Calculation of Layer Quantities in TJ

I. ADDITIONAL PLASMA PARAMETERS

The following additional plasma parameters are required to calculate layer quantities:

- B_0 - the toroidal magnetic field-strength at the magnetic axis
- R_0 - the plasma major radius
- n_0 - the electron number density at the magnetic axis
- α - the electron density profile is assumed to be $n_e = n_0\,(1-\hat{r}^2)^\alpha$
- $Z_{\rm eff}$ - the (assumed spatially uniform) effective ion charge number
- M - the ion mass number
- $\chi_\perp$ - the (assumed spatially uniform) perpendicular momentum/energy diffusivity

The following derived parameter is also required:

- $a = \epsilon\,R_0$ - the plasma minor radius

II. RATIONAL SURFACE PARAMETERS

Let the kth rational surface be resonant with poloidal mode number m_k and lie at normalized radius $\hat{r}_k$. We can define

$$q_k = q(\hat{r}_k), \tag{1}$$

$$s_k = s(\hat{r}_k), \tag{2}$$

$$n_{e\,k} = n_0\,(1-\hat{r}_k^{\,2})^\alpha, \tag{3}$$

$$p_k = \frac{\epsilon^2\,B_0^{\,2}\,p_2(\hat{r}_k),}{\mu_0} \tag{4}$$

$$T_{e\,k} = \frac{p_k}{2\,n_{e\,k}\,e}, \tag{5}$$

$$\ln \mathit{\Lambda}_k = 24 + 3\,\ln 10 - \frac{1}{2}\,\ln n_{e\,k} + \ln T_{e\,k}, \tag{6}$$

$$\tau_{ee\,k} = \frac{6\sqrt{2}\,\pi^{3/2}\,\epsilon_0^2\,m_e^{1/2}\,T_{e\,k}^{3/2}}{\ln \mathit{\Lambda}_k\,e^{5/2}\,n_{e\,k}}, \tag{7}$$

$$\sigma_{\parallel\,k} = \frac{\sqrt{2} + 13\,Z_{\rm eff}/4}{Z_{\rm eff}\,(\sqrt{2} + Z_{\rm eff})}\,\frac{n_{e\,k}\,e^2\,\tau_{ee\,k}}{m_e}, \tag{8}$$

$$g_k = 1 + \epsilon^2\,g_2(\hat{r}_k), \tag{9}$$

$$L_{s\,k} = \frac{R_0\,q_k}{s_k}, \tag{10}$$

$$V_{A\,k} = \frac{B_0\,g_k}{(\mu_0\,n_{e\,k}\,M\,m_p)^{1/2}}, \tag{11}$$

$$d_{i\,k} = \left(\frac{M\,m_p}{n_{e\,k}\,e^2\,\mu_0}\right)^{1/2}, \tag{12}$$

$$\beta_k = \frac{5\,\epsilon^2\,p_2(\hat{r}_k)}{3\,g_k^2}, \tag{13}$$

$$\hat{d}_{\beta\,k} = \left(\frac{\beta_k}{1+\beta_k}\right)^{1/2}\,\frac{d_{i\,k}}{a\,\hat{r}_k}, \tag{14}$$

$$\omega_{*\,k} = \frac{m_k\,B_0\,p_2'(\hat{r}_k)}{\mu_0\,R_0^2\,e\,n_{e\,k}\,g_k\,\hat{r}_k}, \tag{15}$$

$$\tau_{H\,k} = \frac{L_{s\,k}}{m_k\,V_{A\,k}}, \tag{16}$$

$$\tau_{R\,k} = \mu_0\,a^2\,\hat{r}_k^{\,2}\,\sigma_\parallel(\hat{r}_k), \tag{17}$$

$$\tau_{\perp\,k} = \frac{a^2\,\hat{r}_k^{\,2}}{\chi_\perp}. \tag{18}$$

Here, we are assuming that the electrons and ions have the same temperature.

III. LAYER PARAMETERS

Layer physics at the kth rational surface is governed by the following normalized parameters:

$$S_k^{1/3} = \left(\frac{\tau_{R\,k}}{\tau_{H\,k}}\right)^{1/3}, \tag{19}$$

$$\tau_k = S_k^{1/3}\,\tau_{H\,k}, \tag{20}$$

$$\iota_{e\,k} = \frac{1}{2}, \tag{21}$$

$$Q_{e\,k} = -\iota_{e\,k}\,\tau_k\,\omega_{\ast\,k}, \tag{22}$$

$$Q_{i\,k} = (1-\iota_{e\,k})\,\tau_k\,\omega_{\ast\,k}, \tag{23}$$

$$D_k = S_k^{1/3}\,\iota_{e\,k}^{1/2}\,\hat{d}_{\beta\,k}, \tag{24}$$

$$P_{\varphi\,k} = \frac{\tau_{R\,k}}{\tau_{\perp\,k}}, \tag{25}$$

$$P_{\perp\,k} = \frac{\tau_{R\,k}}{\tau_{\perp\,k}}. \tag{26}$$

IV. CRITICAL $\varDelta'$

The critical $\varDelta'$, due to magnetic field-line curvature effects at the rational surface, that must be exceeded before an electron-branch tearing mode becomes unstable, can be written

$$\varDelta_{c\,k} = -\frac{\sqrt{2}\,\pi^{3/2}\,D_{R\,k}}{\hat{W}_{d\,k}}, \tag{27}$$

where

$$D_{R\,k} = -\frac{2\,\epsilon^2\,\hat{r}_k\,p_2'(\hat{r}_k)\,(1-q_k^2)}{s_k^2} - \frac{2\,\epsilon^2\,p_2'(\hat{r}_k)\,q_k^2\,H_1'(\hat{r}_k)}{s_k}, \tag{28}$$

and

$$\hat{W}_{d\,k} = \sqrt{8}\left(\frac{\chi_\perp}{\chi_\parallel}\right)^{1/4}\frac{1}{(\epsilon\,\hat{r}_k\,s_k\,n)^{1/2}}, \tag{29}$$

$$\chi_\parallel = \frac{\chi_\parallel^{\rm smfp}\,\chi_\parallel^{\rm lmfp}}{\chi_\parallel^{\rm smfp}+\chi_\parallel^{\rm lmfp}}, \tag{30}$$

$$\chi_\parallel^{\rm smfp} = \frac{1.581\,\tau_{ee\,k}\,v_{t\,e\,k}}{1+0.2535\,Z_{\rm eff}}, \tag{31}$$

$$\chi_\parallel^{\rm lmfp} = \frac{2\,R_0\,v_{t\,e\,k}}{\pi^{1/2}\,n\,s_k\,\hat{W}_{d\,k}}, \tag{32}$$

$$v_{t\,e\,k} = \left(\frac{2\,T_{e\,k}}{m_e}\right)^{1/2}. \tag{33}$$