Calculation of Layer Quantities in TJ

I. ADDITIONAL PLASMA PARAMETERS

The following additional plasma parameters are required to calculate layer quantities:

- B_0 - the toroidal magnetic field-strength at the magnetic axis
- R_0 - the plasma major radius
- n_0 - the electron number density at the magnetic axis
- α - the electron density profile is assumed to be $n_e = n_0\,(1-\hat{r}^2)^\alpha$
- $Z_{\rm eff}$ - the (assumed spatially uniform) effective ion charge number
- M - the ion mass number
- $\chi_\perp$ - the (assumed spatially uniform) perpendicular momentum/energy diffusivity

The following derived parameter is also required:

- $a = \epsilon\,R_0$ - the plasma minor radius

II. RATIONAL SURFACE PARAMETERS

Let the kth rational surface be resonant with poloidal mode number m_k and lie at normalized radius $\hat{r}_k$. We can define

$$n_{e\,k} = n_0\,(1-\hat{r}_k^{\,2})^\alpha, \tag{1}$$

$$p_k = \frac{\epsilon^2\,B_0^{\,2}\,p_2(\hat{r}_k),}{\mu_0} \tag{2}$$

$$T_{e\,k} = \frac{p_k}{2\,n_{e\,k}\,e}, \tag{3}$$

$$\ln\Lambda_k = 24 + 3\,\ln 10 - \frac{1}{2}\,\ln n_{e\,k} + \ln T_{e\,k}, \tag{4}$$

$$\tau_{ee\,k} = \frac{6\sqrt{2}\,\pi^{3/2}\,\epsilon_0^2\,m_e^{1/2}\,T_{e\,k}^{3/2}}{\ln\Lambda_k\,e^{5/2}\,n_{e\,k}}, \tag{5}$$

$$\sigma_{\parallel\,k} = \frac{\sqrt{2}+13\,Z_{\rm eff}/4}{Z_{\rm eff}\,(\sqrt{2}+Z_{\rm eff})}\,\frac{n_{e\,k}\,e^2\,\tau_{ee\,k}}{m_e}, \tag{6}$$

$$g_k = 1+\epsilon^2\,g_2(\hat{r}_k), \tag{7}$$

$$L_{s\,k} = \frac{R_0\,q(\hat{r}_k)}{s(\hat{r}_k)}, \tag{8}$$

$$V_{A\,k} = \frac{B_0\,g_k}{(\mu_0\,n_{e\,k}\,M\,m_p)^{1/2}}, \tag{9}$$

$$d_{i\,k} = \left(\frac{M\,m_p}{n_{e\,k}\,e^2\,\mu_0}\right)^{1/2}, \tag{10}$$

$$\beta_k = \frac{5\,\epsilon^2\,p_2(\hat{r}_k)}{3\,g_k^2}, \tag{11}$$

$$\hat{d}_{\beta\,k} = \left(\frac{\beta_k}{1+\beta_k}\right)^{1/2}\frac{d_{i\,k}}{a\,\hat{r}_k}, \tag{12}$$

$$\omega_{*\,k} = \frac{m_k\,B_0\,p_2'(\hat{r}_k)}{\mu_0\,R_0^2\,e\,n_{e\,k}\,g_k\,\hat{r}_k}, \tag{13}$$

$$\tau_{H\,k} = \frac{L_{s\,k}}{m_k\,V_{A\,k}}, \tag{14}$$

$$\tau_{R\,k} = \mu_0\,a^2\,\hat{r}_k^2\,\sigma_\parallel(\hat{r}_k), \tag{15}$$

$$\tau_{\perp\,k} = \frac{a^2\,\hat{r}_k^2}{\chi_\perp}. \tag{16}$$

Here, we are assuming that the electrons and ions have the same temperature.

III. LAYER PARAMETERS

Layer physics at the kth rational surface is governed by the following normalized parameters:

$$S_k^{1/3} = \left(\frac{\tau_{R\,k}}{\tau_{H\,k}}\right)^{1/3}, \tag{17}$$

$$\tau_k = S_k^{1/3}\,\tau_{H\,k}, \tag{18}$$

$$\iota_{e\,k} = \frac{1}{2}, \tag{19}$$

$$Q_{e\,k} = -\iota_{e\,k}\,\tau_k\,\omega_{*\,k}, \tag{20}$$

$$Q_{i\,k} = (1-\iota_{e\,k})\,\tau_k\,\omega_{*\,k}, \tag{21}$$

$$D_k = S_k^{1/3}\,\iota_{e\,k}^{1/2}\,\hat{d}_{\beta\,k}, \tag{22}$$

$$P_{\varphi\,k} = \frac{\tau_{R\,k}}{\tau_{\perp\,k}}, \tag{23}$$

$$P_{\perp\,k} = \frac{\tau_{R\,k}}{\tau_{\perp\,k}}. \tag{24}$$