Plasma Flow Shifts

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1 Plasma Equilibrium

We have

$$\mathbf{B} = \nabla\phi\times\nabla\psi + I(\psi)\,\nabla\phi = B_0\,R_0\,[f(r)\,\nabla\phi\times\nabla r + g(r)\,\nabla\phi]\,. \tag{1}$$

So

$$\frac{d\psi}{dr} = B_0\,R_0\,f(r), \tag{2}$$

$$I(\psi) = B_0\,R_0\,g(r). \tag{3}$$

We also have

$$|\nabla\phi| = \frac{1}{R}, \tag{4}$$

$$\nabla r\times\nabla\theta\cdot\nabla\phi = \frac{R_0}{r\,R^{\,2}}, \tag{5}$$

$$q(r) = \frac{r\,g}{R_0\,f}. \tag{6}$$

2 Plasma Flows

We have

$$\mathbf{E} = -\nabla\Phi(\psi) + \frac{E_\parallel}{B_0}\,\mathbf{B}, \tag{7}$$

and

$$\mathbf{V}^{\,a} = \mathbf{V}_\perp^{\,a} + \frac{V_\parallel^{\,a}(\psi)}{B_0}\,\mathbf{B}, \tag{8}$$

$$\mathbf{V}_\perp^{\,a} = \frac{\mathbf{E}\times\mathbf{B}}{B^{\,2}} + \frac{\mathbf{B}\times\nabla p_a}{n_a\,e_a\,B^{\,2}}. \tag{9}$$

Furthermore,

$$\omega_E(r) = -\frac{d\Phi}{d\psi}, \tag{10}$$

$$\omega_*^{\,a}(r) = -\frac{T_a}{e_a}\,\frac{d\ln p_a}{d\psi}. \tag{11}$$

It follows that

$$\mathbf{V}_\perp^{\,a} = (\omega_E+\omega_*^{\,a})\left(\frac{B_0}{B}\right)^2\left(\frac{r\,g^{\,2}}{q}\right)\left(-R_0\,\nabla\phi\times\nabla r + \frac{r\,|\nabla r|^{\,2}}{q}\,\nabla\phi\right), \tag{12}$$

$$\frac{V_\parallel^{\,a}}{B_0}\,\mathbf{B} = V_\parallel^{\,a}\left(\frac{r\,g}{q}\,\nabla\phi\times\nabla r + R_0\,g\,\nabla\phi\right). \tag{13}$$

Let

$$\Omega_\theta^{a} = \mathbf{V}^{\,a}\cdot\nabla\theta, \tag{14}$$

$$\Omega_\phi^{a} = \mathbf{V}^{\,a}\cdot\nabla\phi. \tag{15}$$

It follows that

$$q\,\Omega_\theta^{a} = \frac{g\,V_\parallel^{\,a}}{R_0}\left(\frac{R_0}{R}\right)^2 - (\omega_E+\omega_*^{\,a})\,g^{\,2}\left(\frac{R_0}{R}\right)^2\left(\frac{B_0}{B}\right)^2, \tag{16}$$

$$\Omega_\phi^{a} = \frac{g\,V_\parallel^{\,a}}{R_0}\left(\frac{R_0}{R}\right)^2 + (\omega_E+\omega_*^{\,a})\,g^{\,2}\left(\frac{R_0}{R}\right)^2\left(\frac{B_0}{B}\right)^2\left(\frac{r}{R_0\,q}\right)^2|\nabla r|^{\,2}. \tag{17}$$

Let

$$\omega_\parallel^{a} = \frac{g\,V_\parallel^{\,a}}{R_0}, \tag{18}$$

$$C_1 = \left\langle\left(\frac{R_0}{R}\right)^2\right\rangle, \tag{19}$$

$$C_2 = g^{\,2}\left\langle\left(\frac{R_0}{R}\right)^2\left(\frac{B_0}{B}\right)^2\right\rangle = \left\langle\left[1+\left(\frac{r}{R_0\,q}\right)^2|\nabla r|^{\,2}\right]^{-1}\right\rangle. \tag{20}$$

It follows that

$$q\,\langle\Omega_\theta^{a}\rangle = C_1\,\omega_\parallel^{a} - C_2\,(\omega_E+\omega_*^{\,a}), \tag{21}$$

$$\langle\Omega_\phi^{a}\rangle = C_1\,\omega_\parallel^{a} + (1-C_2)\,(\omega_E+\omega_*^{\,a}). \tag{22}$$

Thus,

$$\Delta\omega_E = \Delta\Omega_\phi^{a} - q\,\Delta\Omega_\theta^{a}, \tag{23}$$

$$\Delta\omega_\parallel^{\,a} = \frac{C_2\,\Delta\Omega_\phi^a + (1-C_2)\,q\,\Delta\Omega_\theta^a}{C_1}. \tag{24}$$

Finally,

$$\Delta V_\parallel^{\,a} = \frac{R_0}{g}\,\Delta\omega_\parallel^{\,a}, \tag{25}$$

$$\Delta V_{E\,\theta} = -C_2\,\frac{r}{q}\,\Delta\omega_E. \tag{26}$$