Calculation of Layer Quantities in TJ

I. ADDITIONAL PLASMA PARAMETERS

The following additional plasma parameters are required to calculate layer quantities:

- B_0 - the toroidal magnetic field-strength at the magnetic axis

- R_0 - the plasma major radius

- n_0 - the electron number density at the magnetic axis

- $n_{e\,\mathrm{ped}}$ - the electron number density at the plasma boundary

- $T_{e\,\mathrm{ped}}$ - the electron temperature at the plasma boundary

- α - the electron density profile is assumed to be $n_e = n_0\,(1 - \hat{r}^2)^\alpha$

- Z_{eff} - the (assumed spatially uniform) effective ion charge number

- M - the ion mass number

- $\chi_\perp$ - the (assumed spatially uniform) perpendicular momentum/energy diffusivity

The following derived parameter is also required:

- $a = \epsilon\,R_0$ - the plasma minor radius

II. RATIONAL SURFACE PARAMETERS

Let the kth rational surface be resonant with poloidal mode number m_k and lie at normalized radius $\hat{r}_k$. We can define

$$q_k = q(\hat{r}_k), \tag{1}$$

$$s_k = s(\hat{r}_k), \tag{2}$$

$$n_{e\,k} = n_0\,(1 - \hat{r}_k^2)^\alpha + n_{e\,\mathrm{ped}}, \tag{3}$$

$$p_k = \frac{\epsilon^2 \, B_0^2 \, p_2(\hat{r}_k)}{\mu_0}, \tag{4}$$

$$T_{e\,k} = \frac{p_k}{2 \, (n_{e\,k} - n_{e\,\text{ped}}) \, e} + T_{e\,\text{ped}}, \tag{5}$$

$$\ln \Lambda_k = 24 + 3 \ln 10 - \frac{1}{2} \ln n_{e\,k} + \ln T_{e\,k}, \tag{6}$$

$$\tau_{ee\,k} = \frac{6\sqrt{2} \, \pi^{3/2} \, \epsilon_0^2 \, m_e^{1/2} \, T_{e\,k}^{3/2}}{\ln \Lambda_k \, e^{5/2} \, n_{e\,k}}, \tag{7}$$

$$\sigma_{\|\,k} = \frac{\sqrt{2} + 13 \, Z_{\text{eff}}/4}{Z_{\text{eff}} \, (\sqrt{2} + Z_{\text{eff}})} \, \frac{n_{e\,k} \, e^2 \, \tau_{ee\,k}}{m_e}, \tag{8}$$

$$g_k = 1 + \epsilon^2 \, g_2(\hat{r}_k), \tag{9}$$

$$L_{s\,k} = \frac{R_0 \, q_k}{s_k}, \tag{10}$$

$$V_{A\,k} = \frac{B_0 \, g_k}{(\mu_0 \, n_{e\,k} \, M \, m_p)^{1/2}}, \tag{11}$$

$$d_{i\,k} = \left(\frac{M \, m_p}{n_{e\,k} \, e^2 \, \mu_0} \right)^{1/2}, \tag{12}$$

$$\beta_k = \frac{5 \, \epsilon^2 \, p_2(\hat{r}_k)}{3 \, g_k^2}, \tag{13}$$

$$\hat{d}_{\beta\,k} = \left(\frac{\beta_k}{1 + \beta_k} \right)^{1/2} \frac{d_{i\,k}}{a \, \hat{r}_k}, \tag{14}$$

$$\omega_{*\,k} = \frac{m_k \, B_0 \, p_2'(\hat{r}_k)}{\mu_0 \, R_0^2 \, e \, n_{e\,k} \, g_k \, \hat{r}_k}, \tag{15}$$

$$\tau_{H\,k} = \frac{L_{s\,k}}{m_k \, V_{A\,k}}, \tag{16}$$

$$\tau_{R\,k} = \mu_0 \, a^2 \, \hat{r}_k^2 \, \sigma_{\|}(\hat{r}_k), \tag{17}$$

$$\tau_{\perp\,k} = \frac{a^2 \, \hat{r}_k^2}{\chi_{\perp}}. \tag{18}$$

Here, we are assuming that the electrons and ions have the same temperature.

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III. LAYER PARAMETERS

Layer physics at the kth rational surface is governed by the following normalized parameters:

$$S_k^{1/3} = \left(\frac{\tau_{Rk}}{\tau_{Hk}}\right)^{1/3}, \tag{19}$$

$$\tau_k = S_k^{1/3}\,\tau_{Hk}, \tag{20}$$

$$\iota_{ek} = \frac{1}{2}, \tag{21}$$

$$Q_{ek} = -\iota_{ek}\,\tau_k\,\omega_{*k}, \tag{22}$$

$$Q_{ik} = (1-\iota_{ek})\,\tau_k\,\omega_{*k}, \tag{23}$$

$$D_k = S_k^{1/3}\,\iota_{ek}^{1/2}\,\hat{d}_{\beta k}, \tag{24}$$

$$P_{\varphi k} = \frac{\tau_{Rk}}{\tau_{\perp k}}, \tag{25}$$

$$P_{\perp k} = \frac{\tau_{Rk}}{\tau_{\perp k}}. \tag{26}$$

IV. CRITICAL Δ'

The critical Δ', due to magnetic field-line curvature effects at the rational surface, that must be exceeded before an electron-branch tearing mode becomes unstable, can be written

$$\Delta_{ck} = -\frac{\sqrt{2}\,\pi^{3/2}\,D_{Rk}}{\hat{W}_{dk}}, \tag{27}$$

where

$$D_{Rk} = -\frac{2\,\epsilon^2\,\hat{r}_k\,p_2'(\hat{r}_k)\,(1-q_k^2)}{s_k^2} - \frac{2\,\epsilon^2\,p_2'(\hat{r}_k)\,q_k^2\,H_1'(\hat{r}_k)}{s_k}, \tag{28}$$

and

$$\hat{W}_{dk} = \sqrt{8}\left(\frac{\chi_\perp}{\chi_\parallel}\right)^{1/4}\frac{1}{(\epsilon\,\hat{r}_k\,s_k\,n)^{1/2}}, \tag{29}$$

$$\chi_\parallel = \frac{\chi_\parallel^{\mathrm{smfp}}\,\chi_\parallel^{\mathrm{lmfp}}}{\chi_\parallel^{\mathrm{smfp}} + \chi_\parallel^{\mathrm{lmfp}}}, \tag{30}$$

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$$\chi_\parallel^{\text{smfp}} = \frac{1.581\,\tau_{ee\,k}\,v_{te\,k}^2}{1 + 0.2535\,Z_{\text{eff}}}, \tag{31}$$

$$\chi_\parallel^{\text{lmfp}} = \frac{2\,R_0\,v_{te\,k}}{\pi^{1/2}\,n\,s_k\,\hat{W}_{d\,k}}, \tag{32}$$

$$v_{te\,k} = \left(\frac{2\,T_{e\,k}}{m_e}\right)^{1/2}. \tag{33}$$