Program RESCALE

Richard Fitzpatrick

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1 Rescaling gFile

1.1 Grad-Shafranov Equation

Let R, ϕ, Z be a set of cylindrical coordinates. The equilibrium magnetic field is written

$$\mathbf{B} = \nabla\phi \times \nabla\psi + T(\psi)\,\nabla\phi. \tag{1}$$

The Grad-Shafranov equation takes the form

$$\frac{\partial^2\psi}{\partial R^2} - \frac{1}{R}\frac{\partial\psi}{\partial R} + \frac{\partial^2\psi}{\partial Z^2} = -R^2\frac{dP}{d\psi} - \frac{1}{2}\frac{dT^2}{d\psi}, \tag{2}$$

where

$$j_\phi = -R\frac{dP}{d\psi} - \frac{1}{2\,R}\frac{dT^2}{d\psi}, \tag{3}$$

$$q(\psi) = \frac{T}{2\pi}\oint_\psi \frac{dl}{R\,|\nabla\psi|}. \tag{4}$$

1.2 Type I Rescaling

The following rescaling of variables leaves the Grad-Shafranov equation invariant:

$$R_{\rm new} = R_{\rm old}, \tag{5}$$

$$Z_{\rm new} = Z_{\rm old}, \tag{6}$$

$$\psi_{\rm new} = a_1\,\psi_{\rm old}, \tag{7}$$

$$T_{\rm new} = a_1\,T_{\rm old}, \tag{8}$$

$$P_{\rm new} = a_1^2\,P_{\rm old}. \tag{9}$$

It follows that

$$I_{\phi\,{\rm new}} = a_1\,I_{\phi\,{\rm old}}, \tag{10}$$

$$q_{\rm new} = q_{\rm old}. \tag{11}$$

1.3 Type II Rescaling

The following rescaling of variables also leaves the Grad-Shafranov equation invariant:

$$R_{\rm new} = R_{\rm old}, \tag{12}$$

$$Z_{\rm new} = Z_{\rm old}, \tag{13}$$

$$\psi_{\rm new} = \psi_{\rm old}, \tag{14}$$

$$T_{\rm new} = {\rm sgn}(T_{\rm old})\sqrt{T_{\rm old}^2 + a_2}, \tag{15}$$

$$P_{\rm new} = P_{\rm old}. \tag{16}$$

It follows that

$$I_{\phi\,{\rm new}} = I_{\phi\,{\rm old}}, \tag{17}$$

$$q_{\rm new} = \frac{T_{\rm new}}{T_{\rm old}}\, q_{\rm old}. \tag{18}$$

1.4 Type III Recaling

The following rescaling of variables also leaves the Grad-Shafranov equation invariant:

$$R_{\rm new} = a_3\, R_{\rm old}, \tag{19}$$

$$Z_{\rm new} = a_3\, Z_{\rm old}, \tag{20}$$

$$\psi_{\rm new} = a_3^2\, \psi_{\rm old}, \tag{21}$$

$$T_{\rm new} = a_3\, T_{\rm old}, \tag{22}$$

$$P_{\rm new} = P_{\rm old}. \tag{23}$$

It follows that

$$I_{\phi\,{\rm new}} = a_3^{-1}\, I_{\phi\,{\rm old}}, \tag{24}$$

$$q_{\rm new} = q_{\rm old}. \tag{25}$$

2 Rescaling Classes

2.1 q_{95} Rescaling

Let

$$\Psi_N = \frac{\psi - \psi_{\rm axis}}{\psi_{\rm separatrix} - \psi_{\rm axis}}. \tag{26}$$

First, perform a Type II rescaling such that

$$a_2 = \left(\frac{q_{95\,{\rm target}}^2}{q_{95\,{\rm old}}^2} - 1\right) T_{95\,{\rm old}}^2, \tag{27}$$

where

$$q_{95} \equiv q(\Psi_N = 0.95), \tag{28}$$

$$T_{95} \equiv T(\Psi_N = 0.95). \tag{29}$$

It follows that

$$\psi_{\rm new} = \psi_{\rm old}, \tag{30}$$

$$T_{\rm new} = T_{\rm old}\sqrt{1 + \left(\frac{q^2_{95\,\rm target}}{q^2_{95\,\rm old}} - 1\right)\frac{T^2_{95\,\rm old}}{T^2_{\rm old}}}, \tag{31}$$

$$P_{\rm new} = P_{\rm old}, \tag{32}$$

and

$$I_{\phi\,\rm new} = I_{\phi\,\rm old}, \tag{33}$$

$$q_{\rm new} = q_{\rm old}\sqrt{1 + \left(\frac{q^2_{95\,\rm target}}{q^2_{95\,\rm old}} - 1\right)\frac{T^2_{95\,\rm old}}{T^2_{\rm old}}}. \tag{34}$$

Next, perform a Type I rescaling such that

$$a_1 = \frac{T_{1\,\rm old}}{T_{1\,\rm new}}, \tag{35}$$

where

$$T_1 \equiv T(\Psi_N = 1). \tag{36}$$

It follows that

$$\psi_{\rm new\,new} = a_1\,\psi_{\rm old}, \tag{37}$$

$$T_{\rm new\,new} = a_1\,T_{\rm new} = T_{1\,\rm old}\sqrt{\frac{T^2_{\rm old} + (q^2_{95\,\rm target}/q^2_{95\,\rm old} - 1)\,T^2_{95\,\rm old}}{T^2_{1\,\rm old} + (q^2_{95\,\rm target}/q^2_{95\,\rm old} - 1)\,T^2_{95\,\rm old}}}, \tag{38}$$

$$P_{\rm new\,new} = a_1^2\,P_{\rm old}, \tag{39}$$

and

$$I_{\phi\,\rm new\,new} = a_1\,I_{\phi\,\rm old}, \tag{40}$$

$$q_{\rm new\,new} = q_{\rm new} = q_{\rm old}\sqrt{1 + \left(\frac{q^2_{95\,\rm target}}{q^2_{95\,\rm old}} - 1\right)\frac{T^2_{95\,\rm old}}{T^2_{\rm old}}}. \tag{41}$$

Note that

$$q_{95\,\rm new\,new} = q_{95\,\rm target}, \tag{42}$$

$$T^2_{\rm new\,new} = a_1^2\,T^2_{\rm old} + (1 - a_1^2)\,T^2_{1\,\rm old}, \tag{43}$$

$$P_{\rm new\,new} = a_1^2\,P_{\rm old}. \tag{44}$$

2.2 n_e Rescaling

Perform Type I rescaling with $\alpha_1 = a_n^{1/2}$. So,

$$R \to R, \tag{45}$$
$$P \to a_n\, P, \tag{46}$$
$$T \to a_n^{1/2}\, T, \tag{47}$$
$$\psi \to a_n^{1/2}\, \psi. \tag{48}$$

Follows that

$$B_{p,t} \to a_n^{1/2}\, B_{p,t}, \tag{49}$$
$$q_{95} \to q_{95}. \tag{50}$$

Let

$$n_a \to a_n\, n_a, \tag{51}$$
$$T_a \to T_a. \tag{52}$$

Here, $a \equiv e, i, b, I$. Follows that

$$\omega_{*\,a} \to a_n^{-1/2}\, \omega_{*\,a}. \tag{53}$$

Assuming that

$$\omega_{\theta\,a} \to a_n^{-1/2}\, \omega_{\theta\,a}, \tag{54}$$
$$\omega_E \to \omega_E, \tag{55}$$

we require

$$\omega_{\phi\,a} \to \omega_{\phi\,a} + (a_n^{-1/2} - 1)\, (\omega_{*\,a} + \omega_{\theta\,a}). \tag{56}$$

2.3 T_e Rescaling

Perform Type I rescaling with $\alpha_1 = a_T^{1/2}$. So,

$$R \to R, \tag{57}$$
$$P \to a_T\, P, \tag{58}$$
$$T \to a_T^{1/2}\, T, \tag{59}$$
$$\psi \to a_T^{1/2}\, \psi. \tag{60}$$

Follows that

$$B_{p,t} \to a_T^{1/2}\, B_{p,t}, \tag{61}$$

$$q_{95} \to q_{95}. \tag{62}$$

Let

$$n_a \to n_a, \tag{63}$$

$$T_a \to a_T\, T_a. \tag{64}$$

Follows that

$$\omega_{*\,a} \to a_T^{1/2}\, \omega_{*\,a}. \tag{65}$$

Assuming that

$$\omega_{\theta\,a} \to a_T^{1/2}\, \omega_{\theta\,a}, \tag{66}$$

$$\omega_E \to \omega_E, \tag{67}$$

we require

$$\omega_{\phi\,a} \to \omega_{\phi\,a} + (a_T^{1/2} - 1)\,(\omega_{*\,a} + \omega_{\theta\,a}). \tag{68}$$

2.4 R Rescaling

Perform a Type 3 rescaling with $a_3 = a_R$. It follows that

$$R \to a_R\, R, \tag{69}$$

$$P \to P, \tag{70}$$

$$R \to a_R\, T, \tag{71}$$

$$\psi \to a_R^2. \tag{72}$$

Follows that

$$B_{t,p} \to a_R\, B_{t,p}, \tag{73}$$

$$q_{95} \to q_{95}. \tag{74}$$

Let

$$n_a \to n_a, \tag{75}$$

$$T_a \to T_a, \tag{76}$$

so

$$\omega_{*\,a} \to a_R^{-2}\, \omega_{*\,a}. \tag{77}$$

Assuming that

$$\omega_{\theta\,a} \to a_R^{-2}\, \omega_{\theta\,a}, \tag{78}$$

$$\omega_E \to \omega_E, \tag{79}$$

we require

$$\omega_{\phi\,a} \to \omega_{\phi\,a} + (a_R^{-2} - 1)\,(\omega_{*\,a} + \omega_{\theta\,a}). \tag{80}$$

2.5 P Rescaling

Let

$$R \to a_R\, R, \tag{81}$$
$$P \to P + a_P, \tag{82}$$
$$T \to T, \tag{83}$$
$$\psi \to a_R^2. \tag{84}$$

$$B_{t,p} \to a_R\, B_{t,p}, \tag{85}$$
$$q_{95} \to q_{95}. \tag{86}$$

Let

$$n_a \to n_a + \frac{a_P}{T_a}, \tag{87}$$
$$T_a \to T_a, \tag{88}$$

so

$$\omega_{*\,a} \to \omega_{*\,a}. \tag{89}$$

Assuming that

$$\omega_{\theta\,a} \to \omega_{\theta\,a}, \tag{90}$$
$$\omega_E \to \omega_E, \tag{91}$$

we require

$$\omega_{\phi\,a} \to \omega_{\phi\,a}. \tag{92}$$

2.6 ω_E Rescaling

Do not rescale equilibrium. So,

$$R \to R, \tag{93}$$
$$P \to P, \tag{94}$$
$$T \to T, \tag{95}$$
$$\psi \to \psi. \tag{96}$$

Follows that

$$B_{p,t} \to B_{p,t}, \tag{97}$$
$$q_{95} \to q_{95}. \tag{98}$$

Let

$$n_a \to n_a, \tag{99}$$

$$T_a \to T_a, \tag{100}$$

so

$$\omega_{*a} \to \omega_{*a}. \tag{101}$$

Assuming that

$$\omega_{\theta a} \to \omega_{\theta a}, \tag{102}$$

$$\omega_E \to a_E \, \omega_E, \tag{103}$$

we require

$$\omega_{\phi a} \to \omega_{\phi a} + (a_E - 1) \, \omega_E. \tag{104}$$

2.7 χ_ϕ Rescaling

Let everything stay the same, except

$$\chi_{\phi,e,i} =\!\!\to a_\phi \, \chi_{\phi e i}, \tag{105}$$

$$D_\perp =\!\!\to a_\phi \, D_\perp. \tag{106}$$

3 Rescaling pFile

3.1 q_{95} Rescaling

The variables in pFiles are rescaled as follows:

$$\mathsf{ne} \to a_1 \, \mathsf{ne}, \tag{107}$$

$$\mathsf{te} \to a_1 \, \mathsf{te}, \tag{108}$$

$$\mathsf{ni} \to a_1 \, \mathsf{ni}, \tag{109}$$

$$\mathsf{ti} \to a_1 \, \mathsf{ti}, \tag{110}$$

$$\mathsf{nb} \to a_1 \, \mathsf{nb}, \tag{111}$$

$$\mathsf{pb} \to a_1^2 \, \mathsf{pb}, \tag{112}$$

$$\mathsf{ptot} \to a_1^2 \, \mathsf{ne}, \tag{113}$$

$$\mathsf{omeg} \to \mathsf{omeg}, \tag{114}$$

$$\mathsf{omegp} \to \mathsf{omegp}, \tag{115}$$

$$\mathsf{omegvb} \to \mathsf{omegvb}, \tag{116}$$

$$\mathsf{omegpp} \to \mathsf{omegpp}, \tag{117}$$

$$\mathsf{omegeb} \to \mathsf{omegeb}, \tag{118}$$
$$\mathsf{er} \to a_1\,\mathsf{er}, \tag{119}$$
$$\mathsf{ommvb} \to \mathsf{ommvb}, \tag{120}$$
$$\mathsf{ommpp} \to \mathsf{ommpp}, \tag{121}$$
$$\mathsf{omevb} \to \mathsf{omevb}, \tag{122}$$
$$\mathsf{omepp} \to \mathsf{omepp}, \tag{123}$$
$$\mathsf{kpol} \to \mathsf{kpol}, \tag{124}$$
$$\mathsf{omghb} \to a_1\,\mathsf{omghb}, \tag{125}$$
$$\mathsf{nz1} \to a_1\,\mathsf{nz1}, \tag{126}$$
$$\mathsf{vtor1} \to \mathsf{vtor1}, \tag{127}$$
$$\mathsf{vpol1} \to a_1\,\mathsf{vpol1}, \tag{128}$$
$$\mathsf{N\,Z\,A} \to \mathsf{N\,Z\,A}. \tag{129}$$