I. INTRODUCTION

In the context of tokamak plasmas, a resonant magnetic perturbation (RMP) refers to an externally generated, (usually) static, helical magnetic perturbation that resonates at one or more rational surfaces within the plasma. Resonant magnetic perturbations have been successfully employed to compensate error fields (i.e., accidentally produced RMP), and, thereby, to prevent the formation of locked modes in a wide variety of different types of tokamak discharge.[1–3] RMPs have also been used to either control or mitigate edge-localized-modes (ELMs) in H-mode tokamak discharges.[4–9]

By far the most efficient manner in which to model the response of a tokamak plasma to an RMP is to employ an "asymptotic matching" approach.[10–14] According to such an approach, the response of the plasma to the applied RMP is governed by a combination of flux-freezing and perturbed force balance—this combination is usually referred to as "marginally-stable ideal-magnetohydrodynamics (MHD)", which is a misnomer because MHD *per se* plays no role—everywhere in the plasma apart from a number of relatively narrow (in the radial direction) regions in which the applied perturbation resonates with the equilibrium magnetic field. Magnetic reconnection can take place within the resonant regions to produce relatively thin magnetic island chains. Within the resonant regions, the plasma response is governed by linear or nonlinear two-fluid resistive-MHD, depending on whether the induced magnetic island widths are smaller or larger than the corresponding linear layer widths. Thus, when employing the asymptotic matching approach, the equations of marginally-stable ideal-MHD are solved in the so-called "outer region" that comprises most of the plasma (and the surrounding vacuum), the equations of linear/nonlinear two-fluid resistive-MHD are solved in the various resonant layers that constitute the so-called "inner region", and the two sets of solutions are then asymptotically matched to one another.

In the nonlinear resonant response regime, the evolution of the island width in a given resonant layer is governed by the familiar Rutherford island width evolution equation.[15,16] On the other hand, the phase evolution of the island chain is conventionally governed by the well-known no-slip constraint.[11] According to this constraint, plasma is trapped inside the magnetic separatrix of the magnetic island chain, which forces the chain to co-rotate with the local plasma flow. In reality, however, the plasma is able to diffuse resistively across the separatrix to some extent.[17,18]

The primary aim of this paper is to investigate what happens when the no-slip constraint is relaxed in a recently-developed model of RMP-induced suppression of ELMs in H-mode tokamak discharges.[19] The secondary aim is to examine what happens when an improvement is made to the manner in which the resonant plasma response component of the model interpolates between the linear and the nonlinear regime.

This paper is organized as follows. In Sect. II, we develop a cylindrical version of our new resonant plasma response model. In Sect. III, we use this cylindrical version as a guide for producing a toroidal version of the model.

II. CYLINDRICAL RESONANT RESPONSE MODEL

A. Plasma Equilibrium

Consider a large aspect-ratio, low-β, tokamak plasma whose magnetic flux surfaces map out (almost) concentric circles in the poloidal plane. Such a plasma is well approximated as a periodic cylinder. Suppose that the minor radius of the plasma is a. Standard cylindrical coordinates (r, θ, z) are adopted. The system is assumed to be periodic in the z-direction, with periodicity length $2\pi\,R_0$, where $R_0 \gg a$ is the simulated plasma major radius. It is convenient to define the simulated toroidal angle $\phi = z/R_0$.

The equilibrium magnetic field is written $\mathbf{B} = [0, B_\theta(r), B_\phi]$. The associated equilibrium plasma current density takes the form $\mathbf{j} = [0, 0, j_\phi(r)]$, where $\mu_0\, j_\phi = (1/r)\, d(r\, B_\theta)/dr$. The so-called "safety-factor", $q(r) = r\, B_\phi/(R_0\, B_\theta)$, parameterizes the helical pitches of equilibrium magnetic field-lines.

B. Plasma Response to RMP

Consider the response of the plasma to an externally generated, static, helical, RMP. Suppose that the RMP has m periods in the poloidal direction, and n periods in the toroidal direction. It is convenient to express the perturbed magnetic field and the perturbed plasma current density in terms of a perturbed poloidal magnetic flux, $\psi(r, \theta, \phi, t)$. In fact, $\delta\mathbf{B} = \nabla\psi \times \nabla z$, and $\mu_0\,\delta\mathbf{j} = -\nabla^2\psi\,\nabla z$, where $\psi(r, \theta, \phi, t) = \hat{\psi}(r, t)\, \exp[{\rm i}\,(m\,\theta - n\,\phi)]$, and $\hat{\psi}$ is real. This particular representation is valid provided that $m/n \gg a/R_0$.[11]

The response of the plasma to the RMP is governed by the equations of marginally-stable, ideal-MHD everywhere in the plasma, apart from a relatively narrow (in r) region in the vicinity of the so-called "rational surface", minor radius r_s, at which $q(r_s) = m/n$.[11]

It is convenient to parameterize the RMP in terms of the so-called "vacuum magnetic flux", $\Psi_v(t) = |\Psi_v|\,{\rm e}^{-{\rm i}\,\varphi_v}$, which is defined to be the value of $\hat{\psi}(r,t)$ at radius r_s in the presence of the RMP, but in the absence of plasma. Here, φ_v is the helical phase of the RMP. Likewise, the response of the plasma in the vicinity of the rational surface is parameterized in terms of the so-called "reconnected magnetic flux", $\Psi_s(t) = |\Psi_s|\,{\rm e}^{\,{\rm i}\,\varphi_s}$, which is the actual value of $\hat{\psi}(r,t)$ at radius r_s. Here, φ_s is the helical phase of the reconnected magnetic flux.

The intrinsic stability of the m, n tearing mode is governed by the dimensionless parameter $E_{ss} = [d\hat{\psi}_s/d\ln r]_{r_{s-}}^{r_{s+}}$, where $\hat{\psi}_s(r)$ is a solution of the marginally-stable, ideal-MHD equations, for the case of an m, n helical perturbation, that satisfies physical boundary conditions at $r = 0$ and $r = a$, in the absence of the RMP, and is such that $\hat{\psi}_s(r_s) = 1$.[10,12] According to resistive-MHD theory,[10,15] if $E_{ss} > 0$ then the m, n tearing mode spontaneously reconnects magnetic flux at the rational surface to form a helical magnetic island chain. In this paper, it is assumed that $E_{ss} < 0$, so that the m, n tearing mode is intrinsically stable. In this case, any magnetic reconnection that takes place at the rational surface is due solely to the action of the RMP.

The ideal-MHD response of the plasma to the RMP is governed by the dimensionless parameter $E_{sv} = [d\hat{\psi}_v/d\ln r]_{r_{s-}}^{r_{s+}}$, where $\hat{\psi}_v(r)$ is a solution of the marginally-stable, ideal-MHD equations, for the case of an m, n helical perturbation, that satisfies physical boundary conditions at $r = 0$ and $r = a$, in the presence of the RMP, and is such that $\hat{\psi}_v(r_s) = 0$.[12]

C. Linear Response Regime

In the linear response regime, the reconnected magnetic flux induced by the RMP at the rational surface is governed by [11,21,22]

$$\frac{\delta_s}{r_s}\,\tau_R\left(\frac{d}{dt} + {\rm i}\,\omega_s\right)\Psi_s = E_{ss}\,\Psi_s + E_{sv}\,\Psi_v, \tag{1}$$

where δ_s is the linear layer width, $\tau_R = \mu_0\,r_s^{\,2}\,\sigma(r_s)$ the global resistive diffusion timescale, and

$$\omega_s(t) = m\,\Omega_\theta(r_s,t) - n\,\Omega_\phi(r_s,t). \tag{2}$$

Here, $\sigma(r)$, $\Omega_\theta(r,t)$, and $\Omega_\phi(r,t)$ are the plasma conductivity, poloidal angular velocity, and toroidal angular velocity profiles, respectively. Note that, in this paper, we are assuming that the constant-ψ approximation [10] holds at the rational surface. (It is demonstrated in Ref. 23 that the appropriate linear response regime for an RMP resonant in the pedestal of a typical DIII-D H-mode discharge is the so-called "first semi-collisional regime", which is indeed a constant-ψ response regime. Moreover, if the appropriate linear response regime is constant-ψ then so is the appropriate nonlinear response regime, because the constant-ψ constraint is easier to satisfy in the nonlinear regime than in the linear regime.[21,24])

Equation (1) can be rewritten as an "amplitude evolution equation",

$$\frac{\delta_s}{r_s}\,\tau_R\,\frac{d|\Psi_s|}{dt} = E_{ss}\,|\Psi_s| + E_{sv}\,|\Psi_v|\,\cos(\varphi_s-\varphi_v), \tag{3}$$

combined with a "phase evolution equation",

$$\frac{d\varphi_s}{dt} = \omega_s - \frac{E_{sv}}{\tau_R}\,\frac{r_s}{\delta_s}\,\frac{|\Psi_v|}{|\Psi_s|}\,\sin(\varphi_s-\varphi_v). \tag{4}$$

The final term on the right-hand side of Eq. (4) is termed the "slip-frequency", and is the difference between the helical phase velocity of the reconnected magnetic flux and that expected on the assumption that the flux is convected by the plasma flow at the rational surface (i.e., $d\varphi_s/dt = \omega_s$). In general, the slip-frequency is non-zero because the plasma in the vicinity of the rational surface is capable of diffusing resistively through the linear layer structure.

D. Nonlinear Response Regime

In the nonlinear response regime, Eq. (3) generalizes to the "Rutherford island width evolution equation": [15,16]

$$\frac{(\mathcal{I}/2)\,W_s}{r_s}\,\tau_R\,\frac{d|\Psi_s|}{dt} = E_{ss}\,|\Psi_s| + E_{sv}\,|\Psi_v|\,\cos(\varphi_s-\varphi_v), \tag{5}$$

where

$$W_s = 4\left(\frac{|\Psi_s|}{s_s\,r_s\,B_\theta(r_s)}\right)^{1/2} r_s \tag{6}$$

is the full radial width of the magnetic island chain, $\mathcal{I} = 0.8227$, $s(r) = d\ln q/d\ln r$, and $s_s = s(r_s)$. The nonlinear regime holds when $W_s \gg \delta_s$, whereas the linear regime holds when $\delta_s \ll W_s$.

Equation (5) is usually coupled with the so-called "no-slip constraint":[11]

$$\frac{d\varphi_s}{dt} = \omega_s. \tag{7}$$

The reasoning behind the imposition of this constraint is that the plasma is trapped inside the magnetic separatrix of the magnetic island chain that forms in the vicinity of the rational surface, which forces the chain to co-rotate with the local plasma flow. However, in reality, the plasma is able to diffuse resistively across the separatrix to some extent. Hence, by analogy with Eqs. (3), (4), and (5), and in accordance with Refs. 17 and 18, in this paper we shall modify Eq. (7) such that

$$\frac{d\varphi_s}{dt} = \omega_s - \frac{E_{sv}}{\tau_R}\,\frac{r_s}{(\mathcal{I}/2)\,W_s}\,\frac{|\Psi_v|}{|\Psi_s|}\,\sin(\varphi_s-\varphi_v). \tag{8}$$

The final term on the right-hand side of the previous equation is the "nonlinear slip-frequency". In general, we would expect this frequency to be relatively small (because it is generally the case that $|\omega_s|\,\tau_R\,(W_s/r_s) \gg 1$ for fully-developed magnetic islands in high-temperature tokamak plasmas[18,23]). Nevertheless, the nonlinear slip-frequency may be non-negligible for developing island chains, which is why we are including it in our analysis. (Note that the nonlinear slip-frequency does not appear in rigorous island calculations, such as that described in Ref. 25, and references therein, because such calculations depend crucially on complicated hierarchical ordering assumptions, according to which the nonlinear slip-frequency is negligible.)

E. Composite Model

We can combine Eqs. (3), (4), (5), and (8) to give the following composite resonant response model that interpolates between the linear and the nonlinear regimes:

$$\left(\frac{(\mathcal{I}/2)\,W_s+\delta_s}{r_s}\right)\tau_R\,\frac{d|\Psi_s|}{dt} = E_{ss}\,|\Psi_s| + E_{sv}\,|\Psi_v|\,\cos(\varphi_s-\varphi_v), \tag{9}$$

$$\frac{d\varphi_s}{dt} = \omega_s - \frac{E_{sv}}{\tau_R}\left(\frac{r_s}{(\mathcal{I}/2)\,W_s+\delta_s}\right)\frac{|\Psi_v|}{|\Psi_s|}\,\sin(\varphi_s-\varphi_v). \tag{10}$$

If we define $X_s = |\Psi_s|\,\cos\varphi_s$, $Y_s = |\Psi_s|\,\sin\varphi_s$, $X_v = |\Psi_v|\,\cos\varphi_v$, and $Y_v = |\Psi_v|\,\sin\varphi_v$ then the previous two equations are more conveniently written in the nonsingular (when $|\Psi_s| = 0$)

forms: [17,18,26]

$$\left(\frac{(\mathcal{I}/2)\,W_s+\delta_s}{r_s}\right)\tau_R\left(\frac{dX_s}{dt}+\omega_s\,Y_s\right)=E_{ss}\,X_s+E_{sv}\,X_v, \tag{11}$$

$$\left(\frac{(\mathcal{I}/2)\,W_s+\delta_s}{r_s}\right)\tau_R\left(\frac{dY_s}{dt}-\omega_s\,X_s\right)=E_{ss}\,Y_s+E_{sv}\,Y_v, \tag{12}$$

where

$$W_s=4\left(\frac{\sqrt{X_s^{\,2}+Y_s^{\,2}}}{s_s\,r_s\,B_\theta(r_s)}\right)^{1/2} r_s. \tag{13}$$

With the benefit of hindsight, Eqs. (11) and (12) could have been derived more directly from the following heuristic generalization of Eq. (1): [18]

$$\left(\frac{(\mathcal{I}/2)\,W_s+\delta_s}{r_s}\right)\tau_R\left(\frac{d}{dt}+{\rm i}\,\omega_s\right)\Psi_s=E_{ss}\,\Psi_s+E_{sv}\,\Psi_v. \tag{14}$$

III. MODIFIED TOROIDAL RESONANT RESPONSE MODEL

Our toroidal model of the response of a tokamak plasma to a static, externally generated, RMP is described in detail in Ref. 19. In the light of the composite cylindrical resonant response model developed in Sect. II, we shall modify the resonant response component of the aforementioned toroidal model; this component is described in Appendix D of Ref. 19.

Equations (D1) and (D2) in Ref. 19 are modified such that

$$\left(\hat{W}_k+\hat{\delta}_k\right)\mathcal{S}_k\,\frac{d\hat{\Psi}_k}{d\hat{t}}=\sum_{k'=1,K}\hat{E}_{kk'}\hat{\Psi}_{k'}\,\cos(\varphi_k-\varphi_{k'}-\xi_{kk'})+\hat{E}_{kk}\,\hat{\chi}_k\,\cos(\varphi_k-\zeta_k), \tag{15}$$

$$\mathcal{S}_k=\frac{\tau_R(\hat{r}_k)}{\tau_A}, \tag{16}$$

respectively. Here,

$$\hat{W}_k=\frac{2\,\mathcal{I}}{\hat{a}\,\hat{r}_k}\left(\frac{q}{g\,s}\right)^{1/2}_{\hat{r}_k}|\hat{\Psi}_k|^{\,1/2}, \tag{17}$$

$$\hat{\delta}_k=\frac{\delta_{\rm SC}(\hat{r}_k)}{R_0\,\hat{a}\,\hat{r}_k}, \tag{18}$$

where $\delta_{\rm SC}(\hat{r})$ is the semi-collisional linear layer width specified in Ref. 23.

In the previous expressions, R_0 is the major radius of the magnetic axis, r is a magnetic flux surface label with dimensions of length (which, roughly speaking, is the average minor radius of the flux surface), $q(r)$ is the safety-factor profile, $s(r)=d\ln q/d\ln r$, $g(r)\,B_0$ is the toroidal magnetic field-strength at major radius R_0, B_0 is the vacuum toroidal magnetic field-strength on the magnetic axis, a is the value of r on the last closed magnetic-flux surface, r_k is the value of r on the kth rational surface (resonant with poloidal mode number m_k, and toroidal mode number n), $\hat{a} = a/R_0$, $\hat{r} = r/a$, and $\hat{r}_k = r_k/a$. Furthermore, $\tau_R(\hat{r}) = \mu_0\, r^2\, \sigma_{ee}\, Q_{00}^{ee}$, where σ_{ee} is the classical plasma parallel electrical conductivity, and Q_{00}^{ee} describes the corrections to this conductivity due to neoclassical effects and the presence of plasma impurities. Also, τ_A is a convenient scale time, and $\hat{t} = t/\tau_A$. Finally, $\Psi_k = B_0\, R_0\, \hat{\Psi}_k\, \mathrm{e}^{-\mathrm{i}\,\varphi_k}$ is the reconnected magnetic flux at the kth rational surface, $\chi_k = B_0\, R_0\, \hat{\chi}_k\, \mathrm{e}^{-\mathrm{i}\,\zeta_k}$ is a measure of the vacuum magnetic flux at the kth rational surface (in fact, $\hat{E}_{kk}\, \chi_k$ is equivalent to $E_{sv}\, \Psi_v$ in Sect. II), and $E_{kk'} = \hat{E}_{kk'}\, \mathrm{e}^{-\mathrm{i}\,\xi_{kk'}}$ is the toroidal tearing stability matrix.[12] Here, $\hat{\Psi}_k > 0$, φ_k, $\hat{\chi}_k > 0$, ζ_k, $\hat{E}_{kk'} > 0$, and $\xi_{kk'}$ are all real quantities.

Equation (15) represents a minor improvement to Eq. (D1) of Ref. 19 in which the interpolation between the linear and nonlinear regimes is performed in a more systematic manner.

Equation (D6) of Ref. 19 is modified such that

$$\left(\hat{W}_k + \hat{\delta}_k\right) \mathcal{S}_k \left(\frac{d\varphi_k}{d\hat{t}} - \hat{\varpi}_k\right) = -\sum_{k'=1,K} \hat{E}_{kk'}\, \hat{\Psi}_{k'}\, \sin(\varphi_k - \varphi_{k'} - \xi_{kk'}) - \hat{E}_{kk}\, \hat{\chi}_k\, \sin(\varphi_k - \zeta_k), \tag{19}$$

where

$$\hat{\varpi}_k = \hat{\varpi}_{k0} + m_k\, \Delta\hat{\Omega}_\theta(\hat{r}_k, \hat{t}) - n\, \Delta\hat{\Omega}_\phi(\hat{r}_k, \hat{t}), \tag{20}$$

and $\hat{\varpi}_{k0} = \varpi_{k0}\, \tau_A$, $\hat{\Omega}_\theta = \Delta\Omega_\theta\, \tau_A$, and $\hat{\Omega}_\phi = \Delta\Omega_\phi\, \tau_A$. Here, ϖ_{k0} is the so-called "natural frequency" at the kth rational surface; this quantity is defined as the helical phase velocity of a naturally unstable island chain, resonant at the surface, in the absence of an RMP (or any other island chains). On the other hand, $\Delta\Omega_\theta$ and $\Delta\Omega_\phi$ are the changes in the plasma poloidal and toroidal velocity profiles, respectively, induced by the electromagnetic torques that develop in the vicinities of the various rational surfaces in the plasma. Equation (19) represents a major improvement to Eq. (D6) that takes into account the finite slip-frequencies of the magnetic island chains induced at the various rational surfaces in the plasma. In the absence of a finite slip-frequency, the right-hand side of Eq. (19) would be zero.

If we define $X_k = \hat{\Psi}_k\, \cos\varphi_k$ and $Y_k = \hat{\Psi}_k\, \sin\varphi_k$ then Eqs. (15) and (19) are more conveniently written in the nonsingular (when $\hat{\Psi}_k = 0$) forms

$$\left(\hat{W}_k + \hat{\delta}_k\right) \mathcal{S}_k \left(\frac{dX_k}{d\hat{t}} + \hat{\varpi}_k\, Y_k\right) = \sum_{k'=1,K} \hat{E}_{kk'} \left(\cos\xi_{kk'}\, X_{k'} - \sin\xi_{kk'}\, Y_{k'}\right) + \hat{E}_{kk}\, \hat{\chi}_k\, \cos\zeta_k, \tag{21}$$

$$\left(\hat{W}_k + \hat{\delta}_k\right) \mathcal{S}_k \left(\frac{dY_k}{d\hat{t}} - \hat{\varpi}_k\, X_k\right) = \sum_{k'=1,K} \hat{E}_{kk'} \left(\cos\xi_{kk'}\, Y_{k'} + \sin\xi_{kk'}\, X_{k'}\right) + \hat{E}_{kk}\, \hat{\chi}_k\, \sin\zeta_k, \tag{22}$$

where

$$\hat{W}_k = \frac{2\,\mathcal{I}}{\hat{a}\,\hat{r}_k} \left(\frac{q}{g\,s}\right)_{\hat{r}_k}^{1/2} (X_k^{\,2} + Y_k^{\,2})^{\,1/4}. \tag{23}$$

Equations (15), (19), (21), (22) and (23) are the toroidal generalizations of Eqs. (9), (10), (11), (12), and (13), respectively.

IV. DISCUSSION AND CONCLUSIONS

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DISCLAIMER

thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DATA AVAILABITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

[1] J.T. Scoville, and R.J. La Haye, Nucl. Fusion **43**, 250 (2003).

[2] J.-K. Park, M.J. Schaffer, J.E. Menard, and A.H. Boozer Phys. Rev. Lett. **99**, 195003 (2007).

[3] N.M. Ferraro, J.-K. Park, C.E. Myers, A. Brooks, S.P. Gerhardt, J.E. Menard, S. Munaretto, and M.L. Reinke, Nucl. Fusion **59**, 086021 (2019).

[4] T.E. Evans, R.A. Moyer, J.G. Watkins, P.R. Thomas, T.H. Osborne, J.A. Boedo, M.E. Fenstermacher, K.H. Finken, R.J. Groebner, M. Groth, *et al.*, Phys. Rev. Lett. **92**, 235003 (2004).

[5] Y. Liang, H.R. Koslowski, P.R. Thomas, E. Nardon, B. Alper, P. Andrew, Y. Andrew, G. Arnoux, Y. Baranov, M. Bécoulet, *et al.*, Phys. Rev. Lett. **98**, 265004 (2007).

[6] W. Suttrop, T. Eich, J.C. Fuchs, S. Günter, A. Janzer, A. Herrmann, A. Kallenbach, P.T. Lang, T. Lunt, M. Maraschek, *et al.*, Phys. Rev. Lett. **106**, 225004 (2011).

[7] Y.M. Jeon, J.-K. Park, S.W. Yoon, W.H. Ko, S.G. Lee, K.D. Lee, G.S. Yun, Y.U. Nam, W.C. Kim, J.-G. Kwak, K.S. Lee, H.K. Kim, and H.L. Yang, *et al.*, Phys. Rev. Lett. **109**, 035004 (2012).

[8] A. Kirk, I.T. Chapman, Y. Liu, P. Cahyna, P. Denner, G. Fishpool, C.J. Ham, J.R. Harrison, Y. Liang, E. Nardon, S. Saarelma, R. Scannell, A.J. Thornton, and MAST Team, Nucl. Fusion **53**, 043007 (2013).

[9] T. Sun, Y. Liang, Y.Q. Liu, S. Gu, X. Yang, W. Guo, T. Shi, M. Jia, L. Wang, B. Lyu, *et al.*, Phys. Rev. Lett. **117**, 115001 (2016).

[10] H.P. Furth, J. Killeen, and M.N. Rosenbluth, Phys. Fluids **6**, 459 (1963).

[11] R. Fitzpatrick, Nucl. Fusion **33**, 1049 (1993).

[12] R. Fitzpatrick, R.J. Hastie, T.J. Martin, and C.M. Roach, Nucl. Fusion **33**, 1533 (1993).

[13] A.H. Glasser, Z.R. Wang, and J.-K. Park, Phys. Plasmas **23**, 112506 (2016).

[14] R. Fitzpatrick, Phys. Plasmas **24**, 072506 (2017).

[15] P.H. Rutherford, Phys. Fluids **16**, 1903 (1973).

[16] P.H. Rutherford, in *Basic Physical Processes of Toroidal Fusion Plasmas*, Proc. Course and Workshop, Varenna, 1985. (Commission of the European Communities, Brussels, 1985.) Vol. 2, p. 531.

[17] L.G. Eliseev, N.V. Ivanov, A.M. Kakurin, A.V. Melnikov, and S.V. Perfilov, Phys. Plasmas **22**, 052504 (2015).

[18] W. Huang, and P. Zhu, Phys. Plasmas **27**, 022514 (2020).

[19] R. Fitzpatrick, and A.O. Nelson, Phys. Plasmas **27**, 072501 (2020).

[20] R. Fitzpatrick, "Modelling q_{95} Windows for the Suppression of Edge Localized Modes by Resonant Magnetic Perturbations in the DIII-D Tokamak", submitted to Physics of Plasmas (2020).

[21] R. Fitzpatrick, Phys. Plasmas **5**, 3325 (1998).

[22] R. Fitzpatrick, Phys. Plasmas **21**, 092513 (2014).

[23] R. Fitzpatrick, Phys. Plasmas **27**, 042506 (2020).

[24] R. Fitzpatrick, Phys. Plasmas **10**, 2304 (2003).

[25] R. Fitzpatrick, Phys. Plasmas **25**, 082513 (2018).

[26] A.N. Chudnovskiy, Y.V. Gvozdkov, N.V. Ivanov, A.M. Kakurin, A.A. Medvedev, I.I. Orlovskiy, Y.D. Pavlov, V.V. Piterskiy, V.D. Pustovitov, M.B. Safonova, V.V. Volkov, and the T-10 Team, Nucl. Fusion **43** 681 (2003).