Relaxation of No-Slip Constraint

Richard Fitzpatrick

August 18, 2020

Fundamental equation:

$$S_k\left(\hat{\Psi}_k^{\,1/2}+\hat{\delta}_k\right)\left(\frac{d}{d\hat{t}}+{\rm i}\,\hat{\varpi}_k\right)\Psi_k=\sum_{k'=1,K}E_{kk'}\,\Psi_{k'}+\hat{E}_{kk}\,\chi_k, \tag{1}$$

where

$$\Psi_k=\hat{\Psi}_k\,{\rm e}^{-{\rm i}\,\varphi_k}, \tag{2}$$

$$\chi_k=\hat{\chi}_k\,{\rm e}^{-{\rm i}\,\zeta_k}, \tag{3}$$

$$E_{kk'}=\hat{E}_{kk'}\,{\rm e}^{-{\rm i}\,\xi_{kk'}}. \tag{4}$$

Follows from Eq. (1) that

$$S_k\left(\hat{\Psi}_k^{\,1/2}+\hat{\delta}_k\right)\frac{d\hat{\Psi}_k}{d\hat{t}}=\sum_{k'=1,K}\hat{E}_{kk'}\,\hat{\Psi}_{k'}\,\cos(\varphi_k-\varphi_{k'}-\xi_{kk'})+\hat{E}_{kk}\,\hat{\chi}_k\,\cos(\varphi_k-\zeta_k), \tag{5}$$

$$S_k\left(\hat{\Psi}_k^{\,1/2}+\hat{\delta}_k\right)\hat{\Psi}_k\left(\frac{d\varphi_k}{d\hat{t}}-\hat{\varpi}_k\right)=-\sum_{k'=1,K}\hat{E}_{kk'}\,\hat{\Psi}_{k'}\,\sin(\varphi_k-\varphi_{k'}-\xi_{kk'})-\hat{E}_{kk}\,\hat{\chi}_k\,\sin(\varphi_k-\zeta_k). \tag{6}$$

Alternatively, let

$$X_k=\hat{\Psi}_k\,\cos\varphi_k, \tag{7}$$

$$Y_k=\hat{\Psi}_k\,\sin\varphi_k. \tag{8}$$

So, Eq. (1) gives

$$\begin{aligned}S_k\left(\hat{\Psi}_k^{\,1/2}+\hat{\delta}_k\right)\left(\frac{d}{d\hat{t}}+{\rm i}\,\hat{\varpi}_k\right)(X_k-{\rm i}\,Y_k)&=\sum_{k'=1,K}\hat{E}_{kk'}\,(\cos\xi_{kk'}-{\rm i}\,\sin\xi_{kk'})\,(X_{k'}-{\rm i}\,Y_{k'})\\&\quad+\hat{E}_{kk}\,\hat{\chi}_k\,(\cos\zeta_k-{\rm i}\,\sin\zeta_k),\end{aligned} \tag{9}$$

or

$$S_k\left(\hat{\Psi}_k^{1/2}+\hat{\delta}_k\right)\left(\frac{dX_k}{d\hat{t}}-\mathrm{i}\,\frac{dY_k}{d\hat{t}}+\mathrm{i}\,\hat{\varpi}_k\,X_k+\hat{\varpi}_k\,Y_k\right)$$
$$=\sum_{k'=1,K}\hat{E}_{kk'}\left(\cos\xi_{kk'}\,X_{k'}-\mathrm{i}\,\cos\xi_{kk'}\,Y_{k'}-\mathrm{i}\,\sin\xi_{kk'}\,X_{k'}-\sin\xi_{kk'}\,Y_{k'}\right)$$
$$+\hat{E}_{kk'}\,\hat{\chi}_k\,(\cos\zeta_k-\mathrm{i}\,\sin\zeta_k). \tag{10}$$

Follows that

$$S_k\left(\hat{\Psi}_k^{1/2}+\hat{\delta}_k\right)\left(\frac{dX_k}{d\hat{t}}+\hat{\varpi}_k\,Y_k\right)=\sum_{k'=1,K}\hat{E}_{kk'}\left(\cos\xi_{kk'}\,X_{k'}-\sin\xi_{kk'}\,Y_{k'}\right)+\hat{E}_{kk}\,\hat{\chi}_k\,\cos\zeta_k, \tag{11}$$

$$S_k\left(\hat{\Psi}_k^{1/2}+\hat{\delta}_k\right)\left(\frac{dY_k}{d\hat{t}}-\hat{\varpi}_k\,X_k\right)=\sum_{k'=1,K}\hat{E}_{kk'}\left(\cos\xi_{kk'}\,Y_{k'}+\sin\xi_{kk'}\,X_{k'}\right)+\hat{E}_{kk}\,\hat{\chi}_k\,\sin\zeta_k. \tag{12}$$

Finally,

$$\hat{\varpi}_k=\hat{\varpi}_{k\,\mathrm{ln}}+(\hat{\varpi}_{k\,\mathrm{nl}}-\hat{\varpi}_{k\,\mathrm{ln}})\tanh\left(\frac{\hat{\Psi}_k^{1/2}}{\hat{\delta}_k}\right). \tag{13}$$