A Modern Almagest:
An Updated Version of Ptolemy's Model of the Solar System

Richard Fitzpatrick

Chapter 1

Introduction

1.1 Euclid's Elements and Ptolemy's Almagest

The modern world inherited two major scientific treaties from the civilization of ancient Greece. The first of these, the *Elements* (Στοιχεῖα) of Euclid (Εὐκλείδης), is a large compendium of mathematical theorems concerning geometry, proportion, and number theory. These theorems were not necessarily discovered by Euclid himself—being largely the work of earlier mathematicians, such as Eudoxos of Cnidus (Εὔδοξος ὁ Κνίδιος), and Theaetetus of Athens (Θεαίτητος ὁ Ἀθηναῖος) —but were arranged by him in a logical manner, so as to demonstrate that they can all ultimately be derived from five simple axioms. The Elements is rightly regarded as the first, largely successful, attempt to construct an axiomatic system in mathematics, and is still held in high esteem within the scientific community.

The second treatise, the *Almagest* [1] of Claudius Ptolemy (Κλαύδιος Πτολεμαῖος), is an attempt to find a simple geometric explanation for the apparent motions of the Sun, the Moon, and the five visible planets in the Earth's sky. On the basis of his own naked-eye observations, combined with those of earlier astronomers such as Hipparchus of Nicaea (Ἱππάρχος ὁ Νικαεύς), Ptolemy proposed a model of the Solar System in which the Earth is stationary. According to this model, the Sun moves in a circular orbit, (nearly) centered on the Earth, that maintains a fixed inclination of about 23° to the terrestrial equator. Furthermore, each planet moves on the rim of a small circle called an epicycle (ἐπίκυκλος in Greek), whose center revolves around the Earth on a large eccentric circle called a deferent (ἀποφορά in Greek). (See Figure 8.2.) The planetary deferents and epicycles also maintain fixed inclinations,[2] which are all fairly close to 23°, to the terrestrial equator.

The scientific reputation of the Almagest has not fared as well as that of Euclid's Elements. Nowadays, it is a commonly held belief, even amongst scientists, that Ptolemy's mistaken adherence to the tenets of Aristotelian philosophy—in particular, the immovability of the Earth, and the necessity for heavenly bodies to move uniformly in circles—led him to construct an overcomplicated, unwieldy, and faintly ridiculous model of planetary motion. As is well known, Ptolemy's model was superseded in 1543 AD by the heliocentric model of Nicolaus Copernicus, in which the planets revolve about the Sun in circular orbits.[3] The Copernican model was, in turn, superseded in the early 1600's AD by the, ultimately correct, model of Johannes Kepler, in which the planets revolve about the Sun in eccentric elliptical orbits.

[1] The true title of this work is Μαθηματικὴ Σύνταξις, which means "Mathematical Treatise". The treatise was later called Ἡ Μεγάλη Σύνταξις, meaning "The Great Treatise", and the superlative form of the adjective, μεγίστη, with the arabic article "al" prepended, lies behind the Arabic name from which the English name Almagest derives.

[2] Actually, Ptolemy erroneously allowed the inclinations of the deferents and epicycles to vary slightly.

[3] In fact, the planets revolve on small circular epicycles, whose centers revolve around the Sun on eccentric circular deferents.

The aim of this treatise is to re-examine the scientific merits of Ptolemy's Almagest.

1.2 Ptolemy's Model of the Solar System

Claudius Ptolemy lived and worked in the city of Alexandria, capital of the Roman province of Egypt, during the reigns of the later Flavian and the Antonine emperors. Ptolemy was heir—via the writings of Euclid, and later mathematicians such as Apollonius of Perga (Ἀπολλώνιος ὁ Περγαῖος), and Archimedes of Syracuse (Ἀρχιμήδης ὁ Συρακούσιος)—to the considerable mathematical knowledge of geometry and arithmetic acquired by the civilization of ancient Greece. Ptolemy also inherited an extensive ancient Greek tradition of observational and theoretical astronomy. The most important astronomer prior to Ptolemy was undoubtedly Hipparchus of Nicaea (second century BC), who developed the theory of solar motion used by Ptolemy, discovered the precession of the equinoxes, and collected an extensive set of astronomical observations—some of which he made himself, and some of which dated back to Babylonian times—which were available to Ptolemy (possibly via the famous Library of Alexandria). Other astronomers who made significant contributions prior to Ptolemy include Meton of Athens (Μέτων ὁ Ἀθηναῖος, 5th century BC), Eudoxos of Cnidus (5th/4th century BC), Callipus of Cyzicus (Καλλίππος ὁ Κυζίκιος, 4th century BC), Aristarchus of Samos (Ἀρίσταρχος ὁ Σάμιος, 4th/3rd century BC), Eratosthenes of Cyrene (Ἐρατοσθένους τοῦ Κυρηναίου, 3rd/2nd century BC), and Menelaus of Alexandria (Μενέλαος ὁ Ἀλεξανδρεύς, 1st century AD).

Ptolemy's aim in the Almagest is to construct a kinematic model of the Solar System, as seen from the Earth. In other words, the Almagest outlines a relatively simple geometric model that describes the apparent motions of the Sun, Moon, and planets, relative to the Earth, but does not attempt to explain why these motions occur. (The models of Copernicus and Kepler are similar to that of Ptolemy in this respect.) As such, the fact that the model described in the Almagest is geocentric in nature is a non-issue, because the Earth is stationary in its own frame of reference. This is not to say that the heliocentric hypothesis is without advantages. As we shall see, the assumption of heliocentricity allowed Copernicus to determine, for the first time, the ratios of the mean radii of the various planets in the Solar System.

We now know, from the work of Kepler, that planetary orbits are actually ellipses that are confocal with the Sun. Such orbits possess two main properties. First, they are eccentric; that is, the Sun is displaced from the geometric center of the orbit. Second, they are elliptical; that is, the orbit is elongated along a particular axis. Now, Keplerian orbits are characterized by a quantity, e, called the eccentricity, which measures their deviation from circularity. It is easily demonstrated that the eccentricity of a Keplerian orbit scales as e, whereas the corresponding degree of elongation scales as e^2. Because the orbits of the visible planets in the Solar System all possess relatively small values of e (that is, $e \leq 0.21$), it follows that, to an excellent approximation, these orbits can be represented as eccentric circles; that is, circles that are not quite concentric with the Sun. In other words, we can neglect the ellipticities of planetary orbits compared to their eccentricities. This is exactly what Ptolemy does in the Almagest. It follows that Ptolemy's assumption that heavenly bodies move in circles is actually one of the main strengths of his model, rather than being the main weakness, as is commonly supposed.

Kepler's second law of planetary motion states that the radius vector connecting a planet to the Sun sweeps out equal areas in equal time intervals. In the approximation in which planetary orbits are represented as eccentric circles, this law implies that a typical planet revolves around the Sun at a non-uniform rate. However, it is easily demonstrated that the non-uniform rotation of the radius vector connecting the planet to the Sun implies a uniform rotation of the radius vector connecting the planet to the so-called equant; that is, the point diagrammatically opposite to the Sun with respect to the geometric center of the orbit. (See Figure 1.1.) Ptolemy discovered the equant scheme empirically, and used it to control the non-uniform rotation of the planets in his model. In fact, this discovery is one of Ptolemy's main claims to fame.

It follows, from the previous discussion, that the geocentric model of Ptolemy is equivalent to a heliocentric model in which the various planetary orbits are represented as eccentric circles, and in which the radius vector connecting a given planet to its corresponding equant revolves at a uniform rate. In fact, Ptolemy's model of planetary motion can be thought of as a version of Kepler's model that is accurate to first order in the planetary eccentricities. (See Chapter 4.) According to the Ptolemaic scheme, from the point of view of the Earth, the orbit of the Sun is described by a single circular motion, whereas that of a planet is described by a combination of two circular motions. In reality, the single circular motion of the Sun represents the (approximately) circular motion of the Earth around the Sun, whereas the two circular motions of a typical planet represent a combination of the planet's (approximately) circular motion around the Sun, and the Earth's motion around the Sun. Incidentally, the popular myth that Ptolemy's scheme requires an absurdly large number of circles in order to fit the observational data to any degree of accuracy has no basis in fact. Actually, Ptolemy's model of the Sun and the planets, which fits the data very well, only contains 13 circles (that is, 6 deferents and 7 epicycles). (There are 7 epicycles because Mercury possesses an addional spurious epicycle.)

Ptolemy is often accused of slavish adherence to the tenants of Aristotelian philosophy, to the overall detriment of his model. However, despite Ptolemy's conventional geocentrism, his model of the Solar System deviates from orthodox Aristotelism in a number of crucially important respects. In his treatise Περὶ Οὐρανοῦ (On the Heavens), Aristotle (Ἀριστοτέλης) argues, from a purely philosophical standpoint, that heavenly bodies should move in single uniform circles. However, in the Ptolemaic system, the motion of the planets is a combination of two circular motions. Moreover, at least one of these motions is non-uniform. Aristotle also argues, again from purely philosophical grounds, that the Earth is located at the exact center of the universe, about which all heavenly bodies orbit in concentric circles. However, in the Ptolemaic system, the Earth is slightly displaced from the center of the universe. Indeed, there is no unique center of the universe, because the circular orbit of the Sun and the circular planetary deferents all have slightly different geometric centers, none of which coincide with the Earth. As described in the Almagest, the non-orthodox (from the point of view of Aristolelian philosophy) aspects of Ptolemy's model were ultimately dictated by observations. This suggests that, although Ptolemy's world-view was based on Aristolelian philosophy, he did not hesitate to deviate from this standpoint when required to by observational data.

From our heliocentric point of view, it is easily appreciated that the epicycles of the superior planets (that is, the planets further from the Sun than the Earth) in Ptolemy's model actually represent the Earth's orbit around the Sun, whereas the deferents represent the planets' orbits around the Sun. (See Figure 8.1.) It follows that the epicycles of the superior planets should all be the same size (that is, the size of the Earth's orbit), and that the radius vectors connecting the centers of the epicycles to the planets should always all point in the same direction as the vector connecting the Earth to the Sun.

We can also appreciate that the deferents of the inferior planets (that is, the planets closer to the Sun than the Earth) in Ptolemy's model actually represent the Earth's orbit around the Sun, whereas the epicycles represent the planets' orbits around the Sun. (See Figure 9.1.) It follows that the deferents of the inferior planets should all be the same size (that is, the size of the Earth's orbit), and that the centers of the epicycles (relative to the Earth) should all correspond to the position of the Sun (relative to the Earth).

The geocentric model of the Solar System outlined previously represents a perfected version of Ptolemy's model, constructed with a knowledge of the true motions of the planets around the Sun. Not surprisingly, the model actually described in the Almagest deviates somewhat from this ideal form. In the following, we shall refer to these deviations as "errors", but this should not be understood in a pejorative sense.

Ptolemy's first error lies in his model of the Sun's apparent motion around the Earth, which he inherited

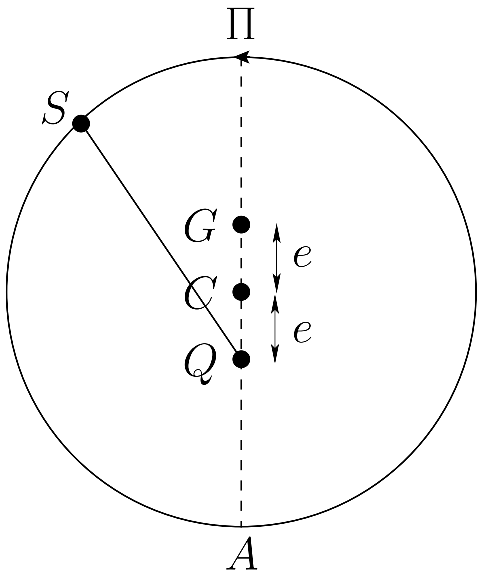

Figure 1.1: Hipparchus' (and Ptolemy's) model of the Sun's apparent orbit about the Earth (right) compared to the optimal model (left). The radius vectors in both models rotate uniformly. Here, S is the Sun, G the Earth, C the geometric center of the orbit, Q the equant, Π the perigee, and A the apogee. The radius of the orbit is normalized to unity.

from Hipparchus. Figure 1.1 compares what Ptolemy actually did, in this respect, compared to what he should have done in order to be completely consistent with the rest of his model. Let us normalize the mean radius of the Sun's apparent orbit to unity, for the sake of clarity. Ptolemy should have adopted the model shown on the left in Figure 1.1, in which the Earth is displaced from the center of the Sun's orbit a distance $e = 0.0167$ (the eccentricity of the Earth's orbit around the Sun) towards the perigee (the point of the Sun's closest approach to the Earth), and the equant is displaced the same distance in the opposite direction. The instantaneous angular position of the Sun is then obtained by allowing the radius vector connecting the equant to the Sun to rotate uniformly at the Sun's mean orbital angular velocity. Of course, this implies that the Sun rotates non-uniformly about the Earth. Ptolemy actually adopted the Hipparchian model shown on the right in Figure 1.1. In this model, the Earth is displaced a distance $2e$ from the center of the Sun's orbit in the direction of the perigee, and the Sun rotates at a uniform rate (that is, the radius vector CS rotates uniformly). It turns out that, to first order in e, these two models are equivalent in terms of their ability to predict the angular position of the Sun relative to the Earth. (See Chapter 4.) Nevertheless, the Hippachian model is incorrect, because it predicts too large (by a factor of two) a variation in the radial distance of the Sun from the Earth (and, hence, the angular size of the Sun) during the course of a year. (See Chaper 4.) Ptolemy probably adopted the Hipparchian model because his Aristotelian leanings prejudiced him in favor of uniform circular motion whenever this was consistent with observations. (It should be noted that Ptolemy was not interested in explaining the relatively small variations in the angular size of the Sun during the year; presumably, because this effect would have been very difficult for him to accurately measure.)

Ptolemy's next error was to neglect the non-uniform rotation of the superior planets on their epicycles. This is equivalent to neglecting the orbital eccentricity of the Earth (recall that the epicycles of the superior planets actually represent the Earth's orbit) compared to those of the superior planets. It turns out that this is a fairly good approximation, because the superior planets all have significantly greater orbital eccentricities than the Earth. Nevertheless, neglecting the non-uniform rotation of the superior planets on their epicycles has the unfortunate effect of obscuring the tight coupling between the apparent motions of these planets, and that of the Sun. The radius vectors connecting the epicycle centers of the superior planets to the planets themselves should always all point exactly in the same direction as that of the Sun relative to the Earth. When the aforementioned non-uniform rotation is neglected, the radius vectors instead point in the direction of the mean Sun relative to the Earth. The mean Sun is a fictitious body that has the same apparent orbit around the Earth as the real Sun, but that circles the Earth at a uniform rate. The mean Sun only coincides with the real Sun twice a year.

Ptolemy's third error is associated with his treatment of the inferior planets. As we have seen, in going from the superior to the inferior planets, deferents and epicycles effectively swap roles. For instance, it is the deferents of the inferior planets, rather than the epicycles, that represent the Earth's orbit. Hence, for the sake of consistency with his treatment of the superior planets, Ptolemy should have neglected the non-uniform rotation of the epicycle centers around the deferents of the inferior planets, and retained the non-uniform rotation of the planets themselves around the epicycle centers. Instead, he did exactly the opposite. This is equivalent to neglecting the inferior planets' orbital eccentricities relative to that of the Earth. It follows that this approximation only works when an inferior planet has a significantly smaller orbital eccentricity than that of the Earth. It turns out that this is indeed the case for Venus, which has the smallest eccentricity of any planet in the Solar System. Thus, Ptolemy was able to successfully account for the apparent motion of Venus. Mercury, on the other hand, has a much larger orbital eccentricity than the Earth. Moreover, it is particularly difficult to obtain good naked-eye positional data for Mercury, because this planet always appears very close to the Sun in the sky. Consequently, Ptolemy's Mercury data was highly inaccurate. Not surprisingly, Ptolemy was not able to account for the apparent motion of Mercury using his standard deferent-epicycle approach. Instead, in order to fit the data, he was forced to introduce an additional, and entirely spurious, epicycle into his model of Mercury's orbit.

Ptolemy's fourth, and possibly largest, error is associated with his treatment of the Moon. It should be noted that the Moon's motion around the Earth is extremely complicated in nature, because it is strongly perturbed by the Sun, and was not fully understood until the early 20th century AD. Ptolemy constructed an ingenious geometric model of the Moon's orbit that was capable of predicting the lunar ecliptic longitude to reasonable accuracy. Unfortunately, this model necessitates a monthly variation in the Earth-Moon distance by a factor of about two, which implies a similarly large variation in the Moon's angular diameter. However, the observed variation in the Moon's diameter is very much smaller than this. Hence, Ptolemy's model of lunar motion is not even approximately correct.

Ptolemy's fifth error is associated with his treatment of planetary ecliptic latitudes. Given that the deferents and epicycles of the superior planets represent the orbits of the planets themselves around the Sun, and the Sun's apparent orbit around the Earth, respectively, it follows that one should take the slight inclination of planetary orbits to the ecliptic plane (that is, the plane of the Sun's apparent orbit) into account by tilting the deferents of superior planets, while keeping their epicycles parallel to the ecliptic. Similarly, given that the epicycles and deferents of inferior planets represent the orbits of the planets themselves around the Sun, and the Sun's apparent orbit around the Earth, respectively, one should tilt the epicycles of inferior planets, while keeping their deferents parallel to the ecliptic. Finally, because the inclination of planetary orbits are all essentially constant in time, the inclinations of the epicycles and deferents should also be constant. Unfortunately, when Ptolemy constructed his theory of planetary latitudes, he tilted the both deferents and epicycles of all of the planets. Even worse, he allowed the inclinations of the epicycles to the ecliptic plane to vary in time. The net result is a theory which is far more complicated than is necessary.

The final failing in Ptolemy's model of the Solar System lies in its scale invariance. Using angular position data alone, Ptolemy was able to determine the ratio of the epicycle radius to that of the deferent for each planet, but was not able to determine the relative sizes of the deferents of different planets. In order to break this scale invariance it is necessary to make an additional assumption; namely, that the Earth orbits the Sun. This brings us to Copernicus.

1.3 Copernicus's Model of the Solar System

The Polish astronomer Nicolaus Copernicus (1473–1543 AD) studied the Almagest assiduously, but eventually became dissatisfied with Ptolomy's approach. The main reason for this dissatisfaction was not the geocentric nature of Ptolomy's model, but rather the fact that it mandates that heavenly bodies execute non-uniform circular motion. Copernicus, like Aristotle, was convinced that the supposed perfection of the heavens requires such bodies to execute uniform circular motion only. Copernicus was thus spurred to construct his own model of the Solar System, which was described in his book *De Revolutionibus Orbium Coelestium* (On the Revolutions of the Heavenly Spheres), published in the year of his death.

The most well-known aspect of Copernicus's model is the fact that it is heliocentric. As has already been mentioned, when describing the motion of the Sun, Moon, and planets relative to the Earth, it makes little practical difference whether one adopts a geocentric or a heliocentric model of the Solar System. Having said this, the heliocentric approach does have one large advantage. If we accept that the Sun, and not the Earth, is stationary, then it immediately follows that the epicycles of the superior planets, and the deferents of the inferior planets, represent the Earth's orbit around the Sun. Hence, all of these circles must be the same size. This realization allows us to break the scale invariance which is one of the main failings of Ptolemy's model. Thus, the ratio of the deferent radius to that of the epicycle for a superior planet, which is easily inferred from observations, actually corresponds to the ratio of planet's orbital radius to that of the Earth. Likewise, the ratio of the epicycle radius to that of the deferent for an inferior planet, which is again easily determined observationally, also corresponds to the ratio of the planet's orbital radius to that of the Earth. Using this type of reasoning, Copernicus was able to construct the first accurate scale model of the Solar System, and to firmly establish the order in which the planets orbit the Sun. In some sense, this was his main achievement.

Copernicus's insistence that heavenly bodies should only move in uniform circles lead him to reject Ptolemy's equant scheme, and to replace it with the scheme illustrated in Figure 1.2. According to Copernicus, a heliocentric planetary orbit is a combination of two circular motions. The first is the motion of the planet around a small circular epicycle, and the second is the motion of the center of the epicycle around the Sun on a circular deferent. Both motions are uniform, and in the same direction. However, the former motion is twice as fast as the latter. In addition, the Sun is displaced from the center of the deferent in the direction of the perihelion, the displacement being proportional to the orbital eccentricity. Furthermore, the Sun's displacement is three times greater than the radius of the epicycle. Finally, the radius of the deferent is equal to the major radius of the planetary orbit. It turns out that Copernicus' scheme is a marginally less accurate approximation than Ptolemy's to a low eccentricity Keplerian orbit. (See Chapter 4.)

Copernicus modeled the orbit of the Earth around the Sun using an Hippachian scheme (see Figure 1.1) in which the Earth moves uniformly around an eccentric circle. Unfortunately, such a scheme exaggerates the variation in the radial distance between the Earth and the Sun during the course of a year by a factor of two, and so introduces significant errors into the calculation of the parallax of the planets due to the motion of the Earth. On the other hand, Copernicus' model of the Moon's orbit around the Earth is a considerable improvement on Ptolemy's, because it does not grossly exaggerate the monthly variation in the Earth-Moon distance. Like Ptolemy, Copernicus introduced an additional spurious epicycle into his model of Mercury's

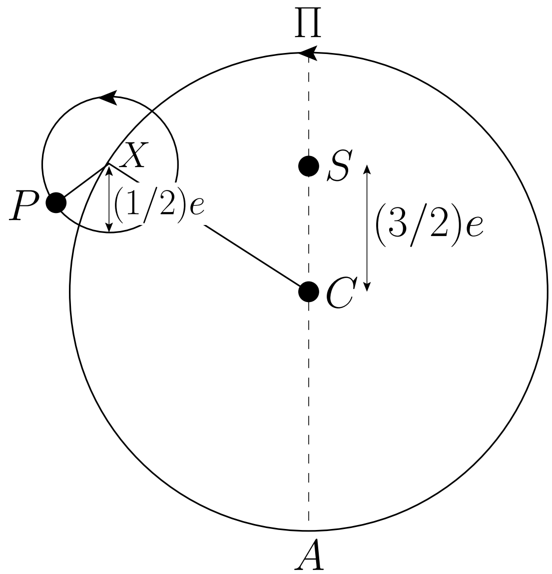

Figure 1.2: Copernicus' model of a heliocentric planetary orbit. Here, S is the Sun, P the planet, C the geometric center of the deferent, X the center of the epicycle, Π the perihelion, and A the aphelion. The radius vectors CX and XP both rotate uniformly in the same direction, but XP rotates twice as fast as CX. The major radius of the orbit is normalized to unity.

orbit, and erroneously allowed the inclination of his planetary orbits to vary slightly in time.

In summary, Copernicus's model of the Solar System contains approximately the same number of epicycles as Ptolemy's, the only difference being that Copernicus' epicycles are much smaller than Ptolemy's. Indeed, the model of Copernicus is about as complicated, and not appreciably more accurate, than that described in the Almagest. In this respect, Copernicus cannot be said to have demonstrated the correctness of his heliocentric approach on the basis of observational data.

1.4 Kepler's Model of the Solar System

Johannes Kepler (1571–1630 AD) was fortunate to inherit an extensive set of naked-eye solar, lunar, and planetary angular position data from the Danish astronomer Tycho Brahe (1546–1601 AD). This data extended over many decades, and was of unprecedented accuracy.

Although Kepler adopted the heliocentric approach of Copernicus, what he effectively first did was to perfect Ptolemy's model of the Solar System (or, rather, its heliocentric equivalent). Thus, Kepler replaced Ptolemy's erroneous equantless model of the Sun's apparent orbit around the Earth with a corrected version containing an equant; in the process, halving the eccentricity of the orbit. (See Figure 1.1.) Kepler also introduced equants into the epicycles of the superior and inferior planets. Once he had perfected Ptolemy's model, the heliocentric nature of the Solar System became manifestly apparent to Kepler. For instance, he found that the epicycles of the superior planets, the Sun's apparent orbit around the Earth, and the deferents of the inferior planets, all had exactly the same eccentricity. The obvious implication is that these circles all correspond to some common motion within the Solar System; in fact, the motion of the Earth around the Sun.

Once Kepler had corrected the Almagest model, he compared its predictions with his observational data.

In particular, Kepler investigated the apparent motion of Mars in the night sky. Kepler found that his model performed extremely well, but that there remained small differences between its predictions and the observational data. The maximum discrepancy was about $8'$; that is, about one quarter of the apparent size of the Sun. By the standards of naked-eye astronomy, this was a very small discrepancy. Nevertheless, given the incredible accuracy of Tycho Brahe's observations, the discrepancy was still significant. Thus, Kepler embarked on an epic new series of calculations which eventually lead him to the conclusion that the planetary orbits are actually eccentric ellipses, rather than eccentric circles. Kepler published the results of his research in his treatise *Astronomia Nova* (New Astronomy) in 1609 AD. It is interesting to note that had Tycho's data been a little less accurate, or had the orbit of Mars been a little less eccentric, Kepler might well have settled for a model which was kinematically equivalent to a perfected version of the model described in the Almagest. We can also appreciate that, given the far less accurate observational data available to Ptolemy, there was no way in which he could have discerned the very small difference between elliptical planetary orbits and the eccentric circular orbits employed in the Almagest.

1.5 Purpose of Treatise

As we have seen, misconceptions abound regarding the details of Ptolemy's model of the Solar System, as well as its scientific merit. Part of the reason for this is that the Almagest is an extremely difficult book for a modern reader to comprehend. For instance, virtually all of its theoretical results are justified via lengthy and opaque geometric proofs. Moreover, the plane and spherical trigonometry employed by Ptolemy is of a rather primitive nature, and, consequently, somewhat unwieldy. Dates are also a major stumbling block, because three different systems are used in the Almagest, all of which are archaic, and essentially meaningless to the modern reader. Another difficulty is the unfamiliar, and far from optimal, ancient Greek method of representing numbers and fractions. Finally, the terminology employed in the Almagest is, in many instances, significantly different to that used in modern astronomy textbooks.

The aim of this treatise is to reconstruct Ptolemy's model of the Solar System employing modern mathematical methods, standard dates, and conventional astronomical terminology. It is hoped that the resulting model will enable the reader to comprehend the full extent of Ptolemy's scientific achievement. In fact, the model described in this work is a somewhat improved version of Ptolemy's, in that all of the previously mentioned deficiencies have been corrected. Furthermore, Ptolemy's equant scheme has been replaced by a Keplerian scheme, expanded to second order in the planetary eccentricities. It should be noted, however, that these two schemes are essentially indistinguishable for small eccentricity orbits. Certain aspects of the Almagest have not been reproduced. For instance, it was not thought necessary to instruct the reader on how to construct trigonometric tables, or primitive astronomical instruments. Furthermore, no attempted has been made to derive any of the model parameters directly from observational data, because the orbital elements and physical properties of the Sun, Moon, and planets are, by now, extremely well established. Any detailed discussion of the fixed stars has also been omitted, because stellar positions are also very well established, and the apparent motion of the stars in the sky is comparatively straightforward compared to those of the Sun, the Moon, and the planets. What remains is a mathematical model of the Solar System that is surprisingly accurate (the maximum errors in the ecliptic longitudes of the Sun, Moon, Mercury, Venus, Mars, Jupiter, and Saturn during the years 1995–2006 AD are $0.7'$, $14'$, $28'$, $10'$, $14'$, $4'$, and $1'$, respectively), yet sufficiently simple that all of the necessary calculations can be performed by hand, with the aid of tables. The form of the calculations, as well as the layout of the tables, is, for the most part, fairly similar to those found in the Almagest. Many examples of the use of the tables are provided.

Chapter 2

Spherical Astronomy

2.1 Celestial Sphere

It is often helpful to imagine that celestial objects are attached to a vast sphere centered on the Earth. This fictitious construction is known as the *celestial sphere*. The Earth's dimensions are assumed to be infinitesimally small compared to those of the sphere. (Because the distance of a typical celestial object from the Earth is very much larger than the Earth's radius.) It follows that only half of the sphere is visible from any particular observation site on the Earth's surface. Furthermore, the angular position of a given celestial object (relative to some fixed celestial reference) is the same at all such sites. In other words, there is negligible parallax associated with viewing the same celestial object from different observation sites on the surface of the Earth.[1]

2.2 Celestial Motions

Celestial objects exhibit two distinct types of motion. The first motion is such that the whole celestial sphere, and all of the celestial objects attached to it, rotates uniformly from east to west once every 24 (sidereal) hours, about a fixed axis passing through the Earth's north and south poles. This type of motion is called *diurnal motion*, and is a consequence of the Earth's daily rotation. Diurnal motion preserves the relative angular positions of all celestial objects. However, certain celestial objects, such as the Sun, the Moon, and the planets, possess a second motion, superimposed on the first, which causes their angular positions to slowly change relative to one another, and to the fixed stars. This *intrinsic motion* of objects in the Solar System is due to a combination of the Earth's orbital motion about the Sun, and the orbital motions of the Moon and the planets about the Earth and the Sun, respectively.

2.3 Celestial Coordinates

Consider Figure 2.1. The celestial sphere rotates about the celestial axis, PP', which is the imagined extension of the Earth's axis of rotation. This axis intersects the celestial sphere at the *north celestial pole*, P, and the *south celestial pole*, P'. It follows that the two celestial poles are unaffected by diurnal motion, and remain fixed in the sky.

[1] The one exception to this rule is the Moon, which is sufficiently close to the Earth that its parallax is significant. See Section 6.4.

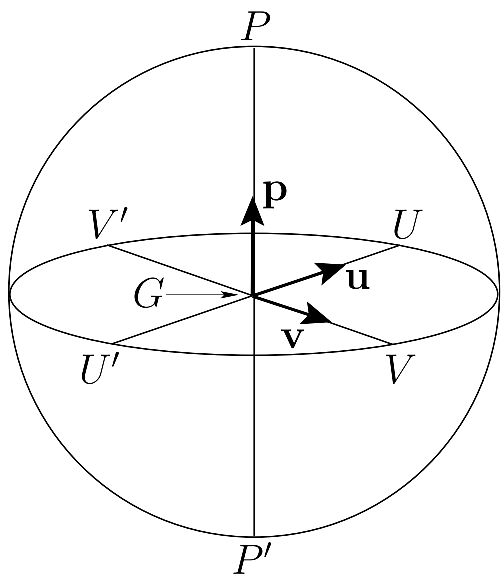

Figure 2.1: The celestial sphere. Here, G, P, P', V, and V' represent the Earth, north celestial pole, south celestial pole, vernal equinox, and autumnal equinox, respectively. Moreover, $VUV'U'$ is the celestial equator, and PP' the celestial axis.

The *celestial equator*, $VUV'U'$, is the intersection of the Earth's equatorial plane with the celestial sphere, and is therefore perpendicular to the celestial axis. The so-called *vernal equinox*, V, is a particular point on the celestial equator that is used as the origin of celestial longitude. Furthermore, the *autumnal equinox*, V', is a point that lies directly opposite the vernal equinox on the celestial equator. Let the line UU' lie in the plane of the celestial equator such that it is perpendicular to VV', as shown in the figure.

It is helpful to define three, right-handed, mutually perpendicular, unit vectors; **v**, **u**, and **p**. Here, **v** is directed from the Earth to the vernal equinox, **u** from the Earth to point U, and **p** from the Earth to the north celestial pole. See Figure 2.1.

Consider a general celestial object, R. See Figure 2.2. The location of R on the celestial sphere is conveniently specified by two angular coordinates, δ and α. Let GR' be the projection of GR onto the equatorial plane. The coordinate δ, which is known as *declination*, is the angle subtended between GR' and GR. Objects north of the celestial equator have positive declinations, and vice versa. It follows that objects on the celestial equator have declinations of 0°, whereas the north and south celestial poles have declinations of $+90^\circ$ and -90°, respectively. The coordinate α, which is known as *right ascension*, is the angle subtended between GV and GR'. Right ascension increases from west to east (that is, in the opposite direction to the celestial sphere's diurnal rotation). Thus, the vernal and autumnal equinoxes have right ascensions of 0° and 180°, respectively. Note that α lies in the range 0° to 360°. Right ascension is sometimes measured in hours, instead of degrees, with one hour corresponding to 15° (because it takes 24 hours for the celestial sphere to complete one diurnal rotation). In this scheme, the vernal and autumnal equinoxes have right ascensions of 0 hours and 12 hours, respectively. Moreover, α lies in the range 0 to 24 hours. (Incidentally, in this treatise, α is measured relative to the mean equinox at date, unless otherwise specified.) Finally, let **r** be a unit vector which is directed from the Earth to R. See Figure 2.2. It is easily demonstrated that

$$\mathbf{r} = \cos\delta\,\cos\alpha\,\mathbf{v} + \cos\delta\,\sin\alpha\,\mathbf{u} + \sin\delta\,\mathbf{p}, \tag{2.1}$$

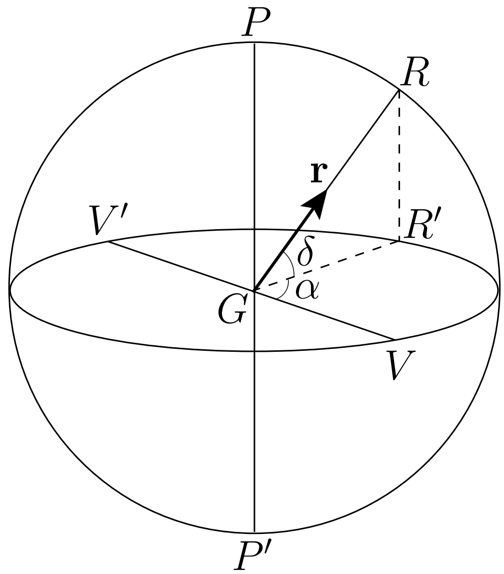

Figure 2.2: Celestial coordinates. Here, R is a celestial object, and R' its projection onto the plane of the celestial equator, $VR'V'$.

and

$$\sin\delta = \mathbf{r}\cdot\mathbf{p}, \tag{2.2}$$

$$\tan\alpha = \frac{\mathbf{r}\cdot\mathbf{u}}{\mathbf{r}\cdot\mathbf{v}}. \tag{2.3}$$

2.4 Ecliptic Circle

During the course of a year, the Sun's intrinsic motion causes it to trace out a fixed circle that bisects the celestial sphere. This circle is known as the *ecliptic*. The Sun travels around the ecliptic from west to east (that is, in the opposite direction to the celestial sphere's diurnal rotation). Moreover, the ecliptic circle is inclined at a fixed angle of $\epsilon = 23^\circ 26'$ to the celestial equator. This angle actually represents the fixed inclination of the Earth's axis of rotation to the normal to its orbital plane.[2]

The vernal equinox, V, is defined as the point at which the ecliptic crosses the celestial equator from south to north (in the direction of the Sun's ecliptic motion). See Figure 2.3. Likewise, the autumnal equinox, V', is the point at which the ecliptic crosses the celestial equator from north to south. In addition, the *summer solstice*, S, is the point on the ecliptic which is furthest north of the celestial equator, whereas the *winter solstice*, S', is the point which is furthest south. It follows that the lines VV' and SS' are perpendicular. Let QQ' be the normal to the plane of the ecliptic which passes through the Earth, as shown in Figure 2.3. Here, Q is termed the *northern ecliptic pole*, and Q' the *southern ecliptic pole*. It is easily demonstrated that

$$\mathbf{s} = \cos\epsilon\,\mathbf{u} + \sin\epsilon\,\mathbf{p}, \tag{2.4}$$

$$\mathbf{q} = -\sin\epsilon\,\mathbf{u} + \cos\epsilon\,\mathbf{p}, \tag{2.5}$$

[2] In fact, ϵ is very slowly decreasing in time. The value of ϵ used in the Almagest is $23^\circ 51'$. However, the true value of ϵ in Ptolemy's day was $23^\circ 41'$.

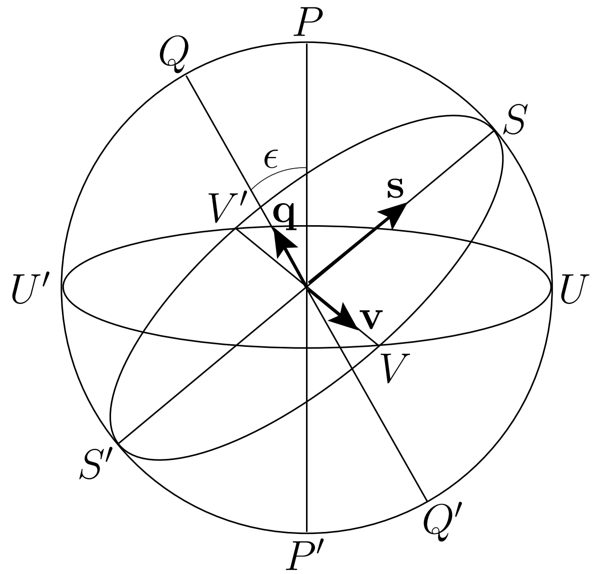

Figure 2.3: The ecliptic circle. Here, P, P', Q, Q', V, V', S, and S' denote the north celestial pole, south celestial pole, north ecliptic pole, south ecliptic pole, vernal equinox, autumnal equinox, summer solstice, and winter solstice, respectively. Moreover, $VUV'U'$ is the celestial equator, $VSV'S'$ the ecliptic, and PP' the celestial axis.

where **s** is a unit vector that is directed from the Earth to the summer solstice, and **q** a unit vector that is directed from the Earth to the north ecliptic pole. See Figure 2.3. We can also write

$$\mathbf{u} = \cos\epsilon\,\mathbf{s} - \sin\epsilon\,\mathbf{q}, \tag{2.6}$$

$$\mathbf{p} = \sin\epsilon\,\mathbf{s} + \cos\epsilon\,\mathbf{q}. \tag{2.7}$$

Thus, **v**, **s**, and **q** constitute another right-handed, mutually perpendicular, set of unit vectors.

2.5 Ecliptic Coordinates

It is convenient to specify the positions of the Sun, Moon, and planets in the sky using a pair of angular coordinates, β and λ, that are measured with respect to the ecliptic, rather than the celestial equator. Let R denote a celestial object, and GR' the projection of the line GR onto the plane of the ecliptic, $VR'V'$. See Figure 2.4. The coordinate β, which is known as *ecliptic latitude*, is the angle subtended between GR' and GR. Objects north of the ecliptic plane have positive ecliptic latitudes, and vice versa. The coordinate λ, which is known as *ecliptic longitude*, is the angle subtended between GV and GR'. Ecliptic longitude increases from west to east (that is, in the same direction that the Sun travels around the ecliptic). (Again, in this treatise, λ is measured relative to the mean equinox at date, unless specified otherwise.) Note that the basis vectors in the ecliptic coordinate system are **v**, **s**, and **q**, whereas the corresponding basis vectors in the celestial coordinate system are **v**, **u**, and **p**. See Figures 2.1 and 2.3. By analogy with Equations (2.1)–(2.3),

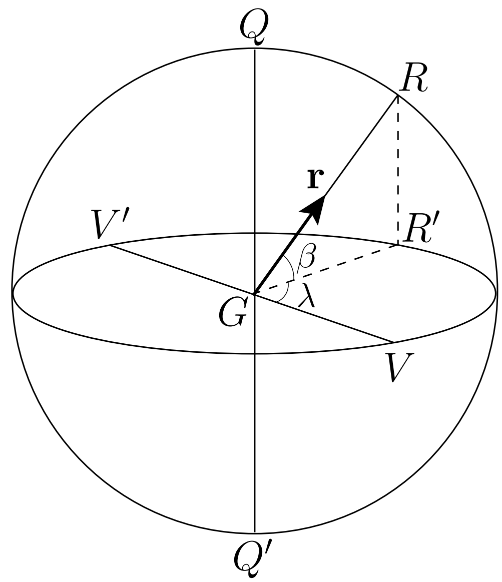

Figure 2.4: Ecliptic coordinates. Here, G is the Earth, R a celestial object, and R' its projection onto the ecliptic plane, $VR'V'$.

we can write

$$\mathbf{r} = \cos\beta\,\cos\lambda\,\mathbf{v} + \cos\beta\,\sin\lambda\,\mathbf{s} + \sin\beta\,\mathbf{q}, \tag{2.8}$$

$$\sin\beta = \mathbf{r}\cdot\mathbf{q}, \tag{2.9}$$

$$\tan\lambda = \frac{\mathbf{r}\cdot\mathbf{s}}{\mathbf{r}\cdot\mathbf{v}}, \tag{2.10}$$

where **r** is a unit vector that is directed from G to R. Hence, it follows from Equations (2.1), (2.4), and (2.5) that

$$\sin\beta = \cos\epsilon\,\sin\delta - \sin\epsilon\,\cos\delta\,\sin\alpha, \tag{2.11}$$

$$\tan\lambda = \frac{\cos\epsilon\,\cos\delta\,\sin\alpha + \sin\epsilon\,\sin\delta}{\cos\delta\,\cos\alpha}. \tag{2.12}$$

These expressions specify the transformation from celestial to ecliptic coordinates. The inverse transformation follows from Equations (2.2), (2.3), and (2.6)–(2.8):

$$\sin\delta = \cos\epsilon\,\sin\beta + \sin\epsilon\,\cos\beta\,\sin\lambda, \tag{2.13}$$

$$\tan\alpha = \frac{\cos\epsilon\,\cos\beta\,\sin\lambda - \sin\epsilon\,\sin\beta}{\cos\beta\,\cos\lambda}. \tag{2.14}$$

Figures 2.13 and 2.14 show all stars of visible magnitude less than $+6$ lying within 15° of the ecliptic. Table 2.1 gives the ecliptic longitudes (relative to the start of the zodiacal sign—see next section), ecliptic latitudes, and visible magnitudes of a selection of these stars which lie within 10° of the ecliptic. The figures and the table can be used to convert ecliptic longitude and latitude into approximate position in the sky against the backdrop of the fixed stars.

2.6 Signs of the Zodiac

The *signs of the zodiac* are a well-known set of names given to 30° long segments of the ecliptic circle. Thus, the sign of Aries extends over the range of ecliptic longitudes 0°–30°, the sign of Taurus over the range 30°–60°, and so on. Note that, as a consequence of the precession of the equinoxes, the signs of the zodiac no longer coincide with the constellations of the same name. See Figures 2.13 and 2.14. (Incidentally, Ptolemy was well aware of this fact because, even in his day, the signs of the zodiac did not coincide with the constellations. The signs coincided with the constellations in the time of the ancient Mesopotamians; that is, about 1500 BC.) The 12 zodiacal signs are listed in the table below. It can be seen from the table that ecliptic longitude 72° corresponds to the twelfth degree of Gemini, and ecliptic longitude 242° to the second degree of Sagittarius, et cetera

Sign	Abbr.	Longitude	Sign	Abbr.	Longitude	Sign	Abbr.	Longitude
Aries	AR	0°–30°	Leo	LE	120°–150°	Sagittarius	SG	240°–270°
Taurus	TA	30°–60°	Virgo	VI	150°–180°	Capricorn	CP	270°–300°
Gemini	GE	60°–90°	Libra	LI	180°–210°	Aquarius	AQ	300°–330°
Cancer	CN	90°–120°	Scorpio	SC	210°–240°	Pisces	PI	330°–360°

2.7 Ecliptic Declinations and Right Ascenesions.

According to Equations (2.13) and (2.14), the celestial coordinates of a point on the ecliptic circle (which corresponds to $\beta = 0$) that has ecliptic longitude λ are specified by

$$\sin\,\delta = \sin\epsilon\,\sin\lambda, \tag{2.15}$$

$$\tan\,\alpha = \cos\epsilon\,\tan\lambda. \tag{2.16}$$

The previous formulae have been used to construct Tables 2.2 and 2.3, which list the declinations and right ascensions of a set of equally spaced points on the ecliptic circle. Note that λ is measured relative to the start of the appropriate zodiacal sign.

2.8 Local Horizon and Meridian

Consider a general observation site X located on the surface of the Earth. (Note that, in the following, it is tacitly assumed that the site lies the Earth's northern hemisphere. However, the analysis also applies to sites situated in the the southern hemisphere.) The local *zenith*, Z, is the point on the celestial sphere that is directly overhead at X, whereas the *nadir*, Z', is the point that is directly underfoot. See Figure 2.5. The *horizon* is the tangent plane to the Earth at X, and divides the celestial sphere into two halves. The upper half, containing the zenith, is visible from site X, whereas the lower half is invisible.

Figure 2.6 shows the visible half of the celestial sphere at observation site X. Here, $NESW$ is the local horizon, and N, E, S, and W are the north, east, south, and west compass points, respectively. The plane $NPZS$, which passes through the north and south compass points, as well as the zenith, is known as the local *meridian*. The meridian is perpendicular to the horizon. The north celestial pole lies in the meridian plane, and is elevated an angular distance L above the north compass point. See Figures 2.5 and 2.6. Here, L is the terrestrial *latitude* of observation site X. It is helpful to define three, right-handed, mutually perpendicular,

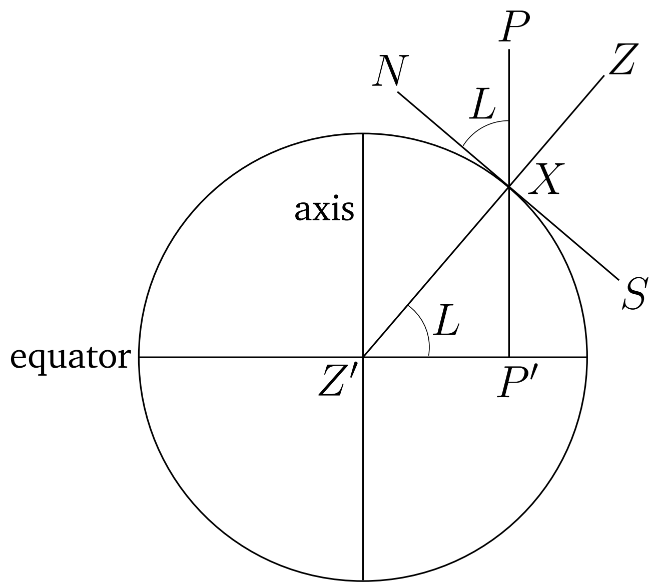

Figure 2.5: A general observation site X, of latitude L, on the surface of the Earth. Here, P, P', Z, and Z' denote the directions to the north celestial pole, south celestial pole, zenith, and nadir, respectively. The line NS represents the local horizon.

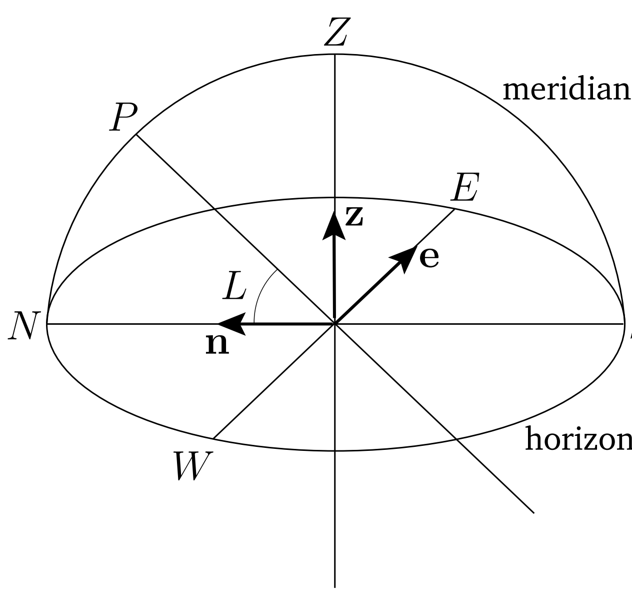

Figure 2.6: The local horizon and meridian. Here, N, S, E, W denote the north. south, east, and west compass points, Z the zenith, and P the north celestial pole. Moreover, $NESW$ is the horizon, and $NPZS$ the meridian.

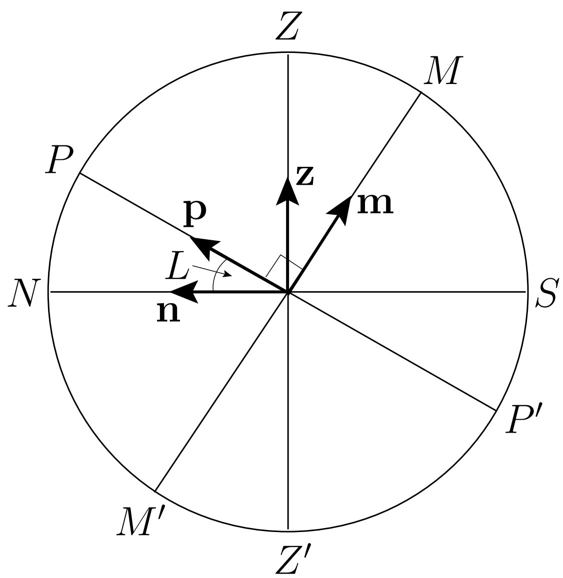

Figure 2.7: The local meridian.

local unit vectors: **e**, **n**, and **z**. Here, **e** is directed toward the east compass point, **n** toward the north compass point, and **z** toward the zenith. See Figures 2.6.

Figure 2.7 shows the meridian plane at X. Let the line MM' lie in this plane such that it is perpendicular to the celestial axis, PP'. Moreover, let M lie in the visible hemisphere. It is helpful to define the unit vector **m** which is directed toward M, as shown in the diagram. It is easily seen that

$$\mathbf{n} = \cos L\,\mathbf{p} - \sin L\,\mathbf{m}, \tag{2.17}$$

$$\mathbf{z} = \sin L\,\mathbf{p} + \cos L\,\mathbf{m}. \tag{2.18}$$

Figure 2.8 shows the celestial equator viewed from observation site X. Here, α_0 is the right ascension of the celestial objects culminating (that is, attaining their highest altitude in the sky) on the meridian at the time of observation. Incidentally, it is easily demonstrated that all objects culminating on the meridian at any instant in time have the same right ascension. Note that the angle α_0 increases uniformly in time, at the rate of 15° a (sidereal) hour, due to the diurnal motion of the celestial sphere. It can be seen from the diagram that

$$\mathbf{m} = \sin\alpha_0\,\mathbf{u} + \cos\alpha_0\,\mathbf{v}, \tag{2.19}$$

$$\mathbf{e} = \cos\alpha_0\,\mathbf{u} - \sin\alpha_0\,\mathbf{v}. \tag{2.20}$$

Thus, from Equations (2.17) and (2.18),

$$\mathbf{e} = -\sin\alpha_0\,\mathbf{v} + \cos\alpha_0\,\mathbf{u}, \tag{2.21}$$

$$\mathbf{n} = -\sin L\,\cos\alpha_0\,\mathbf{v} - \sin L\,\sin\alpha_0\,\mathbf{u} + \cos L\,\mathbf{p}, \tag{2.22}$$

$$\mathbf{z} = \cos L\,\cos\alpha_0\,\mathbf{v} + \cos L\,\sin\alpha_0\,\mathbf{u} + \sin L\,\mathbf{p}. \tag{2.23}$$

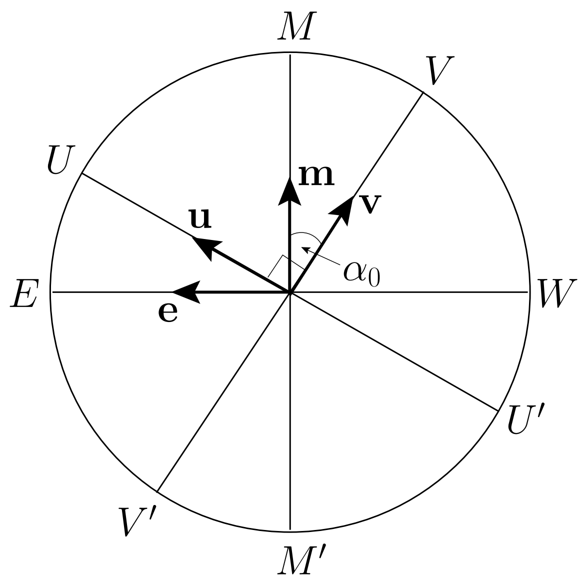

Figure 2.8: The local celestial equator.

Similarly, from Equations (2.6) and (2.7),

$$\mathbf{e} = -\sin\alpha_0\,\mathbf{v} + \cos\epsilon\,\cos\alpha_0\,\mathbf{s} - \sin\epsilon\,\cos\alpha_0\,\mathbf{q}, \tag{2.24}$$

$$\mathbf{n} = -\sin L\,\cos\alpha_0\,\mathbf{v} + (\cos L\,\sin\epsilon - \sin L\,\cos\epsilon\,\sin\alpha_0)\,\mathbf{s} + (\cos L\,\cos\epsilon + \sin L\,\sin\epsilon\,\sin\alpha_0)\,\mathbf{q}, \tag{2.25}$$

$$\mathbf{z} = \cos L\,\cos\alpha_0\,\mathbf{v} + (\sin L\,\sin\epsilon + \cos L\,\cos\epsilon\,\sin\alpha_0)\,\mathbf{s} + (\sin L\,\cos\epsilon - \cos L\,\sin\epsilon\,\sin\alpha_0)\,\mathbf{q}. \tag{2.26}$$

2.9 Horizontal Coordinates

It is convenient to specify the positions of celestial objects in the sky, when viewed from a particular observation site, X, on the Earth's surface, using a pair of angular coordinates, a and A, that are measured with respect to the local horizon. Let R denote a celestial object, and XR' the projection of the line XR onto the horizontal plane, $NESW$. See Figure 2.9. The coordinate a, which is known as *altitude*, is the angle subtended between XR' and XR. Objects above the horizon have positive altitudes, whereas objects below the horizon have negative altitudes. The zenith has altitude 90°, and the horizon altitude 0°. The coordinate A, which is known as *azimuth*, is the angle subtended between XN and XR'. Azimuth increases from the north towards the east. Thus, the north, east, south, and west compass points have azimuths of 0°, 90°, 180°, and 270°, respectively. Note that the basis vectors in the horizontal coordinate system are **e**, **n**, and **z**, whereas the corresponding basis vectors in the celestial coordinate system are **v**, **u**, and **p**. See Figures 2.1 and 2.6. By analogy with Equations (2.1)–(2.3), we can write

$$\mathbf{r} = \cos a\,\sin A\,\mathbf{e} + \cos a\,\cos A\,\mathbf{n} + \sin a\,\mathbf{z}, \tag{2.27}$$

$$\sin a = \mathbf{r}\cdot\mathbf{z}, \tag{2.28}$$

$$\tan A = \frac{\mathbf{r}\cdot\mathbf{e}}{\mathbf{r}\cdot\mathbf{n}}, \tag{2.29}$$

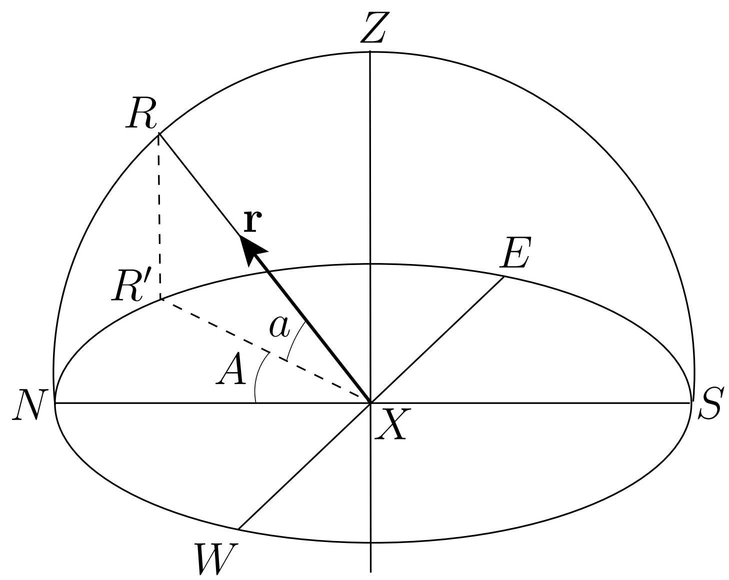

Figure 2.9: Horizontal coordinates. Here, R is a celestial object, and R' its projection onto the horizontal plane, $NESW$.

where **r** is a unit vector directed from X to R. Hence, it follows from Equations (2.1), and (2.22)–(2.23), that

$$\sin a = \sin L\,\sin\delta + \cos L\,\cos\delta\,\cos(\alpha - \alpha_0), \tag{2.30}$$

$$\tan A = \frac{\cos\delta\,\sin(\alpha-\alpha_0)}{\cos L\,\sin\delta - \sin L\,\cos\delta\,\cos(\alpha-\alpha_0)}. \tag{2.31}$$

These expressions allow us to calculate the altitude and azimuth of a celestial object of declination δ and right ascension α that is viewed from an observation site on the Earth's surface of terrestrial latitude L at an instance in time when celestial objects of right ascension α_0 are culminating at the meridian.. According to Equations (2.8), and (2.25)–(2.26), the altitude and azimuth of a similarly viewed point on the ecliptic (which corresponds to $\beta = 0$) of ecliptic longitude λ are given by

$$\sin a = \cos L\,\cos\lambda\,\cos\alpha_0 + \sin L\,\sin\epsilon\,\sin\lambda + \cos L\,\cos\epsilon\,\sin\lambda\,\sin\alpha_0, \tag{2.32}$$

$$\tan A = \frac{\cos\epsilon\,\sin\lambda\,\cos\alpha_0 - \cos\lambda\,\sin\alpha_0}{\cos L\,\sin\epsilon\,\sin\lambda - \sin L\,\cos\lambda\,\cos\alpha_0 - \sin L\,\cos\epsilon\,\sin\lambda\,\sin\alpha_0}. \tag{2.33}$$

2.10 Meridian Transits

Consider a celestial object, of declination δ and right ascension α, that is viewed from an observation site on the Earth's surface of terrestrial latitude L. According to Equation (2.30), the object culminates, or attains its highest altitude in the sky, when $\alpha_0 = \alpha$. This event is known as an *upper transit*. Furthermore, the object attains its lowest altitude in the sky when $\alpha_0 = 180^\circ + \alpha$. This event is known as a *lower transit*. Both upper and lower transits take place as the object in question passes through the meridian plane.

According to Equation (2.30), the altitude of a celestial object at its upper transit satisfies $\sin a_+ = \cos(L - \delta)$, implying that

$$a_+ = 90^\circ - |L - \delta|. \tag{2.34}$$

Likewise, the altitude at its lower transit satisfies $\sin a_- = -\cos(L + \delta)$, giving

$$a_- = |L + \delta| - 90^\circ. \tag{2.35}$$

The previous two expressions allow us to group celestial objects into three classes. Objects with declinations satisfying $|L+\delta| > 90^\circ$ never set; that is, their lower transits lie above the horizon. Objects with declinations satisfying $|L - \delta| > 90^\circ$ never rise; that is, their upper transits lie below the horizon. Finally, objects with declinations which satisfy neither of the two previous inequalities both rise and set during the course of a day. It follows that all celestial objects appear to rise and set when viewed from an observation site on the terrestrial equator (which corresponds to $L = 0^\circ$) . On the other hand, when viewed from an observation site at the north pole (which corresponds to $L = 90^\circ$), objects north of the celestial equator never set, while objects south of the celestial equator never rise, and vice versa for objects viewed from the south pole. All three classes of celestial object are present when the sky is viewed from an observation site on the Earth's surface of intermediate latitude.

2.11 Principal Terrestrial Latitude Circles

According to Equation (2.15), the Sun's declination varies between $-\epsilon$ and $+\epsilon$ during the course of a year. It follows from Equation (2.34) that it is only possible for the Sun to have an upper transit at the zenith in a region of the Earth whose latitude lies between $-\epsilon$ and ϵ. The circles of latitude bounding this region are known as the *tropics*. Thus, the *tropic of Capricorn*—so-called because the Sun is at the winter solstice, and, therefore, at the first point of Capricorn (that is, the zeroth degree of Capricorn), when it culminates at the zenith at this latitude—lies at $L = -23^\circ 26'$. Moreover, the *tropic of Cancer*—so-called because the Sun is at the summer solstice, and, therefore, at the first point of Cancer, when it culminates at the zenith at this latitude—lies at $L = +23^\circ 26'$.

Equations (2.34) and (2.35) imply that the Sun does not rise for part of the year, and does not set for part of the year, in two regions of the Earth whose terrestrial latitudes satisfy $|L| > 90^\circ - \epsilon$. These two regions are bounded by the poles and two circles of latitude known as the *arctic circles*. The *south arctic circle* lies at $L = -66^\circ 34'$. Likewise, the *north arctic circle* lies at $L = +66^\circ 34'$.

The equator, the two tropics, and the two arctic circles constitute the five *principal latitude circles* of the Earth, and are shown in Figure 2.10.

2.12 Equinoxes and Solstices

The ecliptic longitude of the Sun when it reaches the vernal equinox is $\lambda = 0^\circ$. It follows, from Equation (2.32), that the altitude of the Sun on the day of the equinox is given by $\sin a = \cos L \cos \alpha_0$. Thus, the Sun rises when $\alpha_0 = -90^\circ$, culminates at an altitude of $90^\circ - |L|$ when $\alpha_0 = 0^\circ$, and sets when $\alpha_0 = 90^\circ$. We conclude that the length of the equinoctial day is 180 time-degrees, which is equivalent to 12 hours (because 15° of right ascension cross the meridian in one hour). Thus, day and night are equally long on the day of the vernal equinox. (The word equinox is derived from the Latin word *aequinoctium*, which means "equal night".) It is easily demonstrated that the same is true on the day of the autumnal equinox.

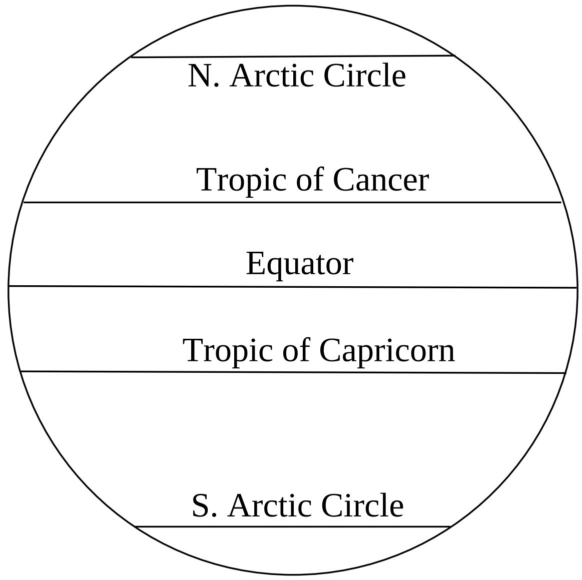

Figure 2.10: The principal latitude circles of the Earth.

The ecliptic longitude of the Sun when it reaches the summer solstice is $\lambda = 90^\circ$. It follows that the altitude of the Sun on the day of the solstice is given by $\sin a = \sin L \, \sin\epsilon + \cos L \, \cos\epsilon \, \sin\alpha_0$. Thus, the Sun rises when $\alpha_0 = -\sin^{-1}(\tan L \, \tan\epsilon)$, culminates at an altitude of $90^\circ - |L - \epsilon|$ when $\alpha_0 = 90^\circ$, and sets when $\alpha_0 = 180^\circ + \sin^{-1}(\tan L \, \tan\epsilon)$. We conclude that the length of the longest day of the year in the Earth's northern hemisphere (which, of course, occurs when the Sun reaches the summer solstice) is $180 + 2 \, \sin^{-1}(\tan L \, \tan\epsilon)$ time-degrees. Likewise, the length of the shortest night (which also occurs at the summer solstice) is $180 - 2 \, \sin^{-1}(\tan L \, \tan\epsilon)$ time-degrees. These formulae are only valid for northern latitudes below the arctic circle. At higher latitudes, the Sun never sets for part of the year, and the longest "day" is consequently longer than 24 hours. It is easily demonstrated that the shortest day in the Earth's northern hemisphere, which takes place when the Sun reaches the winter solstice, is equal to the shortest night, and the longest night (which also occurs at the winter solstice) to the longest day. Moreover, the Sun culminates at an altitude of $90^\circ - |L + \epsilon|$ on day of the winter solstice. Again, at latitudes above the arctic circle, the Sun never rises for part of the year, and the longest "night" is consequently longer than 24 hours.

Consider an observation site on the Earth's surface of latitude L that lies above the northern arctic circle. The declination of the Sun on the first day after the spring equinox on which it fails to set is $\delta = 90^\circ - L$. According to Equation (2.15), the Sun's ecliptic longitude on this day is $\sin^{-1}(\cos L/\sin\epsilon)$. Likewise, the declination of the Sun on the day when it starts to set again is $\delta = 90^\circ - L$, and its ecliptic longitude is $180^\circ - \sin^{-1}(\cos L/\sin\epsilon)$. Assuming that the Sun travels around the ecliptic circle at a uniform rate (which is approximately true), the fraction of a year that the Sun stays above the horizon in summer is $0.5 - \sin^{-1}(\cos L/\sin\epsilon)/180^\circ$. It is easily demonstrated that the fraction of a year that the Sun stays below the horizon in winter is also $0.5 - \sin^{-1}(\cos L/\sin\epsilon)/180^\circ$.

2.13 Terrestrial Climes

Table 2.4 specifies the length of the longest day, as well as the altitude of the Sun when it culminates at the meridian on the days of the equinoxes and solstices, calculated for a set of observation sites in the northern hemisphere with equally spaced terrestrial latitudes. This table was constructed using the formulae in the previous section. The table can be adapted to observation sites in the Earth's southern hemisphere via the following simple transformation: $L \to -L$, Summer $\leftrightarrow$ Winter, N $\leftrightarrow$ S. For instance, at a latitude of -10°, the longest day, which corresponds to the winter solstice, is of length $12^h 35^m$. Moreover, on this day, the Sun's upper transit is south of the zenith, at an altitude of $+76^\circ 34'$.

2.14 Ecliptic Ascensions

Consider the rising, or *ascension*, of celestial objects at the eastern horizon, as viewed from a particular observation site on the Earth's surface. If the observation site lies on the terrestrial equator then all celestial objects appear to ascend at right angles to the horizon. This process is known as *right ascension*. On the other hand, if the observation site does not lie at the equator then celestial objects appear to ascend at an oblique angle to the horizon. This process is known as *oblique ascension*. For the case of right ascension, it is easily demonstrated that all celestial objects with the same celestial longitude ascend simultaneously. Indeed, celestial longitude is generally known as "right ascension" because, in the case of right ascension, the celestial longitude of an object (in hours) is simply the time elapsed between the ascension of the vernal equinox, and the ascension of the object in question.

Let us now consider the ascension of points on the ecliptic. Applying Equation (2.30) to a point on the celestial equator (which corresponds to $\delta = 0$) of right ascension α, we obtain

$$\sin a = \cos L\, \cos(\alpha - \alpha_0) = \cos L\, \sin(\alpha_0 - \alpha + 90^\circ). \tag{2.36}$$

It follows that we can write

$$\alpha_0 = \alpha - 90^\circ, \tag{2.37}$$

where α is the right ascension of the point on the celestial equator that ascends at the eastern horizon (that is, $a = 0$ and $da/dt > 0$) at the same time that celestial objects of right ascension α_0 are culminating at the meridian. Substituting this result into Equation (2.32), we get

$$\sin a = \cos L\, \cos\lambda\, \sin\alpha + \sin L\, \sin\epsilon\, \sin\lambda - \cos L\, \cos\epsilon\, \sin\lambda\, \cos\alpha, \tag{2.38}$$

which implies that if $a = 0$ and $da/dt > 0$ then

$$\tan\lambda = \frac{\cos L\, \sin\alpha}{\cos L\, \cos\epsilon\, \cos\alpha - \sin\epsilon\, \sin L}. \tag{2.39}$$

This expression specifies the ecliptic longitude, λ, of the point on the ecliptic circle that ascends simultaneously with a point on the celestial equator of right ascension α. Note, incidentally, that points on the celestial equator ascend at a uniform rate of 15° an hour at all viewing sites on the Earth's surface (except the poles, where the celestial equator does not ascend at all). The same is not true of points on the ecliptic. Expression (2.39) can be inverted to give

$$\alpha = \tan^{-1}(\tan\lambda\, \cos\epsilon) - \sin^{-1}\left[\frac{\sin\lambda\, \sin\epsilon\, \tan L}{(1 - \sin^2\lambda\, \sin^2\epsilon)^{1/2}}\right]. \tag{2.40}$$

The solution of Equation (2.40) for observation sites lying above the arctic circle is complicated by the fact that, at such sites, a section of the ecliptic never sets, or descends, and a section never ascends. It is easily demonstrated that the section that never descends lies between ecliptic longitudes λ_c and $180^\circ - \lambda_c$, whereas the section that never ascends lies between longitudes $180^\circ + \lambda_c$ and $360^\circ - \lambda_c$. Here, $\lambda_c = \sin^{-1}(\cos L/\sin\epsilon)$. Points on the ecliptic of longitude λ_c, $180^\circ - \lambda_c$, $180^\circ + \lambda_c$, and $360^\circ - \lambda_c$ ascend simultaneously with points on the celestial equator of right ascension $360^\circ - \alpha_c$, α_c, $360^\circ - \alpha_c$, and α_c, respectively. Here, $\alpha_c = \cos^{-1}(1/\tan L\,\tan\epsilon)$.

Tables 2.5–2.17 list the ascensions of a series of equally spaced points on the ecliptic circle, as viewed from a set of observation sites in the Earth's northern hemisphere with different terrestrial latitudes. The tables were calculated with the aid of formula (2.40). Let us now illustrate the use of these tables.

Consider a day on which the Sun is at ecliptic longitude 14LE00 (that is, $14^\circ 00'$ into the sign of Leo). What is the length of the day (that is, the period between sunrise and sunset) at an observation site on the Earth's surface of latitude $+30^\circ$? Consulting Table 2.8, we find that the Sun ascends simultaneously with a point on the celestial equator of right ascension $126^\circ 32'$. Now, the ecliptic is a great circle on the celestial sphere. Hence, exactly half of the ecliptic is visible from any observation site on the Earth's surface. This implies that when a given point on the ecliptic circle is ascending, the point directly opposite it on the circle is descending, and vice versa. Let us term the directly opposite point the *complimentary point*. By definition, the difference in ecliptic longitude between a given point on the ecliptic circle and its complementary point is 180°. Thus, the complimentary point to 14LE00 is 14AQ00. It follows that 14AQ00 ascends at the same time that 14LE00 descends. In other words, the Sun sets when 14AQ00 ascends. Consulting Table 2.8, we find that the Sun sets at the same time that a point on the celestial equator of right ascension $326^\circ 23'$ rises. Thus, in the time interval between the rising and setting of the Sun, a $326^\circ 23' - 126^\circ 32' = 199^\circ 51'$ section of the celestial equator ascends at the eastern horizon. However, points on the celestial equator ascend at the uniform rate of 15° an hour. Thus, the length, in hours, of the period between the rising and setting of the Sun is $199^\circ 51'/15^\circ = 13^h 14^m$. In other words, the length of the day in question is $13^h 14^m$.

The previous calculation is slightly inaccurate for a number of reasons. Firstly, it neglects the fact that the Sun is continuously moving on the ecliptic circle at the rate of about 1° a day. Secondly, it neglects the fact that the celestial equator ascends at the rate of 15° per sidereal, rather than solar, hour. A sidereal hour is $1/24$th of a sidereal day, which is the time between successive upper transits of a fixed celestial object, such as a star. On the other hand, a solar hour is $1/24$th of a solar day, which is the mean time between successive upper transits of the Sun. A sidereal day is shorter than a solar day by 4 minutes. Fortunately, it turns out that these first two inaccuracies largely cancel one another out. Another source of inaccuracy is the fact that, due to refraction of light by the atmosphere, the Sun is actually 1° below the horizon when it appears to rise or set. The final source of inaccuracy is the fact that the Sun has a finite angular extent (of about half a degree), and that, strictly speaking, dawn and dusk commence when the Sun's upper limb rises and sets, respectively. Of course, our calculation only deals with the rising and setting of the center of the Sun. Taken together, the previously mentioned inaccuracies can cause the true length of a day differ from that calculated from the ascension tables by up to 15 minutes.

Tables 2.5–2.17 also effectively list the descents of a series of equally spaced points on the ecliptic circle, as viewed from a set of observation sites in the Earth's southern hemisphere with different terrestrial latitudes (which are minus those specified in the various tables). For instance, Table 2.6 gives the right ascensions of points on the celestial equator that set simultaneously with points on the ecliptic, as seen from an observation site at latitude -10°.

Consider a day on which the Sun is at ecliptic longitude 08SC00. Let us calculate the length of the day at an observation site on the Earth's surface of latitude -50°? Consulting Table 2.10, we find that the Sun sets simultaneously with a point on the celestial equator of right ascension $233^\circ 09'$. Now, the complementary point on the ecliptic to 08SC00 is 08TA00. Consulting Table 2.10 again, we find that this point sets simultaneously with a point on the celestial equator of right ascension $18^\circ 07'$. It follows that the Sun rises simultaneously with the latter point. Thus, the time interval between the rising and setting of the Sun is $233°09' - 18°07' = 215°02'$ time-degrees, or 14^h20^m.

The *ascendent*, or *horoscope*, is defined as the point on the ecliptic that is ascending at the eastern horizon. Suppose that we wish to find the ascendent 2.6 hours after sunrise, as seen from an observation site of latitude $+55°$, on a day on which the Sun has ecliptic longitude 16SC00. Of course, knowledge of the ascendent at the time of birth is key to drawing up a natal chart in astrology. Hence, this type of calculation was of great importance to the ancient Greeks. (It was of particular importance to Ptolemy, because he wrote the standard treatise on Greek natal astrology—the so-called *Tetrabiblos*.) Consulting Table 2.11, we find that, on the day in question, the Sun rises simultaneously with a point on the celestial equator of right ascension $248°46'$. Now, 2.6 hours corresponds to $39°00'$. Thus, the ascendent rises simultaneously with a point on the celestial equator of right ascension $248°46' + 39°00' = 287°46'$. Consulting Table 2.11 again, we find that, to the nearest degree, the ascendent at the time in question has ecliptic longitude 13SG00.

Suppose, next, that we wish to find the right ascension, α, of the point on the celestial equator that culminates simultaneously with a given point on the ecliptic of ecliptic longitude λ. From Equation (2.33), we can see that if $A = 180°$ then $\tan A = 0$, and $\tan\lambda\,\cos\epsilon = \sin\alpha$, or

$$\alpha = \sin^{-1}(\tan\lambda\,\cos\epsilon). \tag{2.41}$$

However, this expression is identical to expression (2.40), when the latter is evaluated for the special case $L = 0°$. It follows that our problem can be solved by consulting Table 2.5, which is the ascension table for the case of right ascension. For instance, on a day on which the ecliptic longitude of the Sun is 08TA00, we find from Table 2.5 that the right ascension of the point on the celestial equator that culminates simultaneously with the Sun (in other words, that culminates at local noon) is $35°38'$. Moreover, this is the case for observation sites at all terrestrial latitudes. Note that we have effectively calculated the right ascension of the Sun on the day in question.

Suppose, finally, that we wish to find the point on the ecliptic that culminates 7 hours after local noon on the aforementioned day. Because 7 hours corresponds to $105°$, the right ascension of the point on the celestial equator that culminates simultaneously with the point is question is $35°38' + 105°00' = 143°38'$. Consulting Table 2.5 again, we find that, to the nearest degree, the ecliptic longitude of the point in question is 21LE00.

2.15 Azimuth of Ecliptic Ascension Point

Consider the azimuth of the point on the ecliptic circle that is ascending at the eastern horizon. According to Equation (2.27), the azimuth of any point on the horizon (which corresponds to $a = 0°$) satisfies $\cos A = \mathbf{r}\cdot\mathbf{n}$. It follows from Equations (2.8) and (2.25) that

$$\cos A = -\cos\lambda\,\sin L\,\sin\alpha + \sin\lambda\,\cos L\,\sin\epsilon + \sin\lambda\,\sin L\,\cos\epsilon\,\cos\alpha. \tag{2.42}$$

Here, we have made use of the fact that the point in question also lies on the ecliptic (which corresponds to $\beta = 0$), as well as the fact that $\alpha_0 = \alpha - 90°$, where α is the right ascension of the simultaneously rising point on the celestial equator. Here, λ is the ecliptic longitude of the point in question, and L the terrestrial latitude of the observation site. Now, λ and α satisfy Equation (2.39), as well as the previous equation. Thus, eliminating α between these two equations, we obtain

$$\cos A = \frac{\sin\lambda\,\sin\epsilon}{\cos L}. \tag{2.43}$$

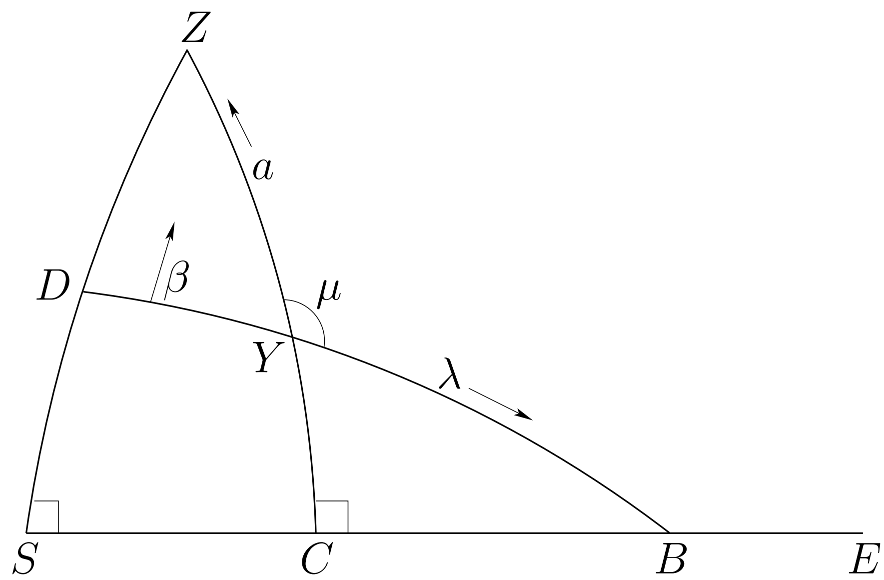

Figure 2.11: Parallactic angle in the case where increasing altitude corresponds to increasing ecliptic latitude. Here, $SCBE$ is the southern horizon, with S and E the south and east compass points, respectively. Moreover, DYB is the ecliptic, ZDS the meridian, Z the zenith, and ZYC an altitude circle.

This expression gives the azimuth, A, of the ascending point of the ecliptic as a function of its ecliptic longitude, λ, and the latitude, L, of the observation site.

For instance, suppose that we wish to find the azimuth of the point at which the Sun rises on the eastern horizon at an observation site of terrestrial latitude $+60^\circ$, on a day on which the Sun's ecliptic longitude is 08PI00. It follows from Equation (2.43) that $A = \cos^{-1}[\sin(338^\circ)\ \sin(23^\circ 26')/\cos(60^\circ)] = 107^\circ 20'$. We conclude that the Sun rises $17^\circ 20'$ to the south of the east compass point on the day in question. It is easily demonstrated that the Sun sets $17^\circ 20'$ south of the west compass point on the same day (neglecting the slight change in the Sun's ecliptic latitude during the course of the day.) Likewise, it can easily be shown that, at an observation site of terrestrial latitude -60°, the Sun also rises $17^\circ 20'$ to the south of the east compass point on the day in question, and sets $17^\circ 20'$ to the south of the west compass point.

2.16 Ecliptic Altitude and Orientation

Consider a point on the ecliptic circle of ecliptic longitude λ. We wish to determine the altitude of this point, as well as the angle subtended there between the ecliptic and the vertical, t hours before or after it culminates at the meridian, as seen from an observation site on the Earth's surface of latitude L.

The situation is as shown in Figure 2.11. Here, Y is the point in question, and ZYC an altitude circle (that is, a great circle passing through the zenith) drawn through it. We wish to determine the altitude $a \equiv CY$ of point Y, as well as the angle $\mu \equiv ZYB$. Note that μ is defined such that it lies between the ecliptic in the direction of increasing ecliptic longitude and the altitude circle in the direction of increasing altitude. Moreover, μ is acute when increasing altitude, a, corresponds to increasing ecliptic latitude, β, and obtuse when increasing a corresponds to decreasing β. See Figures 2.11 and 2.12. Incidentally, this definition is adopted in order to simplify the calculation of lunar parallax. See Section 6.4. In the following, we shall refer to μ as the *parallactic angle*. However, it should be noted that, according to the modern definition, the

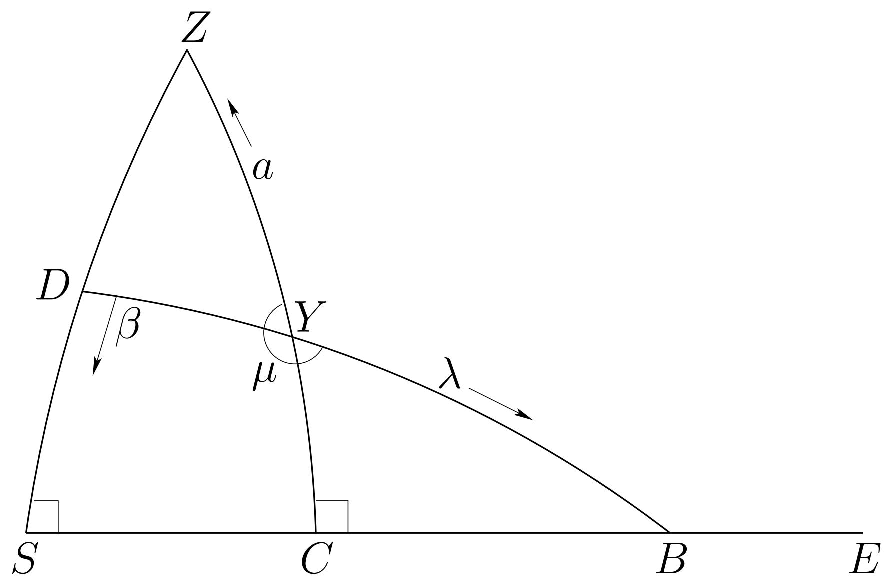

Figure 2.12: Parallactic angle in the case where increasing altitude corresponds to decreasing ecliptic latitude. Here, $SCBE$ is the southern horizon, with S and E the south and east compass points, respectively. Moreover, DYB is the ecliptic, ZDS the meridian, Z the zenith, and ZYC an altitude circle.

parallactic angle is $90^\circ - \mu$.

From Equations (2.15) and (2.16), the declination and right ascension of point Y are given by

$$\sin\delta = \sin\epsilon\,\sin\lambda, \tag{2.44}$$

$$\tan\,\alpha = \cos\epsilon\,\tan\lambda, \tag{2.45}$$

respectively. We can also write $\alpha_0 = \alpha - t$, where α_0 is the right ascension of the point on the ecliptic that is culminating (that is, point D in the diagram), and t is measured in time-degrees. Note that if t is positive then it measures time before culmination, whereas if it is negative then its magnitude measures time after culmination. It follows from Equations (2.30) and (2.31) that the altitude and azimuth of point Y satisfy

$$\sin a = \sin L\,\sin\delta + \cos L\,\cos\delta\,\cos t, \tag{2.46}$$

$$\tan A = \frac{\cos\delta\,\sin t}{\cos L\,\sin\delta - \sin L\,\cos\delta\,\cos t}, \tag{2.47}$$

respectively.

From Equation (2.8), the unit vector

$$\mathbf{r} = \cos\lambda\,\mathbf{v} + \sin\lambda\,\mathbf{s} \tag{2.48}$$

is directed from the observation site to point Y. Furthermore, the unit vector

$$\frac{\partial\mathbf{r}}{\partial\lambda} = -\sin\lambda\,\mathbf{v} + \cos\lambda\,\mathbf{s} \tag{2.49}$$

is tangent to the ecliptic circle, at point Y, in the direction of increasing ecliptic longitude. From Equation (2.27), the unit vector

$$\mathbf{r} = \cos a\,\sin A\,\mathbf{e} + \cos a\,\cos A\,\mathbf{n} + \sin a\,\mathbf{z} \tag{2.50}$$

is directed from the observation site to point Y. Here, a and A are the altitude and azimuth, respectively, of this point in the sky. Moreover, the unit vector

$$\frac{\partial \mathbf{r}}{\partial a} = -\sin a\,\sin A\,\mathbf{e} - \sin a\,\cos A\,\mathbf{n} - \cos a\,\mathbf{z} \equiv (\cos A\,\mathbf{e} - \sin A\,\mathbf{n}) \times \mathbf{r} \tag{2.51}$$

is a tangent to the altitude circle passing through point Y in the direction of increasing altitude. It follows from the definition of parallactic angle, and elementary vector algebra, that

$$\begin{aligned}\cos\mu = \frac{\partial \mathbf{r}}{\partial \lambda}\cdot\frac{\partial \mathbf{r}}{\partial a} &= -\sin\lambda\,\cos A\,\mathbf{v}\times\mathbf{e}\cdot\mathbf{r} + \sin\lambda\,\sin A\,\mathbf{v}\times\mathbf{n}\cdot\mathbf{r}\\ &\quad + \cos\lambda\,\cos A\,\mathbf{s}\times\mathbf{e}\cdot\mathbf{r} - \cos\lambda\,\sin A\,\mathbf{s}\times\mathbf{n}\cdot\mathbf{r}.\end{aligned} \tag{2.52}$$

However, according to Equations (2.24), (2.25), and (2.48),

$$\mathbf{v}\times\mathbf{n}\cdot\mathbf{r} = -\sin L\,(\cos L\,\cos\epsilon + \sin L\,\sin\epsilon\,\sin\alpha_0), \tag{2.53}$$

$$\mathbf{v}\times\mathbf{e}\cdot\mathbf{r} = \sin\lambda\,\sin\epsilon\,\cos\alpha_0, \tag{2.54}$$

$$\mathbf{s}\times\mathbf{e}\cdot\mathbf{r} = -\cos\lambda\,\sin\epsilon\,\cos\alpha_0, \tag{2.55}$$

$$\mathbf{s}\times\mathbf{n}\cdot\mathbf{r} = \cos\lambda\,(\cos L\,\cos\epsilon + \sin L\,\sin\epsilon\,\sin\alpha_0). \tag{2.56}$$

The previous five equations can be combined to give

$$\cos\mu = -\cos A\,\sin\epsilon\,\cos(\alpha - t) - \sin A\,[\cos L\,\cos\epsilon + \sin L\,\sin\epsilon\,\sin(\alpha - t)]\,. \tag{2.57}$$

Now, it follows from Equation (2.26) that

$$\mathbf{z}\cdot\mathbf{q} = \sin L\,\cos\epsilon - \cos L\,\sin\epsilon\,\sin(\alpha - t). \tag{2.58}$$

This quantity is significant because if $\mathbf{z}\cdot\mathbf{q} > 0$ then increasing altitude corresponds to increasing ecliptic latitude, whereas if $\mathbf{z}\cdot\mathbf{q} < 0$ then increasing altitude corresponds to decreasing ecliptic latitude. Thus, in the former case, μ is the solution of Equation (2.57) that lies in the range $0^\circ \leq \mu \leq 180^\circ$, whereas in the latter case it is the solution that lies in the range $180^\circ \leq \mu \leq 360^\circ$.

According to Equation (2.46), the critical value of t at which point Y reaches the horizon is given by

$$\cos t_h = -\tan L\,\tan\delta. \tag{2.59}$$

Of course, the previous equation is only soluble if $|\tan L\,\tan\delta| < 1$. However, it is easily demonstrated that if $\tan L\,\tan\delta < -1$ then point Y never sets, whereas if $\tan L\,\tan\delta > 1$ then point Y never rises.

Note that the value of μ at $t = 0$ represents the inclination of the ecliptic to the vertical as point Y culminates. Furthermore, the values of μ at $t = t_h$ (corresponding to $a = 0^\circ$) represent the inclination of the ecliptic to the vertical as point Y rises and sets.

Tables 2.18–2.26 show the altitudes of twelve equally spaced points on the ecliptic, as well as the parallactic angle at these points, as functions of time, calculated for a series of observation sites in the Earth's northern hemisphere with equally spaced terrestrial latitudes. The twelve points correspond to the start of the twelve zodiacal signs, and are named accordingly. Thus, "Aries" corresponds to ecliptic longitude 0°, "Taurus" to ecliptic longitude 30°, et cetera. For each point, four columns of data are provided. The first column corresponds to the time (in hours and minutes) either before or after the culmination of the point, the second column gives the altitude of the point (which is the same in both cases), the third column gives the parallactic angle, μ, for the case in which the first column indicates time prior to the culmination of the point, and the fourth column gives the parallactic angle for the opposite case. Data is only provided for cases in which the various points on the ecliptic lie on or above the horizon.

Now, it can be seen, from the previous analysis, that if $L \to -L$, $t \to t$, $\lambda \to \lambda + 180^\circ$ then $\delta \to -\delta$, $\alpha \to \alpha + 180^\circ$, $A \to 180^\circ - A$, $\cos\mu \to \cos\mu$, $\mathbf{z} \cdot \mathbf{q} \to -\mathbf{z} \cdot \mathbf{q}$, and so $a \to a$, $\mu \to 360^\circ - \mu$. It follows that Tables 2.18–2.26 can also be used to calculate altitudes and parallactic angles of points on the ecliptic, as functions of time, for observation sites in the Earth's southern hemisphere. For example, suppose that we wish to determine the altitude and parallactic angle of the first point of Gemini (that is, $\lambda = 60^\circ$), as seen from an observation site of terrestrial latitude -10°, 3 hours before and after it culminates at the meridian. In order to do this, we must examine the Sagittarius (that is, $\lambda = 240^\circ$) entry in the $L = +10^\circ$ ecliptic altitude table; that is, Table 2.19 (because $\lambda \to \lambda + 180^\circ$ when $L \to -L$). The fourth row of this entry tells us that, $t = 03{:}00$ hours before culmination, the altitude and parallactic angle of the first point of Gemini are $a = 36^\circ 26'$ and $\mu = 360^\circ - 162^\circ 11' = 197^\circ 49'$, respectively (because $a \to a$ and $\mu \to 360^\circ - \mu$ as $L \to -L$). This row also tells us that, $t = 03{:}00$ hours after culmination, the altitude and parallactic angle of the first point of Gemini are $a = 36^\circ 26'$ and $\mu = 360^\circ - 042^\circ 17' = 317^\circ 43'$, respectively

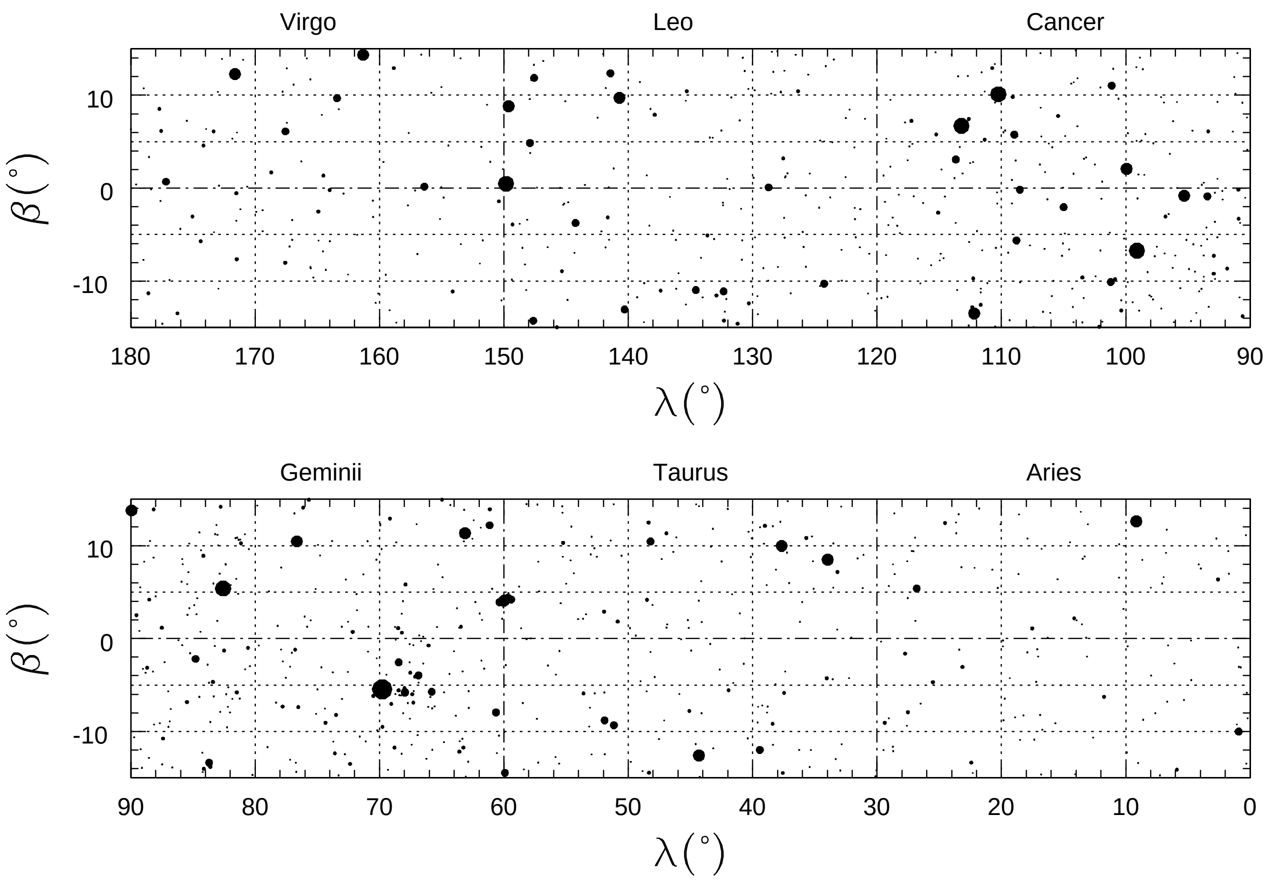

Figure 2.13: Map showing all stars of visual magnitude less than $+6$ lying within 15° of the ecliptic plane.

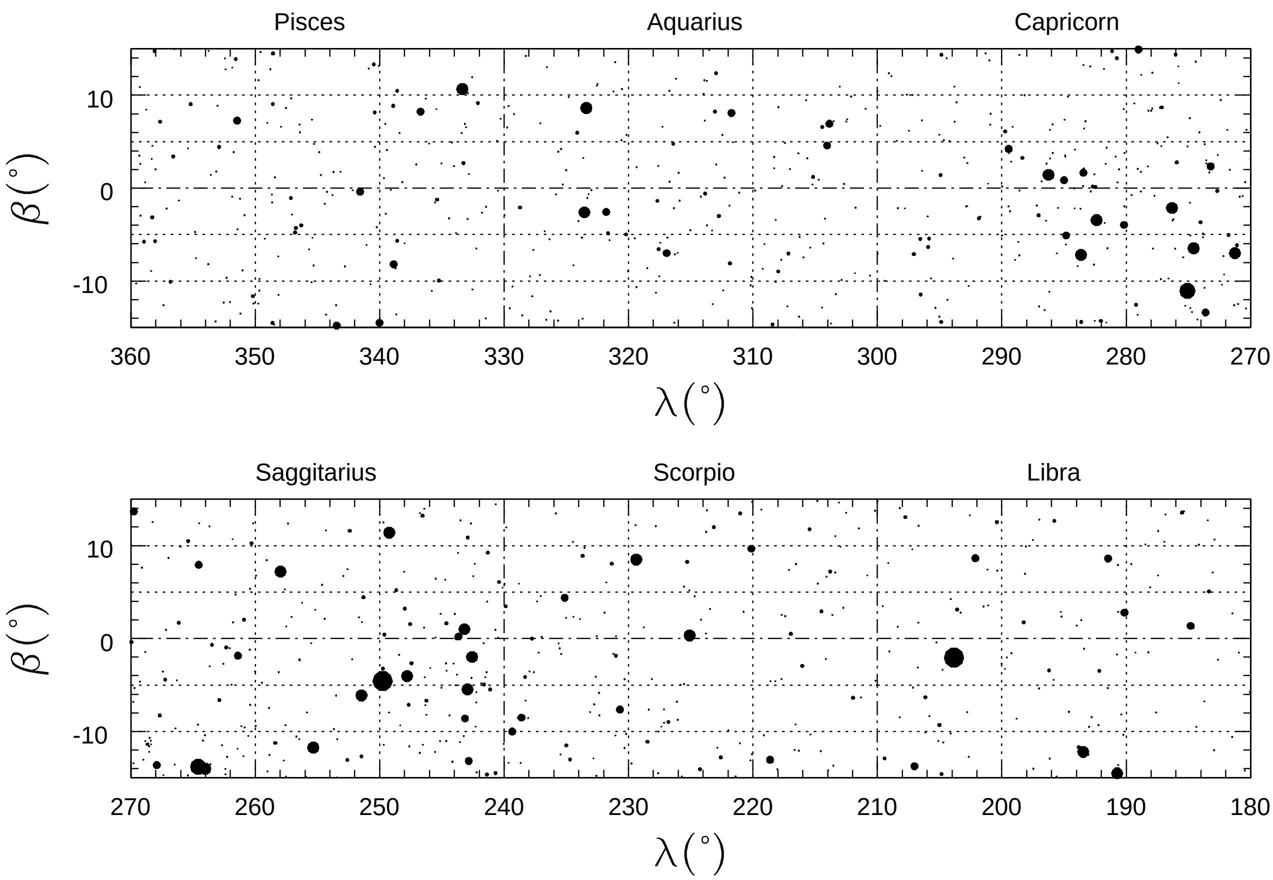

Figure 2.14: Map showing all stars of visual magnitude less than +6 lying within 15° of the ecliptic plane.

Aries

λ	β	Mag.	Name
$09°09'$	$+12°36'$	$+2.8$	γ PEG
$26°49'$	$+5°23'$	$+3.6$	η PSC

Libra

λ	β	Mag.	Name
$04°50'$	$+1°22'$	$+3.9$	η VIR
$10°08'$	$+2°46'$	$+3.5$	γ VIR
$11°28'$	$+8°37'$	$+3.4$	δ VIR
$22°08'$	$+8°38'$	$+3.4$	ζ VIR
$23°51'$	$-2°03'$	$+1.0$	α VIR

Taurus

λ	β	Mag.	Name
$03°58'$	$+8°29'$	$+2.6$	β ARI
$07°39'$	$+9°58'$	$+2.0$	α ARI

Scorpio

λ	β	Mag.	Name
$15°5'$	$+0°20'$	$+2.8$	α LIB
$19°22'$	$+8°30'$	$+2.6$	β LIB

Gemini

λ	β	Mag.	Name
$00°00'$	$+4°03'$	$+2.9$	η TAU
$09°47'$	$-5°28'$	$+0.9$	α TAU
$22°34'$	$+5°23'$	$+1.7$	β TAU

Sagittarius

λ	β	Mag.	Name
$02°34'$	$-1°59'$	$+2.3$	δ SCO
$02°56'$	$-5°29'$	$+2.9$	π SCO
$03°11'$	$+1°00'$	$+2.6$	β SCO
$07°48'$	$-4°02'$	$+2.9$	σ SCO
$09°46'$	$-4°34'$	$+1.0$	α SCO
$11°27'$	$-6°08'$	$+2.8$	τ SCO
$17°58'$	$+7°12'$	$+2.4$	η OPH

Cancer

λ	β	Mag.	Name
$05°18'$	$-0°49'$	$+2.9$	μ GEM
$09°06'$	$-6°44'$	$+1.9$	γ GEM
$09°56'$	$+2°04'$	$+3.0$	ϵ GEM
$20°14'$	$+10°06'$	$+2.0$	α GEM
$23°13'$	$+6°41'$	$+1.1$	β GEM

Capricorn

λ	β	Mag.	Name
$01°16'$	$-7°00'$	$+3.0$	γ SGR
$04°34'$	$-6°28'$	$+2.7$	δ SGR
$06°19'$	$-2°08'$	$+2.8$	λ SGR
$12°23'$	$-3°27'$	$+2.0$	σ SGR
$13°38'$	$-7°11'$	$+2.6$	ζ SGR
$16°15'$	$+1°26'$	$+2.9$	π SGR

Leo

λ	β	Mag.	Name
$20°47'$	$+9°43'$	$+3.0$	ϵ LEO
$29°37'$	$+8°49'$	$+2.6$	γ LEO
$29°50'$	$+0°28'$	$+1.4$	α LEO

Aquarius

λ	β	Mag.	Name
$23°24'$	$+8°37'$	$+2.9$	β AQR
$23°33'$	$-2°36'$	$+2.9$	δ CAP

Virgo

λ	β	Mag.	Name
$06°23'$	$+0°09'$	$+3.9$	ρ LEO
$13°25'$	$+9°40'$	$+3.3$	θ LEO
$17°34'$	$+6°06'$	$+3.9$	ι LEO
$27°10'$	$+0°42'$	$+3.6$	β VIR

Pisces

λ	β	Mag.	Name
$06°43'$	$+8°14'$	$+3.8$	γ AQR
$08°52'$	$-8°12'$	$+3.3$	δ AQR
$11°34'$	$-0°23'$	$+3.7$	λ AQR
$21°27'$	$+7°15'$	$+3.7$	γ PSC

Table 2.1: Ecliptic longitudes (relative to the mean equinox at the J2000 epoch), ecliptic latitudes, and visual magnitudes of selected bright stars lying within $10°$ of the ecliptic plane.

Aries			Taurus			Gemini		
λ	δ	α	λ	δ	α	λ	δ	α
00°	+00°00′	000°00′	00°	+11°28′	027°55′	00°	+20°09′	057°49′
02°	+00°48′	001°50′	02°	+12°10′	029°50′	02°	+20°33′	059°54′
04°	+01°35′	003°40′	04°	+12°51′	031°45′	04°	+20°57′	062°00′
06°	+02°23′	005°30′	06°	+13°31′	033°41′	06°	+21°18′	064°07′
08°	+03°10′	007°21′	08°	+14°10′	035°38′	08°	+21°38′	066°14′
10°	+03°58′	009°11′	10°	+14°49′	037°36′	10°	+21°57′	068°22′
12°	+04°45′	011°02′	12°	+15°26′	039°34′	12°	+22°13′	070°30′
14°	+05°31′	012°53′	14°	+16°02′	041°33′	14°	+22°28′	072°39′
16°	+06°18′	014°44′	16°	+16°37′	043°32′	16°	+22°42′	074°48′
18°	+07°04′	016°36′	18°	+17°11′	045°32′	18°	+22°54′	076°57′
20°	+07°49′	018°28′	20°	+17°44′	047°33′	20°	+23°03′	079°07′
22°	+08°34′	020°20′	22°	+18°16′	049°35′	22°	+23°12′	081°17′
24°	+09°19′	022°13′	24°	+18°46′	051°38′	24°	+23°18′	083°28′
26°	+10°02′	024°07′	26°	+19°15′	053°41′	26°	+23°22′	085°39′
28°	+10°46′	026°00′	28°	+19°43′	055°45′	28°	+23°25′	087°49′
30°	+11°28′	027°55′	30°	+20°09′	057°49′	30°	+23°26′	090°00′

Cancer			Leo			Virgo		
λ	δ	α	λ	δ	α	λ	δ	α
00°	+23°26′	090°00′	00°	+20°09′	122°11′	00°	+11°28′	152°05′
02°	+23°25′	092°11′	02°	+19°43′	124°15′	02°	+10°46′	153°60′
04°	+23°22′	094°21′	04°	+19°15′	126°19′	04°	+10°02′	155°53′
06°	+23°18′	096°32′	06°	+18°46′	128°22′	06°	+09°19′	157°47′
08°	+23°12′	098°43′	08°	+18°16′	130°25′	08°	+08°34′	159°40′
10°	+23°03′	100°53′	10°	+17°44′	132°27′	10°	+07°49′	161°32′
12°	+22°54′	103°03′	12°	+17°11′	134°28′	12°	+07°04′	163°24′
14°	+22°42′	105°12′	14°	+16°37′	136°28′	14°	+06°18′	165°16′
16°	+22°28′	107°21′	16°	+16°02′	138°27′	16°	+05°31′	167°07′
18°	+22°13′	109°30′	18°	+15°26′	140°26′	18°	+04°45′	168°58′
20°	+21°57′	111°38′	20°	+14°49′	142°24′	20°	+03°58′	170°49′
22°	+21°38′	113°46′	22°	+14°10′	144°22′	22°	+03°10′	172°39′
24°	+21°18′	115°53′	24°	+13°31′	146°19′	24°	+02°23′	174°30′
26°	+20°57′	117°60′	26°	+12°51′	148°15′	26°	+01°35′	176°20′
28°	+20°33′	120°06′	28°	+12°10′	150°10′	28°	+00°48′	178°10′
30°	+20°09′	122°11′	30°	+11°28′	152°05′	30°	+00°00′	180°00′

Table 2.2: Declinations and right ascensions of points on the ecliptic circle.

Libra			Scorpio			Sagittarius		
λ	δ	α	λ	δ	α	λ	δ	α
00°	-00°00′	180°00′	00°	-11°28′	207°55′	00°	-20°09′	237°49′
02°	-00°48′	181°50′	02°	-12°10′	209°50′	02°	-20°33′	239°54′
04°	-01°35′	183°40′	04°	-12°51′	211°45′	04°	-20°57′	242°00′
06°	-02°23′	185°30′	06°	-13°31′	213°41′	06°	-21°18′	244°07′
08°	-03°10′	187°21′	08°	-14°10′	215°38′	08°	-21°38′	246°14′
10°	-03°58′	189°11′	10°	-14°49′	217°36′	10°	-21°57′	248°22′
12°	-04°45′	191°02′	12°	-15°26′	219°34′	12°	-22°13′	250°30′
14°	-05°31′	192°53′	14°	-16°02′	221°33′	14°	-22°28′	252°39′
16°	-06°18′	194°44′	16°	-16°37′	223°32′	16°	-22°42′	254°48′
18°	-07°04′	196°36′	18°	-17°11′	225°32′	18°	-22°54′	256°57′
20°	-07°49′	198°28′	20°	-17°44′	227°33′	20°	-23°03′	259°07′
22°	-08°34′	200°20′	22°	-18°16′	229°35′	22°	-23°12′	261°17′
24°	-09°19′	202°13′	24°	-18°46′	231°38′	24°	-23°18′	263°28′
26°	-10°02′	204°07′	26°	-19°15′	233°41′	26°	-23°22′	265°39′
28°	-10°46′	206°00′	28°	-19°43′	235°45′	28°	-23°25′	267°49′
30°	-11°28′	207°55′	30°	-20°09′	237°49′	30°	-23°26′	270°00′

Capricorn			Aquarius			Pisces		
λ	δ	α	λ	δ	α	λ	δ	α
00°	-23°26′	270°00′	00°	-20°09′	302°11′	00°	-11°28′	332°05′
02°	-23°25′	272°11′	02°	-19°43′	304°15′	02°	-10°46′	333°60′
04°	-23°22′	274°21′	04°	-19°15′	306°19′	04°	-10°02′	335°53′
06°	-23°18′	276°32′	06°	-18°46′	308°22′	06°	-09°19′	337°47′
08°	-23°12′	278°43′	08°	-18°16′	310°25′	08°	-08°34′	339°40′
10°	-23°03′	280°53′	10°	-17°44′	312°27′	10°	-07°49′	341°32′
12°	-22°54′	283°03′	12°	-17°11′	314°28′	12°	-07°04′	343°24′
14°	-22°42′	285°12′	14°	-16°37′	316°28′	14°	-06°18′	345°16′
16°	-22°28′	287°21′	16°	-16°02′	318°27′	16°	-05°31′	347°07′
18°	-22°13′	289°30′	18°	-15°26′	320°26′	18°	-04°45′	348°58′
20°	-21°57′	291°38′	20°	-14°49′	322°24′	20°	-03°58′	350°49′
22°	-21°38′	293°46′	22°	-14°10′	324°22′	22°	-03°10′	352°39′
24°	-21°18′	295°53′	24°	-13°31′	326°19′	24°	-02°23′	354°30′
26°	-20°57′	297°60′	26°	-12°51′	328°15′	26°	-01°35′	356°20′
28°	-20°33′	300°06′	28°	-12°10′	330°10′	28°	-00°48′	358°10′
30°	-20°09′	302°11′	30°	-11°28′	332°05′	30°	-00°00′	360°00′

Table 2.3: Declinations and right ascensions of points on the ecliptic circle.

L	Longest Day	Summer Solstice Noon Altitude of Sun	Equinoctial Noon Altitude of Sun	Winter Solstice Noon Altitude of Sun
$+00^\circ$	12^h00^m	$+66^\circ34'$ N	$+90^\circ00'$ S	$+66^\circ34'$ S
$+05^\circ$	12^h17^m	$+71^\circ34'$ N	$+85^\circ00'$ S	$+61^\circ34'$ S
$+10^\circ$	12^h35^m	$+76^\circ34'$ N	$+80^\circ00'$ S	$+56^\circ34'$ S
$+15^\circ$	12^h53^m	$+81^\circ34'$ N	$+75^\circ00'$ S	$+51^\circ34'$ S
$+20^\circ$	13^h13^m	$+86^\circ34'$ N	$+70^\circ00'$ S	$+46^\circ34'$ S
$+25^\circ$	13^h33^m	$+88^\circ26'$ N	$+65^\circ00'$ S	$+41^\circ34'$ S
$+30^\circ$	13^h56^m	$+83^\circ26'$ S	$+60^\circ00'$ S	$+36^\circ34'$ S
$+35^\circ$	14^h21^m	$+78^\circ26'$ S	$+55^\circ00'$ S	$+31^\circ34'$ S
$+40^\circ$	14^h51^m	$+73^\circ26'$ S	$+50^\circ00'$ S	$+26^\circ34'$ S
$+45^\circ$	15^h25^m	$+68^\circ26'$ S	$+45^\circ00'$ S	$+21^\circ34'$ S
$+50^\circ$	16^h09^m	$+63^\circ26'$ S	$+40^\circ00'$ S	$+16^\circ34'$ S
$+55^\circ$	17^h06^m	$+58^\circ26'$ S	$+35^\circ00'$ S	$+11^\circ34'$ S
$+60^\circ$	18^h29^m	$+53^\circ26'$ S	$+30^\circ00'$ S	$+06^\circ34'$ S
$+65^\circ$	21^h07^m	$+48^\circ26'$ S	$+25^\circ00'$ S	$+01^\circ34'$ S
$+70^\circ$	61^d06^h	$+43^\circ26'$ S	$+20^\circ00'$ S	$-03^\circ26'$ S
$+75^\circ$	100^d06^h	$+38^\circ26'$ S	$+15^\circ00'$ S	$-08^\circ26'$ S
$+80^\circ$	130^d02^h	$+33^\circ26'$ S	$+10^\circ00'$ S	$-13^\circ26'$ S
$+85^\circ$	156^d22^h	$+28^\circ26'$ S	$+05^\circ00'$ S	$-18^\circ26'$ S
$+90^\circ$	182^d15^h	$+23^\circ26'$ S	$+00^\circ00'$ S	$-23^\circ26'$ S

Table 2.4: Terrestrial climes in the Earth's northern hemisphere. The superscripts h, m, and d stand for hours, minutes, and days, respectively. The symbols S and N indicated that the upper transit of the Sun occurs to the south and north of the zenith, respectively.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	027°55′	00°	057°49′	00°	090°00′	00°	122°11′	00°	152°05′
02°	001°50′	02°	029°50′	02°	059°54′	02°	092°11′	02°	124°15′	02°	153°60′
04°	003°40′	04°	031°45′	04°	062°00′	04°	094°21′	04°	126°19′	04°	155°53′
06°	005°30′	06°	033°41′	06°	064°07′	06°	096°32′	06°	128°22′	06°	157°47′
08°	007°21′	08°	035°38′	08°	066°14′	08°	098°43′	08°	130°25′	08°	159°40′
10°	009°11′	10°	037°36′	10°	068°22′	10°	100°53′	10°	132°27′	10°	161°32′
12°	011°02′	12°	039°34′	12°	070°30′	12°	103°03′	12°	134°28′	12°	163°24′
14°	012°53′	14°	041°33′	14°	072°39′	14°	105°12′	14°	136°28′	14°	165°16′
16°	014°44′	16°	043°32′	16°	074°48′	16°	107°21′	16°	138°27′	16°	167°07′
18°	016°36′	18°	045°32′	18°	076°57′	18°	109°30′	18°	140°26′	18°	168°58′
20°	018°28′	20°	047°33′	20°	079°07′	20°	111°38′	20°	142°24′	20°	170°49′
22°	020°20′	22°	049°35′	22°	081°17′	22°	113°46′	22°	144°22′	22°	172°39′
24°	022°13′	24°	051°38′	24°	083°28′	24°	115°53′	24°	146°19′	24°	174°30′
26°	024°07′	26°	053°41′	26°	085°39′	26°	117°60′	26°	148°15′	26°	176°20′
28°	026°00′	28°	055°45′	28°	087°49′	28°	120°06′	28°	150°10′	28°	178°10′
30°	027°55′	30°	057°49′	30°	090°00′	30°	122°11′	30°	152°05′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	207°55′	00°	237°49′	00°	270°00′	00°	302°11′	00°	332°05′
02°	181°50′	02°	209°50′	02°	239°54′	02°	272°11′	02°	304°15′	02°	333°60′
04°	183°40′	04°	211°45′	04°	242°00′	04°	274°21′	04°	306°19′	04°	335°53′
06°	185°30′	06°	213°41′	06°	244°07′	06°	276°32′	06°	308°22′	06°	337°47′
08°	187°21′	08°	215°38′	08°	246°14′	08°	278°43′	08°	310°25′	08°	339°40′
10°	189°11′	10°	217°36′	10°	248°22′	10°	280°53′	10°	312°27′	10°	341°32′
12°	191°02′	12°	219°34′	12°	250°30′	12°	283°03′	12°	314°28′	12°	343°24′
14°	192°53′	14°	221°33′	14°	252°39′	14°	285°12′	14°	316°28′	14°	345°16′
16°	194°44′	16°	223°32′	16°	254°48′	16°	287°21′	16°	318°27′	16°	347°07′
18°	196°36′	18°	225°32′	18°	256°57′	18°	289°30′	18°	320°26′	18°	348°58′
20°	198°28′	20°	227°33′	20°	259°07′	20°	291°38′	20°	322°24′	20°	350°49′
22°	200°20′	22°	229°35′	22°	261°17′	22°	293°46′	22°	324°22′	22°	352°39′
24°	202°13′	24°	231°38′	24°	263°28′	24°	295°53′	24°	326°19′	24°	354°30′
26°	204°07′	26°	233°41′	26°	265°39′	26°	297°60′	26°	328°15′	26°	356°20′
28°	206°00′	28°	235°45′	28°	267°49′	28°	300°06′	28°	330°10′	28°	358°10′
30°	207°55′	30°	237°49′	30°	270°00′	30°	302°11′	30°	332°05′	30°	360°00′

Table 2.5: Right ascensions of the ecliptic for latitude 0°.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	025°52′	00°	054°07′	00°	085°37′	00°	118°28′	00°	150°02′
02°	001°42′	02°	027°39′	02°	056°07′	02°	087°48′	02°	120°38′	02°	152°04′
04°	003°23′	04°	029°27′	04°	058°08′	04°	089°59′	04°	122°47′	04°	154°06′
06°	005°05′	06°	031°16′	06°	060°10′	06°	092°11′	06°	124°56′	06°	156°07′
08°	006°47′	08°	033°05′	08°	062°13′	08°	094°23′	08°	127°05′	08°	158°08′
10°	008°29′	10°	034°55′	10°	064°17′	10°	096°34′	10°	129°13′	10°	160°09′
12°	010°12′	12°	036°46′	12°	066°22′	12°	098°46′	12°	131°20′	12°	162°09′
14°	011°55′	14°	038°38′	14°	068°28′	14°	100°58′	14°	133°27′	14°	164°09′
16°	013°38′	16°	040°31′	16°	070°34′	16°	103°10′	16°	135°33′	16°	166°08′
18°	015°21′	18°	042°25′	18°	072°41′	18°	105°22′	18°	137°39′	18°	168°08′
20°	017°05′	20°	044°19′	20°	074°49′	20°	107°34′	20°	139°44′	20°	170°07′
22°	018°49′	22°	046°15′	22°	076°58′	22°	109°45′	22°	141°49′	22°	172°06′
24°	020°34′	24°	048°11′	24°	079°07′	24°	111°57′	24°	143°53′	24°	174°04′
26°	022°19′	26°	050°09′	26°	081°16′	26°	114°07′	26°	145°57′	26°	176°03′
28°	024°05′	28°	052°07′	28°	083°26′	28°	116°18′	28°	148°00′	28°	178°01′
30°	025°52′	30°	054°07′	30°	085°37′	30°	118°28′	30°	150°02′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	209°58′	00°	241°32′	00°	274°23′	00°	305°53′	00°	334°08′
02°	181°59′	02°	212°00′	02°	243°42′	02°	276°34′	02°	307°53′	02°	335°55′
04°	183°57′	04°	214°03′	04°	245°53′	04°	278°44′	04°	309°51′	04°	337°41′
06°	185°56′	06°	216°07′	06°	248°03′	06°	280°53′	06°	311°49′	06°	339°26′
08°	187°54′	08°	218°11′	08°	250°15′	08°	283°02′	08°	313°45′	08°	341°11′
10°	189°53′	10°	220°16′	10°	252°26′	10°	285°11′	10°	315°41′	10°	342°55′
12°	191°52′	12°	222°21′	12°	254°38′	12°	287°19′	12°	317°35′	12°	344°39′
14°	193°52′	14°	224°27′	14°	256°50′	14°	289°26′	14°	319°29′	14°	346°22′
16°	195°51′	16°	226°33′	16°	259°02′	16°	291°32′	16°	321°22′	16°	348°05′
18°	197°51′	18°	228°40′	18°	261°14′	18°	293°38′	18°	323°14′	18°	349°48′
20°	199°51′	20°	230°47′	20°	263°26′	20°	295°43′	20°	325°05′	20°	351°31′
22°	201°52′	22°	232°55′	22°	265°37′	22°	297°47′	22°	326°55′	22°	353°13′
24°	203°53′	24°	235°04′	24°	267°49′	24°	299°50′	24°	328°44′	24°	354°55′
26°	205°54′	26°	237°13′	26°	270°01′	26°	301°52′	26°	330°33′	26°	356°37′
28°	207°56′	28°	239°22′	28°	272°12′	28°	303°53′	28°	332°21′	28°	358°18′
30°	209°58′	30°	241°32′	30°	274°23′	30°	305°53′	30°	334°08′	30°	360°00′

Table 2.6: Oblique ascensions of the ecliptic for latitude $+10°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	023°41′	00°	050°09′	00°	080°55′	00°	114°30′	00°	147°51′
02°	001°33′	02°	025°20′	02°	052°04′	02°	083°07′	02°	116°46′	02°	150°02′
04°	003°06′	04°	026°59′	04°	054°00′	04°	085°18′	04°	119°01′	04°	152°12′
06°	004°38′	06°	028°40′	06°	055°57′	06°	087°31′	06°	121°16′	06°	154°22′
08°	006°12′	08°	030°22′	08°	057°56′	08°	089°44′	08°	123°31′	08°	156°31′
10°	007°45′	10°	032°04′	10°	059°56′	10°	091°58′	10°	125°46′	10°	158°40′
12°	009°18′	12°	033°48′	12°	061°57′	12°	094°12′	12°	128°00′	12°	160°49′
14°	010°52′	14°	035°32′	14°	063°59′	14°	096°27′	14°	130°14′	14°	162°58′
16°	012°26′	16°	037°18′	16°	066°02′	16°	098°42′	16°	132°27′	16°	165°06′
18°	014°01′	18°	039°04′	18°	068°07′	18°	100°57′	18°	134°40′	18°	167°14′
20°	015°36′	20°	040°52′	20°	070°13′	20°	103°12′	20°	136°53′	20°	169°22′
22°	017°12′	22°	042°41′	22°	072°19′	22°	105°28′	22°	139°06′	22°	171°30′
24°	018°48′	24°	044°31′	24°	074°27′	24°	107°44′	24°	141°18′	24°	173°37′
26°	020°25′	26°	046°23′	26°	076°35′	26°	109°59′	26°	143°29′	26°	175°45′
28°	022°02′	28°	048°15′	28°	078°45′	28°	112°15′	28°	145°40′	28°	177°53′
30°	023°41′	30°	050°09′	30°	080°55′	30°	114°30′	30°	147°51′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	212°09′	00°	245°30′	00°	279°05′	00°	309°51′	00°	336°19′
02°	182°07′	02°	214°20′	02°	247°45′	02°	281°15′	02°	311°45′	02°	337°58′
04°	184°15′	04°	216°31′	04°	250°01′	04°	283°25′	04°	313°37′	04°	339°35′
06°	186°23′	06°	218°42′	06°	252°16′	06°	285°33′	06°	315°29′	06°	341°12′
08°	188°30′	08°	220°54′	08°	254°32′	08°	287°41′	08°	317°19′	08°	342°48′
10°	190°38′	10°	223°07′	10°	256°48′	10°	289°47′	10°	319°08′	10°	344°24′
12°	192°46′	12°	225°20′	12°	259°03′	12°	291°53′	12°	320°56′	12°	345°59′
14°	194°54′	14°	227°33′	14°	261°18′	14°	293°58′	14°	322°42′	14°	347°34′
16°	197°02′	16°	229°46′	16°	263°33′	16°	296°01′	16°	324°28′	16°	349°08′
18°	199°11′	18°	232°00′	18°	265°48′	18°	298°03′	18°	326°12′	18°	350°42′
20°	201°20′	20°	234°14′	20°	268°02′	20°	300°04′	20°	327°56′	20°	352°15′
22°	203°29′	22°	236°29′	22°	270°16′	22°	302°04′	22°	329°38′	22°	353°48′
24°	205°38′	24°	238°44′	24°	272°29′	24°	304°03′	24°	331°20′	24°	355°22′
26°	207°48′	26°	240°59′	26°	274°42′	26°	306°00′	26°	333°01′	26°	356°54′
28°	209°58′	28°	243°14′	28°	276°53′	28°	307°56′	28°	334°40′	28°	358°27′
30°	212°09′	30°	245°30′	30°	279°05′	30°	309°51′	30°	336°19′	30°	360°00′

Table 2.7: Oblique ascensions of the ecliptic for latitude $+20°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	021°11′	00°	045°36′	00°	075°30′	00°	109°57′	00°	145°22′
02°	001°23′	02°	022°41′	02°	047°24′	02°	077°42′	02°	112°19′	02°	147°42′
04°	002°45′	04°	024°11′	04°	049°14′	04°	079°55′	04°	114°41′	04°	150°01′
06°	004°08′	06°	025°43′	06°	051°06′	06°	082°08′	06°	117°04′	06°	152°21′
08°	005°31′	08°	027°15′	08°	053°00′	08°	084°23′	08°	119°26′	08°	154°40′
10°	006°54′	10°	028°49′	10°	054°55′	10°	086°39′	10°	121°48′	10°	156°59′
12°	008°17′	12°	030°23′	12°	056°51′	12°	088°56′	12°	124°10′	12°	159°18′
14°	009°41′	14°	031°59′	14°	058°50′	14°	091°14′	14°	126°32′	14°	161°37′
16°	011°05′	16°	033°37′	16°	060°49′	16°	093°32′	16°	128°54′	16°	163°55′
18°	012°30′	18°	035°15′	18°	062°51′	18°	095°51′	18°	131°16′	18°	166°13′
20°	013°55′	20°	036°55′	20°	064°54′	20°	098°11′	20°	133°38′	20°	168°31′
22°	015°21′	22°	038°36′	22°	066°58′	22°	100°32′	22°	135°59′	22°	170°49′
24°	016°47′	24°	040°19′	24°	069°04′	24°	102°52′	24°	138°20′	24°	173°07′
26°	018°15′	26°	042°03′	26°	071°12′	26°	105°14′	26°	140°41′	26°	175°25′
28°	019°42′	28°	043°48′	28°	073°20′	28°	107°35′	28°	143°01′	28°	177°42′
30°	021°11′	30°	045°36′	30°	075°30′	30°	109°57′	30°	145°22′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	214°38′	00°	250°03′	00°	284°30′	00°	314°24′	00°	338°49′
02°	182°18′	02°	216°59′	02°	252°25′	02°	286°40′	02°	316°12′	02°	340°18′
04°	184°35′	04°	219°19′	04°	254°46′	04°	288°48′	04°	317°57′	04°	341°45′
06°	186°53′	06°	221°40′	06°	257°08′	06°	290°56′	06°	319°41′	06°	343°13′
08°	189°11′	08°	224°01′	08°	259°28′	08°	293°02′	08°	321°24′	08°	344°39′
10°	191°29′	10°	226°22′	10°	261°49′	10°	295°06′	10°	323°05′	10°	346°05′
12°	193°47′	12°	228°44′	12°	264°09′	12°	297°09′	12°	324°45′	12°	347°30′
14°	196°05′	14°	231°06′	14°	266°28′	14°	299°11′	14°	326°23′	14°	348°55′
16°	198°23′	16°	233°28′	16°	268°46′	16°	301°10′	16°	328°01′	16°	350°19′
18°	200°42′	18°	235°50′	18°	271°04′	18°	303°09′	18°	329°37′	18°	351°43′
20°	203°01′	20°	238°12′	20°	273°21′	20°	305°05′	20°	331°11′	20°	353°06′
22°	205°20′	22°	240°34′	22°	275°37′	22°	307°00′	22°	332°45′	22°	354°29′
24°	207°39′	24°	242°56′	24°	277°52′	24°	308°54′	24°	334°17′	24°	355°52′
26°	209°59′	26°	245°19′	26°	280°05′	26°	310°46′	26°	335°49′	26°	357°15′
28°	212°18′	28°	247°41′	28°	282°18′	28°	312°36′	28°	337°19′	28°	358°37′
30°	214°38′	30°	250°03′	30°	284°30′	30°	314°24′	30°	338°49′	30°	360°00′

Table 2.8: Oblique ascensions of the ecliptic for latitude $+30°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	018°07′	00°	039°54′	00°	068°40′	00°	104°15′	00°	142°17′
02°	001°10′	02°	019°24′	02°	041°34′	02°	070°52′	02°	106°46′	02°	144°49′
04°	002°20′	04°	020°43′	04°	043°16′	04°	073°06′	04°	109°17′	04°	147°21′
06°	003°30′	06°	022°03′	06°	045°01′	06°	075°21′	06°	111°48′	06°	149°52′
08°	004°41′	08°	023°24′	08°	046°48′	08°	077°38′	08°	114°20′	08°	152°24′
10°	005°52′	10°	024°46′	10°	048°36′	10°	079°57′	10°	116°53′	10°	154°55′
12°	007°03′	12°	026°10′	12°	050°27′	12°	082°18′	12°	119°25′	12°	157°26′
14°	008°14′	14°	027°35′	14°	052°20′	14°	084°39′	14°	121°57′	14°	159°57′
16°	009°26′	16°	029°02′	16°	054°15′	16°	087°03′	16°	124°30′	16°	162°28′
18°	010°38′	18°	030°30′	18°	056°12′	18°	089°27′	18°	127°03′	18°	164°58′
20°	011°51′	20°	031°59′	20°	058°12′	20°	091°53′	20°	129°35′	20°	167°29′
22°	013°05′	22°	033°31′	22°	060°13′	22°	094°19′	22°	132°08′	22°	169°59′
24°	014°19′	24°	035°04′	24°	062°17′	24°	096°47′	24°	134°40′	24°	172°29′
26°	015°34′	26°	036°38′	26°	064°23′	26°	099°16′	26°	137°13′	26°	175°00′
28°	016°50′	28°	038°15′	28°	066°31′	28°	101°45′	28°	139°45′	28°	177°30′
30°	018°07′	30°	039°54′	30°	068°40′	30°	104°15′	30°	142°17′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	217°43′	00°	255°45′	00°	291°20′	00°	320°06′	00°	341°53′
02°	182°30′	02°	220°15′	02°	258°15′	02°	293°29′	02°	321°45′	02°	343°10′
04°	185°00′	04°	222°47′	04°	260°44′	04°	295°37′	04°	323°22′	04°	344°26′
06°	187°31′	06°	225°20′	06°	263°13′	06°	297°43′	06°	324°56′	06°	345°41′
08°	190°01′	08°	227°52′	08°	265°41′	08°	299°47′	08°	326°29′	08°	346°55′
10°	192°31′	10°	230°25′	10°	268°07′	10°	301°48′	10°	328°01′	10°	348°09′
12°	195°02′	12°	232°57′	12°	270°33′	12°	303°48′	12°	329°30′	12°	349°22′
14°	197°32′	14°	235°30′	14°	272°57′	14°	305°45′	14°	330°58′	14°	350°34′
16°	200°03′	16°	238°03′	16°	275°21′	16°	307°40′	16°	332°25′	16°	351°46′
18°	202°34′	18°	240°35′	18°	277°42′	18°	309°33′	18°	333°50′	18°	352°57′
20°	205°05′	20°	243°07′	20°	280°03′	20°	311°24′	20°	335°14′	20°	354°08′
22°	207°36′	22°	245°40′	22°	282°22′	22°	313°12′	22°	336°36′	22°	355°19′
24°	210°08′	24°	248°12′	24°	284°39′	24°	314°59′	24°	337°57′	24°	356°30′
26°	212°39′	26°	250°43′	26°	286°54′	26°	316°44′	26°	339°17′	26°	357°40′
28°	215°11′	28°	253°14′	28°	289°08′	28°	318°26′	28°	340°36′	28°	358°50′
30°	217°43′	30°	255°45′	30°	291°20′	30°	320°06′	30°	341°53′	30°	360°00′

Table 2.9: Oblique ascensions of the ecliptic for latitude $+40°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	013°55′	00°	031°54′	00°	058°54′	00°	096°15′	00°	138°06′
02°	000°53′	02°	014°56′	02°	033°22′	02°	061°06′	02°	098°59′	02°	140°54′
04°	001°47′	04°	015°59′	04°	034°52′	04°	063°21′	04°	101°44′	04°	143°43′
06°	002°40′	06°	017°02′	06°	036°25′	06°	065°40′	06°	104°29′	06°	146°31′
08°	003°34′	08°	018°07′	08°	038°01′	08°	068°00′	08°	107°15′	08°	149°19′
10°	004°27′	10°	019°13′	10°	039°40′	10°	070°24′	10°	110°02′	10°	152°07′
12°	005°22′	12°	020°21′	12°	041°22′	12°	072°50′	12°	112°50′	12°	154°55′
14°	006°16′	14°	021°31′	14°	043°06′	14°	075°18′	14°	115°37′	14°	157°42′
16°	007°11′	16°	022°42′	16°	044°54′	16°	077°49′	16°	118°26′	16°	160°30′
18°	008°07′	18°	023°54′	18°	046°45′	18°	080°22′	18°	121°14′	18°	163°17′
20°	009°03′	20°	025°09′	20°	048°38′	20°	082°57′	20°	124°02′	20°	166°05′
22°	010°00′	22°	026°26′	22°	050°35′	22°	085°33′	22°	126°51′	22°	168°52′
24°	010°57′	24°	027°44′	24°	052°35′	24°	088°12′	24°	129°40′	24°	171°39′
26°	011°56′	26°	029°05′	26°	054°38′	26°	090°51′	26°	132°28′	26°	174°26′
28°	012°55′	28°	030°28′	28°	056°45′	28°	093°33′	28°	135°17′	28°	177°13′
30°	013°55′	30°	031°54′	30°	058°54′	30°	096°15′	30°	138°06′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	221°54′	00°	263°45′	00°	301°06′	00°	328°06′	00°	346°05′
02°	182°47′	02°	224°43′	02°	266°27′	02°	303°15′	02°	329°32′	02°	347°05′
04°	185°34′	04°	227°32′	04°	269°09′	04°	305°22′	04°	330°55′	04°	348°04′
06°	188°21′	06°	230°20′	06°	271°48′	06°	307°25′	06°	332°16′	06°	349°03′
08°	191°08′	08°	233°09′	08°	274°27′	08°	309°25′	08°	333°34′	08°	350°00′
10°	193°55′	10°	235°58′	10°	277°03′	10°	311°22′	10°	334°51′	10°	350°57′
12°	196°43′	12°	238°46′	12°	279°38′	12°	313°15′	12°	336°06′	12°	351°53′
14°	199°30′	14°	241°34′	14°	282°11′	14°	315°06′	14°	337°18′	14°	352°49′
16°	202°18′	16°	244°23′	16°	284°42′	16°	316°54′	16°	338°29′	16°	353°44′
18°	205°05′	18°	247°10′	18°	287°10′	18°	318°38′	18°	339°39′	18°	354°38′
20°	207°53′	20°	249°58′	20°	289°36′	20°	320°20′	20°	340°47′	20°	355°33′
22°	210°41′	22°	252°45′	22°	292°00′	22°	321°59′	22°	341°53′	22°	356°26′
24°	213°29′	24°	255°31′	24°	294°20′	24°	323°35′	24°	342°58′	24°	357°20′
26°	216°17′	26°	258°16′	26°	296°39′	26°	325°08′	26°	344°01′	26°	358°13′
28°	219°06′	28°	261°01′	28°	298°54′	28°	326°38′	28°	345°04′	28°	359°07′
30°	221°54′	30°	263°45′	30°	301°06′	30°	328°06′	30°	346°05′	30°	360°00′

Table 2.10: Oblique ascensions of the ecliptic for latitude $+50°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	011°04′	00°	026°14′	00°	051°45′	00°	090°35′	00°	135°15′
02°	000°42′	02°	011°54′	02°	027°31′	02°	053°58′	02°	093°29′	02°	138°15′
04°	001°24′	04°	012°44′	04°	028°52′	04°	056°15′	04°	096°24′	04°	141°15′
06°	002°06′	06°	013°36′	06°	030°16′	06°	058°35′	06°	099°21′	06°	144°15′
08°	002°48′	08°	014°30′	08°	031°44′	08°	060°59′	08°	102°18′	08°	147°14′
10°	003°31′	10°	015°24′	10°	033°14′	10°	063°27′	10°	105°16′	10°	150°14′
12°	004°14′	12°	016°21′	12°	034°48′	12°	065°57′	12°	108°14′	12°	153°13′
14°	004°57′	14°	017°18′	14°	036°26′	14°	068°31′	14°	111°14′	14°	156°12′
16°	005°41′	16°	018°18′	16°	038°07′	16°	071°08′	16°	114°13′	16°	159°11′
18°	006°25′	18°	019°19′	18°	039°52′	18°	073°48′	18°	117°13′	18°	162°10′
20°	007°10′	20°	020°23′	20°	041°41′	20°	076°31′	20°	120°13′	20°	165°08′
22°	007°55′	22°	021°28′	22°	043°34′	22°	079°16′	22°	123°14′	22°	168°07′
24°	008°41′	24°	022°36′	24°	045°31′	24°	082°03′	24°	126°14′	24°	171°05′
26°	009°28′	26°	023°46′	26°	047°32′	26°	084°52′	26°	129°14′	26°	174°03′
28°	010°15′	28°	024°58′	28°	049°37′	28°	087°43′	28°	132°14′	28°	177°02′
30°	011°04′	30°	026°14′	30°	051°45′	30°	090°35′	30°	135°15′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	224°45′	00°	269°25′	00°	308°15′	00°	333°46′	00°	348°56′
02°	182°58′	02°	227°46′	02°	272°17′	02°	310°23′	02°	335°02′	02°	349°45′
04°	185°57′	04°	230°46′	04°	275°08′	04°	312°28′	04°	336°14′	04°	350°32′
06°	188°55′	06°	233°46′	06°	277°57′	06°	314°29′	06°	337°24′	06°	351°19′
08°	191°53′	08°	236°46′	08°	280°44′	08°	316°26′	08°	338°32′	08°	352°05′
10°	194°52′	10°	239°47′	10°	283°29′	10°	318°19′	10°	339°37′	10°	352°50′
12°	197°50′	12°	242°47′	12°	286°12′	12°	320°08′	12°	340°41′	12°	353°35′
14°	200°49′	14°	245°47′	14°	288°52′	14°	321°53′	14°	341°42′	14°	354°19′
16°	203°48′	16°	248°46′	16°	291°29′	16°	323°34′	16°	342°42′	16°	355°03′
18°	206°47′	18°	251°46′	18°	294°03′	18°	325°12′	18°	343°39′	18°	355°46′
20°	209°46′	20°	254°44′	20°	296°33′	20°	326°46′	20°	344°36′	20°	356°29′
22°	212°46′	22°	257°42′	22°	299°01′	22°	328°16′	22°	345°30′	22°	357°12′
24°	215°45′	24°	260°39′	24°	301°25′	24°	329°44′	24°	346°24′	24°	357°54′
26°	218°45′	26°	263°36′	26°	303°45′	26°	331°08′	26°	347°16′	26°	358°36′
28°	221°45′	28°	266°31′	28°	306°02′	28°	332°29′	28°	348°06′	28°	359°18′
30°	224°45′	30°	269°25′	30°	308°15′	30°	333°46′	30°	348°56′	30°	360°00′

Table 2.11: Oblique ascensions of the ecliptic for latitude $+55°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	007°20′	00°	018°22′	00°	041°21′	00°	082°44′	00°	131°31′
02°	000°27′	02°	007°54′	02°	019°24′	02°	043°34′	02°	085°54′	02°	134°47′
04°	000°55′	04°	008°29′	04°	020°29′	04°	045°54′	04°	089°06′	04°	138°02′
06°	001°23′	06°	009°05′	06°	021°38′	06°	048°18′	06°	092°19′	06°	141°17′
08°	001°50′	08°	009°42′	08°	022°50′	08°	050°48′	08°	095°33′	08°	144°32′
10°	002°18′	10°	010°20′	10°	024°07′	10°	053°23′	10°	098°48′	10°	147°47′
12°	002°46′	12°	011°00′	12°	025°27′	12°	056°03′	12°	102°04′	12°	151°01′
14°	003°15′	14°	011°41′	14°	026°52′	14°	058°47′	14°	105°20′	14°	154°15′
16°	003°44′	16°	012°24′	16°	028°22′	16°	061°35′	16°	108°36′	16°	157°29′
18°	004°13′	18°	013°08′	18°	029°57′	18°	064°27′	18°	111°52′	18°	160°42′
20°	004°43′	20°	013°55′	20°	031°38′	20°	067°23′	20°	115°09′	20°	163°55′
22°	005°13′	22°	014°43′	22°	033°23′	22°	070°22′	22°	118°26′	22°	167°09′
24°	005°44′	24°	015°34′	24°	035°14′	24°	073°24′	24°	121°42′	24°	170°22′
26°	006°15′	26°	016°28′	26°	037°11′	26°	076°28′	26°	124°59′	26°	173°34′
28°	006°47′	28°	017°23′	28°	039°13′	28°	079°35′	28°	128°15′	28°	176°47′
30°	007°20′	30°	018°22′	30°	041°21′	30°	082°44′	30°	131°31′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	228°29′	00°	277°16′	00°	318°39′	00°	341°38′	00°	352°40′
02°	183°13′	02°	231°45′	02°	280°25′	02°	320°47′	02°	342°37′	02°	353°13′
04°	186°26′	04°	235°01′	04°	283°32′	04°	322°49′	04°	343°32′	04°	353°45′
06°	189°38′	06°	238°18′	06°	286°36′	06°	324°46′	06°	344°26′	06°	354°16′
08°	192°51′	08°	241°34′	08°	289°38′	08°	326°37′	08°	345°17′	08°	354°47′
10°	196°05′	10°	244°51′	10°	292°37′	10°	328°22′	10°	346°05′	10°	355°17′
12°	199°18′	12°	248°08′	12°	295°33′	12°	330°03′	12°	346°52′	12°	355°47′
14°	202°31′	14°	251°24′	14°	298°25′	14°	331°38′	14°	347°36′	14°	356°16′
16°	205°45′	16°	254°40′	16°	301°13′	16°	333°08′	16°	348°19′	16°	356°45′
18°	208°59′	18°	257°56′	18°	303°57′	18°	334°33′	18°	349°00′	18°	357°14′
20°	212°13′	20°	261°12′	20°	306°37′	20°	335°53′	20°	349°40′	20°	357°42′
22°	215°28′	22°	264°27′	22°	309°12′	22°	337°10′	22°	350°18′	22°	358°10′
24°	218°43′	24°	267°41′	24°	311°42′	24°	338°22′	24°	350°55′	24°	358°37′
26°	221°58′	26°	270°54′	26°	314°06′	26°	339°31′	26°	351°31′	26°	359°05′
28°	225°13′	28°	274°06′	28°	316°26′	28°	340°36′	28°	352°06′	28°	359°33′
30°	228°29′	30°	277°16′	30°	318°39′	30°	341°38′	30°	352°40′	30°	360°00′

Table 2.12: Oblique ascensions of the ecliptic for latitude $+60°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	000°00′	00°	002°07′	00°	005°56′	00°	021°39′	00°	070°18′	00°	126°18′
02°	000°08′	02°	002°17′	02°	006°22′	02°	023°56′	02°	074°04′	02°	129°57′
04°	000°16′	04°	002°28′	04°	006°51′	04°	026°25′	04°	077°50′	04°	133°35′
06°	000°23′	06°	002°39′	06°	007°22′	06°	029°06′	06°	081°36′	06°	137°12′
08°	000°31′	08°	002°51′	08°	007°57′	08°	031°58′	08°	085°22′	08°	140°49′
10°	000°39′	10°	003°03′	10°	008°36′	10°	034°59′	10°	089°08′	10°	144°25′
12°	000°47′	12°	003°16′	12°	009°19′	12°	038°09′	12°	092°54′	12°	148°00′
14°	000°55′	14°	003°29′	14°	010°07′	14°	041°26′	14°	096°39′	14°	151°35′
16°	001°04′	16°	003°44′	16°	011°02′	16°	044°50′	16°	100°24′	16°	155°09′
18°	001°12′	18°	003°59′	18°	012°04′	18°	048°19′	18°	104°08′	18°	158°43′
20°	001°21′	20°	004°15′	20°	013°14′	20°	051°52′	20°	107°52′	20°	162°16′
22°	001°29′	22°	004°32′	22°	014°33′	22°	055°29′	22°	111°35′	22°	165°50′
24°	001°38′	24°	004°51′	24°	016°02′	24°	059°08′	24°	115°17′	24°	169°22′
26°	001°48′	26°	005°11′	26°	017°42′	26°	062°50′	26°	118°58′	26°	172°55′
28°	001°57′	28°	005°33′	28°	019°34′	28°	066°33′	28°	122°38′	28°	176°28′
30°	002°07′	30°	005°56′	30°	021°39′	30°	070°18′	30°	126°18′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	233°42′	00°	289°42′	00°	338°21′	00°	354°04′	00°	357°53′
02°	183°32′	02°	237°22′	02°	293°27′	02°	340°26′	02°	354°27′	02°	358°03′
04°	187°05′	04°	241°02′	04°	297°10′	04°	342°18′	04°	354°49′	04°	358°12′
06°	190°38′	06°	244°43′	06°	300°52′	06°	343°58′	06°	355°09′	06°	358°22′
08°	194°10′	08°	248°25′	08°	304°31′	08°	345°27′	08°	355°28′	08°	358°31′
10°	197°44′	10°	252°08′	10°	308°08′	10°	346°46′	10°	355°45′	10°	358°39′
12°	201°17′	12°	255°52′	12°	311°41′	12°	347°56′	12°	356°01′	12°	358°48′
14°	204°51′	14°	259°36′	14°	315°10′	14°	348°58′	14°	356°16′	14°	358°56′
16°	208°25′	16°	263°21′	16°	318°34′	16°	349°53′	16°	356°31′	16°	359°05′
18°	212°00′	18°	267°06′	18°	321°51′	18°	350°41′	18°	356°44′	18°	359°13′
20°	215°35′	20°	270°52′	20°	325°01′	20°	351°24′	20°	356°57′	20°	359°21′
22°	219°11′	22°	274°38′	22°	328°02′	22°	352°03′	22°	357°09′	22°	359°29′
24°	222°48′	24°	278°24′	24°	330°54′	24°	352°38′	24°	357°21′	24°	359°37′
26°	226°25′	26°	282°10′	26°	333°35′	26°	353°09′	26°	357°32′	26°	359°44′
28°	230°03′	28°	285°56′	28°	336°04′	28°	353°38′	28°	357°43′	28°	359°52′
30°	233°42′	30°	289°42′	30°	338°21′	30°	354°04′	30°	357°53′	30°	360°00′

Table 2.13: Oblique ascensions of the ecliptic for latitude $+65°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	360°00′	00°	354°02′	00°	—	00°	—	00°41′	032°54′	00°	118°13′
02°	359°39′	02°	353°30′	02°	—	02°	—	02°	044°26′	02°	122°31′
04°	359°18′	04°	352°57′	04°	—	04°	—	04°	052°41′	04°	126°47′
06°	358°57′	06°	352°21′	06°	—	06°	—	06°	059°22′	06°	131°01′
08°	358°35′	08°	351°42′	08°	—	08°	—	08°	065°22′	08°	135°13′
10°	358°14′	10°	351°00′	10°	—	10°	—	10°	070°57′	10°	139°22′
12°	357°52′	12°	350°14′	12°	—	12°	—	12°	076°15′	12°	143°31′
14°	357°29′	14°	349°23′	14°	—	14°	—	14°	081°21′	14°	147°37′
16°	357°06′	16°	348°26′	16°	—	16°	—	16°	086°18′	16°	151°43′
18°	356°43′	18°	347°20′	18°	—	18°	—	18°	091°07′	18°	155°47′
20°	356°18′	20°	346°04′	20°	—	20°	—	20°	095°49′	20°	159°51′
22°	355°53′	22°	344°32′	22°	—	22°	—	22°	100°26′	22°	163°54′
24°	355°27′	24°	342°37′	24°	—	24°	—	24°	104°58′	24°	167°56′
26°	355°00′	26°	340°03′	26°	—	26°	—	26°	109°27′	26°	171°57′
28°	354°32′	28°	335°55′	28°	—	28°	—	28°	113°51′	28°	175°59′
30°	354°02′	29°19′	327°07′	30°	—	30°	—	30°	118°13′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	241°47′	00°	—	00°	—	00°41′	032°52′	00°	005°58′
02°	184°01′	02°	246°09′	02°	—	02°	—	02°	024°05′	02°	005°28′
04°	188°03′	04°	250°33′	04°	—	04°	—	04°	019°57′	04°	005°00′
06°	192°04′	06°	255°02′	06°	—	06°	—	06°	017°23′	06°	004°33′
08°	196°06′	08°	259°34′	08°	—	08°	—	08°	015°28′	08°	004°07′
10°	200°09′	10°	264°11′	10°	—	10°	—	10°	013°56′	10°	003°42′
12°	204°13′	12°	268°53′	12°	—	12°	—	12°	012°40′	12°	003°17′
14°	208°17′	14°	273°42′	14°	—	14°	—	14°	011°34′	14°	002°54′
16°	212°23′	16°	278°39′	16°	—	16°	—	16°	010°37′	16°	002°31′
18°	216°29′	18°	283°45′	18°	—	18°	—	18°	009°46′	18°	002°08′
20°	220°38′	20°	289°03′	20°	—	20°	—	20°	009°00′	20°	001°46′
22°	224°47′	22°	294°38′	22°	—	22°	—	22°	008°18′	22°	001°25′
24°	228°59′	24°	300°38′	24°	—	24°	—	24°	007°39′	24°	001°03′
26°	233°13′	26°	307°19′	26°	—	26°	—	26°	007°03′	26°	000°42′
28°	237°29′	28°	315°34′	28°	—	28°	—	28°	006°30′	28°	000°21′
30°	241°47′	29°19′	327°07′	30°	—	30°	—	30°	005°58′	30°	000°00′

Table 2.14: Oblique ascensions of the ecliptic for latitude $+70°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	360°00′	00°	338°42′	00°	—	00°	—	00°	—	00°	102°52′
02°	358°52′	02°	336°16′	02°	—	02°	—	02°	—	02°	108°49′
04°	357°44′	04°	333°24′	04°	—	04°	—	04°	—	04°	114°32′
06°	356°35′	06°	329°54′	06°	—	06°	—	06°	—	06°	120°04′
08°	355°25′	08°	325°10′	08°	—	08°	—	08°	—	08°	125°27′
10°	354°13′	10°	316°55′	10°	—	10°	—	10°	—	10°	130°43′
12°	353°00′	11°36′	308°12′	12°	—	12°	—	12°	—	12°	135°52′
14°	351°44′	14°	—	14°	—	14°	—	14°	—	14°	140°57′
16°	350°26′	16°	—	16°	—	16°	—	16°	—	16°	145°58′
18°	349°05′	18°	—	18°	—	18°	—	19°24′	051°50′	18°	150°56′
20°	347°39′	20°	—	20°	—	20°	—	20°	061°44′	20°	155°50′
22°	346°08′	22°	—	22°	—	22°	—	22°	073°54′	22°	160°43′
24°	344°30′	24°	—	24°	—	24°	—	24°	082°31′	24°	165°34′
26°	342°45′	26°	—	26°	—	26°	—	26°	089°54′	26°	170°23′
28°	340°50′	28°	—	28°	—	28°	—	28°	096°36′	28°	175°12′
30°	338°42′	30°	—	30°	—	30°	—	30°	102°52′	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	257°08′	00°	—	00°	—	00°	—	00°	021°18′
02°	184°48′	02°	263°24′	02°	—	02°	—	02°	—	02°	019°10′
04°	189°37′	04°	270°06′	04°	—	04°	—	04°	—	04°	017°15′
06°	194°26′	06°	277°29′	06°	—	06°	—	06°	—	06°	015°30′
08°	199°17′	08°	286°06′	08°	—	08°	—	08°	—	08°	013°52′
10°	204°10′	10°	298°16′	10°	—	10°	—	10°	—	10°	012°21′
12°	209°04′	11°36′	308°12′	12°	—	12°	—	12°	—	12°	010°55′
14°	214°02′	14°	—	14°	—	14°	—	14°	—	14°	009°34′
16°	219°03′	16°	—	16°	—	16°	—	16°	—	16°	008°16′
18°	224°08′	18°	—	18°	—	18°	—	19°24′	051°50′	18°	007°00′
20°	229°17′	20°	—	20°	—	20°	—	20°	043°05′	20°	005°47′
22°	234°33′	22°	—	22°	—	22°	—	22°	034°50′	22°	004°35′
24°	239°56′	24°	—	24°	—	24°	—	24°	030°06′	24°	003°25′
26°	245°28′	26°	—	26°	—	26°	—	26°	026°36′	26°	002°16′
28°	251°11′	28°	—	28°	—	28°	—	28°	023°44′	28°	001°08′
30°	257°08′	30°	—	30°	—	30°	—	30°	021°18′	30°	000°00′

Table 2.15: Oblique ascensions of the ecliptic for latitude $+75°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	360°00′	00°	—	00°	—	00°	—	00°	—	00°	—
02°	357°19′	02°	—	02°	—	02°	—	02°	—	02°	—
04°	354°37′	04°	—	04°	—	04°	—	04°	—	04°07′	066°01′
06°	351°52′	06°	—	06°	—	06°	—	06°	—	06°	089°25′
08°	349°02′	08°	—	08°	—	08°	—	08°	—	08°	100°58′
10°	346°04′	10°	—	10°	—	10°	—	10°	—	10°	110°24′
12°	342°58′	12°	—	12°	—	12°	—	12°	—	12°	118°47′
14°	339°39′	14°	—	14°	—	14°	—	14°	—	14°	126°33′
16°	336°02′	16°	—	16°	—	16°	—	16°	—	16°	133°52′
18°	331°59′	18°	—	18°	—	18°	—	18°	—	18°	140°54′
20°	327°20′	20°	—	20°	—	20°	—	20°	—	20°	147°42′
22°	321°39′	22°	—	22°	—	22°	—	22°	—	22°	154°20′
24°	313°51′	24°	—	24°	—	24°	—	24°	—	24°	160°51′
25°53′	294°01′	26°	—	26°	—	26°	—	26°	—	26°	167°16′
28°	—	28°	—	28°	—	28°	—	28°	—	28°	173°39′
30°	—	30°	—	30°	—	30°	—	30°	—	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	—	00°	—	00°	—	00°	—	00°	—
02°	186°21′	02°	—	02°	—	02°	—	02°	—	02°	—
04°	192°44′	04°	—	04°	—	04°	—	04°	—	04°07′	066°01′
06°	199°09′	06°	—	06°	—	06°	—	06°	—	06°	046°09′
08°	205°40′	08°	—	08°	—	08°	—	08°	—	08°	038°21′
10°	212°18′	10°	—	10°	—	10°	—	10°	—	10°	032°40′
12°	219°06′	12°	—	12°	—	12°	—	12°	—	12°	028°01′
14°	226°08′	14°	—	14°	—	14°	—	14°	—	14°	023°58′
16°	233°27′	16°	—	16°	—	16°	—	16°	—	16°	020°21′
18°	241°13′	18°	—	18°	—	18°	—	18°	—	18°	017°02′
20°	249°36′	20°	—	20°	—	20°	—	20°	—	20°	013°56′
22°	259°02′	22°	—	22°	—	22°	—	22°	—	22°	010°58′
24°	270°35′	24°	—	24°	—	24°	—	24°	—	24°	008°08′
25°53′	294°01′	26°	—	26°	—	26°	—	26°	—	26°	005°23′
28°	—	28°	—	28°	—	28°	—	28°	—	28°	002°41′
30°	—	30°	—	30°	—	30°	—	30°	—	30°	000°00′

Table 2.16: Oblique ascensions of the ecliptic for latitude $+80°$.

Aries		Taurus		Gemini		Cancer		Leo		Virgo	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	360°00′	00°	—	00°	—	00°	—	00°	—	00°	—
02°	352°42′	02°	—	02°	—	02°	—	02°	—	02°	—
04°	345°11′	04°	—	04°	—	04°	—	04°	—	04°	—
06°	337°07′	06°	—	06°	—	06°	—	06°	—	06°	—
08°	328°02′	08°	—	08°	—	08°	—	08°	—	08°	—
10°	316°53′	10°	—	10°	—	10°	—	10°	—	10°	—
12°	299°32′	12°	—	12°	—	12°	—	12°	—	12°	—
12°40′	281°40′	14°	—	14°	—	14°	—	14°	—	14°	—
16°	—	16°	—	16°	—	16°	—	16°	—	17°20′	078°23′
18°	—	18°	—	18°	—	18°	—	18°	—	18°	097°28′
20°	—	20°	—	20°	—	20°	—	20°	—	20°	118°31′
22°	—	22°	—	22°	—	22°	—	22°	—	22°	133°20′
24°	—	24°	—	24°	—	24°	—	24°	—	24°	146°06′
26°	—	26°	—	26°	—	26°	—	26°	—	26°	157°50′
28°	—	28°	—	28°	—	28°	—	28°	—	28°	169°02′
30°	—	30°	—	30°	—	30°	—	30°	—	30°	180°00′

Libra		Scorpio		Sagittarius		Capricorn		Aquarius		Pisces	
λ	α	λ	α	λ	α	λ	α	λ	α	λ	α
00°	180°00′	00°	—	00°	—	00°	—	00°	—	00°	—
02°	190°58′	02°	—	02°	—	02°	—	02°	—	02°	—
04°	202°10′	04°	—	04°	—	04°	—	04°	—	04°	—
06°	213°54′	06°	—	06°	—	06°	—	06°	—	06°	—
08°	226°40′	08°	—	08°	—	08°	—	08°	—	08°	—
10°	241°29′	10°	—	10°	—	10°	—	10°	—	10°	—
12°	262°32′	12°	—	12°	—	12°	—	12°	—	12°	—
12°40′	281°40′	14°	—	14°	—	14°	—	14°	—	14°	—
16°	—	16°	—	16°	—	16°	—	16°	—	17°20′	078°23′
18°	—	18°	—	18°	—	18°	—	18°	—	18°	060°28′
20°	—	20°	—	20°	—	20°	—	20°	—	20°	043°07′
22°	—	22°	—	22°	—	22°	—	22°	—	22°	031°58′
24°	—	24°	—	24°	—	24°	—	24°	—	24°	022°53′
26°	—	26°	—	26°	—	26°	—	26°	—	26°	014°49′
28°	—	28°	—	28°	—	28°	—	28°	—	28°	007°18′
30°	—	30°	—	30°	—	30°	—	30°	—	30°	000°00′

Table 2.17: Oblique ascensions of the ecliptic for latitude $+85°$.

Aries

00:00	90°00′	314°53′	178°15′
01:00	75°00′	156°34′	336°34′
02:00	60°00′	156°34′	336°34′
03:00	45°00′	156°34′	336°34′
04:00	30°00′	156°34′	336°34′
05:00	15°00′	156°34′	336°34′
06:00	00°00′	156°34′	336°34′

Taurus

00:00	78°32′	249°26′	249°26′
01:00	71°12′	196°00′	302°51′
02:00	58°04′	178°26′	320°25′
03:00	43°52′	170°40′	328°11′
04:00	29°20′	165°58′	332°53′
05:00	14°42′	162°29′	336°23′
06:00	00°00′	159°26′	339°26′

Gemini

00:00	69°51′	257°46′	257°46′
01:00	65°04′	219°53′	295°39′
02:00	54°24′	198°35′	316°57′
03:00	41°36′	186°47′	328°46′
04:00	28°00′	179°01′	336°32′
05:00	14°04′	173°03′	342°30′
06:00	00°00′	167°46′	347°46′

Cancer

00:00	69°51′	282°14′	282°14′
01:00	65°04′	244°21′	320°07′
02:00	54°24′	223°03′	341°25′
03:00	41°36′	211°14′	353°13′
04:00	28°00′	203°28′	000°59′
05:00	14°04′	197°30′	006°57′
06:00	00°00′	192°14′	012°14′
06:00	00°00′	192°14′	012°14′

Leo

00:00	69°51′	282°14′	282°14′
01:00	65°04′	244°21′	320°07′
02:00	54°24′	223°03′	341°25′
03:00	41°36′	211°14′	353°13′
04:00	28°00′	203°28′	000°59′
05:00	14°04′	197°30′	006°57′
06:00	00°00′	192°14′	012°14′

Virgo

00:00	78°32′	290°34′	290°34′
01:00	71°12′	237°09′	344°00′
02:00	58°04′	219°35′	001°34′
03:00	43°52′	211°49′	009°20′
04:00	29°20′	207°07′	014°02′
05:00	14°42′	203°37′	017°31′
06:00	00°00′	200°34′	020°34′

Libra

00:00	90°00′	045°07′	181°45′
01:00	75°00′	203°26′	023°26′
02:00	60°00′	203°26′	023°26′
03:00	45°00′	203°26′	023°26′
04:00	30°00′	203°26′	023°26′
05:00	15°00′	203°26′	023°26′
06:00	00°00′	203°26′	023°26′

Scorpio

00:00	78°32′	110°34′	110°34′
01:00	71°12′	164°00′	057°09′
02:00	58°04′	181°34′	039°35′
03:00	43°52′	189°20′	031°49′
04:00	29°20′	194°02′	027°07′
05:00	14°42′	197°31′	023°37′
06:00	00°00′	200°34′	020°34′

Sagittarius

00:00	69°51′	102°14′	102°14′
01:00	65°04′	140°07′	064°21′
02:00	54°24′	161°25′	043°03′
03:00	41°36′	173°13′	031°14′
04:00	28°00′	180°59′	023°28′
05:00	14°04′	186°57′	017°30′
06:00	00°00′	192°14′	012°14′

Capricorn

00:00	66°34′	090°00′	090°00′
01:00	62°24′	123°58′	056°02′
02:00	52°37′	145°26′	034°34′
03:00	40°27′	158°19′	021°41′
04:00	27°18′	167°04′	012°56′
05:00	13°44′	173°55′	006°05′
06:00	00°00′	180°00′	360°00′

Aquarius

00:00	69°51′	077°46′	077°46′
01:00	65°04′	115°39′	039°53′
02:00	54°24′	136°57′	018°35′
03:00	41°36′	148°46′	006°47′
04:00	28°00′	156°32′	359°01′
05:00	14°04′	162°30′	353°03′
06:00	00°00′	167°46′	347°46′

Pisces

00:00	78°32′	069°26′	069°26′
01:00	71°12′	122°51′	016°00′
02:00	58°04′	140°25′	358°26′
03:00	43°52′	148°11′	350°40′
04:00	29°20′	152°53′	345°58′
05:00	14°42′	156°23′	342°29′
06:00	00°00′	159°26′	339°26′

Table 2.18: Ecliptic altitude and parallactic angle for latitude 0°.

Aries

00:00	80°00′	066°34′	066°34′
01:00	72°02′	122°18′	010°50′
02:00	58°32′	137°08′	356°00′
03:00	44°08′	142°34′	350°34′
04:00	29°30′	145°03′	348°05′
05:00	14°46′	146°13′	346°55′
06:00	00°00′	146°34′	346°34′

Taurus

00:00	88°32′	249°26′	249°26′
01:00	75°11′	163°41′	335°10′
02:00	60°30′	159°21′	339°30′
03:00	45°48′	156°49′	342°02′
04:00	31°08′	154°35′	344°16′
05:00	16°31′	152°16′	346°35′
06:00	01°59′	149°37′	349°14′
06:08	00°00′	149°13′	349°38′

Gemini

00:00	79°51′	257°46′	257°46′
01:00	72°20′	200°37′	314°55′
02:00	59°22′	182°38′	332°54′
03:00	45°32′	174°04′	341°29′
04:00	31°28′	168°13′	347°20′
05:00	17°24′	163°15′	352°18′
06:00	03°26′	158°22′	357°10′
06:14	00°00′	157°07′	358°26′

Cancer

00:00	76°34′	270°00′	270°00′
01:00	70°22′	220°40′	319°20′
02:00	58°23′	200°04′	339°56′
03:00	45°04′	189°35′	350°25′
04:00	31°23′	182°27′	357°33′
05:00	17°38′	176°31′	003°29′
06:00	03°58′	170°49′	009°11′
06:17	00°00′	169°05′	010°55′

Leo

00:00	79°51′	282°14′	282°14′
01:00	72°20′	225°05′	339°23′
02:00	59°22′	207°06′	357°22′
03:00	45°32′	198°31′	005°56′
04:00	31°28′	192°40′	011°47′
05:00	17°24′	187°42′	016°45′
06:00	03°26′	182°50′	021°38′
06:14	00°00′	181°34′	022°53′

Virgo

00:00	88°32′	290°34′	290°34′
01:00	75°11′	204°50′	016°19′
02:00	60°30′	200°30′	020°39′
03:00	45°48′	197°58′	023°11′
04:00	31°08′	195°44′	025°25′
05:00	16°31′	193°25′	027°44′
06:00	01°59′	190°46′	030°23′
06:08	00°00′	190°22′	030°47′

Libra

00:00	80°00′	113°26′	113°26′
01:00	72°02′	169°10′	057°42′
02:00	58°32′	184°00′	042°52′
03:00	44°08′	189°26′	037°26′
04:00	29°30′	191°55′	034°57′
05:00	14°46′	193°05′	033°47′
06:00	00°00′	193°26′	033°26′

Scorpio

00:00	68°32′	110°34′	110°34′
01:00	63°52′	145°55′	075°13′
02:00	53°15′	165°58′	055°11′
03:00	40°23′	176°40′	044°29′
04:00	26°37′	183°07′	038°01′
05:00	12°26′	187°30′	033°39′
05:51	00°00′	190°22′	030°47′

Sagittarius

00:00	59°51′	102°14′	102°14′
01:00	56°26′	129°41′	074°47′
02:00	47°48′	149°23′	055°05′
03:00	36°26′	162°11′	042°17′
04:00	23°44′	170°55′	033°32′
05:00	10°20′	177°27′	027°00′
05:45	00°00′	181°34′	022°53′

Capricorn

00:00	56°34′	090°00′	090°00′
01:00	53°29′	115°22′	064°38′
02:00	45°31′	134°39′	045°21′
03:00	34°44′	147°56′	032°04′
04:00	22°30′	157°24′	022°36′
05:00	09°29′	164°40′	015°20′
05:42	00°00′	169°05′	010°55′

Aquarius

00:00	59°51′	077°46′	077°46′
01:00	56°26′	105°13′	050°19′
02:00	47°48′	124°55′	030°37′
03:00	36°26′	137°43′	017°49′
04:00	23°44′	146°28′	009°05′
05:00	10°20′	153°00′	002°33′
05:45	00°00′	157°07′	358°26′

Pisces

00:00	68°32′	069°26′	069°26′
01:00	63°52′	104°47′	034°05′
02:00	53°15′	124°49′	014°02′
03:00	40°23′	135°31′	003°20′
04:00	26°37′	141°59′	356°53′
05:00	12°26′	146°21′	352°30′
05:51	00°00′	149°13′	349°38′

Table 2.19: Ecliptic altitude and parallactic angle for latitude $+10°$.

	Aries		
00:00	70°00′	066°34′	066°34′
01:00	65°11′	101°59′	031°09′
02:00	54°28′	120°31′	012°37′
03:00	41°38′	129°20′	003°48′
04:00	28°01′	133°46′	359°22′
05:00	14°05′	135°55′	357°13′
06:00	00°00′	136°34′	356°34′
	Taurus		
00:00	81°28′	069°26′	069°26′
01:00	73°15′	126°58′	011°53′
02:00	59°57′	139°10′	359°41′
03:00	45°59′	142°26′	356°25′
04:00	31°54′	142°53′	355°58′
05:00	17°50′	141°53′	356°58′
06:00	03°54′	139°48′	359°03′
06:16	00°00′	139°00′	359°51′
	Gemini		
00:00	89°51′	257°46′	257°46′
01:00	75°55′	165°46′	349°46′
02:00	61°52′	162°48′	352°44′
03:00	47°52′	159°52′	355°41′
04:00	33°59′	156°42′	358°51′
05:00	20°15′	153°07′	002°26′
06:00	06°46′	148°54′	006°38′
06:30	00°00′	146°24′	009°08′
	Cancer		
00:00	86°34′	270°00′	270°00′
01:00	75°39′	190°58′	349°02′
02:00	61°58′	181°12′	358°48′
03:00	48°13′	175°44′	004°16′
04:00	34°33′	171°08′	008°52′
05:00	21°03′	166°33′	013°27′
06:00	07°49′	161°32′	018°28′
06:36	00°00′	158°07′	021°53′
	Leo		
00:00	89°51′	282°14′	282°14′
01:00	75°55′	190°14′	014°14′
02:00	61°52′	187°16′	017°12′
03:00	47°52′	184°19′	020°08′
04:00	33°59′	181°09′	023°18′
05:00	20°15′	177°34′	026°53′
06:00	06°46′	173°22′	031°06′
06:30	00°00′	170°52′	033°36′
	Virgo		
00:00	81°28′	110°34′	110°34′
01:00	73°15′	168°07′	053°02′
02:00	59°57′	180°19′	040°50′
03:00	45°59′	183°35′	037°34′
04:00	31°54′	184°02′	037°07′
05:00	17°50′	183°02′	038°07′
06:00	03°54′	180°57′	040°12′
06:16	00°00′	180°09′	041°00′
	Libra		
00:00	70°00′	113°26′	113°26′
01:00	65°11′	148°51′	078°01′
02:00	54°28′	167°23′	059°29′
03:00	41°38′	176°12′	050°40′
04:00	28°01′	180°38′	046°14′
05:00	14°05′	182°47′	044°05′
06:00	00°00′	183°26′	043°26′
	Scorpio		
00:00	58°32′	110°34′	110°34′
01:00	55°14′	135°49′	085°19′
02:00	46°51′	153°58′	067°11′
03:00	35°41′	165°27′	055°42′
04:00	23°06′	172°48′	048°21′
05:00	09°48′	177°40′	043°29′
05:43	00°00′	180°09′	041°00′
	Sagittarius		
00:00	49°51′	102°14′	102°14′
01:00	47°15′	123°13′	081°14′
02:00	40°15′	140°14′	064°14′
03:00	30°24′	152°37′	051°50′
04:00	18°52′	161°33′	042°55′
05:00	06°21′	168°11′	036°16′
05:29	00°00′	170°52′	033°36′
	Capricorn		
00:00	46°34′	090°00′	090°00′
01:00	44°10′	109°49′	070°11′
02:00	37°38′	126°24′	053°36′
03:00	28°16′	138°59′	041°01′
04:00	17°10′	148°24′	031°36′
05:00	05°00′	155°40′	024°20′
05:23	00°00′	158°07′	021°53′
	Aquarius		
00:00	49°51′	077°46′	077°46′
01:00	47°15′	098°46′	056°47′
02:00	40°15′	115°46′	039°46′
03:00	30°24′	128°10′	027°23′
04:00	18°52′	137°05′	018°27′
05:00	06°21′	143°44′	011°49′
05:29	00°00′	146°24′	009°08′
	Pisces		
00:00	58°32′	069°26′	069°26′
01:00	55°14′	094°41′	044°11′
02:00	46°51′	112°49′	026°02′
03:00	35°41′	124°18′	014°33′
04:00	23°06′	131°39′	007°12′
05:00	09°48′	136°31′	002°20′
05:43	00°00′	139°00′	359°51′

Table 2.20: Ecliptic altitude and parallactic angle for latitude $+20°$.

Aries			
00:00	60°00′	066°34′	066°34′
01:00	56°46′	090°43′	042°25′
02:00	48°35′	107°28′	025°40′
03:00	37°46′	117°20′	015°48′
04:00	25°40′	122°53′	010°15′
05:00	12°57′	125°42′	007°26′
06:00	00°00′	126°34′	006°34′
Taurus			
00:00	71°28′	069°26′	069°26′
01:00	66°49′	104°08′	034°43′
02:00	56°33′	121°13′	017°38′
03:00	44°24′	128°24′	010°27′
04:00	31°35′	131°07′	007°44′
05:00	18°36′	131°23′	007°28′
06:00	05°42′	129°55′	008°56′
06:26	00°00′	128°45′	010°06′
Gemini			
00:00	80°09′	077°46′	077°46′
01:00	73°15′	128°48′	026°45′
02:00	61°12′	141°47′	013°46′
03:00	48°20′	144°53′	010°40′
04:00	35°22′	144°39′	010°54′
05:00	22°30′	142°39′	012°54′
06:00	09°55′	139°19′	016°14′
06:48	00°00′	135°36′	019°57′
Cancer			
00:00	83°26′	090°00′	090°00′
01:00	75°06′	150°38′	029°22′
02:00	62°30′	159°40′	020°20′
03:00	49°32′	160°38′	019°22′
04:00	36°36′	159°05′	020°55′
05:00	23°52′	156°10′	023°50′
06:00	11°28′	152°05′	027°55′
06:57	00°00′	146°59′	033°01′
Leo			
00:00	80°09′	102°14′	102°14′
01:00	73°15′	153°15′	051°12′
02:00	61°12′	166°14′	038°13′
03:00	48°20′	169°20′	035°07′
04:00	35°22′	169°06′	035°21′
05:00	22°30′	167°06′	037°21′
06:00	09°55′	163°46′	040°41′
06:48	00°00′	160°03′	044°24′
Virgo			
00:00	71°28′	110°34′	110°34′
01:00	66°49′	145°17′	075°52′
02:00	56°33′	162°22′	058°47′
03:00	44°24′	169°33′	051°36′
04:00	31°35′	172°16′	048°53′
05:00	18°36′	172°32′	048°37′
06:00	05°42′	171°04′	050°05′
06:26	00°00′	169°54′	051°15′
Libra			
00:00	60°00′	113°26′	113°26′
01:00	56°46′	137°35′	089°17′
02:00	48°35′	154°20′	072°32′
03:00	37°46′	164°12′	062°40′
04:00	25°40′	169°45′	057°07′
05:00	12°57′	172°34′	054°18′
06:00	00°00′	173°26′	053°26′
Scorpio			
00:00	48°32′	110°34′	110°34′
01:00	46°05′	129°26′	091°43′
02:00	39°28′	144°41′	076°27′
03:00	30°03′	155°36′	065°33′
04:00	18°58′	163°03′	058°06′
05:00	06°54′	168°00′	053°09′
05:33	00°00′	169°54′	051°15′
Sagittarius			
00:00	39°51′	102°14′	102°14′
01:00	37°49′	118°43′	085°45′
02:00	32°08′	132°59′	071°28′
03:00	23°45′	144°13′	060°14′
04:00	13°33′	152°43′	051°44′
05:00	02°11′	159°04′	045°23′
05:11	00°00′	160°03′	044°24′
Capricorn			
00:00	36°34′	090°00′	090°00′
01:00	34°40′	105°49′	074°11′
02:00	29°18′	119°46′	060°14′
03:00	21°17′	131°05′	048°55′
04:00	11°27′	139°56′	040°04′
05:00	00°23′	146°47′	033°13′
05:02	00°00′	146°59′	033°01′
Aquarius			
00:00	39°51′	077°46′	077°46′
01:00	37°49′	094°15′	061°17′
02:00	32°08′	108°32′	047°01′
03:00	23°45′	119°46′	035°47′
04:00	13°33′	128°16′	027°17′
05:00	02°11′	134°37′	020°56′
05:11	00°00′	135°36′	019°57′
Pisces			
00:00	48°32′	069°26′	069°26′
01:00	46°05′	088°17′	050°34′
02:00	39°28′	103°33′	035°19′
03:00	30°03′	114°27′	024°24′
04:00	18°58′	121°54′	016°57′
05:00	06°54′	126°51′	012°00′
05:33	00°00′	128°45′	010°06′

Table 2.21: Ecliptic altitude and parallactic angle for latitude +30°.

Aries			
00:00	50°00′	066°34′	066°34′
01:00	47°44′	083°43′	049°25′
02:00	41°34′	097°21′	035°47′
03:00	32°48′	106°41′	026°27′
04:00	22°31′	112°28′	020°40′
05:00	11°26′	115°35′	017°33′
06:00	00°00′	116°34′	016°34′
Taurus			
00:00	61°28′	069°26′	069°26′
01:00	58°32′	091°45′	047°06′
02:00	51°05′	106°59′	031°52′
03:00	41°12′	115°28′	023°23′
04:00	30°13′	119°34′	019°17′
05:00	18°47′	120°50′	018°01′
06:00	07°21′	120°00′	018°51′
06:39	00°00′	118°26′	020°25′
Gemini			
00:00	70°09′	077°46′	077°46′
01:00	66°21′	107°24′	048°09′
02:00	57°35′	123°23′	032°10′
03:00	46°53′	130°11′	025°21′
04:00	35°31′	132°22′	023°11′
05:00	24°03′	131°54′	023°39′
06:00	12°47′	129°33′	026°00′
07:00	02°01′	125°32′	030°00′
07:11	00°00′	124°34′	030°59′
Cancer			
00:00	73°26′	090°00′	090°00′
01:00	69°09′	123°52′	056°08′
02:00	59°48′	139°36′	040°24′
03:00	48°49′	145°21′	034°39′
04:00	37°23′	146°36′	033°24′
05:00	25°57′	145°23′	034°37′
06:00	14°49′	142°24′	037°36′
07:00	04°14′	137°54′	042°06′
07:25	00°00′	135°32′	044°28′
Leo			
00:00	70°09′	102°14′	102°14′
01:00	66°21′	131°51′	072°36′
02:00	57°35′	147°50′	056°37′
03:00	46°53′	154°39′	049°49′
04:00	35°31′	156°49′	047°38′
05:00	24°03′	156°21′	048°06′
06:00	12°47′	154°00′	050°27′
07:00	02°01′	150°00′	054°28′
07:11	00°00′	149°01′	055°26′
Virgo			
00:00	61°28′	110°34′	110°34′
01:00	58°32′	132°54′	088°15′
02:00	51°05′	148°08′	073°01′
03:00	41°12′	156°37′	064°32′
04:00	30°13′	160°43′	060°26′
05:00	18°47′	161°59′	059°10′
06:00	07°21′	161°09′	060°00′
06:39	00°00′	159°35′	061°34′
Libra			
00:00	50°00′	113°26′	113°26′
01:00	47°44′	130°35′	096°17′
02:00	41°34′	144°13′	082°39′
03:00	32°48′	153°33′	073°19′
04:00	22°31′	159°20′	067°32′
05:00	11°26′	162°27′	064°25′
06:00	00°00′	163°26′	063°26′
Scorpio			
00:00	38°32′	110°34′	110°34′
01:00	36°41′	124°53′	096°16′
02:00	31°29′	137°16′	083°53′
03:00	23°46′	146°52′	074°17′
04:00	14°20′	153°47′	067°22′
05:00	03°49′	158°26′	062°42′
05:20	00°00′	159°35′	061°34′
Sagittarius			
00:00	29°51′	102°14′	102°14′
01:00	28°15′	115°14′	089°13′
02:00	23°40′	126°57′	077°30′
03:00	16°41′	136°40′	067°47′
04:00	07°57′	144°17′	060°10′
04:48	00°00′	149°01′	055°26′
Capricorn			
00:00	26°34′	090°00′	090°00′
01:00	25°03′	102°38′	077°22′
02:00	20°41′	114°10′	065°50′
03:00	13°58′	123°56′	056°04′
04:00	05°30′	131°48′	048°12′
04:34	00°00′	135°32′	044°28′
Aquarius			
00:00	29°51′	077°46′	077°46′
01:00	28°15′	090°47′	064°46′
02:00	23°40′	102°30′	053°03′
03:00	16°41′	112°13′	043°20′
04:00	07°57′	119°50′	035°43′
04:48	00°00′	124°34′	030°59′
Pisces			
00:00	38°32′	069°26′	069°26′
01:00	36°41′	083°44′	055°07′
02:00	31°29′	096°07′	042°44′
03:00	23°46′	105°43′	033°08′
04:00	14°20′	112°38′	026°13′
05:00	03°49′	117°18′	021°34′
05:20	00°00′	118°26′	020°25′

Table 2.22: Ecliptic altitude and parallactic angle for latitude $+40°$.

Aries			
00:00	40°00′	066°34′	066°34′
01:00	38°23′	078°49′	054°19′
02:00	33°50′	089°20′	043°48′
03:00	27°02′	097°15′	035°53′
04:00	18°45′	102°34′	030°34′
05:00	09°35′	105°36′	027°32′
06:00	00°00′	106°34′	026°34′
Taurus			
00:00	51°28′	069°26′	069°26′
01:00	49°32′	084°17′	054°34′
02:00	44°15′	096°05′	042°46′
03:00	36°43′	103°58′	034°53′
04:00	27°52′	108°27′	030°24′
05:00	18°23′	110°17′	028°34′
06:00	08°46′	110°00′	028°51′
06:55	00°00′	108°01′	030°50′
Gemini			
00:00	60°09′	077°46′	077°46′
01:00	57°51′	096°00′	059°33′
02:00	51°51′	109°08′	046°25′
03:00	43°40′	116°42′	038°50′
04:00	34°26′	120°14′	035°19′
05:00	24°50′	120°57′	034°36′
06:00	15°18′	119°34′	035°59′
07:00	06°11′	116°25′	039°08′
07:43	00°00′	113°05′	042°27′
Cancer			
00:00	63°26′	090°00′	090°00′
01:00	60°58′	110°03′	069°57′
02:00	54°38′	123°43′	056°17′
03:00	46°12′	131°02′	048°58′
04:00	36°50′	134°04′	045°56′
05:00	27°13′	134°17′	045°43′
06:00	17°44′	132°27′	047°33′
07:00	08°45′	128°55′	051°05′
08:00	00°34′	123°50′	056°10′
08:04	00°00′	123°24′	056°36′
Leo			
00:00	60°09′	102°14′	102°14′
01:00	57°51′	120°27′	084°00′
02:00	51°51′	133°35′	070°52′
03:00	43°40′	141°10′	063°18′
04:00	34°26′	144°41′	059°46′
05:00	24°50′	145°24′	059°03′
06:00	15°18′	144°01′	060°26′
07:00	06°11′	140°52′	063°35′
07:43	00°00′	137°33′	066°55′
Virgo			
00:00	51°28′	110°34′	110°34′
01:00	49°32′	125°26′	095°43′
02:00	44°15′	137°14′	083°55′
03:00	36°43′	145°07′	076°02′
04:00	27°52′	149°36′	071°33′
05:00	18°23′	151°26′	069°43′
06:00	08°46′	151°09′	070°00′
06:55	00°00′	149°10′	071°59′
Libra			
00:00	40°00′	113°26′	113°26′
01:00	38°23′	125°41′	101°11′
02:00	33°50′	136°12′	090°40′
03:00	27°02′	144°07′	082°45′
04:00	18°45′	149°26′	077°26′
05:00	09°35′	152°28′	074°24′
06:00	00°00′	153°26′	073°26′
Scorpio			
00:00	28°32′	110°34′	110°34′
01:00	27°08′	121°21′	099°48′
02:00	23°09′	131°02′	090°07′
03:00	17°03′	138°58′	082°11′
04:00	09°22′	144°55′	076°14′
05:00	00°37′	148°57′	072°11′
05:04	00°00′	149°10′	071°59′
Sagittarius			
00:00	19°51′	102°14′	102°14′
01:00	18°36′	112°20′	092°07′
02:00	15°00′	121°40′	082°48′
03:00	09°22′	129°39′	074°48′
04:00	02°10′	136°05′	068°22′
04:16	00°00′	137°33′	066°55′
Capricorn			
00:00	16°34′	090°00′	090°00′
01:00	15°22′	099°56′	080°04′
02:00	11°54′	109°10′	070°50′
03:00	06°27′	117°13′	062°47′
03:55	00°00′	123°24′	056°36′
Aquarius			
00:00	19°51′	077°46′	077°46′
01:00	18°36′	087°53′	067°40′
02:00	15°00′	097°12′	058°20′
03:00	09°22′	105°12′	050°21′
04:00	02°10′	111°38′	043°55′
04:16	00°00′	113°05′	042°27′
Pisces			
00:00	28°32′	069°26′	069°26′
01:00	27°08′	080°12′	058°39′
02:00	23°09′	089°53′	048°58′
03:00	17°03′	097°49′	041°02′
04:00	09°22′	103°46′	035°05′
05:00	00°37′	107°49′	031°03′
05:04	00°00′	108°01′	030°50′

Table 2.23: Ecliptic altitude and parallactic angle for latitude $+50°$.

	Aries		
00:00	30°00′	066°34′	066°34′
01:00	28°53′	075°04′	058°04′
02:00	25°40′	082°40′	050°28′
03:00	20°42′	088°46′	044°22′
04:00	14°29′	093°08′	040°00′
05:00	07°26′	095°43′	037°25′
06:00	00°00′	096°34′	036°34′
	Taurus		
00:00	41°28′	069°26′	069°26′
01:00	40°12′	079°11′	059°40′
02:00	36°37′	087°35′	051°17′
03:00	31°15′	093°51′	045°00′
04:00	24°40′	097°53′	040°58′
05:00	17°24′	099°50′	039°01′
06:00	09°55′	099°56′	038°55′
07:00	02°36′	098°20′	040°31′
07:22	00°00′	097°20′	041°31′
	Gemini		
00:00	50°09′	077°46′	077°46′
01:00	48°44′	089°05′	066°27′
02:00	44°49′	098°24′	057°08′
03:00	39°04′	104°52′	050°41′
04:00	32°12′	108°33′	046°59′
05:00	24°49′	109°55′	045°37′
06:00	17°21′	109°22′	046°11′
07:00	10°11′	107°09′	048°23′
08:00	03°39′	103°29′	052°03′
08:37	00°00′	100°29′	055°04′
	Cancer		
00:00	53°26′	090°00′	090°00′
01:00	51°57′	102°07′	077°53′
02:00	47°53′	111°53′	068°07′
03:00	41°58′	118°24′	061°36′
04:00	35°01′	121°55′	058°05′
05:00	27°35′	123°01′	056°59′
06:00	20°09′	122°11′	057°49′
07:00	13°03′	119°43′	060°17′
08:00	06°36′	115°51′	064°09′
09:00	01°09′	110°43′	069°17′
09:14	00°00′	109°17′	070°43′
	Leo		
00:00	50°09′	102°14′	102°14′
01:00	48°44′	113°33′	090°55′
02:00	44°49′	122°52′	081°36′
03:00	39°04′	129°19′	075°08′
04:00	32°12′	133°01′	071°27′
05:00	24°49′	134°23′	070°05′
06:00	17°21′	133°49′	070°38′
07:00	10°11′	131°37′	072°51′
08:00	03°39′	127°57′	076°31′
08:37	00°00′	124°56′	079°31′
	Virgo		
00:00	41°28′	110°34′	110°34′
01:00	40°12′	120°20′	100°49′
02:00	36°37′	128°43′	092°25′
03:00	31°15′	135°00′	086°09′
04:00	24°40′	139°02′	082°07′
05:00	17°24′	140°59′	080°10′
06:00	09°55′	141°05′	080°04′
07:00	02°36′	139°29′	081°40′
07:22	00°00′	138°29′	082°40′
	Libra		
00:00	30°00′	113°26′	113°26′
01:00	28°53′	121°56′	104°56′
02:00	25°40′	129°32′	097°20′
03:00	20°42′	135°38′	091°14′
04:00	14°29′	140°00′	086°52′
05:00	07°26′	142°35′	084°17′
06:00	00°00′	143°26′	083°26′
	Scorpio		
00:00	18°32′	110°34′	110°34′
01:00	17°31′	118°22′	102°46′
02:00	14°36′	125°33′	095°36′
03:00	10°02′	131°37′	089°32′
04:00	04°11′	136°18′	084°51′
04:37	00°00′	138°29′	082°40′
	Sagittarius		
00:00	09°51′	102°14′	102°14′
01:00	08°56′	109°45′	094°42′
02:00	06°13′	116°48′	087°40′
03:00	01°56′	122°57′	081°31′
03:22	00°00′	124°56′	079°31′
	Capricorn		
00:00	06°34′	090°00′	090°00′
01:00	05°40′	097°28′	082°32′
02:00	03°02′	104°30′	075°30′
02:45	00°00′	109°17′	070°43′
	Aquarius		
00:00	09°51′	077°46′	077°46′
01:00	08°56′	085°18′	070°15′
02:00	06°13′	092°20′	063°12′
03:00	01°56′	098°29′	057°03′
03:22	00°00′	100°29′	055°04′
	Pisces		
0:00	18°32′	069°26′	069°26′
01:00	17°31′	077°14′	061°38′
02:00	14°36′	084°24′	054°27′
03:00	10°02′	090°28′	048°23′
04:00	04°11′	095°09′	043°42′
04:37	00°00′	097°20′	041°31′

Table 2.24: Ecliptic altitude and parallactic angle for latitude $+60°$.

Aries

00:00	20°00′	066°34′	066°34′
01:00	19°17′	071°57′	061°11′
02:00	17°14′	076°53′	056°15′
03:00	14°00′	081°00′	052°08′
04:00	09°51′	084°04′	049°04′
05:00	05°05′	085°56′	047°12′
06:00	00°00′	086°34′	046°34′

Taurus

00:00	31°28′	069°26′	069°26′
01:00	30°42′	075°20′	063°31′
02:00	28°30′	080°39′	058°12′
03:00	25°05′	084°55′	053°56′
04:00	20°46′	087°54′	050°58′
05:00	15°53′	089°31′	049°20′
06:00	10°46′	089°48′	049°03′
07:00	05°45′	088°49′	050°02′
08:00	01°06′	086°40′	052°12′
08:15	00°00′	085°55′	052°56′

Gemini

00:00	40°09′	077°46′	077°46′
01:00	39°20′	084°21′	071°12′
02:00	37°00′	090°08′	065°25′
03:00	33°25′	094°37′	060°56′
04:00	28°58′	097°34′	057°59′
05:00	24°00′	098°58′	056°34′
06:00	18°53′	098°58′	056°35′
07:00	13°55′	097°40′	057°52′
08:00	09°23′	095°15′	060°18′
09:00	05°33′	091°50′	063°43′
10:00	02°37′	087°38′	067°55′
11:00	00°46′	082°51′	072°42′
12:00	00°09′	077°46′	077°46′

Cancer

00:00	43°26′	090°00′	090°00′
01:00	42°36′	096°54′	083°06′
02:00	40°12′	102°56′	077°04′
03:00	36°33′	107°31′	072°29′
04:00	32°03′	110°27′	069°33′
05:00	27°04′	111°47′	068°13′
06:00	21°57′	111°38′	068°22′
07:00	17°00′	110°13′	069°47′
08:00	12°31′	107°40′	072°20′
09:00	08°44′	104°10′	075°50′
10:00	05°51′	099°54′	080°06′
11:00	04°03′	095°05′	084°55′
12:00	03°26′	090°00′	090°00′

Leo

00:00	40°09′	102°14′	102°14′
01:00	39°20′	108°48′	095°39′
02:00	37°00′	114°35′	089°52′
03:00	33°25′	119°04′	085°23′
04:00	28°58′	122°01′	082°26′
05:00	24°00′	123°26′	081°02′
06:00	18°53′	123°25′	081°02′
07:00	13°55′	122°08′	082°20′
08:00	09°23′	119°42′	084°45′
09:00	05°33′	116°17′	088°10′
10:00	02°37′	112°05′	092°22′
11:00	00°46′	107°18′	097°09′
12:00	00°09′	102°14′	102°14′

Virgo

00:00	31°28′	110°34′	110°34′
01:00	30°42′	116°29′	104°40′
02:00	28°30′	121°48′	099°21′
03:00	25°05′	126°04′	095°05′
04:00	20°46′	129°02′	092°06′
05:00	15°53′	130°40′	090°29′
06:00	10°46′	130°57′	090°12′
07:00	05°45′	129°58′	091°11′
08:00	01°06′	127°48′	093°20′
08:15	00°00′	127°04′	094°05′

Libra

00:00	20°00′	113°26′	113°26′
01:00	19°17′	118°49′	108°03′
02:00	17°14′	123°45′	103°07′
03:00	14°00′	127°52′	099°00′
04:00	09°51′	130°56′	095°56′
05:00	05°05′	132°48′	094°04′
06:00	00°00′	133°26′	093°26′

Scorpio

00:00	08°32′	110°34′	110°34′
01:00	07°52′	115°42′	105°27′
02:00	05°56′	120°28′	100°40′
03:00	02°53′	124°35′	096°34′
03:44	00°00′	127°04′	094°05′

Pisces

00:00	08°32′	069°26′	069°26′
01:00	07°52′	074°33′	064°18′
02:00	05°56′	079°20′	059°32′
03:00	02°53′	083°26′	055°25′
03:44	00°00′	085°55′	052°56′

Table 2.25: Ecliptic altitude and parallactic angle for latitude $+70°$.

Aries			
00:00	10°00′	066°34′	066°34′
01:00	09°39′	069°11′	063°57′
02:00	08°39′	071°36′	061°32′
03:00	07°03′	073°40′	059°28′
04:00	04°59′	075°15′	057°53′
05:00	02°35′	076°14′	056°54′
06:00	00°00′	076°34′	056°34′
Taurus			
00:00	21°28′	069°26′	069°26′
01:00	21°07′	072°11′	066°40′
02:00	20°04′	074°44′	064°07′
03:00	18°26′	076°52′	061°59′
04:00	16°19′	078°26′	060°25′
05:00	13°53′	079°23′	059°29′
06:00	11°18′	079°38′	059°14′
07:00	08°44′	079°12′	059°39′
08:00	06°21′	078°08′	060°43′
09:00	04°20′	076°30′	062°21′
10:00	02°47′	074°25′	064°26′
11:00	01°48′	072°00′	066°51′
12:00	01°28′	069°26′	069°26′
Gemini			
00:00	30°09′	077°46′	077°46′
01:00	29°47′	080°44′	074°48′
02:00	28°43′	083°27′	072°05′
03:00	27°02′	085°42′	069°51′
04:00	24°53′	087°19′	068°14′
05:00	22°25′	088°14′	067°19′
06:00	19°50′	088°25′	067°08′
07:00	17°17′	087°53′	067°39′
08:00	14°56′	086°44′	068°49′
09:00	12°56′	085°01′	070°32′
10:00	11°25′	082°51′	072°41′
11:00	10°28′	080°24′	075°09′
12:00	10°09′	077°46′	077°46′
Cancer			
00:00	33°26′	090°00′	090°00′
01:00	33°04′	093°04′	086°56′
02:00	31°59′	095°53′	084°07′
03:00	30°17′	098°10′	081°50′
04:00	28°07′	099°49′	080°11′
05:00	25°39′	100°43′	079°17′
06:00	23°03′	100°53′	079°07′
07:00	20°31′	100°19′	079°41′
08:00	18°11′	099°06′	080°54′
09:00	16°12′	097°21′	082°39′
10:00	14°42′	095°09′	084°51′
11:00	13°45′	092°39′	087°21′
12:00	13°26′	090°00′	090°00′
Leo			
00:00	30°09′	102°14′	102°14′
01:00	29°47′	105°12′	099°16′
02:00	28°43′	107°55′	096°33′
03:00	27°02′	110°09′	094°18′
04:00	24°53′	111°46′	092°41′
05:00	22°25′	112°41′	091°46′
06:00	19°50′	112°52′	091°35′
07:00	17°17′	112°21′	092°07′
08:00	14°56′	111°11′	093°16′
09:00	12°56′	109°28′	094°59′
10:00	11°25′	107°19′	097°09′
11:00	10°28′	104°51′	099°36′
12:00	10°09′	102°14′	102°14′
Virgo			
00:00	21°28′	110°34′	110°34′
01:00	21°07′	113°20′	107°49′
02:00	20°04′	115°53′	105°16′
03:00	18°26′	118°01′	103°08′
04:00	16°19′	119°35′	101°34′
05:00	13°53′	120°31′	100°37′
06:00	11°18′	120°46′	100°22′
07:00	08°44′	120°21′	100°48′
08:00	06°21′	119°17′	101°52′
09:00	04°20′	117°39′	103°30′
10:00	02°47′	115°34′	105°35′
11:00	01°48′	113°09′	108°00′
12:00	01°28′	110°34′	110°34′
Libra			
00:00	10°00′	113°26′	113°26′
01:00	09°39′	116°03′	110°49′
02:00	08°39′	118°28′	108°24′
03:00	07°03′	120°32′	106°20′
04:00	04°59′	122°07′	104°45′
05:00	02°35′	123°06′	103°46′
06:00	00°00′	123°26′	103°26′

Table 2.26: Ecliptic altitude and parallactic angle for latitude $+80°$.