

Linear Layer Physics

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1 Fundamental Quantities

Layer width - δ

Layer tearing stability index - Δ

Poloidal mode number - m

Toroidal mode number - n

Major radius - R_0

Minor radius - r

Toroidal magnetic field-strength - B_0

Magnetic shear - s

Electrostatic potential - Φ

Poloidal magnetic flux - ψ

Magnitude of electron charge - e

Electron mass - m_e

Electron number density - n_e

Electron temperature - T_e

Ion mass - m_i

Ion number density - n_i

Ion temperature - T_i

Parallel electrical resistivity - η_{\parallel}

Perpendicular particle diffusivity - D_{\perp}

Perpendicular energy diffusivity - χ_E

Perpendicular momentum diffusivity - χ_{ϕ}

2 Derived Quantities

Safety-factor - $q = m/n$

Scale length - $l = R_0 q/s$

Electron pressure - $p_e = n_e T_e$

Ion pressure - $p_i = n_i T_i$

Alfvén velocity - $v_A = B_0/\sqrt{\mu_0 n_e m_i}$

Alfvén time - $\tau_A = l/v_A = m \tau_H$

Hydromagnetic time - $\tau_H = R_0/(n s v_A)$

Resistive time - $\tau_R = \mu_0 r^2/\eta_{\parallel}$

Particle/energy confinement time - $\tau_E = r^2/[D_{\perp} + (2/3) \chi_E]$

Effective momentum confinement time - $\tau_M = (R_0 q)^2/\chi_{\phi}$

Ion sound radius - $\rho_s = \sqrt{m_i T_e}/(e B_0)$

$\mathbf{E} \times \mathbf{B}$ frequency - $\omega_E = d\Phi/d\psi$

Electron diamagnetic frequency - $\omega_{*e} = (T_e/e) d \ln p_e/d\psi$

Ion diamagnetic frequency - $\omega_{*i} = -(T_i/e) d \ln p_i/d\psi$

$\mathbf{E} \times \mathbf{B}$ velocity - $v_E = (r/q) \omega_E$

Electron diamagnetic velocity - $v_{*e} = (r/q) \omega_{*e}$

Ion diamagnetic velocity - $v_{*i} = (r/q) \omega_{*i}$

3 Non-dimensional Quantities

Lundqvist number - $S = \tau_R/\tau_H$

Magnetic Prandtl number - $P_M = \tau_R/\tau_M$

Energy confinement number - $P_E = \tau_R/\tau_E$

4 Layer Theory Quantities

$$\left(\frac{k}{\eta}\right)^{1/3} = S^{1/3} \frac{l}{r}$$

$$\tau = -\frac{\omega_{*i}}{\omega_{*e}}$$

$$c_\beta^2 = P_E$$

$$P = P_M$$

$$D = \frac{5}{3} S^{1/3} \frac{\rho_s}{r}$$

$$Q_E = S^{1/3} n \omega_E \tau_H$$

$$Q_{e,i} = -S^{1/3} n \omega_{*e,i} \tau_H$$

$$\Delta = \frac{\delta}{r} \tau_R \text{i} n (\omega_E - \omega_{*i})$$

$$\hat{\Delta} = \left(\frac{\eta}{k}\right)^{1/3} \frac{l}{r} \Delta$$

5 Constant- ψ Layer Physics

Layer equation:

$$\frac{d^2 Y}{dp^2} - \left[\frac{-Q_E (Q_E - Q_i) + \text{i} (Q_E - Q_i) (P_M + P_E) p^2 + P_M P_E p^4}{\text{i} (Q_E - Q_e) + \{P_E + \text{i} (Q_E - Q_i) D^2\} p^2 + (1 + \tau) P_M D^2 p^4} \right] p^2 Y = 0.$$

Suppose that small- p behavior of solution that is well-behaved as $p \rightarrow \infty$ is

$$Y(p) = Y_0 [1 - c p + \mathcal{O}(p^2)] .$$

Layer width is

$$\frac{\delta}{r} = \frac{\pi |c|}{S^{1/3}}.$$

Large- p limit of layer equation is

$$\frac{d^2 Y}{dp^2} - C p^2 Y = 0,$$

where

$$C = \frac{P_E}{(1 + \tau) D^2}.$$

Solutions are

$$p^{1/2} I_{1/4} \left(\frac{C^{1/2} p^2}{2} \right), \quad p^{1/2} K_{1/4} \left(\frac{C^{1/2} p^2}{2} \right).$$

Should need to integrate to $p \sim C^{-1/4}$.