

Benchmark of TJ Code Against STRIDE Code

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I. SYMMETRY RELATIONS

Let all lengths be normalized to the major radius of the magnetic axis, R_0 , and let all magnetic field-strengths be normalized to the toroidal magnetic field-strength at the axis, B_0 . Consider an axisymmetric toroidal plasma equilibrium. Let R, ϕ, Z be a conventional cylindrical coordinate system that is co-axial with the toroidal symmetry axis of the plasma. The equilibrium magnetic field is written

$$\mathbf{B} = \nabla\phi \times \nabla\psi + g(\psi) \nabla\psi, \quad (1)$$

where ψ is the poloidal magnetic flux. We can also define

$$\Psi_N = \frac{\psi}{\psi_a}, \quad (2)$$

where ψ_a is the poloidal flux at the plasma boundary. Thus, $\Psi_N = 0$ at the magnetic axis, and $\Psi_N = 1$ on the plasma boundary. In the following, it is assumed that Ψ_N is the ‘radial’ coordinate in STRIDE.

Let

$$L_m^m(\Psi_N) = I(\Psi_N) \left(J(\Psi_N) \left[\frac{m g(\Psi_N)}{q(\Psi_N)} \right]^2 + [n \psi_a]^2 \right), \quad (3)$$

$$I(\Psi_N) = \int_0^{\Psi_N} \frac{2 q(\Psi'_N)}{g(\Psi'_N)} d\Psi'_N, \quad (4)$$

$$J(\Psi_N) = \oint |\nabla\Psi_N|^{-2} \frac{d\theta}{2\pi}, \quad (5)$$

$$\rho(\Psi_N) = \frac{J(\Psi_N) g(\Psi_N)}{q(\Psi_N)}, \quad (6)$$

$$s(\Psi_N) = \frac{d \ln q}{d\Psi_N}, \quad (7)$$

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where m is a poloidal mode number, n is a toroidal mode number, $q(\Psi_N)$ is the safety-factor, and θ is a ‘straight’ poloidal angle defined such that

$$(\nabla\phi \times \nabla\theta \cdot \nabla\phi)^{-1} = \frac{q R^2}{g}, \quad (8)$$

Note that we are working in PEST coordinates.

Let m_j be the poloidal mode number of the j th rational surface, which is located at $\Psi_N = \Psi_{Nj}$. Let D_{Ij} be the ideal Mercier index at the j th rational surface, and let

$$\nu_{Lj} = \frac{1}{2} - \sqrt{-D_{Ij}}, \quad (9)$$

$$\nu_{Sj} = \frac{1}{2} + \sqrt{-D_{Ij}}. \quad (10)$$

Let

$$f_{Lj} = \left[\rho^{\nu_{Lj}} \left(\frac{\nu_{Sj} - \nu_{Lj}}{L_{m_j}^{m_j}} \right)^{1/2} s m_j \right]_{\Psi_{Nj}}, \quad (11)$$

$$f_{Sj} = \left[\rho^{\nu_{Sj}} \left(\frac{\nu_{Sj} - \nu_{Lj}}{L_{m_j}^{m_j}} \right)^{1/2} s m_j \right]_{\Psi_{Nj}}. \quad (12)$$

Finally, let

$$\hat{A}_{ij} = f_{Sj}^{-1} A_{ij} f_{Lj'}, \quad (13)$$

$$\hat{B}_{ij} = f_{Sj}^{-1} B_{ij} f_{Lj'}, \quad (14)$$

$$\hat{\Gamma}_{ij} = f_{Sj}^{-1} \Gamma_{ij} f_{Lj'}, \quad (15)$$

$$\hat{\Delta}_{ij} = f_{Sj}^{-1} \Delta_{ij} f_{Lj'}, \quad (16)$$

where A_{ij} , B_{ij} , Γ_{ij} , and Δ_{ij} are the elements of the outer matching matrices calculated by STRIDE, whereas the hatted quantities are the corresponding matching matrices calculated by TJ. Equations (77) and (100) of Ref. 1 imply that

$$\hat{A}_{ji}^* = \hat{A}_{ij}, \quad (17)$$

$$\hat{\Delta}_{ji}^* = \hat{\Delta}_{ij}, \quad (18)$$

$$\hat{B}_{ji}^* = \hat{\Gamma}_{ij}. \quad (19)$$

These symmetries are ultimately due to the self-adjoint nature of the ideal-MHD force operator. However, as explained in Ref. 2, the symmetries can also be related to the conservation

of toroidal electromagnetic angular momentum. The symmetries have to be respected by a toroidal tearing mode code, otherwise the code would predict that an isolated plasma could exert a net toroidal electromagnetic torque on itself. Note that, in the cylindrical limit, $\hat{\Delta}_{jj}$ is equivalent to $r_s \Delta'$: i.e., the tearing stability index normalized to the minor radius of the rational surface.

II. TJ/STRIDE/TEAR EQUILIBRIUM

The TJ/STRIDE/TEAR large-aspect ratio, circular, plasma equilibrium has a Wesson-like current profile³ characterized by the safety-factor on the magnetic axis, q_0 , and the safety-factor at the plasma boundary, q_a . The parallel current profile, $\sigma = \mathbf{J} \cdot \mathbf{B}/B^2$, is written

$$\sigma(r) = \frac{2}{q_0} \left[1 - \left(\frac{r}{a} \right)^2 \right]^{p_\sigma}. \quad (20)$$

whereas the a pressure profile takes the form

$$p(r) = \frac{\beta_0}{2} \left[1 - \left(\frac{r}{a} \right)^2 \right]^{p_p}. \quad (21)$$

Here, r is the flux surface label defined in Ref. 2. Moreover, the magnetic axis corresponds to $r = 0$, and the plasma boundary to $r = a$. A STRIDE equilibrium is defined by the parameters q_0 , p_σ , β_0 , and p_p . In TJ equilibria, q_0 , β_0 , and p_p are the same as in STRIDE, but p_σ is adjusted to obtain the same q_a value as in STRIDE. TEAR equilibria are specified by q_0 and q_a . The TJ and STRIDE equilibria are also characterized by the inverse aspect-ratio of the plasma, ϵ_a . The TEAR equilibria are such that $\epsilon_a = 0$.

III. BENCHMARK TESTS

A. Single Rational Surface

We consider the stability of $n = 1$ tearing modes in a zero pressure equilibrium that only contain a single $n = 1$ rational surface: namely, the 2/1 surface.

1. Test 1

The first test has equilibria characterized by $q_0 = 1.1$ and $p_\sigma = 1.36$ (in STRIDE), with a range of different inverse aspect-ratios, ϵ_a . Figure 1 compares the $\hat{\Delta}_{11}$ (i.e., the tearing stability index of the 2/1 tearing mode) values calculated by the TJ code,⁴ the TEAR code (which is a cylindrical tearing mode code), and the STRIDE code. It can be seen that the tearing stability indices calculated by the TJ and STRIDE are in good agreement, and asymptote to that calculated by the TEAR code in the cylindrical limit, $\epsilon_a \rightarrow 0$.

2. Test 2

The second test has equilibria characterized by $q_0 = 1.1$ and $\epsilon_a = 0.2$, with a range of edge safety-factor values, q_a . Figure 2 compares the $\hat{\Delta}_{11}$ (i.e., the tearing stability index of the 2/1 tearing mode) values calculated by the TJ code, the TEAR code, and the STRIDE code. It can be seen that the tearing stability indices calculated by the TJ and STRIDE are in good agreement, except when the $q = 3$ surface closely approaches the plasma boundary (i.e., $q_a \rightarrow 3$).

B. Two Rational Surfaces

We consider the stability of $n = 1$ tearing modes in a zero pressure equilibrium that contains two $n = 1$ rational surface: namely, the 2/1 surface and the 3/1 surface.

1. Test 3

The third test has equilibria characterized by $q_0 = 1.1$ and $\epsilon_a = 0.2$, with a range of edge safety-factor values, q_a . Figure 3 compares the elements of the tearing stability matrix calculated by the TJ and the STRIDE codes. It can be seen that the elements calculated by TJ and STRIDE are in good agreement.

REFERENCES

- ¹ A. Pletzer, A. Bondeson, and R.L. Dewar, J. Comp. Phys. **115**, 530 (1994).
- ² R. Fitzpatrick, Phys. Plasmas **1**, 3308 (1994).
- ³ J.A. Wesson, Nucl. Fusion **18**, 87 (1978).
- ⁴ R. Fitzpatrick, Phys. Plasmas **31**, 102507 (2024).

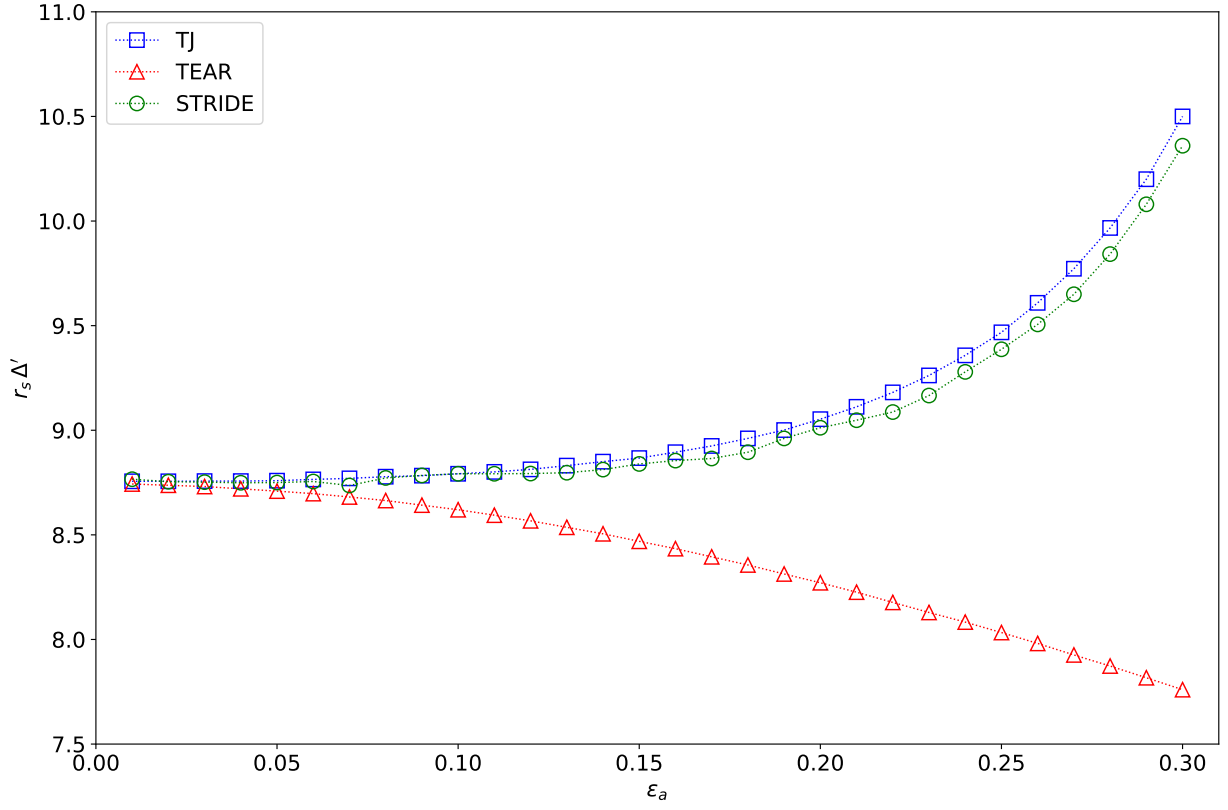


FIG. 1. Test 1: $q_0 = 1.1$, $p_\sigma = 1.36$ (STRIDE), $\beta_0 = 0.0$. Variation of 2/1 tearing stability index with inverse-aspect ratio, ϵ_a .

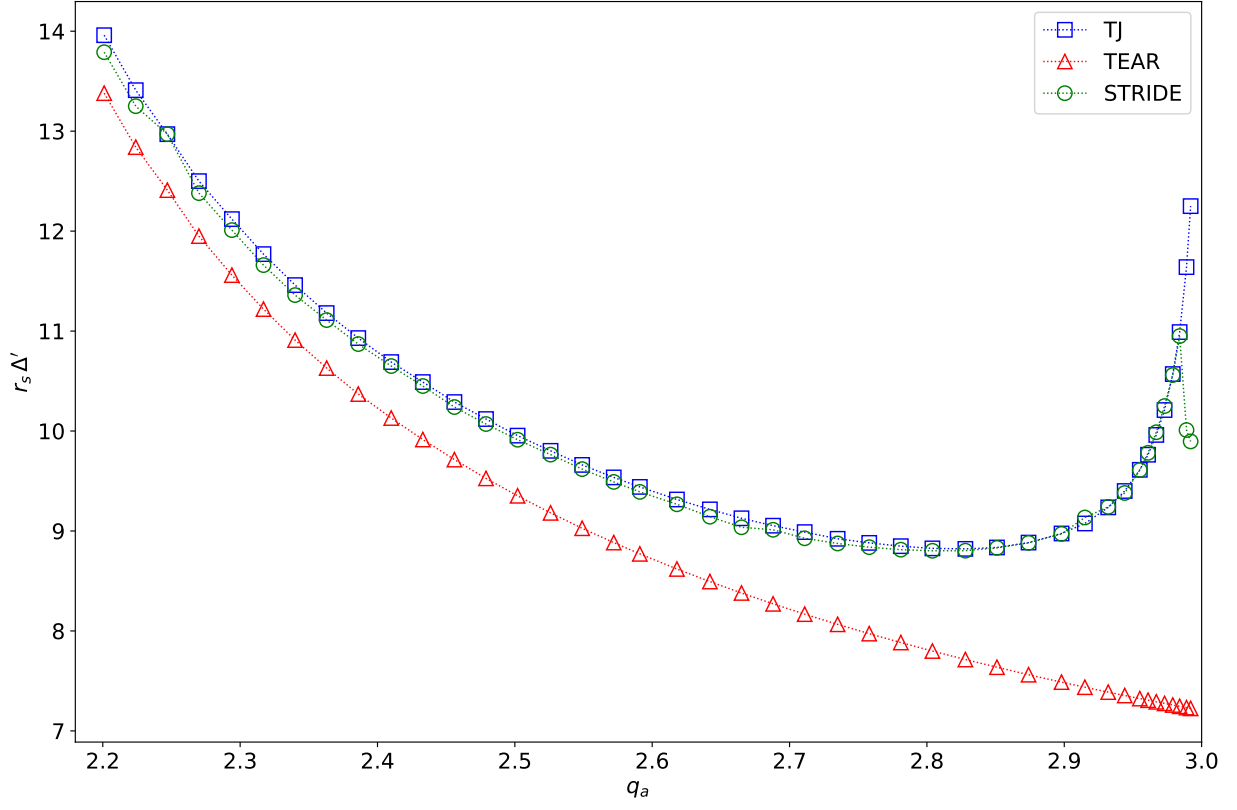


FIG. 2. Test 2: $q_0 = 1.1$, $\epsilon_a = 0.2$, $\beta_0 = 0.0$. Variation of 2/1 tearing stability index with q_a .

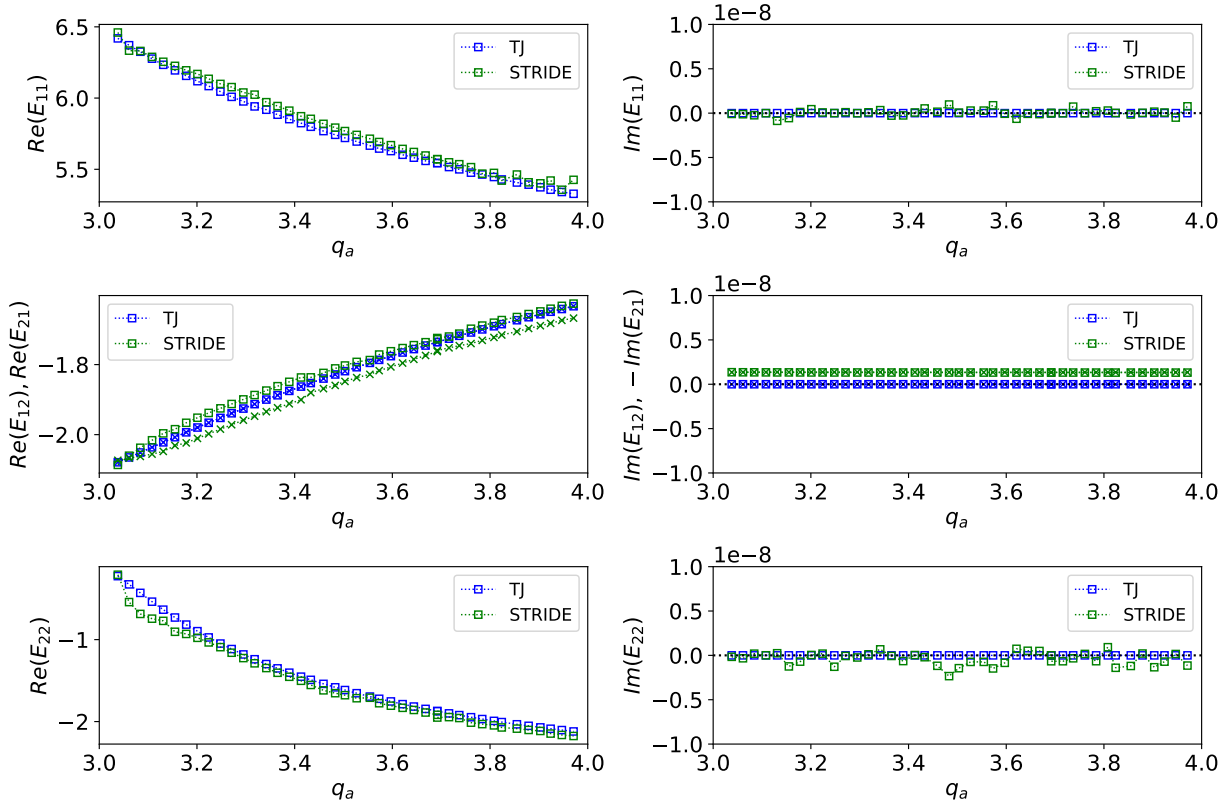


FIG. 3. Test 3: $q_0 = 1.1$, $\epsilon_a = 0.2$, $\beta_0 = 0.0$. Variation of elements of tearing stability matrix with q_a . In the middle panels the squares indicate E_{12} and the crosses indicate E_{21} .