Neoclassical Physics

1 Lowest-Order Force Balance

Let

$$\vec{B} = I(\psi)\,\vec{\nabla}\zeta + \vec{\nabla}\zeta \times \vec{\nabla}\psi, \tag{1}$$

$$\vec{E} = -\vec{\nabla}\Phi(\psi) + E_{\parallel}\,\vec{b}, \tag{2}$$

where $\vec{b} = \vec{B}/B$. Expect $n_a = n_a(\psi)$, $p_a = p_a(\psi)$, and $T_a = T_a(\psi)$. We can write

$$\vec{u}_j^a = u_{\parallel j}^a\,\vec{b} + \vec{u}_{\perp j}^a, \tag{3}$$

and

$$\vec{u}_0^a = \vec{V}_a, \tag{4}$$

$$\vec{u}_1^a = -\frac{2}{5}\frac{\vec{q}_a}{p_a}, \tag{5}$$

with

$$\vec{u}_{\perp 0}^a = \frac{\vec{E} \times \vec{B}}{B^2} + \frac{\vec{B} \times \vec{\nabla}p_a}{n_a\,e_a\,B^2}, \tag{6}$$

$$\vec{u}_{\perp 1}^a = -\frac{\vec{B} \times \vec{\nabla}T_a}{e_a\,B^2}. \tag{7}$$

It follows that

$$u_{\theta j}^a = \frac{u_{\parallel j}^a}{B} - \frac{V_{\psi j}^a}{B^2}, \tag{8}$$

where

$$u_{\theta j}^a = \frac{\vec{u}_j^a \cdot \vec{\nabla}\theta}{\vec{B} \cdot \vec{\nabla}\theta}, \tag{9}$$

$$V_{\psi 0}^a(\psi) = -\left(\frac{\vec{u}_{\perp 0} \cdot \vec{\nabla}\theta}{\vec{B} \cdot \vec{\nabla}\theta}\right)B^2 = -\frac{I\,T_a}{e_a}\left(\frac{d\ln p_a}{d\psi} + \frac{e_a}{T_a}\frac{d\Phi}{d\psi}\right), \tag{10}$$

$$V_{\psi 1}^a(\psi) = -\left(\frac{\vec{u}_{\perp 1} \cdot \vec{\nabla}\theta}{\vec{B} \cdot \vec{\nabla}\theta}\right)B^2 = \frac{I\,T_a}{e_a}\frac{d\ln T_a}{d\psi}. \tag{11}$$

2 Friction Forces

$$\begin{pmatrix} \vec{F}_{a\,0} \\ \vec{F}_{a\,1} \end{pmatrix} = \sum_b \begin{pmatrix} l_{00}^{ab}, & l_{01}^{ab} \\ l_{10}^{ab}, & l_{11}^{ab} \end{pmatrix} \begin{pmatrix} \vec{u}_0^b \\ \vec{u}_1^b \end{pmatrix}, \tag{12}$$

$$l_{jk}^{ab} = \left(\sum_c \frac{n_a\, m_a}{\tau_{ac}} M_{ac}^{jk} \right) \delta_{ab} + \frac{n_a\, m_a}{\tau_{ab}} N_{ab}^{jk}, \tag{13}$$

$$\frac{1}{\tau_{ab}} = \frac{4}{3\sqrt{\pi}} \frac{4\pi\, n_b\, e_a^2\, e_b^2\, \ln\Lambda}{(4\pi\,\epsilon_0)^2\, m_a^2\, v_{T\,a}^3}, \tag{14}$$

$$v_{T\,a} = \sqrt{\frac{2\,T_a}{m_a}}, \tag{15}$$

$$M_{ab}^{00} = -\frac{1 + m_a/m_b}{\left(1 + x_{ab}^2\right)^{3/2}}, \tag{16}$$

$$M_{ab}^{01} = -\frac{3}{2} \frac{1 + m_a/m_b}{\left(1 + x_{ab}^2\right)^{5/2}} = M_{ab}^{10}, \tag{17}$$

$$M_{ab}^{11} = -\frac{13/4 + 4\,x_{ab}^2 + (15/2)\,x_{ab}^4}{\left(1 + x_{ab}^2\right)^{5/2}}, \tag{18}$$

$$N_{ab}^{00} = \frac{1 + m_a/m_b}{\left(1 + x_{ab}^2\right)^{3/2}}, \tag{19}$$

$$N_{ab}^{01} = \frac{3}{2} \frac{T_a}{T_b} \frac{1 + m_b/m_a}{x_{ab}\left(1 + x_{ba}^2\right)^{5/2}}, \tag{20}$$

$$N_{ab}^{10} = \frac{3}{2} \frac{1 + m_a/m_b}{\left(1 + x_{ab}^2\right)^{5/2}}, \tag{21}$$

$$N_{ab}^{11} = \frac{27}{4} \frac{T_a}{T_b} \frac{x_{ab}^2}{\left(1 + x_{ab}^2\right)^{5/2}}, \tag{22}$$

where

$$x_{ab} = \frac{v_{T\,b}}{v_{T\,a}}. \tag{23}$$

Species are e, i, and I. Let

$$Z_{\text{eff}} = \frac{n_i\, e_i^2 + n_I\, e_I^2}{n_e\, e_e^2}. \tag{24}$$

Follows that

$$\frac{n_e}{n_i} = \frac{Z_I - 1}{Z_I - Z_{\text{eff}}}, \tag{25}$$

$$\frac{n_I}{n_i} = \frac{Z_{\text{eff}} - 1}{Z_I\,(Z_I - Z_{\text{eff}})}, \tag{26}$$

$$\frac{n_I}{n_e} = \frac{Z_{\text{eff}} - 1}{Z_I (Z_I - 1)}. \tag{27}$$

Thus,

$$M_{aa}^{00} = -\frac{1}{\sqrt{2}}, \tag{28}$$

$$M_{aa}^{01} = -\frac{3}{4\sqrt{2}} = M_{aa}^{10}, \tag{29}$$

$$M_{aa}^{11} = -\frac{59}{16\sqrt{2}}, \tag{30}$$

$$N_{aa}^{00} = \frac{1}{\sqrt{2}}, \tag{31}$$

$$N_{aa}^{01} = \frac{3}{4\sqrt{2}}, \tag{32}$$

$$N_{aa}^{10} = \frac{3}{4\sqrt{2}}, \tag{33}$$

$$N_{aa}^{11} = \frac{27}{16\sqrt{2}}, \tag{34}$$

for $a = e, i, I$. Now,

$$M_{iI}^{00} = -\frac{1 + m_i/m_I}{(1 + x_{iI}^2)^{3/2}}, \tag{35}$$

$$M_{iI}^{01} = -\frac{3}{2} \frac{1 + m_i/m_I}{(1 + x_{iI}^2)^{5/2}} = M_{iI}^{10}, \tag{36}$$

$$M_{iI}^{11} = -\frac{13/4 + 4\, x_{iI}^2 + (15/2)\, x_{iI}^4}{(1 + x_{iI}^2)^{5/2}}, \tag{37}$$

$$N_{iI}^{00} = \frac{1 + m_i/m_I}{(1 + x_{iI}^2)^{3/2}}, \tag{38}$$

$$N_{iI}^{01} = \frac{3}{2} \frac{T_i}{T_I} \frac{1 + m_I/m_i}{x_{iI}\,(1 + x_{Ii}^2)^{5/2}}, \tag{39}$$

$$N_{iI}^{10} = \frac{3}{2} \frac{1 + m_i/m_I}{(1 + x_{iI}^2)^{5/2}}, \tag{40}$$

$$N_{iI}^{11} = \frac{27}{4} \frac{T_i}{T_I} \frac{x_{iI}^2}{(1 + x_{iI}^2)^{5/2}}. \tag{41}$$

Furthermore,

$$M_{Ii}^{00} = -\frac{1 + m_I/m_i}{(1 + x_{Ii}^2)^{3/2}}, \tag{42}$$

$$M_{Ii}^{01} = -\frac{3}{2} \frac{1 + m_I/m_i}{(1 + x_{Ii}^2)^{5/2}} = M_{Ii}^{10}, \tag{43}$$

$$M_{Ii}^{11} = -\frac{13/4 + 4\,x_{Ii}^2 + (15/2)\,x_{Ii}^4}{(1 + x_{Ii}^2)^{5/2}}, \tag{44}$$

$$N_{Ii}^{00} = \frac{1 + m_I/m_i}{(1 + x_{Ii}^2)^{3/2}}, \tag{45}$$

$$N_{Ii}^{01} = \frac{3}{2} \frac{T_I}{T_i} \frac{1 + m_i/m_I}{x_{Ii}\,(1 + x_{iI}^2)^{5/2}}, \tag{46}$$

$$N_{Ii}^{10} = \frac{3}{2} \frac{1 + m_I/m_i}{(1 + x_{Ii}^2)^{5/2}}, \tag{47}$$

$$N_{Ii}^{11} = \frac{27}{4} \frac{T_I}{T_i} \frac{x_{Ii}^2}{(1 + x_{Ii}^2)^{5/2}}. \tag{48}$$

Also,

$$M_{ea}^{00} = -1, \tag{49}$$

$$M_{ea}^{01} = -\frac{3}{2} = M_{ea}^{10}, \tag{50}$$

$$M_{ea}^{11} = -\frac{13}{4}, \tag{51}$$

$$N_{ea}^{00} = 1, \tag{52}$$

$$N_{ea}^{01} = 0, \tag{53}$$

$$N_{ea}^{10} = \frac{3}{2}, \tag{54}$$

$$N_{ea}^{11} = 0, \tag{55}$$

for $a = i, I$, where we have neglected terms of order $(m_e/m_i)^{1/2}$. Finally,

$$M_{ae}^{00} = 0, \tag{56}$$

$$M_{ae}^{01} = 0 = M_{ae}^{10}, \tag{57}$$

$$M_{ae}^{11} = 0, \tag{58}$$

$$N_{ae}^{00} = 0, \tag{59}$$

$$N_{ae}^{01} = 0, \tag{60}$$

$$N_{ae}^{10} = 0 \tag{61}$$

$$N_{ae}^{11} = 0, \tag{62}$$

for $a = i, I$.

Now,

$$l_{jk}^{ii} = \frac{n_i \, m_i}{\tau_{ii}} \left[\left(\sum_a \frac{n_a \, e_a^2}{n_i \, e_i^2} \, M_{ia}^{jk} \right) + N_{ii}^{jk} \right] \tag{63}$$

$$= \frac{n_i \, m_i}{\tau_{ii}} \left(M_{ii}^{jk} + N_{ii}^{jk} + \alpha \, M_{iI}^{jk} \right), \tag{64}$$

where

$$\alpha = \frac{n_I \, e_I^2}{n_i \, e_i^2} = (Z_{\text{eff}} - 1) \left(\frac{Z_I}{Z_I - Z_{\text{eff}}} \right). \tag{65}$$

Thus,

$$l_{00}^{ii} = -\frac{n_i \, m_i}{\tau_{ii}} \frac{\alpha \, (1 + m_i/m_I)}{(1 + x_{iI}^2)^{3/2}}, \tag{66}$$

$$l_{01}^{ii} = -\frac{n_i \, m_i}{\tau_{ii}} \frac{3}{2} \frac{\alpha \, (1 + m_i/m_I)}{(1 + x_{iI}^2)^{5/2}}, \tag{67}$$

$$l_{10}^{ii} = -\frac{n_i \, m_i}{\tau_{ii}} \frac{3}{2} \frac{\alpha \, (1 + m_i/m_I)}{(1 + x_{iI}^2)^{5/2}}, \tag{68}$$

$$l_{11}^{ii} = -\frac{n_i \, m_i}{\tau_{ii}} \left\{ \sqrt{2} + \frac{\alpha \, [13/4 + 4 \, x_{iI}^2 + (15/2) \, x_{iI}^4]}{(1 + x_{iI}^2)^{5/2}} \right\}. \tag{69}$$

Now,

$$l_{jk}^{iI} = \frac{n_i \, m_i}{\tau_{ii}} \, \alpha \, N_{iI}^{jk}, \tag{70}$$

so

$$l_{00}^{iI} = \frac{n_i \, m_i}{\tau_{ii}} \frac{\alpha \, (1 + m_i/m_I)}{(1 + x_{iI}^2)^{3/2}}, \tag{71}$$

$$l_{01}^{iI} = \frac{n_i \, m_i}{\tau_{ii}} \frac{3}{2} \frac{T_i}{T_I} \frac{\alpha \, (1 + m_I/m_i)}{x_{iI} \, (1 + x_{Ii}^2)^{5/2}}, \tag{72}$$

$$l_{10}^{iI} = \frac{n_i \, m_i}{\tau_{ii}} \frac{3}{2} \frac{\alpha \, (1 + m_i/m_I)}{(1 + x_{iI}^2)^{5/2}}, \tag{73}$$

$$l_{11}^{iI} = \frac{n_i \, m_i}{\tau_{ii}} \frac{27}{4} \frac{T_i}{T_I} \frac{\alpha \, x_{iI}^2}{(1 + x_{iI}^2)^{5/2}}. \tag{74}$$

Now,

$$l_{jk}^{II} = \frac{n_I \, m_I}{\tau_{II}} \left[\left(\sum_a \frac{n_a \, e_a^2}{n_I \, e_I^2} \, M_{Ia}^{jk} \right) + N_{II}^{jk} \right] \tag{75}$$

$$= \frac{n_I \, m_I}{\tau_{II}} \left(M_{II}^{jk} + N_{II}^{jk} + \alpha^{-1} \, M_{Ii}^{jk} \right)$$

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$$= \frac{n_i \, m_i}{\tau_{ii}} \, \alpha^2 \, \frac{m_i}{m_I} \, x_{Ii}^3 \left(M_{II}^{jk} + N_{II}^{jk} + \alpha^{-1} \, M_{Ii}^{jk} \right), \tag{76}$$

so

$$l_{00}^{II} = -\frac{n_i \, m_i}{\tau_{ii}} \, \frac{m_i}{m_I} \, x_{Ii}^3 \frac{\alpha \, (1 + m_I/m_i)}{(1 + x_{Ii}^2)^{3/2}}, \tag{77}$$

$$l_{01}^{II} = -\frac{n_i \, m_i}{\tau_{ii}} \, \frac{m_i}{m_I} \, x_{Ii}^3 \frac{3}{2} \frac{\alpha \, (1 + m_I/m_i)}{(1 + x_{Ii}^2)^{5/2}}, \tag{78}$$

$$l_{10}^{II} = -\frac{n_i \, m_i}{\tau_{ii}} \, \frac{m_i}{m_I} \, x_{Ii}^3 \frac{3}{2} \frac{\alpha \, (1 + m_I/m_i)}{(1 + x_{Ii}^2)^{5/2}}, \tag{79}$$

$$l_{11}^{II} = -\frac{n_i \, m_i}{\tau_{ii}} \, \frac{m_i}{m_I} \, x_{Ii}^3 \left\{ \sqrt{2} \, \alpha^2 + \frac{\alpha \, [13/4 + 4 \, x_{Ii}^2 + (15/2) \, x_{Ii}^4]}{(1 + x_{Ii}^2)^{5/2}} \right\}. \tag{80}$$

Finally,

$$l_{jk}^{Ii} = \frac{n_i \, m_i}{\tau_{ii}} \, \alpha \, \frac{m_i}{m_I} \, x_{Ii}^3 \, N_{Ii}^{jk}, \tag{81}$$

so

$$l_{00}^{Ii} = \frac{n_i \, m_i}{\tau_{ii}} \, \frac{m_i}{m_I} \, x_{Ii}^3 \frac{\alpha \, (1 + m_I/m_i)}{(1 + x_{Ii}^2)^{3/2}}, \tag{82}$$

$$l_{01}^{Ii} = \frac{n_i \, m_i}{\tau_{ii}} \, \frac{m_i}{m_I} \, x_{Ii}^3 \frac{3}{2} \frac{T_I}{T_i} \frac{\alpha \, (1 + m_i/m_I)}{x_{Ii} \, (1 + x_{iI}^2)^{5/2}}, \tag{83}$$

$$l_{10}^{Ii} = \frac{n_i \, m_i}{\tau_{ii}} \, \frac{m_i}{m_I} \, x_{Ii}^3 \frac{3}{2} \frac{\alpha \, (1 + m_I/m_i)}{(1 + x_{Ii}^2)^{5/2}}, \tag{84}$$

$$l_{11}^{Ii} = \frac{n_i \, m_i}{\tau_{ii}} \, \frac{m_i}{m_I} \, x_{Ii}^3 \frac{27}{4} \frac{T_I}{T_i} \frac{\alpha \, x_{Ii}^2}{(1 + x_{Ii}^2)^{5/2}}. \tag{85}$$

Thus,

$$(F^i) = \frac{n_i \, m_i}{\tau_{ii}} \left\{ - \left[F^{ii} \right] (u^i) + \left[F^{iI} \right] (u^I) \right\}, \tag{86}$$

$$(F^I) = \frac{n_i \, m_i}{\tau_{ii}} \left\{ \left[F^{Ii} \right] (u^i) - \left[F^{II} \right] (u^I) \right\}, \tag{87}$$

where

$$(F^a) = \begin{pmatrix} \vec{F}_{a\,0} \\ \vec{F}_{a\,1} \end{pmatrix}, \tag{88}$$

$$(u^a) = \begin{pmatrix} \vec{u}_0^a \\ \vec{u}_1^a \end{pmatrix}. \tag{89}$$

Here,

$$F_{00}^{ii} = \frac{\alpha \, (1 + m_i/m_I)}{(1 + x_{iI})^{3/2}}, \tag{90}$$

$$F_{01}^{ii} = \frac{3}{2} \frac{\alpha \left(1 + m_i/m_I\right)}{\left(1 + x_{iI}^2\right)^{5/2}}, \tag{91}$$

$$F_{10}^{ii} = \frac{3}{2} \frac{\alpha \left(1 + m_i/m_I\right)}{\left(1 + x_{iI}^2\right)^{5/2}} = F_{01}^{ii}, \tag{92}$$

$$F_{11}^{ii} = \sqrt{2} + \frac{\alpha \left[13/4 + 4\, x_{iI}^2 + (15/2)\, x_{iI}^4\right]}{\left(1 + x_{iI}^2\right)^{5/2}}, \tag{93}$$

$$F_{00}^{iI} = \frac{\alpha \left(1 + m_i/m_I\right)}{\left(1 + x_{iI}^2\right)^{3/2}} = F_{00}^{ii}, \tag{94}$$

$$F_{01}^{iI} = \frac{3}{2} \frac{T_i}{T_I} \frac{\alpha \left(1 + m_I/m_i\right)}{x_{iI}\left(1 + x_{Ii}^2\right)^{5/2}}, \tag{95}$$

$$F_{10}^{iI} = \frac{3}{2} \frac{\alpha \left(1 + m_i/m_I\right)}{\left(1 + x_{iI}^2\right)^{5/2}} = F_{01}^{ii}, \tag{96}$$

$$F_{11}^{iI} = \frac{27}{4} \frac{T_i}{T_I} \frac{\alpha\, x_{iI}^2}{\left(1 + x_{iI}^2\right)^{5/2}}, \tag{97}$$

$$F_{00}^{Ii} = \frac{m_i}{m_I} x_{Ii}^3 \frac{\alpha \left(1 + m_I/m_i\right)}{\left(1 + x_{Ii}^2\right)^{3/2}} = \frac{\alpha \left(1 + m_i/m_I\right)}{\left(1 + x_{iI}^2\right)^{3/2}} = F_{00}^{ii}, \tag{98}$$

$$F_{01}^{Ii} = \frac{m_i}{m_I} x_{Ii}^3 \frac{3}{2} \frac{T_I}{T_i} \frac{\alpha \left(1 + m_I/m_I\right)}{x_{Ii}\left(1 + x_{iI}^2\right)^{5/2}} = \frac{3}{2} \frac{\alpha \left(1 + m_i/m_I\right)}{\left(1 + x_{iI}^2\right)^{5/2}} = F_{01}^{ii}, \tag{99}$$

$$F_{10}^{Ii} = \frac{m_i}{m_I} x_{Ii}^3 \frac{3}{2} \frac{\alpha \left(1 + m_I/m_i\right)}{\left(1 + x_{Ii}^2\right)^{5/2}} = \frac{3}{2} \frac{T_i}{T_I} \frac{\alpha \left(1 + m_I/m_i\right)}{x_{iI}\left(1 + x_{Ii}^2\right)^{5/2}} = F_{01}^{iI}, \tag{100}$$

$$F_{11}^{Ii} = \frac{m_i}{m_I} x_{Ii}^3 \frac{27}{4} \frac{T_I}{T_i} \frac{\alpha\, x_{Ii}^2}{\left(1 + x_{Ii}^2\right)^{5/2}} = \frac{27}{4} \frac{\alpha\, x_{iI}^2}{\left(1 + x_{iI}^2\right)^{5/2}}, \tag{101}$$

$$F_{00}^{II} = \frac{m_i}{m_I} x_{Ii}^3 \frac{\alpha \left(1 + m_I/m_i\right)}{\left(1 + x_{Ii}^2\right)^{3/2}} = \frac{\alpha \left(1 + m_i/m_I\right)}{\left(1 + x_{iI}^2\right)^{3/2}} = F_{00}^{ii}, \tag{102}$$

$$F_{01}^{II} = \frac{m_i}{m_I} x_{Ii}^3 \frac{3}{2} \frac{\alpha \left(1 + m_I/m_i\right)}{\left(1 + x_{Ii}^2\right)^{5/2}} = \frac{3}{2} \frac{T_i}{T_I} \frac{\alpha \left(1 + m_I/m_i\right)}{x_{iI}\left(1 + x_{Ii}^2\right)^{5/2}} = F_{01}^{iI}, \tag{103}$$

$$F_{10}^{II} = \frac{m_i}{m_I} x_{Ii}^3 \frac{3}{2} \frac{\alpha \left(1 + m_I/m_i\right)}{\left(1 + x_{Ii}^2\right)^{5/2}} = \frac{3}{2} \frac{T_i}{T_I} \frac{\alpha \left(1 + m_I/m_i\right)}{x_{iI}\left(1 + x_{Ii}^2\right)^{5/2}} = F_{01}^{iI}, \tag{104}$$

$$F_{11}^{II} = \frac{m_i}{m_I} x_{Ii}^3 \left\{ \sqrt{2}\, \alpha^2 + \frac{\alpha \left[13/4 + 4\, x_{Ii}^2 + (15/2)\, x_{Ii}^4\right]}{\left(1 + x_{Ii}^2\right)^{5/2}} \right\}$$

$$= \frac{T_i}{T_I} \left\{ \sqrt{2}\, \alpha^2\, x_{Ii} + \frac{\alpha \left[15/2 + 4\, x_{iI}^2 + (13/4)\, x_{iI}^4\right]}{\left(1 + x_{iI}^2\right)^{5/2}} \right\}. \tag{105}$$

Note that $F_{0j}^{ii} = F_{0j}^{Ii}$ and $F_{0j}^{iI} = F_{0j}^{II}$, for $j = 0, 1$, which implies that $(F^I)_0 + (F^i)_0 = 0$. This is a statement of collisional momentum conservation. Furthermore, $F_{j0}^{ii} = F_{j0}^{iI}$ and $F_{j0}^{Ii} = F_{j0}^{II}$.

Now,

$$l_{jk}^{ee} = \frac{n_e\,m_e}{\tau_{ee}} \left[\left(\sum_a \frac{n_a\,e_a^2}{n_e\,e_e^2}\,M_{ea}^{jk}\right) + N_{ee}^{jk}\right] \tag{106}$$

$$= \frac{n_e\,m_e}{\tau_{ee}} \left(M_{ee}^{jk} + N_{ee}^{jk} + Z_{\text{eff}}\,M_{ei}^{jk}\right), \tag{107}$$

where $a = e,\,i,\,I$. It follows that

$$l_{00}^{ee} = -\frac{n_e\,m_e}{\tau_{ee}}\,Z_{\text{eff}}, \tag{108}$$

$$l_{01}^{ee} = -\frac{n_e\,m_e}{\tau_{ee}}\,\frac{3}{2}\,Z_{\text{eff}}, \tag{109}$$

$$l_{10}^{ee} = -\frac{n_e\,m_e}{\tau_{ee}}\,\frac{3}{2}\,Z_{\text{eff}}, \tag{110}$$

$$l_{11}^{ee} = -\frac{n_e\,m_e}{\tau_{ee}} \left(\sqrt{2} + \frac{13}{4}\,Z_{\text{eff}}\right). \tag{111}$$

Likewise,

$$l_{jk}^{ea} = \frac{n_e\,m_e}{\tau_{ee}}\,Z_{\text{eff}\,a}\,N_{ea}^{jk}, \tag{112}$$

where

$$Z_{\text{eff}\,a} = \frac{n_a\,e_a^2}{n_e\,e_e^2}, \tag{113}$$

for $a = i,\,I$. In particular,

$$Z_{\text{eff}\,i} = \frac{Z_I - Z_{\text{eff}}}{Z_I - 1}, \tag{114}$$

$$Z_{\text{eff}\,I} = \frac{Z_I\,(Z_{\text{eff}} - 1)}{Z_I - 1}. \tag{115}$$

Hence,

$$l_{00}^{ei} = \frac{n_e\,m_e}{\tau_{ee}}\,Z_{\text{eff}\,i}, \tag{116}$$

$$l_{01}^{ei} = 0, \tag{117}$$

$$l_{10}^{ei} = \frac{n_e\,m_e}{\tau_{ee}}\,\frac{3}{2}\,Z_{\text{eff}\,i}, \tag{118}$$

$$l_{11}^{ei} = 0, \tag{119}$$

and

$$l_{00}^{eI} = \frac{n_e\,m_e}{\tau_{ee}}\,Z_{\text{eff}\,I}, \tag{120}$$

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$$l_{01}^{eI} = 0, \tag{121}$$

$$l_{10}^{eI} = \frac{n_e\,m_e}{\tau_{ee}}\,\frac{3}{2}\,Z_{\text{eff}\,I}, \tag{122}$$

$$l_{11}^{eI} = 0. \tag{123}$$

Thus,

$$(F^e) = \frac{n_e\,m_e}{\tau_{ee}}\left\{-\left[F^{ee}\right](u^e) + \left[F^{ei}\right](u^i) + \left[F^{eI}\right](u^I)\right\}, \tag{124}$$

where

$$F_{00}^{ee} = Z_{\text{eff}}, \tag{125}$$

$$F_{01}^{ee} = \frac{3}{2}\,Z_{\text{eff}}, \tag{126}$$

$$F_{10}^{ee} = \frac{3}{2}\,Z_{\text{eff}}, \tag{127}$$

$$F_{11}^{ee} = \sqrt{2} + \frac{13}{4}\,Z_{\text{eff}}, \tag{128}$$

$$F_{00}^{ei} = Z_{\text{eff}\,i}, \tag{129}$$

$$F_{01}^{ei} = 0, \tag{130}$$

$$F_{10}^{ei} = \frac{3}{2}\,Z_{\text{eff}\,i}, \tag{131}$$

$$F_{11}^{ei} = 0, \tag{132}$$

$$F_{00}^{eI} = Z_{\text{eff}\,I}, \tag{133}$$

$$F_{01}^{eI} = 0, \tag{134}$$

$$F_{10}^{eI} = \frac{3}{2}\,Z_{\text{eff}\,I}, \tag{135}$$

$$F_{11}^{eI} = 0. \tag{136}$$

Note that $F_{j0}^{ee} = F_{j0}^{ei} + F_{j0}^{eI}$.

3   Neoclassical Viscosities

$$\left\langle \vec{B} \cdot \vec{\nabla} \cdot (\overleftrightarrow{\Pi}^a) \right\rangle = \frac{n_a\,m_a}{\tau_{aa}}\,[\mu^a]\,(u_\theta^a), \tag{137}$$

where

$$\langle A \rangle = \oint \frac{A(\theta)\,d\theta}{\vec{B}\cdot\vec{\nabla}\theta} \bigg/ \oint \frac{d\theta}{\vec{B}\cdot\vec{\nabla}\theta}, \tag{138}$$

$$(\overleftrightarrow{\Pi}^a) = \begin{pmatrix} \overleftrightarrow{\Pi}_{\|\,a} \\ \overleftrightarrow{\Theta}_{\|\,a} \end{pmatrix}, \tag{139}$$

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$$(u_\theta^a) = \begin{pmatrix} u_{\theta 0}^a \langle B^2 \rangle \\ u_{\theta 1}^a \langle B^2 \rangle \end{pmatrix}, \tag{140}$$

$$[\mu^a] = \begin{pmatrix} \mu_{00}^a, & \mu_{01}^a \\ \mu_{10}^a, & \mu_{11}^a \end{pmatrix}. \tag{141}$$

Let

$$\mu_{00}^a = K_{00}^a, \tag{142}$$

$$\mu_{01}^a = \frac{5}{2} K_{00}^a - K_{01}^a = \mu_{10}^a, \tag{143}$$

$$\mu_{11}^a = K_{11}^a - 5 K_{01}^a + \frac{25}{4} K_{00}^a, \tag{144}$$

and

$$\omega_{ta} = \frac{v_{Ta}}{R_0 q}, \tag{145}$$

$$\nu_{*a} = \frac{8}{3\pi} \frac{\langle B^2 \rangle}{\langle (\vec{b} \cdot \vec{\nabla} B)^2 \rangle} \frac{g \, \omega_{ta}}{v_{Ta}^2 \, \tau_{aa}} \simeq \frac{16}{3\pi} \frac{g}{\epsilon^2 \, \omega_{ta} \, \tau_{aa}}, \tag{146}$$

$$g = \frac{1 - f_c}{f_c}, \tag{147}$$

$$f_c = \frac{3}{4} \langle B^2 \rangle \int_0^{1/B_{\max}} \frac{\lambda \, d\lambda}{\langle \sqrt{1 - \lambda B} \rangle} \simeq 1 - 1.46 \, \epsilon^{1/2} + 0.46 \, \epsilon^{3/2}. \tag{148}$$

We have

$$K_{ij}^a = g \frac{4}{3\sqrt{\pi}} \int_0^\infty \frac{e^{-x} \, x^{4+i+j} \, \hat{\nu}_D^a(x) \, dx}{[x^2 + \nu_{*a} \, \hat{\nu}_D^a(x)] \, [x^2 + (5\pi/8) \, (\omega_{ta} \, \tau_{aa})^{-1} \, \hat{\nu}_T^a(x)]}, \tag{149}$$

and

$$\hat{\nu}_D^a(x) = \frac{3\sqrt{\pi}}{4} \sum_b \frac{n_b \, e_b^2}{n_a \, e_a^2} \left[\left(1 - \frac{x_{ab}}{2x} \right) \psi \left(\frac{x}{x_{ab}} \right) + \psi' \left(\frac{x}{x_{ab}} \right) \right] \frac{1}{x}, \tag{150}$$

$$\hat{\nu}_\epsilon^a(x) = \frac{3\sqrt{\pi}}{2} \sum_b \frac{n_b \, e_b^2}{n_a \, e_a^2} \left[\frac{m_a}{m_b} \psi \left(\frac{x}{x_{ab}} \right) - \psi' \left(\frac{x}{x_{ab}} \right) \right] \frac{1}{x}, \tag{151}$$

$$\hat{\nu}_T^a(x) = 3 \, \hat{\nu}_D^a(x) + \hat{\nu}_\epsilon^a(x), \tag{152}$$

$$\psi(x) = \frac{2}{\sqrt{\pi}} \int_0^{x^2} \sqrt{t} \, e^{-t} \, dt = \mathrm{erf}(x) - \frac{2}{\sqrt{\pi}} x \, e^{-x^2}, \tag{153}$$

$$\psi'(x) = \frac{2}{\sqrt{\pi}} x \, e^{-x^2}. \tag{154}$$

In particular,

$$K_{ij}^i = g \frac{4}{3\sqrt{\pi}} \int_0^\infty \frac{e^{-x} \, x^{2+i+j} \, \hat{\nu}_D^i(x) \, dx}{[x + \nu_{*i} \, \hat{\nu}_D^i(x)] \, [x + (5\pi/8) \, (\omega_{ti} \, \tau_{ii})^{-1} \, \hat{\nu}_T^i(x)]}, \tag{155}$$

$$\hat{\nu}_D^i = \frac{3\sqrt{\pi}}{4}\left[\left(1 - \frac{1}{2\,x}\right)\psi(x) + \psi'(x)\right]\frac{1}{x}$$

$$+ \frac{3\sqrt{\pi}}{4}\,\alpha\left[\left(1 - \frac{x_{iI}}{2\,x}\right)\psi\left(\frac{x}{x_{iI}}\right) + \psi'\left(\frac{x}{x_{iI}}\right)\right]\frac{1}{x}, \tag{156}$$

$$\hat{\nu}_\epsilon^i = \frac{3\sqrt{\pi}}{2}\left[\psi(x) - \psi'(x)\right]\frac{1}{x}$$

$$+ \frac{3\sqrt{\pi}}{2}\,\alpha\left[\frac{m_i}{m_I}\,\psi\left(\frac{x}{x_{iI}}\right) - \psi'\left(\frac{x}{x_{iI}}\right)\right]\frac{1}{x}, \tag{157}$$

and

$$K_{ij}^I = g\,\frac{4}{3\sqrt{\pi}}\int_0^\infty \frac{e^{-x}\,x^{2+i+j}\,\hat{\nu}_D^I(x)\,dx}{\left[x + \nu_{*I}\,\hat{\nu}_D^I(x)\right]\left[x + (5\pi/8)\,(\omega_{tI}\,\tau_{II})^{-1}\,\hat{\nu}_T^I(x)\right]}, \tag{158}$$

$$\hat{\nu}_D^I = \frac{3\sqrt{\pi}}{4}\left[\left(1 - \frac{1}{2\,x}\right)\psi(x) + \psi'(x)\right]\frac{1}{x}$$

$$+ \frac{3\sqrt{\pi}}{4}\,\alpha^{-1}\left[\left(1 - \frac{x_{Ii}}{2\,x}\right)\psi\left(\frac{x}{x_{Ii}}\right) + \psi'\left(\frac{x}{x_{Ii}}\right)\right]\frac{1}{x}, \tag{159}$$

$$\hat{\nu}_\epsilon^I = \frac{3\sqrt{\pi}}{2}\left[\psi(x) - \psi'(x)\right]\frac{1}{x}$$

$$+ \frac{3\sqrt{\pi}}{2}\,\alpha^{-1}\left[\frac{m_I}{m_i}\,\psi\left(\frac{x}{x_{Ii}}\right) - \psi'\left(\frac{x}{x_{Ii}}\right)\right]\frac{1}{x}, \tag{160}$$

and

$$K_{ij}^e = g\,\frac{4}{3\sqrt{\pi}}\int_0^\infty \frac{e^{-x}\,x^{4+i+j}\,\hat{\nu}_D^e(x)\,dx}{\left[x^2 + \nu_{*e}\,\hat{\nu}_D^e(x)\right]\left[x^2 + (5\pi/8)\,(\omega_{te}\,\tau_{ee})^{-1}\,\hat{\nu}_T^e(x)\right]}, \tag{161}$$

$$\hat{\nu}_D^e = \frac{3\sqrt{\pi}}{4}\left[\left(1 - \frac{1}{2\,x}\right)\psi(x) + \psi'(x)\right] + \frac{3\sqrt{\pi}}{4}\,Z_{\text{eff}}, \tag{162}$$

$$\hat{\nu}_\epsilon^e = \frac{3\sqrt{\pi}}{2}\left[\psi(x) - \psi'(x)\right]. \tag{163}$$

4 Parallel Force and Heat Balance Equations

$$\langle \vec{B} \cdot \vec{\nabla} \cdot \overleftrightarrow{\Pi}_a \rangle = \langle \vec{B} \cdot \vec{F}_{a\,0} \rangle + \langle \vec{B} \cdot \vec{F}_{a\,0}^{cx} \rangle + n_a\,e_a\,\langle \vec{E} \cdot \vec{B} \rangle, \tag{164}$$

$$\langle \vec{B} \cdot \vec{\nabla} \cdot \overleftrightarrow{\Theta}_a \rangle = \langle \vec{B} \cdot \vec{F}_{a\,1} \rangle + \langle \vec{B} \cdot \vec{F}_{a\,1}^{cx} \rangle. \tag{165}$$

Now,

$$\langle \vec{B} \cdot \vec{F}^i \rangle = \frac{n_i\,m_i}{\tau_{ii}}\left\{-\left[F^{ii}\right](u_\parallel^i) + \left[F^{iI}\right](u_\parallel^I)\right\}, \tag{166}$$

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$$\langle \vec{B} \cdot \vec{F}^I \rangle = \frac{n_i\, m_i}{\tau_{ii}} \left\{ [F^{Ii}]\,(u_\parallel^i) - [F^{II}]\,(u_\parallel^I) \right\}, \tag{167}$$

$$\langle \vec{B} \cdot \vec{F}^e \rangle = \frac{n_e\, m_e}{\tau_{ee}} \left\{ -[F^{ee}]\,(u_\parallel^e) + [F^{ei}]\,(u_\parallel^i) + [F^{eI}]\,(u_\parallel^I) \right\}, \tag{168}$$

$$\langle \vec{B} \cdot \vec{F}^{i\,cx} \rangle = -n_i\, m_i\, \langle \sigma\, v \rangle_i^{cx}\, [X^i](\tilde{u}_\parallel^i), \tag{169}$$

$$\langle \vec{B} \cdot \vec{F}^{I\,cx} \rangle = 0, \tag{170}$$

$$\langle \vec{B} \cdot \vec{F}^{e\,cx} \rangle = 0, \tag{171}$$

where

$$(u_\parallel^a) = \begin{pmatrix} \langle u_{\parallel 0}^a\, B \rangle \\ \langle u_{\parallel 1}^a\, B \rangle \end{pmatrix}, \tag{172}$$

$$(\tilde{u}_\parallel^a) = \begin{pmatrix} \langle n_n\, u_{\parallel 0}^a\, B \rangle \\ \langle n_n\, u_{\parallel 1}^a\, B \rangle \end{pmatrix}, \tag{173}$$

$$[X^i] = \begin{pmatrix} 1, & 0 \\ 0, & E_n/T_i \end{pmatrix}. \tag{174}$$

Here, E_n is the neutral kinetic energy, n_n the neutral number density, and $\langle \sigma\, v \rangle_i^{cx}$ the majority ion charge exchange rate coefficient.

Let

$$(E_\parallel) = \begin{pmatrix} \langle E_\parallel\, B \rangle / \langle B \rangle \\ 0 \end{pmatrix}, \tag{175}$$

$$\Omega_a = \frac{e_a\, \langle B \rangle}{m_a}, \tag{176}$$

$$[\tilde{\mu}^I] = \alpha^2\, \frac{m_i}{m_I}\, x_{Ii}^3\, [\mu^I] = \alpha^2\, \frac{T_i}{T_I}\, x_{Ii}\, [\mu^I]. \tag{177}$$

Follows that

$$\begin{bmatrix} [F^{ii}], & -[F^{iI}] \\ -[F^{Ii}], & [F^{II}] \end{bmatrix} \begin{bmatrix} (u_\parallel^i) \\ (u_\parallel^I) \end{bmatrix} = - \begin{bmatrix} [\mu^i]\,(u_\theta^i) \\ [\tilde{\mu}^I]\,(u_\theta^I) \end{bmatrix} + \begin{bmatrix} \tau_{ii}\, \langle \sigma\, v \rangle_i^{cx}\, [X^i]\,(\tilde{u})_{\parallel i}, \\ 0 \end{bmatrix}, \tag{178}$$

and

$$[\mu^e]\,(u_\theta^e) = -[F^{ee}]\,(u_\parallel^e) + [F^{ei}]\,(u_\parallel^i) + [F^{eI}]\,(u_\parallel^I) + \Omega_e\, \tau_{ee}\,(E_\parallel). \tag{179}$$

Follows from Eq. (8) that

$$(u_\parallel^a) = (u_\theta^a) + (V_\psi^a). \tag{180}$$

where

$$V_\psi^a = \begin{pmatrix} V_{\psi 0}^a \\ V_{\psi 1}^a \end{pmatrix}. \tag{181}$$

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Thus,

$$
\begin{bmatrix} [F^{ii} + \mu^i + Y^i/y], & -[F^{iI}] \\ -[F^{Ii}], & [F^{II} + \tilde{\mu}^I] \end{bmatrix} \begin{bmatrix} (u_\theta^i) \\ (u_\theta^I) \end{bmatrix} = - \begin{bmatrix} [F^{ii} + Y^i], & -[F^{iI}] \\ -[F^{Ii}], & [F^{II}] \end{bmatrix} \begin{bmatrix} (V_\psi^i) \\ (V_\psi^I) \end{bmatrix},
$$
(182)

where

$$
[Y^i] = \tau_{ii} \langle \sigma v \rangle_i^{cx} \langle n_n \rangle [X^i],
$$
(183)

$$
y = \frac{\langle n_n \rangle \langle B^2 \rangle}{\langle n_n B^2 \rangle}.
$$
(184)

Can write

$$
(V_\psi^a) = (V_*^a) + (V_E),
$$
(185)

where

$$
V_{*0}^a(\psi) = -\frac{I\,T_a}{e_a} \frac{d\ln p_a}{d\psi},
$$
(186)

$$
V_{*1}^a(\psi) = \frac{I\,T_a}{e_a} \frac{d\ln T_a}{d\psi},
$$
(187)

$$
V_{E0}(\psi) = -I \frac{d\Phi}{d\psi},
$$
(188)

$$
V_{E1}(\psi) = 0.
$$
(189)

Fact that $F_{j0}^{ii} = F_{j0}^{iI}$ and $F_{j0}^{Ii} = F_{j0}^{II}$ means that we can rewrite Eq. (182) as

$$
\begin{bmatrix} [F^{ii} + \mu^i + Y^i/y], & -[F^{iI}] \\ -[F^{Ii}], & [F^{II} + \tilde{\mu}^I] \end{bmatrix} \begin{bmatrix} (u_\theta^i) \\ (u_\theta^I) \end{bmatrix} = - \begin{bmatrix} [F^{ii} + Y^i], & -[F^{iI}] \\ -[F^{Ii}], & [F^{II}] \end{bmatrix} \begin{bmatrix} (V_*^i) \\ (V_*^I) \end{bmatrix}.
$$
(190)

Let

$$
\begin{bmatrix} [L^{ii}], & [L^{iI}] \\ [L^{Ii}], & [L^{II}] \end{bmatrix} = \begin{bmatrix} [F^{ii} + \mu^i + Y^i/y], & -[F^{iI}] \\ -[F^{Ii}], & [F^{II} + \tilde{\mu}^I] \end{bmatrix}^{-1} \begin{bmatrix} [F^{ii} + Y^i], & -[F^{iI}] \\ -[F^{Ii}], & [F^{II}] \end{bmatrix}.
$$
(191)

Follows that

$$
(u_\theta^i) = -[L^{ii}](V_*^i) - [L^{iI}](V_*^I),
$$
(192)

$$
(u_\theta^I) = -[L^{II}](V_*^I) - [L^{Ii}](V_*^i),
$$
(193)

$$
(u_\parallel^i) = (V_E) + (1 - [L^{ii}])(V_*^i) - [L^{iI}](V_*^I),
$$
(194)

$$
(u_\parallel^I) = (V_E) + (1 - [L^{II}])(V_*^I) - [L^{Ii}](V_*^i).
$$
(195)

Equation (179) becomes

$$
[F^{ee} + \mu^e](u_\theta^e) = - \left([F^{ee}] - [F^{ei}] - [F^{eI}]\right)(V_E) - [F^{ee}](V_*^e)
$$

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$$+ \left\{ [F^{ei}] \left(1 - [L^{ii}]\right) - [F^{eI}] [L^{Ii}] \right\} (V_*^i)$$
$$+ \left\{ [F^{eI}] \left(1 - [L^{II}]\right) - [F^{ei}] [L^{iI}] \right\} (V_*^I) + \Omega_{ee} \, \tau_{ee} \, (E_\parallel). \tag{196}$$

However, because $F_{j0}^{ee} = F_{j0}^{ei} + F_{j0}^{eI}$, the previous equation becomes

$$[F^{ee} + \mu^e] (u_\theta^e) = -[F^{ee}] (V_*^e) + \left\{ [F^{eI}] \left(1 - [L^{ii}]\right) - [F^{II}] [L^{Ii}] \right\} (V_*^i)$$
$$+ \left\{ [F^{eI}] \left(1 - [L^{II}]\right) - [F^{ei}] [L^{iI}] \right\} (V_*^I) + \Omega_{ee} \, \tau_{ee} \, (E_\parallel). \tag{197}$$

Let

$$[Q^{ee}] = [F^{ee} + \mu^e]^{-1}, \tag{198}$$
$$[L^{ee}] = [Q^{ee}] [F^{ee}], \tag{199}$$
$$[L^{ei}] = -[Q^{ee}] \left\{ [F^{ei}] \left(1 - [L^{ii}]\right) - [F^{eI}] [L^{Ii}] \right\}, \tag{200}$$
$$[L^{eI}] = -[Q^{ee}] \left\{ [F^{eI}] \left(1 - [L^{II}]\right) - [F^{ei}] [L^{iI}] \right\}. \tag{201}$$

Follows that

$$(u_\theta^e) = -[L^{ee}] (V_*^e) - [L^{ei}] (V_*^i) - [L^{eI}] (V_*^I) + \Omega_e \, \tau_{ee} \, [Q^{ee}] (E_\parallel), \tag{202}$$
$$(u_\parallel^e) = (V_E) + (1 - [L^{ee}]) (V_*^e) - [L^{ei}] (V_*^i) - [L^{eI}] (V_*^I) + \Omega_e \, \tau_{ee} \, [Q^{ee}] (E_\parallel), \tag{203}$$

Finally,

$$(j_\parallel) = \sum_a n_a \, e_a \, (u_\parallel^a)$$
$$= \left\{ n_i \, e_i \left(1 - [L^{ii}]\right) - n_I \, e_I \, [L^{Ii}] - n_e \, e_e \, [L^{ei}] \right\} (V_*^i)$$
$$+ \left\{ n_I \, e_I \left(1 - [L^{II}]\right) - n_i \, e_i \, [L^{iI}] - n_e \, e_e \, [L^{eI}] \right\} (V_*^I)$$
$$+ n_e \, e_e \left(1 - [L^{ee}]\right) (V_*^e) + n_e \, e_e \, \Omega_e \, \tau_{ee} \, [Q^{ee}] (E_\parallel), \tag{204}$$

where we have made use of the fact that $n_i \, e_i + n_I \, e_I + n_e \, e_e = 0$.

5 Velocities and Current Density

Let

$$\omega_E = -\frac{d\Phi}{d\psi}, \tag{205}$$

$$\omega_{*a} = -\frac{T_a}{e_a} \frac{d \ln p_a}{d\psi}, \tag{206}$$

$$\eta_a = \frac{d \ln T_a}{d \ln n_a}. \tag{207}$$

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It follows that

$$V_{*0}^a = I\,\omega_{*a}, \tag{208}$$

$$V_{*1}^a = -I\left(\frac{\eta_a}{1+\eta_a}\right)\omega_{*a}, \tag{209}$$

$$V_{E0} = I\,\omega_E, \tag{210}$$

$$V_{E1} = 0. \tag{211}$$

We have $e_e = -e$, $e_i = e$, $e_I = Z_I\,e$. Thus,

$$
\frac{V_\theta^{\,i}}{B_\theta}\frac{\langle B^2\rangle}{I} = -L_{00}^{ii}\,\omega_{*i} - L_{00}^{iI}\,\omega_{*I} + L_{01}^{ii}\left(\frac{\eta_i}{1+\eta_i}\right)\omega_{*i}
$$
$$
+ L_{01}^{iI}\left(\frac{\eta_I}{1+\eta_I}\right)\omega_{*I}, \tag{212}
$$

$$
\frac{V_\theta^{\,I}}{B_\theta}\frac{\langle B^2\rangle}{I} = -L_{00}^{II}\,\omega_{*I} - L_{00}^{Ii}\,\omega_{*i} + L_{01}^{II}\left(\frac{\eta_I}{1+\eta_I}\right)\omega_{*I}
$$
$$
+ L_{01}^{Ii}\left(\frac{\eta_i}{1+\eta_i}\right)\omega_{*i}, \tag{213}
$$

$$
\frac{\langle V_\parallel^{\,i}\,B\rangle}{I} = \omega_E + \omega_{*i} - L_{00}^{ii}\,\omega_{*i} - L_{00}^{iI}\,\omega_{*I} + L_{01}^{ii}\left(\frac{\eta_i}{1+\eta_i}\right)\omega_{*i}
$$
$$
+ L_{01}^{iI}\left(\frac{\eta_I}{1+\eta_I}\right)\omega_{*I}, \tag{214}
$$

$$
\frac{\langle V_\parallel^{\,I}B\rangle}{I} = \omega_E + \omega_{*I} - L_{00}^{II}\,\omega_{*I} - L_{00}^{Ii}\,\omega_{*i} + L_{01}^{II}\left(\frac{\eta_I}{1+\eta_I}\right)\omega_{*I}
$$
$$
+ L_{01}^{Ii}\left(\frac{\eta_i}{1+\eta_i}\right)\omega_{*i}. \tag{215}
$$

Can write

$$j_\parallel = j_{\text{bootstrap}} + j_{\text{ohmic}}, \tag{216}$$

where

$$
\frac{\langle j_{\text{bootstrap}}\,B\rangle}{I} = -\left\{1 - L_{00}^{ii} - \left(\frac{Z_{\text{eff}}-1}{Z_I - Z_{\text{eff}}}\right)L_{00}^{Ii} + \left(\frac{Z_I-1}{Z_I - Z_{\text{eff}}}\right)L_{00}^{ei}\right\}\frac{dp_i}{d\psi}
$$
$$
- \left\{1 - L_{00}^{II} - \left(\frac{Z_I - Z_{\text{eff}}}{Z_{\text{eff}}-1}\right)L_{00}^{iI} + \left(\frac{Z_I-1}{Z_{\text{eff}}-1}\right)L_{00}^{eI}\right\}\frac{dp_I}{d\psi}
$$
$$
- (1 - L_{00}^{ee})\frac{dp_e}{d\psi}
$$
$$
- \left\{L_{01}^{ii} + \left(\frac{Z_{\text{eff}}-1}{Z_I - Z_{\text{eff}}}\right)L_{01}^{Ii} - \left(\frac{Z_I-1}{Z_I - Z_{\text{eff}}}\right)L_{01}^{ei}\right\}n_i\frac{dT_i}{d\psi}
$$

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$$- \left\{ L_{01}^{II} + \left(\frac{Z_I - Z_{\text{eff}}}{Z_{\text{eff}} - 1} \right) L_{01}^{iI} - \left(\frac{Z_I - 1}{Z_{\text{eff}} - 1} \right) L_{01}^{eI} \right\} n_I \frac{dT_I}{d\psi}$$

$$- L_{01}^{ee} \, n_e \, \frac{dT_e}{d\psi} \tag{217}$$

and

$$\langle j_{\text{ohmic}} \, B \rangle = Q_{00}^{ee} \, \sigma_{ee} \, \langle E_\parallel \, B \rangle, \tag{218}$$

where

$$\sigma_{ee} = \frac{n_e \, e^2 \, \tau_{ee}}{m_e}. \tag{219}$$