars after the epoch. Our first estimate of the time at which this event takes place is

$$t \simeq t_0 + (540 - 284.998)/0.46157617 \simeq t_0 + 552\,\text{JD}.$$

A calculation of the epicyclic anomaly at this time yields $\mu = 188.649^\circ$. Now, the actual opposition occurs when $\mu = 180^\circ$. Hence, our second estimate is

$$t \simeq t_0 + 552 - 8.649/0.46157617 \simeq t_0 + 533\,\text{JD}.$$

A calculation of the epicyclic anomaly at this time gives 181.455°. Thus, our third estimate is

$$t \simeq t_0 + 533 - 1.455/0.46157617 \simeq t_0 + 530\,\text{JD}.$$

A calculation of the epicyclic anomaly at this time yields 180.244°. Hence, our final estimate is

$$t = t_0 + 530 - 0.244/0.46157617 = t_0 + 529.5\,\text{JD},$$

which corresponds to June 13, 2001 AD. Incidentally, it is clear from the previous analysis that the mean time period between successive conjunctions, or oppositions, of Mars is

$$360/0.46157617 = 779.9\,\text{JD},$$

which is equivalent to 2.14 years.

Let us now consider the stations of Mars. We can approximate the ecliptic longitude of a superior planet as

$$\lambda \simeq \bar{\lambda} + \bar{\theta}, \tag{8.30}$$

where

$$\bar{\theta} = \tan^{-1}\left(\frac{\sin\bar{\mu}}{\bar{a} + \cos\bar{\mu}}\right), \tag{8.31}$$

and $\bar{a} = a\,\bar{z}$. Note that $d\bar{\lambda}/dt = n$ and $d\bar{\mu}/dt = n_S - n$. It follows that

$$\frac{d\lambda}{dt} \simeq n + \left(\frac{\bar{a}\,\cos\bar{\mu} + 1}{1 + 2\,\bar{a}\,\cos\bar{\mu} + \bar{a}^2}\right)(n_S - n). \tag{8.32}$$

Now, a station corresponds to $d\lambda/dt = 0$ (that is, a local maximum or minimum of λ), which gives

$$\cos\bar{\mu} \simeq -\frac{(\bar{a}^2 + n_S/n)}{\bar{a}\,(1 + n_S/n)}. \tag{8.33}$$

For the case of Mars, we find that $\bar{\mu} = 163.3^\circ$ or 196.7°. The first solution corresponds to the so-called *retrograde station*, at which the planet switches from direct to retrograde motion. The second solution corresponds to the so-called *direct station*, at which the planet switches from retrograde to direct motion. The mean time interval between a retrograde station and the following opposition, or between an opposition and the following direct station, is $(180 - 163.3)/0.46157617 \simeq 36$ JD. Unfortunately, the only option for accurately determining the dates at which the stations occur is to calculate the ecliptic longitude of Mars over a range of days centered 36 days before and after its opposition.

Table 8.6 shows the conjunctions, oppositions, and stations of Mars for the years 2000–2020 AD, calculated using the previously described techniques.

8.5 *Determination of ecliptic longitude of Jupiter*

The ecliptic longitude of Jupiter can be determined with the aid of Tables 8.7–8.9. Table 8.7 allows the mean longitude, $\bar{\lambda}$, and the mean anomaly, M, of Jupiter to be calculated as functions of time. Next, Table 8.8 permits the equation of center, q, and the radial anomaly, ζ, to be determined as functions of the mean anomaly. Finally, Table 8.9 allows the quantities $\delta\theta_-$, $\bar{\theta}$, and $\delta\theta_+$ to be calculated as functions of the epicyclic anomaly, μ. The procedure for using the tables is analogous to the previously described procedure for using the Mars tables. One example of this procedure is given in the following.

Example: May 5, 2005 AD, 00:00 UT:

From before, $t - t_0 = 1\,950.5$ JD, $\lambda_S = 44.602^\circ$, $M_S \simeq 120^\circ$, and $\zeta_S = -8.56 \times 10^{-3}$. Making use of Table 8.7, we find:

t(JD)	$\bar{\lambda}(^\circ)$	$M(^\circ)$
+1000	83.125	83.081
+900	74.813	74.773
+50	4.156	4.154
+.5	0.042	0.042
Epoch	34.365	19.348
	196.501	181.398
Modulus	196.501	181.398

Given that $M \simeq 181^\circ$, Table 8.8 yields

$$q(181^\circ) = -0.091^\circ, \qquad \zeta(181^\circ) = -4.838 \times 10^{-2}.$$

Thus,

$$\mu = \lambda_S - \bar{\lambda} - q = 44.602 - 196.501 + 0.091 = -151.808 \simeq 208^\circ,$$

where we have rounded the epicyclic anomaly to the nearest degree. It follows from Table 8.9 that

$$\delta\theta_-(208^\circ) = -0.447^\circ, \qquad \bar{\theta}(208^\circ) = -6.194^\circ, \qquad \delta\theta_+(208^\circ) = -0.522^\circ.$$

Now,

$$z = (1 - \zeta)/(1 - \zeta_S) = (1 + 4.838 \times 10^{-2})/(1 + 8.56 \times 10^{-3}) = 1.0395.$$

However, from Table 8.1, $\bar{z} = 1.00109$ and $\delta z = 0.06512$, so

$$\xi = (\bar{z} - z)/\delta z = (1.00109 - 1.0395)/0.06512 \simeq -0.59.$$

According to Table 8.2,

$$\Theta_-(-0.59) = -0.469, \qquad \Theta_+(-0.59) = -0.121,$$

so

$$\begin{aligned}
\theta &= \Theta_-\,\delta\theta_- + \bar{\theta} + \Theta_+\,\delta\theta_+ \\
&= 0.469 \times 0.447 - 6.194 + 0.121 \times 0.522 \\
&= -5.921^\circ.
\end{aligned}$$

Finally,

$$\lambda = \bar{\lambda} + q + \theta = 196.501 - 0.091 - 5.921 = 190.489 \simeq 190^\circ 29'.$$

Thus, the ecliptic longitude of Jupiter at 00:00 UT on May 5, 2005 AD was 10LI29.

The conjunctions, oppositions, and stations of Jupiter can be investigated using analogous methods to those employed earlier to examine the conjunctions, oppositions, and stations of Mars. We find that the mean time period between successive oppositions or conjunctions of Jupiter is 1.09 yr. Furthermore, on average, the retrograde and direct stations of Jupiter occur when the epicyclic anomaly takes the values $\mu = 125.6^\circ$ and 234.4°, respectively. Finally, the mean time period between a retrograde station and the following opposition, or between the opposition and the following direct station, is 60 JD. The conjunctions, oppositions, and stations of Jupiter during the years 2000–2010 AD are shown in Table 8.10.

8.6 Determination of ecliptic longitude of Saturn

The ecliptic longitude of Saturn can be determined with the aid of Tables 8.11–8.13. Table 8.11 allows the mean longitude, $\bar{\lambda}$, and the mean anomaly, M, of Saturn to be calculated as functions of time. Next, Table 8.12 permits the equation of center, q, and the radial anomaly, ζ, to be determined as functions of the mean anomaly. Finally, Table 8.13 allows the quantities $\delta\theta_-$, $\bar{\theta}$, and $\delta\theta_+$ to be calculated as functions of the epicyclic anomaly, μ. The procedure for using the tables is analogous to the previously described procedure for using the Mars tables. One example of this procedure is given in the following.

Example: May 5, 2005 AD, 00:00 UT:

From before, $t - t_0 = 1\,950.5$ JD, $\lambda_S = 44.602^\circ$, $M_S \simeq 120^\circ$, and $\zeta_S = -8.56 \times 10^{-3}$. Making use of Table 8.11, we find:

t(JD)	$\bar{\lambda}(^\circ)$	$M(^\circ)$
+1000	33.508	33.482
+900	30.157	30.133
+50	1.675	1.674
+.5	0.017	0.017
Epoch	50.059	317.857
	115.416	383.163
Modulus	115.416	23.163

Given that $M \simeq 23^\circ$, Table 8.12 yields

$$q(23^\circ) = 2.561^\circ, \qquad \zeta(23^\circ) = 4.913 \times 10^{-2}.$$

Thus,

$$\mu = \lambda_S - \bar{\lambda} - q = 44.602 - 115.416 - 2.561 = -73.375 \simeq 287^\circ,$$

where we have rounded the epicyclic anomaly to the nearest degree. It follows from Table 8.13 that

$$\delta\theta_-(287^\circ) = -0.353^\circ, \qquad \bar{\theta}(287^\circ) = -5.551^\circ, \qquad \delta\theta_+(287^\circ) = -0.405^\circ.$$

Now,

$$z = (1-\zeta)/(1-\zeta_S) = (1-4.913\times 10^{-2})/(1+8.56\times 10^{-3}) = 0.9428.$$

However, from Table 8.1, $\bar{z} = 1.00118$ and $\delta z = 0.07059$, so

$$\xi = (\bar{z}-z)/\delta z = (1.00118-0.9428)/0.07059 \simeq 0.83.$$

According to Table 8.2,

$$\Theta_-(0.83) = 0.071, \qquad \Theta_+(0.83) = 0.759,$$

so

$$\begin{aligned}\theta &= \Theta_-\,\delta\theta_- + \bar{\theta} + \Theta_+\,\delta\theta_+\\ &= -0.071\times 0.353 - 5.551 - 0.759\times 0.405\\ &= -5.883^\circ.\end{aligned}$$

Finally,

$$\lambda = \bar{\lambda} + q + \theta = 115.416 + 2.561 - 5.883 = 112.094 \simeq 112^\circ 06'.$$

Thus, the ecliptic longitude of Saturn at 00:00 UT on May 5, 2005 AD was 22CN06.

The conjunctions, oppositions, and stations of Saturn can be investigated using analogous methods to those employed earlier to examine the conjunctions, oppositions, and stations of Mars. We find that the mean time period between successive oppositions or conjunctions of Saturn is 1.035 yr. Furthermore, on average, the retrograde and direct stations of Saturn occur when the epicyclic anomaly takes the values $\mu = 114.5^\circ$ and 245.5°, respectively. Finally, the mean time period between a retrograde station and the following opposition, or between the opposition and the following direct station, is 69 JD. The conjunctions, oppositions, and stations of Saturn during the years 2000–2010 AD are shown in Table 8.14.

Incidentally, the information contained in the mean motion tables, 8.3, 8.7, and 8.11, as well as the deferential anomaly tables, 8.4, 8.8, and 8.12, is essentially equivalent to that contained in the "Tables of the mean longitudinal motion and anomalies of the five stars" (*Κανόνες μέσων κινήσεων μήκους τε καὶ ἀνωμαλίας τῶν πέντε ἀστέρων*) that appear in Section 4 of Book IX of the Almagest. Likewise, the information contained in the epicyclic anomaly tables, 8.5, 8.9, and 8.13, is equivalent to that contained in the "Tables of the longitudinal corrections of the five planets" (*Κανόνες τῆς κατὰ μῆκος τῶν πέντε πλανωμένων διευκρινήσεως*) that appear in Section 11 of Book XI of the Almagest. The computation of the stations of the superior planets is discussed in Book XII of the Almagest.

8.7 Tables

Planet	$\bar{z}$	δz	$z_{\rm min}$	$z_{\rm max}$
Mercury	1.04774	0.23216	0.81558	1.27990
Venus	1.00016	0.02349	0.97667	1.02365
Mars	1.00184	0.11014	0.89170	1.11198
Jupiter	1.00109	0.06512	0.93597	1.06602
Saturn	1.00118	0.07059	0.93059	1.07177

Table 8.1: Constants associated with the epicycles of the inferior and superior planets.

ξ	Θ_-	Θ_+	ξ	Θ_-	Θ_+	ξ	Θ_-	Θ_+	ξ	Θ_-	Θ_+
0.00	0.000	0.000	0.25	0.094	0.156	0.50	0.125	0.375	0.75	0.094	0.656
0.01	0.005	0.005	0.26	0.096	0.164	0.51	0.125	0.385	0.76	0.091	0.669
0.02	0.010	0.010	0.27	0.099	0.171	0.52	0.125	0.395	0.77	0.089	0.681
0.03	0.015	0.015	0.28	0.101	0.179	0.53	0.125	0.405	0.78	0.086	0.694
0.04	0.019	0.021	0.29	0.103	0.187	0.54	0.124	0.416	0.79	0.083	0.707
0.05	0.024	0.026	0.30	0.105	0.195	0.55	0.124	0.426	0.80	0.080	0.720
0.06	0.028	0.032	0.31	0.107	0.203	0.56	0.123	0.437	0.81	0.077	0.733
0.07	0.033	0.037	0.32	0.109	0.211	0.57	0.123	0.447	0.82	0.074	0.746
0.08	0.037	0.043	0.33	0.111	0.219	0.58	0.122	0.458	0.83	0.071	0.759
0.09	0.041	0.049	0.34	0.112	0.228	0.59	0.121	0.469	0.84	0.067	0.773
0.10	0.045	0.055	0.35	0.114	0.236	0.60	0.120	0.480	0.85	0.064	0.786
0.11	0.049	0.061	0.36	0.115	0.245	0.61	0.119	0.491	0.86	0.060	0.800
0.12	0.053	0.067	0.37	0.117	0.253	0.62	0.118	0.502	0.87	0.057	0.813
0.13	0.057	0.073	0.38	0.118	0.262	0.63	0.117	0.513	0.88	0.053	0.827
0.14	0.060	0.080	0.39	0.119	0.271	0.64	0.115	0.525	0.89	0.049	0.841
0.15	0.064	0.086	0.40	0.120	0.280	0.65	0.114	0.536	0.90	0.045	0.855
0.16	0.067	0.093	0.41	0.121	0.289	0.66	0.112	0.548	0.91	0.041	0.869
0.17	0.071	0.099	0.42	0.122	0.298	0.67	0.111	0.559	0.92	0.037	0.883
0.18	0.074	0.106	0.43	0.123	0.307	0.68	0.109	0.571	0.93	0.033	0.897
0.19	0.077	0.113	0.44	0.123	0.317	0.69	0.107	0.583	0.94	0.028	0.912
0.20	0.080	0.120	0.45	0.124	0.326	0.70	0.105	0.595	0.95	0.024	0.926
0.21	0.083	0.127	0.46	0.124	0.336	0.71	0.103	0.607	0.96	0.019	0.941
0.22	0.086	0.134	0.47	0.125	0.345	0.72	0.101	0.619	0.97	0.015	0.955
0.23	0.089	0.141	0.48	0.125	0.355	0.73	0.099	0.631	0.98	0.010	0.970
0.24	0.091	0.149	0.49	0.125	0.365	0.74	0.096	0.644	0.99	0.005	0.985
0.25	0.094	0.156	0.50	0.125	0.375	0.75	0.094	0.656	1.00	0.000	1.000

Table 8.2: Epicyclic interpolation coefficients. Note that $\Theta_\pm(\xi) = -\Theta_\mp(-\xi)$.

Δt(JD)	$\Delta\bar{\lambda}(^\circ)$	$\Delta M(^\circ)$	$\Delta\bar{F}(^\circ)$	Δt(JD)	$\Delta\bar{\lambda}(^\circ)$	$\Delta M(^\circ)$	$\Delta\bar{F}(^\circ)$
10 000	200.712	200.208	200.409	1 000	164.071	164.021	164.041
20 000	41.424	40.415	40.819	2 000	328.142	328.042	328.082
30 000	242.135	240.623	241.228	3 000	132.214	132.062	132.123
40 000	82.847	80.830	81.638	4 000	296.285	296.083	296.164
50 000	283.559	281.038	282.047	5 000	100.356	100.104	100.205
60 000	124.271	121.246	122.456	6 000	264.427	264.125	264.246
70 000	324.983	321.453	322.866	7 000	68.498	68.145	68.287
80 000	165.694	161.661	163.275	8 000	232.569	232.166	232.328
90 000	6.406	1.868	3.685	9 000	36.641	36.187	36.368
100	52.407	52.402	52.404	10	5.241	5.240	5.240
200	104.814	104.804	104.808	20	10.481	10.480	10.481
300	157.221	157.206	157.212	30	15.722	15.721	15.721
400	209.628	209.608	209.616	40	20.963	20.961	20.962
500	262.036	262.010	262.020	50	26.204	26.201	26.202
600	314.443	314.412	314.425	60	31.444	31.441	31.442
700	6.850	6.815	6.829	70	36.685	36.681	36.683
800	59.257	59.217	59.233	80	41.926	41.922	41.923
900	111.664	111.619	111.637	90	47.166	47.162	47.164
1	0.524	0.524	0.524	0.1	0.052	0.052	0.052
2	1.048	1.048	1.048	0.2	0.105	0.105	0.105
3	1.572	1.572	1.572	0.3	0.157	0.157	0.157
4	2.096	2.096	2.096	0.4	0.210	0.210	0.210
5	2.620	2.620	2.620	0.5	0.262	0.262	0.262
6	3.144	3.144	3.144	0.6	0.314	0.314	0.314
7	3.668	3.668	3.668	0.7	0.367	0.367	0.367
8	4.193	4.192	4.192	0.8	0.419	0.419	0.419
9	4.717	4.716	4.716	0.9	0.472	0.472	0.472

Table 8.3: Mean motion of Mars. Here, $\Delta t = t - t_0$, $\Delta\bar{\lambda} = \bar{\lambda} - \bar{\lambda}_0$, $\Delta M = M - M_0$, and $\Delta\bar{F} = \bar{F} - \bar{F}_0$. At epoch ($t_0 = 2\,451\,545.0$ JD), $\bar{\lambda}_0 = 355.460^\circ$, $M_0 = 19.388^\circ$, and $\bar{F}_0 = 305.796^\circ$.

$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$
0	0.000	9.339	90	10.702	−0.872	180	0.000	−9.339	270	−10.702	−0.872
2	0.417	9.333	92	10.652	−1.197	182	−0.330	−9.335	272	−10.739	−0.545
4	0.833	9.312	94	10.589	−1.519	184	−0.660	−9.321	274	−10.763	−0.217
6	1.249	9.279	96	10.514	−1.839	186	−0.989	−9.298	276	−10.773	0.114
8	1.662	9.232	98	10.426	−2.155	188	−1.317	−9.265	278	−10.770	0.444
10	2.072	9.171	100	10.326	−2.468	190	−1.645	−9.224	280	−10.753	0.776
12	2.479	9.098	102	10.214	−2.776	192	−1.971	−9.173	282	−10.722	1.107
14	2.882	9.011	104	10.091	−3.081	194	−2.296	−9.113	284	−10.678	1.438
16	3.281	8.911	106	9.957	−3.380	196	−2.619	−9.044	286	−10.619	1.768
18	3.674	8.799	108	9.811	−3.675	198	−2.940	−8.966	288	−10.546	2.097
20	4.062	8.674	110	9.655	−3.964	200	−3.259	−8.878	290	−10.458	2.424
22	4.443	8.537	112	9.489	−4.248	202	−3.575	−8.782	292	−10.357	2.749
24	4.817	8.388	114	9.313	−4.527	204	−3.889	−8.676	294	−10.241	3.071
26	5.184	8.227	116	9.127	−4.799	206	−4.199	−8.562	296	−10.111	3.389
28	5.542	8.054	118	8.932	−5.065	208	−4.506	−8.438	298	−9.967	3.705
30	5.892	7.870	120	8.727	−5.324	210	−4.810	−8.306	300	−9.809	4.016
32	6.233	7.675	122	8.514	−5.576	212	−5.110	−8.165	302	−9.637	4.322
34	6.564	7.470	124	8.293	−5.822	214	−5.405	−8.015	304	−9.452	4.623
36	6.885	7.254	126	8.064	−6.060	216	−5.696	−7.857	306	−9.252	4.919
38	7.195	7.029	128	7.827	−6.292	218	−5.983	−7.690	308	−9.040	5.208
40	7.494	6.794	130	7.583	−6.515	220	−6.264	−7.515	310	−8.814	5.491
42	7.782	6.550	132	7.332	−6.731	222	−6.540	−7.331	312	−8.575	5.768
44	8.059	6.297	134	7.074	−6.939	224	−6.810	−7.139	314	−8.323	6.036
46	8.323	6.036	136	6.810	−7.139	226	−7.074	−6.939	316	−8.059	6.297
48	8.575	5.768	138	6.540	−7.331	228	−7.332	−6.731	318	−7.782	6.550
50	8.814	5.491	140	6.264	−7.515	230	−7.583	−6.515	320	−7.494	6.794
52	9.040	5.208	142	5.983	−7.690	232	−7.827	−6.292	322	−7.195	7.029
54	9.252	4.919	144	5.696	−7.857	234	−8.064	−6.060	324	−6.885	7.254
56	9.452	4.623	146	5.405	−8.015	236	−8.293	−5.822	326	−6.564	7.470
58	9.637	4.322	148	5.110	−8.165	238	−8.514	−5.576	328	−6.233	7.675
60	9.809	4.016	150	4.810	−8.306	240	−8.727	−5.324	330	−5.892	7.870
62	9.967	3.705	152	4.506	−8.438	242	−8.932	−5.065	332	−5.542	8.054
64	10.111	3.389	154	4.199	−8.562	244	−9.127	−4.799	334	−5.184	8.227
66	10.241	3.071	156	3.889	−8.676	246	−9.313	−4.527	336	−4.817	8.388
68	10.357	2.749	158	3.575	−8.782	248	−9.489	−4.248	338	−4.443	8.537
70	10.458	2.424	160	3.259	−8.878	250	−9.655	−3.964	340	−4.062	8.674
72	10.546	2.097	162	2.940	−8.966	252	−9.811	−3.675	342	−3.674	8.799
74	10.619	1.768	164	2.619	−9.044	254	−9.957	−3.380	344	−3.281	8.911
76	10.678	1.438	166	2.296	−9.113	256	−10.091	−3.081	346	−2.882	9.011
78	10.722	1.107	168	1.971	−9.173	258	−10.214	−2.776	348	−2.479	9.098
80	10.753	0.776	170	1.645	−9.224	260	−10.326	−2.468	350	−2.072	9.171
82	10.770	0.444	172	1.317	−9.265	262	−10.426	−2.155	352	−1.662	9.232
84	10.773	0.114	174	0.989	−9.298	264	−10.514	−1.839	354	−1.249	9.279
86	10.763	−0.217	176	0.660	−9.321	266	−10.589	−1.519	356	−0.833	9.312
88	10.739	−0.545	178	0.330	−9.335	268	−10.652	−1.197	358	−0.417	9.333
90	10.702	−0.872	180	0.000	−9.339	270	−10.702	−0.872	360	−0.000	9.339

Table 8.4: Deferential anomalies of Mars.

μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$	μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$	μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$	μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$
0	0.000	0.000	0.000	45	1.159	17.566	1.329	90	2.679	33.228	3.125	135	5.180	40.793	6.547
1	0.025	0.396	0.028	46	1.187	17.945	1.362	91	2.721	33.527	3.176	136	5.246	40.716	6.658
2	0.049	0.792	0.056	47	1.216	18.322	1.394	92	2.764	33.822	3.228	137	5.312	40.619	6.771
3	0.074	1.187	0.084	48	1.244	18.699	1.427	93	2.807	34.114	3.281	138	5.378	40.503	6.885
4	0.099	1.583	0.113	49	1.273	19.075	1.461	94	2.851	34.403	3.335	139	5.442	40.366	7.001
5	0.123	1.979	0.141	50	1.302	19.450	1.494	95	2.895	34.688	3.390	140	5.506	40.206	7.118
6	0.148	2.374	0.169	51	1.331	19.824	1.528	96	2.940	34.969	3.445	141	5.568	40.024	7.235
7	0.173	2.770	0.197	52	1.360	20.196	1.562	97	2.985	35.246	3.501	142	5.628	39.817	7.354
8	0.197	3.165	0.226	53	1.390	20.568	1.596	98	3.031	35.519	3.558	143	5.687	39.584	7.474
9	0.222	3.560	0.254	54	1.419	20.939	1.630	99	3.078	35.788	3.616	144	5.744	39.325	7.594
10	0.247	3.955	0.282	55	1.449	21.309	1.665	100	3.125	36.053	3.675	145	5.797	39.038	7.714
11	0.272	4.350	0.311	56	1.479	21.677	1.700	101	3.173	36.313	3.735	146	5.848	38.721	7.833
12	0.297	4.745	0.339	57	1.510	22.045	1.735	102	3.221	36.568	3.796	147	5.895	38.373	7.952
13	0.322	5.140	0.368	58	1.540	22.411	1.771	103	3.270	36.819	3.857	148	5.938	37.992	8.069
14	0.347	5.534	0.396	59	1.571	22.776	1.807	104	3.320	37.065	3.920	149	5.976	37.577	8.184
15	0.372	5.928	0.425	60	1.602	23.139	1.843	105	3.370	37.306	3.984	150	6.009	37.126	8.297
16	0.397	6.322	0.453	61	1.633	23.502	1.879	106	3.421	37.541	4.049	151	6.036	36.638	8.405
17	0.422	6.716	0.482	62	1.665	23.863	1.916	107	3.472	37.771	4.115	152	6.056	36.110	8.509
18	0.447	7.110	0.511	63	1.696	24.222	1.953	108	3.525	37.996	4.182	153	6.069	35.541	8.607
19	0.472	7.503	0.540	64	1.728	24.581	1.991	109	3.578	38.214	4.251	154	6.072	34.929	8.698
20	0.497	7.896	0.568	65	1.761	24.938	2.029	110	3.631	38.426	4.321	155	6.066	34.271	8.780
21	0.523	8.288	0.597	66	1.793	25.293	2.067	111	3.686	38.632	4.391	156	6.050	33.567	8.852
22	0.548	8.680	0.626	67	1.826	25.647	2.106	112	3.741	38.831	4.464	157	6.022	32.813	8.911
23	0.573	9.072	0.656	68	1.859	25.999	2.145	113	3.796	39.023	4.537	158	5.980	32.007	8.955
24	0.599	9.464	0.685	69	1.893	26.349	2.184	114	3.853	39.209	4.612	159	5.925	31.149	8.982
25	0.625	9.855	0.714	70	1.927	26.698	2.224	115	3.910	39.386	4.688	160	5.854	30.235	8.988
26	0.650	10.246	0.744	71	1.961	27.045	2.264	116	3.968	39.556	4.765	161	5.766	29.265	8.972
27	0.676	10.636	0.773	72	1.995	27.390	2.305	117	4.026	39.718	4.844	162	5.660	28.236	8.929
28	0.702	11.026	0.803	73	2.030	27.734	2.346	118	4.086	39.872	4.925	163	5.535	27.146	8.855
29	0.728	11.415	0.833	74	2.065	28.075	2.387	119	4.146	40.017	5.007	164	5.389	25.996	8.747
30	0.754	11.804	0.863	75	2.100	28.415	2.429	120	4.206	40.153	5.091	165	5.221	24.783	8.601
31	0.780	12.192	0.893	76	2.136	28.753	2.472	121	4.268	40.279	5.176	166	5.030	23.506	8.411
32	0.806	12.580	0.923	77	2.172	29.088	2.515	122	4.330	40.396	5.262	167	4.815	22.167	8.174
33	0.833	12.968	0.953	78	2.209	29.421	2.558	123	4.393	40.502	5.351	168	4.576	20.764	7.886
34	0.859	13.354	0.984	79	2.246	29.752	2.602	124	4.456	40.598	5.441	169	4.311	19.299	7.541
35	0.886	13.741	1.014	80	2.283	30.081	2.647	125	4.520	40.683	5.533	170	4.021	17.774	7.138
36	0.913	14.126	1.045	81	2.321	30.408	2.692	126	4.584	40.756	5.626	171	3.707	16.189	6.673
37	0.939	14.511	1.076	82	2.359	30.732	2.737	127	4.649	40.816	5.721	172	3.368	14.549	6.145
38	0.966	14.896	1.107	83	2.397	31.054	2.784	128	4.715	40.864	5.818	173	3.005	12.857	5.553
39	0.994	15.279	1.138	84	2.436	31.373	2.830	129	4.780	40.899	5.917	174	2.621	11.116	4.899
40	1.021	15.662	1.169	85	2.475	31.689	2.878	130	4.847	40.920	6.018	175	2.217	9.333	4.186
41	1.048	16.045	1.201	86	2.515	32.003	2.926	131	4.913	40.926	6.120	176	1.796	7.513	3.420
42	1.076	16.426	1.233	87	2.555	32.314	2.975	132	4.980	40.918	6.224	177	1.360	5.662	2.608
43	1.103	16.807	1.265	88	2.596	32.622	3.024	133	5.047	40.893	6.330	178	0.913	3.788	1.760
44	1.131	17.187	1.297	89	2.637	32.927	3.074	134	5.113	40.851	6.438	179	0.458	1.898	0.886
45	1.159	17.566	1.329	90	2.679	33.228	3.125	135	5.180	40.793	6.547	180	0.000	0.000	0.000

Table 8.5: Epicyclic anomalies of Mars. All quantities are in degrees. Note that $\bar{\theta}(360^\circ - \mu) = -\bar{\theta}(\mu)$, and $\delta\theta_\pm(360^\circ - \mu) = -\delta\theta_\pm(\mu)$.

Event	Date	λ	Event	Date	λ
Conjunction	01/07/2000	10CN13	Conjunction	04/02/2011	15AQ42
Station (R)	12/05/2001	29SG00	Station (R)	24/01/2012	23VI01
Opposition	13/06/2001	22SG44	Opposition	03/03/2012	13VI42
Station (D)	19/07/2001	15SG02	Station (D)	14/04/2012	03VI51
Conjunction	10/08/2002	18LE05	Conjunction	17/04/2013	28AR06
Station (R)	29/07/2003	10PI20	Station (R)	01/03/2014	27LI31
Opposition	28/08/2003	05PI03	Opposition	08/04/2014	19LI00
Station (D)	27/09/2003	29AQ55	Station (D)	20/05/2014	09LI04
Conjunction	15/09/2004	23VI06	Conjunction	14/06/2015	23GE28
Station (R)	02/10/2005	23TA31	Station (R)	17/04/2016	08SG45
Opposition	07/11/2005	15TA06	Opposition	22/05/2016	01SG43
Station (D)	09/12/2005	08TA24	Station (D)	29/06/2016	22SC55
Conjunction	23/10/2006	29LI44	Conjunction	27/07/2017	04LE11
Station (R)	15/11/2007	12CN36	Station (R)	27/06/2018	09AQ35
Opposition	24/12/2007	02CN45	Opposition	27/07/2018	04AQ22
Station (D)	31/01/2008	24GE15	Station (D)	27/08/2018	28CP43
Conjunction	05/12/2008	14SG08	Conjunction	02/09/2019	09VI43
Station (R)	20/12/2009	19LE35	Station (R)	10/09/2020	28AR08
Opposition	29/01/2010	09LE45	Opposition	13/10/2020	20AR59
Station (D)	10/03/2010	00LE20	Station (D)	13/11/2020	15AR05

Table 8.6: The conjunctions, oppositions, and stations of Mars during the years 2000–2020 AD. (R) indicates a retrograde station, and (D) a direct station.

Δt(JD)	$\Delta\bar{\lambda}(^\circ)$	$\Delta M(^\circ)$	$\Delta\bar{F}(^\circ)$	Δt(JD)	$\Delta\bar{\lambda}(^\circ)$	$\Delta M(^\circ)$	$\Delta\bar{F}(^\circ)$
10 000	111.251	110.810	110.812	1 000	83.125	83.081	83.081
20 000	222.501	221.620	221.624	2 000	166.250	166.162	166.162
30 000	333.752	332.430	332.437	3 000	249.375	249.243	249.244
40 000	85.003	83.240	83.249	4 000	332.500	332.324	332.325
50 000	196.253	194.050	194.061	5 000	55.625	55.405	55.406
60 000	307.504	304.860	304.873	6 000	138.750	138.486	138.487
70 000	58.755	55.670	55.685	7 000	221.875	221.567	221.569
80 000	170.006	166.480	166.498	8 000	305.001	304.648	304.650
90 000	281.256	277.290	277.310	9 000	28.126	27.729	27.731
100	8.313	8.308	8.308	10	0.831	0.831	0.831
200	16.625	16.616	16.616	20	1.663	1.662	1.662
300	24.938	24.924	24.924	30	2.494	2.492	2.492
400	33.250	33.232	33.232	40	3.325	3.323	3.323
500	41.563	41.541	41.541	50	4.156	4.154	4.154
600	49.875	49.849	49.849	60	4.988	4.985	4.985
700	58.188	58.157	58.157	70	5.819	5.816	5.816
800	66.500	66.465	66.465	80	6.650	6.646	6.646
900	74.813	74.773	74.773	90	7.481	7.477	7.477
1	0.083	0.083	0.083	0.1	0.008	0.008	0.008
2	0.166	0.166	0.166	0.2	0.017	0.017	0.017
3	0.249	0.249	0.249	0.3	0.025	0.025	0.025
4	0.333	0.332	0.332	0.4	0.033	0.033	0.033
5	0.416	0.415	0.415	0.5	0.042	0.042	0.042
6	0.499	0.498	0.498	0.6	0.050	0.050	0.050
7	0.582	0.582	0.582	0.7	0.058	0.058	0.058
8	0.665	0.665	0.665	0.8	0.067	0.066	0.066
9	0.748	0.748	0.748	0.9	0.075	0.075	0.075

Table 8.7: Mean motion of Jupiter. Here, $\Delta t = t - t_0$, $\Delta\bar{\lambda} = \bar{\lambda} - \bar{\lambda}_0$, $\Delta M = M - M_0$, and $\Delta\bar{F} = \bar{F} - \bar{F}_0$. At epoch ($t_0 = 2\,451\,545.0$ JD), $\bar{\lambda}_0 = 34.365^\circ$, $M_0 = 19.348^\circ$, and $\bar{F}_0 = 293.660^\circ$.

$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$
0	0.000	4.839	90	5.545	−0.234	180	0.000	−4.839	270	−5.545	−0.234
2	0.205	4.835	92	5.530	−0.403	182	−0.182	−4.836	272	−5.553	−0.065
4	0.410	4.826	94	5.508	−0.571	184	−0.363	−4.828	274	−5.554	0.105
6	0.614	4.810	96	5.479	−0.737	186	−0.545	−4.815	276	−5.549	0.274
8	0.818	4.787	98	5.444	−0.903	188	−0.725	−4.796	278	−5.537	0.444
10	1.020	4.758	100	5.403	−1.067	190	−0.905	−4.772	280	−5.518	0.613
12	1.221	4.723	102	5.355	−1.230	192	−1.085	−4.743	282	−5.492	0.782
14	1.420	4.681	104	5.301	−1.391	194	−1.263	−4.709	284	−5.459	0.950
16	1.617	4.633	106	5.241	−1.550	196	−1.439	−4.669	286	−5.419	1.117
18	1.812	4.579	108	5.175	−1.707	198	−1.615	−4.624	288	−5.372	1.283
20	2.004	4.519	110	5.102	−1.862	200	−1.789	−4.574	290	−5.318	1.448
22	2.194	4.453	112	5.024	−2.014	202	−1.961	−4.519	292	−5.257	1.611
24	2.380	4.382	114	4.941	−2.163	204	−2.131	−4.459	294	−5.190	1.773
26	2.563	4.304	116	4.851	−2.310	206	−2.298	−4.394	296	−5.116	1.932
28	2.742	4.221	118	4.757	−2.454	208	−2.464	−4.324	298	−5.035	2.089
30	2.918	4.132	120	4.657	−2.595	210	−2.627	−4.249	300	−4.947	2.244
32	3.089	4.038	122	4.551	−2.732	212	−2.787	−4.169	302	−4.853	2.396
34	3.256	3.938	124	4.441	−2.867	214	−2.945	−4.085	304	−4.752	2.545
36	3.419	3.834	126	4.326	−2.997	216	−3.100	−3.995	306	−4.645	2.691
38	3.576	3.724	128	4.207	−3.124	218	−3.251	−3.902	308	−4.532	2.834
40	3.729	3.610	130	4.082	−3.248	220	−3.399	−3.803	310	−4.413	2.973
42	3.877	3.491	132	3.954	−3.367	222	−3.543	−3.701	312	−4.287	3.108
44	4.019	3.368	134	3.821	−3.482	224	−3.684	−3.594	314	−4.156	3.240
46	4.156	3.240	136	3.684	−3.594	226	−3.821	−3.482	316	−4.019	3.368
48	4.287	3.108	138	3.543	−3.701	228	−3.954	−3.367	318	−3.877	3.491
50	4.413	2.973	140	3.399	−3.803	230	−4.082	−3.248	320	−3.729	3.610
52	4.532	2.834	142	3.251	−3.902	232	−4.207	−3.124	322	−3.576	3.724
54	4.645	2.691	144	3.100	−3.995	234	−4.326	−2.997	324	−3.419	3.834
56	4.752	2.545	146	2.945	−4.085	236	−4.441	−2.867	326	−3.256	3.938
58	4.853	2.396	148	2.787	−4.169	238	−4.551	−2.732	328	−3.089	4.038
60	4.947	2.244	150	2.627	−4.249	240	−4.657	−2.595	330	−2.918	4.132
62	5.035	2.089	152	2.464	−4.324	242	−4.757	−2.454	332	−2.742	4.221
64	5.116	1.932	154	2.298	−4.394	244	−4.851	−2.310	334	−2.563	4.304
66	5.190	1.773	156	2.131	−4.459	246	−4.941	−2.163	336	−2.380	4.382
68	5.257	1.611	158	1.961	−4.519	248	−5.024	−2.014	338	−2.194	4.453
70	5.318	1.448	160	1.789	−4.574	250	−5.102	−1.862	340	−2.004	4.519
72	5.372	1.283	162	1.615	−4.624	252	−5.175	−1.707	342	−1.812	4.579
74	5.419	1.117	164	1.439	−4.669	254	−5.241	−1.550	344	−1.617	4.633
76	5.459	0.950	166	1.263	−4.709	256	−5.301	−1.391	346	−1.420	4.681
78	5.492	0.782	168	1.085	−4.743	258	−5.355	−1.230	348	−1.221	4.723
80	5.518	0.613	170	0.905	−4.772	260	−5.403	−1.067	350	−1.020	4.758
82	5.537	0.444	172	0.725	−4.796	262	−5.444	−0.903	352	−0.818	4.787
84	5.549	0.274	174	0.545	−4.815	264	−5.479	−0.737	354	−0.614	4.810
86	5.554	0.105	176	0.363	−4.828	266	−5.508	−0.571	356	−0.410	4.826
88	5.553	−0.065	178	0.182	−4.836	268	−5.530	−0.403	358	−0.205	4.835
90	5.545	−0.234	180	0.000	−4.839	270	−5.545	−0.234	360	−0.000	4.839

Table 8.8: Deferential anomalies of Jupiter.

μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$	μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$	μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$	μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$
0	0.000	0.000	0.000	45	0.366	6.816	0.410	90	0.649	10.868	0.736	135	0.616	8.927	0.713
1	0.008	0.161	0.009	46	0.374	6.948	0.418	91	0.653	10.902	0.741	136	0.609	8.796	0.705
2	0.017	0.322	0.019	47	0.381	7.077	0.427	92	0.657	10.933	0.746	137	0.601	8.661	0.697
3	0.025	0.483	0.028	48	0.389	7.206	0.436	93	0.661	10.961	0.750	138	0.593	8.522	0.688
4	0.033	0.644	0.037	49	0.396	7.333	0.444	94	0.664	10.986	0.755	139	0.585	8.380	0.679
5	0.042	0.805	0.046	50	0.404	7.459	0.453	95	0.668	11.008	0.759	140	0.576	8.233	0.669
6	0.050	0.965	0.056	51	0.411	7.583	0.461	96	0.671	11.026	0.763	141	0.567	8.083	0.659
7	0.058	1.126	0.065	52	0.419	7.705	0.470	97	0.674	11.041	0.766	142	0.558	7.929	0.649
8	0.067	1.286	0.074	53	0.426	7.826	0.478	98	0.677	11.053	0.770	143	0.548	7.771	0.638
9	0.075	1.446	0.084	54	0.434	7.946	0.486	99	0.680	11.062	0.773	144	0.538	7.610	0.627
10	0.083	1.606	0.093	55	0.441	8.063	0.495	100	0.682	11.067	0.777	145	0.528	7.445	0.615
11	0.092	1.766	0.102	56	0.448	8.180	0.503	101	0.684	11.069	0.780	146	0.517	7.276	0.603
12	0.100	1.925	0.111	57	0.455	8.294	0.511	102	0.686	11.068	0.782	147	0.506	7.104	0.590
13	0.108	2.084	0.121	58	0.462	8.407	0.519	103	0.688	11.063	0.785	148	0.495	6.929	0.577
14	0.116	2.242	0.130	59	0.470	8.517	0.527	104	0.690	11.054	0.787	149	0.484	6.750	0.564
15	0.125	2.400	0.139	60	0.477	8.626	0.535	105	0.692	11.042	0.789	150	0.472	6.568	0.550
16	0.133	2.558	0.148	61	0.484	8.734	0.543	106	0.693	11.027	0.791	151	0.459	6.383	0.536
17	0.141	2.715	0.158	62	0.490	8.839	0.551	107	0.694	11.008	0.793	152	0.447	6.194	0.522
18	0.149	2.872	0.167	63	0.497	8.942	0.559	108	0.695	10.985	0.794	153	0.434	6.003	0.507
19	0.158	3.028	0.176	64	0.504	9.044	0.567	109	0.695	10.959	0.795	154	0.421	5.808	0.492
20	0.166	3.184	0.185	65	0.511	9.143	0.574	110	0.696	10.929	0.796	155	0.407	5.610	0.476
21	0.174	3.339	0.194	66	0.517	9.240	0.582	111	0.696	10.895	0.796	156	0.393	5.410	0.460
22	0.182	3.494	0.204	67	0.524	9.336	0.590	112	0.696	10.858	0.797	157	0.379	5.206	0.444
23	0.191	3.648	0.213	68	0.531	9.429	0.597	113	0.695	10.817	0.797	158	0.365	5.000	0.427
24	0.199	3.801	0.222	69	0.537	9.520	0.605	114	0.695	10.772	0.796	159	0.350	4.792	0.410
25	0.207	3.954	0.231	70	0.543	9.609	0.612	115	0.694	10.723	0.796	160	0.336	4.581	0.393
26	0.215	4.106	0.240	71	0.550	9.696	0.619	116	0.693	10.671	0.795	161	0.320	4.367	0.375
27	0.223	4.257	0.249	72	0.556	9.780	0.626	117	0.691	10.614	0.794	162	0.305	4.151	0.357
28	0.231	4.407	0.258	73	0.562	9.862	0.633	118	0.690	10.554	0.792	163	0.289	3.933	0.339
29	0.239	4.557	0.268	74	0.568	9.942	0.640	119	0.688	10.490	0.790	164	0.274	3.713	0.321
30	0.248	4.705	0.277	75	0.574	10.019	0.647	120	0.686	10.422	0.788	165	0.258	3.491	0.302
31	0.256	4.853	0.286	76	0.580	10.094	0.654	121	0.683	10.350	0.786	166	0.241	3.267	0.283
32	0.264	5.000	0.295	77	0.585	10.167	0.661	122	0.680	10.274	0.783	167	0.225	3.041	0.264
33	0.272	5.146	0.304	78	0.591	10.237	0.667	123	0.677	10.194	0.780	168	0.208	2.814	0.244
34	0.280	5.292	0.313	79	0.596	10.304	0.674	124	0.674	10.110	0.776	169	0.192	2.585	0.225
35	0.288	5.436	0.322	80	0.602	10.369	0.680	125	0.670	10.023	0.772	170	0.175	2.354	0.205
36	0.296	5.579	0.331	81	0.607	10.431	0.686	126	0.666	9.931	0.768	171	0.158	2.123	0.185
37	0.304	5.721	0.339	82	0.612	10.491	0.692	127	0.662	9.835	0.764	172	0.140	1.890	0.165
38	0.311	5.862	0.348	83	0.617	10.548	0.698	128	0.657	9.736	0.759	173	0.123	1.656	0.145
39	0.319	6.002	0.357	84	0.622	10.602	0.704	129	0.652	9.632	0.753	174	0.106	1.421	0.124
40	0.327	6.141	0.366	85	0.627	10.654	0.710	130	0.647	9.524	0.748	175	0.088	1.185	0.104
41	0.335	6.278	0.375	86	0.632	10.702	0.715	131	0.641	9.413	0.742	176	0.071	0.949	0.083
42	0.343	6.415	0.384	87	0.636	10.748	0.721	132	0.636	9.297	0.735	177	0.053	0.712	0.062
43	0.351	6.550	0.392	88	0.641	10.791	0.726	133	0.629	9.178	0.728	178	0.035	0.475	0.042
44	0.358	6.684	0.401	89	0.645	10.831	0.731	134	0.623	9.055	0.721	179	0.018	0.238	0.021
45	0.366	6.816	0.410	90	0.649	10.868	0.736	135	0.616	8.927	0.713	180	0.000	0.000	0.000

Table 8.9: Epicyclic anomalies of Jupiter. All quantities are in degrees. Note that $\bar{\theta}(360^\circ - \mu) = -\bar{\theta}(\mu)$, and $\delta\theta_\pm(360^\circ - \mu) = -\delta\theta_\pm(\mu)$.

Event	Date	λ	Event	Date	λ
Conjunction	08/05/2000	17TA53	Conjunction	22/10/2005	29LI16
Station (R)	29/09/2000	11GE13	Station (R)	04/03/2006	18SC54
Opposition	28/11/2000	06GE08	Opposition	04/05/2006	14SC03
Station (D)	25/01/2001	01GE10	Station (D)	06/07/2006	09SC02
Conjunction	14/06/2001	23GE30	Conjunction	22/11/2006	29SC34
Station (R)	02/11/2001	15CN41	Station (R)	06/04/2007	19SG49
Opposition	01/01/2002	10CN37	Opposition	06/06/2007	14SG57
Station (D)	01/03/2002	05CN37	Station (D)	07/08/2007	09SG58
Conjunction	20/07/2002	27CN11	Conjunction	23/12/2007	01CP03
Station (R)	04/12/2002	18LE06	Station (R)	09/05/2008	22CP23
Opposition	02/02/2003	13LE06	Opposition	09/07/2008	17CP30
Station (D)	04/04/2003	08LE03	Station (D)	08/09/2008	12CP33
Conjunction	22/08/2003	28LE55	Conjunction	24/01/2009	04AQ23
Station (R)	04/01/2004	18VI54	Station (R)	15/06/2009	27AQ01
Opposition	04/03/2004	13VI58	Opposition	14/08/2009	22AQ04
Station (D)	05/05/2004	08VI55	Station (D)	13/10/2009	17AQ10
Conjunction	22/09/2004	29VI21	Conjunction	28/02/2010	09PI43
Station (R)	02/02/2005	18LI53	Station (R)	23/07/2010	03AR20
Opposition	03/04/2005	14LI00	Opposition	21/09/2010	28PI19
Station (D)	05/06/2005	08LI58	Station (D)	18/11/2010	23PI26

Table 8.10: The conjunctions, oppositions, and stations of Jupiter during the years 2000–2010 AD. (R) indicates a retrograde station, and (D) a direct station.

Δt(JD)	$\Delta\bar{\lambda}(^\circ)$	$\Delta M(^\circ)$	$\Delta\bar{F}(^\circ)$	Δt(JD)	$\Delta\bar{\lambda}(^\circ)$	$\Delta M(^\circ)$	$\Delta\bar{F}(^\circ)$
10 000	335.083	334.815	334.779	1 000	33.508	33.482	33.478
20 000	310.166	309.630	309.559	2 000	67.017	66.963	66.956
30 000	285.249	284.446	284.338	3 000	100.525	100.445	100.434
40 000	260.332	259.261	259.118	4 000	134.033	133.926	133.912
50 000	235.415	234.076	233.897	5 000	167.541	167.408	167.390
60 000	210.498	208.891	208.677	6 000	201.050	200.889	200.868
70 000	185.581	183.706	183.456	7 000	234.558	234.371	234.346
80 000	160.664	158.522	158.236	8 000	268.066	267.852	267.824
90 000	135.747	133.337	133.015	9 000	301.575	301.334	301.302
100	3.351	3.348	3.348	10	0.335	0.335	0.335
200	6.702	6.696	6.696	20	0.670	0.670	0.670
300	10.052	10.044	10.043	30	1.005	1.004	1.004
400	13.403	13.393	13.391	40	1.340	1.339	1.339
500	16.754	16.741	16.739	50	1.675	1.674	1.674
600	20.105	20.089	20.087	60	2.010	2.009	2.009
700	23.456	23.437	23.435	70	2.346	2.344	2.343
800	26.807	26.785	26.782	80	2.681	2.679	2.678
900	30.157	30.133	30.130	90	3.016	3.013	3.013
1	0.034	0.033	0.033	0.1	0.003	0.003	0.003
2	0.067	0.067	0.067	0.2	0.007	0.007	0.007
3	0.101	0.100	0.100	0.3	0.010	0.010	0.010
4	0.134	0.134	0.134	0.4	0.013	0.013	0.013
5	0.168	0.167	0.167	0.5	0.017	0.017	0.017
6	0.201	0.201	0.201	0.6	0.020	0.020	0.020
7	0.235	0.234	0.234	0.7	0.023	0.023	0.023
8	0.268	0.268	0.268	0.8	0.027	0.027	0.027
9	0.302	0.301	0.301	0.9	0.030	0.030	0.030

Table 8.11: Mean motion of Saturn. Here, $\Delta t = t - t_0$, $\Delta\bar{\lambda} = \bar{\lambda} - \bar{\lambda}_0$, $\Delta M = M - M_0$, and $\Delta\bar{F} = \bar{F} - \bar{F}_0$. At epoch ($t_0 = 2\,451\,545.0$ JD), $\bar{\lambda}_0 = 50.059^\circ$, $M_0 = 317.857^\circ$, and $\bar{F}_0 = 296.482^\circ$.

$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$
0	0.000	5.386	90	6.172	−0.290	180	0.000	−5.386	270	−6.172	−0.290
2	0.230	5.383	92	6.154	−0.478	182	−0.201	−5.383	272	−6.183	−0.102
4	0.459	5.372	94	6.128	−0.664	184	−0.402	−5.374	274	−6.186	0.087
6	0.688	5.354	96	6.095	−0.850	186	−0.602	−5.360	276	−6.182	0.276
8	0.916	5.328	98	6.055	−1.034	188	−0.802	−5.339	278	−6.169	0.465
10	1.143	5.296	100	6.007	−1.217	190	−1.001	−5.313	280	−6.149	0.654
12	1.368	5.256	102	5.953	−1.397	192	−1.199	−5.281	282	−6.122	0.842
14	1.591	5.209	104	5.891	−1.576	194	−1.396	−5.243	284	−6.086	1.030
16	1.811	5.156	106	5.823	−1.753	196	−1.591	−5.200	286	−6.043	1.217
18	2.029	5.095	108	5.748	−1.927	198	−1.785	−5.150	288	−5.992	1.402
20	2.245	5.027	110	5.666	−2.098	200	−1.977	−5.095	290	−5.933	1.586
22	2.456	4.953	112	5.578	−2.267	202	−2.168	−5.035	292	−5.867	1.768
24	2.665	4.873	114	5.484	−2.433	204	−2.356	−4.969	294	−5.793	1.949
26	2.869	4.785	116	5.384	−2.596	206	−2.542	−4.897	296	−5.711	2.127
28	3.070	4.692	118	5.277	−2.755	208	−2.725	−4.820	298	−5.622	2.302
30	3.266	4.592	120	5.165	−2.911	210	−2.906	−4.737	300	−5.525	2.476
32	3.457	4.486	122	5.048	−3.063	212	−3.084	−4.649	302	−5.421	2.646
34	3.644	4.375	124	4.924	−3.211	214	−3.259	−4.556	304	−5.310	2.813
36	3.825	4.257	126	4.796	−3.356	216	−3.430	−4.458	306	−5.191	2.976
38	4.002	4.134	128	4.662	−3.496	218	−3.598	−4.354	308	−5.065	3.136
40	4.172	4.006	130	4.524	−3.632	220	−3.763	−4.246	310	−4.933	3.292
42	4.337	3.873	132	4.380	−3.764	222	−3.923	−4.133	312	−4.793	3.444
44	4.495	3.735	134	4.232	−3.892	224	−4.080	−4.015	314	−4.648	3.591
46	4.648	3.591	136	4.080	−4.015	226	−4.232	−3.892	316	−4.495	3.735
48	4.793	3.444	138	3.923	−4.133	228	−4.380	−3.764	318	−4.337	3.873
50	4.933	3.292	140	3.763	−4.246	230	−4.524	−3.632	320	−4.172	4.006
52	5.065	3.136	142	3.598	−4.354	232	−4.662	−3.496	322	−4.002	4.134
54	5.191	2.976	144	3.430	−4.458	234	−4.796	−3.356	324	−3.825	4.257
56	5.310	2.813	146	3.259	−4.556	236	−4.924	−3.211	326	−3.644	4.375
58	5.421	2.646	148	3.084	−4.649	238	−5.048	−3.063	328	−3.457	4.486
60	5.525	2.476	150	2.906	−4.737	240	−5.165	−2.911	330	−3.266	4.592
62	5.622	2.302	152	2.725	−4.820	242	−5.277	−2.755	332	−3.070	4.692
64	5.711	2.127	154	2.542	−4.897	244	−5.384	−2.596	334	−2.869	4.785
66	5.793	1.949	156	2.356	−4.969	246	−5.484	−2.433	336	−2.665	4.873
68	5.867	1.768	158	2.168	−5.035	248	−5.578	−2.267	338	−2.456	4.953
70	5.933	1.586	160	1.977	−5.095	250	−5.666	−2.098	340	−2.245	5.027
72	5.992	1.402	162	1.785	−5.150	252	−5.748	−1.927	342	−2.029	5.095
74	6.043	1.217	164	1.591	−5.200	254	−5.823	−1.753	344	−1.811	5.156
76	6.086	1.030	166	1.396	−5.243	256	−5.891	−1.576	346	−1.591	5.209
78	6.122	0.842	168	1.199	−5.281	258	−5.953	−1.397	348	−1.368	5.256
80	6.149	0.654	170	1.001	−5.313	260	−6.007	−1.217	350	−1.143	5.296
82	6.169	0.465	172	0.802	−5.339	262	−6.055	−1.034	352	−0.916	5.328
84	6.182	0.276	174	0.602	−5.360	264	−6.095	−0.850	354	−0.688	5.354
86	6.186	0.087	176	0.402	−5.374	266	−6.128	−0.664	356	−0.459	5.372
88	6.183	−0.102	178	0.201	−5.383	268	−6.154	−0.478	358	−0.230	5.383
90	6.172	−0.290	180	0.000	−5.386	270	−6.172	−0.290	360	−0.000	5.386

Table 8.12: Deferential anomalies of Saturn.

μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$	μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$	μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$	μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$
0	0.000	0.000	0.000	45	0.242	3.944	0.276	90	0.391	5.979	0.450	135	0.322	4.573	0.375
1	0.006	0.095	0.006	46	0.247	4.017	0.282	91	0.393	5.989	0.452	136	0.318	4.499	0.370
2	0.011	0.190	0.013	47	0.252	4.089	0.287	92	0.394	5.997	0.453	137	0.313	4.423	0.364
3	0.017	0.284	0.019	48	0.256	4.160	0.292	93	0.395	6.004	0.454	138	0.308	4.346	0.358
4	0.023	0.379	0.026	49	0.261	4.230	0.298	94	0.396	6.008	0.456	139	0.302	4.267	0.352
5	0.028	0.474	0.032	50	0.265	4.299	0.303	95	0.397	6.011	0.457	140	0.297	4.186	0.346
6	0.034	0.568	0.039	51	0.270	4.367	0.308	96	0.397	6.012	0.458	141	0.292	4.104	0.340
7	0.040	0.662	0.045	52	0.274	4.433	0.313	97	0.398	6.011	0.459	142	0.286	4.020	0.333
8	0.045	0.757	0.052	53	0.279	4.499	0.318	98	0.399	6.008	0.459	143	0.280	3.935	0.327
9	0.051	0.851	0.058	54	0.283	4.564	0.323	99	0.399	6.004	0.460	144	0.274	3.848	0.320
10	0.057	0.945	0.064	55	0.287	4.627	0.328	100	0.399	5.997	0.460	145	0.269	3.760	0.313
11	0.062	1.038	0.071	56	0.292	4.689	0.333	101	0.399	5.989	0.461	146	0.262	3.670	0.306
12	0.068	1.132	0.077	57	0.296	4.750	0.338	102	0.399	5.979	0.461	147	0.256	3.578	0.299
13	0.074	1.225	0.084	58	0.300	4.810	0.343	103	0.399	5.966	0.461	148	0.250	3.486	0.292
14	0.079	1.318	0.090	59	0.304	4.869	0.347	104	0.399	5.952	0.460	149	0.243	3.392	0.284
15	0.085	1.410	0.096	60	0.308	4.926	0.352	105	0.399	5.937	0.460	150	0.237	3.296	0.276
16	0.090	1.502	0.103	61	0.312	4.982	0.356	106	0.398	5.919	0.460	151	0.230	3.199	0.269
17	0.096	1.594	0.109	62	0.316	5.037	0.361	107	0.397	5.899	0.459	152	0.223	3.101	0.261
18	0.102	1.686	0.115	63	0.320	5.091	0.365	108	0.397	5.877	0.458	153	0.216	3.002	0.253
19	0.107	1.777	0.122	64	0.323	5.143	0.370	109	0.396	5.854	0.457	154	0.209	2.901	0.244
20	0.113	1.868	0.128	65	0.327	5.194	0.374	110	0.395	5.828	0.456	155	0.202	2.800	0.236
21	0.118	1.958	0.134	66	0.330	5.243	0.378	111	0.394	5.801	0.455	156	0.195	2.697	0.228
22	0.124	2.048	0.141	67	0.334	5.292	0.382	112	0.392	5.772	0.454	157	0.187	2.593	0.219
23	0.129	2.138	0.147	68	0.337	5.338	0.386	113	0.391	5.740	0.452	158	0.180	2.488	0.210
24	0.134	2.227	0.153	69	0.341	5.384	0.390	114	0.389	5.707	0.450	159	0.172	2.382	0.202
25	0.140	2.315	0.159	70	0.344	5.428	0.394	115	0.387	5.672	0.448	160	0.165	2.275	0.193
26	0.145	2.403	0.165	71	0.347	5.470	0.398	116	0.386	5.635	0.446	161	0.157	2.167	0.184
27	0.151	2.490	0.171	72	0.350	5.511	0.401	117	0.384	5.596	0.444	162	0.149	2.059	0.175
28	0.156	2.577	0.178	73	0.353	5.551	0.405	118	0.381	5.555	0.442	163	0.142	1.949	0.166
29	0.161	2.663	0.184	74	0.356	5.589	0.408	119	0.379	5.512	0.439	164	0.134	1.839	0.156
30	0.167	2.749	0.190	75	0.359	5.625	0.412	120	0.377	5.467	0.437	165	0.126	1.727	0.147
31	0.172	2.834	0.196	76	0.362	5.660	0.415	121	0.374	5.421	0.434	166	0.118	1.616	0.138
32	0.177	2.918	0.202	77	0.365	5.694	0.418	122	0.371	5.372	0.431	167	0.109	1.503	0.128
33	0.182	3.002	0.208	78	0.367	5.726	0.421	123	0.368	5.322	0.427	168	0.101	1.390	0.118
34	0.188	3.085	0.214	79	0.370	5.756	0.424	124	0.365	5.270	0.424	169	0.093	1.276	0.109
35	0.193	3.167	0.219	80	0.372	5.784	0.427	125	0.362	5.215	0.420	170	0.085	1.162	0.099
36	0.198	3.248	0.225	81	0.375	5.811	0.430	126	0.359	5.159	0.417	171	0.076	1.047	0.089
37	0.203	3.329	0.231	82	0.377	5.837	0.433	127	0.355	5.101	0.413	172	0.068	0.932	0.080
38	0.208	3.409	0.237	83	0.379	5.861	0.435	128	0.352	5.042	0.409	173	0.060	0.816	0.070
39	0.213	3.488	0.243	84	0.381	5.883	0.438	129	0.348	4.980	0.404	174	0.051	0.700	0.060
40	0.218	3.566	0.248	85	0.383	5.903	0.440	130	0.344	4.917	0.400	175	0.043	0.584	0.050
41	0.223	3.644	0.254	86	0.385	5.922	0.442	131	0.340	4.851	0.395	176	0.034	0.467	0.040
42	0.228	3.720	0.260	87	0.387	5.939	0.444	132	0.336	4.784	0.390	177	0.026	0.351	0.030
43	0.233	3.796	0.265	88	0.388	5.954	0.446	133	0.331	4.716	0.386	178	0.017	0.234	0.020
44	0.238	3.871	0.271	89	0.390	5.967	0.448	134	0.327	4.645	0.380	179	0.009	0.117	0.010
45	0.242	3.944	0.276	90	0.391	5.979	0.450	135	0.322	4.573	0.375	180	0.000	0.000	0.000

Table 8.13: Epicyclic anomalies of Saturn. All quantities are in degrees. Note that $\bar{\theta}(360^\circ - \mu) = -\bar{\theta}(\mu)$, and $\delta\theta_\pm(360^\circ - \mu) = -\delta\theta_\pm(\mu)$.

Event	Date	λ	Event	Date	λ
Conjunction	10/05/2000	20TA26	Station (R)	22/11/2005	11LE19
Station (R)	12/09/2000	00GE59	Opposition	27/01/2006	07LE51
Opposition	19/11/2000	27TA29	Station (D)	05/04/2006	04LE22
Station (D)	24/01/2001	24TA03	Conjunction	07/08/2006	14LE51
Conjunction	25/05/2001	04GE22	Station (R)	06/12/2006	25LE04
Station (R)	26/09/2001	14GE59	Opposition	10/02/2007	21LE38
Opposition	03/12/2001	11GE29	Station (D)	19/04/2007	18LE09
Station (D)	08/02/2002	08GE02	Conjunction	22/08/2007	28LE32
Conjunction	09/06/2002	18GE28	Station (R)	19/12/2007	08VI34
Station (R)	11/10/2002	29GE06	Opposition	24/02/2008	05VI10
Opposition	17/12/2002	25GE36	Station (D)	03/05/2008	01VI41
Station (D)	22/02/2003	22GE08	Conjunction	04/09/2008	11VI56
Conjunction	24/06/2003	02CN39	Station (R)	31/12/2008	21VI46
Station (R)	25/10/2003	13CN15	Opposition	08/03/2009	18VI23
Opposition	31/12/2003	09CN46	Station (D)	17/05/2009	14VI56
Station (D)	07/03/2004	06CN17	Conjunction	17/09/2009	25VI01
Conjunction	08/07/2004	16CN50	Station (R)	13/01/2010	04LI40
Opposition	22/03/2010	01LI18	Station (R)	08/11/2004	27CN21
Opposition	13/01/2005	23CN52	Station (D)	30/05/2010	27VI51
Station (D)	22/03/2005	20CN23	Conjunction	01/10/2010	07LI46
Conjunction	23/07/2005	00LE56			

Table 8.14: The conjunctions, oppositions, and stations of Saturn during the years 2000–2010 AD. (R) indicates a retrograde station, and (D) a direct station.

9. The inferior planets

9.1 Determination of ecliptic longitude

Figure 9.1 compares and contrasts heliocentric and geocentric models of the motion of an inferior planet (that is, a planet that is closer to the Sun than the Earth), P, as seen from the Earth, G. The Sun is at S. As before, in the heliocentric model the Earth-planet displacement vector, **P**, is the sum of the Earth-Sun displacement vector, **S**, and the Sun-planet displacement vector, $\mathbf{P}'$. On the other hand, in the geocentric model **S** gives the displacement of the guide-point, G', from the Earth. Because **S** is also the displacement of the Sun, S, from the Earth, G, it is clear that G' executes a Keplerian orbit about the Earth whose elements are the same as those of the apparent orbit of the Sun about the Earth. This implies that the Sun is coincident with G'. The ellipse traced out by G' is termed the deferent. The vector $\mathbf{P}'$ gives the displacement of the planet, P, from the guide-point, G'. Because $\mathbf{P}'$ is also the displacement of the planet, P, from the Sun, S, it is clear that P executes a Keplerian orbit about the guide-point whose elements are the same as those of the orbit of the planet about the Sun. The ellipse traced out by P about G' is termed the epicycle.

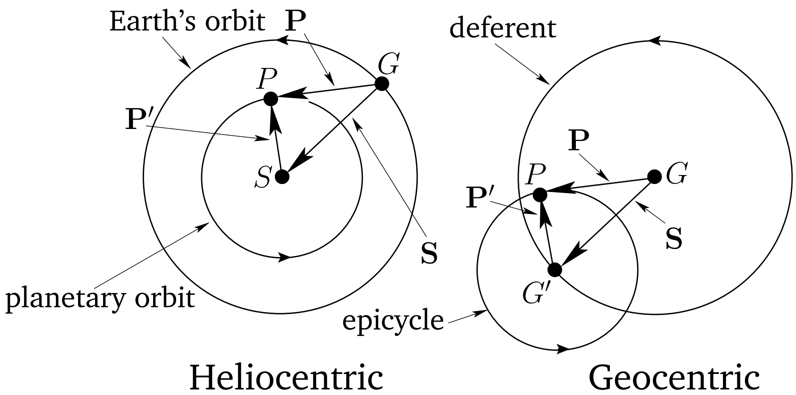

Figure 9.1: Heliocentric and geocentric models of the motion of an inferior planet. Here, S is the Sun, G the Earth, and P the planet. View is from the northern ecliptic pole.

As we have seen, the deferent of a superior planet has the same elements as the planet's orbit about the Sun, whereas the epicycle has the same elements as the Sun's apparent orbit about the Earth. On the other hand, the deferent of an inferior planet has the same elements as the Sun's apparent orbit about the Earth, whereas the epicycle has the same elements as the planet's orbit about the Sun. It follows that we can formulate a procedure for determining the ecliptic longitude of an inferior planet by simply taking the procedure used in the previous chapter for determining the ecliptic longitude of a superior planet and exchanging the roles of the Sun and the planet.

Our procedure is described in the following. As before, it is assumed that the ecliptic longitude, λ_S, and the radial anomaly, ζ_S, of the Sun have already been calculated. In the following, a, e, n, $\tilde{n}$, $\bar{\lambda}_0$, and M_0 represent elements of the orbit of the planet in question about the Sun, whereas e_S is the eccentricity of the Sun's apparent orbit about the Earth. Again, a is the major radius of the planetary orbit in units in which the major radius of the Sun's apparent orbit about the Earth is unity. The requisite elements for all of the inferior planets at the J2000 epoch ($t_0 = 2\,451\,545.0$ JD) are listed in Table 5.1. The ecliptic longitude of an inferior planet is specified by the following formulae:

$$\bar{\lambda} = \bar{\lambda}_0 + n\,(t - t_0), \tag{9.1}$$

$$M = M_0 + \tilde{n}\,(t - t_0), \tag{9.2}$$

$$q = 2\,e\,\sin\,M + (5/4)\,e^2\,\sin\,2M, \tag{9.3}$$

$$\zeta = e\,\cos M - e^2\,\sin^2 M, \tag{9.4}$$

$$\mu = \bar{\lambda} + q - \lambda_S, \tag{9.5}$$

$$\bar{\theta} = \theta(\mu, \bar{z}) \equiv \tan^{-1}\left(\frac{\sin\mu}{a^{-1}\,\bar{z} + \cos\mu}\right), \tag{9.6}$$

$$\delta\theta_- = \theta(\mu, \bar{z}) - \theta(\mu, z_{\rm max}), \tag{9.7}$$

$$\delta\theta_+ = \theta(\mu, z_{\rm min}) - \theta(\mu, \bar{z}), \tag{9.8}$$

$$z = \frac{1 - \zeta_S}{1 - \zeta}, \tag{9.9}$$

$$\xi = \frac{\bar{z} - z}{\delta z}, \tag{9.10}$$

$$\theta = \Theta_-(\xi)\,\delta\theta_- + \bar{\theta} + \Theta_+(\xi)\,\delta\theta_+, \tag{9.11}$$

$$\lambda = \lambda_S + \theta. \tag{9.12}$$

Here, $\bar{z} = (1+e\,e_S)/(1-e^2)$, $\delta z = (e+e_S)/(1-e^2)$, $z_{\rm min} = \bar{z}-\delta z$, and $z_{\rm max} = \bar{z}+\delta z$. The constants $\bar{z}$, δz, $z_{\rm min}$, and $z_{\rm max}$ for each of the inferior planets are listed in Table. 2.4. Finally, the functions $\Theta_\pm$ are tabulated in Table 2.16.

For the case of Venus, the previous formulae are capable of matching NASA ephemeris data during the years 1995–2006 AD with a mean error of $2'$ and a maximum error of $10'$. For the case of Mercury, given its relatively large eccentricity of 0.205636, it is necessary to modify the formulae slightly by expressing q and ζ to third-order in the eccentricity:

$$q = [2\,e - (1/4)\,e^3]\ \sin\,M + (5/4)\,e^2\ \sin\,2M + (13/12)\,e^3\ \sin\,3M, \tag{9.13}$$

$$\zeta = -(1/2)\,e^2 + [e - (3/8)\,e^3]\ \cos\,M + (1/2)\,e^2\ \cos\,2M + (3/8)\,e^3\ \cos\,3M. \tag{9.14}$$

With this modification, the mean error is $6'$ and the maximum error $28'$. Our modification of the planetary longitude model for the case of Mercury can be thought of as being equivalent to the additional epicycle that Ptolemy introduced into his model of Mercury's orbit. In both cases, the amendment to the model is necessitated by Mercury's particularly large orbital eccentricity.

9.2 Determination of ecliptic longitude of Venus

The ecliptic longitude of Venus can be determined with the aid of Tables 9.1–9.3. Table 9.1 allows the mean longitude, $\bar{\lambda}$, and the mean anomaly, M, of Venus to be calculated as functions of time. Next, Table 9.2 permits the equation of center, q, and the radial anomaly, ζ, to be determined as functions of the mean anomaly. Finally, Table 9.3 allows the quantities $\delta\theta_-$, $\bar{\theta}$, and $\delta\theta_+$ to be calculated as functions of the epicyclic anomaly, μ.

The procedure for using the tables is as follows:

1. Determine the fractional Julian day number, t, corresponding to the date and time at which the ecliptic longitude is to be calculated with the aid of Tables 3.1–3.3. Form $\Delta t = t - t_0$, where $t_0 = 2\,451\,545.0$ is the epoch.

2. Calculate the ecliptic longitude, λ_S, and radial anomaly, ζ_S, of the Sun using the procedure set out in Section 5.2.

3. Enter Table 9.1 with the digit for each power of 10 in Δt and take out the corresponding values of $\Delta\bar{\lambda}$ and ΔM. If Δt is negative then the corresponding values are also negative. The value of the mean longitude, $\bar{\lambda}$, is the sum of all the $\Delta\bar{\lambda}$ values plus value of $\bar{\lambda}$ at the epoch. Likewise, the value of the mean anomaly, M, is the sum of all the ΔM values plus the value of M at the epoch. Add as many multiples of 360° to $\bar{\lambda}$ and M as is required to make them both fall in the range 0° to 360°. Round M to the nearest degree.

4. Enter Table 9.2 with the value of M and take out the corresponding value of the equation of center, q, and the radial anomaly, ζ. It is necessary to interpolate if M is odd.

5. Form the epicyclic anomaly, $\mu = \bar{\lambda} + q - \lambda_S$. Add as many multiples of 360° to μ as is required to make it fall in the range 0° to 360°. Round μ to the nearest degree.

6. Enter Table 9.3 with the value of μ and take out the corresponding values of $\delta\theta_-$, $\bar{\theta}$, and $\delta\theta_+$. If $\mu > 180^\circ$ then it is necessary to make use of the identities $\delta\theta_\pm(360^\circ - \mu) = -\delta\theta_\pm(\mu)$ and $\bar{\theta}(360^\circ - \mu) = -\bar{\theta}(\mu)$.

7. Form $z = (1 - \zeta_S)/(1 - \zeta)$.
8. Obtain the values of $\bar{z}$ and δz from Table 2.4. Form $\xi = (\bar{z} - z)/\delta z$.
9. Enter Table 2.16 with the value of ξ and take out the corresponding values of Θ_- and Θ_+. If $\xi < 0$ then it is necessary to use the identities $\Theta_+(\xi) = -\Theta_-(-\xi)$ and $\Theta_-(\xi) = -\Theta_+(-\xi)$.
10. Form the equation of the epicycle, $\theta = \Theta_-\,\delta\theta_- + \bar{\theta} + \Theta_+\,\delta\theta_+$.
11. The ecliptic longitude, λ, is the sum of the ecliptic longitude of the Sun, λ_S, and the equation of the epicycle, θ. If necessary convert λ into an angle in the range 0° to 360°. The decimal fraction can be converted into arc minutes using Table 5.2. Round to the nearest arc minute. The final result can be written in terms of the signs of the zodiac using the table in Section 2.6.

Two examples of this procedure are given in the following.

Example 1: May 5, 2005 AD, 00:00 UT:

From Chapter 8, $t - t_0 = 1\,950.5$ JD, $\lambda_S = 44.602^\circ$, and $\zeta_S = -8.56 \times 10^{-3}$. Making use of Table 9.1, we find:

t(JD)	$\bar{\lambda}(^\circ)$	$M(^\circ)$
+1000	162.169	162.130
+900	1.952	1.917
+50	80.108	80.107
+.5	0.801	0.801
Epoch	181.973	49.237
	427.003	294.192
Modulus	67.003	294.192

Given that $M \simeq 294^\circ$, Table 9.2 yields

$$q(294^\circ) = -0.712^\circ, \qquad \zeta(294^\circ) = 2.72 \times 10^{-3},$$

so

$$\mu = \bar{\lambda} + q - \bar{\lambda}_S = 67.003 - 0.712 - 44.602 = 21.689 \simeq 22^\circ.$$

It follows from Table 9.3 that

$$\delta\theta_-(22^\circ) = 0.126^\circ, \qquad \bar{\theta}(22^\circ) = 9.212^\circ, \qquad \delta\theta_+(22^\circ) = 0.129^\circ.$$

Now,

$$z = (1 - \zeta_S)/(1 - \zeta) = (1 + 8.56 \times 10^{-3})/(1 - 2.72 \times 10^{-3}) = 1.01131.$$

However, from Table 2.4, $\bar{z} = 1.00016$ and $\delta z = 0.02349$, so

$$\xi = (\bar{z} - z)/\delta z = (1.00016 - 1.01131)/0.02349 \simeq -0.48.$$

According to Table 2.16,

$$\Theta_-(-0.48) = -0.355, \qquad \Theta_+(-0.48) = -0.125,$$

so

$$\begin{aligned}\theta &= \Theta_-\,\delta\theta_- + \bar{\theta} + \Theta_+\,\delta\theta_+\\ &= -0.355 \times 0.126 + 9.212 - 0.125 \times 0.129\\ &= 9.151^\circ.\end{aligned}$$

Finally,

$$\lambda = \bar{\lambda}_S + \theta = 44.602 + 9.151 = 53.753 \simeq 53^\circ 45'.$$

Thus, the ecliptic longitude of Venus at 00:00 UT on May 5, 2005 AD was 23TA45.

Example 2: December 25, 1800 AD, 00:00 UT:

From Chapter 8, $t - t_0 = -72\,690.5$ JD, $\lambda_S = 273.055^\circ$, and $\zeta_S = 1.662 \times 10^{-2}$. Making use of Table 9.1, we find:

t(JD)	$\bar{\lambda}(^\circ)$	$M(^\circ)$
-70,000	-191.810	-189.128
-2,000	-324.337	-324.261
-600	-241.301	-241.278
-90	-144.195	-144.192
-.5	-0.801	-0.801
Epoch	181.973	49.237
	-720.471	-850.423
Modulus	359.529	229.577

Given that $M \simeq 230^\circ$, Table 9.2 yields

$$q(230^\circ) = -0.592^\circ, \qquad \zeta(230^\circ) = -4.38 \times 10^{-3},$$

so

$$\mu = \bar{\lambda} + q - \bar{\lambda}_S = 359.529 - 0.592 - 273.055 = 85.882 \simeq 86^\circ.$$

It follows from Table 9.3 that

$$\delta\theta_-(86^\circ) = 0.589^\circ, \qquad \bar{\theta}(86^\circ) = 34.482^\circ, \qquad \delta\theta_+(86^\circ) = 0.607^\circ.$$

Now,

$$z = (1 - \zeta_S)/(1 - \zeta) = (1 - 1.662 \times 10^{-2})/(1 + 4.38 \times 10^{-3}) = 0.97909,$$

so

$$\xi = (\bar{z} - z)/\delta z = (1.00016 - 0.97909)/0.02349 \simeq 0.90.$$

According to Table 2.16,

$$\Theta_-(0.90) = 0.045, \qquad \Theta_+(0.90) = 0.855,$$

so

$$\begin{aligned}\theta &= \Theta_-\,\delta\theta_- + \bar{\theta} + \Theta_+\,\delta\theta_+\\ &= 0.045 \times 0.589 + 34.482 + 0.855 \times 0.607\\ &= 35.027^\circ.\end{aligned}$$

Finally,

$$\lambda = \bar{\lambda}_S + \theta = 273.055 + 35.027 = 308.082 \simeq 308^\circ 5'.$$

Thus, the ecliptic longitude of Venus at 00:00 UT on December 25, 1800 AD was 8AQ5.

9.3 Conjunction and greatest elongation dates

The geocentric orbit of an inferior planet is similar to that of the superior planet shown in Figure 8.4, except for the fact that the Sun is coincident with guide-point G' in the former case. It follows that it is impossible for an inferior planet to have an opposition with the Sun (that is, for the Earth to lie directly between the planet and the Sun). However, inferior planets do have two different kinds of conjunctions with the Sun. A *superior conjunction* takes place when the Sun lies directly between the planet and the Earth. Conversely, an *inferior conjunction* takes place when the planet lies directly between the Sun and the Earth. It is clear from Figure 8.4 that a superior conjunction corresponds to $\mu = 0^\circ$, and an inferior conjunction to $\mu = 180^\circ$. Now, the equation of the epicycle, θ, measures the angular separation between the planet and the Sun (because the Sun lies at the guide-point). It is evident from Figure 8.4 that θ attains a maximum and a minimum value each time the planet revolves around its epicycle. In other words, there is a limit to how large the angular separation between an inferior planet and the Sun can become. The maximum value is termed the *greatest eastern elongation* of the planet, whereas the modulus of the minimum value is termed the *greatest western elongation*.

Tables 9.1–9.3 can be used to determine the dates of the conjunctions and greatest elongations of Venus. Consider the first superior conjunction after the epoch (January 1, 2000 AD). We can estimate the time at which this event occurs by approximating the epicyclic anomaly as the mean epicyclic anomaly:

$$\begin{aligned}\mu \simeq \bar{\mu} = \bar{\lambda} - \bar{\lambda}_S &= \bar{\lambda}_0 - \bar{\lambda}_{0\,S} + (n - n_S)\,(t - t_0)\\ &= 261.515 + 0.61652137\,(t - t_0).\end{aligned}$$

Thus,

$$t \simeq t_0 + (360 - 261.515)/0.61652137 \simeq t_0 + 160\,\text{JD}.$$

A calculation of the epicyclic anomaly at this time, using Tables 9.1–9.3, yields $\mu = -1.267^\circ$. Now, the actual conjunction takes place when $\mu = 0^\circ$. Hence, our final estimate is

$$t = t_0 + 160 + 1.267/0.61652137 = t_0 + 162.1\,\text{JD},$$

which corresponds to June 11, 2000 AD.

Consider the first inferior conjunction of Venus after the epoch. Our first estimate of the time at which this event takes place is

$$t \simeq t_0 + (540 - 261.515)/0.61652137 \simeq t_0 + 452\,\text{JD}.$$

A calculation of the epicyclic anomaly at this time yields $\mu = 178.900^\circ$. Now, the actual conjunction takes place when $\mu = 180^\circ$. Hence, our final estimate is

$$t = t_0 + 452 + 1.100/0.61652137 = t_0 + 453.8\,\text{JD},$$

which corresponds to March 30, 2001 AD. Incidentally, it is clear from the previous analysis that the mean time period between successive superior, or inferior, conjunctions of Venus is $360/0.61652137 = 583.9$ JD, which is equivalent to 1.60 years.

Consider the greatest elongations of Venus. We can approximate the equation of the epicycle as

$$\theta \simeq \bar{\theta} = \tan^{-1}\left(\frac{\sin\bar{\mu}}{\bar{a}^{-1} + \cos\bar{\mu}}\right), \tag{9.15}$$

where $\bar{\mu}$ is the mean epicyclic anomaly, and $\bar{a} = a/\bar{z}$. It follows that

$$\frac{d\bar{\theta}}{d\bar{\mu}} = \frac{\bar{a}^{-1}\,\cos\bar{\mu} + 1}{1 + 2\,\bar{a}^{-1}\,\cos\bar{\mu} + \bar{a}^{-2}}. \tag{9.16}$$

Now, $\bar{\theta}$ attains its maximum or minimum value when $d\bar{\theta}/d\bar{\mu} = 0$: *i.e.*, when

$$\bar{\mu} = \cos^{-1}(-\bar{a}). \tag{9.17}$$

For the case of Venus, we obtain $\bar{\mu} = 136.3^\circ$ or 223.7°. The first solution corresponds to the greatest eastern elongation, and the second to the greatest western elongation. Substituting back into Equation (9.15), we find that $\bar{\theta} = \pm 46.3^\circ$. Hence, the mean value of the greatest eastern or western elongation of Venus is 46.3°. The mean time period between a greatest eastern elongation and the following inferior conjunction, or between an inferior conjunction and the following greatest western elongation, is $(180-136.3)/0.61652137 \simeq 71$ JD. Unfortunately, the only option for accurately determining the dates at which the greatest elongations occur is to calculate the equation of the epicycle of Venus over a range of days centered 71 days before and after an inferior conjunction.

Table 9.4 shows the conjunctions, and greatest elongations of Venus for the years 2000–2015 AD, calculated using the previously described techniques.

9.4 *Determination of ecliptic longitude of Mercury*

The ecliptic longitude of Mercury can be determined with the aid of Tables 9.5–9.7. Table 9.5 allows the mean longitude, $\bar{\lambda}$, and the mean anomaly, M, of Mercury to be calculated as functions of time. Next, Table 9.6 permits the equation of center, q, and the radial anomaly, ζ, to be determined as functions of the mean anomaly. Finally, Table 9.7 allows the quantities $\delta\theta_-$, $\bar{\theta}$, and $\delta\theta_+$ to be calculated as functions of the epicyclic anomaly, μ. The procedure for using the tables is analogous to the previously described procedure for using the Venus tables. One example of this procedure is given in the following.

Example: May 5, 2005 AD, 00:00 UT:

From Chapter 8, $t - t_0 = 1\,950.5$ JD, $\lambda_S = 44.602°$, and $\zeta_S = -8.56 \times 10^{-3}$. Making use of Table 9.5, we find:

t(JD)	$\bar{\lambda}(°)$	$M(°)$
+1000	132.377	132.334
+900	83.139	83.101
+50	204.619	204.617
+.5	2.046	2.046
Epoch	252.087	174.693
	647.268	596.791
Modulus	314.268	236.791

Given that $M \simeq 237°$, Table 9.6 yields

$$q(237°) = -16.974°, \qquad \zeta(237°) = -1.367 \times 10^{-1},$$

so

$$\mu = \bar{\lambda} + q - \bar{\lambda}_S = 314.268 - 16.974 - 44.602 = 252.692 \simeq 253°.$$

It follows from Table 9.7 that

$$\delta\theta_-(253°) = -4.005°, \qquad \bar{\theta}(253°) = -21.609°, \qquad \delta\theta_+(253°) = -6.182°.$$

Now,

$$z = (1 - \zeta_S)/(1 - \zeta) = (1 + 8.56 \times 10^{-3})/(1 + 1.367 \times 10^{-1}) = 0.8873.$$

However, from Table 2.4, $\bar{z} = 1.04774$ and $\delta z = 0.23216$, so

$$\xi = (\bar{z} - z)/\delta z = (1.04774 - 0.8873)/0.23216 \simeq 0.69.$$

According to Table 2.16,

$$\Theta_-(0.69) = 0.107, \qquad \Theta_+(0.69) = 0.583,$$

so

$$\begin{aligned}\theta &= \Theta_-\,\delta\theta_- + \bar{\theta} + \Theta_+\,\delta\theta_+\\ &= -0.107\times 4.005 - 21.609 - 0.583\times 6.182\\ &= -25.642^\circ.\end{aligned}$$

Finally,

$$\lambda = \bar{\lambda}_S + \theta = 44.602 - 25.642 = 18.960 \simeq 18^\circ 58'.$$

Thus, the ecliptic longitude of Mercury at 00:00 UT on May 5, 2005 AD was 18AR58.

The conjunctions and elongations of Mercury can be investigated using analogous methods to those employed earlier to examine the conjunctions and elongations of Venus. We find that the mean time period between successive superior, or inferior, conjunctions of Mercury is 116 days. On average, the greatest eastern and western elongations of Mercury occur when the epicyclic anomaly takes the values $\mu = 111.7^\circ$ and 248.3°, respectively. Furthermore, the mean value of the greatest eastern or western elongation is 21.7°. Finally, the mean time period between a greatest eastern elongation and the following inferior conjunction, or between the inferior conjunction and the following greatest western elongation, is 22 JD. The conjunctions and elongations of Mercury during the years 2000–2002 AD are shown in Table 9.8.

As before, the information contained in the mean motion tables, 9.1 and 9.5, and the deferential anomaly tables, 9.2 and 9.6, is essentially equivalent to that contained in the "Tables of the mean longitudinal motion and anomalies of the five stars" (*Κανόνες μέσων κινήσεων μήκους τε καὶ ἀνωμαλίας τῶν πέντε ἀστέρων*) that appear in Section 4 of Book IX of the Almagest. Likewise, the information contained in the epicyclic anomaly tables, 9.3 and 9.7, is equivalent to that contained in the "Tables of the longitudinal corrections of the five planets" (*Κανόνες τῆς κατὰ μῆκος τῶν πέντε πλανωμένων διευκρινήσεως*) that appear in Section 11 of Book XI of the Almagest. The computation of the greatest elongations of the inferior planets is discussed in Book XII of the Almagest.

9.5 Tables

Δt(JD)	$\Delta\bar{\lambda}(^\circ)$	$\Delta M(^\circ)$	$\Delta\bar{F}(^\circ)$	Δt(JD)	$\Delta\bar{\lambda}(^\circ)$	$\Delta M(^\circ)$	$\Delta\bar{F}(^\circ)$
10 000	181.687	181.304	181.381	1 000	162.169	162.130	162.138
20 000	3.374	2.608	2.761	2 000	324.337	324.261	324.276
30 000	185.062	183.912	184.142	3 000	126.506	126.391	126.414
40 000	6.749	5.216	5.523	4 000	288.675	288.522	288.552
50 000	188.436	186.520	186.904	5 000	90.844	90.652	90.690
60 000	10.123	7.824	8.284	6 000	253.012	252.782	252.828
70 000	191.810	189.128	189.665	7 000	55.181	54.913	54.966
80 000	13.498	10.432	11.046	8 000	217.350	217.043	217.105
90 000	195.185	191.736	192.426	9 000	19.518	19.174	19.243
100	160.217	160.213	160.214	10	16.022	16.021	16.021
200	320.434	320.426	320.428	20	32.043	32.043	32.043
300	120.651	120.639	120.641	30	48.065	48.064	48.064
400	280.867	280.852	280.855	40	64.087	64.085	64.086
500	81.084	81.065	81.069	50	80.108	80.107	80.107
600	241.301	241.278	241.283	60	96.130	96.128	96.128
700	41.518	41.491	41.497	70	112.152	112.149	112.150
800	201.735	201.704	201.710	80	128.173	128.170	128.171
900	1.952	1.917	1.924	90	144.195	144.192	144.192
1	1.602	1.602	1.602	0.1	0.160	0.160	0.160
2	3.204	3.204	3.204	0.2	0.320	0.320	0.320
3	4.807	4.806	4.806	0.3	0.481	0.481	0.481
4	6.409	6.409	6.409	0.4	0.641	0.641	0.641
5	8.011	8.011	8.011	0.5	0.801	0.801	0.801
6	9.613	9.613	9.613	0.6	0.961	0.961	0.961
7	11.215	11.215	11.215	0.7	1.122	1.121	1.121
8	12.817	12.817	12.817	0.8	1.282	1.282	1.282
9	14.420	14.419	14.419	0.9	1.442	1.442	1.442

Table 9.1: Mean motion of Venus. Here, $\Delta t = t - t_0$, $\Delta\bar{\lambda} = \bar{\lambda} - \bar{\lambda}_0$, $\Delta M = M - M_0$, and $\Delta\bar{F} = \bar{F} - \bar{F}_0$. At epoch ($t_0 = 2\,451\,545.0$ JD), $\bar{\lambda}_0 = 181.973^\circ$, $M_0 = 49.237^\circ$, and $\bar{F}_0 = 105.253^\circ$.

$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$
0	0.000	0.678	90	0.777	−0.005	180	0.000	−0.678	270	−0.777	−0.005
2	0.027	0.677	92	0.776	−0.028	182	−0.027	−0.677	272	−0.776	0.019
4	0.055	0.676	94	0.774	−0.052	184	−0.054	−0.676	274	−0.775	0.043
6	0.082	0.674	96	0.772	−0.075	186	−0.080	−0.674	276	−0.773	0.066
8	0.109	0.671	98	0.768	−0.099	188	−0.107	−0.671	278	−0.770	0.090
10	0.136	0.667	100	0.764	−0.122	190	−0.134	−0.668	280	−0.766	0.113
12	0.163	0.663	102	0.758	−0.145	192	−0.160	−0.663	282	−0.761	0.137
14	0.189	0.657	104	0.752	−0.168	194	−0.186	−0.658	284	−0.755	0.160
16	0.216	0.651	106	0.745	−0.191	196	−0.212	−0.652	286	−0.748	0.183
18	0.242	0.644	108	0.737	−0.214	198	−0.238	−0.645	288	−0.741	0.205
20	0.268	0.636	110	0.728	−0.236	200	−0.263	−0.637	290	−0.732	0.228
22	0.293	0.628	112	0.718	−0.258	202	−0.289	−0.629	292	−0.722	0.250
24	0.318	0.618	114	0.707	−0.279	204	−0.313	−0.620	294	−0.712	0.272
26	0.343	0.608	116	0.695	−0.301	206	−0.338	−0.610	296	−0.701	0.293
28	0.367	0.597	118	0.683	−0.322	208	−0.362	−0.599	298	−0.688	0.315
30	0.391	0.586	120	0.670	−0.342	210	−0.385	−0.588	300	−0.675	0.335
32	0.414	0.573	122	0.656	−0.362	212	−0.409	−0.576	302	−0.662	0.356
34	0.437	0.560	124	0.641	−0.382	214	−0.431	−0.563	304	−0.647	0.376
36	0.460	0.547	126	0.625	−0.401	216	−0.453	−0.550	306	−0.631	0.395
38	0.481	0.532	128	0.609	−0.420	218	−0.475	−0.536	308	−0.615	0.414
40	0.502	0.517	130	0.592	−0.438	220	−0.496	−0.521	310	−0.598	0.433
42	0.523	0.502	132	0.574	−0.456	222	−0.516	−0.506	312	−0.580	0.451
44	0.543	0.485	134	0.555	−0.473	224	−0.536	−0.490	314	−0.562	0.468
46	0.562	0.468	136	0.536	−0.490	226	−0.555	−0.473	316	−0.543	0.485
48	0.580	0.451	138	0.516	−0.506	228	−0.574	−0.456	318	−0.523	0.502
50	0.598	0.433	140	0.496	−0.521	230	−0.592	−0.438	320	−0.502	0.517
52	0.615	0.414	142	0.475	−0.536	232	−0.609	−0.420	322	−0.481	0.532
54	0.631	0.395	144	0.453	−0.550	234	−0.625	−0.401	324	−0.460	0.547
56	0.647	0.376	146	0.431	−0.563	236	−0.641	−0.382	326	−0.437	0.560
58	0.662	0.356	148	0.409	−0.576	238	−0.656	−0.362	328	−0.414	0.573
60	0.675	0.335	150	0.385	−0.588	240	−0.670	−0.342	330	−0.391	0.586
62	0.688	0.315	152	0.362	−0.599	242	−0.683	−0.322	332	−0.367	0.597
64	0.701	0.293	154	0.338	−0.610	244	−0.695	−0.301	334	−0.343	0.608
66	0.712	0.272	156	0.313	−0.620	246	−0.707	−0.279	336	−0.318	0.618
68	0.722	0.250	158	0.289	−0.629	248	−0.718	−0.258	338	−0.293	0.628
70	0.732	0.228	160	0.263	−0.637	250	−0.728	−0.236	340	−0.268	0.636
72	0.741	0.205	162	0.238	−0.645	252	−0.737	−0.214	342	−0.242	0.644
74	0.748	0.183	164	0.212	−0.652	254	−0.745	−0.191	344	−0.216	0.651
76	0.755	0.160	166	0.186	−0.658	256	−0.752	−0.168	346	−0.189	0.657
78	0.761	0.137	168	0.160	−0.663	258	−0.758	−0.145	348	−0.163	0.663
80	0.766	0.113	170	0.134	−0.668	260	−0.764	−0.122	350	−0.136	0.667
82	0.770	0.090	172	0.107	−0.671	262	−0.768	−0.099	352	−0.109	0.671
84	0.773	0.066	174	0.080	−0.674	264	−0.772	−0.075	354	−0.082	0.674
86	0.775	0.043	176	0.054	−0.676	266	−0.774	−0.052	356	−0.055	0.676
88	0.776	0.019	178	0.027	−0.677	268	−0.776	−0.028	358	−0.027	0.677
90	0.777	−0.005	180	0.000	−0.678	270	−0.777	−0.005	360	−0.000	0.678

Table 9.2: Deferential anomalies of Venus.

μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$	μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$	μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$	μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$
0	0.000	0.000	0.000	45	0.267	18.694	0.274	90	0.629	35.875	0.649	135	1.344	46.305	1.408
1	0.006	0.420	0.006	46	0.273	19.100	0.281	91	0.640	36.217	0.660	136	1.369	46.320	1.434
2	0.011	0.839	0.012	47	0.280	19.505	0.288	92	0.650	36.557	0.671	137	1.393	46.317	1.461
3	0.017	1.259	0.017	48	0.286	19.910	0.294	93	0.661	36.893	0.682	138	1.418	46.294	1.489
4	0.023	1.679	0.023	49	0.293	20.314	0.301	94	0.672	37.227	0.693	139	1.444	46.252	1.517
5	0.028	2.098	0.029	50	0.300	20.717	0.308	95	0.683	37.558	0.705	140	1.470	46.188	1.546
6	0.034	2.518	0.035	51	0.307	21.119	0.315	96	0.694	37.886	0.717	141	1.496	46.102	1.575
7	0.040	2.937	0.041	52	0.313	21.521	0.322	97	0.706	38.210	0.729	142	1.523	45.992	1.605
8	0.045	3.357	0.046	53	0.320	21.921	0.329	98	0.718	38.531	0.741	143	1.550	45.857	1.636
9	0.051	3.776	0.052	54	0.327	22.321	0.336	99	0.729	38.849	0.753	144	1.577	45.695	1.667
10	0.057	4.195	0.058	55	0.334	22.720	0.344	100	0.742	39.164	0.766	145	1.605	45.505	1.698
11	0.062	4.614	0.064	56	0.341	23.119	0.351	101	0.754	39.474	0.779	146	1.632	45.284	1.730
12	0.068	5.033	0.070	57	0.348	23.516	0.358	102	0.766	39.781	0.792	147	1.660	45.032	1.762
13	0.074	5.452	0.076	58	0.355	23.912	0.366	103	0.779	40.084	0.805	148	1.688	44.745	1.794
14	0.079	5.870	0.082	59	0.363	24.308	0.373	104	0.792	40.383	0.818	149	1.715	44.422	1.827
15	0.085	6.289	0.087	60	0.370	24.702	0.381	105	0.805	40.677	0.832	150	1.742	44.060	1.859
16	0.091	6.707	0.093	61	0.377	25.095	0.388	106	0.818	40.968	0.846	151	1.769	43.657	1.892
17	0.097	7.125	0.099	62	0.385	25.487	0.396	107	0.832	41.253	0.860	152	1.795	43.210	1.924
18	0.102	7.543	0.105	63	0.392	25.879	0.404	108	0.846	41.534	0.875	153	1.820	42.716	1.955
19	0.108	7.960	0.111	64	0.400	26.269	0.411	109	0.860	41.810	0.890	154	1.844	42.173	1.986
20	0.114	8.378	0.117	65	0.407	26.658	0.419	110	0.874	42.081	0.905	155	1.867	41.577	2.015
21	0.120	8.795	0.123	66	0.415	27.045	0.427	111	0.889	42.346	0.920	156	1.888	40.923	2.043
22	0.126	9.212	0.129	67	0.423	27.432	0.435	112	0.904	42.606	0.936	157	1.906	40.210	2.069
23	0.131	9.628	0.135	68	0.431	27.817	0.443	113	0.919	42.860	0.952	158	1.921	39.433	2.092
24	0.137	10.045	0.141	69	0.439	28.201	0.452	114	0.934	43.108	0.968	159	1.933	38.587	2.112
25	0.143	10.461	0.147	70	0.447	28.583	0.460	115	0.950	43.349	0.985	160	1.942	37.669	2.128
26	0.149	10.876	0.153	71	0.455	28.964	0.468	116	0.966	43.585	1.002	161	1.945	36.675	2.140
27	0.155	11.292	0.159	72	0.463	29.344	0.477	117	0.983	43.813	1.019	162	1.943	35.599	2.146
28	0.161	11.707	0.165	73	0.471	29.722	0.485	118	1.000	44.034	1.037	163	1.934	34.437	2.145
29	0.167	12.121	0.172	74	0.480	30.099	0.494	119	1.017	44.248	1.055	164	1.918	33.186	2.137
30	0.173	12.536	0.178	75	0.488	30.474	0.503	120	1.034	44.453	1.074	165	1.893	31.840	2.119
31	0.179	12.950	0.184	76	0.497	30.847	0.512	121	1.052	44.651	1.093	166	1.859	30.396	2.091
32	0.185	13.363	0.190	77	0.506	31.219	0.521	122	1.070	44.840	1.112	167	1.814	28.850	2.050
33	0.191	13.776	0.196	78	0.514	31.589	0.530	123	1.089	45.021	1.132	168	1.758	27.200	1.996
34	0.197	14.189	0.203	79	0.523	31.958	0.539	124	1.108	45.191	1.152	169	1.688	25.442	1.926
35	0.203	14.601	0.209	80	0.532	32.324	0.548	125	1.127	45.353	1.173	170	1.604	23.577	1.840
36	0.209	15.013	0.215	81	0.541	32.689	0.558	126	1.147	45.503	1.194	171	1.505	21.604	1.735
37	0.216	15.424	0.222	82	0.551	33.052	0.567	127	1.167	45.644	1.216	172	1.391	19.526	1.612
38	0.222	15.834	0.228	83	0.560	33.412	0.577	128	1.188	45.772	1.238	173	1.261	17.347	1.468
39	0.228	16.245	0.235	84	0.569	33.771	0.587	129	1.209	45.889	1.260	174	1.116	15.071	1.305
40	0.234	16.654	0.241	85	0.579	34.127	0.597	130	1.230	45.994	1.284	175	0.956	12.707	1.123
41	0.241	17.063	0.248	86	0.589	34.482	0.607	131	1.252	46.085	1.307	176	0.783	10.266	0.922
42	0.247	17.472	0.254	87	0.599	34.834	0.617	132	1.275	46.163	1.331	177	0.598	7.760	0.707
43	0.254	17.880	0.261	88	0.609	35.183	0.628	133	1.297	46.226	1.356	178	0.404	5.202	0.478
44	0.260	18.287	0.267	89	0.619	35.530	0.638	134	1.321	46.274	1.382	179	0.204	2.610	0.242
45	0.267	18.694	0.274	90	0.629	35.875	0.649	135	1.344	46.305	1.408	180	0.000	0.000	0.000

Table 9.3: Epicyclic anomalies of Venus. All quantities are in degrees. Note that $\bar{\theta}(360^\circ - \mu) = -\bar{\theta}(\mu)$, and $\delta\theta_\pm(360^\circ - \mu) = -\delta\theta_\pm(\mu)$.

Event	Date	λ	Elongation
Superior Conjunction	11/06/2000	20GE46	
Greatest Elongation	17/01/2001	14PI23	47.1° E
Inferior Conjunction	30/03/2001	09AR36	
Greatest Elongation	08/06/2001	01TA39	45.8° W
Superior Conjunction	14/01/2002	23CP59	
Greatest Elongation	22/08/2002	15LI08	46.0° E
Inferior Conjunction	31/10/2002	07SC58	
Greatest Elongation	11/01/2003	03SG31	47.0° W
Superior Conjunction	18/08/2003	25LE20	
Greatest Elongation	29/03/2004	25TA04	46.0° E
Inferior Conjunction	08/06/2004	17GE52	
Greatest Elongation	17/08/2004	09CN29	45.8° W
Superior Conjunction	31/03/2005	10AR33	
Greatest Elongation	03/11/2005	28SG27	47.1° E
Inferior Conjunction	13/01/2006	23CP36	
Greatest Elongation	25/03/2006	18AQ07	46.5° W
Superior Conjunction	27/10/2006	04SC13	
Greatest Elongation	09/06/2007	03LE20	45.4° E
Inferior Conjunction	18/08/2007	24LE45	
Greatest Elongation	28/10/2007	18VI19	46.5° W
Superior Conjunction	09/06/2008	18GE41	
Greatest Elongation	14/01/2009	12PI03	47.1° E
Inferior Conjunction	27/03/2009	07AR19	
Greatest Elongation	05/06/2009	29AR25	45.8° W
Superior Conjunction	11/01/2010	21CP24	
Greatest Elongation	19/08/2010	12LI49	46.0° E
Inferior Conjunction	29/10/2010	05SC34	
Greatest Elongation	08/01/2011	01SG06	47.0° W
Superior Conjunction	16/08/2011	23LE13	
Greatest Elongation	27/03/2012	22TA50	46.0° E
Inferior Conjunction	06/06/2012	15GE43	
Greatest Elongation	15/08/2012	07CN18	45.8° W
Superior Conjunction	28/03/2013	08AR12	
Greatest Elongation	01/11/2013	26SG02	47.1° E
Inferior Conjunction	11/01/2014	21CP08	
Greatest Elongation	23/03/2014	15AQ44	46.5° W
Superior Conjunction	25/10/2014	01SC51	
Greatest Elongation	06/06/2015	01LE09	45.4° E
Inferior Conjunction	15/08/2015	22LE33	
Greatest Elongation	26/10/2015	16VI02	46.4° W

Table 9.4: The conjunctions and greatest elongations of Venus during the years 2000–2015 AD.

Δt(JD)	$\Delta\bar{\lambda}(^\circ)$	$\Delta M(^\circ)$	$\Delta\bar{F}(^\circ)$	Δt(JD)	$\Delta\bar{\lambda}(^\circ)$	$\Delta M(^\circ)$	$\Delta\bar{F}(^\circ)$
10 000	243.770	243.344	243.422	1 000	132.377	132.334	132.342
20 000	127.541	126.688	126.844	2 000	264.754	264.669	264.684
30 000	11.311	10.032	10.266	3 000	37.131	37.003	37.027
40 000	255.081	253.376	253.688	4 000	169.508	169.338	169.369
50 000	138.852	136.720	137.110	5 000	301.885	301.672	301.711
60 000	22.622	20.063	20.533	6 000	74.262	74.006	74.053
70 000	266.392	263.407	263.955	7 000	206.639	206.341	206.395
80 000	150.162	146.751	147.377	8 000	339.016	338.675	338.738
90 000	33.933	30.095	30.799	9 000	111.393	111.010	111.080
100	49.238	49.233	49.234	10	40.924	40.923	40.923
200	98.475	98.467	98.468	20	81.848	81.847	81.847
300	147.713	147.700	147.703	30	122.771	122.770	122.770
400	196.951	196.934	196.937	40	163.695	163.693	163.694
500	246.189	246.167	246.171	50	204.619	204.617	204.617
600	295.426	295.401	295.405	60	245.543	245.540	245.541
700	344.664	344.634	344.640	70	286.466	286.463	286.464
800	33.902	33.868	33.874	80	327.390	327.387	327.387
900	83.139	83.101	83.108	90	8.314	8.310	8.311
1	4.092	4.092	4.092	0.1	0.409	0.409	0.409
2	8.185	8.185	8.185	0.2	0.818	0.818	0.818
3	12.277	12.277	12.277	0.3	1.228	1.228	1.228
4	16.370	16.369	16.369	0.4	1.637	1.637	1.637
5	20.462	20.462	20.462	0.5	2.046	2.046	2.046
6	24.554	24.554	24.554	0.6	2.455	2.455	2.455
7	28.647	28.646	28.646	0.7	2.865	2.865	2.865
8	32.739	32.739	32.739	0.8	3.274	3.274	3.274
9	36.831	36.831	36.831	0.9	3.683	3.683	3.683

Table 9.5: Mean motion of Mercury. Here, $\Delta t = t - t_0$, $\Delta\bar{\lambda} = \bar{\lambda} - \bar{\lambda}_0$, $\Delta M = M - M_0$, and $\Delta\bar{F} = \bar{F} - \bar{F}_0$. At epoch ($t_0 = 2\,451\,545.0$ JD), $\bar{\lambda}_0 = 252.087^\circ$, $M_0 = 174.693^\circ$, and $\bar{F}_0 = 204.436^\circ$.

$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$	$M(^\circ)$	$q(^\circ)$	$100\,\zeta$
0	0.000	20.564	90	22.900	−4.229	180	0.000	−20.564	270	−22.900	−4.229
2	1.086	20.544	92	22.677	−4.896	182	−0.663	−20.555	272	−23.100	−3.551
4	2.169	20.487	94	22.433	−5.552	184	−1.326	−20.528	274	−23.276	−2.864
6	3.247	20.391	96	22.168	−6.197	186	−1.987	−20.483	276	−23.428	−2.168
8	4.316	20.257	98	21.884	−6.831	188	−2.647	−20.420	278	−23.553	−1.463
10	5.376	20.085	100	21.580	−7.452	190	−3.304	−20.340	280	−23.652	−0.750
12	6.422	19.876	102	21.259	−8.062	192	−3.959	−20.242	282	−23.723	−0.030
14	7.454	19.631	104	20.920	−8.659	194	−4.610	−20.126	284	−23.764	0.697
16	8.467	19.350	106	20.566	−9.243	196	−5.257	−19.993	286	−23.775	1.429
18	9.460	19.035	108	20.195	−9.815	198	−5.900	−19.842	288	−23.755	2.165
20	10.431	18.685	110	19.809	−10.373	200	−6.538	−19.675	290	−23.703	2.905
22	11.377	18.303	112	19.409	−10.918	202	−7.170	−19.490	292	−23.617	3.648
24	12.298	17.889	114	18.996	−11.450	204	−7.796	−19.288	294	−23.497	4.392
26	13.190	17.445	116	18.569	−11.969	206	−8.417	−19.070	296	−23.342	5.137
28	14.052	16.971	118	18.129	−12.473	208	−9.030	−18.835	298	−23.150	5.880
30	14.882	16.469	120	17.677	−12.964	210	−9.637	−18.583	300	−22.922	6.621
32	15.680	15.941	122	17.212	−13.441	212	−10.236	−18.316	302	−22.656	7.359
34	16.443	15.388	124	16.737	−13.904	214	−10.827	−18.032	304	−22.353	8.091
36	17.171	14.811	126	16.250	−14.353	216	−11.410	−17.733	306	−22.010	8.818
38	17.862	14.212	128	15.752	−14.787	218	−11.985	−17.418	308	−21.629	9.536
40	18.517	13.593	130	15.243	−15.207	220	−12.552	−17.087	310	−21.208	10.245
42	19.133	12.954	132	14.724	−15.613	222	−13.109	−16.741	312	−20.748	10.942
44	19.710	12.299	134	14.196	−16.004	224	−13.657	−16.380	314	−20.249	11.628
46	20.249	11.628	136	13.657	−16.380	226	−14.196	−16.004	316	−19.710	12.299
48	20.748	10.942	138	13.109	−16.741	228	−14.724	−15.613	318	−19.133	12.954
50	21.208	10.245	140	12.552	−17.087	230	−15.243	−15.207	320	−18.517	13.593
52	21.629	9.536	142	11.985	−17.418	232	−15.752	−14.787	322	−17.862	14.212
54	22.010	8.818	144	11.410	−17.733	234	−16.250	−14.353	324	−17.171	14.811
56	22.353	8.091	146	10.827	−18.032	236	−16.737	−13.904	326	−16.443	15.388
58	22.656	7.359	148	10.236	−18.316	238	−17.212	−13.441	328	−15.680	15.941
60	22.922	6.621	150	9.637	−18.583	240	−17.677	−12.964	330	−14.882	16.469
62	23.150	5.880	152	9.030	−18.835	242	−18.129	−12.473	332	−14.052	16.971
64	23.342	5.137	154	8.417	−19.070	244	−18.569	−11.969	334	−13.190	17.445
66	23.497	4.392	156	7.796	−19.288	246	−18.996	−11.450	336	−12.298	17.889
68	23.617	3.648	158	7.170	−19.490	248	−19.409	−10.918	338	−11.377	18.303
70	23.703	2.905	160	6.538	−19.675	250	−19.809	−10.373	340	−10.431	18.685
72	23.755	2.165	162	5.900	−19.842	252	−20.195	−9.815	342	−9.460	19.035
74	23.775	1.429	164	5.257	−19.993	254	−20.566	−9.243	344	−8.467	19.350
76	23.764	0.697	166	4.610	−20.126	256	−20.920	−8.659	346	−7.454	19.631
78	23.723	−0.030	168	3.959	−20.242	258	−21.259	−8.062	348	−6.422	19.876
80	23.652	−0.750	170	3.304	−20.340	260	−21.580	−7.452	350	−5.376	20.085
82	23.553	−1.463	172	2.647	−20.420	262	−21.884	−6.831	352	−4.316	20.257
84	23.428	−2.168	174	1.987	−20.483	264	−22.168	−6.197	354	−3.247	20.391
86	23.276	−2.864	176	1.326	−20.528	266	−22.433	−5.552	356	−2.169	20.487
88	23.100	−3.551	178	0.663	−20.555	268	−22.677	−4.896	358	−1.086	20.544
90	22.900	−4.229	180	0.000	−20.564	270	−22.900	−4.229	360	−0.000	20.564

Table 9.6: Deferential anomalies of Mercury.

μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$	μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$	μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$	μ	$\delta\theta_-$	$\bar{\theta}$	$\delta\theta_+$
0	0.000	0.000	0.000	45	1.711	11.702	2.403	90	3.450	20.277	5.113	135	4.257	19.475	7.325
1	0.038	0.270	0.052	46	1.749	11.941	2.459	91	3.486	20.395	5.177	136	4.237	19.267	7.327
2	0.075	0.540	0.104	47	1.788	12.179	2.515	92	3.522	20.509	5.241	137	4.214	19.048	7.324
3	0.113	0.809	0.156	48	1.827	12.415	2.572	93	3.557	20.618	5.305	138	4.188	18.818	7.317
4	0.150	1.079	0.208	49	1.866	12.650	2.628	94	3.592	20.722	5.368	139	4.159	18.578	7.304
5	0.188	1.348	0.260	50	1.905	12.882	2.685	95	3.627	20.822	5.432	140	4.127	18.326	7.286
6	0.225	1.618	0.313	51	1.944	13.114	2.742	96	3.662	20.917	5.496	141	4.092	18.064	7.262
7	0.263	1.887	0.365	52	1.983	13.343	2.799	97	3.696	21.007	5.559	142	4.053	17.791	7.233
8	0.301	2.156	0.417	53	2.022	13.571	2.857	98	3.729	21.091	5.623	143	4.011	17.506	7.197
9	0.338	2.425	0.469	54	2.061	13.797	2.914	99	3.762	21.171	5.686	144	3.966	17.210	7.155
10	0.376	2.693	0.521	55	2.100	14.021	2.972	100	3.795	21.246	5.749	145	3.917	16.903	7.106
11	0.414	2.961	0.574	56	2.139	14.244	3.030	101	3.827	21.315	5.812	146	3.865	16.585	7.050
12	0.451	3.229	0.626	57	2.178	14.464	3.088	102	3.858	21.378	5.874	147	3.809	16.255	6.986
13	0.489	3.497	0.678	58	2.217	14.683	3.146	103	3.889	21.436	5.937	148	3.749	15.914	6.915
14	0.527	3.764	0.731	59	2.257	14.899	3.205	104	3.919	21.488	5.999	149	3.686	15.561	6.836
15	0.564	4.031	0.783	60	2.296	15.113	3.263	105	3.949	21.534	6.060	150	3.619	15.197	6.749
16	0.602	4.298	0.836	61	2.335	15.326	3.322	106	3.977	21.575	6.121	151	3.548	14.822	6.654
17	0.640	4.564	0.889	62	2.374	15.536	3.381	107	4.005	21.609	6.182	152	3.473	14.436	6.549
18	0.678	4.829	0.941	63	2.413	15.743	3.441	108	4.032	21.636	6.242	153	3.394	14.039	6.436
19	0.716	5.094	0.994	64	2.453	15.949	3.500	109	4.059	21.658	6.301	154	3.311	13.630	6.314
20	0.753	5.359	1.047	65	2.492	16.152	3.560	110	4.084	21.673	6.360	155	3.224	13.211	6.182
21	0.791	5.622	1.100	66	2.531	16.353	3.620	111	4.109	21.681	6.418	156	3.133	12.780	6.041
22	0.829	5.886	1.153	67	2.570	16.551	3.680	112	4.132	21.682	6.475	157	3.039	12.339	5.890
23	0.867	6.148	1.206	68	2.609	16.747	3.740	113	4.155	21.676	6.532	158	2.940	11.888	5.729
24	0.905	6.410	1.259	69	2.649	16.940	3.801	114	4.176	21.663	6.587	159	2.838	11.426	5.558
25	0.943	6.672	1.312	70	2.688	17.131	3.862	115	4.197	21.643	6.641	160	2.732	10.955	5.377
26	0.981	6.932	1.366	71	2.727	17.319	3.923	116	4.216	21.616	6.695	161	2.622	10.474	5.186
27	1.019	7.192	1.419	72	2.766	17.504	3.984	117	4.233	21.580	6.747	162	2.508	9.983	4.985
28	1.057	7.451	1.473	73	2.805	17.686	4.045	118	4.250	21.538	6.797	163	2.391	9.483	4.774
29	1.095	7.709	1.526	74	2.844	17.865	4.107	119	4.265	21.487	6.846	164	2.271	8.974	4.554
30	1.133	7.967	1.580	75	2.882	18.042	4.169	120	4.278	21.428	6.894	165	2.147	8.457	4.324
31	1.172	8.223	1.634	76	2.921	18.215	4.231	121	4.291	21.361	6.940	166	2.019	7.932	4.084
32	1.210	8.479	1.688	77	2.960	18.385	4.293	122	4.301	21.286	6.984	167	1.889	7.399	3.835
33	1.248	8.734	1.742	78	2.999	18.552	4.355	123	4.310	21.202	7.026	168	1.756	6.859	3.578
34	1.286	8.987	1.796	79	3.037	18.716	4.417	124	4.317	21.109	7.067	169	1.620	6.312	3.312
35	1.325	9.240	1.851	80	3.075	18.876	4.480	125	4.322	21.008	7.105	170	1.481	5.759	3.038
36	1.363	9.491	1.905	81	3.114	19.033	4.543	126	4.326	20.898	7.140	171	1.340	5.200	2.757
37	1.402	9.742	1.960	82	3.152	19.186	4.606	127	4.327	20.778	7.173	172	1.197	4.636	2.469
38	1.440	9.991	2.015	83	3.190	19.336	4.669	128	4.326	20.649	7.204	173	1.052	4.067	2.174
39	1.479	10.240	2.070	84	3.227	19.482	4.732	129	4.323	20.511	7.231	174	0.905	3.494	1.875
40	1.517	10.487	2.125	85	3.265	19.625	4.795	130	4.318	20.363	7.256	175	0.756	2.917	1.570
41	1.556	10.732	2.180	86	3.302	19.763	4.859	131	4.311	20.206	7.277	176	0.607	2.337	1.261
42	1.594	10.977	2.236	87	3.340	19.898	4.922	132	4.301	20.038	7.295	177	0.456	1.755	0.949
43	1.633	11.220	2.291	88	3.377	20.029	4.986	133	4.289	19.861	7.309	178	0.304	1.171	0.634
44	1.672	11.462	2.347	89	3.413	20.155	5.049	134	4.274	19.673	7.319	179	0.152	0.586	0.317
45	1.711	11.702	2.403	90	3.450	20.277	5.113	135	4.257	19.475	7.325	180	0.000	0.000	0.000

Table 9.7: Epicyclic anomalies of Mercury. All quantities are in degrees. Note that $\bar{\theta}(360^\circ - \mu) = -\bar{\theta}(\mu)$, and $\delta\theta_\pm(360^\circ - \mu) = -\delta\theta_\pm(\mu)$.

Event	Date	λ	Elongation
Superior Conjunction	15/01/2000	25CP08	
Greatest Elongation	15/02/2000	13PI44	18.1° E
Inferior Conjunction	01/03/2000	11PI23	
Greatest Elongation	28/03/2000	10PI35	27.9° W
Superior Conjunction	09/05/2000	18TA59	
Greatest Elongation	09/06/2000	13CN27	24.2° E
Inferior Conjunction	06/07/2000	14CN39	
Greatest Elongation	27/07/2000	15CN03	19.7° W
Superior Conjunction	22/08/2000	29LE16	
Greatest Elongation	06/10/2000	09SC17	25.6° E
Inferior Conjunction	30/10/2000	06SC58	
Greatest Elongation	15/11/2000	04SC02	19.3° W
Superior Conjunction	25/12/2000	04CP18	
Greatest Elongation	28/01/2001	27AQ07	18.4° E
Inferior Conjunction	13/02/2001	24AQ23	
Greatest Elongation	11/03/2001	23AQ05	27.5° W
Superior Conjunction	23/04/2001	03TA22	
Greatest Elongation	22/05/2001	24GE13	22.6° E
Inferior Conjunction	16/06/2001	25GE26	
Greatest Elongation	09/07/2001	26GE18	21.1° W
Superior Conjunction	05/08/2001	13LE32	
Greatest Elongation	19/09/2001	22LI50	26.6° E
Inferior Conjunction	14/10/2001	20LI51	
Greatest Elongation	29/10/2001	17LI50	18.5° W
Superior Conjunction	04/12/2001	12SG45	
Greatest Elongation	12/01/2002	10AQ33	18.9° E
Inferior Conjunction	27/01/2002	07AQ42	
Greatest Elongation	21/02/2002	05AQ55	26.6° W
Superior Conjunction	07/04/2002	17AR27	
Greatest Elongation	04/05/2002	04GE54	21.0° E
Inferior Conjunction	27/05/2002	05GE48	
Greatest Elongation	21/06/2002	07GE04	22.8° W
Superior Conjunction	21/07/2002	28CN06	
Greatest Elongation	01/09/2002	06LI10	27.3° E
Inferior Conjunction	27/09/2002	04LI35	
Greatest Elongation	13/10/2002	01LI44	18.0° W
Superior Conjunction	14/11/2002	21SC39	
Greatest Elongation	26/12/2002	24CP01	19.8° E

Table 9.8: The conjunctions and greatest elongations of Mercury during the years 2000–2002 AD.

10. Planetary latitudes

10.1 Determination of ecliptic latitude of superior planet

Up to now, we have neglected the fact that the orbits of the five visible planets about the Sun are all slightly inclined to the plane of the ecliptic. Of course, these inclinations cause the ecliptic latitudes of the said planets to take small, but non-zero, values. In the following, we shall outline a model that is capable of predicting these values. Incidentally, the calculation of planetary latitudes is discussed in Book XIII of the Almagest, which is also the last book in the treatise. The planetary latitude model outlined in Book XIII is rather unwieldy, and not particularly accurate. Ptolemy grearly improved this model in his later works *Handy Tables* (Πρόχειροι κανόνες) and *Planetary Hypotheses* (Ὑποθέσεις τῶν πλανωμένων).

Figure 10.1 shows the orbit of a superior planet. As we have already mentioned, the deferent and epicycle of such a planet have the same elements as the orbit of the planet in question around the Sun, and the apparent orbit of the Sun around the Earth, respectively. It follows that the deferent and epicycle of a superior planet are, respectively, inclined and parallel to the ecliptic plane. (Recall that the ecliptic plane corresponds to the plane of the Sun's apparent orbit about the Earth.) Let the plane of the deferent cut the ecliptic plane along the line NGN'. Here, N is the point at which the deferent passes through the plane of the ecliptic from south to north, in the direction of the mean planetary motion. This point is called the *ascending node*. Note that the line NGN' must pass through point G, because the Earth is common to the plane of the deferent and the ecliptic plane.

Now, it follows from simple geometry that the elevation of the guide-point G' above the ecliptic plane satisfies

$$v = r\ \sin i\ \sin F, \tag{10.1}$$

where r is the length GG', i the fixed inclination of the planetary orbit (and, hence, of the deferent) to the ecliptic plane, and F the angle NGG'. The angle F is termed the *argument of latitude*. We can write (see Chapter 8)

$$F = \bar{F} + q, \tag{10.2}$$

where $\bar{F}$ is the *mean argument of latitude*, and q the equation of center of the deferent. Note that $\bar{F}$ increases uniformly in time; that is,

$$\bar{F} = \bar{F}_0 + \breve{n}\,(t - t_0). \tag{10.3}$$

Because the epicycle is parallel to the ecliptic plane, the elevation of the planet above the said plane is the same as that of the guide-point. Hence, from simple geometry, the ecliptic latitude of the planet satisfies

$$\beta = \frac{v}{r''}, \tag{10.4}$$

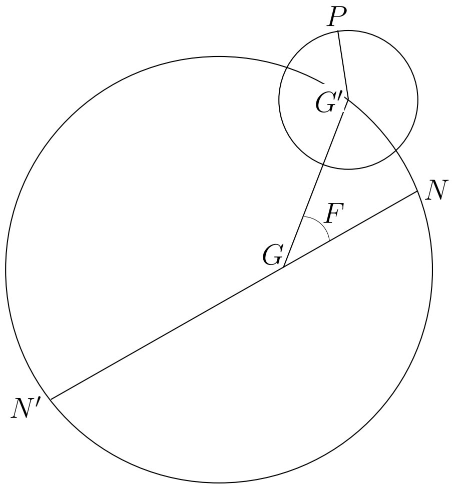

Figure 10.1: Orbit of a superior planet. Here, G, G', P, N, N', and F represent the Earth, the guide-point, the planet, the ascending node, the descending node, and the argument of latitude, respectively. The view is from northern ecliptic pole.

where r'' is the length GP, and we have used the small angle approximation. However, it is apparent from Figure 8.3 that

$$r'' = (r^2 + 2\,r\,r'\,\cos\mu + r'^{\,2})^{1/2}, \tag{10.5}$$

where r' the length $G'P$, and μ the equation of the epicycle. But, according to the analysis in Chapter 8, $r/r' = a\,z$, where a is the planetary major radius in units in which the major radius of the Sun's apparent orbit about the Earth is unity, and z is defined in Equation (8.4). Thus, we obtain

$$\beta = h\,\beta_0, \tag{10.6}$$

where

$$\beta_0(F) = \sin i\,\sin F \tag{10.7}$$

is termed the *deferential latitude*, and

$$h(\mu, z) = \left[1 + 2\,(a\,z)^{-1}\,\cos\mu + (a\,z)^{-2}\right]^{-1/2} \tag{10.8}$$

the *epicyclic latitude correction factor*.

In the following, a, e, n, $\tilde{n}$, $\breve{n}$, $\bar{\lambda}_0$, M_0, $\bar{F}_0$, and i are elements of the orbit of the planet in question about the Sun, and e_S, ζ_S, and λ_S are elements of the Sun's apparent orbit about the Sarth. The requisite elements for all of the superior planets at the J2000 epoch (t_0 = 2 451 545.0 JD) are listed in Tables 5.1 and 10.1. Employing a quadratic interpolation scheme to represent $F(\mu, z)$ (see Chapter 8), our procedure for determining the ecliptic latitude of a superior planet is summed up by the following formulae:

$$\bar{\lambda} = \bar{\lambda}_0 + n\,(t - t_0), \tag{10.9}$$

$$M = M_0 + \tilde{n}\,(t - t_0), \tag{10.10}$$

$$\bar{F} = \bar{F}_0 + \breve{n}\,(t - t_0), \tag{10.11}$$

$$q = 2\,e\,\sin\,M + (5/4)\,e^2\,\sin\,2M, \tag{10.12}$$

$$\zeta = e\,\cos M - e^2\,\sin^2 M, \tag{10.13}$$

$$F = \bar{F} + q, \tag{10.14}$$

$$\beta_0 = \sin i\,\sin F, \tag{10.15}$$

$$\mu = \lambda_S - \bar{\lambda} - q, \tag{10.16}$$

$$\bar{h} = h(\mu, \bar{z}) \equiv \left[1 + 2\,(a\,\bar{z})^{-1}\,\cos\mu + (a\,\bar{z})^{-2}\right]^{-1/2}, \tag{10.17}$$

$$\delta h_- = h(\mu, \bar{z}) - h(\mu, z_{\rm max}), \tag{10.18}$$

$$\delta h_+ = h(\mu, z_{\rm min}) - h(\mu, \bar{z}), \tag{10.19}$$

$$z = \frac{1 - \zeta}{1 - \zeta_S}, \tag{10.20}$$

$$\xi = \frac{\bar{z} - z}{\delta z}, \tag{10.21}$$

$$h = \Theta_-(\xi)\,\delta h_- + \bar{h} + \Theta_+(\xi)\,\delta\,h_+, \tag{10.22}$$

$$\beta = h\,\beta_0. \tag{10.23}$$

Here, $\bar{z} = (1+e\,e_S)/(1-e_S^2)$, $\delta z = (e+e_S)/(1-e_S^2)$, $z_{\rm min} = \bar{z}-\delta z$, and $z_{\rm max} = \bar{z}+\delta z$. The constants $\bar{z}$, δz, $z_{\rm min}$, and $z_{\rm max}$ for each of the superior planets are listed in Table 8.1. Finally, the functions $\Theta_\pm$ are tabulated in Table 8.2.

For the case of Mars, the previous formulae are capable of matching NASA ephemeris data during the years 1995–2006 AD with a mean error of $0.3'$ and a maximum error of $1.5'$. For the case of Jupiter, the mean error is $0.2'$ and the maximum error $0.5'$. Finally, for the case of Saturn, the mean error is $0.05'$ and the maximum error $0.08'$.

10.2 Determination of ecliptic latitude of Mars

The ecliptic latitude of Mars can be determined with the aid of Tables 8.3, 10.2, and 10.3. Table 8.3 allows the mean argument of latitude, $\bar{F}$, of Mars to be calculated as a function of time. Next, Table 10.2 permits the deferential latitude, β_0, to be determined as a function of the true argument of latitude, F. Finally, Table 10.3 allows the quantities δh_-, $\bar{h}$, and δh_+ to be calculated as functions of the epicyclic anomaly, μ.

The procedure for using the tables is as follows:

1. Determine the fractional Julian day number, t, corresponding to the date and time at which the ecliptic latitude is to be calculated with the aid of Tables 3.1–3.3. Form $\Delta t = t - t_0$, where $t_0 = 2\,451\,545.0$ is the epoch.

2. Calculate the planetary equation of center, q, ecliptic anomaly, μ, and interpolation parameters Θ_+ and Θ_- using the procedure set out in Chapter 8.

3. Enter Table 8.3 with the digit for each power of 10 in Δt and take out the corresponding values of $\Delta\bar{F}$. If Δt is negative then the corresponding values are also negative. The value of the mean argument of latitude, $\bar{F}$, is the sum of all the $\Delta\bar{F}$ values plus the value of $\bar{F}$ at the epoch.

4. Form the true argument of latitude, $F = \bar{F} + q$. Add as many multiples of 360° to F as is required to make it fall in the range 0° to 360°. Round F to the nearest degree.

5. Enter Table 10.2 with the value of F and take out the corresponding value of the deferential latitude, β_0. It is necessary to interpolate if F is odd.

6. Enter Table 10.3 with the value of μ and take out the corresponding values of δh_-, $\bar{h}$, and δh_+. If $\mu > 180^\circ$ then it is necessary to make use of the identities $\delta h_\pm(360^\circ - \mu) = \delta h_\pm(\mu)$ and $\bar{h}(360^\circ - \mu) = \bar{h}(\mu)$.

7. Form the epicyclic latitude correction factor, $h = \Theta_-\,\delta h_- + \bar{h} + \Theta_+\,\delta h_+$.

8. The ecliptic latitude, β, is the product of the deferential latitude, β_0, and the epicyclic latitude correction factor, h. The decimal fraction can be converted into arc minutes using Table 5.2. Round to the nearest arc minute.

One example of this procedure is given in the following.

Example: May 5, 2005 AD, 00:00 UT:

From Chapter 8, $t - t_0 = 1\,950.5$ JD, $q = -7.345^\circ$, $\mu = 114.286^\circ$, $\Theta_- = 0.101$, and $\Theta_+ = 0.619$. Making use of Table 8.3, we find:

t(JD)	$\bar{F}(^\circ)$
+1000	164.041
+900	111.637
+50	26.202
+.5	0.262
Epoch	305.796
	607.938
Modulus	247.938

Thus,

$$F = \bar{F} + q = 247.938 - 7.345 = 240.593 \simeq 241^\circ.$$

It follows from Table 10.2 that

$$\beta_0(241^\circ) = -1.615^\circ.$$

Because $\mu \simeq 114^\circ$, Table 10.3 yields

$$\delta h_-(114^\circ) = -0.017, \qquad \bar{h}(114^\circ) = 1.056, \qquad \delta h_+(114^\circ) = -0.027,$$

so

$$\begin{aligned} h &= \Theta_- \, \delta h_- + \bar{h} + \Theta_+ \, \delta h_+ \\ &= -0.101 \times 0.017 + 1.056 - 0.619 \times 0.027 \\ &= 1.038. \end{aligned}$$

Finally,

$$\beta = h \, \beta_0 = -1.038 \times 1.615 = -1.676 \simeq -1^\circ 41'.$$

Thus, the ecliptic latitude of Mars at 00:00 UT on May 5, 2005 AD was $-1^\circ 41'$.

10.3 Determination of ecliptic latitude of Jupiter

The ecliptic latitude of Jupiter can be determined with the aid of Tables 8.7, 10.4, and 10.5. Table 8.7 allows the mean argument of latitude, $\bar{F}$, of Jupiter to be calculated as a function of time. Next, Table 10.4 permits the deferential latitude, β_0, to be determined as a function of the true argument of latitude, F. Finally, Table 10.5 allows the quantities δh_-, $\bar{h}$, and δh_+ to be calculated as functions of the epicyclic anomaly, μ. The procedure for using these tables is analogous to the previously described procedure for using the Mars tables. One example of this procedure is given in the following.

Example: May 5, 2005 AD, 00:00 UT:

From Chapter 8, $t - t_0 = 1\,950.5$ JD, $q = -0.091^\circ$, $\mu = 208.192^\circ$, $\Theta_- = -0.469$, and $\Theta_+ = -0.121$. Making use of Table 8.7, we find:

t(JD)	$\bar{F}(^\circ)$
+1000	83.081
+900	74.773
+50	4.154
+.5	0.042
Epoch	293.660
	455.710
Modulus	95.710

Thus,

$$F = \bar{F} + q = 95.710 - 0.091 = 95.619 \simeq 96^\circ.$$

It follows from Table 10.4 that

$$\beta_0(96^\circ) = 1.297^\circ.$$

Because $\mu \simeq 208^\circ$, Table 10.5 yields

$$\delta h_-(208^\circ) = 0.014, \qquad \bar{h}(208^\circ) = 1.197, \qquad \delta h_+(208^\circ) = 0.016,$$

so

$$\begin{aligned} h &= \Theta_-\,\delta h_- + \bar{h} + \Theta_+\,\delta h_+ \\ &= -0.469 \times 0.014 + 1.197 - 0.121 \times 0.016 \\ &= 1.188. \end{aligned}$$

Finally,

$$\beta = h\,\beta_0 = 1.188 \times 1.297 = 1.541 \simeq 1^\circ 32'.$$

Thus, the ecliptic latitude of Jupiter at 00:00 UT on May 5, 2005 CE was $1^\circ 32'$.

10.4 Determination of ecliptic latitude of Saturn

The ecliptic latitude of Saturn can be determined with the aid of Tables 8.11, 10.6, and 10.7. Table 8.11 allows the mean argument of latitude, $\bar{F}$, of Saturn to be calculated as a function of time. Next, Table 10.6 permits the deferential latitude, β_0, to be determined as a function of the true argument of latitude, F. Finally, Table 10.7 allows the quantities δh_-, $\bar{h}$, and δh_+ to be calculated as functions of the epicyclic anomaly, μ. The procedure for using these tables is analogous to the previously described procedure for using the Mars tables. One example of this procedure is given in the following.

Example: May 5, 2005 AD, 00:00 UT:

From Chapter 8, $t - t_0 = 1\,950.5$ JD, $q = 2.561^\circ$, $\mu = 286.625^\circ$, $\Theta_- = 0.071$, and $\Theta_+ = 0.759$. Making use of Table 8.11, we find:

t(JD)	$\bar{F}(^\circ)$
+1000	33.478
+900	30.130
+50	1.674
+.5	0.017
Epoch	296.482
	361.781
Modulus	1.781

Thus,

$$F = \bar{F} + q = 1.781 + 2.561 = 4.342 \simeq 4^\circ.$$

It follows from Table 10.6 that

$$\beta_0(4^\circ) = 0.173^\circ.$$

Because $\mu \simeq 287^\circ$, Table 10.7 yields

$$\delta h_-(287^\circ) = -0.002, \qquad \bar{h}(287^\circ) = 0.966, \qquad \delta h_+(287^\circ) = -0.003,$$

so

$$\begin{aligned} h &= \Theta_-\,\delta h_- + \bar{h} + \Theta_+\,\delta h_+ \\ &= -0.071 \times 0.002 + 0.966 - 0.759 \times 0.003 \\ &= 0.964. \end{aligned}$$

Finally,

$$\beta = h\,\beta_0 = 0.964 \times 0.173 = 0.167 \simeq 0^\circ 10'.$$

Thus, the ecliptic latitude of Saturn at 00:00 UT on May 5, 2005 CE was $0^\circ 10'$.

10.5 Determination of ecliptic latitude of inferior planet

Figure 10.2 shows the orbit of an inferior planet. As we have already mentioned, the epicycle and deferent of such a planet have the same elements as the orbit of the planet in question around the Sun, and the apparent orbit of the Sun around the Earth, respectively. It follows that the epicycle and deferent of an inferior planet are, respectively, inclined and parallel to the ecliptic plane. Let the plane of the epicycle cut the ecliptic plane along the line $NG'N'$. Here, N is the point at which the epicycle passes through the plane of the ecliptic from south to north, in the direction of the mean planetary motion. This point is called the *ascending node*. Note that the

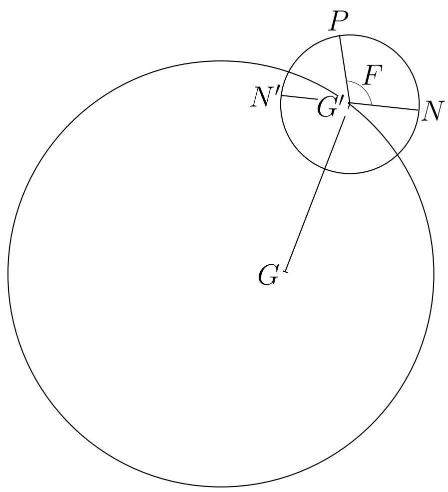

Figure 10.2: Orbit of an inferior planet. Here, G, G', P, N, N', and F represent the earth, the guide-point, the planet, the ascending node, the descending node, and the argument of latitude, respectively. The view is from northern ecliptic pole.

line $NG'N'$ must pass through the guide-point, G', because the Sun (which is coincident with the guide-point) is common to the plane of the planetary orbit and the ecliptic plane.

Now, it follows from simple geometry that the elevation of the planet P above the guide-point, G', satisfies

$$v = r' \sin i \sin F, \tag{10.24}$$

where r' is the length $G'P$, i the fixed inclination of the planetary orbit (and, hence, of the epicycle) to the ecliptic plane, and F the angle $NG'P$. The angle F is termed the argument of latitude. We can write (see Chapter 9)

$$F = \bar{F} + q, \tag{10.25}$$

where $\bar{F}$ is the mean argument of latitude, and q the equation of center of the epicycle. Note that $\bar{F}$ increases uniformly in time; that is,

$$\bar{F} = \bar{F}_0 + \breve{n}\,(t - t_0). \tag{10.26}$$

Because the deferent is parallel to the ecliptic plane, the elevation of the planet above the said plane is the same as that of the planet above the guide-point. Hence, from simple geometry, the ecliptic latitude of the planet satisfies

$$\beta = \frac{v}{r''}, \tag{10.27}$$

where r'' is the length GP, and we have used the small angle approximation. However, it is apparent from Fig. 8.3 that

$$r'' = (r^2 + 2\,r\,r'\,\cos\mu + r'^{\,2})^{1/2}, \tag{10.28}$$

where r the length GG', and μ the equation of the epicycle. But, according to the analysis in Chapter 9, $r'/r = a/z$, where a is the planetary major radius in units in which the major radius of the Sun's apparent orbit about the Earth is unity, and z is defined in Equation (9.9). Thus, we obtain

$$\beta = h\,\beta_0, \tag{10.29}$$

where

$$\beta_0(F) = a\,\sin i\,\sin F \tag{10.30}$$

is termed the *epicyclic latitude*, and

$$h(\mu, z) = \left[z^2 + 2\,a\,z\,\cos\mu + a^2\right]^{-1/2} \tag{10.31}$$

the *deferential latitude correction factor*.

In the following, a, e, n, $\tilde{n}$, $\breve{n}$, $\bar{\lambda}_0$, M_0, $\bar{F}_0$, and i are elements of the orbit of the planet in question about the Sun, and e_S, ζ_S, and λ_S are elements of the Sun's apparent orbit about the Sarth. The requisite elements for all of the superior planets at the J2000 epoch ($t_0 = 2\,451\,545.0$ JD) are listed in Tables 5.1 and 10.1. Employing a quadratic interpolation scheme to represent $F(\mu, z)$ (see Chapter 8), our procedure for determining the ecliptic latitude of a superior planet is summed up by the following formulae:

$$\bar{\lambda} = \bar{\lambda}_0 + n\,(t - t_0), \tag{10.32}$$

$$M = M_0 + \tilde{n}\,(t - t_0), \tag{10.33}$$

$$\bar{F} = \bar{F}_0 + \breve{n}\,(t - t_0), \tag{10.34}$$

$$q = 2\,e\,\sin M + (5/4)\,e^2\,\sin 2M, \tag{10.35}$$

$$\zeta = e\,\cos M - e^2\,\sin^2 M, \tag{10.36}$$

$$F = \bar{F} + q, \tag{10.37}$$

$$\beta_0 = a\,\sin i\,\sin F, \tag{10.38}$$

$$\mu = \bar{\lambda} + q - \bar{\lambda}_S, \tag{10.39}$$

$$\bar{h} = h(\mu, \bar{z}) \equiv \left[\bar{z}^2 + 2\,a\,\bar{z}\,\cos\mu + a^2\right]^{-1/2}, \tag{10.40}$$

$$\delta h_- = h(\mu, \bar{z}) - h(\mu, z_{\rm max}), \tag{10.41}$$

$$\delta h_+ = h(\mu, z_{\rm min}) - h(\mu, \bar{z}), \tag{10.42}$$

$$z = \frac{1 - \zeta_S}{1 - \zeta}, \tag{10.43}$$

$$\xi = \frac{\bar{z} - z}{\delta z}, \tag{10.44}$$

$$h = \Theta_-(\xi)\,\delta h_- + \bar{h} + \Theta_+(\xi)\,\delta\,h_+, \tag{10.45}$$

$$\beta = h\,\beta_0. \tag{10.46}$$

Here, $\bar{z} = (1+e\,e_S)/(1-e^2)$, $\delta z = (e+e_S)/(1-e^2)$, $z_{\rm min} = \bar{z}-\delta z$, and $z_{\rm max} = \bar{z}+\delta z$. The constants $\bar{z}$, δz, $z_{\rm min}$, and $z_{\rm max}$ for each of the inferior planets are listed in Table 8.1. Finally, the functions $\Theta_\pm$ are tabulated in Table 8.2.

For the case of Venus, the previous formulae are capable of matching NASA ephemeris data during the years 1995–2006 AD with a mean error of $0.7'$ and a maximum error of $1.8'$. For the case of Mercury, with the augmentations to the theory described in Chapter 9, the mean error is $1.6'$ and the maximum error $5'$.

10.6 Determination of ecliptic latitude of Venus

The ecliptic latitude of Venus can be determined with the aid of Tables 9.1, 10.8, and 10.9. Table 9.1 allows the mean argument of latitude, $\bar{F}$, of Venus to be calculated as a function of time. Next, Table 10.8 permits the epicyclic latitude, β_0, to be determined as a function of the true argument of latitude, F. Finally, Table 10.9 allows the quantities δh_-, $\bar{h}$, and δh_+ to be calculated as functions of the epicyclic anomaly, μ.

The procedure for using the tables is as follows:

1. Determine the fractional Julian day number, t, corresponding to the date and time at which the ecliptic latitude is to be calculated with the aid of Tables 3.1–3.3. Form $\Delta t = t - t_0$, where $t_0 = 2\,451\,545.0$ is the epoch.

2. Calculate the planetary equation of center, q, ecliptic anomaly, μ, and interpolation parameters Θ_+ and Θ_- using the procedure set out in Chapter 9.

3. Enter Table 9.1 with the digit for each power of 10 in Δt and take out the corresponding values of $\Delta\bar{F}$. If Δt is negative then the corresponding values are also negative. The value of the mean argument of latitude, $\bar{F}$, is the sum of all the $\Delta\bar{F}$ values plus the value of $\bar{F}$ at the epoch.

4. Form the true argument of latitude, $F = \bar{F} + q$. Add as many multiples of 360° to F as is required to make it fall in the range 0° to 360°. Round F to the nearest degree.

5. Enter Table 10.8 with the value of F and take out the corresponding value of the epicyclic latitude, β_0. It is necessary to interpolate if F is odd.

6. Enter Table 10.9 with the value of μ and take out the corresponding values of δh_-, $\bar{h}$, and δh_+. If $\mu > 180^\circ$ then it is necessary to make use of the identities $\delta h_\pm(360^\circ - \mu) = \delta h_\pm(\mu)$ and $\bar{h}(360^\circ - \mu) = \bar{h}(\mu)$.

7. Form the deferential latitude correction factor, $h = \Theta_-\,\delta h_- + \bar{h} + \Theta_+\,\delta h_+$.

8. The ecliptic latitude, β, is the product of the epicyclic latitude, β_0, and the deferential latitude correction factor, h. The decimal fraction can be converted into arc minutes using Table 5.2. Round to the nearest arc minute.

One example of this procedure is given in the following.

Example: May 5, 2005 AD, 00:00 UT:

From Chapter 9, $t - t_0 = 1\,950.5$ JD, $q = -0.712^\circ$, $\mu = 21.689^\circ$, $\Theta_- = -0.355$, and $\Theta_+ = -0.125$. Making use of Table 9.1, we find:

t(JD)	$\bar{F}(^\circ)$
+1000	162.138
+900	1.924
+50	80.107
+.5	0.801
Epoch	105.253
	350.223
Modulus	350.223

Thus,

$$F = \bar{F} + q = 350.223 - 0.712 = 349.511 \simeq 350^\circ.$$

It follows from Table 10.8 that

$$\beta_0(350^\circ) = -0.423^\circ.$$

Because $\mu \simeq 22^\circ$, Table 10.9 yields

$$\delta h_-(22^\circ) = 0.008, \qquad \bar{h}(22^\circ) = 0.591, \qquad \delta h_+(22^\circ) = 0.008,$$

so

$$\begin{aligned} h &= \Theta_-\,\delta h_- + \bar{h} + \Theta_+\,\delta h_+ \\ &= -0.355 \times 0.008 + 0.591 - 0.125 \times 0.008 \\ &= 0.587. \end{aligned}$$

Finally,

$$\beta = h\,\beta_0 = -0.587 \times 0.423 = -0.248 \simeq -0^\circ 15'.$$

Thus, the ecliptic latitude of Venus at 00:00 UT on May 5, 2005 AD was $-0^\circ 15'$.

10.7 Determination of ecliptic latitude of Mercury

The ecliptic latitude of Mercury can be determined with the aid of Tables 9.5, 10.10, and 10.11. Table 9.5 allows the mean argument of latitude, $\bar{F}$, of Mercury to be calculated as a function of time. Next, Table 10.10 permits the epicyclic latitude, β_0, to be determined as a function of the true argument of latitude, F. Finally, Table 10.11 allows the quantities δh_-, $\bar{h}$, and δh_+ to be calculated as functions of the epicyclic anomaly, μ. The procedure for using the tables is analogous to the previously described procedure for using the Venus tables. One example of this procedure is given in the following.

Example: May 5, 2005 AD, 00:00 UT:

From Chapter 9, $t - t_0 = 1\,950.5$ JD, $q = -16.974^\circ$, $\mu = 252.692^\circ$, $\Theta_- = 0.107$, and $\Theta_+ = 0.583$. Making use of Table 9.5, we find:

t(JD)	$\bar{F}(^\circ)$
+1000	132.342
+900	83.108
+50	204.617
+.5	2.046
Epoch	204.436
	626.549
Modulus	266.549

Thus,

$$F = \bar{F} + q = 266.549 - 16.974 = 249.575 \simeq 250^\circ.$$

It follows from Table 10.10 that

$$\beta_0(250^\circ) = -2.511^\circ.$$

Because $\mu \simeq 253^\circ$, Table 10.11 yields

$$\delta h_-(253^\circ) = 0.184, \qquad \bar{h}(253^\circ) = 1.037, \qquad \delta h_+(253^\circ) = 0.272,$$

so

$$\begin{aligned} h &= \Theta_-\,\delta h_- + \bar{h} + \Theta_+\,\delta h_+ \\ &= 0.107 \times 0.184 + 1.037 + 0.583 \times 0.272 \\ &= 1.215. \end{aligned}$$

Finally,

$$\beta = h\,\beta_0 = -1.215 \times 2.511 = -3.051 \simeq -3^\circ 03'.$$

Thus, the ecliptic latitude of Mercury at 00:00 UT on May 5, 2005 CE was $-3^\circ 03'$.

10.8 Tables

Object	$i(^\circ)$	$\breve{n}\,(^\circ/\text{day})$	$\bar{F}_0\,(^\circ)$
Mercury	6.9190	4.09234221	204.436
Venus	3.3692	1.60213807	105.253
Mars	1.8467	0.52404094	305.796
Jupiter	1.3044	0.08308122	293.660
Saturn	2.4860	0.03347795	296.482

Table 10.1: Additional Keplerian orbital elements for the five visible planets at the J2000 epoch (that is, 12:00 UT, January 1, 2000 CE, which corresponds to $t_0 = 2\,451\,545.0$ JD). The elements are optimized for use in the time period 1800 CE to 2050 AD. Source: Jet Propulsion Laboratory (NASA), `http://ssd.jpl.nasa.gov/`.

$F(^\circ)$	$\beta_0(^\circ)$	$F(^\circ)$	$F(^\circ)$	$\beta_0(^\circ)$	$F(^\circ)$
000/180	0.000	(180)/(360)	046/134	1.328	(226)/(314)
002/178	0.064	(182)/(358)	048/132	1.372	(228)/(312)
004/176	0.129	(184)/(356)	050/130	1.414	(230)/(310)
006/174	0.193	(186)/(354)	052/128	1.455	(232)/(308)
008/172	0.257	(188)/(352)	054/126	1.494	(234)/(306)
010/170	0.321	(190)/(350)	056/124	1.531	(236)/(304)
012/168	0.384	(192)/(348)	058/122	1.566	(238)/(302)
014/166	0.447	(194)/(346)	060/120	1.599	(240)/(300)
016/164	0.509	(196)/(344)	062/118	1.630	(242)/(298)
018/162	0.571	(198)/(342)	064/116	1.660	(244)/(296)
020/160	0.631	(200)/(340)	066/114	1.687	(246)/(294)
022/158	0.692	(202)/(338)	068/112	1.712	(248)/(292)
024/156	0.751	(204)/(336)	070/110	1.735	(250)/(290)
026/154	0.809	(206)/(334)	072/108	1.756	(252)/(288)
028/152	0.867	(208)/(332)	074/106	1.775	(254)/(286)
030/150	0.923	(210)/(330)	076/104	1.792	(256)/(284)
032/148	0.978	(212)/(328)	078/102	1.806	(258)/(282)
034/146	1.032	(214)/(326)	080/100	1.818	(260)/(280)
036/144	1.085	(216)/(324)	082/098	1.828	(262)/(278)
038/142	1.137	(218)/(322)	084/096	1.836	(264)/(276)
040/140	1.187	(220)/(320)	086/094	1.842	(266)/(274)
042/138	1.235	(222)/(318)	088/092	1.845	(268)/(272)
044/136	1.283	(224)/(316)	090/090	1.846	(270)/(270)

Table 10.2: Deferential ecliptic latitude of Mars. The latitude is minus the value shown in the table if the argument is in parentheses.

μ	δh_-	$\bar{h}$	δh_+	μ	δh_-	$\bar{h}$	δh_+	μ	δh_-	$\bar{h}$	δh_+	μ	δh_-	$\bar{h}$	δh_+
0	−0.025	0.604	−0.028	45	−0.025	0.652	−0.029	90	−0.025	0.836	−0.031	135	0.015	1.410	0.003
1	−0.025	0.604	−0.028	46	−0.025	0.654	−0.029	91	−0.025	0.843	−0.031	136	0.018	1.433	0.006
2	−0.025	0.604	−0.028	47	−0.025	0.656	−0.029	92	−0.024	0.850	−0.031	137	0.021	1.457	0.009
3	−0.025	0.604	−0.028	48	−0.025	0.659	−0.029	93	−0.024	0.857	−0.031	138	0.025	1.482	0.013
4	−0.025	0.605	−0.028	49	−0.025	0.661	−0.029	94	−0.024	0.865	−0.031	139	0.028	1.507	0.017
5	−0.025	0.605	−0.028	50	−0.025	0.664	−0.030	95	−0.024	0.872	−0.031	140	0.032	1.533	0.021
6	−0.025	0.605	−0.028	51	−0.025	0.666	−0.030	96	−0.024	0.880	-0.031	141	0.037	1.560	0.026
7	−0.025	0.605	−0.028	52	−0.025	0.669	−0.030	97	−0.024	0.888	−0.031	142	0.041	1.588	0.031
8	−0.025	0.606	−0.028	53	−0.025	0.672	−0.030	98	−0.023	0.896	−0.031	143	0.046	1.616	0.036
9	−0.025	0.606	−0.028	54	−0.025	0.674	−0.030	99	−0.023	0.904	−0.031	144	0.051	1.646	0.043
10	−0.025	0.606	−0.028	55	−0.025	0.677	−0.030	100	−0.023	0.912	−0.030	145	0.057	1.676	0.049
11	−0.025	0.607	−0.028	56	−0.025	0.680	−0.030	101	−0.023	0.921	−0.030	146	0.063	1.708	0.056
12	−0.025	0.607	−0.028	57	−0.026	0.683	−0.030	102	−0.022	0.930	−0.030	147	0.069	1.740	0.064
13	−0.025	0.608	−0.028	58	−0.026	0.686	−0.030	103	−0.022	0.939	−0.030	148	0.076	1.773	0.073
14	−0.025	0.609	−0.028	59	−0.026	0.689	−0.030	104	−0.022	0.948	−0.030	149	0.084	1.807	0.083
15	−0.025	0.609	−0.028	60	−0.026	0.693	−0.030	105	−0.021	0.958	−0.030	150	0.092	1.843	0.093
16	−0.025	0.610	−0.028	61	−0.026	0.696	−0.030	106	−0.021	0.968	−0.029	151	0.100	1.879	0.104
17	−0.025	0.611	−0.028	62	−0.026	0.699	−0.030	107	−0.021	0.978	−0.029	152	0.109	1.916	0.117
18	−0.025	0.611	−0.028	63	−0.026	0.703	−0.030	108	−0.020	0.988	−0.029	153	0.118	1.955	0.130
19	−0.025	0.612	−0.028	64	−0.026	0.706	−0.030	109	−0.020	0.999	−0.029	154	0.128	1.994	0.145
20	−0.025	0.613	−0.028	65	−0.026	0.710	−0.030	110	−0.019	1.010	−0.028	155	0.139	2.034	0.161
21	−0.025	0.614	−0.028	66	−0.026	0.714	−0.030	111	−0.019	1.021	−0.028	156	0.151	2.075	0.178
22	−0.025	0.615	−0.028	67	−0.026	0.718	−0.030	112	−0.018	1.032	−0.027	157	0.163	2.117	0.197
23	−0.025	0.616	−0.028	68	−0.026	0.722	−0.030	113	−0.018	1.044	−0.027	158	0.175	2.160	0.218
24	−0.025	0.617	−0.028	69	−0.026	0.726	−0.031	114	−0.017	1.056	−0.027	159	0.189	2.203	0.240
25	−0.025	0.618	−0.029	70	−0.026	0.730	−0.031	115	−0.016	1.069	−0.026	160	0.202	2.247	0.265
26	−0.025	0.619	−0.029	71	−0.026	0.734	−0.031	116	−0.015	1.082	−0.025	161	0.217	2.292	0.291
27	−0.025	0.621	−0.029	72	−0.026	0.738	−0.031	117	−0.015	1.095	−0.025	162	0.232	2.337	0.319
28	−0.025	0.622	−0.029	73	−0.026	0.743	−0.031	118	−0.014	1.108	−0.024	163	0.248	2.382	0.349
29	−0.025	0.623	−0.029	74	−0.026	0.747	−0.031	119	−0.013	1.122	−0.023	164	0.264	2.427	0.382
30	−0.025	0.625	−0.029	75	−0.026	0.752	−0.031	120	−0.012	1.137	−0.023	165	0.280	2.472	0.416
31	−0.025	0.626	−0.029	76	−0.026	0.757	−0.031	121	−0.011	1.151	−0.022	166	0.297	2.517	0.453
32	−0.025	0.627	−0.029	77	−0.026	0.762	−0.031	122	−0.010	1.167	−0.021	167	0.314	2.560	0.491
33	−0.025	0.629	−0.029	78	−0.025	0.767	−0.031	123	−0.008	1.182	−0.020	168	0.331	2.603	0.530
34	−0.025	0.631	−0.029	79	−0.025	0.772	−0.031	124	−0.007	1.198	−0.019	169	0.348	2.644	0.571
35	−0.025	0.632	−0.029	80	−0.025	0.777	−0.031	125	−0.006	1.215	−0.017	170	0.364	2.683	0.612
36	−0.025	0.634	−0.029	81	−0.025	0.782	−0.031	126	−0.004	1.232	−0.016	171	0.380	2.721	0.654
37	−0.025	0.636	−0.029	82	−0.025	0.788	−0.031	127	−0.003	1.249	−0.015	172	0.395	2.755	0.694
38	−0.025	0.637	−0.029	83	−0.025	0.793	−0.031	128	−0.001	1.267	−0.013	173	0.408	2.787	0.734
39	−0.025	0.639	−0.029	84	−0.025	0.799	−0.031	129	0.001	1.286	−0.011	174	0.421	2.816	0.770
40	−0.025	0.641	−0.029	85	−0.025	0.805	−0.031	130	0.003	1.305	−0.009	175	0.432	2.840	0.804
41	−0.025	0.643	−0.029	86	−0.025	0.811	−0.031	131	0.005	1.325	−0.007	176	0.442	2.861	0.833
42	−0.025	0.645	−0.029	87	−0.025	0.817	−0.031	132	0.007	1.345	−0.005	177	0.449	2.878	0.857
43	−0.025	0.647	−0.029	88	−0.025	0.823	−0.031	133	0.010	1.366	−0.003	178	0.455	2.890	0.874
44	−0.025	0.649	−0.029	89	−0.025	0.830	−0.031	134	0.012	1.388	−0.000	179	0.458	2.897	0.885
45	−0.025	0.652	−0.029	90	−0.025	0.836	−0.031	135	0.015	1.410	0.003	180	0.459	2.899	0.889

Table 10.3: Epicyclic latitude correction factor for Mars. μ is in degrees. Note that $\bar{h}(360^\circ - \mu) = \bar{h}(\mu)$, and $\delta h_\pm(360^\circ - \mu) = \delta h_\pm(\mu)$.

$F(^\circ)$	$\beta_0(^\circ)$	$F(^\circ)$	$F(^\circ)$	$\beta_0(^\circ)$	$F(^\circ)$
000/180	0.000	(180)/(360)	046/134	0.938	(226)/(314)
002/178	0.046	(182)/(358)	048/132	0.969	(228)/(312)
004/176	0.091	(184)/(356)	050/130	0.999	(230)/(310)
006/174	0.136	(186)/(354)	052/128	1.028	(232)/(308)
008/172	0.182	(188)/(352)	054/126	1.055	(234)/(306)
010/170	0.226	(190)/(350)	056/124	1.081	(236)/(304)
012/168	0.271	(192)/(348)	058/122	1.106	(238)/(302)
014/166	0.316	(194)/(346)	060/120	1.130	(240)/(300)
016/164	0.360	(196)/(344)	062/118	1.152	(242)/(298)
018/162	0.403	(198)/(342)	064/116	1.172	(244)/(296)
020/160	0.446	(200)/(340)	066/114	1.192	(246)/(294)
022/158	0.489	(202)/(338)	068/112	1.209	(248)/(292)
024/156	0.531	(204)/(336)	070/110	1.226	(250)/(290)
026/154	0.572	(206)/(334)	072/108	1.240	(252)/(288)
028/152	0.612	(208)/(332)	074/106	1.254	(254)/(286)
030/150	0.652	(210)/(330)	076/104	1.266	(256)/(284)
032/148	0.691	(212)/(328)	078/102	1.276	(258)/(282)
034/146	0.729	(214)/(326)	080/100	1.284	(260)/(280)
036/144	0.767	(216)/(324)	082/098	1.292	(262)/(278)
038/142	0.803	(218)/(322)	084/096	1.297	(264)/(276)
040/140	0.838	(220)/(320)	086/094	1.301	(266)/(274)
042/138	0.873	(222)/(318)	088/092	1.303	(268)/(272)
044/136	0.906	(224)/(316)	090/090	1.304	(270)/(270)

Table 10.4: Deferential ecliptic latitude of Jupiter. The latitude is minus the value shown in the table if the argument is in parentheses.

μ	δh_-	$\bar{h}$	δh_+	μ	δh_-	$\bar{h}$	δh_+	μ	δh_-	$\bar{h}$	δh_+	μ	δh_-	$\bar{h}$	δh_+
0	−0.008	0.839	−0.009	45	−0.007	0.874	−0.008	90	−0.002	0.982	−0.003	135	0.009	1.143	0.010
1	−0.008	0.839	−0.009	46	−0.007	0.876	−0.008	91	−0.002	0.985	−0.002	136	0.009	1.147	0.011
2	−0.008	0.839	−0.009	47	−0.007	0.877	−0.008	92	−0.002	0.988	−0.002	137	0.010	1.150	0.011
3	−0.008	0.839	−0.009	48	−0.007	0.879	−0.008	93	−0.002	0.992	−0.002	138	0.010	1.154	0.011
4	−0.008	0.839	−0.009	49	−0.007	0.881	−0.008	94	−0.001	0.995	−0.002	139	0.010	1.157	0.012
5	−0.008	0.839	−0.009	50	−0.007	0.883	−0.008	95	−0.001	0.998	−0.001	140	0.010	1.160	0.012
6	−0.008	0.840	−0.009	51	−0.007	0.884	−0.008	96	−0.001	1.002	−0.001	141	0.011	1.164	0.012
7	−0.008	0.840	−0.009	52	−0.007	0.886	−0.007	97	−0.001	1.005	−0.001	142	0.011	1.167	0.013
8	−0.008	0.840	−0.009	53	−0.007	0.888	−0.007	98	−0.001	1.008	−0.001	143	0.011	1.170	0.013
9	−0.008	0.840	−0.009	54	−0.006	0.890	−0.007	99	−0.000	1.012	−0.001	144	0.012	1.173	0.013
10	−0.008	0.841	−0.009	55	−0.006	0.892	−0.007	100	−0.000	1.015	−0.000	145	0.012	1.177	0.014
11	−0.008	0.841	−0.009	56	−0.006	0.894	−0.007	101	0.000	1.019	−0.000	146	0.012	1.180	0.014
12	−0.008	0.841	−0.009	57	−0.006	0.896	−0.007	102	0.000	1.022	0.000	147	0.012	1.183	0.014
13	−0.008	0.842	−0.009	58	−0.006	0.898	−0.007	103	0.000	1.026	0.000	148	0.013	1.186	0.015
14	−0.008	0.842	−0.009	59	−0.006	0.900	−0.007	104	0.001	1.029	0.001	149	0.013	1.189	0.015
15	−0.008	0.843	−0.009	60	−0.006	0.902	−0.007	105	0.001	1.033	0.001	150	0.013	1.192	0.015
16	−0.008	0.843	−0.009	61	−0.006	0.904	−0.007	106	0.001	1.036	0.001	151	0.014	1.194	0.016
17	−0.008	0.844	−0.009	62	−0.006	0.906	−0.007	107	0.001	1.040	0.001	152	0.014	1.197	0.016
18	−0.008	0.845	−0.009	63	−0.006	0.909	−0.006	108	0.002	1.044	0.002	153	0.014	1.200	0.016
19	−0.008	0.845	−0.009	64	−0.006	0.911	−0.006	109	0.002	1.047	0.002	154	0.014	1.202	0.017
20	−0.008	0.846	−0.009	65	−0.005	0.913	−0.006	110	0.002	1.051	0.002	155	0.015	1.205	0.017
21	−0.008	0.847	−0.009	66	−0.005	0.916	−0.006	111	0.002	1.055	0.003	156	0.015	1.207	0.017
22	−0.008	0.847	−0.009	67	−0.005	0.918	−0.006	112	0.003	1.058	0.003	157	0.015	1.210	0.017
23	−0.008	0.848	−0.009	68	−0.005	0.920	−0.006	113	0.003	1.062	0.003	158	0.015	1.212	0.018
24	−0.008	0.849	−0.009	69	−0.005	0.923	−0.006	114	0.003	1.066	0.003	159	0.015	1.214	0.018
25	−0.008	0.850	−0.009	70	−0.005	0.925	−0.006	115	0.003	1.069	0.004	160	0.016	1.216	0.018
26	−0.008	0.851	−0.009	71	−0.005	0.928	−0.006	116	0.004	1.073	0.004	161	0.016	1.218	0.018
27	−0.008	0.852	−0.009	72	−0.005	0.930	−0.005	117	0.004	1.077	0.004	162	0.016	1.220	0.019
28	−0.008	0.853	−0.009	73	−0.005	0.933	−0.005	118	0.004	1.080	0.005	163	0.016	1.222	0.019
29	−0.008	0.854	−0.009	74	−0.004	0.935	−0.005	119	0.004	1.084	0.005	164	0.016	1.224	0.019
30	−0.008	0.855	−0.009	75	−0.004	0.938	−0.005	120	0.005	1.088	0.005	165	0.016	1.225	0.019
31	−0.008	0.856	−0.009	76	−0.004	0.941	−0.005	121	0.005	1.092	0.006	166	0.017	1.227	0.019
32	−0.008	0.857	−0.009	77	−0.004	0.944	−0.005	122	0.005	1.095	0.006	167	0.017	1.228	0.020
33	−0.008	0.858	−0.009	78	−0.004	0.946	−0.005	123	0.006	1.099	0.006	168	0.017	1.230	0.020
34	−0.008	0.859	−0.009	79	−0.004	0.949	−0.004	124	0.006	1.103	0.007	169	0.017	1.231	0.020
35	−0.008	0.860	−0.009	80	−0.004	0.952	−0.004	125	0.006	1.107	0.007	170	0.017	1.232	0.020
36	−0.008	0.861	−0.008	81	−0.004	0.955	−0.004	126	0.006	1.110	0.007	171	0.017	1.233	0.020
37	−0.008	0.863	−0.008	82	−0.003	0.958	−0.004	127	0.007	1.114	0.008	172	0.017	1.234	0.020
38	−0.007	0.864	−0.008	83	−0.003	0.961	−0.004	128	0.007	1.118	0.008	173	0.017	1.235	0.020
39	−0.007	0.865	−0.008	84	−0.003	0.964	−0.004	129	0.007	1.121	0.008	174	0.018	1.236	0.021
40	−0.007	0.867	−0.008	85	−0.003	0.967	−0.003	130	0.008	1.125	0.009	175	0.018	1.236	0.021
41	−0.007	0.868	−0.008	86	−0.003	0.970	−0.003	131	0.008	1.129	0.009	176	0.018	1.237	0.021
42	−0.007	0.870	−0.008	87	−0.003	0.973	−0.003	132	0.008	1.132	0.009	177	0.018	1.237	0.021
43	−0.007	0.871	−0.008	88	−0.002	0.976	−0.003	133	0.008	1.136	0.010	178	0.018	1.237	0.021
44	−0.007	0.873	−0.008	89	−0.002	0.979	−0.003	134	0.009	1.140	0.010	179	0.018	1.238	0.021
45	−0.007	0.874	−0.008	90	−0.002	0.982	−0.003	135	0.009	1.143	0.010	180	0.018	1.238	0.021

Table 10.5: Epicyclic latitude correction factor for Jupiter. μ is in degrees. Note that $\bar{h}(360^\circ - \mu) = \bar{h}(\mu)$, and $\delta h_\pm(360^\circ - \mu) = \delta h_\pm(\mu)$.

$F(^\circ)$	$\beta_0(^\circ)$	$F(^\circ)$	$F(^\circ)$	$\beta_0(^\circ)$	$F(^\circ)$
000/180	0.000	(180)/(360)	046/134	1.788	(226)/(314)
002/178	0.087	(182)/(358)	048/132	1.847	(228)/(312)
004/176	0.173	(184)/(356)	050/130	1.904	(230)/(310)
006/174	0.260	(186)/(354)	052/128	1.958	(232)/(308)
008/172	0.346	(188)/(352)	054/126	2.011	(234)/(306)
010/170	0.432	(190)/(350)	056/124	2.060	(236)/(304)
012/168	0.517	(192)/(348)	058/122	2.108	(238)/(302)
014/166	0.601	(194)/(346)	060/120	2.152	(240)/(300)
016/164	0.685	(196)/(344)	062/118	2.194	(242)/(298)
018/162	0.768	(198)/(342)	064/116	2.234	(244)/(296)
020/160	0.850	(200)/(340)	066/114	2.270	(246)/(294)
022/158	0.931	(202)/(338)	068/112	2.304	(248)/(292)
024/156	1.011	(204)/(336)	070/110	2.335	(250)/(290)
026/154	1.089	(206)/(334)	072/108	2.364	(252)/(288)
028/152	1.167	(208)/(332)	074/106	2.389	(254)/(286)
030/150	1.243	(210)/(330)	076/104	2.411	(256)/(284)
032/148	1.317	(212)/(328)	078/102	2.431	(258)/(282)
034/146	1.390	(214)/(326)	080/100	2.447	(260)/(280)
036/144	1.461	(216)/(324)	082/098	2.461	(262)/(278)
038/142	1.530	(218)/(322)	084/096	2.472	(264)/(276)
040/140	1.597	(220)/(320)	086/094	2.479	(266)/(274)
042/138	1.663	(222)/(318)	088/092	2.484	(268)/(272)
044/136	1.726	(224)/(316)	090/090	2.485	(270)/(270)

Table 10.6: Deferential ecliptic latitude of Saturn. The latitude is minus the value shown in the table if the argument is in parentheses.

μ	δh_-	$\bar{h}$	δh_+	μ	δh_-	$\bar{h}$	δh_+	μ	δh_-	$\bar{h}$	δh_+	μ	δh_-	$\bar{h}$	δh_+
0	−0.006	0.905	−0.006	45	−0.005	0.929	−0.005	90	−0.001	0.995	−0.001	135	0.005	1.077	0.006
1	−0.006	0.905	−0.006	46	−0.004	0.930	−0.005	91	−0.001	0.996	−0.001	136	0.005	1.078	0.006
2	−0.006	0.905	−0.006	47	−0.004	0.931	−0.005	92	−0.000	0.998	−0.001	137	0.005	1.080	0.006
3	−0.006	0.905	−0.006	48	−0.004	0.932	−0.005	93	−0.000	1.000	−0.000	138	0.006	1.081	0.006
4	−0.006	0.905	−0.006	49	−0.004	0.933	−0.005	94	−0.000	1.002	−0.000	139	0.006	1.083	0.007
5	−0.006	0.905	−0.006	50	−0.004	0.934	−0.005	95	−0.000	1.004	−0.000	140	0.006	1.084	0.007
6	−0.006	0.906	−0.006	51	−0.004	0.935	−0.005	96	0.000	1.006	−0.000	141	0.006	1.086	0.007
7	−0.006	0.906	−0.006	52	−0.004	0.937	−0.005	97	0.000	1.007	0.000	142	0.006	1.087	0.007
8	−0.006	0.906	−0.006	53	−0.004	0.938	−0.005	98	0.000	1.009	0.000	143	0.006	1.089	0.007
9	−0.006	0.906	−0.006	54	−0.004	0.939	−0.005	99	0.000	1.011	0.000	144	0.006	1.090	0.007
10	−0.006	0.906	−0.006	55	−0.004	0.940	−0.004	100	0.001	1.013	0.001	145	0.006	1.091	0.007
11	−0.006	0.907	−0.006	56	−0.004	0.942	−0.004	101	0.001	1.015	0.001	146	0.006	1.093	0.008
12	−0.006	0.907	−0.006	57	−0.004	0.943	−0.004	102	0.001	1.017	0.001	147	0.007	1.094	0.008
13	−0.006	0.907	−0.006	58	−0.004	0.944	−0.004	103	0.001	1.019	0.001	148	0.007	1.095	0.008
14	−0.006	0.908	−0.006	59	−0.004	0.945	−0.004	104	0.001	1.020	0.001	149	0.007	1.097	0.008
15	−0.006	0.908	−0.006	60	−0.004	0.947	−0.004	105	0.001	1.022	0.001	150	0.007	1.098	0.008
16	−0.006	0.908	−0.006	61	−0.003	0.948	−0.004	106	0.001	1.024	0.001	151	0.007	1.099	0.008
17	−0.006	0.909	−0.006	62	−0.003	0.949	−0.004	107	0.001	1.026	0.002	152	0.007	1.100	0.008
18	−0.006	0.909	−0.006	63	−0.003	0.951	−0.004	108	0.002	1.028	0.002	153	0.007	1.101	0.008
19	−0.005	0.909	−0.006	64	−0.003	0.952	−0.004	109	0.002	1.030	0.002	154	0.007	1.103	0.009
20	−0.005	0.910	−0.006	65	−0.003	0.954	−0.004	110	0.002	1.032	0.002	155	0.007	1.104	0.009
21	−0.005	0.910	−0.006	66	−0.003	0.955	−0.004	111	0.002	1.034	0.002	156	0.007	1.105	0.009
22	−0.005	0.911	−0.006	67	−0.003	0.957	−0.003	112	0.002	1.036	0.002	157	0.008	1.106	0.009
23	−0.005	0.911	−0.006	68	−0.003	0.958	−0.003	113	0.002	1.037	0.003	158	0.008	1.107	0.009
24	−0.005	0.912	−0.006	69	−0.003	0.960	−0.003	114	0.002	1.039	0.003	159	0.008	1.107	0.009
25	−0.005	0.913	−0.006	70	−0.003	0.961	−0.003	115	0.002	1.041	0.003	160	0.008	1.108	0.009
26	−0.005	0.913	−0.006	71	−0.003	0.963	−0.003	116	0.003	1.043	0.003	161	0.008	1.109	0.009
27	−0.005	0.914	−0.006	72	−0.003	0.964	−0.003	117	0.003	1.045	0.003	162	0.008	1.110	0.009
28	−0.005	0.914	−0.006	73	−0.002	0.966	−0.003	118	0.003	1.047	0.003	163	0.008	1.111	0.009
29	−0.005	0.915	−0.006	74	−0.002	0.967	−0.003	119	0.003	1.049	0.003	164	0.008	1.111	0.009
30	−0.005	0.916	−0.006	75	−0.002	0.969	−0.003	120	0.003	1.050	0.004	165	0.008	1.112	0.009
31	−0.005	0.916	−0.006	76	−0.002	0.971	−0.003	121	0.003	1.052	0.004	166	0.008	1.113	0.010
32	−0.005	0.917	−0.006	77	−0.002	0.972	−0.002	122	0.003	1.054	0.004	167	0.008	1.113	0.010
33	−0.005	0.918	−0.006	78	−0.002	0.974	−0.002	123	0.004	1.056	0.004	168	0.008	1.114	0.010
34	−0.005	0.919	−0.006	79	−0.002	0.975	−0.002	124	0.004	1.058	0.004	169	0.008	1.114	0.010
35	−0.005	0.920	−0.006	80	−0.002	0.977	−0.002	125	0.004	1.060	0.004	170	0.008	1.115	0.010
36	−0.005	0.920	−0.006	81	−0.002	0.979	−0.002	126	0.004	1.061	0.005	171	0.008	1.115	0.010
37	−0.005	0.921	−0.006	82	−0.002	0.981	−0.002	127	0.004	1.063	0.005	172	0.008	1.116	0.010
38	−0.005	0.922	−0.006	83	−0.001	0.982	−0.002	128	0.004	1.065	0.005	173	0.008	1.116	0.010
39	−0.005	0.923	−0.005	84	−0.001	0.984	−0.002	129	0.004	1.067	0.005	174	0.008	1.116	0.010
40	−0.005	0.924	−0.005	85	−0.001	0.986	−0.001	130	0.005	1.068	0.005	175	0.008	1.116	0.010
41	−0.005	0.925	−0.005	86	−0.001	0.987	−0.001	131	0.005	1.070	0.005	176	0.009	1.117	0.010
42	−0.005	0.926	−0.005	87	−0.001	0.989	−0.001	132	0.005	1.072	0.006	177	0.009	1.117	0.010
43	−0.005	0.927	−0.005	88	−0.001	0.991	−0.001	133	0.005	1.073	0.006	178	0.009	1.117	0.010
44	−0.005	0.928	−0.005	89	−0.001	0.993	−0.001	134	0.005	1.075	0.006	179	0.009	1.117	0.010
45	−0.005	0.929	−0.005	90	−0.001	0.995	−0.001	135	0.005	1.077	0.006	180	0.009	1.117	0.010

Table 10.7: Epicyclic latitude correction factor for Saturn. μ is in degrees. Note that $\bar{h}(360^\circ - \mu) = \bar{h}(\mu)$, and $\delta h_\pm(360^\circ - \mu) = \delta h_\pm(\mu)$.

$F(^\circ)$	$\beta_0(^\circ)$	$F(^\circ)$	$F(^\circ)$	$\beta_0(^\circ)$	$F(^\circ)$
000/180	0.000	(180)/(360)	046/134	1.752	(226)/(314)
002/178	0.085	(182)/(358)	048/132	1.810	(228)/(312)
004/176	0.170	(184)/(356)	050/130	1.866	(230)/(310)
006/174	0.255	(186)/(354)	052/128	1.919	(232)/(308)
008/172	0.339	(188)/(352)	054/126	1.970	(234)/(306)
010/170	0.423	(190)/(350)	056/124	2.019	(236)/(304)
012/168	0.506	(192)/(348)	058/122	2.066	(238)/(302)
014/166	0.589	(194)/(346)	060/120	2.109	(240)/(300)
016/164	0.671	(196)/(344)	062/118	2.151	(242)/(298)
018/162	0.753	(198)/(342)	064/116	2.189	(244)/(296)
020/160	0.833	(200)/(340)	066/114	2.225	(246)/(294)
022/158	0.912	(202)/(338)	068/112	2.258	(248)/(292)
024/156	0.991	(204)/(336)	070/110	2.289	(250)/(290)
026/154	1.068	(206)/(334)	072/108	2.316	(252)/(288)
028/152	1.143	(208)/(332)	074/106	2.341	(254)/(286)
030/150	1.218	(210)/(330)	076/104	2.363	(256)/(284)
032/148	1.291	(212)/(328)	078/102	2.382	(258)/(282)
034/146	1.362	(214)/(326)	080/100	2.399	(260)/(280)
036/144	1.432	(216)/(324)	082/098	2.412	(262)/(278)
038/142	1.500	(218)/(322)	084/096	2.422	(264)/(276)
040/140	1.566	(220)/(320)	086/094	2.430	(266)/(274)
042/138	1.630	(222)/(318)	088/092	2.434	(268)/(272)
044/136	1.692	(224)/(316)	090/090	2.436	(270)/(270)

Table 10.8: Epicyclic ecliptic latitude of Venus. The latitude is minus the value shown in the table if the argument is in parentheses.

μ	δh_-	$\bar{h}$	δh_+	μ	δh_-	$\bar{h}$	δh_+	μ	δh_-	$\bar{h}$	δh_+	μ	δh_-	$\bar{h}$	δh_+
0	0.008	0.580	0.008	45	0.009	0.627	0.009	90	0.012	0.810	0.013	135	0.032	1.413	0.033
1	0.008	0.580	0.008	46	0.009	0.629	0.009	91	0.013	0.817	0.013	136	0.033	1.439	0.034
2	0.008	0.580	0.008	47	0.009	0.631	0.009	92	0.013	0.824	0.013	137	0.034	1.466	0.035
3	0.008	0.580	0.008	48	0.009	0.633	0.009	93	0.013	0.831	0.013	138	0.036	1.493	0.037
4	0.008	0.580	0.008	49	0.009	0.636	0.009	94	0.013	0.838	0.013	139	0.037	1.522	0.038
5	0.008	0.581	0.008	50	0.009	0.638	0.009	95	0.013	0.846	0.013	140	0.039	1.552	0.040
6	0.008	0.581	0.008	51	0.009	0.641	0.009	96	0.013	0.854	0.014	141	0.040	1.583	0.041
7	0.008	0.581	0.008	52	0.009	0.643	0.009	97	0.014	0.861	0.014	142	0.042	1.615	0.043
8	0.008	0.582	0.008	53	0.009	0.646	0.009	98	0.014	0.870	0.014	143	0.044	1.648	0.045
9	0.008	0.582	0.008	54	0.009	0.649	0.009	99	0.014	0.878	0.014	144	0.046	1.683	0.047
10	0.008	0.582	0.008	55	0.009	0.652	0.009	100	0.014	0.886	0.014	145	0.048	1.719	0.049
11	0.008	0.583	0.008	56	0.009	0.655	0.009	101	0.014	0.895	0.015	146	0.050	1.756	0.052
12	0.008	0.583	0.008	57	0.009	0.658	0.009	102	0.015	0.904	0.015	147	0.053	1.795	0.054
13	0.008	0.584	0.008	58	0.009	0.661	0.009	103	0.015	0.913	0.015	148	0.055	1.836	0.057
14	0.008	0.584	0.008	59	0.009	0.664	0.010	104	0.015	0.923	0.015	149	0.058	1.878	0.060
15	0.008	0.585	0.008	60	0.009	0.667	0.010	105	0.015	0.933	0.016	150	0.061	1.922	0.063
16	0.008	0.586	0.008	61	0.009	0.670	0.010	106	0.016	0.943	0.016	151	0.065	1.968	0.067
17	0.008	0.586	0.008	62	0.010	0.674	0.010	107	0.016	0.953	0.016	152	0.068	2.016	0.071
18	0.008	0.587	0.008	63	0.010	0.677	0.010	108	0.016	0.964	0.017	153	0.072	2.065	0.075
19	0.008	0.588	0.008	64	0.010	0.681	0.010	109	0.016	0.975	0.017	154	0.076	2.117	0.080
20	0.008	0.589	0.008	65	0.010	0.684	0.010	110	0.017	0.986	0.017	155	0.081	2.170	0.085
21	0.008	0.590	0.008	66	0.010	0.688	0.010	111	0.017	0.997	0.017	156	0.086	2.226	0.090
22	0.008	0.591	0.008	67	0.010	0.692	0.010	112	0.017	1.009	0.018	157	0.091	2.283	0.096
23	0.008	0.592	0.008	68	0.010	0.696	0.010	113	0.018	1.021	0.018	158	0.097	2.343	0.102
24	0.008	0.593	0.008	69	0.010	0.700	0.010	114	0.018	1.034	0.019	159	0.103	2.405	0.109
25	0.008	0.594	0.008	70	0.010	0.704	0.010	115	0.019	1.047	0.019	160	0.110	2.469	0.117
26	0.008	0.595	0.008	71	0.010	0.708	0.010	116	0.019	1.060	0.019	161	0.117	2.535	0.125
27	0.008	0.596	0.008	72	0.010	0.712	0.011	117	0.019	1.074	0.020	162	0.125	2.603	0.134
28	0.008	0.597	0.008	73	0.010	0.717	0.011	118	0.020	1.088	0.020	163	0.133	2.673	0.144
29	0.008	0.599	0.008	74	0.010	0.721	0.011	119	0.020	1.103	0.021	164	0.142	2.744	0.155
30	0.008	0.600	0.008	75	0.011	0.726	0.011	120	0.021	1.118	0.021	165	0.151	2.816	0.166
31	0.008	0.601	0.008	76	0.011	0.730	0.011	121	0.021	1.133	0.022	166	0.161	2.890	0.178
32	0.008	0.603	0.008	77	0.011	0.735	0.011	122	0.022	1.149	0.022	167	0.172	2.964	0.191
33	0.008	0.604	0.008	78	0.011	0.740	0.011	123	0.022	1.166	0.023	168	0.183	3.037	0.204
34	0.008	0.606	0.008	79	0.011	0.745	0.011	124	0.023	1.183	0.023	169	0.194	3.111	0.218
35	0.008	0.608	0.009	80	0.011	0.751	0.011	125	0.024	1.200	0.024	170	0.205	3.182	0.232
36	0.008	0.609	0.009	81	0.011	0.756	0.011	126	0.024	1.219	0.025	171	0.217	3.252	0.247
37	0.008	0.611	0.009	82	0.011	0.761	0.012	127	0.025	1.237	0.025	172	0.228	3.318	0.262
38	0.008	0.613	0.009	83	0.011	0.767	0.012	128	0.026	1.257	0.026	173	0.239	3.380	0.276
39	0.008	0.614	0.009	84	0.012	0.773	0.012	129	0.026	1.277	0.027	174	0.249	3.436	0.290
40	0.008	0.616	0.009	85	0.012	0.778	0.012	130	0.027	1.298	0.028	175	0.258	3.486	0.302
41	0.008	0.618	0.009	86	0.012	0.784	0.012	131	0.028	1.319	0.029	176	0.267	3.529	0.313
42	0.009	0.620	0.009	87	0.012	0.791	0.012	132	0.029	1.342	0.030	177	0.273	3.564	0.322
43	0.009	0.622	0.009	88	0.012	0.797	0.012	133	0.030	1.365	0.031	178	0.278	3.589	0.329
44	0.009	0.624	0.009	89	0.012	0.803	0.012	134	0.031	1.389	0.032	179	0.281	3.604	0.333
45	0.009	0.627	0.009	90	0.012	0.810	0.013	135	0.032	1.413	0.033	180	0.282	3.609	0.334

Table 10.9: Deferential latitude correction factor for Venus. μ is in degrees. Note that $\bar{h}(360^\circ - \mu) = \bar{h}(\mu)$, and $\delta h_\pm(360^\circ - \mu) = \delta h_\pm(\mu)$.

$F(^\circ)$	$\beta_0(^\circ)$	$F(^\circ)$	$F(^\circ)$	$\beta_0(^\circ)$	$F(^\circ)$
000/180	0.000	(180)/(360)	046/134	1.922	(226)/(314)
002/178	0.093	(182)/(358)	048/132	1.986	(228)/(312)
004/176	0.186	(184)/(356)	050/130	2.047	(230)/(310)
006/174	0.279	(186)/(354)	052/128	2.105	(232)/(308)
008/172	0.372	(188)/(352)	054/126	2.162	(234)/(306)
010/170	0.464	(190)/(350)	056/124	2.215	(236)/(304)
012/168	0.556	(192)/(348)	058/122	2.266	(238)/(302)
014/166	0.646	(194)/(346)	060/120	2.314	(240)/(300)
016/164	0.736	(196)/(344)	062/118	2.359	(242)/(298)
018/162	0.826	(198)/(342)	064/116	2.401	(244)/(296)
020/160	0.914	(200)/(340)	066/114	2.441	(246)/(294)
022/158	1.001	(202)/(338)	068/112	2.477	(248)/(292)
024/156	1.087	(204)/(336)	070/110	2.511	(250)/(290)
026/154	1.171	(206)/(334)	072/108	2.541	(252)/(288)
028/152	1.254	(208)/(332)	074/106	2.568	(254)/(286)
030/150	1.336	(210)/(330)	076/104	2.592	(256)/(284)
032/148	1.416	(212)/(328)	078/102	2.613	(258)/(282)
034/146	1.494	(214)/(326)	080/100	2.631	(260)/(280)
036/144	1.570	(216)/(324)	082/098	2.646	(262)/(278)
038/142	1.645	(218)/(322)	084/096	2.657	(264)/(276)
040/140	1.717	(220)/(320)	086/094	2.665	(266)/(274)
042/138	1.788	(222)/(318)	088/092	2.670	(268)/(272)
044/136	1.856	(224)/(316)	090/090	2.672	(270)/(270)

Table 10.10: Epicyclic ecliptic latitude of Mercury. The latitude is minus the value shown in the table if the argument is in parentheses.

μ	δh_-	$\bar{h}$	δh_+	μ	δh_-	$\bar{h}$	δh_+	μ	δh_-	$\bar{h}$	δh_+	μ	δh_-	$\bar{h}$	δh_+
0	0.099	0.719	0.137	45	0.110	0.765	0.152	90	0.152	0.930	0.217	135	0.274	1.283	0.451
1	0.099	0.719	0.137	46	0.110	0.768	0.153	91	0.153	0.935	0.220	136	0.278	1.293	0.461
2	0.099	0.719	0.137	47	0.111	0.770	0.154	92	0.155	0.941	0.222	137	0.282	1.303	0.471
3	0.099	0.719	0.137	48	0.111	0.772	0.154	93	0.157	0.946	0.225	138	0.286	1.313	0.481
4	0.099	0.719	0.137	49	0.112	0.774	0.155	94	0.158	0.952	0.228	139	0.290	1.324	0.491
5	0.099	0.720	0.137	50	0.112	0.777	0.156	95	0.160	0.958	0.231	140	0.295	1.334	0.501
6	0.099	0.720	0.137	51	0.113	0.779	0.157	96	0.162	0.964	0.234	141	0.299	1.344	0.512
7	0.099	0.720	0.137	52	0.113	0.782	0.158	97	0.164	0.970	0.237	142	0.304	1.355	0.523
8	0.099	0.720	0.137	53	0.114	0.784	0.159	98	0.165	0.976	0.240	143	0.308	1.365	0.534
9	0.100	0.721	0.137	54	0.115	0.787	0.160	99	0.167	0.983	0.243	144	0.313	1.375	0.546
10	0.100	0.721	0.138	55	0.115	0.790	0.161	100	0.169	0.989	0.246	145	0.317	1.386	0.558
11	0.100	0.722	0.138	56	0.116	0.793	0.162	101	0.171	0.996	0.250	146	0.322	1.396	0.570
12	0.100	0.722	0.138	57	0.117	0.795	0.163	102	0.173	1.002	0.253	147	0.326	1.406	0.582
13	0.100	0.723	0.138	58	0.117	0.798	0.164	103	0.175	1.009	0.257	148	0.331	1.417	0.594
14	0.100	0.723	0.138	59	0.118	0.801	0.165	104	0.178	1.016	0.261	149	0.335	1.427	0.607
15	0.100	0.724	0.138	60	0.119	0.804	0.166	105	0.180	1.023	0.264	150	0.340	1.437	0.620
16	0.100	0.725	0.139	61	0.120	0.807	0.167	106	0.182	1.030	0.268	151	0.345	1.447	0.633
17	0.101	0.725	0.139	62	0.120	0.811	0.168	107	0.184	1.037	0.272	152	0.349	1.457	0.646
18	0.101	0.726	0.139	63	0.121	0.814	0.169	108	0.187	1.044	0.276	153	0.354	1.467	0.659
19	0.101	0.727	0.139	64	0.122	0.817	0.170	109	0.189	1.052	0.281	154	0.358	1.476	0.672
20	0.101	0.728	0.140	65	0.123	0.820	0.172	110	0.192	1.059	0.285	155	0.363	1.486	0.686
21	0.101	0.729	0.140	66	0.124	0.824	0.173	111	0.194	1.067	0.290	156	0.367	1.495	0.699
22	0.101	0.730	0.140	67	0.124	0.827	0.174	112	0.197	1.074	0.294	157	0.371	1.504	0.713
23	0.102	0.731	0.141	68	0.125	0.831	0.176	113	0.199	1.082	0.299	158	0.376	1.513	0.726
24	0.102	0.732	0.141	69	0.126	0.835	0.177	114	0.202	1.090	0.304	159	0.380	1.522	0.739
25	0.102	0.733	0.141	70	0.127	0.838	0.179	115	0.205	1.098	0.309	160	0.384	1.530	0.753
26	0.102	0.734	0.142	71	0.128	0.842	0.180	116	0.208	1.107	0.315	161	0.388	1.538	0.766
27	0.103	0.735	0.142	72	0.129	0.846	0.182	117	0.210	1.115	0.320	162	0.392	1.546	0.779
28	0.103	0.737	0.142	73	0.130	0.850	0.183	118	0.213	1.123	0.326	163	0.396	1.554	0.791
29	0.103	0.738	0.143	74	0.131	0.854	0.185	119	0.216	1.132	0.331	164	0.399	1.561	0.804
30	0.104	0.739	0.143	75	0.132	0.858	0.186	120	0.219	1.141	0.337	165	0.403	1.568	0.816
31	0.104	0.741	0.144	76	0.133	0.862	0.188	121	0.223	1.149	0.343	166	0.406	1.575	0.827
32	0.104	0.742	0.144	77	0.135	0.866	0.190	122	0.226	1.158	0.350	167	0.409	1.581	0.838
33	0.105	0.743	0.145	78	0.136	0.871	0.192	123	0.229	1.167	0.356	168	0.412	1.587	0.849
34	0.105	0.745	0.145	79	0.137	0.875	0.193	124	0.232	1.176	0.363	169	0.415	1.592	0.858
35	0.105	0.747	0.146	80	0.138	0.880	0.195	125	0.236	1.185	0.370	170	0.417	1.597	0.868
36	0.106	0.748	0.146	81	0.139	0.884	0.197	126	0.239	1.195	0.377	171	0.420	1.602	0.876
37	0.106	0.750	0.147	82	0.141	0.889	0.199	127	0.243	1.204	0.384	172	0.422	1.606	0.884
38	0.106	0.752	0.147	83	0.142	0.894	0.201	128	0.246	1.214	0.392	173	0.424	1.610	0.891
39	0.107	0.754	0.148	84	0.143	0.899	0.203	129	0.250	1.223	0.400	174	0.425	1.613	0.897
40	0.107	0.755	0.149	85	0.145	0.904	0.206	130	0.254	1.233	0.408	175	0.427	1.615	0.903
41	0.108	0.757	0.149	86	0.146	0.909	0.208	131	0.258	1.243	0.416	176	0.428	1.618	0.907
42	0.108	0.759	0.150	87	0.147	0.914	0.210	132	0.262	1.253	0.424	177	0.429	1.619	0.911
43	0.109	0.761	0.151	88	0.149	0.919	0.212	133	0.265	1.263	0.433	178	0.429	1.621	0.913
44	0.109	0.763	0.151	89	0.150	0.924	0.215	134	0.269	1.273	0.442	179	0.430	1.621	0.914
45	0.110	0.765	0.152	90	0.152	0.930	0.217	135	0.274	1.283	0.451	180	0.430	1.622	0.915

Table 10.11: Deferential latitude correction factor for Mercury. μ is in degrees. Note that $\bar{h}(360^\circ - \mu) = \bar{h}(\mu)$, and $\delta h_\pm(360^\circ - \mu) = \delta h_\pm(\mu)$.

Technical terms

Altitude: The angle subtended at the observer by the radius vector connecting a celestial object to an observer on the Earth's surface, and the vector's projection onto the **horizontal plane**. Object's above/below the **horizon** have positive/negative altitudes.

Altitude circle: A **great circle** on the celestial sphere that passes through the local **zenith** at a given observation site on the Earth's surface.

Anomaly: Any deviation in an orbit from uniform circular motion that is concentric with the central body. Anomaly is also used as another word for angle.

Apocenter: Point on a **Keplerian orbit** that is farthest from the central body. If the central body is the Sun then the apocenter is generally termed the *aphelion*. Likewise, if the central body is the Earth then the apocenter is termed the *apogee*.

Arctic circles: Two latitude circles on the Earth's surface that are equidistant from the equator. Above the arctic circles, the Sun never sets for part of the year, and never rises for part of the year.

Argument of latitude: Angle subtended at the central body by the radius vectors connecting the central body to the orbiting body, and the central body to the **ascending node**, in a **Keplerian orbit**.

Ascendent: Point on **ecliptic circle** that is ascending at any given time on the eastern **horizon**.

Ascending node: Point on a **Keplerian orbit** at which the orbital plane crosses the **ecliptic plane** from south to north in the direction of motion of the orbiting body.

Autumnal equinox: The point at which the **ecliptic circle** crosses the **celestial equator** from north to south (in the direction of the Sun's apparent motion along the ecliptic).

Azimuth: Angle subtended at the observer by the projection of the vector connecting a celestial object to an observer on the Earth's surface onto the **horizontal plane**, and the vector connecting the north **compass point** to the observer. Azimuth increases clockwise (that is, from the north to the east) looking at the horizontal plane from above.

Celestial axis: An imaginary extension of the Earth's axis of rotation that pierces the **celestial sphere** at the two **celestial poles**. The sphere's **diurnal motion** is about this axis.

Celestial coordinates: Angular coordinate system whose fundamental plane is the **celestial plane**, and whose poles are the **celestial poles**. The polar and azimuthal angles in this system are called **declination** and **right ascension**, respectively.

Celestial equator: The intersection of the imagined extension of the Earth's **equatorial plane** with the **celestial sphere**.

Celestial plane: The plane containing the Earth's equator.

Celestial poles: The two points at which the **celestial axis** pierces the **celestial sphere**. The north celestial pole lies to the north of the **celestial plane**, whereas the south celestial pole lies to the south. The celestial poles are the only two points on the celestial sphere whose positions are unaffected by **diurnal motion**.

Celestial sphere: An imaginary sphere of infinite radius that is concentric with the Earth. All objects in the sky are thought of as attached to this sphere.

Compass points: At a given observation site on the Earth's surface, the north, east, south, and west compass points lie on the local horizon due north, east, south, and west, respectively, of the observer.

Conjunction: Two celestial objects are said to be in conjunction when they have the same **ecliptic longitude**. For an **inferior planet** in conjunction with the Sun, the conjunction is said to be *superior* if the planet is farther from the Earth than the Sun, and *inferior* if the Sun is farther from the Earth than the planet.

Culmination: A celestial object is said to culminate on a given day when it attains its maximum **altitude** in the sky.

Declination: Angle subtended at the Earth's center by the radius vector connecting a celestial object to the Earth's center, and the vector's projection onto the **celestial plane**. Object's to the north/south of the **celestial equator** have positive/negative declinations.

Deferent: Large circle centered on the Sun about which the **guide point** rotates in a **geocentric planetary orbit**.

Deferential latitude: **Ecliptic latitude** a **superior planet** has by virtue of the **inclination** of its **deferent**.

Deferential latitude correction factor: Correction to the **ecliptic latitude** of an **inferior planet** due to the finite size of its **deferent**.

Diurnal motion: Daily rotation of the **celestial sphere**, and the objects attached to it, from east to west (looking south in the Earth's northern hemisphere) about the **celestial axis**.

Eccentricity: Measure of the displacement along the **major axis** of the central body from the geometric center in a **Keplerian orbit**.

Ecliptic axis: Normal to the **ecliptic plane** that passes through the center of the Earth.

Ecliptic circle: Apparent path traced out by the Sun on the **celestial sphere** during the course of a year.

Ecliptic coordinates: Angular coordinate system whose fundamental plane is the **ecliptic plane**, and whose poles are the **ecliptic poles**.

Ecliptic latitude: Angle subtended at the Earth's center by the radius vector connecting a celestial object to the Earth's center, and the vector's projection onto the **ecliptic plane**. Objects to the north/south of the **ecliptic circle** have positive/ecliptic latitudes.

Ecliptic longitude: Angle subtended at the Earth's center by the projection of the vector connecting a celestial object to the Earth's center onto the **ecliptic plane**, and the vector connecting the **vernal equinox** to the Earth's center. Ecliptic longitude increases counterclockwise (that is, from the west to the east) looking at the ecliptic plane from the north.

Ecliptic plane: Plane containing the mean orbit of the Earth about the Sun.

Ecliptic poles: The two points at which the **ecliptic axis** pierces the **celestial sphere**. The north ecliptic pole lies to the north of the ecliptic plane, whereas the south ecliptic pole lies to the south.

Elongation: Difference in **ecliptic longitude** between two celestial objects.

Epicycle: Small circle, centered on the **guide-point**, about which a planet rotates in a **geocentric planetary orbit**.

Epicyclic anomaly: Angle subtended between the radius vectors connecting the Earth to the **guide-point**, and the guide-point to the planet, in a **geocentric planetary orbit**.

Epicyclic latitude: **Ecliptic latitude** an **inferior planet** has by virtue of the **inclination** of its **epicycle**.

Epicyclic latitude correction factor: Correction to the **ecliptic latitude** of a **superior planet** due to the finite size of its **epicycle**.

Epoch: Standard time at which the **orbital elements** of an orbiting body in the Solar System are specified.

Equant: Point about which the orbiting body appears to rotate uniformly in a **Keplerian orbit** of low **eccentricity**. The equant is diagrammatically opposite the central body with respect to the geometric center of the orbit.

Equation of center: Difference between the **true anomaly** and the **mean anomaly** in a **Keplerian orbit**.

Equation of epicycle: **Elongation** of a planet from its **guide-point** in a **geocentric planetary orbit**.

Equation of time: Time interval between **local noon** and **mean local noon**.

Equinoxes: The two opposite points on the **ecliptic circle** that the Sun reaches on the days of the year that day and night are equally long.

Evection: An **anomaly** of the Moon's orbit about the Earth that is associated with the perturbing influence of the Sun.

Geocentric planetary orbit: An orbit in which a planet rotates about a **guide-point** in a small circle called an **epicycle**, and the guide-point rotates about the Earth in a large circle called a **deferent**.

Great circle: A circle on the surface of a sphere produced by the intersection of a plane that bisects the sphere.

Greatest elongation: Greatest **elongation** of an *inferior planet* from the Sun. If the planet is to the east/west of the Sun then the elongation is called the greatest eastern/western elongation.

Guide-point: Center of an epicycle in a **geocentric planetary orbit**.

Horizon: Tangent plane to the Earth's surface, at a given observation site, that divides the **celestial sphere** into visible and invisible hemispheres.

Horizontal coordinates: Angular coordinate system whose fundamental plane is the **horizontal plane**, and whose poles are the **zenith** and **nadir**.

Horizontal plane: Plane containing the local horizon.

Horoscope: Point on the **ecliptic circle** that is ascending at a given time on the eastern **horizon**.

Inclination: Maximum angle subtended between the plane of a **Keplerian orbit** and the **ecliptic plane**.

Inclination of ecliptic: Inclination of the **ecliptic plane** to the **equatorial plane**.

Inferior planet: A planet that is closer to the Sun than the Earth.

Julian day number: Number ascribed to a particular day in a scheme in which days are numbered consecutively from January 1, 4713 BC (in Julian calendar), which is designated day zero. Julian days start at 12:00 UT.

Keplerian orbit: Ellipse that is confocal with the central object. The radius vector connecting the central and orbiting bodies sweeps out equal areas in equal time intervals.

Local mean noon: Instant in time at which the **mean Sun** attains its upper **transit**.

Local noon: Instant in time at which the Sun attains its upper **transit**.

Longitude of ascending node: Angle subtended at the central body by the radius vectors connecting the central body to the **ascending node**, and the central body to the **vernal equinox**, in a **Keplerian orbit**.

Longitude of pericenter: Angle subtended at the central body by the radius vectors connecting the central body to the **pericenter**, and the central body to the **vernal equinox**, in a **Keplerian orbit**.

Major axis: Longest diameter that passes through the geometric center of a **Keplerian orbit**.

Major radius: Half the length of the **major axis** of a **Keplerian orbit**.

Mean anomaly: Angle that would be subtended at the central body by the radius vectors connecting the central body to the orbiting body, and the central body to the **pericenter**, in a **Keplerian orbit**, if the orbiting body were to rotate about the central body with a uniform angular velocity.

Mean argument of latitude: Value the **argument of latitude** would have if the orbiting body in a **Keplerian orbit** were to rotate about the central body at a fixed angular velocity.

Mean argument of latitude at epoch: Value of the **mean argument of latitude** of a **Keplerian orbit** at the **epoch**.

Mean (ecliptic) longitude: Value the **ecliptic longitude** would have if the orbiting body in a **Keplerian orbit** were to rotate about the central body at a fixed angular velocity.

Mean (ecliptic) longitude at epoch: Value of the **mean longitude** of a **Keplerian orbit** at the **epoch**.

Mean Solar day: Time interval between successive **local mean noons**.

Mean Solar time: Time calculated using the **mean Sun**.

Mean sun: Fictitious body that travels around the **celestial equator** (from west to east looking south in the Earth's northern hemisphere) at a uniform rate, and completes one orbit every tropical **year**.

Meridian plane: Plane passing through the **zenith** and the north and south **compass points** at a given observation site on the Earth's surface.

Minor axis: The minor axis of a **Keplerian orbit** is the shortest diameter that passes through the geometric center.

Minor radius: The minor radius of a **Keplerian orbit** is half the length of the **minor axis**.

Month: A *sidereal month* (27.32166 days) is the mean time needed for the Moon to complete an orbital rotation around the Earth relative to the fixed stars. A *tropical month* (27.32158 days) is the mean time needed for the Moon's ecliptic latitude to increase by 360°, and is 6.8 seconds shorter than a sidereal month because of the precession of the equinoxes. A *synodic month* (29.53059 days) is the mean time interval between successive new Moons. An *anomalistic month* (27.55455 days) is the mean time interval between successive passages of the Moon through its perigee. A *draconic month* (27.21222 days) is the mean time interval between successive passages of the Moon thought its ascending node.

Nadir: Point on the **celestial sphere** that is directly underfoot at a given observation site on the Earth's surface.

Opposition: Two celestial objects are said to be in opposition when their **ecliptic longitudes** differ by 180°.

Orbital elements: Eight quantities that completely specify a **Keplerian orbit**: that is, **major radius**, **eccentricity**, **rate of motion of mean longitude**, **rate of motion of mean anomaly**, **mean longitude at epoch**, **mean anomaly at epoch**, **inclination**, **rate of motion in mean argument of latitude**, **mean argument of latitude at epoch**.

Parallactic angle: Angle subtended between the **ecliptic circle** and an **altitude circle**.

Parallax: Apparent change in position of a nearby celestial object in the sky when it is viewed at different points on the Earth's surface.

Pericenter: Point on a **Keplerian orbit** that is closest to the central body. If the central body is the Sun then the pericenter is generally termed the *perihelion*. Likewise, if the central body is the Earth then the pericenter is termed the *perigee*.

Precession of equinoxes: A slow movement of the **vernal equinox** relative to the fixed stars that causes the **ecliptic longitude** of a fixed star to increase steadily at the rate of $50.3''$ per year.

Prograde motion: Motion of a **superior planet** in the sky in the same direction to that of its mean motion.

Radial anomaly: Difference between the length of the radius vector connecting the central body to the orbiting body, in a **Keplerian orbit**, and the **major radius**.

Rate of motion in mean anomaly: Time derivative of the **mean anomaly** of a **Keplerian orbit**.

Rate of motion in mean argument of latitude: Time derivative of the **mean argument of latitude** of a **Keplerian orbit**.

Rate of motion in mean longitude: Time derivative of the **mean longitude** of a **Keplerian orbit**.

Retrograde motion: Motion of a **superior planet** in the sky in the opposite direction to that of its mean motion.

Right ascension: Angle subtended at the Earth's center by the projection of the vector connecting a celestial body to the Earth's center onto the **celestial plane**, and the vector connecting the **vernal equinox** to the Earth's center. Right ascension increases counterclockwise (that is, from the west to the east) looking at the celestial plane from the north.

Seasons: Spring is the time interval between the **vernal equinox** and the **summer solstice**, summer the interval between the summer solstice and the **autumnal equinox**, autumn the interval between the autumnal equinox and the **winter solstice**, and winter the interval between the winter solstice and the next spring equinox.

Sidereal day: Time interval ($23^h 56^m 4^s$) between successive upper **transits** of a fixed star.

Sidereal hour: One twenty-fourth of a **sidereal day**.

Sidereal time: Time calculated using the fixed stars.

Solar day: Time interval between successive **local noons**.

Solar time: Time calculated using the Sun.

Solstices: The two opposite points on the **ecliptic circle** that the Sun reaches on the longest and shortest days of the year.

Station: Point in the orbit of a **superior planet** at which it switches from **prograde** to **retrograde** motion, or vice versa. The former station is called a *retrograde station*, whereas the latter is called a *prograde station*.

Summer solstice: Most northerly point on the **ecliptic circle**.

Superior planet: A planet farther from the Sun than the Earth.

Syzygy: **Conjunction** or **opposition** of the Sun and the Moon.

Transit: On a given day, and at a given observation site on the Earth's surface, a celestial object is said to transit when it crosses the **meridian plane**. The object simultaneously attains either its highest or lowest altitude in the sky. The transit is called an upper/lower transit when the object attains its highest/lowest altitude.

True anomaly: Angle subtended at the central body by the radius vectors connecting the central body to the orbiting body, and the central body to the **pericenter**, in a **Keplerian orbit**.

Tropics: Two latitude circles on the Earth's surface that are equidistant from the equator. Between the tropics the Sun **culminates** both to the north and south of the **zenith** during the course of a year. Outside the tropics, the Sun culminates either only to the north or only to the south of the zenith.

Universal time: Time defined such that **mean local noon** coincides with 12:00 UT every day at an observation site of terrestrial longitude 0°.

Vernal equinox: Point at which the **ecliptic circle** crosses the **celestial equator** from south to north (in the direction of the Sun's apparent motion along the ecliptic).

Winter solstice: Most southerly point on the **ecliptic circle**.

Year: A *sidereal year* (365.256363004 days) is the time required for the Earth to complete an orbital rotation around the Sun relative to the fixed stars. A *tropical year* (365.2421875 days) is the time interval between successive passages of the Sun through the **vernal equinox**, and is 20.4 minutes shorter than a sidereal year.

Zenith: Point on the **celestial sphere** that is directly overhead at a given observation site on the Earth's surface.

Zodiac: The signs of the zodiac are conventional names given to 30° segments of the **ecliptic circle**.

Index of symbols

A: azimuth.

a: major radius, altitude.

a_M: major radius of Moon's orbit.

a_S: major radius of Sun's orbit.

α: right ascension.

$\bar{\alpha}$: right ascension of mean sun.

b: minor radius.

β: ecliptic latitude.

D: lunar-solar elongation.

$\bar{D}$: mean lunar-solar elongation.

$\tilde{D}$: semi-mean lunar-solar elongation.

δ: declination.

δ_M: parallax of Moon.

e: eccentricity.

e_M: eccentricity of Moon's orbit.

e_S: eccentricity of Sun's orbit.

ϵ: inclination of ecliptic to celestial equator.

E: elliptic anomaly.

ζ: radial anomaly.

ζ_i: lunar radial anomalies.

θ: equation of epicycle.

F: argument of latitude.

$\bar{F}$: mean argument of latitude.

$\bar{F}_0$: mean argument of latitude at epoch.

$\bar{F}_M$: mean argument of latitude of Moon.

λ: ecliptic longitude.

λ_S: ecliptic longitude of Sun.

$\bar{\lambda}$: mean longitude.

$\bar{\lambda}_0$: mean longitude at epoch.

$\bar{\lambda}_M$: mean longitude of Moon.

$\bar{\lambda}_S$: mean longitude of Sun.

L: terrestrial latitude.

M: mean anomaly.

M_0: mean anomaly at epoch.

M_M: mean anomaly of Moon.

M_S: mean anomaly of Sun.

μ: epicyclic anomaly, parallactic angle.

n: rate of motion in mean longitude.

$\tilde{n}$: rate of motion in mean anomaly.

$\breve{n}$: rate of motion in mean argument of latitude.

q: equation of center.

q_i: lunar longitudinal anomalies.

R_E: radius of Earth.

R_M: radius of Moon.

R_S: radius of Sun.

r: radial distance.

ρ_S: angular radius of Sun.

ρ_M: angular radius of Moon.

ρ_U: angular radius of Earth's umbra.

t: time.

t_0: epoch.

T: true anomaly.

τ: orbital period.

ϖ: longitude of perigee.

Ω: longitude of ascending node.

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