

Plasma Flow Shifts

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1 Plasma Equilibrium

We have

$$\mathbf{B} = \nabla\phi \times \nabla\psi + I(\psi) \nabla\phi = B_0 R_0 [f(r) \nabla\phi \times \nabla r + g(r) \nabla\phi]. \quad (1)$$

So

$$\frac{d\psi}{dr} = B_0 R_0 f(r), \quad (2)$$

$$I(\psi) = B_0 R_0 g(r). \quad (3)$$

We also have

$$|\nabla\phi| = \frac{1}{R}, \quad (4)$$

$$\nabla r \times \nabla\theta \cdot \nabla\phi = \frac{R_0}{r R^2}, \quad (5)$$

$$q(r) = \frac{r g}{R_0 f}. \quad (6)$$

2 Plasma Flows

We have

$$\mathbf{E} = -\nabla\Phi(\psi) + \frac{E_{\parallel}}{B_0} \mathbf{B}, \quad (7)$$

and

$$\mathbf{V}^a = \mathbf{V}_{\perp}^a + \frac{V_{\parallel}^a(\psi)}{B_0} \mathbf{B}, \quad (8)$$

$$\mathbf{V}_{\perp}^a = \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{\mathbf{B} \times \nabla p_a}{n_a e_a B^2}. \quad (9)$$

Furthermore,

$$\omega_E(r) = -\frac{d\Phi}{d\psi}, \quad (10)$$

$$\omega_*^a(r) = -\frac{T_a}{e_a} \frac{d \ln p_a}{d\psi}. \quad (11)$$

It follows that

$$\mathbf{V}_\perp^a = (\omega_E + \omega_*^a) \left(\frac{B_0}{B} \right)^2 \left(\frac{r g^2}{q} \right) \left(-R_0 \nabla \phi \times \nabla r + \frac{r |\nabla r|^2}{q} \nabla \phi \right), \quad (12)$$

$$\frac{V_\parallel^a}{B_0} \mathbf{B} = V_\parallel^a \left(\frac{r g}{q} \nabla \phi \times \nabla r + R_0 g \nabla \phi \right). \quad (13)$$

Let

$$\Omega_\theta^a = \mathbf{V}^a \cdot \nabla \theta, \quad (14)$$

$$\Omega_\phi^a = \mathbf{V}^a \cdot \nabla \phi. \quad (15)$$

It follows that

$$q \Omega_\theta^a = \frac{g V_\parallel^a}{R_0} \left(\frac{R_0}{R} \right)^2 - (\omega_E + \omega_*^a) g^2 \left(\frac{R_0}{R} \right)^2 \left(\frac{B_0}{B} \right)^2, \quad (16)$$

$$\Omega_\phi^a = \frac{g V_\parallel^a}{R_0} \left(\frac{R_0}{R} \right)^2 + (\omega_E + \omega_*^a) g^2 \left(\frac{R_0}{R} \right)^2 \left(\frac{B_0}{B} \right)^2 \left(\frac{r}{R_0 q} \right)^2 |\nabla r|^2. \quad (17)$$

Let

$$\omega_\parallel^a = \frac{g V_\parallel^a}{R_0}, \quad (18)$$

$$C_1 = \left\langle \left(\frac{R_0}{R} \right)^2 \right\rangle, \quad (19)$$

$$C_2 = g^2 \left\langle \left(\frac{R_0}{R} \right)^2 \left(\frac{B_0}{B} \right)^2 \right\rangle = \left\langle \left[1 + \left(\frac{r}{R_0 q} \right)^2 |\nabla r|^2 \right]^{-1} \right\rangle. \quad (20)$$

It follows that

$$q \langle \Omega_\theta^a \rangle = C_1 \omega_\parallel^a - C_2 (\omega_E + \omega_*^a), \quad (21)$$

$$\langle \Omega_\phi^a \rangle = C_1 \omega_\parallel^a + (1 - C_2) (\omega_E + \omega_*^a). \quad (22)$$

Thus,

$$\Delta \omega_E = \Delta \Omega_\phi^a - q \Delta \Omega_\theta^a, \quad (23)$$

$$\Delta\omega_{\parallel}^a = \frac{C_2 \Delta\Omega_{\phi}^a + (1 - C_2) q \Delta\Omega_{\theta}^a}{C_1}. \quad (24)$$

Finally,

$$\Delta V_{\parallel}^a = \frac{R_0}{g} \Delta\omega_{\parallel}^a, \quad (25)$$

$$\Delta V_{E\theta} = -C_2 \frac{r}{q} \Delta\omega_E, \quad (26)$$

$$\Delta E_r = \frac{d\psi}{dr} \Delta\omega_E. \quad (27)$$