

# Calculation of Layer Quantities in TJ

## I. ADDITIONAL PLASMA PARAMETERS

The following additional plasma parameters are required to calculate layer quantities:

- $B_0$  - the toroidal magnetic field-strength at the magnetic axis
- $R_0$  - the plasma major radius
- $n_0$  - the electron number density at the magnetic axis
- $\alpha$  - the electron density profile is assumed to be  $n_e = n_0 (1 - \hat{r}^2)^\alpha$
- $Z_{\text{eff}}$  - the (assumed spatially uniform) effective ion charge number
- $M$  - the ion mass number
- $\chi_\perp$  - the (assumed spatially uniform) perpendicular momentum/energy diffusivity

The following derived parameter is also required:

- $a = \epsilon R_0$  - the plasma minor radius

## II. RATIONAL SURFACE PARAMETERS

Let the  $k$ th rational surface be resonant with poloidal mode number  $m_k$  and lie at normalized radius  $\hat{r}_k$ . We can define

$$q_k = q(\hat{r}_k), \tag{1}$$

$$s_k = s(\hat{r}_k), \tag{2}$$

$$n_{ek} = n_0 (1 - \hat{r}_k^2)^\alpha, \tag{3}$$

$$p_k = \frac{\epsilon^2 B_0^2 p_2(\hat{r}_k)}{\mu_0}, \tag{4}$$

$$T_{ek} = \frac{p_k}{2 n_{ek} e}, \tag{5}$$

$$\ln \Lambda_k = 24 + 3 \ln 10 - \frac{1}{2} \ln n_{ek} + \ln T_{ek}, \quad (6)$$

$$\tau_{ee k} = \frac{6\sqrt{2} \pi^{3/2} \epsilon_0^2 m_e^{1/2} T_{ek}^{3/2}}{\ln \Lambda_k e^{5/2} n_{ek}}, \quad (7)$$

$$\sigma_{\parallel k} = \frac{\sqrt{2} + 13 Z_{\text{eff}}/4}{Z_{\text{eff}} (\sqrt{2} + Z_{\text{eff}})} \frac{n_{ek} e^2 \tau_{ee k}}{m_e}, \quad (8)$$

$$g_k = 1 + \epsilon^2 g_2(\hat{r}_k), \quad (9)$$

$$L_{sk} = \frac{R_0 q_k}{s_k}, \quad (10)$$

$$V_{Ak} = \frac{B_0 g_k}{(\mu_0 n_{ek} M m_p)^{1/2}}, \quad (11)$$

$$d_{ik} = \left( \frac{M m_p}{n_{ek} e^2 \mu_0} \right)^{1/2}, \quad (12)$$

$$\beta_k = \frac{5 \epsilon^2 p_2(\hat{r}_k)}{3 g_k^2}, \quad (13)$$

$$\hat{d}_{\beta k} = \left( \frac{\beta_k}{1 + \beta_k} \right)^{1/2} \frac{d_{ik}}{a \hat{r}_k}, \quad (14)$$

$$\omega_{*k} = \frac{m_k B_0 p'_2(\hat{r}_k)}{\mu_0 R_0^2 e n_{ek} g_k \hat{r}_k}, \quad (15)$$

$$\tau_{Hk} = \frac{L_{sk}}{m_k V_{Ak}}, \quad (16)$$

$$\tau_{Rk} = \mu_0 a^2 \hat{r}_k^2 \sigma_{\parallel}(\hat{r}_k), \quad (17)$$

$$\tau_{\perp k} = \frac{a^2 \hat{r}_k^2}{\chi_{\perp}}. \quad (18)$$

Here, we are assuming that the electrons and ions have the same temperature.

### III. LAYER PARAMETERS

Layer physics at the  $k$ th rational surface is governed by the following normalized parameters:

$$S_k^{1/3} = \left( \frac{\tau_{Rk}}{\tau_{Hk}} \right)^{1/3}, \quad (19)$$

$$\tau_k = S_k^{1/3} \tau_{Hk}, \quad (20)$$

$$\iota_{ek} = \frac{1}{2}, \quad (21)$$

$$Q_{ek} = -\iota_{ek} \tau_k \omega_{*k}, \quad (22)$$

$$Q_{ik} = (1 - \iota_{ek}) \tau_k \omega_{*k}, \quad (23)$$

$$D_k = S_k^{1/3} \iota_{ek}^{1/2} \hat{d}_{\beta k}, \quad (24)$$

$$P_{\varphi k} = \frac{\tau_{Rk}}{\tau_{\perp k}}, \quad (25)$$

$$P_{\perp k} = \frac{\tau_{Rk}}{\tau_{\perp k}}. \quad (26)$$

#### IV. CRITICAL $\Delta'$

The critical  $\Delta'$ , due to magnetic field-line curvature effects at the rational surface, that must be exceeded before an electron-branch tearing mode becomes unstable, can be written

$$\Delta_{ck} = -\frac{\sqrt{2} \pi^{3/2} D_{Rk}}{\hat{W}_{dk}}, \quad (27)$$

where

$$D_{Rk} = -\frac{2 \epsilon^2 \hat{r}_k p'_2(\hat{r}_k) (1 - q_k^2)}{s_k^2} - \frac{2 \epsilon^2 p'_2(\hat{r}_k) q_k^2 H'_1(\hat{r}_k)}{s_k}, \quad (28)$$

and

$$\hat{W}_{dk} = \sqrt{8} \left( \frac{\chi_{\perp}}{\chi_{\parallel}} \right)^{1/4} \frac{1}{(\epsilon \hat{r}_k s_k n)^{1/2}}, \quad (29)$$

$$\chi_{\parallel} = \frac{\chi_{\parallel}^{\text{smfp}} \chi_{\parallel}^{\text{lmfp}}}{\chi_{\parallel}^{\text{smfp}} + \chi_{\parallel}^{\text{lmfp}}}, \quad (30)$$

$$\chi_{\parallel}^{\text{smfp}} = \frac{1.581 \tau_{ee k} v_{tek}}{1 + 0.2535 Z_{\text{eff}}}, \quad (31)$$

$$\chi_{\parallel}^{\text{lmfp}} = \frac{2 R_0 v_{tek}}{\pi^{1/2} n s_k \hat{W}_{dk}}, \quad (32)$$

$$v_{tek} = \left( \frac{2 T_{ek}}{m_e} \right)^{1/2}. \quad (33)$$